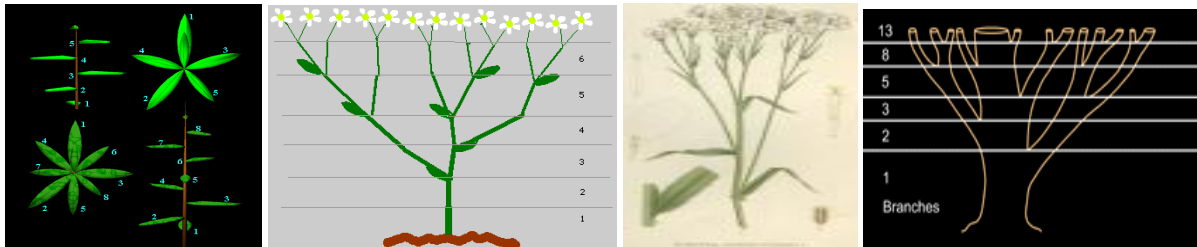




Plants show the Fibonacci numbers in the arrangements of their leaves (Internet access,15). Three clockwise rotations, passing five leaves two counter-clockwise rotations. Sneezewort (*Achillea ptarmica*) also follows the Fibonacci numbers.



Schematic diagram (Sneezewort)

Why do these arrangements occur? In the case of leaf arrangement, or phyllotaxis, some of the cases may be related to maximizing the space for each leaf, or the average amount of light falling on each one.



These pictures are very common to us. We can see the flowers and the patterns of leaves just out of single step of our house. All of these follow the Fibonacci numbers.

## 2.2 Fibonacci spiral

The Fibonacci numbers are found in the arrangement of seeds on flower heads (Internet access, 13). There are 55 spirals spiraling outwards and 34 spirals spiraling inwards in most daisy or sunflower blossoms (Internet access,14). Pinecones clearly show the Fibonacci spirals (Howard, 2004)



Fibonacci spiral can be found in cauliflower. The Fibonacci numbers can also be found in Pineapples and Bananas (Lin and Peng). Bananas have 3 or 5 flat sides and Pineapple scales have Fibonacci spirals in sets of 8, 13, and 21. Inside the fruit of many plants we can observe the presence of Fibonacci order.



Fibonacci spiral (Internet access, 9), (Internet access, 11) are also found in various fields associated in nature. It is seen snail, sea shells, waves, combination of colours; roses etc in so many things created in nature (Internet access 12). But very few of us have time to study this phenomenon.

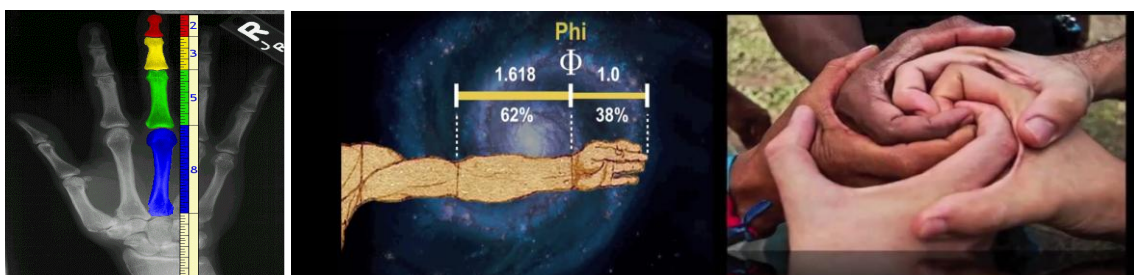


Nature isn't trying to use the Fibonacci numbers: they are appearing as a by-product of a deeper physical process. That is why the spirals are imperfect. The plant is responding to physical constraints, not to a mathematical rule.

The basic idea is that the position of each new growth is about 222.5 degrees away from the previous one, because it provides, on average, the maximum space for all the shoots. This angle is called the **golden angle**, and it divides the complete 360 degree circle in the golden section, 0.618033989 . . . which is described below.

### 2.3 Organs of human body

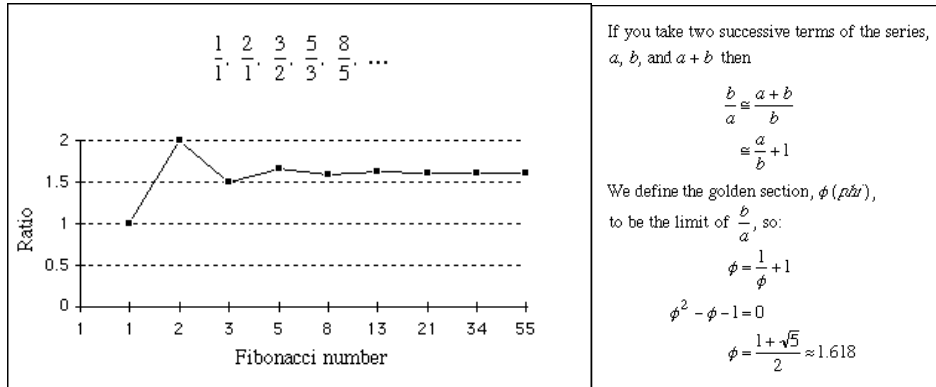
Humans exhibit Fibonacci characteristics. Every human has two hands, each one of these has five fingers and each finger has three parts which are separated by two knuckles (Internet access, 7). All of these numbers fit into the sequence. Moreover the lengths of bones in a hand are in Fibonacci numbers.





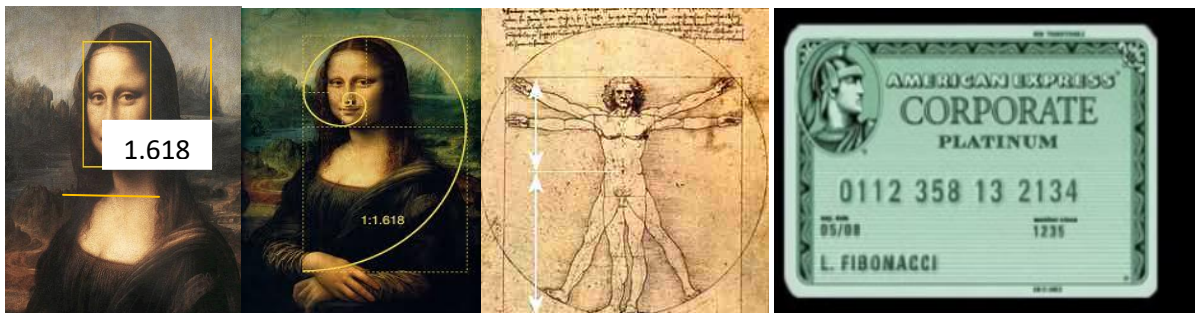
If we keep going, we get an interesting number which mathematicians call “phi” (Golden Ratio or Golden Ratio). It is denoted by  $\phi$  and the value of  $\phi = 1.6180339887$

$$\lim_{n \rightarrow \infty} \frac{F_{n+1}}{F_n} = 1.618$$

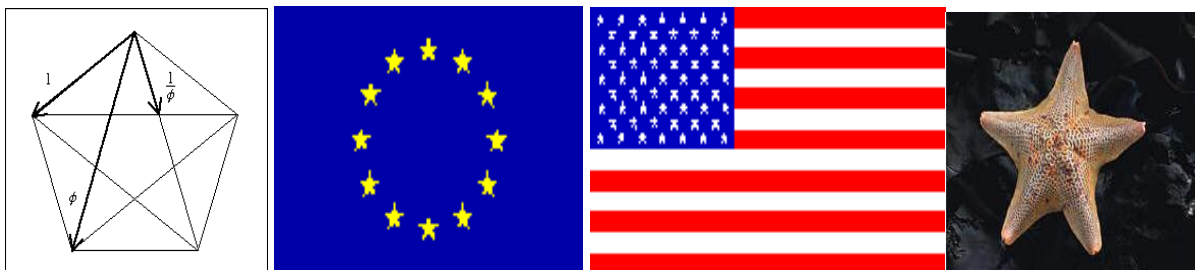


## 2.7 Some applications of Golden ratio

Leonardo da Vinci showed that in a ‘perfect man’ there were lots of measurements that followed the Golden Ratio. The Golden (Divine) Ratio has been talked about for thousands of years.



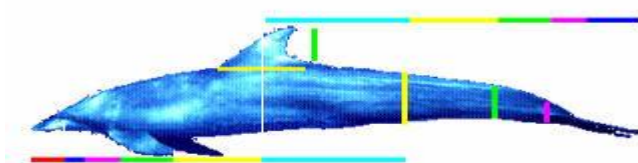
The Golden ratio is widely used in Geometry (Garg et al, 2014). It is the ratio of the side of a regular pentagon to its diagonal. The diagonals cut each other with the golden ratio (Stakov1989). Pentagram describes a star which forms parts of many flags. This five-point symmetry with Golden proportions is found in starfish which has five arms.



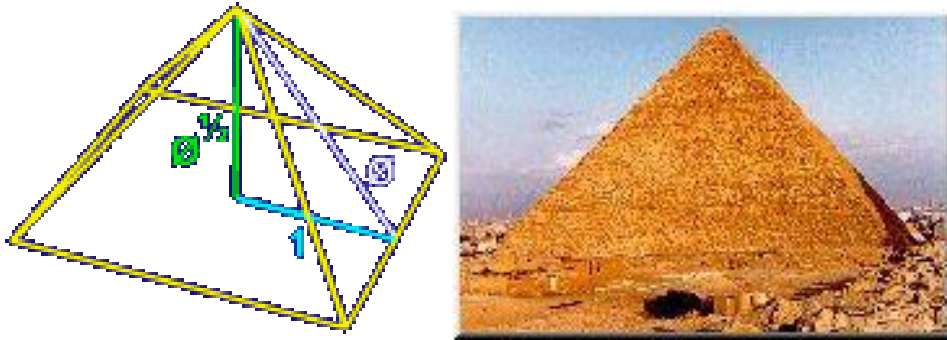
European Union

United States

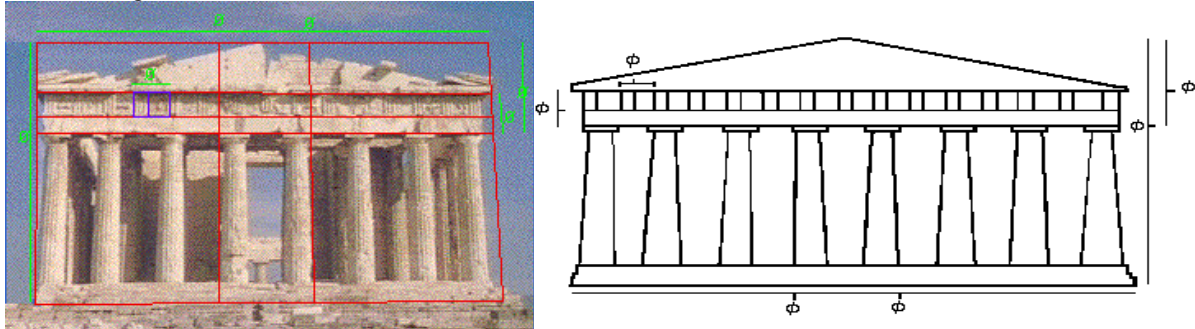
The eyes, fins and tail of the dolphin fall at Golden sections along the body.



The Golden Ratio is also frequently seen in natural architecture also (Internet access, 18). It can be found in the great pyramid in Egypt. Perimeter of the pyramid, divided by twice its vertical height is the value of  $\phi$ .



Golden section (Gend, 2014) appears in many of the proportions of the Parthenon in Greece. Front elevation is built on the golden section (0.618 times as wide as it is tall).



### III. A beautiful example

Take any two consecutive numbers from this series as example 13 and 21 or 34 and 55.

Now smaller number is in miles = the other one in Kilometer or bigger number is in Kilometers = the smaller one in Miles (The other way around).

34 Miles = round (54.72) Kilometers = 55 Kilometers

21 Kilometers = round (13.05) Miles = 13 Miles

For distances which are not exact Fibonacci values you can always proceed by splitting the distance into two or more Fibonacci values.

As example, for converting 15 km into miles we can proceed as following:

15 km = 13 km + 2 km

13 km -> 8 mile

2 km -> 1 mile

15 km -> 8+1 = 9 mile

Another example, for converting 170km into miles we can proceed as:

170 km = 10\*17 km

17 km = 13 km + 2 km + 2 km = 8 + 1 + 1 miles = 10 miles (approximately)

Now, 170 km = 10\*10 miles = 100 miles (approximately)

So, either way we can proceed. For bigger numbers we can proceed as above.

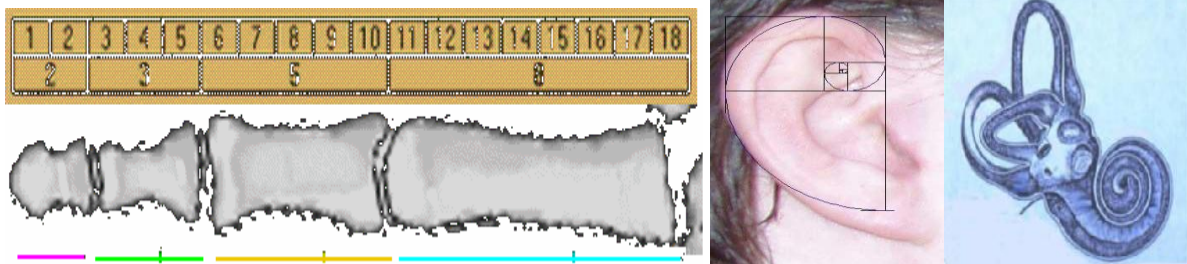
(Ref: Sudip Maji, B.C.Roy Engineering College)

### IV. Fibonacci in Coding

Recently Fibonacci sequence and golden ratio are of great interest to the researchers in many fields of science including high energy physics, quantum mechanics, Cryptography and Coding. Raghu and Ravishankar(2015) developed a paper of application classical encryption techniques for securing data.(Raphael and Sundaram,2012) showed that communication may be secured by the use of Fibonacci numbers. Similar application of Fibonacci in Cryptography is described here by a Simple Illustration.

Suppose that Original Message"CODE" to be Encrypted. It is sent through an unsecured channel. Security key is chosen based on the Fibonacci number. Any one character may be chosen as a first security key to generate cipher text and then Fibonacci sequence can be used. Agarwal et al (2015) used Fibonacci sequence for encryption data.

#### 4.1 Method of Encryption



The cochlea of the inner ear forms a Golden Spiral

## 2.4 Fibonacci in Music

The Fibonacci sequence of numbers and the golden ratio are manifested in music widely.

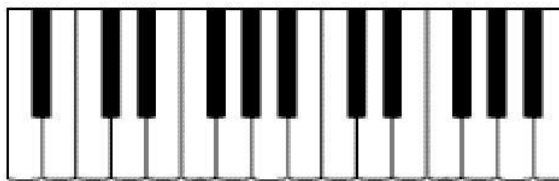
The numbers are present in the octave, the foundational unit of melody and harmony.

Stradivarius used the golden ratio to make the greatest string instruments ever created.

Howat's (1983) research on Debussy's works shows that the composer used the golden ratio and

Fibonacci numbers to structure his music. The *Fibonacci Composition* reveals the inherent aesthetic appeal of this mathematical phenomenon. Fibonacci numbers harmonize naturally and the exponential growth which the Fibonacci sequence typically defines in nature is made present in music by using Fibonacci notes. The intervals between keys on a piano of the same scales are Fibonacci numbers (Gend, 2014).

5 Black  
 3 B      2B

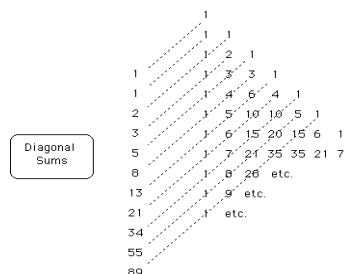


8 W & 5 B, 13 B&W

## 2.5 Fibonacci numbers in Pascal's Triangle

The Fibonacci Numbers are also applied in Pascal's Triangle. Entry is sum of the two numbers either side of it, but in the row above. Diagonal sums in Pascal's Triangle are the Fibonacci numbers. Fibonacci numbers can also be found using a formula

1  
 1    1  
 1    2    1  
 1    3    3    1  
 1    4    6    4    1



## 2.6 The Golden Section

Represented by the Greek letter Phi ( $\phi$ ) = 1.6180339887.

How did 1.6180339887..... come from?

Let's look at the ratio of each number in The Fibonacci sequence to the one before it:

1/1 = 1	13/8 = 1.625	144/89 = 1.61798
2/1 = 2	21/13 = 1.61538	233/144 = 1.61806
3/2 = 1.5	34/21 = 1.61905	
5/3 = 1.666	55/34 = 1.61764	
8/5 = 1.6	89/55 = 1.61861	