

Quiz: 4
MAT 3003 Probability , Statistics & reliability

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Let X and Y have the joint probability density function

$$f(x, y) = \begin{cases} 2, & 0 \leq x \leq y \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

Let $a = E(Y|X = \frac{1}{2})$ and $b = Var(Y|X = \frac{1}{2})$. Then (a, b) is

(A) $(\frac{3}{4}, \frac{7}{12})$

(B) $(\frac{1}{4}, \frac{1}{48})$

(C) $(\frac{1}{4}, \frac{7}{12})$

(D) $(\frac{3}{4}, \frac{1}{48})$

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Let X and Y have the joint probability mass function

$$P(X = m, Y = n) = \begin{cases} \frac{m+n}{21}, & m = 1, 2, 3; n = 1, 2, \\ 0, & \text{otherwise.} \end{cases}$$

Then $P(X = 2|Y = 2)$ equals

(A) $\frac{1}{3}$

(B) $\frac{2}{3}$

(C) $\frac{1}{2}$

(D) $\frac{1}{4}$

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Let X be a Poisson random variable with mean $\frac{1}{2}$. Then $E((X + 1)!)$ equals

(A) $2 e^{-\frac{1}{2}}$

(B) $4 e^{-\frac{1}{2}}$

(C) $4 e^{-1}$

(D) $2 e^{-1}$

34	<p>Let X be a random variable with the distribution function</p> $F(x) = \begin{cases} 0, & x < 0, \\ \frac{1}{4} + \frac{4x - x^2}{8}, & 0 \leq x < 2, \\ 1, & x \geq 2. \end{cases}$ <p>Then</p> $P(X = 0) + P(X = 1.5) + P(X = 2) + P(X \geq 1)$ <p>equals</p> <p>(A) $\frac{3}{8}$ (B) $\frac{5}{8}$ (C) $\frac{7}{8}$ (D) 1</p>
35	<p>Let X be a random variable with Poisson distribution $p(k; \lambda)$. Let $Y = (X + 2)(X + 1)$. What is the value of $E(Y)$?</p> <p>(a) $\lambda^2 + 3\lambda + 1$ (b) $\lambda^2 + 3\lambda + 2$ (c) $\lambda^2 + 4\lambda + 2$ (d) $3\lambda^2 + 3\lambda + 2$ (e) $4\lambda^2 + 4\lambda + 2$</p>
36	<p>Which of the following is equal to $\text{Cov}(X + Y, X - Y)$, where X and Y are random variables on a sample space S?</p> <p>(a) $\text{Var}(X) - \text{Var}(Y)$ (b) $\text{Var}(X^2) - \text{Var}(Y^2)$ (c) $\text{Var}(X^2) + 2\text{Cov}(X, Y) + \text{Var}(Y^2)$ (d) $\text{Var}(X^2) - 2\text{Cov}(X, Y) + \text{Var}(Y^2)$ (e) $(\text{Var}(X))^2 - (\text{Var}(Y))^2$</p>
37	<p>The standard normal curve is symmetric about the value _____</p> <p>a) 0.5 b) 1 c) ∞ d) 0</p>
38	<p>The time to failure of an electric bulb follows a Weibull distribution with scale = 2000 hours and shape = 0.5. What is the probability that the electric bulb will last more than 4000 hours? What is the mean time to failure?</p> <p>a) 25.3%</p>

	<p>b) 24.3%</p> <p>c) 26.3%</p> <p>d) 27.3%</p>
39	<p>Which of the following is not a correct statement?</p> <ul style="list-style-type: none"> a. the exponential distribution describes the Poisson process as a continuous random variable b. the exponential distribution is a family of curves, which are completely described by the mean c. the mean of the exponential distribution is the inverse of the mean of the Poisson d. the Poisson is a probability distribution for a discrete random variable while the exponential distribution is continuous e. the area under the curve for an exponential distribution equals
40	<p>1. A multiple-choice test has 30 questions. There are 4 choices for each question. A student who has not studied for the test decides to answer all the questions randomly by guessing the answer to each question. Which of the following probability distributions can be used to calculate the student's chance of getting at least 20 questions right?</p> <ul style="list-style-type: none"> a. Binomial distribution b. Poisson distribution c. Exponential distribution d. Uniform distribution e. Normal distribution