Taper Functions and their Application in Forest Inventory

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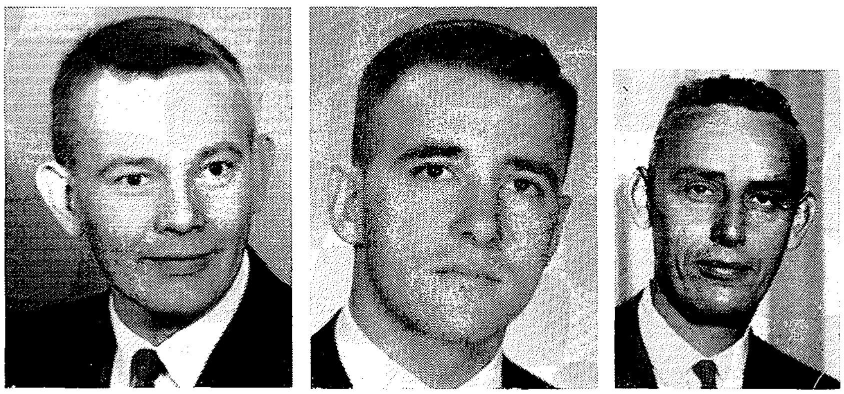

Abstract

Total inventories and methods of volume summary which facilitate compilation of tree and log volumes to any desired standard of utilization are needed. It should be possible to find volume per tree to any specified standard of utilization expressed as stump height, top dib, and/or section height. Furthermore, the technique should be able to provide the fraction of volume per tree located in logs of any specified length and dib and by any system of scaling, be it board feet, cubic feet, or weight. The system also should be usable to describe the influence of biological factors on bole shape and taper, and to estimate component weights in biomass and other studies. The functions developed in this paper and based upon dbh, total height, and section height as a fraction of total height can meet most of these requirements.

Résumé

Les fonctions de défilement et leur application dans les inventaires forestiers

On a besoin d'inventaires forestiers complets et de méthodes pour faciliter la compilation de volumes ligneux d'arbres entiers ou de billes correspondant à des niveaux d'utilisation déterminés. Il devrait être possible d'évaluer le volume total par arbre suivant des niveaux d'utilisation exprimés en terme de hauteur de souche, de diamètre minimum d'utilisation, et/ou de fraction de la hauteur totale. De plus, la technique devrait pouvoir fournir la proportion du volume total de la tige comprise dans les billes de toute longueur ou diamètre, quelque soit le système de mesurage employé (pied mesure de planche, pied cube ou poids). La méthode devrait aussi pouvoir être utilisée pour décrire l'influence des facteurs biologiques sur la forme du fût et son défilement, et pour estimer les composantes du poids dans les études de biomasse et autres. Les fonctions de défilement présentées ici sont basées sur le dhp, la hauteur totale et des fractions de la hauteur totale. Elles peuvent rencontrer la plupart des exigences énumérées.

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Modern manufacturing methods, coupled with the multiplicity of possible end-uses of a single log in an integrated operation, require that the forester provide detailed information regarding the raw material supply. Data for the modern forest inventory must be collected in a manner that permits flexible and comprehensive analyses. Estimates of total volume per acre no longer suffice. Now the volume of material available in certain sizes and qualities must be estimated with high standards of precision and accuracy.

At the 1963 annual meeting of the Canadian Institute of Forestry, Honer and Sayn-Wittgenstein stated:

"We must develop a mathematical tree volume expression which can be efficiently programmed for generally available electronic computing equipment to yield tree and stand volumes from inputs of tree diameter outside bark and total heights (form estimates optional) and for any demanded stump height and top diameter."

The basic total tree volume equation used by the British Columbia Forest Service is logarithmic and reduction factors for various utilization standards and decay classes are usually graphically determined and supplied from tabular summaries. Although well suited to the needs recognized by the British Columbia Forest Service, this system is rather cumbersome to computerize and is difficult to adapt quickly to changing industrial needs for information as to new sources of wood within and among tree classes. Uses of our taper functions in an industrial forest inventory have been described by Clark (1968).

As planning for mechanization of logging and milling accelerates, demands for knowledge about the total lineal footage of logs or number of pieces per unit will increase. That our system can provide such information has been demonstrated already by Lee (1967). We are still exploring the potential uses of these concepts but can draw some useful observations at this stage.

Methods for describing bole shape and taper

Knowledge of the distribution of wood increments within trees has provided information (Smith, Heger and Hejjas, 1966; Heger, 1965, and Larsen, 1953) of value in formulating taper equation hypotheses, however, much remains unknown about biological mechanisms controlling bole shape and taper. As new information becomes available, methods of describing bole shape can be refined. The B.C. Forest Service has prepared taper curves for mature trees representing all of the commercial species and species groups of British Columbia. These are available by two inch dbh classes and ten feet height classes, which provide an excellent graphical basis for definition of tree volume and calculation of the fraction of the total that is utilizable. We have recently converted all of these B.C. Forest Service taper graphs to functions which will be easier to use in the computer.

The B.C. Forest Service also has tree taper curves for immature Coastal Douglas fir, western hemlock, western red cedar, and silver fir. We now are comparing the B.C. Forest Service taper curves with the same data used by Smith and Walters (1964). The graphical methods used by Smith and Walters provided much useful data on dob, dib, and on growth in wood by decades but it is difficult to interpret and to generalize from such graphically summarized data.

An early approach to development of generalized taper functions was made by Newnham (1958) in his M. F. thesis which showed that a quadratic parabola was well suited to description of bole shape. Fries (1965) and Fries and Matern (1965) advocated the use of multivariate methods for the construction of tree taper curves. Our subsequent analyses of multivariate techniques (Kozak and Smith, 1966) convinced us that simple functions, sorting, and graphical methods are adequate for many uses in operations and research. Since then we have found that some rather simple taper functions proposed by Munro (1968) are very useful.

We considered that the systems based upon measurements of upper bole diameters should be avoided, if possible, but still have tested several methods. Concurrently with our study of multivariate techniques Heger (1965b) illustrated the use of Hohenadl's method based on upper stem diameters for determination of stem form and volumes for lodgepole pine and expanded to include Engelmann spruce by Stanek (1966).

When the Barr and Stroud dendrometer became available it seemed at first as if our earlier conclusions about the utility of direct measurements of upper bole diameters might need revision. Although it was expensive, the dendrometer offered many potential advantages through direct measurement and elimination of possible volume table biases. However, our attempts to use the Barr and Stroud operationally under Pacific Coast forest conditions, and the difficulties of securing consistent results among operators, forced us to re-evaluate the dendrometer. Pfeiffer (1967) found that even with easily visible boles the average deviation of Barr and Stroud dib's was from 0.25 to 0.5 inches different from the correct value. If such measurements are not adjusted further for actual variations in bark thickness (Smith and Kozak, 1967) many of them will lead to estimates of dib that are little different from those obtainable indirectly by use of our simple taper functions (Kozak and Smith, 1966).

In 1967 we calculated taper functions for all of the commercial species and species groups in B.C. We could estimate dib at any height with species average standard errors of estimate ranging from 0.73 to 5.32 inches, and averaging about 1.75 inches. Three species gave us most trouble — Sitka spruce, Douglas fir, and western red cedar. Calculated taper lines for each tree were compared with actual data to analyse for possible biases in our estimates. Much of the variation from our taper lines resulted from a few, extremely rapidly tapering, large trees which were neiloidal rather than quadratic paraboloidal in shape. We were able to eliminate bias completely and to reduce our standard errors of estimate substantially by using percentage corrections at each decile within our sample trees, but were not satisfied with the use of such empirical percentage corrections.

A substantial number of new functions in which the influence of dbh, height, and height/dbh could enter directly were postulated and tested. Some of these will be described later, but none of our modifications was significantly better than the simple functions suggested by Munro in 1968.

Concurrently with these studies our functions were tested against stem analysis data from several hundred of each of the major Coast B.C. species that were provided by a large B.C. company. The constants derived from our roughly 100-tree samples of the B.C. Forest Service data fitted these new trees very well, accounting for well over 95 per cent of the variation in dib. However, since some improvements came when our functions were fitted to the new data we now regard our equations as useful generalizations that should be localized if appropriate new data are available.

While working on this aspect we also tried to develop simple methods for calculating taper tables. Since our equations assumed a constant influence of section height on taper, regardless of tree heights, we tested the need to recognize specific height groups. Our analyses showed some advantage to sorting data by classes of dbh and total height and the size of our constants was significantly different for two main classes of total height/dbh. We considered that dense stands would have values of total height/dbh of 7 or more and normal stands would average between 4 and 6, and found significant differences in our constants for these groups of trees. We could not test open grown trees which should have a ratio of total height to dbh of less than 4 because almost all of our B.C. Forest Service data were from forest grown trees. It is apparent that stratification by height classes may result in improved estimates, however, these results using different average classes have been difficult to generalize into an effective estimating system for calculating taper tables. We are continuing work on this aspect.

Our earlier studies stressed lodgepole pine. More recent work involved a thorough investigation of taper in mature western hemlock as part of a system for describing distribution of soundwood volumes.

Munro (1968) found that upper stem diameters, dib, could be estimated with a standard error of estimate of 2.06 inches from a function involving dbh, h/H, and h^2/H^2, which will be described later. Conditioning of this function improved the standard error of estimate to 1.59. Addition of terms up to a fifth degree polynomial reduced the standard error to 1.29, and addition of the complex powers proposed by Bruce **et al.** (1968) further reduced it to 1.20. For practical purposes, however, it appeared as if little real advantage resulted from use of complex equations to estimate tree taper.

Computer programs have been developed in which log volumes determined from dib and length values calculated from our taper equations are adjusted to meet the requirements of a particular volume table or volume function that was applied during inventory.

Derivation of simple yet effective taper functions

The basic relationship to develop the taper function was the parabolic function

$$\frac{d^2}{D^2} = a + b\,\frac{h}{H} + c\,\frac{h^2}{H^2} \qquad (1)$$

where d is diameter inside bark in inches at any given h in feet,
h is height above the ground in feet,
D is diameter breast high, outside bark, in inches, H is total height of the tree in feet, a, b, and c are regression coefficients. (Ancther set of coefficients can be calculated for dob, if desired.)

Estimates of upper bole diameters (d) can easily be obtained by re-arranging equation (1) to the form

$$d = D\sqrt{a + b\,\frac{h}{H} + c\,\frac{h^2}{H^2}} \qquad (2)$$

Least squares solutions of equation (1), obtained from data selected from 19 species and species groups from British Columbia, showed serious bias in estimated upper stem diameters. In most instances diameters in the upper third of tree height were overestimated. To overcome this bias it was necessary to condition the least squares solution to ensure that when h equals H, estimated diameter be exactly zero. This condition was ensured by imposing the restraint a = - b - c. Substitution of this condition into equation (1) yields the equation

$$\frac{d^2}{D^2} = -b - c + b\,\frac{h}{H} + c\,\frac{h^2}{H^2} \qquad (3)$$

which simplifies to

$$\frac{d^2}{D^2} = b\left(\frac{h}{H} - 1\right) + c\left(\frac{h^2}{H^2} - 1\right) \qquad (4)$$

Equation (4) without a constant term can be solved by least squares techniques without difficulty.

With the exception of Coastal spruce and cedar species, bias in diameter estimation throughout the length of the bole was negligible. For spruce and cedar, however, the single condition imposed was not sufficient and negative estimates of diameter occured above 90 per cent of total tree height. The reason for this, and the necessary additional restraints required to remedy the bias are explained in the following paragraphs.

For general discussion purposes, equation (1) can be re-written in the form

$$Y = a + b\,X + c\,X^2 \qquad (5)$$

Since (5) is a quadratic equation, it has two solutions for X when Y is zero. Introducing the first condition (4) forced only one of the two roots to be 1.00. The second root could still be either less than, or greater than, one. For most of the species it was greater than one, which did not matter since the maximum value of X used in the equation to estimate (d) is one. But if the solution for the second root was less than one, as it was for coastal spruce and red cedar, the estimate of (d) became zero and negative around 90 per cent (X = .9) of the total tree height, then returned to zero again for X equals 1 (total tree height) because of the condition a + b + c = 0. This problem was eliminated by forcing the regression constants a, b, and c to take such values that the solution of equation (5) resulted in only one root for X, which was real. To satisfy this condition, the solution formula for quadratic equations, which with our notation in equation (5) is

$$X = \frac{-b \pm \sqrt{b^2 - 4ca}}{2c} \qquad (6)$$

has to be examined.

The first condition, that the sum of the coefficients equals zero, guarantees that one of the two solutions is one. The second condition is that equation (5) should have a unique solution, which occurs from (6) if

$$b^2 - 4ca = 0 \qquad (7)$$

and at the same time — b equals 2c to have the solution equal to one. This condition is satisfied if

$$a = c \text{ and } -b = 2c - 2a \qquad (8)$$

Putting (8) into equation (1) we get

$$\frac{d^2}{D^2} = c + (-2c)\frac{h}{H} + c\frac{h^2}{H^2} \qquad (9)$$

This can be simplified to

$$\frac{d^2}{D^2} = c\left(1 - 2\frac{h}{H} + \frac{h^2}{H^2}\right) \qquad (10)$$

Equation (10) is a simple regression equation without a constant term, where the dependent variable is

$$\frac{d^2}{D^2}$$

and the independent variable is

$$1 - \frac{2h}{H} + \frac{h^2}{H^2}.$$

This new version of conditioning was applied for only those species which had the tendency to estimate a negative diameter inside bark close to the top of the tree. The standard error of estimates, after the modification, ranged from 0.73 to 4.97 with an average of 1.72 for the 19 species.

The final step in our work was to reduce the standard error of estimate by using the diameter outside bark measure at breast height in the position of the diameter inside bark at one foot height. This method eliminated the high variation caused by butt-swell, as required for cubic foot scaling in B.C., and made the trees more uniform within a species close to the ground. Using this procedure the standard errors of estimate ranged from 0.58 to 2.77 with an average of 1.17 for the 19 species studied.

Further possibilities are presently being investigated to improve our taper functions. It looks promising to fit our simple function, with one or two conditions as required, separately for 10 feet height classes within a species, then find some relationship between height, dbh, and the regression constants a, b, and c.

Taper functions for 19 B.C. species and species groups

The method outlined in the previous section was applied to determine the taper functions from representative samples of B.C. Forest Service data for all major groups of the commercial tree species of B.C. The basic data were chosen from trees measured in volume and decay studies to cover a wide range of tree size, site quality, and locality. Trees had been fully described and measured after felling to find diameters inside and outside bark at stump height, breast height, and at deciles of total height above 4.5 feet. The tree sizes represented are shown by species groups in Table 1.

Table 1 Averages and Range of Basic Data

Species group	Dbh (o.b.) In. Min.	Dbh Ave.	Dbh Max.	H Ft. Min.	H Ave.	H Max.	H/Dbh Ft./In. Min.	H/Dbh Ave.	H/Dbh Max.	Age. Yr. Min.	Age Ave.	Age Max.	No. Trees
Alder	5	11	18	53	84	108	5	8	15	29	51	122	101
Aspen	5	10	21	42	78	103	5	8	11	37	95	152	126
Balsam — coastal	6	18	50	32	104	184	3	6	11	66	174	372	86
Balsam — interior	5	10	24	28	70	118	4	7	10	72	151	284	96
Cedar — coastal	6	20	51	31	89	189	3	5	8	73	203	385	85
Cedar — interior	5	16	48	27	79	162	3	6	11	83	191	489	91
Cottonwood	5	12	48	45	80	167	3	8	13	20	55	279	142
Douglas fir — coastal	5	22	71	52	122	231	3	7	11	67	170	409	97
Douglas fir — interior	5	18	42	27	76	120	2	6	9	60	170	460	78
Hemlock — coastal	5	16	42	40	100	176	4	7	12	73	206	509	81
Hemlock — interior	6	12	28	42	73	125	4	7	9	74	186	331	63
Lodgepole pine	5	11	23	42	75	130	4	7	12				456
Maple	6	13	23	49	82	94	4	7	11	20	57	123	52
Spruce — coastal	6	30	85	48	143	246	3	6	8	28	154	396	39
Spruce — interior	5	13	26	37	89	137	4	7	12	60	142	299	121
White birch	5	9	21	45	—	89	4	—	12	—	—	—	73
White pine	6	16	27	45	107	147	5	7	11	74	151	222	74
Yellow cedar	6	14	44	44	65	148	3	5	9	132	250	999	74
Yellow pine	6	18	40	20	78	135	2	5	7	40	186	423	101

Table 2 summarizes the regression constants and standard errors of estimate for the best equations found for the species and species groups discussed. These constants are calculated for equation (1) with single or double conditions, as required, and with dbh outside bark used as the measure of diameter inside bark at one foot height.

The standard errors of estimate (SE_E) listed in Table 2 are computed for diameter inside bark using the following formula

$$SE_E = \sqrt{\frac{\Sigma\,(d_a - d_e)^2}{n - p + k - 1}} \qquad (11)$$

where d_a is actual diameter inside bark,
d_e is estimated diameter inside bark,
n is number of observations used for the least squares fit,
p is number of independent variables used in the least square fit,
k is number of conditions stated,
Σ is the sum.

The equation form selected and the solutions provided in Table 2 permit the estimation of dib at any selected height (h) on the tree stem through equation (2). Merchantable heights to specified top diameters and lineal footage can be estimated by transformation of the basic equation to the form

$$h = \frac{-bH - \sqrt{(bH)^2 - 4c\,(aH^2 - \frac{d^2H^2}{D^2})}}{2c} \qquad (12)$$

Volumes of logs of specified diameters and lengths can be calculated by integrating the basic equation to yield.

$$V = .005454\, D^2H \left[a\,(\beta - \alpha) + \frac{b}{2}\,(\beta^2 - \alpha^2) + \frac{c}{3}\,(\beta^3 - \alpha^3) \right] \qquad (13)$$

where α and β are relative lower and upper limits of integration respectively.

All of the above applications are currently in use in the forest inventory programs of several large integrated forest companies in British Columbia and by several forestry consulting firms.

Table 2. Summary of taper functions

	a	b	c	SE_E
Alder	0.97576	−1.22922	0.25347	0.64
Aspen	0.95806	−1.33682	0.37877	0.72
Balsam — coastal	0.97952	−1.33264	0.35312	1.38
Balsam — interior	1.00965	−1.43135	0.42170	0.67
Cedar — coastal	1.00063	−2.00126	1.00063	1.72
Cedar — interior	1.03508	−2.07016	1.03508	1.28
Cottonwood	0.96150	−1.58271	0.62121	1.54
Douglas fir — coastal	0.85458	−1.29771	0.44313	2.05
Douglas fir — interior	0.87614	−1.48268	0.60654	1.17
Hemlock — coastal	0.97309	−1.43337	0.46028	1.47
Hemlock — interior	0.98060	−1.41272	0.43212	0.88
Lodgepole pine	0.99471	−1.30771	0.31300	0.58
Maple	0.95997	−1.46336	0.50339	0.99
Spruce — coastal	0.99496	−1.98993	0.99496	2.77
Spruce — interior	0.97449	−1.42305	0.44856	0.72
White birch	0.98982	−1.55259	0.56277	0.75
White pine	0.96272	−1,37551	0.41279	0.76
Yellow cedar	0.97982	−1.32688	0.34706	1.14
Yellow pine	0.90178	−1.28594	0.38416	1.21

Tests of taper functions against basic data

Similar taper functions to those in Table 2 were computed for all of the 19 species or species groups from the B.C. Forest Service taper curves. These functions can be considered the basic functions for B.C. They should be more reliable than ours (Table 2) because they cover a wider range of tree size. Our data (Table 1) were tested against the new equations summarizing the taper curves and the results of this test are given in Table 3.

The fact that the equations based on the B.C. Forest Service curves can estimate dib so well for our samples suggests that they can be applied widely. The averages of all standard errors were identical. Only six species yielded standard errors of estimate differing by more than 0.1 between Tables 2 and 3. The standard error of estimate for interior cedar was reduced by .2 by use of the taper curve equation. That for Coastal Douglas fir increased .23, coastal hemlock decreased .17, lodgepole pine increased .22, maple increased .28, and yellow cedar decreased .12 inches when the taper curve equation replaced the sample equation to estimate bole dib. These results suggest a stable estimating system. In addition the small amount of bias is encouraging. The amounts of bias shown indicated the extent to which individual species deviate from the quadratic paraboloidal form assumed in our function. As already mentioned, where it seems worthwhile to do so, bias can be reduced further by use of percentage or other appropriate correction factors.

Uses of taper functions in development of improved inventory systems

The studies that that have just been described evolved out of our search for factors by which total cubic or board foot volumes could be reduced to any desired level of merchantability (Smith and Munro, 1965). Although these factors lacked the flexibility and functional bases we now consider necessary in summary of inventory data, analysis of them directed our attention to a useful tree volume function. The ratio of tree volume to tree basal area is a simple function of tree height (Smith and Breadon, 1964).
It can be calculated from existing equations or tables or used as a basic tree volume equation. The tree volume equation can also be used to estimate stand volume, growth, and yield (Smith, 1967a) and even pulp yield from average annual height growth (Smith, 1967b). To the extent that bark thickness and percentage (Smith and Kozak, 1967) are related to dbh, total height, and section height our taper functions should be able to estimate bark. Also presently non-merchantable branch, bole, and root components should be definable in relation to tree size. We plan to compare our point sampling factor approach with methods described herein to solve directly for volume. Even if a widely useful system can be developed from our basic equation of tree and stand volumes, its utility can be enhanced great-

ly by our functions which define log dib by dbh, total height, and section height as a fraction of total height.

Conclusions

Measurement of upper stem diameters in addition to measurements of dbh and total height has not proven to be necessary or particularly useful for estimates of total volume. Our recent work has confirmed earlier observations (Smith, Ker, and Csizmazia, 1961) that no practical advantage in total volume estimates could be gained from any measurement of form in addition to dbh and total height.

Since our functions based on dbh, total height, and section height can estimate dib for 9 species groups with standard errors of estimate less than 1.0, for 6 species groups with standard errors of estimate less than 1.5, and for black cottonwood 1.54, coastal cedar 1.72, Coastal Douglas fir 2.05, and Sitka spruce 2.77, we have evidence of a good fit. With our functions there is no need to measure or guess at the size of upper bole diameters. Our functions can yield good estimates of bole contents to any desired standard of utilization, and thereby enhance the value of any forest inventory.

Table 3. Test of taper functions

	Average bias of dib at												Computed
	1′	4.5′	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H	SE_E
	Inches												
Alder	0.13	−0.11	−0.24	−0.02	0.19	0.40	0.50	0.45	0.25	−0.03	−0.44	0.00	0.62
Aspen	0.32	−0.19	−0.37	−0.14	0.04	0.17	0.24	0.30	0.10	−0.28	−0.80	0.00	0.70
Balsam — c	0.41	0.03	−1.00	−0.62	−0.41	−0.33	−0.34	−0.42	−0.70	−1.12	−1.47	0.00	1.44
Balsam — i	0.23	0.00	0.02	0.24	0.32	0.38	0.26	0.14	−0.22	−0.49	−0.64	0.00	0.62
Cedar — c	0.97	0.72	−1.22	−0.58	0.04	0.64	1.11	1.35	1.39	1.28	0.73	0.00	1.62
Cedar — i	0.71	0.33	−0.60	−0.19	0.27	0.69	0.91	1.09	0.82	0.53	0.12	0.00	1.08
Cottonwood	0.52	−0.18	−0.48	−0.31	−0.08	−0.06	−0.05	−0.19	−0.51	−0.87	−0.98	0.00	1.54
Douglas fir — c	2.41	−0.33	−1.19	−0.56	0.20	0.86	1.33	1.50	1.31	0.80	−0.19	0.00	1.82
Douglas fir — i	1.58	−0.77	−1.06	−0.52	−0.08	0.21	0.34	0.22	−0.10	−0.46	−1.00	0.00	1.16
Hemlock — c	0.62	0.02	−0.56	−0.11	0.20	0.33	0.38	0.18	−0.15	−0.71	−1.25	0.00	1.30
Hemlock — i	0.45	−0.24	−0.06	0.12	0.28	0.28	0.19	−0.10	−0.40	−0.81	−1.18	0.00	0.87
Lodgepole pine	−0.05	−0.27	−0.52	−0.54	−0.49	−0.44	−0.42	−0.46	−0.60	−0.83	−1.07	0.00	0.80
Maple	0.06	−0.03	−0.62	−0.34	0.05	0.56	1.03	1.07	1.11	1.44	1.37	0.00	1.27
Spruce — c	1.05	0.95	−2.75	−2.05	1.13	−0.33	0.18	0.45	0.41	−0.19	−0.85	0.00	2.75
Spruce — i	0.35	−0.01	−0.30	−0.03	0.27	0.47	0.52	0.44	0.22	−0.13	−0.50	0.00	0.68
White birch	0.09	−0.07	−0.15	0.03	0.18	0.48	0.50	0.52	0.24	−0.05	−0.46	0.00	0.70
White pine	0.48	−0.02	−0.63	−0.39	−0.01	0.19	0.21	0.04	−0.22	−0.61	−1.05	0.00	0.76
Yellow cedar	0.22	0.23	−0.74	−0.47	0.02	0.37	0.70	0.74	0.46	−0.05	−0.73	0.00	1.02
Yellow pine	1.31	−0.61	−0.69	−0.16	0.27	0.42	0.53	0.32	−0.01	−0.63	−1.32	0.00	1.22

c = coastal i = interior H total tree height

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