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A POLYNOMIAL TAPER CURVE FUNCTION FOR ZAMBIAN EXOTIC TREE PLANTATIONS

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HEINONEN, J., SARAMÄKI, J. & SEKELI, P.M. 1996. A polynomial taper curve function for Zambian exotic tree plantations. Defining tree taper gives advantages in deriving the utilisable portions of trees. A high degree polynomial is used to estimate average relative taper. Some of the parameters of the polynomial should be functions of diameter at breast height and total height of the tree, and the remaining parameters can be held constant within a tree species. The study describes a somewhat simplified version of the polynomial taper curve model developed by Laasasenaho and presents a simple method for transforming the overbark taper curve models to underbark ones. Taper curve functions for the main exotic tree species *Pinus kesiya, Pinus oocarpa, Pinus merkusii, Pinus michoacana, Eucalyptus grandis* and *Eucalyptus cloeziana* in Zambia are derived. The polynomial appears to fit equally well with differently tapering tree species and gives good estimates for both over- and underbark diameters. Regression functions for bark thickness are also given.

Keywords: Stem taper - stem volume - *Pinus kesiya - Pinus oocarpa - Pinus merkusii - Pinus michoacana - Eucalyptus grandis - Eucalyptus cloeziana*

HEINONEN, J., SARAMÄKI, J. & SEKELI, P.M. 1996. Fungsi keluk tirus polinomial ladang pokok dagang Zambia. Menentukan keluk pokok memberikan kelebihan untuk mendapatkan bahagian-bahagian pokok yang dapat digunakan. Polinomial yang tinggi darjahnya digunakan untuk menganggarkan purata keluk yang berkaitan. Beberapa parameter bagi polinomial tersebut sepatutnya berfungsi sebagai diameter aras dada dan jumlah ketinggian pokok, dan parameter-parameter yang lain boleh dijadikan konstan di dalam sesuatu spesies pokok. Kajian ini menerangkan mengenai versi keluk tirus polinomial yang dipermudahkan yang dicipta oleh Laasasenoho dan menghasilkan satu kaedah yang mudah untuk memindahkan model keluk tirus atas kulit kepada model bawah kulit. Fungsi keluk tirus bagi spesies pokok dagang utama *Pinus kesiya, Pinus oocarpa, Pirus merkusii, Pinus michoacana, Eucalyptus grandis* dan *Eucalyptus cloeziana* di Zambia telah diperolehi. Polinomial didapati amat sesuai dengan spesies pokok tirus yang berbeza dan memberikan anggaran yang baik bagi diameter atas kulit dan bawah kulit. Fungsi-fungsi regresi untuk ketebalan kulit pokok juga diberikan.

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Introduction

The area covered by fast-growing exotic plantations is increasing. Plantation management needs accurate information about the volume and assortment of trees. One approach is to develop volume functions by means of regression analysis. However, the dimensions for required log size can often change. When dimension requirements change, the functions must be recalculated to match the new dimensions. This is quite laborious and may result in incompatibilities when successive volume estimates are compared. This difficulty can be avoided if a system for describing stem taper is developed.

Taper curves can be expressed mathematically in many different ways, and parameter estimation techniques may likewise differ greatly (e.g. Max & Burkhart 1976, Sterba 1980, Roiko-Jokela 1976, Kilkki *et al.* 1978, Cailliez 1980, Kilkki & Varmola 1981, Laasasenaho 1982, Knoebel *et al.* 1984, Lappi 1986, Kozak 1988, Perez *et al.* 1990). Different measurements of taper may be needed in the application phase. Application programs may utilize different amounts of computing capacity (e.g. Lahtinen & Laasasenaho 1979, Kilkki & Varmola 1981, Laasasenaho 1982).

Because plantations are ubiquitous, and in most cases, each plantation requires its own taper curve functions for each species, the methods to develop these functions need to be simple enough so that functions can be calculated using ordinary microcomputer programs. This is especially important for plantations in developing countries.

The aim of this study was to find a taper curve function system that can, given diameter and height of the tree, produce reasonable and compatible estimates of both over- and underbark volume to any given top diameter or to any given length of the log for Zambian exotic tree plantations.

Taper model development

The polynomial taper curve function originally presented by Laasasenaho (1982) was well suited to this study. The original method was simplified by reducing the number of models needed for the taper curve function from four to three and by developing a simple method in which two additional parameters are needed to produce both overbark and underbark taper curve functions.

The taper model is of the form

$$\frac{d_m}{d_{1.3}} = \frac{P(x_m)}{P(x_{1.3})} \qquad (1)$$

where d_m and $d_{1.3}$ are two diameters (cm) at heights (m) m and 1.3 m respectively, with $0 < m < h$,
$x_m = 1\text{-}m/h$, $x_{1.3} = 1\text{-}1.3/h$, where h is the height of a tree, $h > 1.3$, and
$P(x_k)$ is a polynomial giving the diameter at relative height x_k.

Polynomial P is of the form

$$P(x) = c_1 \cdot x + c_2 \cdot x^2 + c_3 \cdot x^3 + c_4 \cdot x^5 + c_5 \cdot x^8 + c_6 \cdot x^{13} + c_7 \cdot x^{21} + c_8 \cdot x^{34} \tag{2}$$

where $c_1, c_2, \ldots, c_8$ are parameters. At the top of a tree the value of the polynomial = 0. The powers used in the model are in accordance with the so-called Fibonacci-series (Laasasenaho 1982).

Multiplying both sides by $d_{1.3}$ model (1) can be written in the form

$$d_m = c_o \cdot P(x_m)$$

where coefficient $c_o = \dfrac{d_{1.3}}{P(x_{1.3})}$

Ratio of diameters at height m_1 and m_2 is given by

$$\frac{d_{m_1}}{d_{m_1}} = \frac{P(x_{m_1})}{P(x_{m_1})} \tag{3}$$

There are eight parameters in the polynomial P. It is enough, however, if only the three first parameters c_1, c_2 and c_3 are dependent on $d_{1.3}$ and h and parameters $c_4, c_5, \ldots, c_8$ are constants.

Polynomial (2) can then be written in the form

$$\begin{aligned} P(x_m) = {} & (a_1+b_1) \cdot x_m + (a_2+b_2) \cdot x_m^2 + (a_3+b_3) \cdot x_m^2 + b_4 \cdot x_m^2 + b_5 \cdot x_m^8 \\ & + b_6 \cdot x_m^{13} + b_1 \cdot x_m^{21} + b_8 \cdot x_m^{34} \end{aligned} \tag{4}$$

$$= P_a(x_m) + P_b(x_m) \tag{5}$$

where

$P_a(x_m)$ is a polynomial of the third degree, the values of the parameters a_1, a_2 and a_3 depend on $d_{1.3}$ and h, and polynomial $P_b(x_m)$ is of the form (2) and the parameters $b_1, b_2, b_3, \ldots, b_8$ are constant for a tree species.

Polynomial P_b is called here the basic function, and it has been chosen to describe the average taper. The parameters of P_b are estimated using the diameters at different relative heights describing taper.

The polynomial P_b can be scaled in such a way that $P_b(0.8)=1$, in which case equation (3) reduces to

$$\frac{d_m}{d_{.2*h}} = P_b\,(x_m).$$

The parameters of P_b are calculated from equation

$$y_{.i} = P_b\,(x_{.i}) + \varepsilon \tag{6}$$

where

$$y_{.i} = \frac{\bar{d}_{.i}}{\bar{d}_{.2}} = \frac{\frac{1}{n}\sum_{j}^{n} d_{.ij}}{\frac{1}{n}\sum_{j}^{n} d_{.2j}}$$

n = number of trees,
$.i$ refers to relative heights of diameters,
$x_{.i}$ = 1-.i,
.2 refers to the relative height 0.2, and
ε is the error term

Variable $y_{.i}$ can be interpreted as a weighted mean of relative diameters at relative height .i (Laasasenaho 1982).

The parameters can be calculated by the method of least squares. The height of scaling 0.2.h, is such that the estimates of the size independent parameters of the taper curve function are approximately optimal.

In addition to the polynomial P_b, two more models were developed and estimated to impose restrictions on parameters of the size dependent correction polynomial P_a. The models developed were

$$\frac{d_{.4}}{d_{.1}} = f_4\,(d_{1.3}, h) + \varepsilon_4 \tag{7}$$

and

$$\frac{d_{.7}}{d_{.1}} = f_7\,(d_{1.3}, h) + \varepsilon_7 \tag{8}$$

where

f_4 and f_7 are functions, the values of which depend on diameter $d_{1.3}$ and height of the tree, and e_4 and e_7 are error terms.

Equation

$$P_a\,(0.9) = 0 \tag{9}$$

was used as the third requirement for the parameters of polynomial P_a, which means that the value of the correction polynomial equals 0 at relative height 0.1. The purpose of equation (9) is to allow only a small correction to the taper curve defined by the basic polynomial P_b. The form of the final taper curve depends only weakly on the relative height in equation (9) and, for example, relative height 0.2 instead of 0.1 would have given practically as good results.

In his original version of the polynomial taper model Laasasenaho used three regression equations essentially to get estimates of diameters at relative heights 0.1, 0.4 and 0.7 as functions of breast height diameter and height (Laasasenaho 1982). In the first phase, the parameters of the correction polynomial were determined so that, after the first correction, the taper curve function passes through the estimates of diameters at these three relative heights. In the second phase, he calculated a correction coefficient to make the taper curve function pass through the diameter at breast height. Because of the correction coefficient, the final taper curve function does not usually give the same estimates of diameters as the regression models and only the ratios of the three estimates of diameters are compatible with the regression models. The same properties can be obtained more simply by using two regression models (7) and (8) and one restriction (9) which is independent of the tree species.

From (3),(7),(8) and (9) a group of three equations are derived:

$$\begin{aligned}
&\frac{P_a\,(.6) + P_b\,(.6)}{P_a\,(.9) + P_b\,(.9)} = f_4 \\
&\frac{P_a\,(.3) + P_b\,(.3)}{P_a\,(.9) + P_b\,(.9)} = f_7 \\
&P_a\,(0.9) = 0
\end{aligned} \tag{10}$$

where

f_4 and f_7 are values of functions (7) and (8) given $d_{1.3}$ and h of a tree.

Equation (10) can be written further in the form

$$\begin{aligned} a_1^{\cdot} . 6 + a_2^{\cdot} . 6^2 + a_3^{\cdot} . 6^3 &= f_4 \cdot P_b(.9) - P_b(.6) \\ a_1^{\cdot} . 3 + a_2^{\cdot} . 3^2 + a_3^{\cdot} . 3^3 &= f_7 \cdot P_b(.9) - P_b(.3) \\ a_1^{\cdot} . 9 + a_2^{\cdot} . 9^2 + a_3^{\cdot} . 9^3 &= 0 \end{aligned} \tag{11}$$

The solution of (11) can be given, for example, by

$$a_3 = \frac{((f_7 - f_4) \cdot P_b(.9) - P_b(.3) + P_b(.6))}{0.054}$$

$$a_2 = \frac{(P_b(.6) - f_4 \cdot P_b(.9) - .27 \cdot a_3)}{0.18} \tag{12}$$

$$a_1 = -.9 \cdot a_2 - .81 \cdot a_3$$

After parameters a_1, a_2 and a_3 have been calculated, equation (4) provides the final parameters of the taper curve function P. Diameter at height m is estimated by using equation (1). The height at which a given diameter is reached is solved by iteration.

Volume $vol(m_1, m_2)$ between heights m_1 and m_2 is estimated by integrating a squared taper curve function

$$vol(m_1, m_2) = \frac{c_0^2 \cdot \frac{\pi}{h}}{40} \cdot \int_{x_2}^{x_1} (P(x))^2 \, dx \tag{13}$$

where

m_1 and m_2 are given heights, $m_1 < m_2$.

In equation (13) volume is given in litres when $d_{1.3}$ is in centimeters and height in meters.

The overbark model for tapering can be transformed into the underbark model by means of two additional equations

$$\frac{du_{.4}}{du_{.1}} = \left(\frac{d_{.4}}{d_{.1}}\right)^{k_4} \tag{14}$$

and

$$\frac{du_{.7}}{du_{.1}} = \left(\frac{d_{.7}}{d_{.1}}\right)^{k_7} \tag{15}$$

where $du_{.j}$, j =1,4 and 7 are underbark diameters at relative heights 0.1, 0.4 and 0.7 and k_4 and k_7 are parameters.

Parameters k_4 and k_7 can be estimated either by nonlinear regression or by using the logarithmic transformation of models (14) and (15) and linear regression.

For underbark taper curve function, the overbark polynomial P_b is used in equation (5) and in equations (12) f_4 and f_7 are replaced by $f_4^{k_4}$ and $f_7^{k_7}$ respectively. This modification makes the underbark estimates practically unbiased. The diameter $d_{1.3}$ in equation (1) is replaced by underbark diameter $d_{u1.3}$ if known, or otherwise, by an estimate of it.

Taper curve functions for Zambian exotic tree plantations

Material

Material consisted of the six main plantation species in Zambia, namely *Pinus kesiya, P. oocarpa, P. merkusii, P. michoacana, Eucalyptus grandis* and *E. cloeziana.* Table 1 provides general information about the material, all of which was collected from the Copperbelt area - the main plantation area - of Zambia. The data used for *E. grandis* were a random sample of about 1300 felled sample trees. For the rest of the species, all available felled sample tree material was used. All data consisted partly of trees from research plots and partly of random samples from commercial compartments.

The trees had been measured at fixed intervals from ground level. The measuring heights were 0.2, 0.5, 1.0, 1.3, 2.0, 4.0 m and then at two meter intervals to the tip. The parameters of the basic polynomial P_b were estimated using the diameters at the relative heights of 0.01, 0.025, 0.05, 0.075, 0.10, 0.15, 0.20, 0.25, 0.30, ..., 0.85, 0.90 and 0.95. Diameters of each tree were derived by using measured diameters and interpolating. Spline function of the third degree polynomial was used as the interpolating function.

For most of the trees, underbark diameters were measured after peeling, but part of the data from *P. oocarpa* and *P. merkusii* and all data from *P. michoacana* were measured using a bark gauge.

As can be seen in Figure 1, the distribution for *P. michoacana* was especially narrow. To ensure that the function continues logically outside the measured range, 24 of the biggest *P. kesiya* sample trees were also used for deriving the parameters for *P. michoacana.* For the other species, the range approximately covers the existing sizes. Material from *E. cloeziana* contained a few very big trees - breast height diameter over 70 cm - which were excluded from the calculations to avoid introducing bias to smaller trees.

Table 1. Description of the data

Species	Observation	Diameter (cm)	Height (m)	Volume (dm^3)
P. kesiya	506			
mean		21.59	19.13	359.7
std. dev.		7.00	4.54	309.1
minimum		6.7	4.8	15.5
maximum		45.8	32.9	1839.0
P. oocarpa	221			
mean		16.61	15.75	186.2
std. dev.		5.21	4.60	131.9
minimum		5.3	4.0	5.8
maximum		29.1	26.6	697.1
P. merkusii	180			
mean		20.23	14.30	251.4
std. dev.		6.29	5.11	197.7
minimum		4.6	2.9	5.4
maximum		35.5	23.9	863.0
P. michoacana	89			
mean		15.18	7.11	77.7
std. dev.		4.64	1.66	56.6
minimum		5.7	3.7	7.4
maximum		27.7	11.3	281.3
E. grandis	388			
mean		21.61	27.66	584.4
std. dev.		8.71	7.26	589.0
minimum		5.8	11.8	16.9
maximum		54.8	43.6	3741.9
E. cloeziana	202			
mean		29.22	29.48	1065.9
std. dev.		11.36	7.56	1034.1
minimum		9.1	10.4	43.9
maximum		64.6	45.8	5631.1

Functions

The parameters of the basic function P_b in equation (5) are presented in Table 2. Non-significant t values were observed for some of the coefficients but for the sake of similarity they were kept in the model.

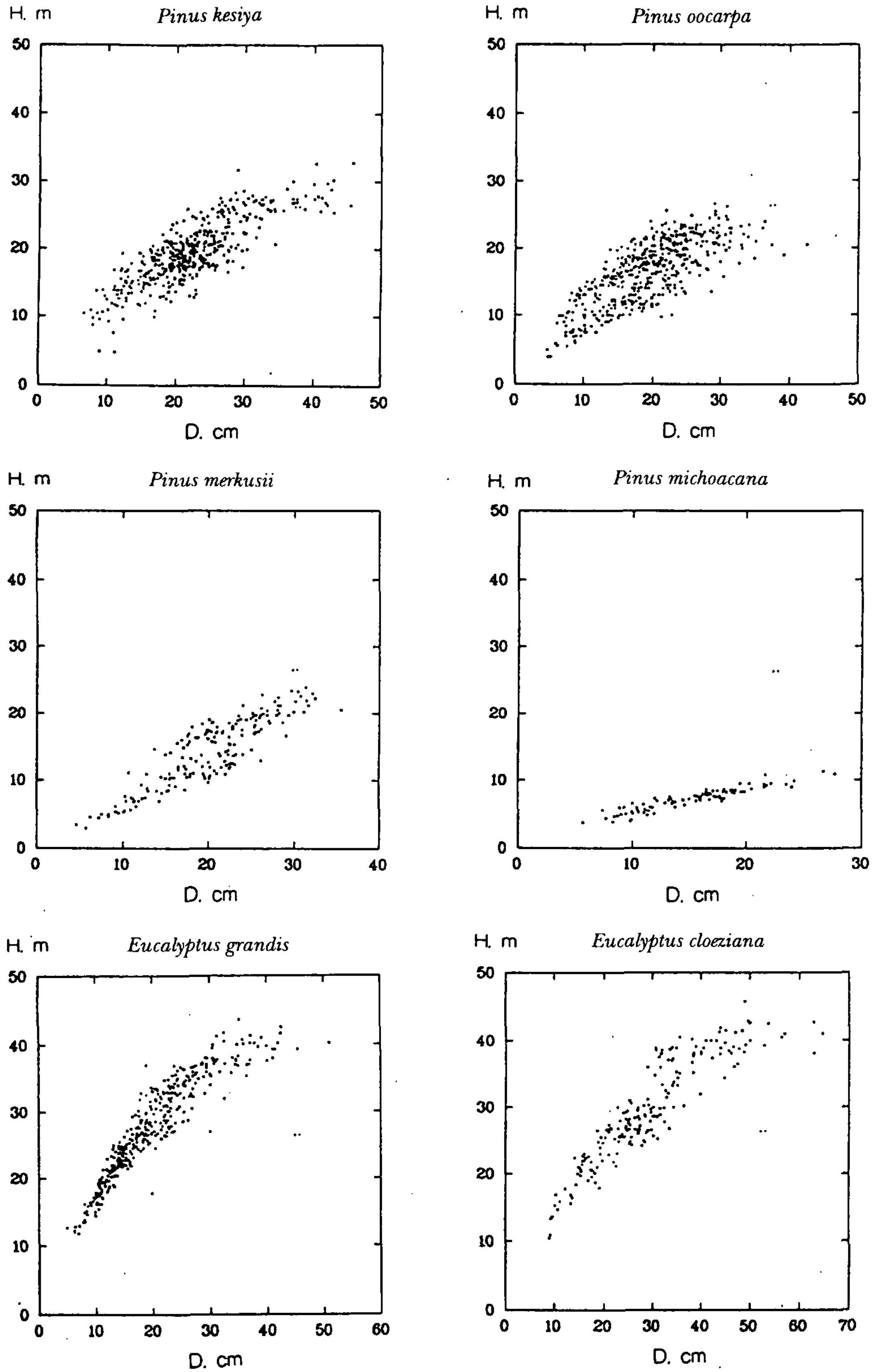

Figure 1. Diameter-height distributions of studied species

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The overbark diameter correction models (7) and (8) were of the form:

$$f_4(d_{.4}/d_{.1}) = p_{41} \cdot h + p_{42} \cdot h^2 + p_{43}/h^{0.5} \tag{16}$$

$$f_7(d_{.7}/d_{.1}) = p_{71} \cdot h + p_{72} \cdot h^2 + p_{73}/h^{0.5} + p_{74} \cdot \ln(h) \cdot \ln(d_{1.3}) \tag{17}$$

Table 2. Parameter values and s_e's (in brackets) of the basic function P_b

Species	b_1	b_2	b_3	b_4
P. kesiya	2.05502	- 0.89331	- 1.50615	3.47354
	(0.08716)	(0.71372)	(1.58429)	(2.28131)
P. oocarpa	1.57420	1.97480	- 6.14061	7.55100
	(0.06889)	(0.56411)	(1.25219)	(1.80310)
P. merkusii	1.34171	2.43816	- 6.71972	8.12709
	(0.04610)	(0.37752)	(0.83801)	(1.20669)
P. michoacana	0.85477	3.35553	- 5.81309	4.40126
	(0.03725)	(0.30504)	(0.67712)	(0.97502)
E. grandis	2.54410	- 3.32322	2.37636	0.14854
	(0.10192)	(0.83460)	(1.85262)	(2.66770)
E. cloeziana	1.88208	- 1.26653	0.92643	- 0.88706
	(0.05884)	(0.48182)	(1.06953)	(1.54008)

Species	b_5	b_6	b_7	b_8
P. kesiya	- 3.10063	1.50246	- 0.05514	0.00070
	(2.55395)	(2.07584)	(1.17985)	(0.34978)
P. oocarpa	- 6.22372	3.70583	- 1.46734	0.47656
	(2.01859)	(1.64070)	(0.93253)	(0.27646)
P. merkusii	- 5.94221	2.68227	- 0.78642	0.15286
	(1.35090)	(1.09801)	(0.62408)	(0.18501)
P. michoacana	- 1.84468	0.23140	0.10631	- 0.02063
	(1.09154)	(0.88720)	(0.50426)	(0.14949)
E. grandis	- 1.65063	2.07501	- 1.53745	0.75849
	(2.93651)	(2.42742)	(1.37968)	(0.40902)
E. cloeziana	0.84461	- 0.37522	- 0.12726	0.43393
	(1.72413)	(1.40137)	(0.79649)	(0.23613)

The parameters of functions (16) and (17) are presented in Table 3.

The exponents k_4 and k_7 in transformation equations (14) and (15) are presented in Table 4.

Table 3. Values for parameters and s_e (in brackets) of functions (16) and (17)

Species	Parameter		
	p_{41}	p_{42}	p_{43}
P. kesiya	0.03975	- 0.000720464	1.02551
	(0.00176)	(0.000056853)	(0.05480)
P. oocarpa	0.05141	- 0.00124492	0.99196
	(0.00234)	(0.00009821)	(0.04518)
P. merkusii	0.04500	- 0.000995659	0.96686
	(0.00305)	(0.000136960)	(0.04844)
P. michoacana	0.04845	- 0.00103268	1.04963
	(0.00370)	(0.00012186)	(0.05167)
E. grandis	0.03230	- 0.000472216	1.30911
	(0.00074)	(0.000017592)	(0.03466)
E. cloeziana	0.03085	- 0.000432485	1.28659
	(0.00136)	(0.000032180)	(0.06346)

Species	Parameter			
	p_{71}	p_{72}	p_{73}	p_{74}
P. kesiya	0.05065	- 0.000806335	0.46317	- 0.03557
	(0.00353)	(0.000064316)	(0.05372)	(0.00525)
P. oocarpa	0.06003	- 0.00112353	0.43268	- 0.04070
	(0.00528)	(0.00012540)	(0.04800)	(0.00722)
P. merkusii	0.04512	- 0.00101906	0.34484	- 0.01262
	(0.01220)	(0.00027148)	(0.06137)	(0.01502)
P. michoacana	0.02821	- 0.000704661	0.38952	0.01107
	(0.01721)	(0.000325987)	(0.05163)	(0.01837)
E. grandis	0.01957	- 0.000319540	0.82998	0.00436
	(0.00260)	(0.000025887)	(0.04080)	(0.00551)
E. cloeziana	0.03825	- 0.000416219	0.86827	- 0.04239
	(0.00610)	(0.000058984)	(0.07705)	(0.01159)

Table 4. The values of exponents k_4 and k_7 and their s_e (in brackets) in logarithmic correction equations in (14) and (15) for underbark relation between relative diameters at 40 and 10 % and at 70 and 10 % relative heights

Species	k_4	k_7
P. kesiya	0.85605	0.96841
	(0.01306)	(0.00909)
P. oocarpa	0.76337	0.89674
	(0.02164)	(0.01369)
P. merkusii	0.86768	0.96284
	(0.02005)	(0.01288)
P. michoacana	0.83770	0.96287
	(0.03993)	(0.02578)
E. grandis	0.93418	1.01400
	(0.00991)	(0.00744)
E. cloeziana	0.99299	1.05088
	(0.02230)	(0.01644)

Regression functions for relative bark thickness at breast height ($b_{1.3}$) were (standard deviations in parentheses)

P. kesiya $b_{1.3}/d_{1.3} = 0.10457 + 1.04924/h - 0.00000101365 \cdot d_{1.3} \cdot h^2$
(0.01205) (0.16468) (0.00000037695)

P. oocarpa $b_{1.3}/d_{1.3} = 0.25217 + 0.94754/h - 0.04507 \cdot \ln(d_{1.3})$
(0.06106) (0.20077) (0.01781)

P. merkusii $b_{1.3}/d_{1.3} = 0.22901 - 0.0000040200 \cdot h \cdot d_{1.3}^2$ (18)
(0.00398) (0.0000004150)

P. michoacana $b_{1.3}/d_{1.3} = 0.27384 - 0.00602964 \cdot d_{1.3} + 0.56221/h$
(0.05974) (0.00188714) (0.22112)

E. grandis $b_{1.3}/d_{1.3} = 0.02223 - 0.00296948 \cdot h + 0.04391 \cdot \ln(d_{1.3})$
(0.017665) (0.00063098) (0.01112)

E. cloeziana $b_{1.3}/d_{1.3} = 0.21038 - 0.00253916 \cdot h$
(0.01453) (0.00047292)

The reliability of the functions

The reliability of the system was tested by comparing both measured diameters and volumes with those of estimated measurements.

Table 5 did not show any marked deviations, even though the average volume deviation for *E. cloeziana* was greater than for the other species. Most of the deviations were positive, which is according to expectations. Diameter deviations were also tested by relative heights (Figure 2). The correction system gives the ratios of diameters at relative heights 40 % and 10 % and the ratios of diameters at relative heights 70 % and 10 % were unbiased. As the stem tapers more at the base and no fixing points lower than at 10 % relative height are present, the greatest deviations were found at the stump level. On average, maximum mean deviations did not exceed 3 mm and in most cases mean deviations were less than one mm. These deviations do not have very much practical value even though they cause bias in tree volumes.

The same trend as for overbark diameters appeared for underbark estimates. The mean deviations for underbark diameters were almost equal to the overbark deviations (Table 5). The variation of deviations was slightly greater. This system of calculating underbark diameters does not always guarantee correct values for bark volume, but the estimates for underbark diameters are reliable and compatible. A few tests were made to compare estimated and measured bark thicknesses. The differences were found to be reasonably small and no contradicting thicknesses were seen along the stem.

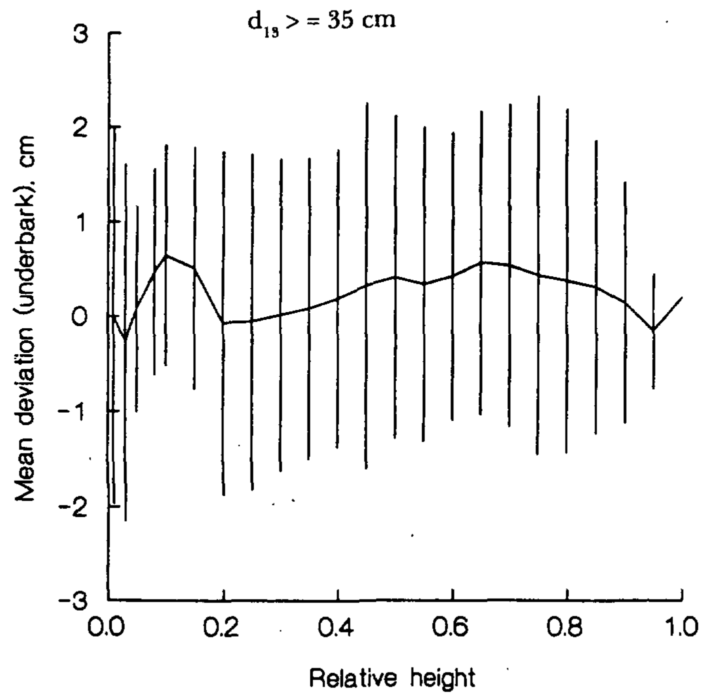

Figure 2. Mean deviation (-) and standard errors of deviations (I) in diameter at different relative heights by diameter class in *Pinus kesiya* material

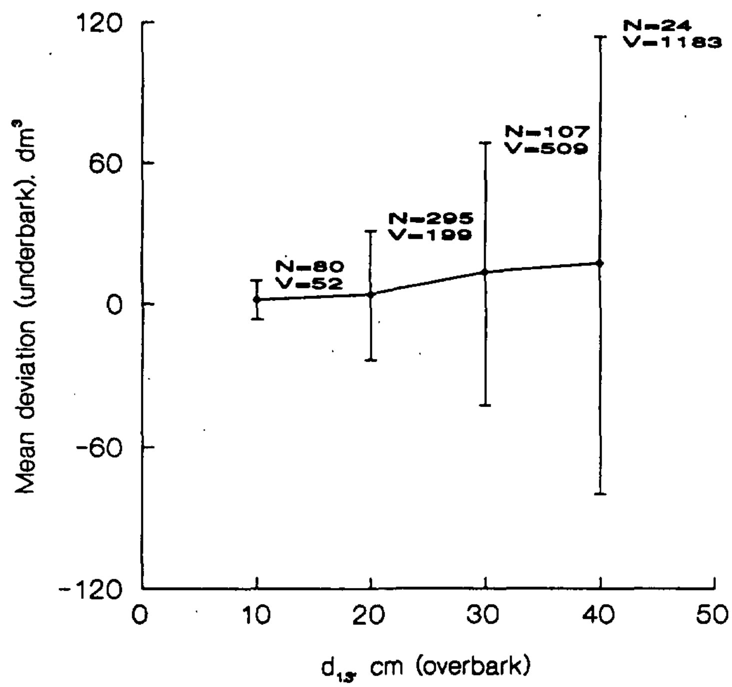

Figure 3. Mean deviations (-) and standard errors of deviations (I) in volume in different diameter classes in *Pinus kesiya* material. Number of stems and mean size (dm^3) are presented at the top of each class

Volume deviations were tested by diameter classes and no marked trend was seen but the variation of deviations - in absolute terms - increases with increasing size of the tree (Figure 3). Relative deviations and their standard deviations remain the same throughout the size range. Underbark volume estimates were as good as overbark volume estimates as can be seen from Table 5. Volume deviations to any log length or top diameter were not separately tested, but these are not expected to be significantly different from total volume deviations.

Table 5. Average differences (upper figure) and standard deviations of differences (lower figure) between measured and estimated values of diameters as a mean of all 23 relative heights and total volumes. d = diameter, v = volume, u = underbark, *est* = predicted, *uest1* = predicted using measured bark thickness at 1.3 m, *uest2* = predicted using equation (18)

Difference	Species		
	E. grandis	*E. cloeziana*	*P. kesiya*
$d - d_{est}$ (cm)	- 0.0376	0.0809	0.1146
	0.9929	1.7175	1.0992
$d_u - d_{uest1}$ (cm)	- 0.0457	- 0.0900	0.0512
	0.9514	1.5526	1.0341
$d_u - d_{uest2}$ (cm)	- 0.0681	- 0.0746	0.0536
	1.1303	1.7513	1.1490
$v - v_{est}$ (dm^3)	0.9739	23.0823	6.0338
	62.3815	184.8664	38.8969
$(v - v_{est})/v_{est}$	- 0.0002	0.0278	0.0234
	0.0819	0.1464	0.1036
$v_u - v_{uest2}$ (dm^3)	5.6409	7.5746	6.1859
	60.9801	165.8456	39.2736
$(v_u - v_{uest2})/v_{uest2}$	0.0018	0.0027	0.0217
	0.0904	0.1603	0.1377

Difference	Species		
	P. oocarpa	*P. merkusii*	*P. michoacana*
$d - d_{est}$ (cm)	0.0868	0.0268	- 0.0413
	0.8561	1.4288	0.8313
$d_u - d_{uest1}$ (cm)	- 0.0417	0.0214	- 0.1540
	0.8685	0.9077	0.8615
$d_u - d_{uest2}$ (cm)	- 0.0389	0.0265	- 0.1540
	0.9910	0.9508	0.8837
$v - v_{est}$ (dm^3)	3.8789	2.8820	- 0.2207
	20.0403	36.2984	6.8864
$(v - v_{est})/v_{est}$	0.0203	0.0186	- 0.0014
	0.0910	0.1818	0.0855
$v_u - v_{uest2}$ (dm^3)	0.9161	0.6964	- 1.9711
	21.8132	24.7322	7.4289
$(v_u - v_{uest2})/v_{uest2}$	0.0113	0.0110	- 0.0229
	0.1479	0.1156	0.1266

Discussion

The presented taper curve system is based on a high degree polynomial which is flexible enough to estimate tree taper reasonably well. As it was first presented by Laasasenaho (1982) the dimensionless relative figures seem to be quite stable within a species and the polynomial of the third degree describes, with reasonable precision, the size dependent differences from the average tapering of a tree species.

In the presented system, diameter and height of a tree must be known. The dependence of tapering on the size of a tree is given by two functions as compared to Laasasenaho's (1982) three functions. Two functions seem to be enough when only diameter at breast height and height of a tree are known. However, more measured information about tree taper would have been more suitable for bigger trees. For instance, some higher diameter as used by Laasasenaho (1982) would have improved the accuracy of the estimation. The use of regression equations for volume estimation would have given at least as good results as the presented system. However, the flexibility and compatibility of assortment calculations is a clear advantage in the present system.

Compatibility of underbark with overbark volumes is also secured. Although the tree species differed remarkably by stem form, the estimated volumes were equally accurate for all species.

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