Digital Communication
Assignment 1

Topic: Prolate Spheroidal Wave Functions.

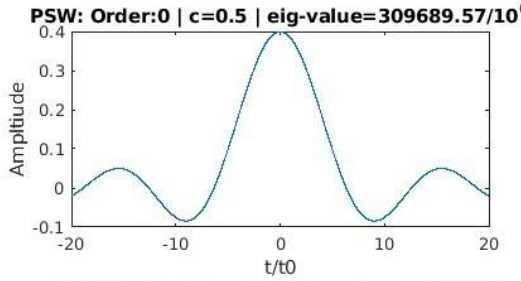
Name: Anand Jhunjhunwala
Roll No: 17EC30041

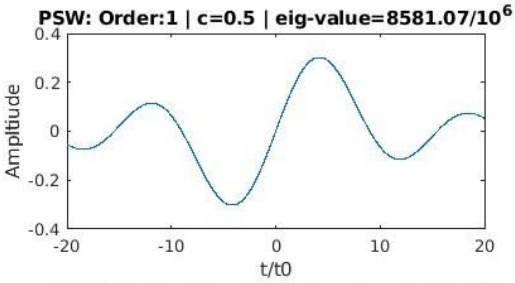
Part 1a: PSW with its eigenvalues.

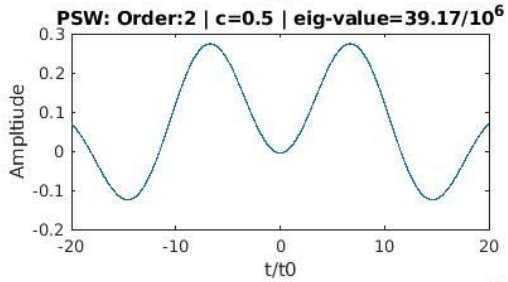
Result plots

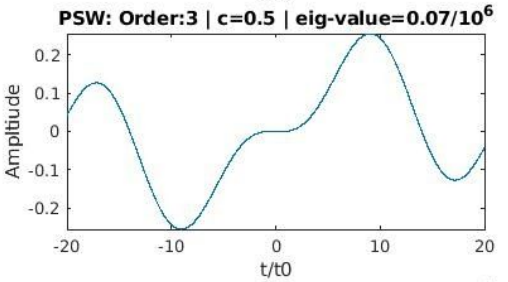
The following plots show PSWs for different values of c, along with its eigenvalues in the range of [-20,20].

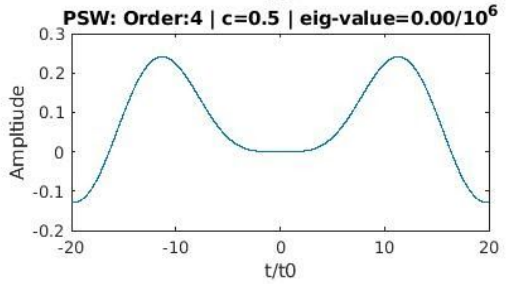
For C = 0.5

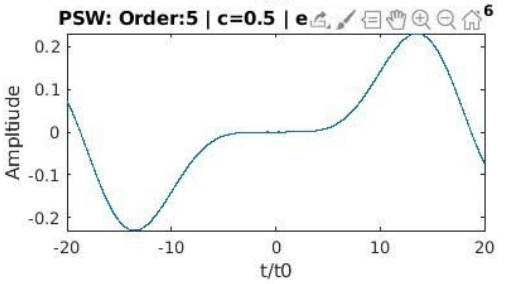




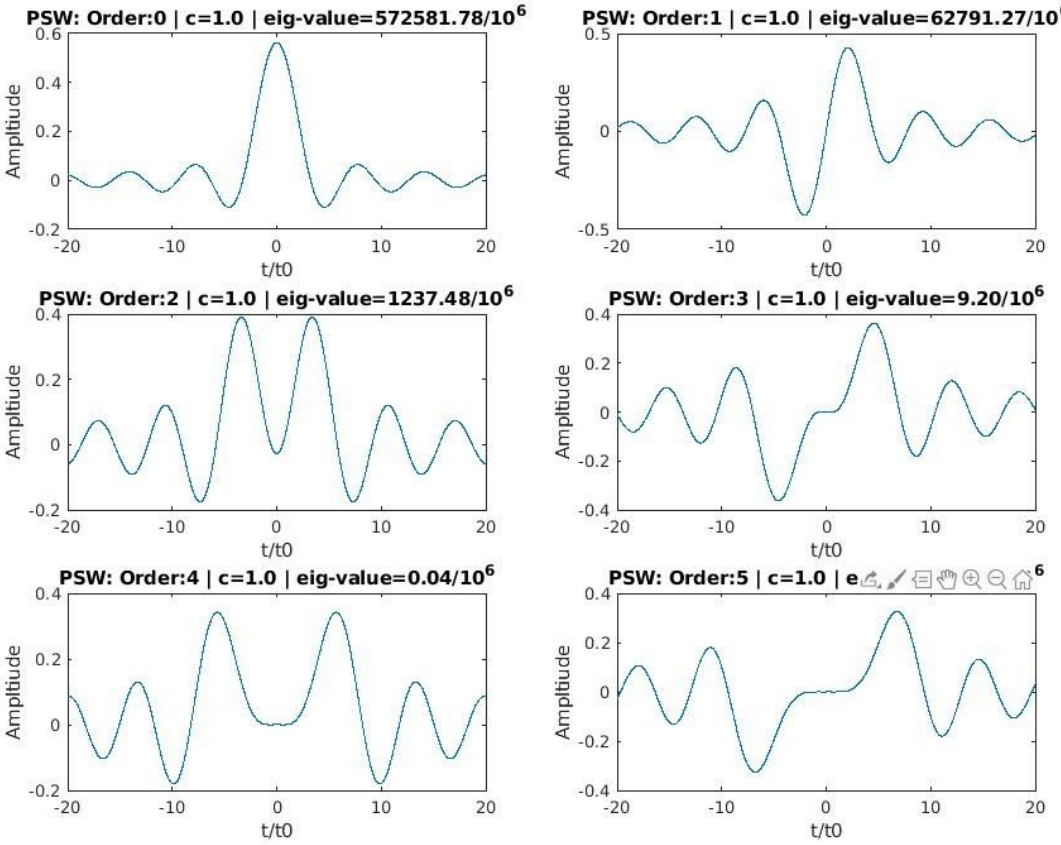




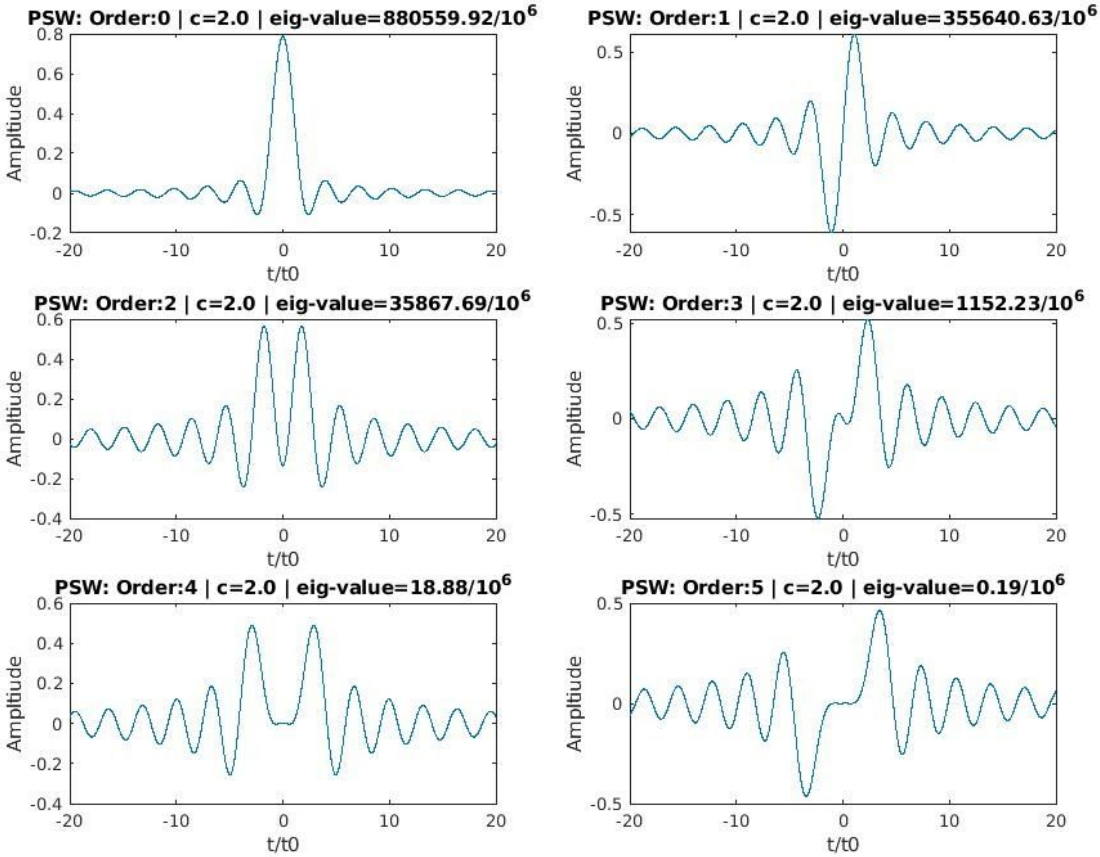




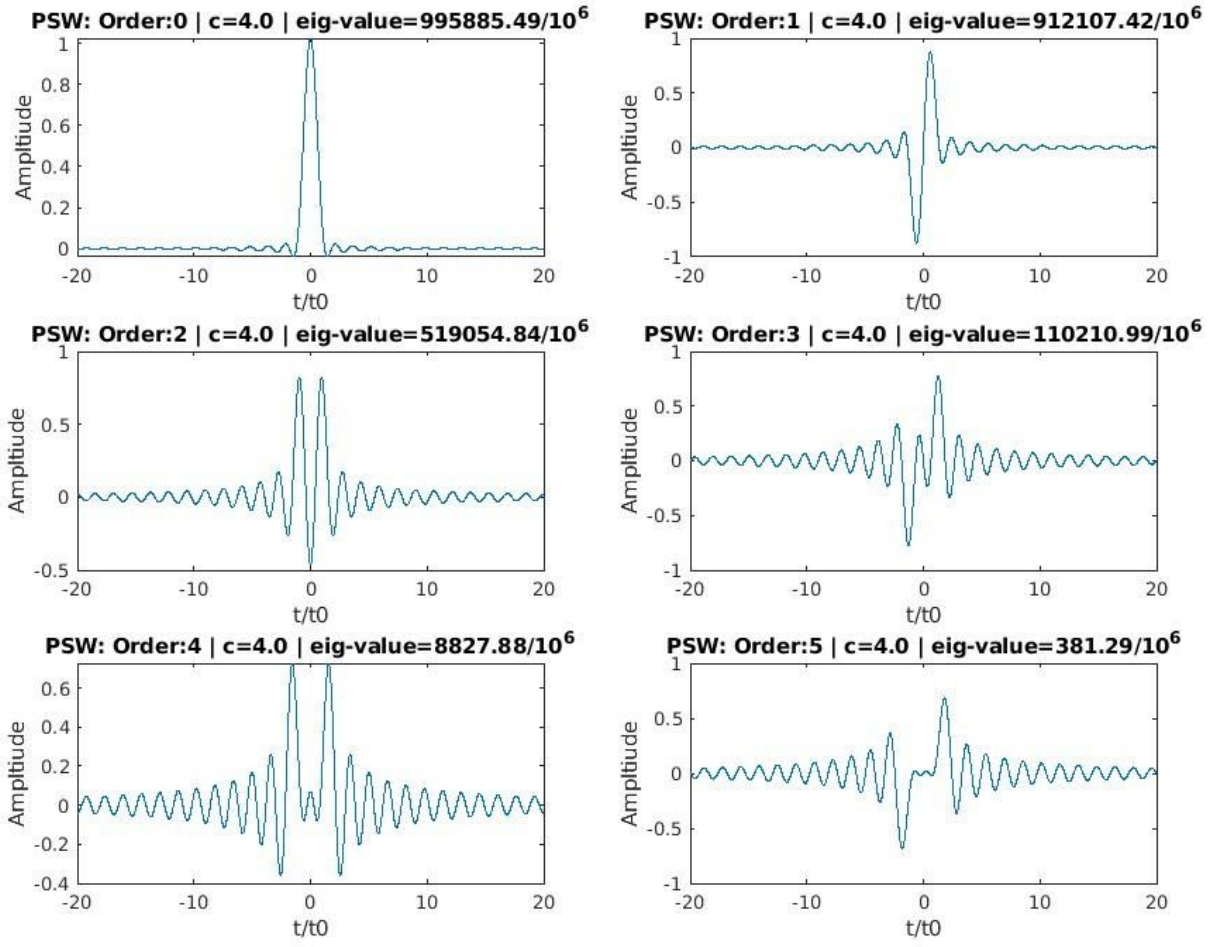
For C = 1


PSW: Order:0 | c=1.0 | eig-value=572581.78/10^6
PSW: Order:1 | c=1.0 | eig-value=62791.27/10^6
PSW: Order:2 | c=1.0 | eig-value=1237.48/10^6
PSW: Order:3 | c=1.0 | eig-value=9.20/10^6
PSW: Order:4 | c=1.0 | eig-value=0.04/10^6
PSW: Order:5 | c=1.0 | e
Amplitude
t/t0

For C =2


PSW: Order:0 | c=2.0 | eig-value=880559.92/10^6
PSW: Order:1 | c=2.0 | eig-value=355640.63/10^6
PSW: Order:2 | c=2.0 | eig-value=35867.69/10^6
PSW: Order:3 | c=2.0 | eig-value=1152.23/10^6
PSW: Order:4 | c=2.0 | eig-value=18.88/10^6
PSW: Order:5 | c=2.0 | eig-value=0.19/10^6
Amplitude
t/t0

For C = 4



The plots are similar to the expected PSWs as seen from the paper. I also observed that with an increase in C, the plot becomes more and more squeezed.

Tabel1: Shows eigenvalues for different order PSW for all 4 values of c.
Note: Each row corresponds to a particular order of PSW (row k correspond to PSW of order k-1) and each column corresponds to a particular c in increasing order.

Eigen_value for infinite time support for all c

0.3097	0.5726	0.8806	0.9959
0.0086	0.0628	0.3556	0.9121
0.0000	0.0012	0.0359	0.5191
0.0000	0.0000	0.0012	0.1102
0.0000	0.0000	0.0000	0.0088
0.0000	0.0000	0.0000	0.0004

Tabel2: Shows eigenvalues for different order PSW for all 4 values of c with finite time.

Note: Each row corresponds to a particular order of PSW (row k correspond to PSW of order k-1) and each column corresponds to a particular c in increasing order.

Eigen_value for finite time support for all c

0.3097	0.5726	0.8806	0.9959
0.0086	0.0628	0.3556	0.9121
0.0000	0.0012	0.0359	0.5191
0.0000	0.0000	0.0012	0.1102
0.0000	0.0000	0.0000	0.0088
0.0000	0.0000	0.0000	0.0004

- Clearly from table1 and 2, I observed that eigenvalues don't change with truncation in time.

Part 1b: Orthogonality and Orthonormality of PSW

For infinite time, i.e range [-20, 20]

Orthonormal matrix for infinite time support for c = 4

0.9999	0	0	0	0	0
0	0.9979	0	0	0	0
0	0	0.9898	0	0	0
0	0	0	0.9771	0	0
0	0	0	0	0.9658	0
0	0	0	0	0	0.9568

Note: Each entry corresponds to the inner product value of two PSW with order given by row_number-1 and column_number-1.

- Clearly, each PSW is orthogonal to all other PSW orders and orthonormal with itself, the inner product with itself is not exactly 1 this is because I have taken value only between -20 to 20 is we increase this range further then this values will tend towards 1.

For a finite time, i.e range [-1, 1]

Orthogonal matrix for infinite time support for c = 4

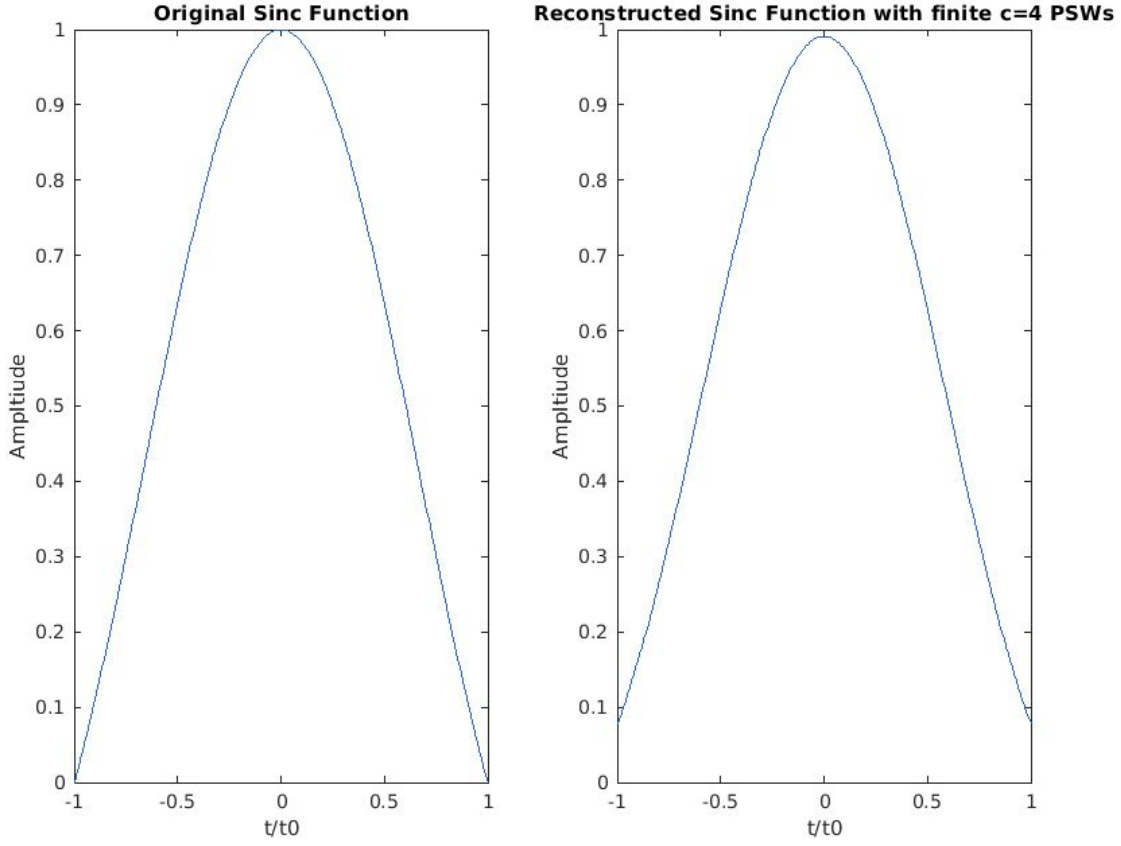
0.9959	0	0	0	0	0
0	0.9121	0	0	0	0.0000
0	0	0.5190	0	0.0000	0
0	0	0	0.1102	0	0.0000
0	0	0.0000	0	0.0088	0
0	0.0000	0	0.0000	0	0.0004

Note: Each entry corresponds to the inner product value of two finite PSW with order given by row_number-1 and column_number-1.

- Clearly, each PSW is orthogonal to all other PSW orders and orthogonal with itself, the inner product with itself is exactly equal to its eigenvalue as expected by the theory. [See table 2, 4th column i.e for c=4].

Part 2a: Approximating Sinc function with 5 finite PSWs.

Results:



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Percentage Energy loss due to finite approximation = 0.597
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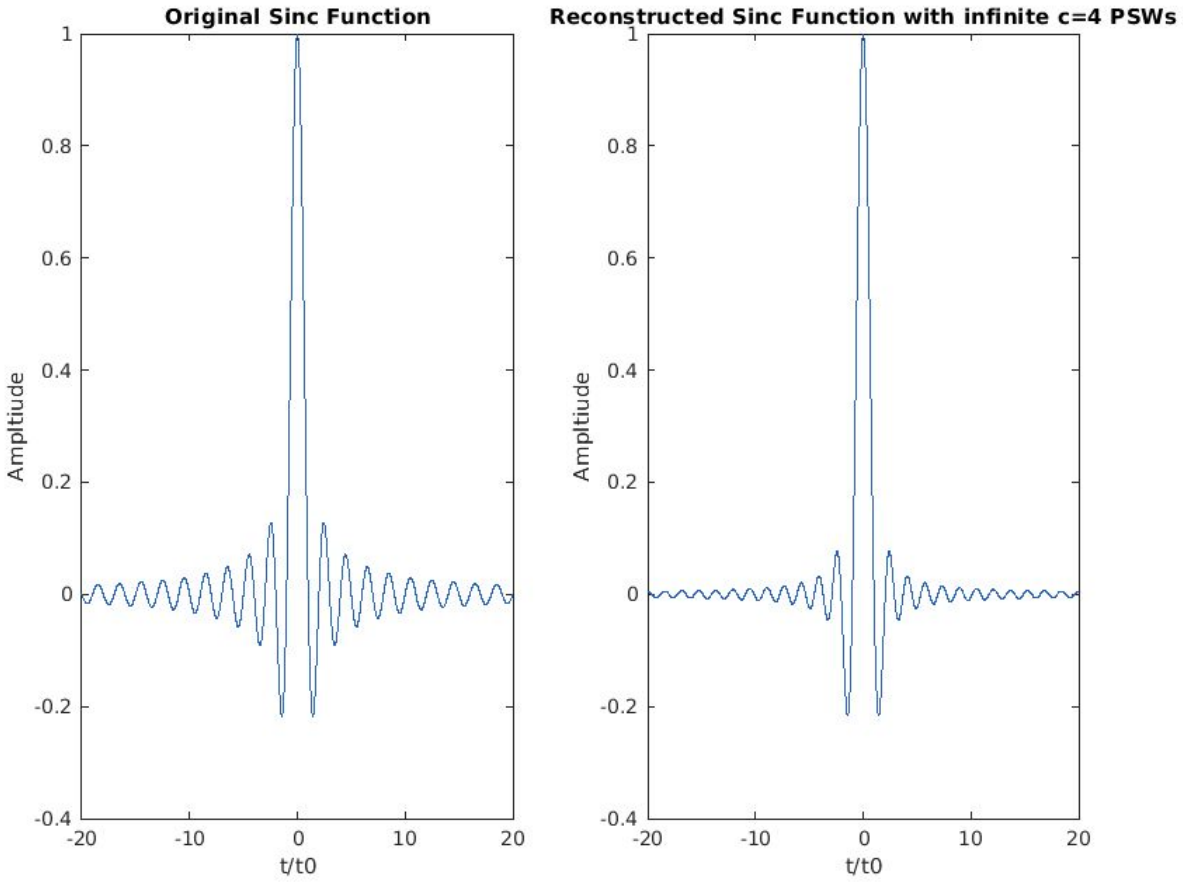
- Clearly approximated Sinc function is very similar to the original function and % loss in energy is very less given by 0.597%. This proves that PSWs can be used to approximate any L2 function.

Part 2b: Approximating Sinc function with 5 infinite PSWs. Results

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Pecentage Energy loss due to infinite approximation = 1.738
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- Clearly approximated Sinc function is very similar to the original function and % loss in energy is very less given by 1.738% in infinite time domain also. This proves that PSWs can be used to approximate any L2 function in any time interval.

Note: Percentage energy loss increase with an increase in time interval this is because, with an increase in the time interval, points approximated also increase which in turn results in more error.