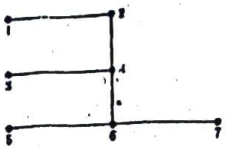
**Master of Computer Applications**
**MCAE 502: Graph Theory**
**Unique Paper Code: 223402502**
**Semester V**
**Nov-Dec 2022**
**Year of admission: 2020**

**Time: Three Hours** **Max. Marks: 70**

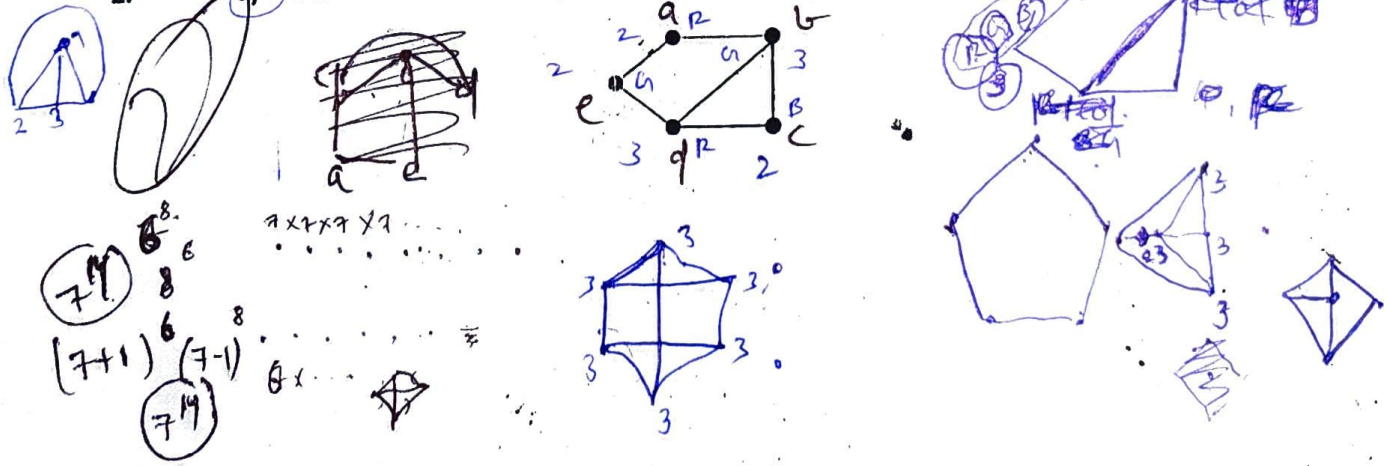
**All questions are compulsory. Attempt parts of a question together.**

1. **Answer the following (attempt any 10)** **(1X10)**
   a) Draw a 3-regular graph having more than 4 vertices.
   b) Differentiate between Euler Circuit and Hamiltonian Circuit with the help of examples.
   c) Draw a $C_5$ graph. Find its complement and draw it.
   d) Give the Prüfer code for the following-

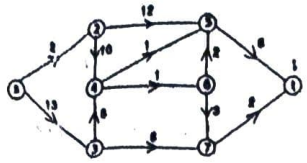


   e) What is the maximum number of edges in a bipartite graph with 14 vertices?
   f) Let G be a simple undirected planar graph on 10 vertices with 15 edges. If G is a connected graph, then compute the number of bounded faces in any embedding of G on the plane.
   g) Define a tournament graph with the help of an example.
   h) What is the relationship between vertex connectivity **K(G)**, edge connectivity **λ(G)** and degree of a Graph **δ(G)** for a graph G (V, E)?
   i) Draw and label the Petersen Graph.
   j) Show that a connected, planar graph with order 22 has no more than 60 edges.
   k) A graph has 26 vertices and 58 edges. There are five vertices of degree 4, six vertices of degree 5, and seven vertices of degree 6. If the remaining vertices all have the same degree, what is this degree?

2. a) Find the Chromatic Polynomial of the following graph – **(4)**



b) Consider the network flow problem with the edge capacities shown below: (4+2)



i. Run the Ford-Fulkerson algorithm to find the maximum flow. Show the residual graph after each iteration.

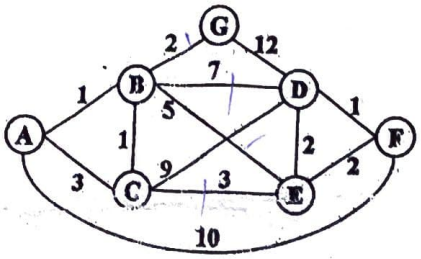
ii. Show the minimum cut.

3. a) Prove that for a planar graph G(V, E) with 'v' vertices and 'e' edges following holds – (3)

$$e \leq 3v-6$$

b) Derive the number of perfect matchings for a complete graph with 'n' vertices. Use it to compute the number of perfect matchings for $K_6$. (2+2)

c) Consider the following undirected weighted graph – (3)

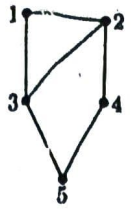



Step through Dijkstra's algorithm to calculate the single-source shortest paths from A to every other vertex. Show your steps in the table below. Cross out old values and write in new ones, from left to right within each cell, as the algorithm proceeds. Also list the vertices in the order which you marked them known. Finally, indicate the lowest-cost path from node A to node F.

| Vertex | Known | Cost | Path |
|---|---|---|---|
| A | | | |
| B | | | |
| C | | | |
| D | | | |
| E | | | |
| F | | | |
| G | | | |

4. a) When is a planar graph said to be triangulated? Draw the maximal planar graph for with 5 vertices. (2)

b) Let G be the following graph: (2+2+4)



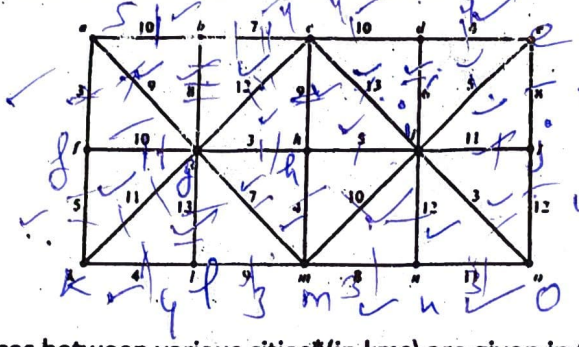

i. What are the radius and diameter of G.

ii. Compute the number of spanning trees of G using the recurrence involving deletion and contraction of an edge (you can choose any edge).

iii. Compute the number of spanning trees of G using the Matrix Tree theorem.

5. a) Draw the tree whose Prüfer code is (1; 1; 1; 1; 6; 5). (2)

b) The telephone company made a mistake and built too many telephone lines between a group of houses. In the graph shown below, the vertices are the houses and the edges are telephone lines. The lengths of the edges are the lengths of the lines. To alleviate the problem, the telephone company wants to remove extra telephone lines so that the sum of remaining lines will be as small as possible, subject to the condition that every house is connected to every other house by a path of telephone lines. Crossing out lines to be removed, determine the total length of remaining lines. (3)



c) The distances between various cities*(in kms) are given in the table below – (1.5+1.5+2)

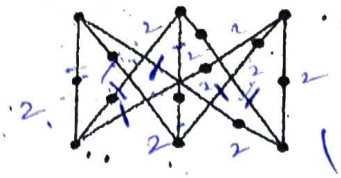
| | Delhi | Jaipur | Bharatpur | Agra | Alwar |
|---|---|---|---|---|---|
| Delhi | | 281 | 220 | 242 | 166 |
| Jaipur | 281 | | 183 | 183 | 160 |
| Bharatpur | 220 | 183 | | 111 | 111 |
| Agra | 242 | 183 | 111 | | 166 |
| Alwar | 166 | 160 | 111 | 166 | |

i. Draw the corresponding graph to the table.

ii. How many Hamiltonian cycles exist in the graph?

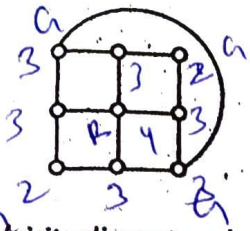
iii. What cycle does the nearest-neighbour algorithm produce for the graph above? (Give the distance also)

6. a) Determine whether the following graph is planar. If so, redraw it in the plane using appropriate procedure. If not, explain why using appropriate procedure. (3)

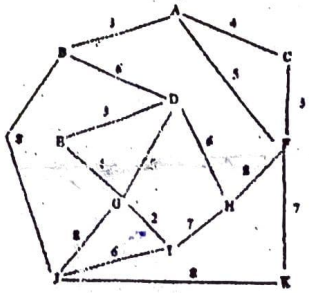


b) A self-complimentary graph is a graph where $G \cong G'$. Construct a self-complementary graph of order 8. Show the two graphs are isomorphic by drawing the complement to look the same as the original graph. (3)

c) What is the Chromatic Index and number of the following graph - (1.5+1.5)



7. a) Consider the graph given below (visit adjacent nodes in alphabetical order)- (6)



Perform the DFS and BFS traversal starting from vertex A. Draw the resulting tree.

b) State true / False. If false, support with an example – (2)

i. Every graph with fewer edges than vertices is planar
ii. All regular graphs are connected

c) Define blocks (wrt. Graph theory). Give the blocks for the following graph- (2)