Fitting Taper Equations from Standing Trees

by

C.A. James and A. Kozak[1]

Abstract

Taper equations from standing trees were developed for 32 major groups of the commercial species of British Columbia by applying outside bark data to an accepted taper model (the Whole-Bole system). Tests of this model on all groups show that estimates of volume, diameters inside bark at different heights, and heights for different diameters are sufficiently accurate and precise for practical use. In some cases, standing tree estimates are more reliable than those produced from the inside bark taper equations currently used in British Columbia. Including a bark thickness prediction function in the model would improve the reliability of estimation.

Taper data can be obtained from standing trees accurately and inexpensively with a Barr & Stroud dendrometer. Because of this method of data collection is more efficient than felled tree measurement, these standing tree taper equations have great potential for use in local situations and where non-destructive samples are required.

Résume

Des équations de défilement, applicables à des arbres sur pied, ont été produites pour 32 groupes d'importance des espèces commerciales de la Colombie-Britannique. Pour ce faire, on a adapté un modèle de défilement accepté (le système "Whole-Bole") en utilisant des données sur l'écorce externe. Des tests effectués sur le nouveau modèle démontrent que pour tous les groupes, les estimations de volume, de diamètre sans écorce à différentes hauteurs, et des hauteurs en fonction de différents diamètres, sont suffisamment précises pour être utilisées à des fins pratiques. Dans certains cas, les estimations pour les arbres sur pied sont plus fiables que celles produites par les équations de défilement sans écorce couramment utilisées en Colombie-Britannique. L'inclusion d'une fonction de prédiction de l'épaisseur de l'écorce dans le modèle améliorerait la fiabilité des estimations.

On peut obtenir à peu de frais des données précises de défilement d'arbres sur pied en utilisant un dendromètre de type "Barr & Stroud". Cette méthode de collecte de données est plus efficace que celle où l'on mesure des arbres abattus. Par conséquent, les équations de défilement, applicables à des arbres sur pied, démontrent un grand potentiel d'utilisation dans des contextes régionaux et lorsque des échantillons non destructifs sont requis.

Introduction

Taper equations give the most complete information concerning tree form (Meyer 1953), and are invaluable tools in forest inventory systems. The search for accurate taper models began late in the 19th century, and since then, many models have been developed. Various authors (Petterson 1927, Bennett and Swindel 1972, Omerod 1973, Max and Burkhart 1976) have recognized that two equations describe the tree profile more accurately than one, and many of these models have resulted in good estimates of taper. However, volume estimates are usually less satisfactory due to bias (Demaerschalk 1973).

Demaerschalk and Kozak (1977) have virtually solved the problem of bias, and have developed a dual-equation model that yields excellent estimates of tree taper and volume. This model is accepted and used in BC by both the British Columbia Ministry of Forests (BCMF) and private industry.

Demaerschalk and Kozak's model, like most of the others, was developed from a set of inside bark diameter measurements, take at intervals along the entire stem. To obtain such data, approximately 100 trees must be felled, bucked, and measured — a costly undertaking. If precise and accurate taper equations could be developed from a set of outside bark measurements from standing trees, data collection would be rapid, relatively low in cost, and non-destructive. The availabil-

[1]C.A. James is a graduate student and A. Kozak is Professor and Associate Dean, both in the Faculty of Forestry at the University of British Columbia. Financial support of the Natural Sciences and Engineering Research Council of Canada and provision of data by the BC Ministry of Forests are gratefully acknowledged.

ity of such equations would make their use in local situations, or where non-destructive samples are required, very appealing.

This paper describes the development of precise and accurate taper equations from standing trees using Demaerschalk and Kozak's (1977) taper system. Advantages and difficulties associated with standing tree equations are also discussed.

The Data Base

The stem analyses data used in this paper were collected from over 34 000 trees by the BCMF Inventory Division. The same data base was used in the development of the BCMF logarithmic volume equations and the taper equations currently used in the province.

The data were stratified into 32 groups according to species, maturity, and locality. For each tree, measurements were made for diameters inside and outside bark and for double bark thickness (DBT) to the nearest 0.1 cm at heights of 0.3, 0.46, 0.60 and 1.3 m, and at each 10th of total height above 1.3 m.

To develop taper equations from standing trees, only the outside bark diameters in the data set are of interest. Although bark thickness can vary considerably among trees of the same species and size, its relationship to diameter at various points on the stem can be predicted from a reference bark thickness at a single point (Mesavage 1969). Breast height was chosen as a convenient reference point, and all outside bark diameters in the data set were reduced by double bark thickness at breast height expressed as a percentage of DBH to obtain the corresponding inside bark diameters. These diameters are the 'standing tree data" used to develop the taper equations in this paper.

Methods

The dual-equation taper system, or 'Whole-Bole system' used to develop taper equations from standing trees has been explained in detail elsewhere (Demaerschalk and Kozak 1977). Briefly, the tree profile is described in terms of diameter inside bark (*d*) as a function of DBH, total height (HT), and distance (*h*) from the top of the tree. The functions of *d* are monotonic, non-decreasing, and strictly non-negative from tree top to ground level. The diameter (*d*) is equal to zero at the tree top, and is equal to diameter inside bark at breast height (DIB). The two equations used to describe the tree profile are linked smoothly together at the inflection point, the point where tree form changes from neiloid to paraboloid (Figure 1).

The shape of all trees within each of the 32 species groups was examined through plotting the relationship between diameters and height from ground on a relative scale, with

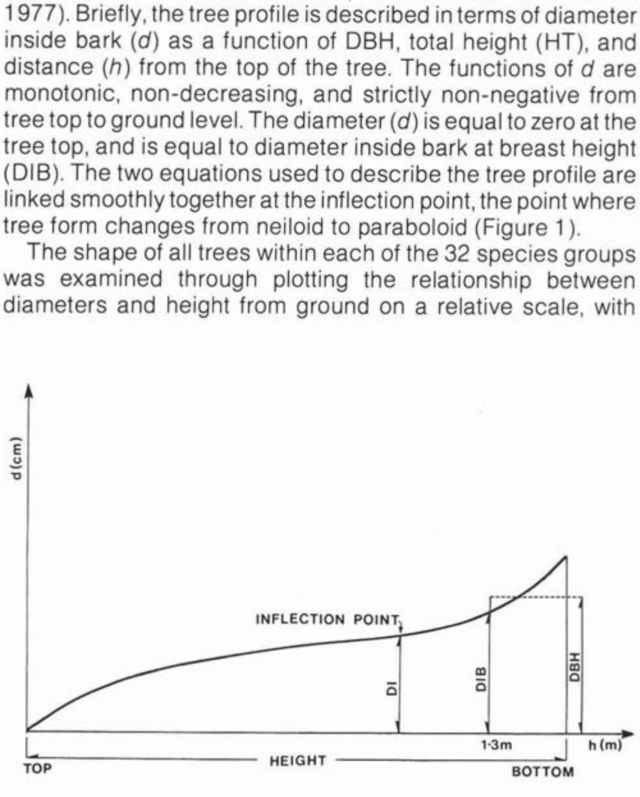

Figure 1. Model of a dual-equation taper system.

d/DBH as the dependent variable and (HT-*h*)/HT as the independent variable. Figure 2 illustrates these plottings for coastal immature Douglas-fir [*Pseudotsuga menziesii* (Mirb.) Franco]. Within each species group, the overall shape of the trees is very similar for all size classes, and the inflection point is at a more or less constant relative height. The relative height of the inflection point above ground was determined for each species group.

In Demaerschalk and Kozak's original model, DIB was predicted by a second-degree polynomial. However, with standing tree data, DBT at breast height is known, and DIB is determined through subtraction of DBT at breast height from DBH. One regression and one source of error are eliminated.

Diameter inside bark at the inflection point (DI) is predicted from DIB by a second-degree polynomial:

$$DI = b_0 + b_1 DIB + b_2 DIB^2.$$

Two mathematical functions are used to describe stem taper. The first function, which describes the tree profile from top to the inflection point, is:

$$Y = a \cdot X^b \cdot c^{(1-X)},$$

and the second function, which describes the tree profile from inflection point to ground level, is:

$$Y = a + b(1-X)^c$$

where $Y = d/DI$, $X = h/HT$, and a, b, and c are constants, different for each species group.

At this point, the equations are properly conditioned to give good estimates. Demaerschalk and Kozak (1977) describe the conditioned equations in detail.

The model described here was run with standing tree data for all 32 species groups. Merchantable heights for various top diameters and total volumes were estimated for each 20 cm DBH class. Estimates of diameters inside bark were determined for 10 separate height classes, each representing 10% of total height. The precision and accuracy of the standing tree taper estimates were determined, respectively, by the standard error of estimate (SE) and the bias in the estimation of these volumes, heights, and diameters.

To determine how efficiently standing trees can be measured, a time study and cost analysis were carried out in which height and diameter measurements were collected from 103 standing trees with a Barr & Stroud optical dendrometer, Type FP-12. The literature indicates that upper stem diameters measured with the Barr & Stroud are highly accurate under favorable field conditions (Bell and Groman 1971, Brickell 1976).

For each tree, a two-man crew recorded the following measurements: total height, diameter outside bark at 0.3, 0.6, and 1.3 m, and at each 10th of total height, and bark thickness at breast height at two different points on the stem. The number of trees measured per day was recorded, and the cost of measuring standing trees was determined. All trees measured were located along stand edges, road cuts, or in open-grown stands on the BC coast, and an even distribution of size classes from 1 to 100 cm DBH was obtained.

Results

Table 1 presents the errors in volume, diameter inside bark, and merchantable height produced by the standing tree taper equations for coastal immature Douglas-fir. Errors produced by the BCMF inside bark taper equations currently used in BC are presented for comparison.

The volume estimates for Douglas-fir produced by the standing tree equations are consistently low, but average bias in volume is only 0.05 m³. Both taper systems produce equally

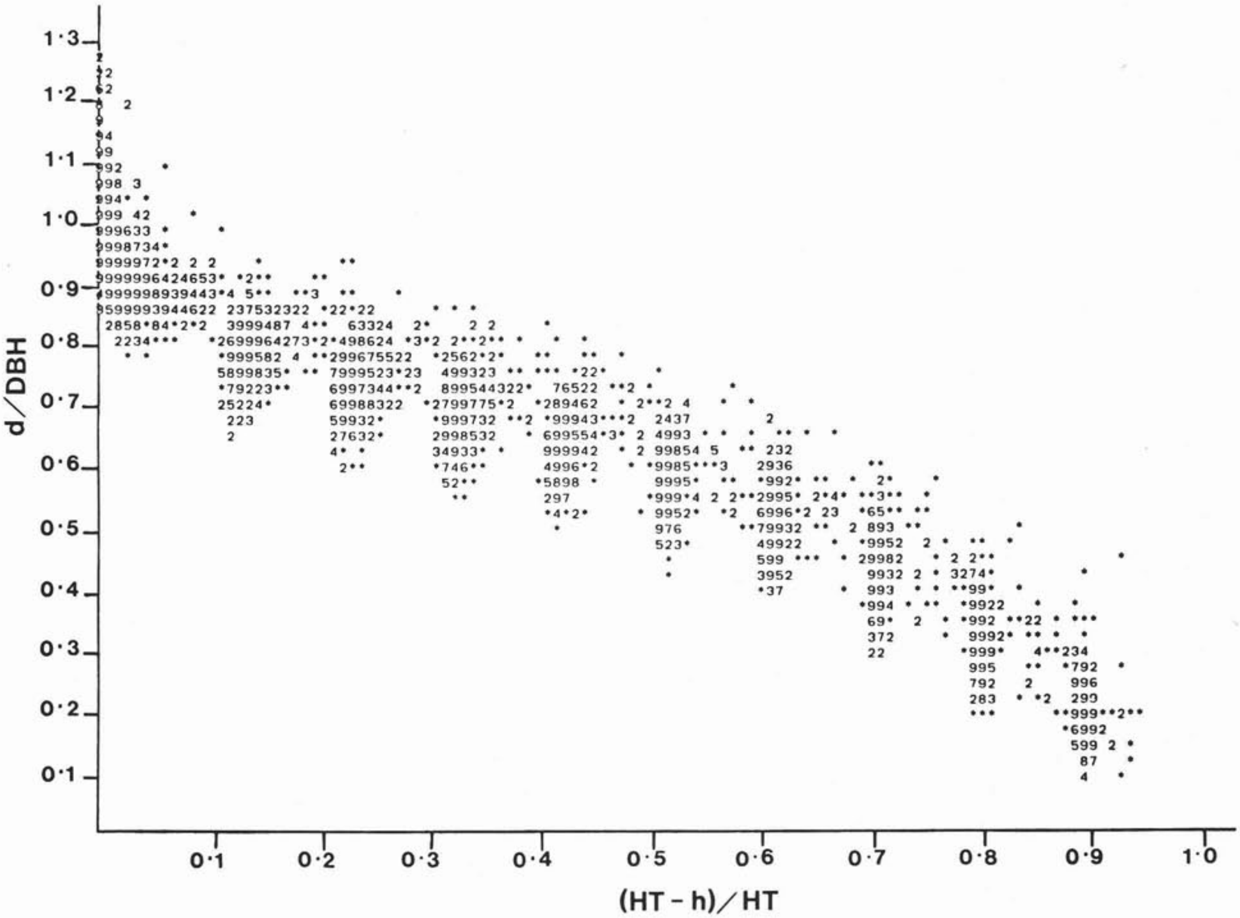

Figure 2. Plotting of tree profile data for coastal immature Douglas-fir.

Table 1. Inside and outside bark values of standard error (SE) and bias in the estimation of volume, diameter inside bark (*d*), and height (*h*) for coastal immature Douglas-fir.

		Standard error of estimate and bias[1] of volume for the following DBH classes:					
		0.0-20.0	20.1-40.0	40.1-60.0	60.1-80.0	80.1-100.0	Total
Outside Bark[2]	Bias (m^3)	0.00	0.03	0.13	0.24	0.26	0.05
	SE (m^3)	0.01	0.08	0.25	0.48	0.26	0.14
Inside Bark	Bias (m^3)	0.00	0.00	0.03	0.09	−0.40	0.01
	SE (m^3)	0.02	0.07	0.23	0.49	0.40	0.14

		Standard error of estimate and bias of *d* at the following heights:										
		.10HT	.20HT	.30HT	.40HT	.50HT	.60HT	.70HT	.80HT	.90HT	1.0HT	Total
Outside Bark	Bias (cm)	0.40	0.60	0.78	0.69	0.63	0.63	0.65	0.58	0.23	0.03	0.47
	SE (cm)	1.88	1.29	1.47	1.45	1.47	1.51	1.66	1.67	1.63	0.83	1.58
Inside Bark	Bias (cm)	1.14	0.03	0.05	−0.09	−0.14	−0.10	0.03	0.12	0.04	0.02	0.32
	SE (cm)	2.46	1.30	1.39	1.42	1.41	1.40	1.49	1.53	1.57	0.81	1.75

		Standard error of estimate and bias of merchantable *h* for the following DBH classes:					
		0.0-20.0	20.1-40.0	40.1-60.0	60.1-80.0	80.1-100.0	Total
Outside Bark	Bias (m)	0.27	0.08	0.02	−0.69	−1.36	0.11
	SE (m)	0.63	0.82	1.17	1.06	1.36	0.85
Inside Bark	Bias (m)	0.26	0.04	−0.03	−0.67	−1.38	0.08
	SE (m)	0.60	0.77	1.09	0.99	1.38	0.79

[1]Positive bias represents underestimation and negative bias represents overestimation.
[2]Outside bark estimates are derived from standing tree taper equations.
Inside bark estimates are derived from BCMF inside bark taper equations.

Table 2. Inside and outside bark values of standard error (SE) and bias in the estimation of volume, diameter inside bark (*d*), and height (*h*) for bigleaf maple.

		Standard error of estimate and bias[1] of volume for the following DBH classes:			
		0.0-20.0	20.1-40.0	40.1-60.0	Total
Outside Bark[2]	Bias (m^3)	0.00	0.01	0.02	0.00
	SE (m^3)	0.01	0.03	0.05	0.02
Inside Bark	Bias (m^3)	0.00	0.01	0.05	0.01
	Se (m^3)	0.01	0.04	0.06	0.02

		Standard error of estimate and bias of *d* at the following heights:										
		.10HT	.20HT	.30HT	.40HT	.50HT	.60HT	.70HT	.80HT	.90HT	1.0HT	TOTAL
Outside Bark	Bias (cm)	1.15	-0.18	-0.07	-0 .06	0.02	-0.01	-0.10	-0.22	-0.30	0.05	0.24
	SE (cm)	1.81	0.61	0.62	0.73	0.89	0.93	1.05	1.04	1.04	0.46	1.20
Inside Bark	Bias (cm)	1.08	-0.13	-0.00	0.02	0.12	0.14	0.10	0.03	-0.03	0.04	0.33
	SE (cm)	1.75	0.60	0.63	0.74	0.93	0.97	1.07	1.05	0.99	0.44	1.17

		Standard error of estimate and bias of merchantable *h* for the following DBH classes:			
		0.0-20.0	20.1-40.0	40.1-60.0	Total
Outside Bark	Bias (m)	0.04	-0.35	-0.86	-0.09
	SE (m)	0.54	0.75	0.87	0.62
Inside Bark	Bias (m)	0.22	-0.14	-0.61	0.10
	SE (m)	0.60	0.71	0.62	0.64

[1]Positive bias represents underestimation and negative bias represents overestimation.
[2]Outside bark estimates are derived from standing tree taper equations.
Inside bark estimates are derived from BCMF inside bark taper equations.

precise volume estimates. Douglas-fir diameters are also consistently underestimated with standing tree data, with an overall bias of 0.47 cm. However, on average, diameter estimates are 10% more precise than those obtained from the inside bark equations. Errors in merchantable height are quite small, with an overall bias of only 0.11 m and a SE of 0.85 m.

Table 2 presents the errors produced by the standing tree equations for bigleaf maple (*Acer macrophyllum* Pursh). Again, errors produced by the BCMF taper equations are presented for comparison.

Excellent results for maple were obtained with the outside bark data. In all cases, the average biases were smaller than those obtained from the inside bark equations. On average, volume of maple can be predicted without bias, and with a SE of only 0.02 m^3, and diameter can be predicted with a bias of only 0.24 cm. Merchantable height estimates for maple are more accurate and more precise than those produced from the inside bark equations. On average, height is overestimated by only 0.09 m with a SE of 0.62 m.

The estimates of volume, diameter and merchantable height based on standing tree data are precise and accurate for all 32 species groups, often more so than the estimates based on inside bark data. Error in the estimation of volume and merchantable height generally increases with increasing size class, and there is often some underestimation of diameter inside bark near ground level, largely due to butt swell. Despite these minor shortcomings, also present in the original model of Demaerschalk and Kozak (1977), the taper equations developed from standing trees produce very reliable estimates.

The cost of measuring standing trees under good conditions in coastal BC with a Barr & Stroud dendrometer is $13.23 per tree[2]. The total cost of measuring 103 trees, the approximate number required for taper equation development, is $1362.58. In comparison, the cost of collecting similar data from felled trees is over twelve times as great, at $165.88 per tree[3]. Thus, dendrometry offers vast savings in time and money over felled tree measurement, and the use of accurate standing tree data for taper equation development is a practical alternative to conventional sampling.

Discussion

Much of the error encountered with standing tree taper estimates is due to the assumption that double bark thickness as a percentage of diameter is constant along the entire stem. Figure 3 illustrates the variation in DBT with increasing tree height for coastal immature Douglas-fir, and shows that the assumption is invalid. In this study, the %DBT at 1.3 m was used to convert all outside bark diameters to inside bark diameters. Clearly, bark thickness was underestimated below 1.3 m and greatly overestimated between 1.3 m and the top of the tree. This in turn led to an overestimation of diameter inside bark (*d*) below 1.3 m and an underestimation of *d* above this point. Volume estimates are consistently low as well, resulting from the underestimation of *d* above 1.3 m.

Errors for many species groups, particularly the thick-barked species such as larch (*Larix* Mill. spp.) and ponderosa pine (*Pinus ponderosa* Laws.) can be explained in this fashion. Very small errors of estimate were obtained for most of the hardwood species, which are relatively thin-barked. The accuracy and precision of the standing tree taper equations could be improved by incorporating a function to account for variable bark thickness along the stem.

Dendrometry is a viable alternative to conventional felled tree analysis as a means of collecting taper data. The Barr & Stroud is an excellent dendrometer when used under favorable conditions, but in some forest types, visibility may pose a problem. Much of the time required for measurement is often spent finding a suitable instrument location and clearing brush that obscures the line of sight.

The cost of dendrometry will vary, depending upon the character of the stand in question. In this study, trees were measured under favorable conditions, and thus, for a very low cost. However, even with increased measurement time in dense stands, it is very likely that standing tree measurement will be efficient and cost-effective when compared with felled tree measurement. For maximum productivity, measurements

[2]Based on the standard BCMF Forest Technician wage rate.
[3]Barker, J. Research Forester, Western Forest Products Ltd., Victoria, BC. 24 June 1982. Personal correspondence.

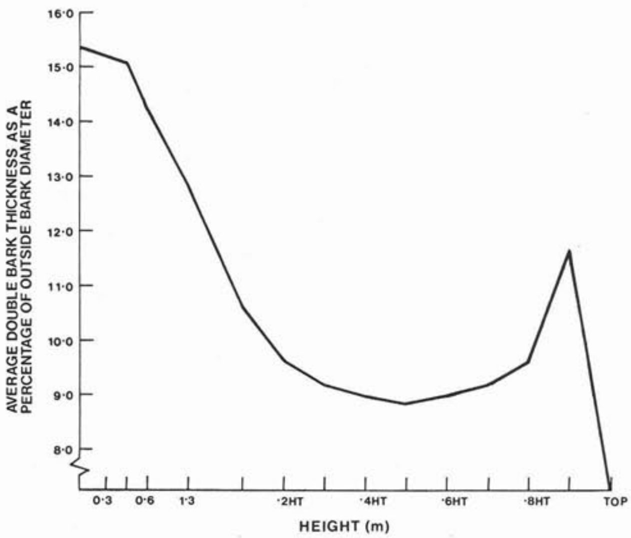

Figure 3. Variation in average double bark thickness with increasing height for coastal immature Douglas-fir.

should be collected on bright, sunny days from trees located along recently exposed road cuts or stand edges. If work in dense stands is unavoidable and parts of a stem are obscured, it is impractical to measure diameters at fixed heights. Rather, diameters can be measured at arbitrary points along visible portions of the stem. Once a suitable instrument location is found, additional measurements are very rapid and are a direct investment in accuracy.

Only one other relatively minor problem was experienced with the Barr & Stroud. The smallest diameter that can be measured with this instrument is 3.8 cm, and occasionally diameters smaller than this are encountered near tops of trees. These diameters can be approximated, based on the operator's judgement. Magnification, a feature of the Barr & Stroud, is an aid in estimating these small diameters and the error introduced by these approximations is negligible.

To accurately convert outside bark diameters to inside bark diameters, a reliable measure of DBT at breast height is required. Unfortunately, the only bark gauges available tody which do not compress the bark core cannot measure bark thicker than 7 cm. The bark of some species, such as Douglas-fir, larch, and ponderosa pine, can greatly exceed 7 cm. In such cases, bark thickness must be estimated by the operator, but this may result in inaccurate inside bark diameters. If measurement of DBT at breast height becomes more common, it is likely that a suitable instrument will become available.

Conclusions

Taper equations derived from standing trees for the commercial species of British Columbia yield precise and accurate estimates of volume, diameter inside bark, and merchantable height. The only field measurement required to use them, in addition to DBH and HT, is double bark thickness at breast height. For several species groups, the standing tree taper equations produce more reliable estimates than do the inside bark equations currently used by the BCMF. They could be further improved by incorporating a bark thickness predicting function into the taper model, to account for variations in DBT with increasing tree height.

Dendrometry is a viable method of collecting taper data, although poor visibility may limit its use in dense stands. Selecting measurement points arbitrarily along visible portions of the stem, rather than at fixed heights, will provide more latitude in selecting trees for measurement and will help minimize measurement time.

Because taper data can be collected accurately and inexpensively with a dendrometer such as the Barr & Stroud, standing tree taper equations like those devloped here have great potential for use in local situations, and where non-destructive samples are required.

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