AM5640 Turbulence Modelling (Jan-May 2022)
Assignment 2

AM5640 Jan-May 2022
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April 8, 2022

Problem Statement

Numerically solve fully-developed turbulent channel flow using RSM with $Re_* = u_* \delta/\nu = 395$, where $u_* = \sqrt{\tau_\omega/\rho}$ is the wall friction velocity. (Set $\rho = u_* = 1$). Use the finite volume method.

Governing Equations

The governing equations to be considered for a stationary flow ($\frac{\partial}{\partial t} <>= 0$) are: The RANS equation:

$$\frac{\partial(\rho U_j U_i)}{\partial x_j} = -\frac{\partial \bar{P}}{\partial x_i} + \frac{\partial}{\partial x_j}\left\{\mu \frac{\partial U_i}{\partial x_j}\right\} - \frac{\partial}{\partial x_j}\left\{\overline{u_i u_j}\right\} \tag{1}$$

The Reynolds' stress equations:

$$\frac{\partial(\rho U_k \overline{u_i u_j})}{\partial x_k} = \mu \frac{\partial^2 \overline{u_i u_j}}{\partial x_k \partial x_k} + P_{ij} + \Phi_{ij} + D_{ij} - \rho\epsilon_{ij} \tag{2}$$

where P_{ij} is the production rate:

$$P_{ij} = -\rho\overline{u_i u_k}\frac{\partial U_j}{\partial x_k} - \rho\overline{u_j u_k}\frac{\partial U_i}{\partial x_k} \tag{3}$$

D_{ij} is the turbulence diffusion rate (pressure diffusion rate is considered negligible) modelled as:

$$D_{ij} = \frac{\partial}{\partial x_m}\left[\frac{\mu_t}{\sigma_k}\frac{\partial \overline{u_i u_j}}{\partial x_m}\right] \tag{4}$$

ϵ_{ij} is the dissipation rate : $\epsilon_{ij} = \frac{2}{3}\epsilon\delta_{ij}$ where ϵ is obtained using another transport equation:

$$\frac{\partial(\rho U_j \epsilon)}{\partial x_j} = \mu\frac{\partial^2 \epsilon}{\partial x_k \partial x_k} + \frac{\partial}{\partial x_m}\left[\frac{\mu_t}{\sigma_\epsilon}\frac{\partial \epsilon}{\partial x_m}\right] + c_{1\epsilon}\rho\frac{\epsilon}{k}P_k - c_{2\epsilon}\rho\frac{\epsilon^2}{k}P_k \tag{5}$$

In equations (4) and (5), $\mu_t = \rho C_\mu k^2/\epsilon$ and k is the turbulence kinetic energy $k = \frac{1}{2}(\overline{u_1 u_1} + \overline{u_2 u_2} + \overline{u_3 u_3})$

Φ represents the dissipation rate terms:

$$\Phi_{ij} = \Phi_{ij,1} + \Phi_{ij,2} + \Phi_{ij,w1} + \Phi_{ij,w2}$$

where $\Phi_{ij,1} = -c_1\rho\frac{\epsilon}{k}\left[\overline{u_i u_j} - \frac{2}{3}\delta_{ij}k\right]$
and $\Phi_{ij,2} = -c_1\left[P_{ij} - \frac{2}{3}\delta_{ij}P_k\right]$
and $\Phi_{ij,w1}$ and $\Phi_{ij,w2}$ are the wall correction terms.

We consider the flow to be fully developed ($\frac{\partial}{\partial x_1} <>= 0$), spanwise homogeneous ($\frac{\partial}{\partial x_3} <>= 0$) and one-dimensional for the mean velocities ($U_2 = U_3 = 0$). Applying these simplifications, the 6 governing equations to be solved are:

$$0 = -\frac{\partial \bar{P}}{\partial x_1} + \frac{\partial}{\partial x_2}\left\{\mu\frac{\partial U_1}{\partial x_2}\right\} - \frac{\partial}{\partial x_2}\left\{\rho\overline{u_1 u_2}\right\} \tag{6}$$

$$0 = -\mu\frac{\partial^2}{\partial x_2^2}\left\{\overline{u_1 u_1}\right\} + P_{11} + \Phi_{11} + D_{11} - \frac{2}{3}\rho\epsilon \tag{7}$$

$$0 = -\mu\frac{\partial^2}{\partial x_2^2}\left\{\overline{u_2 u_2}\right\} + \Phi_{22} + D_{22} - \frac{2}{3}\rho\epsilon \tag{8}$$

$$0 = -\mu \frac{\partial^2}{\partial x_2^2}\{\overline{u_3 u_3}\} + \Phi_{33} + D_{33} - \frac{2}{3}\rho\epsilon \tag{9}$$

$$0 = -\mu \frac{\partial^2}{\partial x_2^2}\{\overline{u_1 u_2}\} + P_{12} + \Phi_{12} + D_{12} \tag{10}$$

$$0 = -\mu \frac{\partial^2 \epsilon}{\partial x_2^2} + D_\epsilon + c_{1\epsilon}\frac{\rho\epsilon}{k}P_k - c_{2\epsilon}\frac{\rho\epsilon^2}{k} \tag{11}$$

And we have 6 unknowns to be solved for: $U, \overline{u_1 u_1}, \overline{u_2 u_2}, \overline{u_3 u_3}, \overline{u_1 u_2}, \epsilon$

Discretized Equations

The finite volume method (FVM) is used to to discretize the governing equations.
For example, the U-velocity equation is discretized as:

$$\frac{2\nu}{\Delta y}U_P = \frac{\nu}{\Delta y}U_S + \frac{\nu}{\Delta y}U_N + \Delta y + \frac{(\overline{u_1 u_2})_S + (\overline{u_1 u_2})_N}{2}$$

Similarly, the other 5 equations are also discretized to the form

$$a_P \phi_P = \sum a_{nb}\phi_{nb} + b$$

The Gauss Seidel method is then used to solve the system of equations.

Boundary Conditions

The first grid point is chosen to be $y^+ = 30$, inside the log-law region. At the first grid point, wall functions are used:

$$U = \frac{u_\tau}{\kappa} ln(y^+ E)$$

$$\overline{u_1 u_1} = 3.67u_\tau^2$$

$$\overline{u_2 u_2} = 0.83u_\tau^2$$

$$\overline{u_3 u_3} = 3.67u_\tau^2$$

$$\overline{u_1 u_2} = -u_\tau^2$$

$$\epsilon = u_\tau^3/(\kappa y)$$

and u_τ is calculated iteratively using the expression:

$$U^+ = \frac{1}{\kappa} ln(y^+) + 5.2$$

A half-channel is used. Hence the symmetry boundary condition $\partial <> /\partial x_2 = 0$ is used for all 6 variables at the channel center-line.

Results

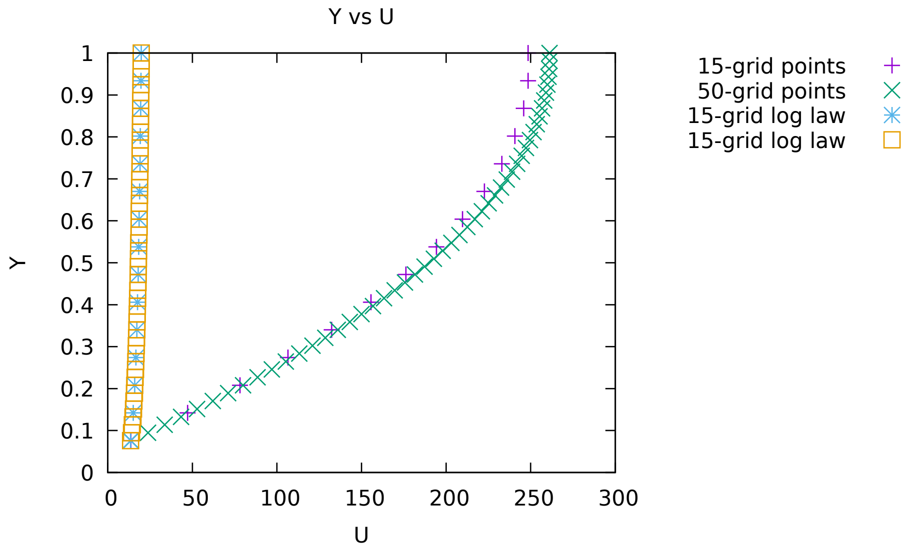

Figure 1: U-velocity vs log-law

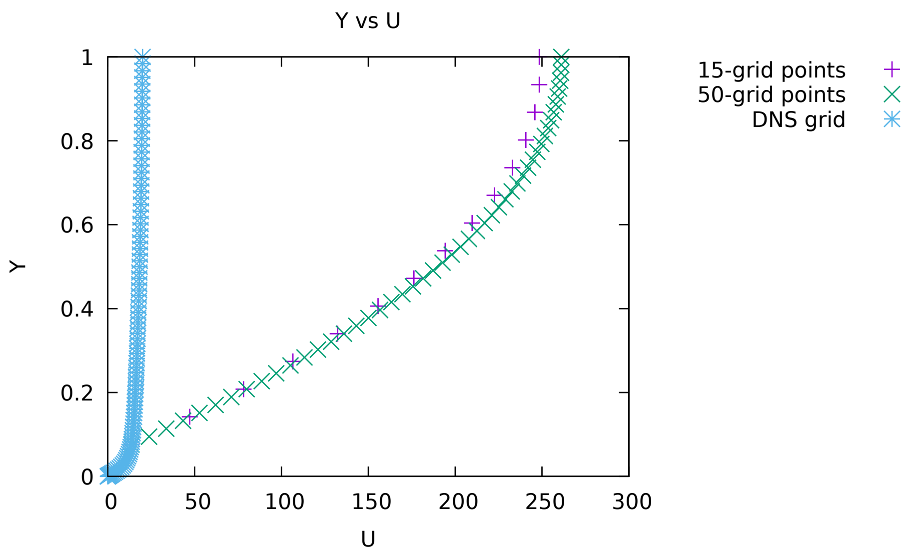

Figure 2: U-velocity vs DNS

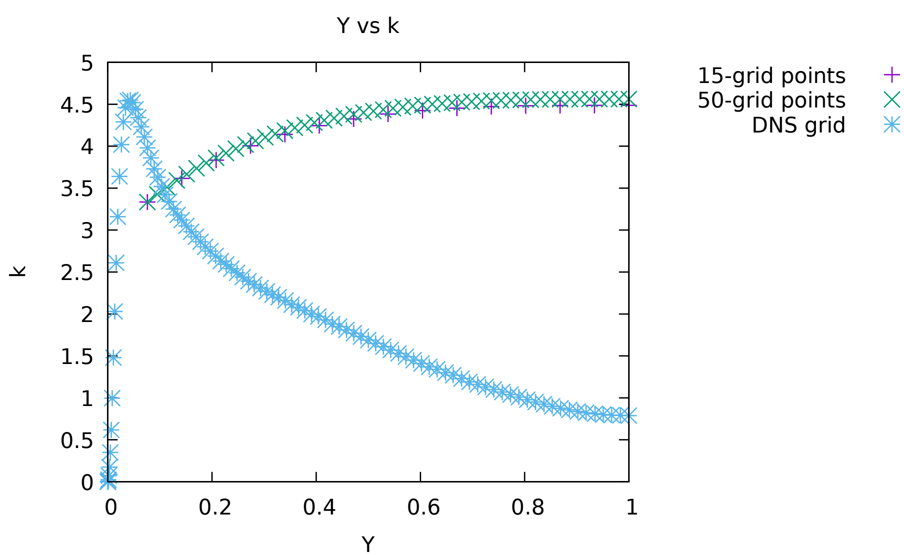

Figure 3: Turbulence Kinetic Energy

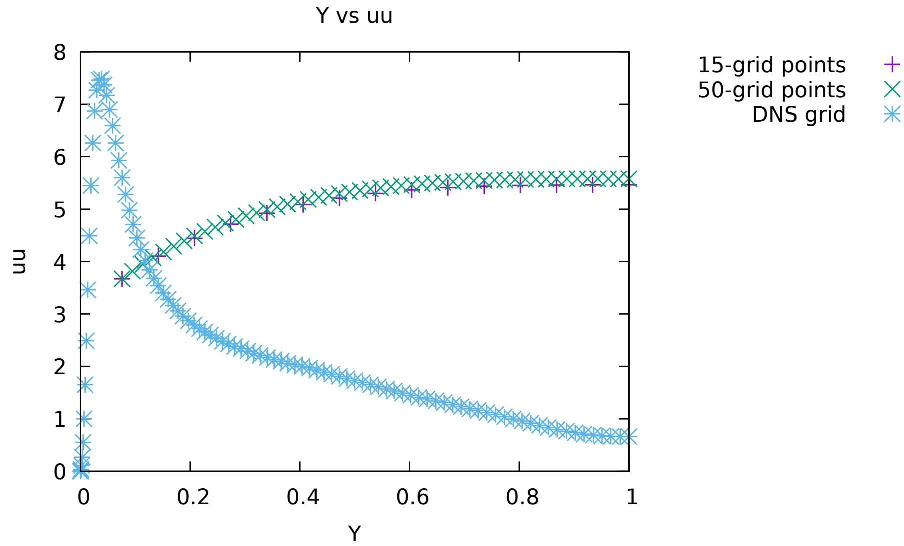

Figure 4: $\overline{u_1 u_1}$-velocity

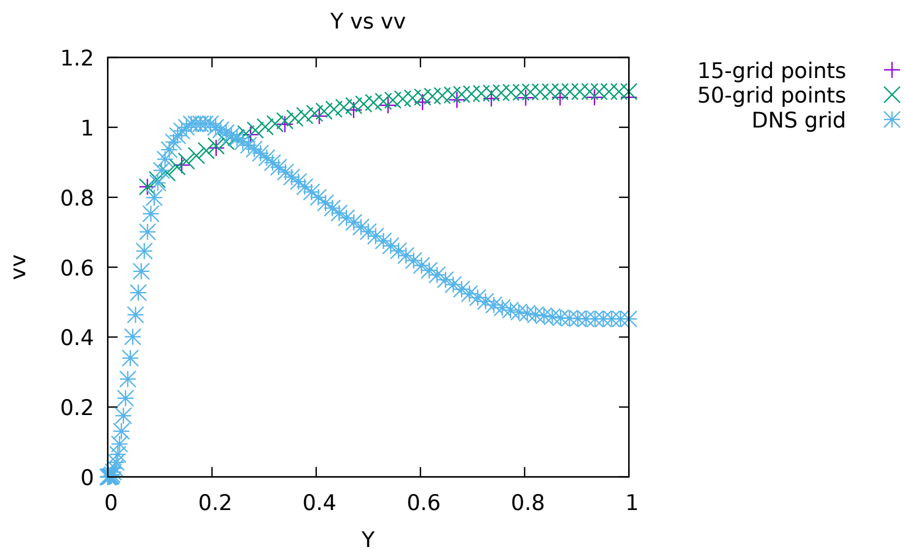

Figure 5: $\overline{u_2 u_2}$-velocity

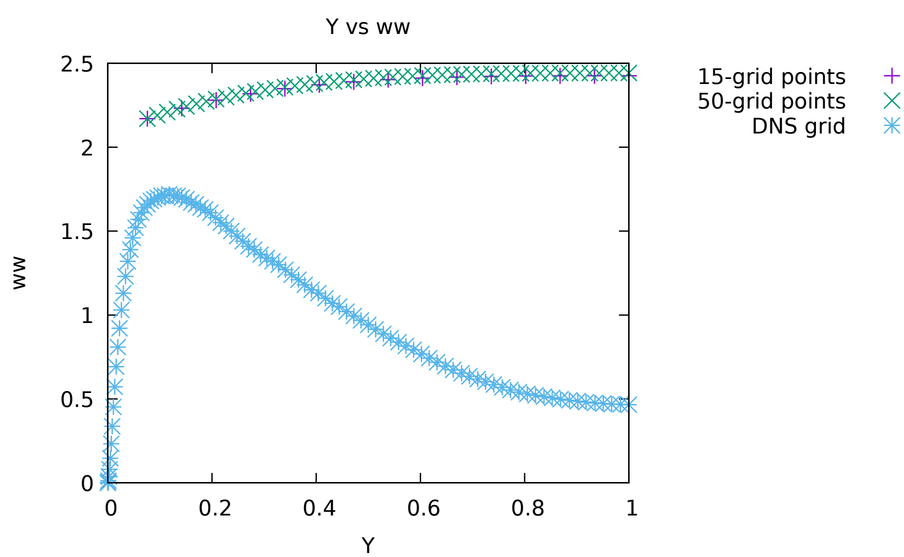

Figure 6: $\overline{u_3 u_3}$-velocity

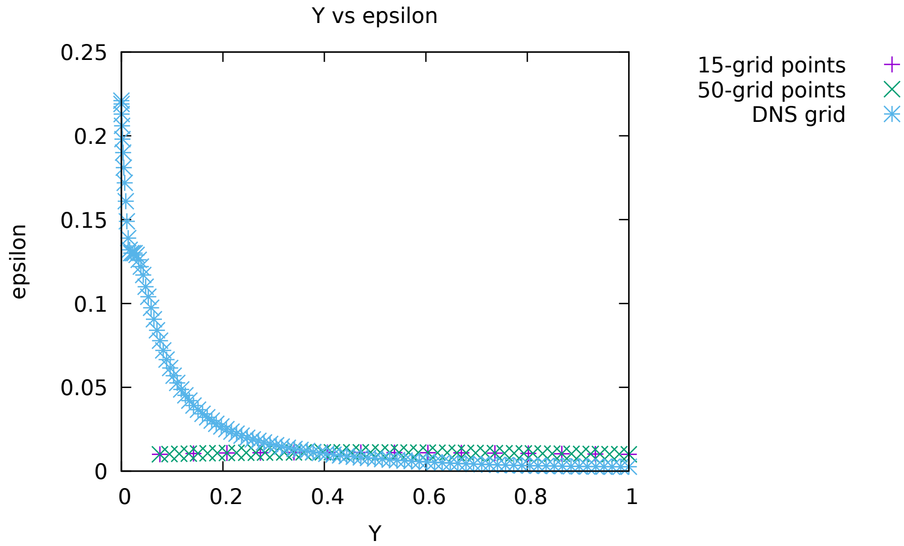

Figure 7: Dissipation rate ϵ