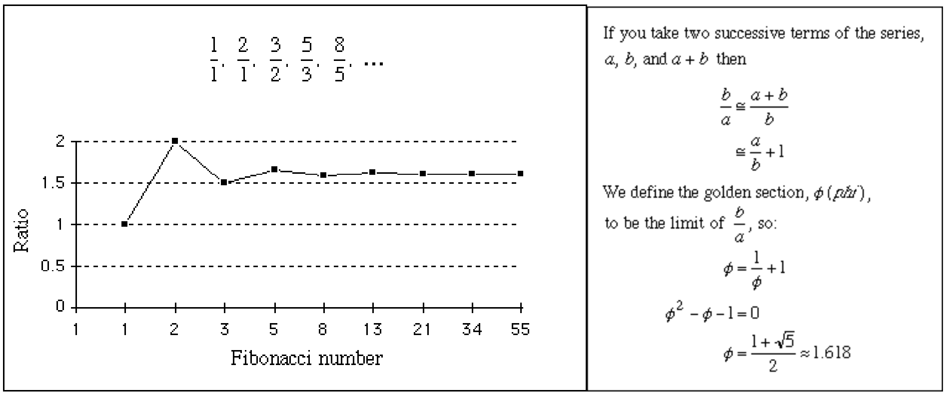
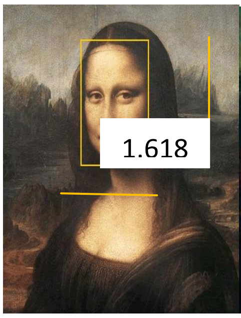
If we keep going, we get an interesting number which mathematicians call "phi" (Golden Ratio or Gollden Ratio). It is denoted by ϕ and the value of ϕ *=1.6180339887*

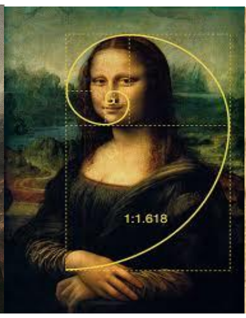
$$\lim_{n\to\infty} \frac{F_{n+1}}{F_n} = 1.618$$

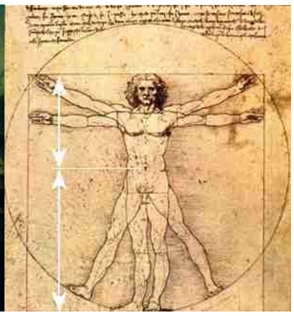


2.7 Some applications of Golden ratio

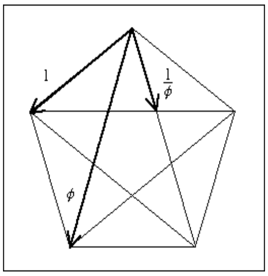
Leonardo da Vinci showed that in a 'perfect man' there were lots of measurements that followed the Golden Ratio. The Golden (Divine) Ratio has been talked about for thousands of years.

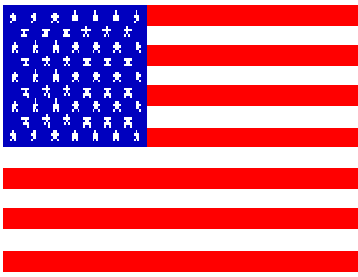







The Golden ratio is widely used in Geometry (Garg et al, 2014). It is the ratio of the side of a regular pentagon to its diagonal. The diagonals cut each other with the golden ratio (Stakov1989). Pentagram describes a star which forms parts of many flags. This five-point symmetry with Golden proportions is found in starfish which has five arms.

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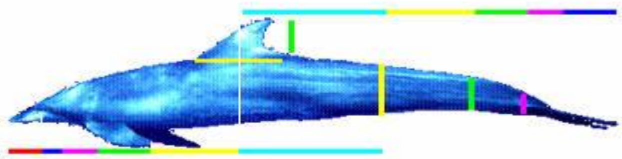
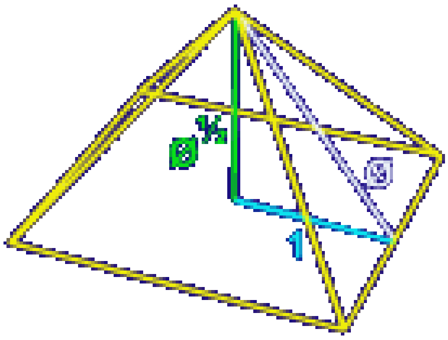
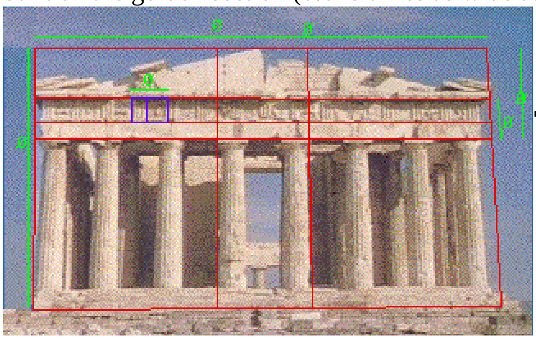
The eyes, fins and tail of the dolphin fall at Golden sections along the body.

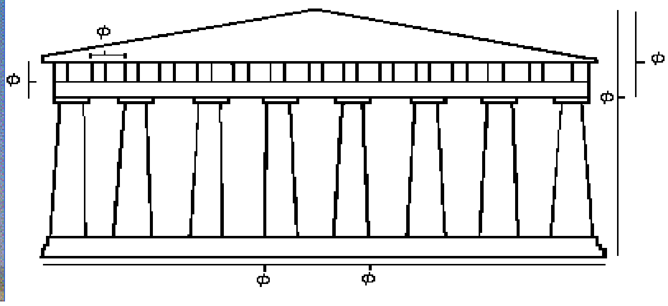


The Golden Ratio is also frequently seen in natural architecture also (Internet access, 18). It can be found in the great pyramid in Egypt. Perimeter of the pyramid, divided by twice its vertical height is the value of ϕ .



Golden section (Gend, 2014) appears in many of the proportions of the Parthenon in Greece. Front elevation is built on the golden section (0.618 times as wide as it is tall).





III. A beautiful example

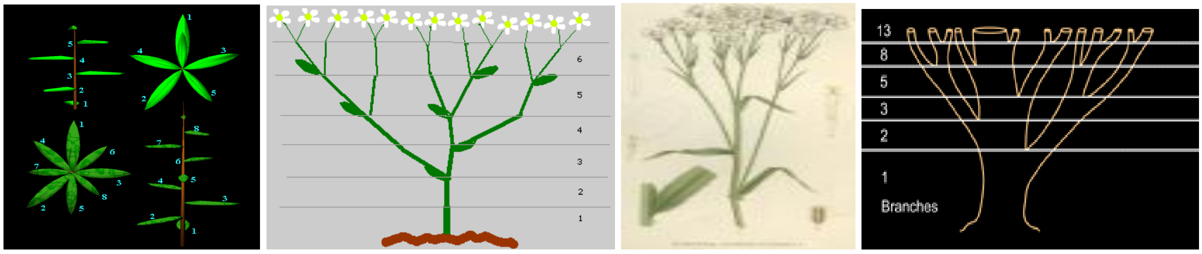
Take any two consecutive numbers from this series as example 13 and 21 or 34 and 55.
Now smaller number is in miles = the other one in Kilometer or bigger number is in Kilometers = the smaller one in Miles (The other way around).
34 Miles = round (54.72) Kilometers = 55 Kilometers
21 Kilometers = round (13.05) Miles = 13 Miles
For distances which are not exact Fibonacci values you can always proceed by splitting the distance into two or more Fibonacci values.
As example, for converting 15 km into miles we can proceed as following:
15 km = 13 km + 2 km
13 km -> 8 mile
2 km -> 1 mile
15 km -> 8+1 = 9 mile
Another example, for converting 170km into miles we can proceed as:
170 km = 10*17 km
17 km = 13 km + 2 km + 2 km = 8 + 1 + 1 miles = 10 miles (approximately)
Now, 170 km = 10*10 miles = 100 miles (approximately)
So, either way we can proceed. For bigger numbers we can proceed as above.
(Ref: Sudip Maji, B.C.Roy Engineering College)

IV. Fibonacci in Coding

Recently Fibonacci sequence and golden ratio are of great interest to the researchers in many fields of science including high energy physics, quantum mechanics, Cryptography and Coding. Raghu and Ravishankar(2015) developed a paper of application classical encryption techniques for securing data.(Raphael and Sundaram,2012) showed that communication may be secured by the use of Fibonacci numbers. Similar application of Fibonacci in Cryptography is described here by a Simple Illustration.
Suppose that Original Message"CODE" to be Encrypted. It is sent through an unsecured channel. Security key is chosen based on the Fibonacci number. Any one character may be chosen as a first security key to generate cipher text and then Fibonacci sequence can be used. Agarwal et al (2015) used Fibonacci sequence for encryption data.

4.1 Method of Encryption

Plants show the Fibonacci numbers in the arrangements of their leaves (Internet access,15). Three clockwise rotations, passing five leaves two counter-clockwise rotations. Sneezewort (Achillea ptarmica) also follows the Fibonacci numbers.



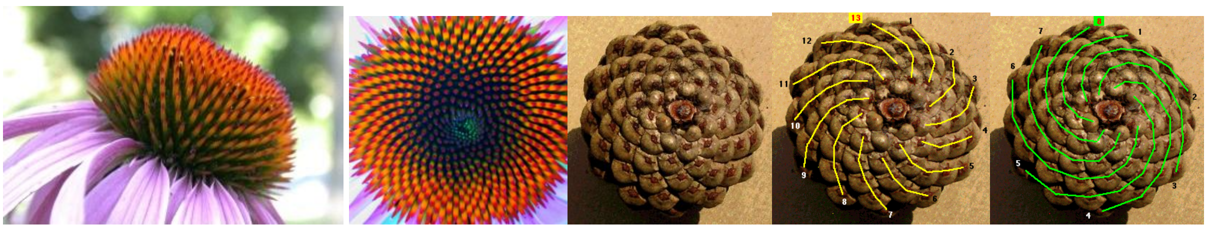
Schematic diagram (Sneezewort)

Why do these arrangements occur? In the case of leaf arrangement, or phyllotaxis, some of the cases may be related to maximizing the space for each leaf, or the average amount of light falling on each one.

These pictures are very common to us. We can see the flowers and the patterns of leaves just out of single step of our house. All of these follow the Fibonacci numbers.

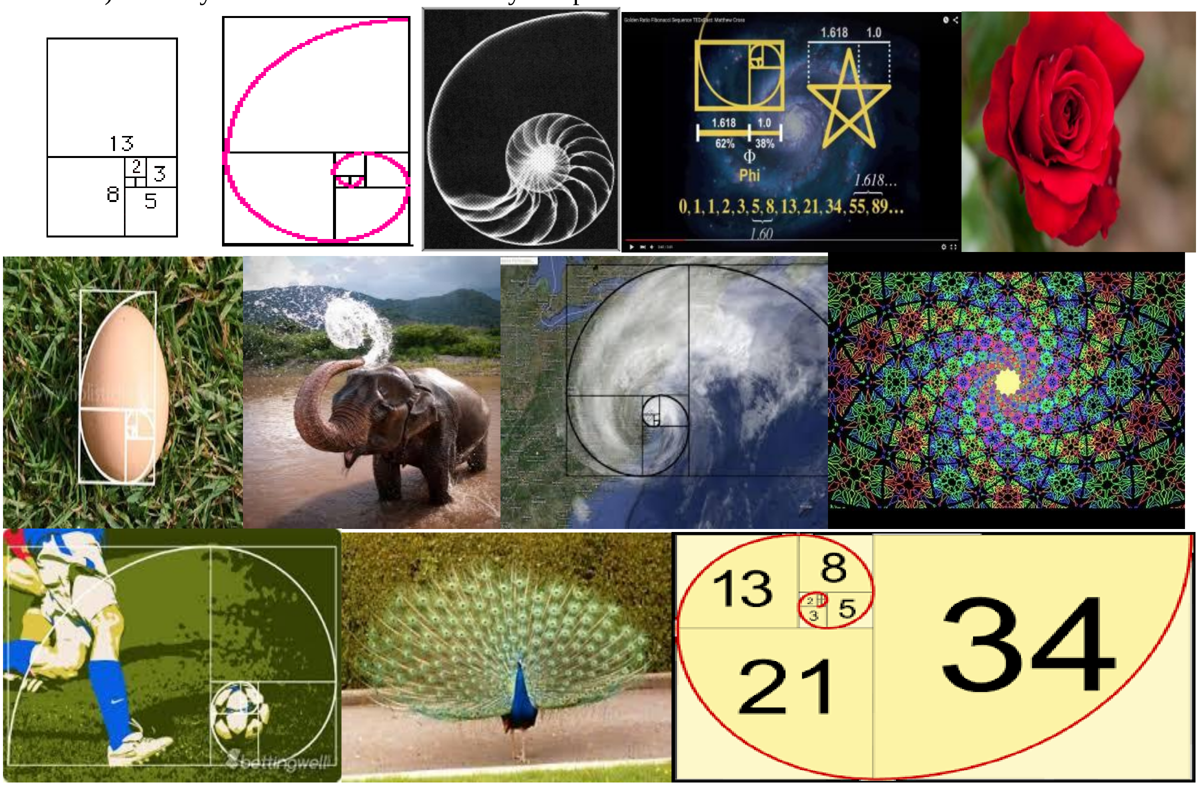
2.2 Fibonacci spiral

The Fibonacci numbers are found in the arrangement of seeds on flower heads (Internet access, 13). There are 55 spirals spiraling outwards and 34 spirals spiraling inwards in most daisy or sunflower blossoms (Internet access,14). Pinecones clearly show the Fibonacci spirals (Howard, 2004)



Fibonacci spiral can be found in cauliflower. The Fibonacci numbers can also be found in Pineapples and Bananas (Lin and Peng). Bananas have 3 or 5 flat sides and Pineapple scales have Fibonacci spirals in sets of 8, 13, and 21. Inside the fruit of many plants we can observe the presence of Fibonacci order.

Fibonacci spiral (Internet access, 9), (Internet access, 11) are also found in various fields associated in nature. It is seen snail, sea shells, waves, combination of colours; roses etc in so many things created in nature (Internet access 12).But very few of us have time to study this phenomenon.

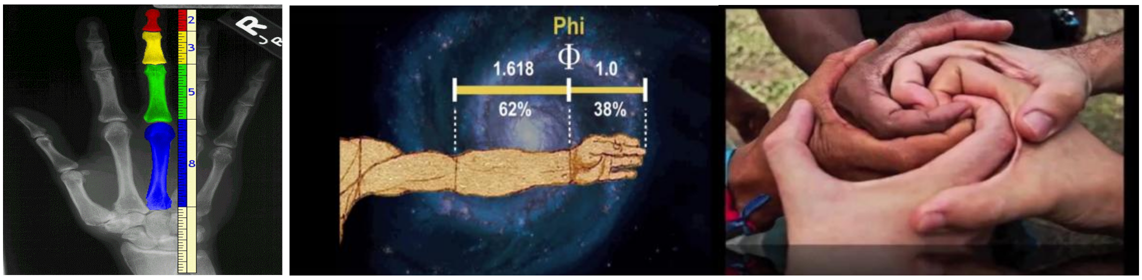


Nature isn't trying to use the Fibonacci numbers: they are appearing as a by-product of a deeper physical process. That is why the spirals are imperfect. The plant is responding to physical constraints, not to a mathematical rule.

The basic idea is that the position of each new growth is about 222.5 degrees away from the previous one, because it provides, on average, the maximum space for all the shoots. This angle is called the **golden angle**, and it divides the complete 360 degree circle in the golden section, 0.618033989 . . . . which is described below.

2.3 Organs of human body

Humans exhibit Fibonacci characteristics. Every human has two hands, each one of these has five fingers and each finger has three parts which are separated by two knuckles (Internet access, 7). All of these numbers fit into the sequence. Moreover the lengths of bones in a hand are in Fibonacci numbers.



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The Fibonacci Numbers and Its Amazing Applications

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Abstract: *Fibonacci sequence of numbers and the associated "Golden Ratio" are manifested in nature and in certain works of art. We observe that many of the natural things follow the Fibonacci sequence. It appears in biological settings such as branching in trees, phyllotaxis (the arrangement of leaves on a stem), the fruit sprouts of a pineapple, the flowering of an artichoke, an uncurling fern and the arrangement of a pine cone's bracts etc. At present Fibonacci numbers plays very important role in coding theory. Fibonacci numbers in different forms are widely applied in constructing security coding.*

Keywords: *Fibonacci Numbers, Golden ratio, Coding, Encryption, Decryption*

Date of Submission: 05-08-2017 Date of acceptance: 13-09-2017

I. Introduction

The Fibonacci numbers were first discovered by a man named Leonardo Pisano. He was known by his nickname, Fibonacci. The Fibonacci sequence is a sequence in which each term is the sum of the 2 numbers preceding it. The Fibonacci Numbers are defined by the recursive relation defined by the equations $F_n = F_{n-1} + F_{n-2}$ for all $n \geq 3$ where $F_1 = 1$; $F_2 = 1$ where Fn represents the nth Fibonacci number (n is called an index).
The Fibonacci sequence can elaborately written as {1,1,2,3,5,8,13,21,34,55,89,144,233…….}.
One of the most common experiments dealing with the Fibonacci sequence is his experiment with rabbits. Fibonacci put one male and one female rabbit in a field. Fibonacci supposed that the rabbits lived infinitely and every month a new pair of one male and one female was produced. Fibonacci asked how many would be formed in a year. Following the Fibonacci sequence perfectly the rabbits reproduction was determined...144 rabbits. Though unrealistic, the rabbit sequence allows people to attach a highly evolved series of complex numbers to an everyday, logical, comprehendible thought.Bortner and Peterson (2016) elaborately described the history and application of Fibonacci numbers.

II. Fibonacci Sequence In Nature

Fibonacci can be found in nature not only in the famous rabbit experiment, but also in beautiful flowers (Internet access, 12). On the head of a sunflower and the seeds are packed in a certain way so that they follow the pattern of the Fibonacci sequence. This spiral prevents the seed of the sunflower from crowding themselves out, thus helping them with survival. The petals of flowers and other plants may also be related to the Fibonacci sequence in the way that they create new petals (Internet access, 10).

2.1 Petals on flowers

Probably most of us have never taken the time to examine very carefully the number or arrangement of petals on a flower. If we were to do so, we would find that the number of petals on a flower that still has all of its petals intact and has not lost any, for many flowers is a Fibonacci number (Internet access,8).
- 1 petal: white cally lily
- 3 petals: lily, iris
- 5 petals: buttercup, wild rose, larkspur, columbine (aquilegia)
- 8 petals: delphiniums
- 13 petals: ragwort, corn marigold, cineraria,
- 21 petals: aster, black-eyed susan, chicory
- 34 petals: plantain, pyrethrum
- 55, 89 petals: michaelmas daisies, the asteraceae family (Internet access,19)