Digital Signal Processing Lab
Experiment 2
Designing low pass filter by windowing method

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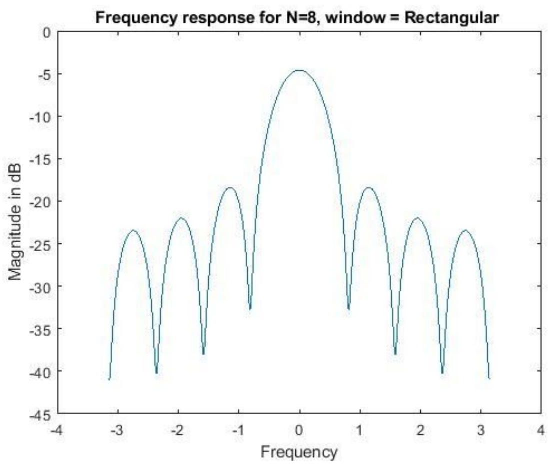
Part A:
First I have defined all the 5 window functions as mentioned in the experiment procedure.
Windows used are:
1) Rectangular Window
2) Triangular Window
3) Hanning Window
4) Hamming Window
5) Blackman Window

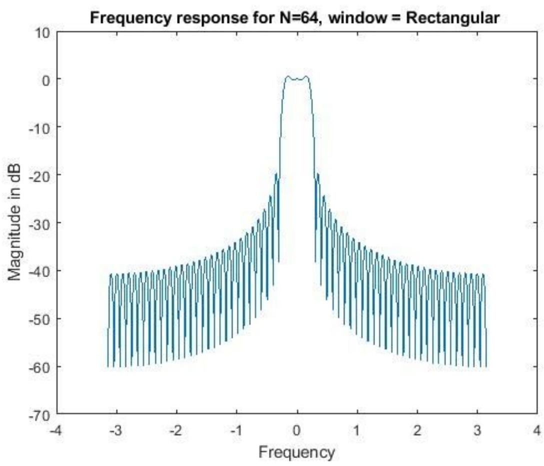
Function of all the above windows are mentioned in the experiment instructions.
Then I have defined Hd[n] which is the time domain low pass filter, each of the above filter and window is defined for different N, i.e N = 8, 64,512.

Basic Idea of Experiment:
The basic idea behind the window design is to choose a proper ideal frequency-selective filter (Which always has a noncausal, infinite-duration impulse response) and then to truncate (or window) its impulse response to obtain a linear-phase and causal FIR filter. Therefore, the emphasis in this method is on selecting an appropriate windowing function and an appropriate ideal filter.
Here we analyse low pass filter design using 5 different window functions.

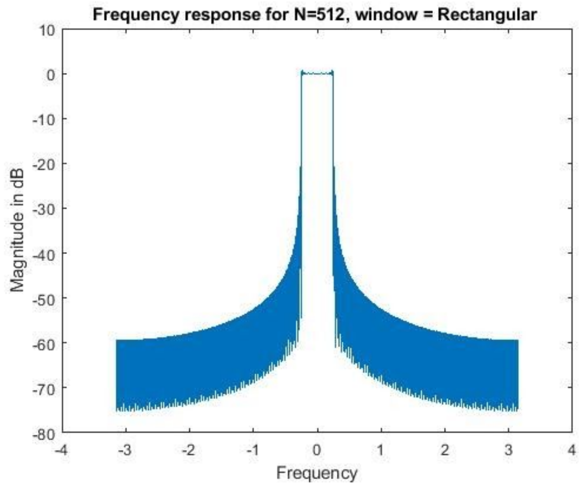
Results:
1) **<u>Using Rectangular Window for different N</u>**



N = 8



N=64



N=512

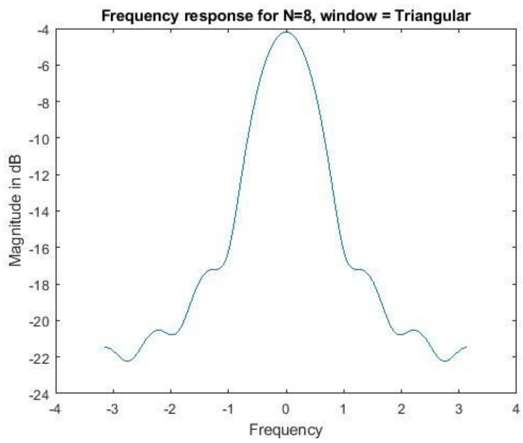
Observations:

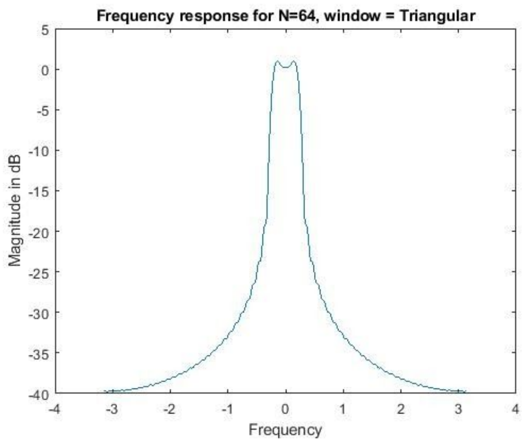
- As N increases, the frequency response approaches the idea behaviour, with decrease in passband ripple.
- As N increases, the peak of the first side lobe decreases.
- As N increases, maximum stop band attenuation increases.
- As N increases, roll-off of the filter increases approaching idea behaviour.

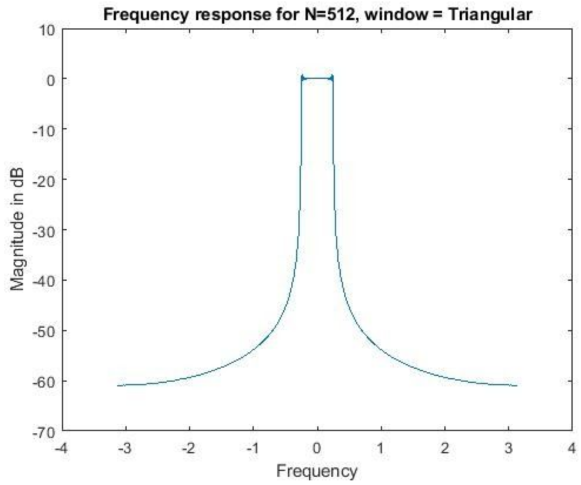
Observation Table:

Rectangular Window			
N	**Transition Width**	**Peak of First Lobe**	**Maximum Stopband Attenuation**
8	0.19	-20.66	37.5
64	0.03	-21.1	50.15
512	0.01	-23.4	64.86

2) <u>Using Triangular Window for different N</u>







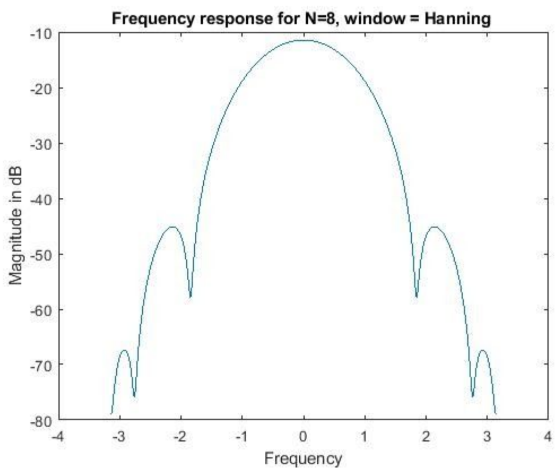
Observations:

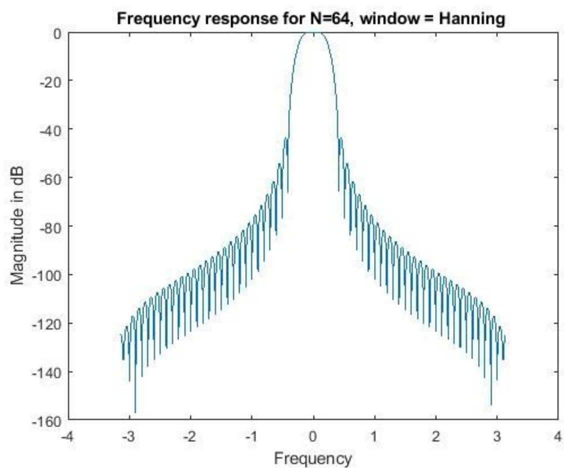
- As N increases, the frequency response approaches the idea behaviour, with decrease in passband ripple.
- As N increases, the peak of the first side lobe decreases.
- As N increases, maximum stop band attenuation increases.
- As N increases, roll-off of the filter increases approaching idea behaviour.

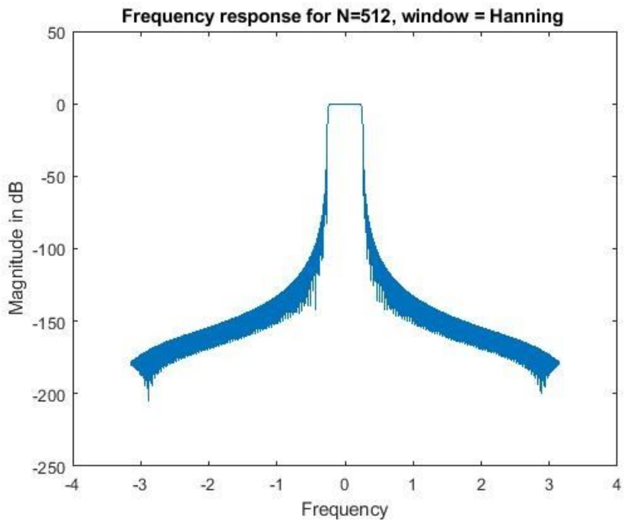
Observation Table:

Triangular Window			
N	**Transition Width**	**Peak of First Lobe (dB)**	**Maximum Stopband Attenuation (dB)**
8	0.28	-17.6	37.37
64	0.03	-19.5	40.62
512	-	-	57.08

3) **Using Hanning Window for different N**







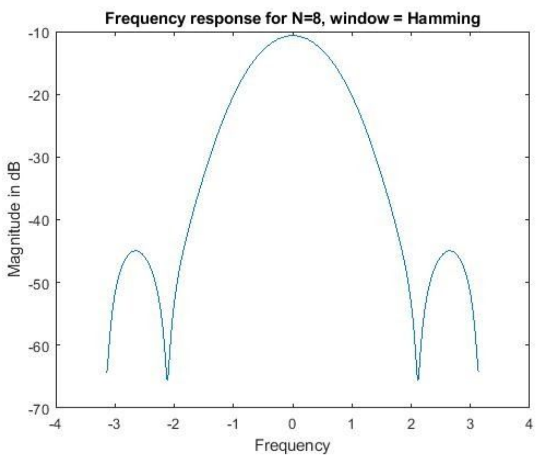
Observations:

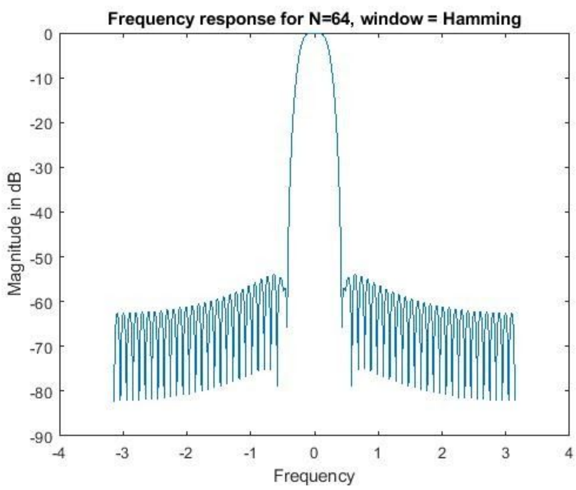
- As N increases, the frequency response approaches the idea behaviour, with decrease in passband ripple.
- As N increases, the peak of the first side lobe decreases.
- As N increases, maximum stop band attenuation increases.
- As N increases, roll-off of the filter increases approaching idea behaviour.

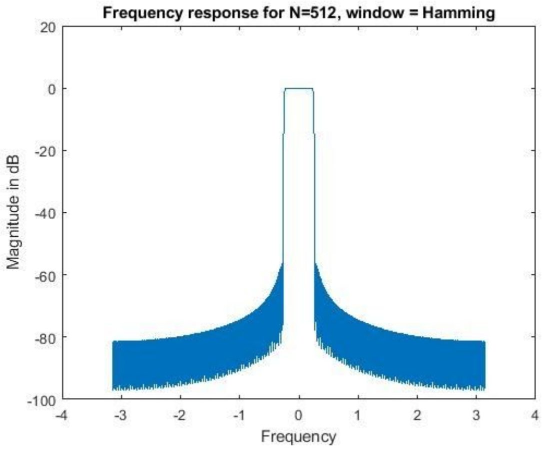
Observation Table:

Hanning Window			
N	**Transition Width**	**Peak of First Lobe**	**Maximum Stopband Attenuation**
8	0.52	-42.4	38.5
64	0.07	-45.5	95.2
512	0.013	-50	182.3

4) **<u>Using Hamming Window for different N</u>**







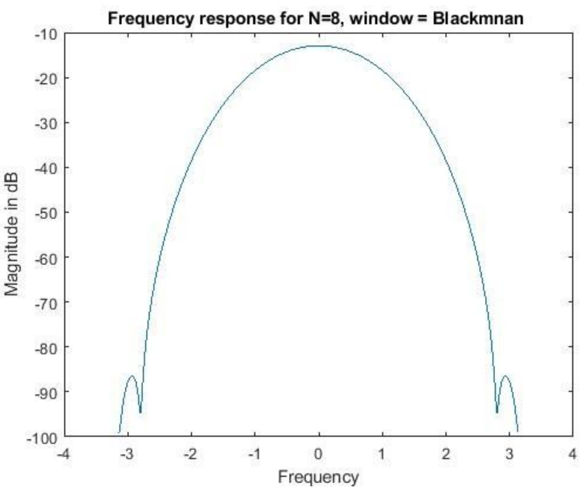
Observations:

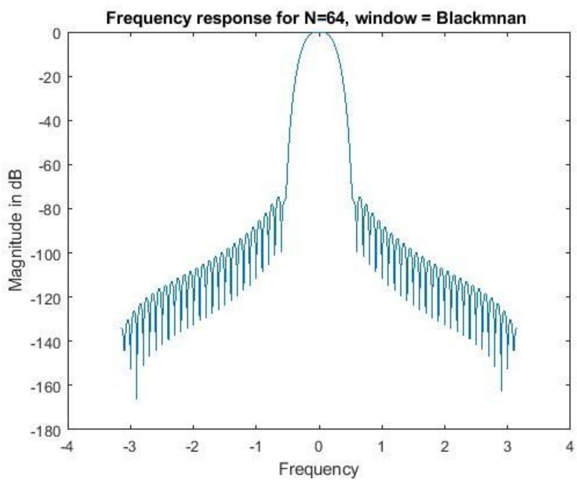
- As N increases, the frequency response approaches the idea behaviour, with decrease in passband ripple.
- As N increases, the peak of the first side lobe decreases.
- As N increases, maximum stop band attenuation increases.
- As N increases, roll-off of the filter increases approaching idea behaviour.

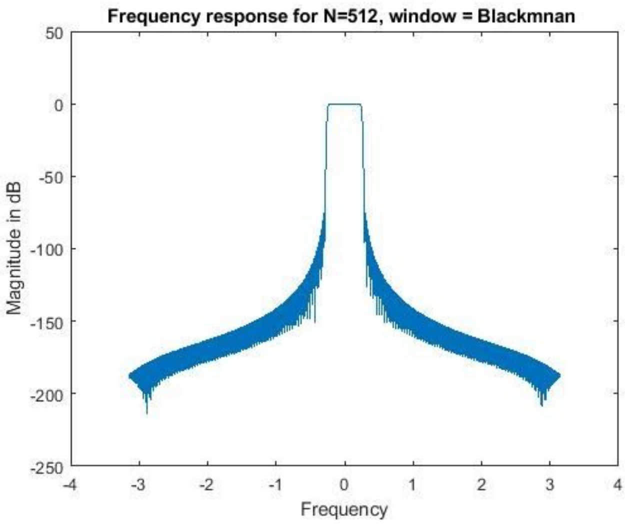
Observation Table:

Hamming Window			
N	**Transition Width**	**Peak of First Lobe**	**Maximum Stopband Attenuation**
8	0.52	-45.3	36.51
64	0.07	-57.3	84.85
512	0.012	-60	87.54

5) <u>Using Blackman Window for different N</u>







Observations:

- As N increases, the frequency response approaches the idea behaviour, with decrease in passband ripple.
- As N increases, the peak of the first side lobe decreases.
- As N increases, maximum stop band attenuation increases.
- As N increases, roll-off of the filter increases approaching idea behaviour.

Observation Table:

Blackman Window			
N	**Transition Width**	**Peak of First Lobe**	**Maximum Stopband Attenuation**
8	0.62	-85	54.82
64	0.12	-80	104
512	0.03	-75.3	190

Discussions:

1) From the results above, with the change in window function the frequency response of filter change significantly, this proves that a proper window function is important for designing the required FIR filter.
2) From the results of the tabel, I found that:
    a) The Blackman Filter has a good stop band attenuation relative to other window filters.
    b) As the length of the window function(N) increases, the main lobe width of W (w) is reduced which in turn reduces the transition width but we have to deal with more ripples.

<u>Part B:</u> Performance of the Filter in the time domain

In this part of the experiment, I made a signal having 2 frequency components, one in the passband and second in the stopband.
This signal is then used to analyze the response of the different filter for all the values of N.

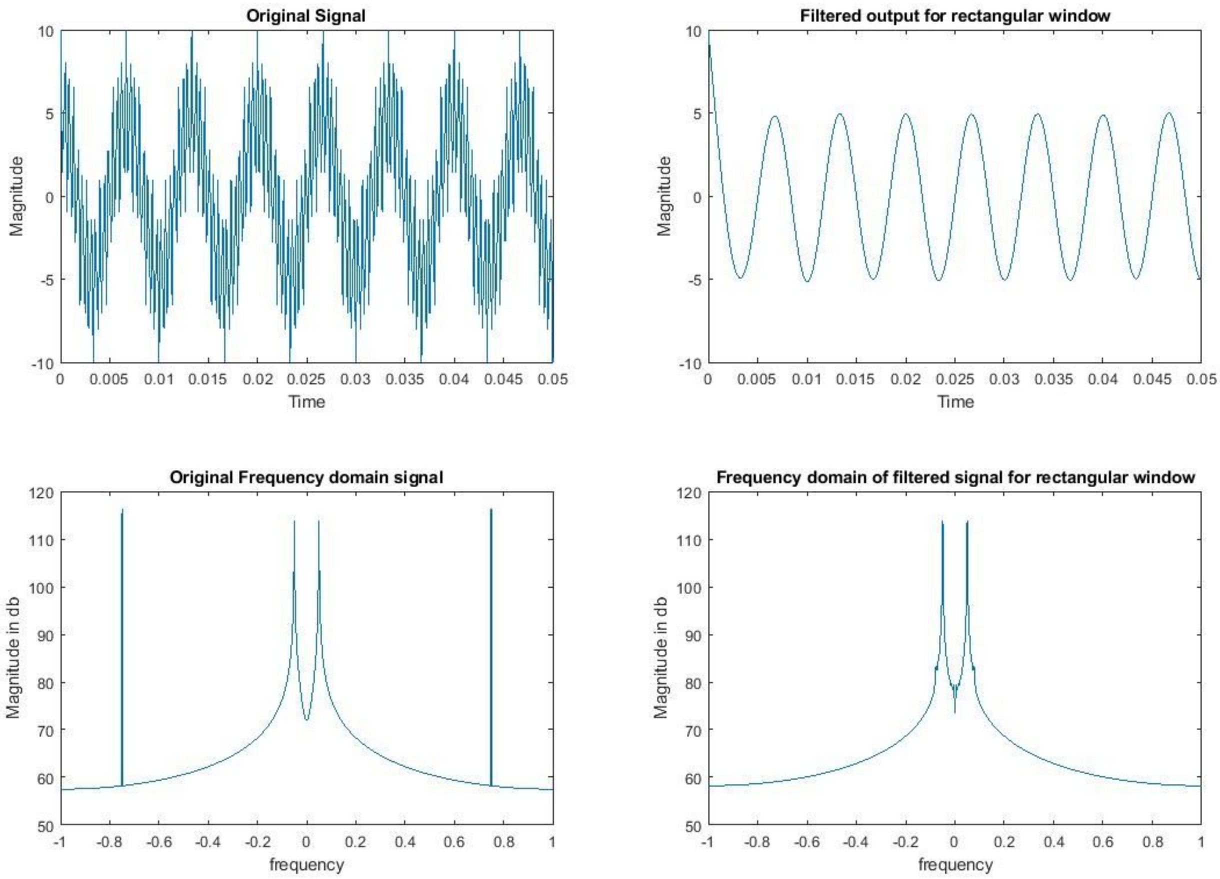
Signal used: **x[n] = 5*cos(2*pi*f_pass*t) + 5*cos(2*pi*f_stop*t);**
Where f_pass = 6000*0.1*Wc, f_stop = 6000*1.5*Wc

In the 1st part, the above x[n] was used without any noise, and filter response was analyzed.

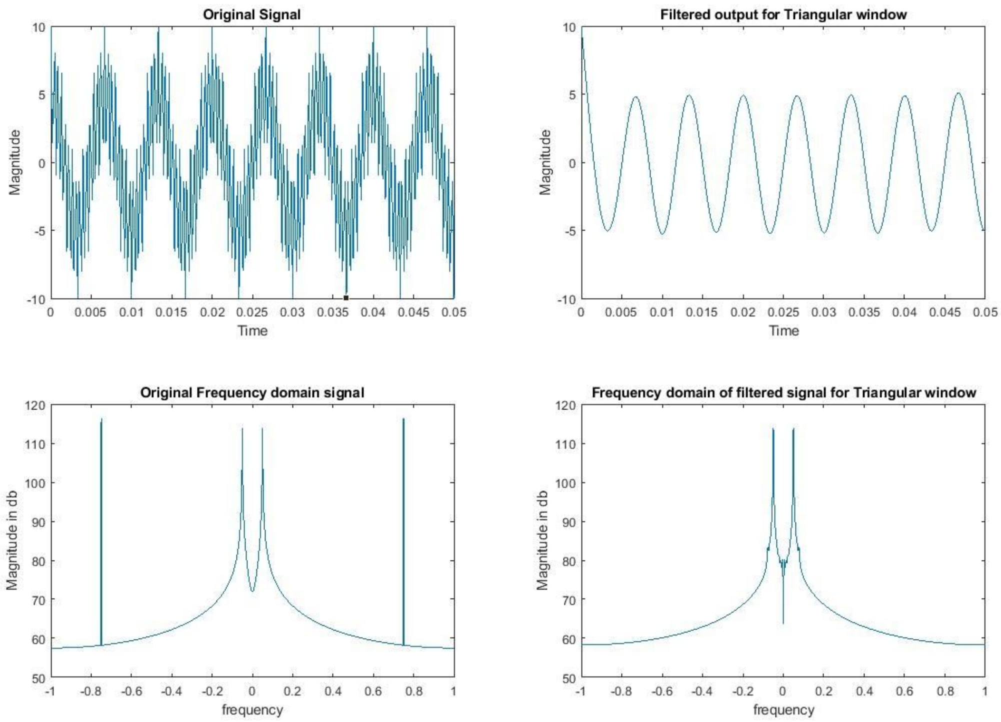
Results:

For N=512

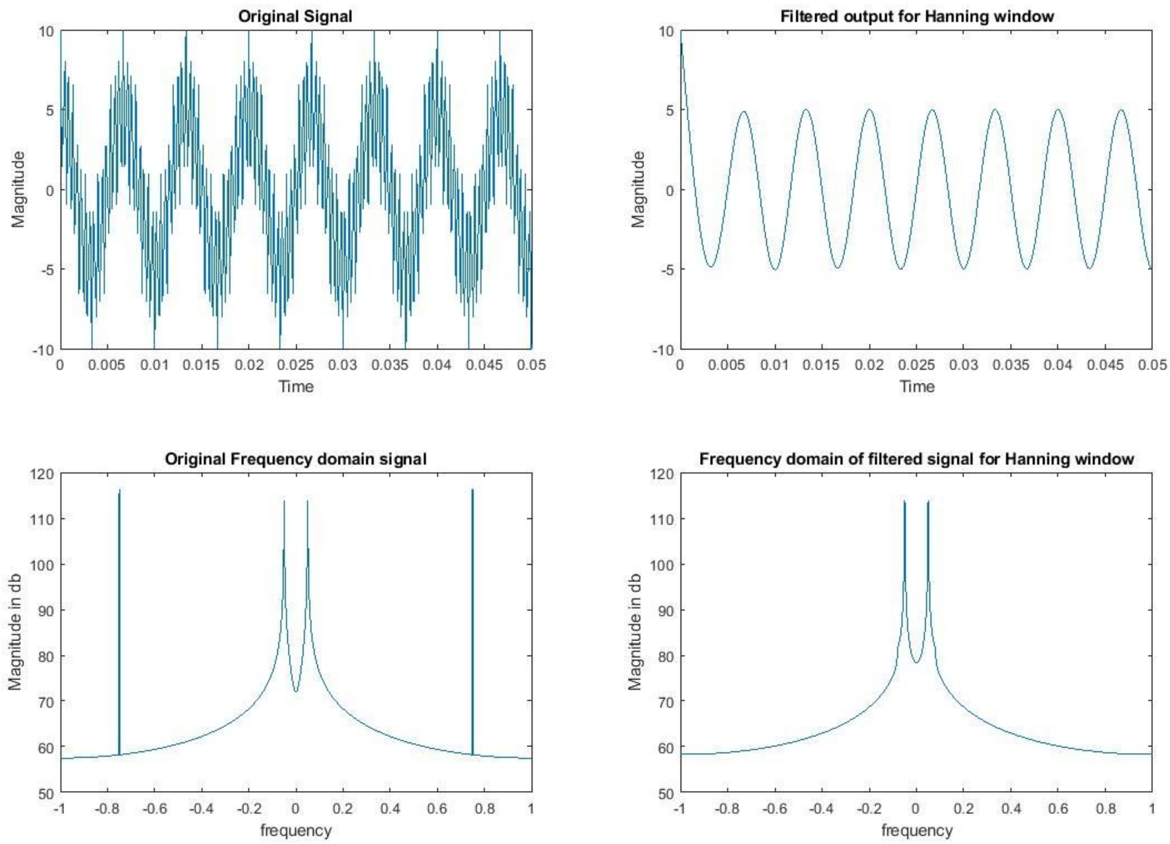
a) **Rectangular Window**



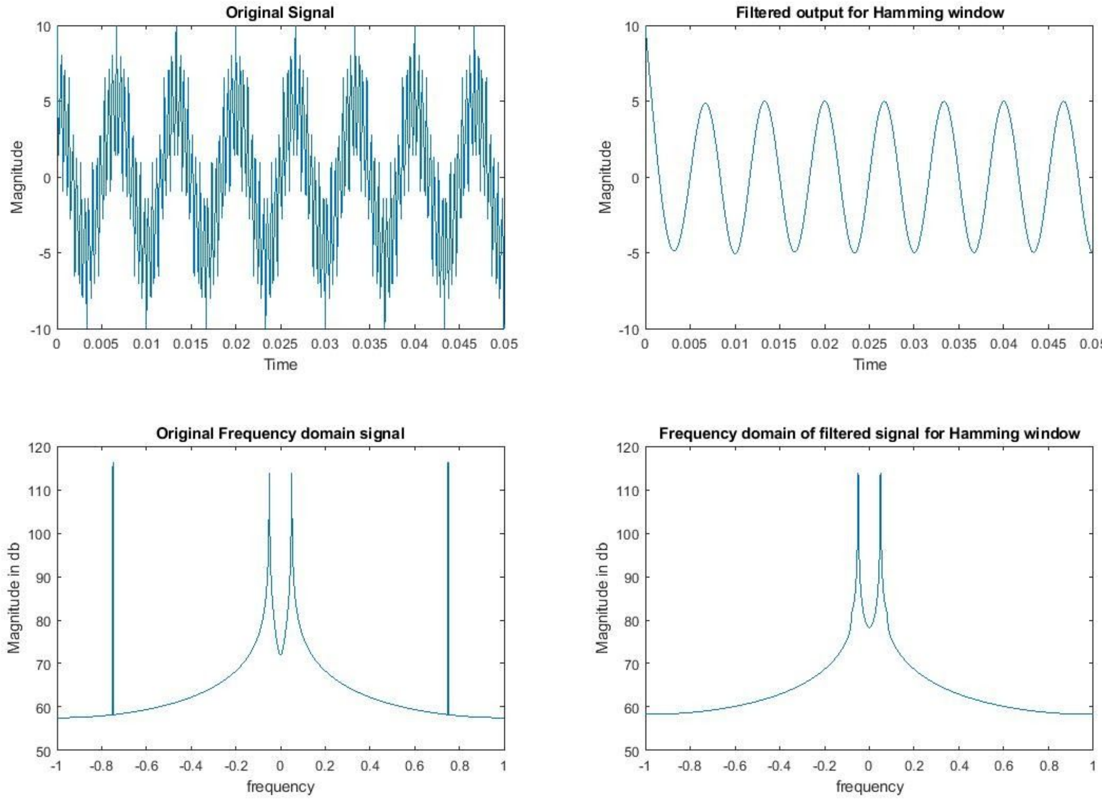
b) **Triangular Window**



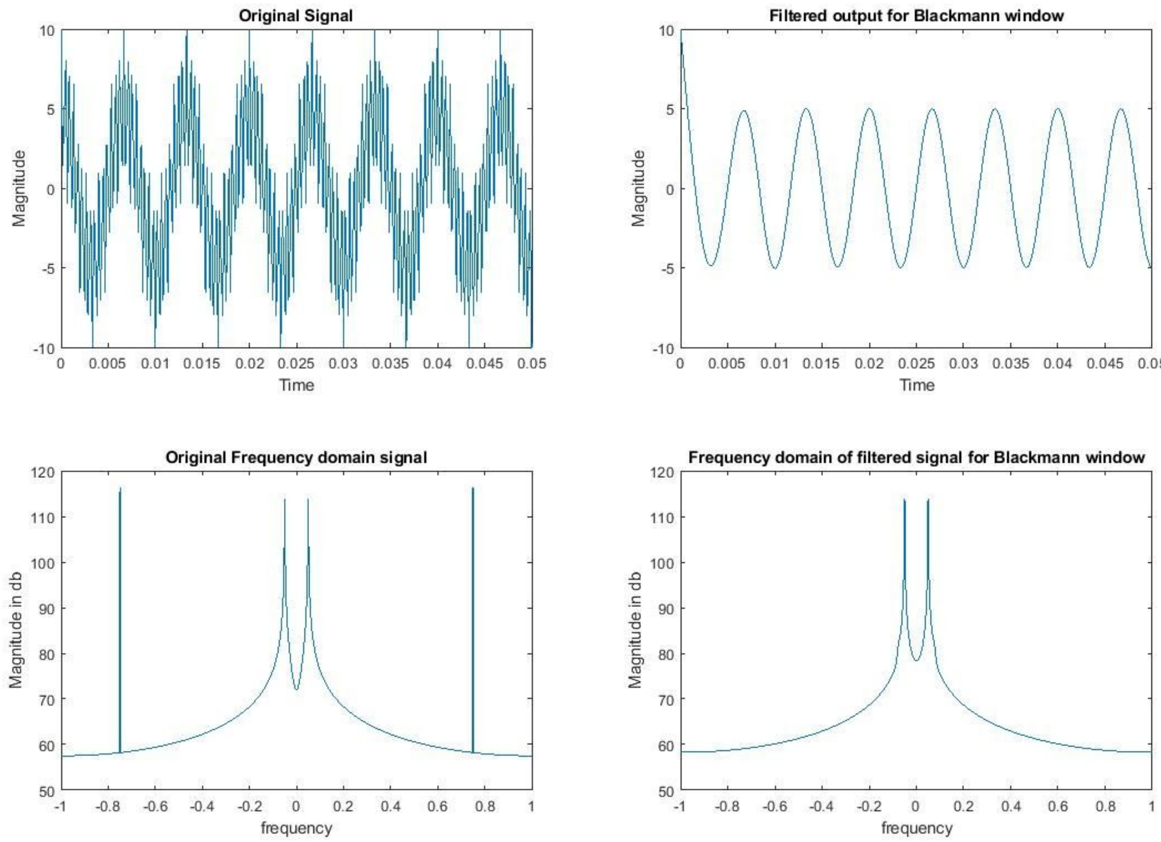
c) Hanning Window



d) Hamming Window



e) Blackman Window



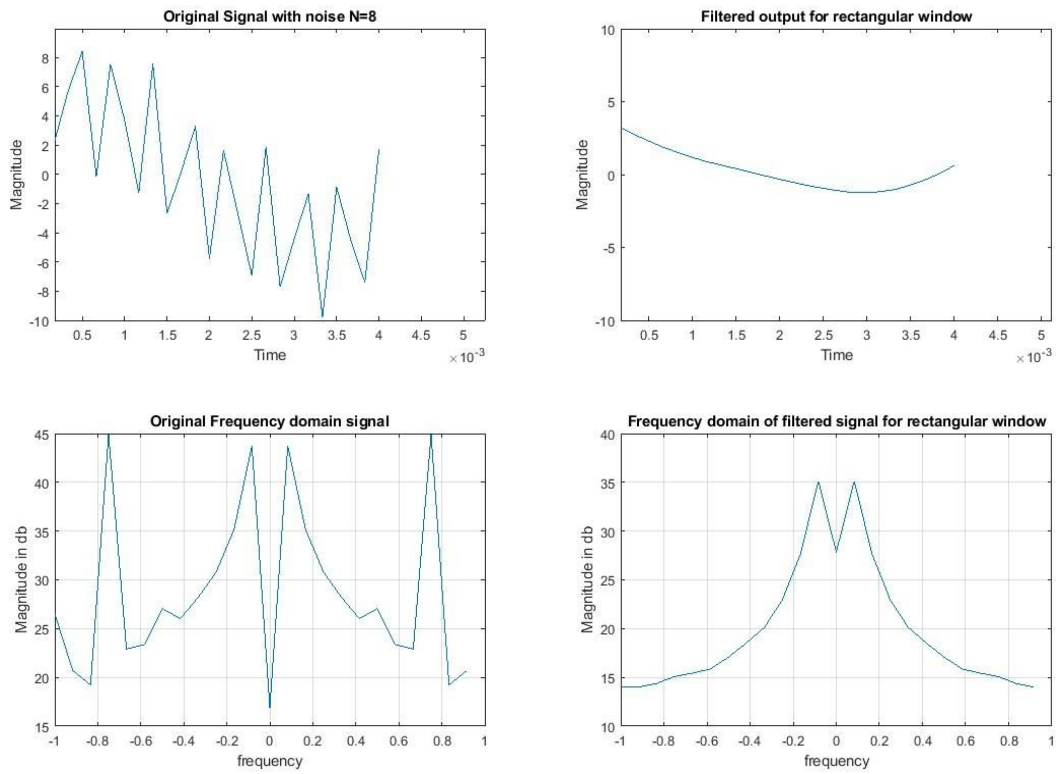
Observations:

1) From the frequency response of different filters, I observed that each filter successfully suppressed the stop band frequency while pass band frequency components remain the same.
2) From the time response of the signal, we can easily see that LPF eliminates the high frequency component, resulting in a smooth signal at the output.
3) From the graph, it is difficult to tell the difference between the window as in case of N=512, passband was almost straight with high roll-off (see Part A), which results in similar results as there is significant difference between passband and stopband frequency.
4) If we place stopband and passband frequency close to each other then we can easily observe the effect of different windows on filter.

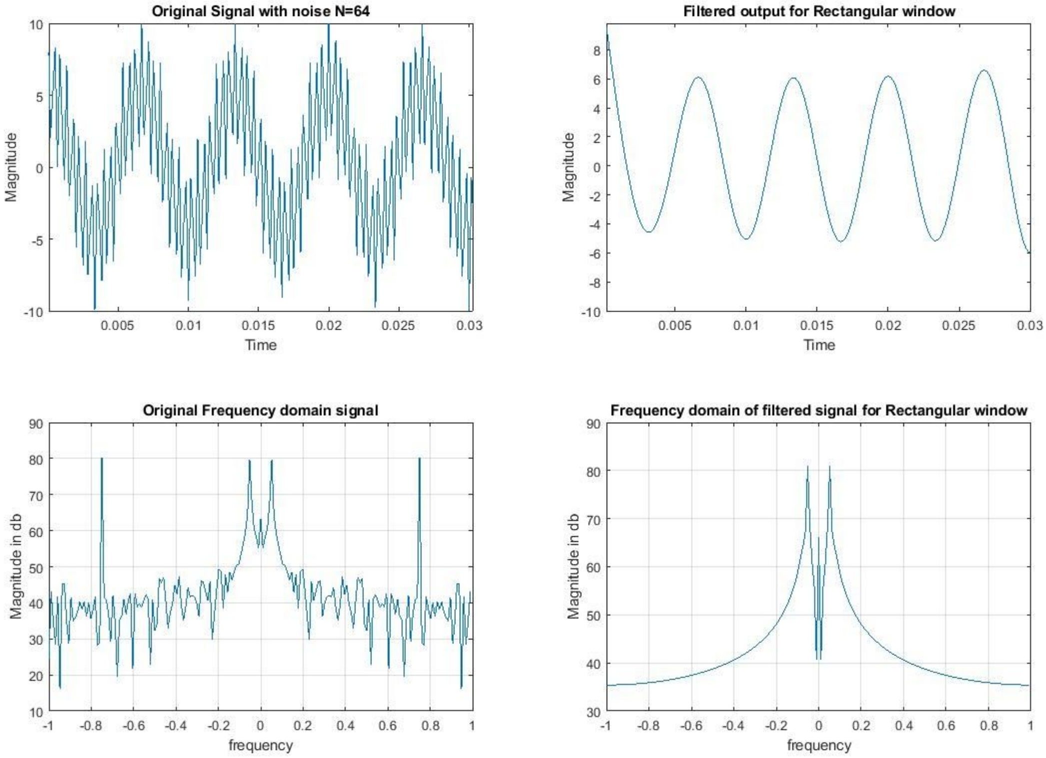
In the second part of the experiment, I added the noise to the x[n], to observe its effect on filtering.

Results:

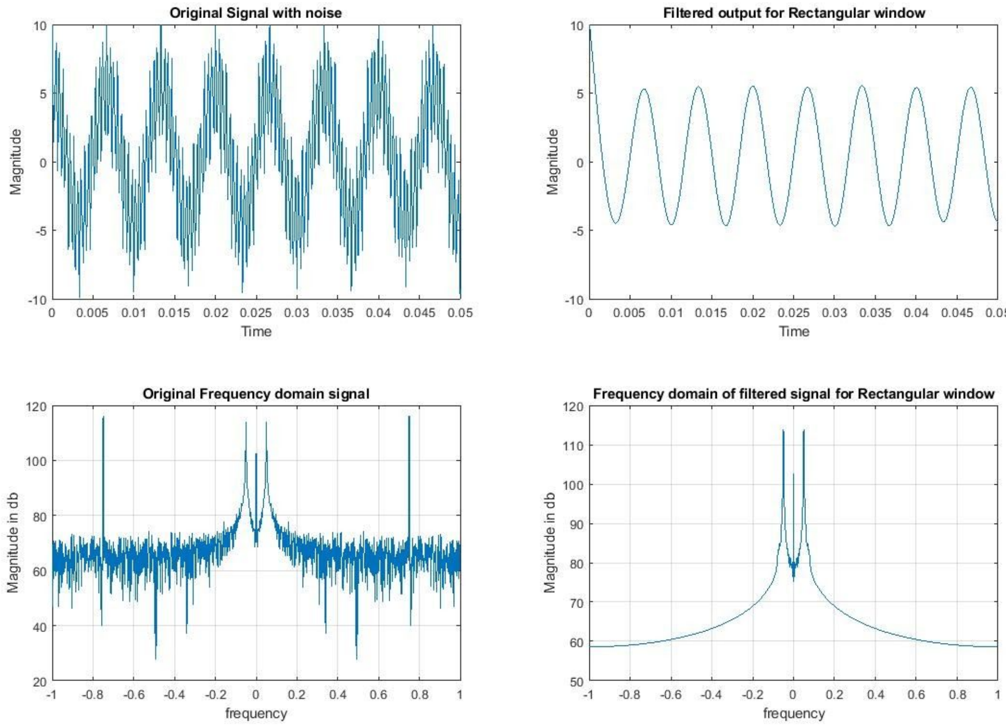
a) **Using Rectangular Window for different N**



N=8


Original Signal with noise N=64
Magnitude
Time
Filtered output for Rectangular window
Magnitude
Time
Original Frequency domain signal
Magnitude in db
frequency
Frequency domain of filtered signal for Rectangular window
Magnitude in db
frequency

N=64


Original Signal with noise
Magnitude
Time
Filtered output for Rectangular window
Magnitude
Time
Original Frequency domain signal
Magnitude in db
frequency
Frequency domain of filtered signal for Rectangular window
Magnitude in db
frequency

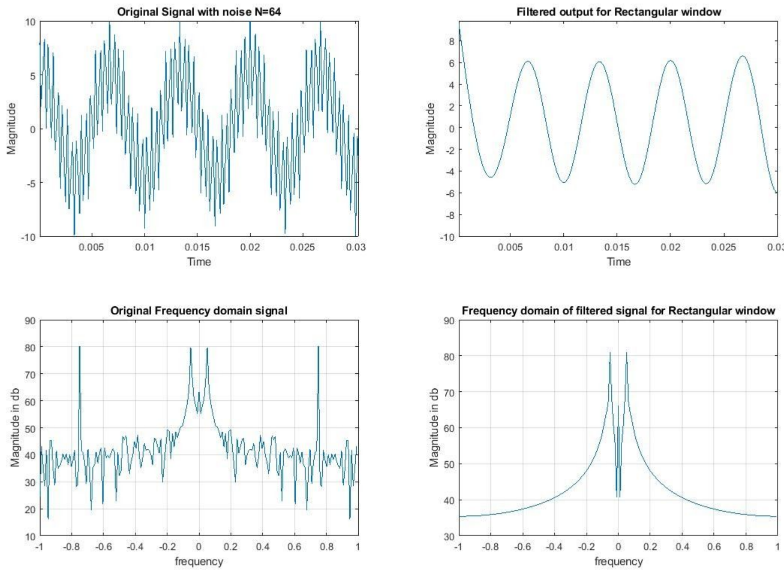
N=512

Observation Table:

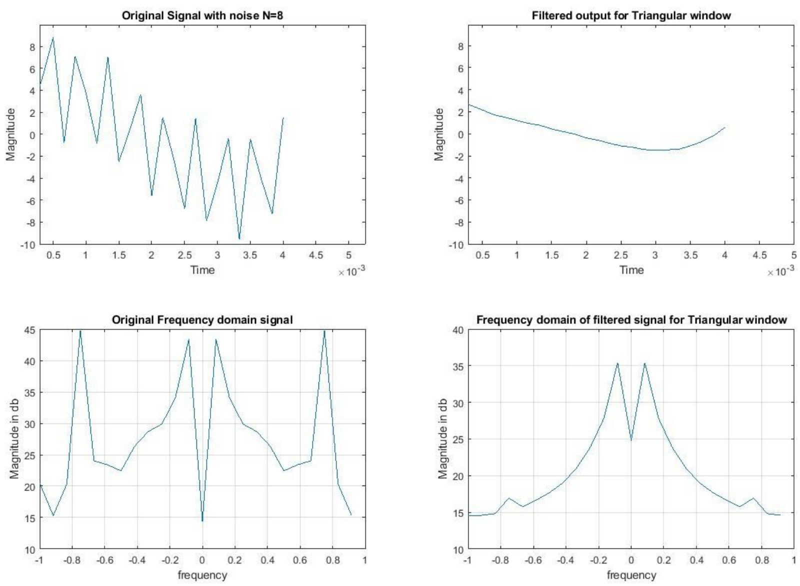
Rectangular Window			
N	**Input SNR**	**Output SNR**	**SNR Gain (Output - Input)**
8	2.99	3.271	0.719
64	2.01	4.553	2.543
512	1.32	3.662	2.342

b) <u>Using Triangular Window for different N</u>

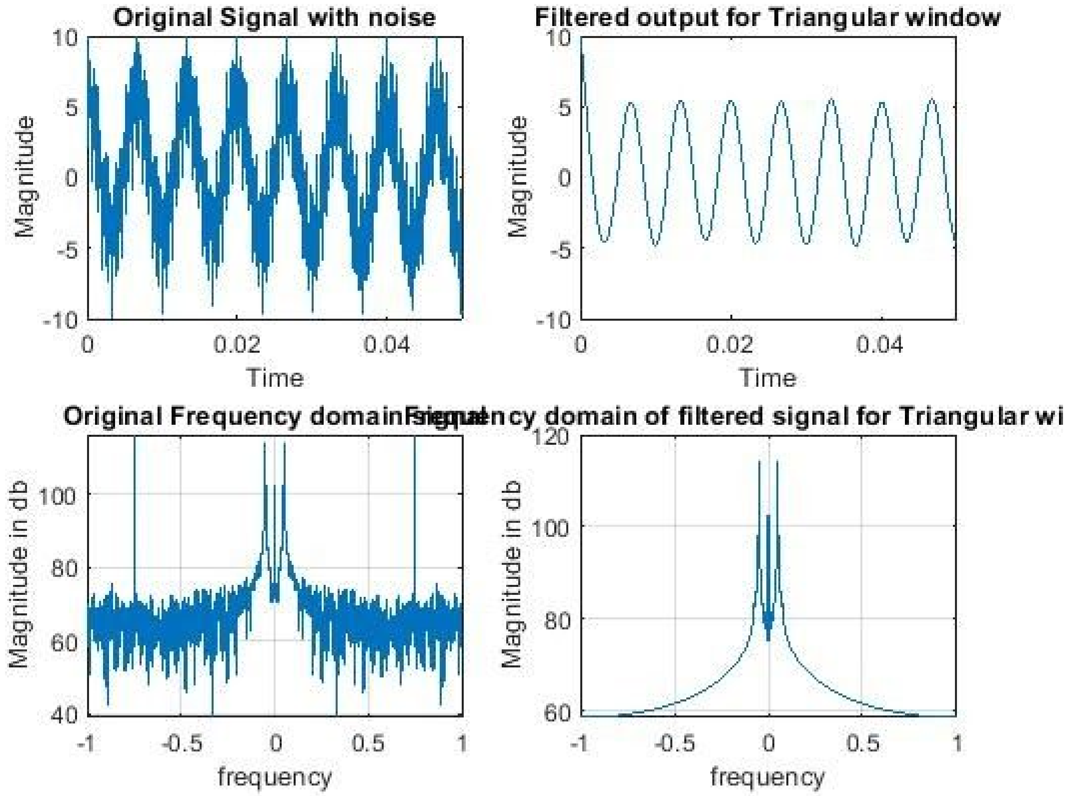
<u>N=64</u>



N=8



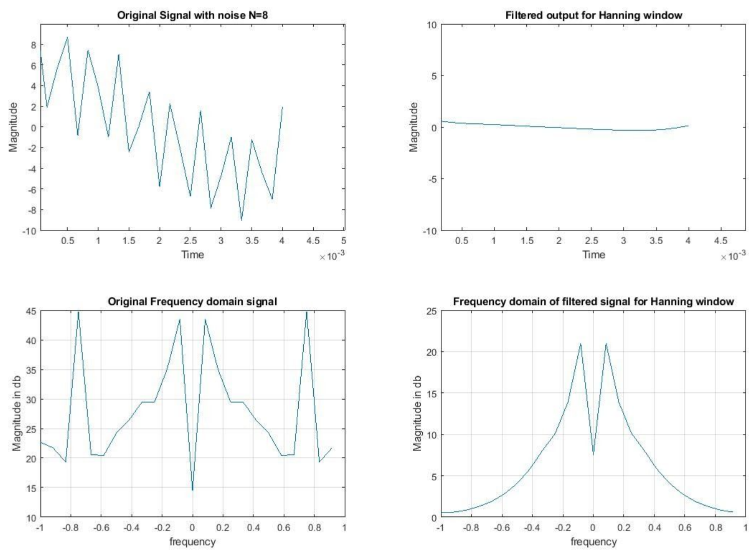
N=512



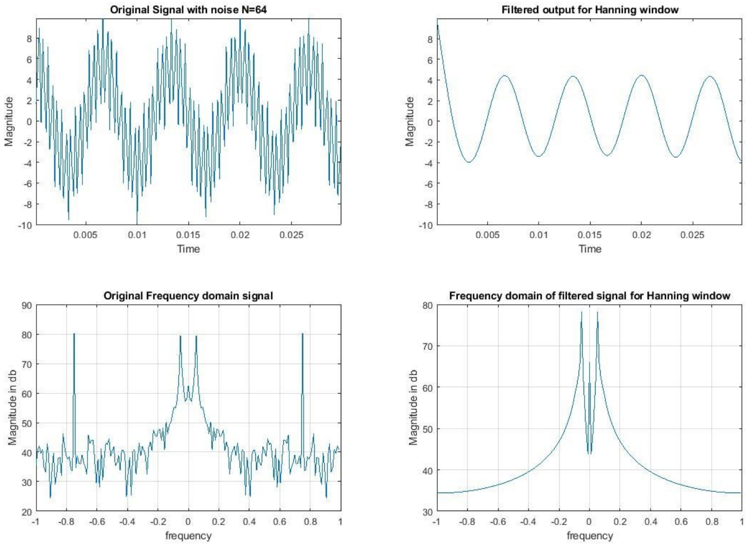
Observation Table:

Triangular Window			
N	**Input SNR**	**Output SNR**	**SNR Gain (Output - Input)**
8	2.99	5.704	2.714
64	2.01	4.547	2.537
512	1.32	4.856	3.536

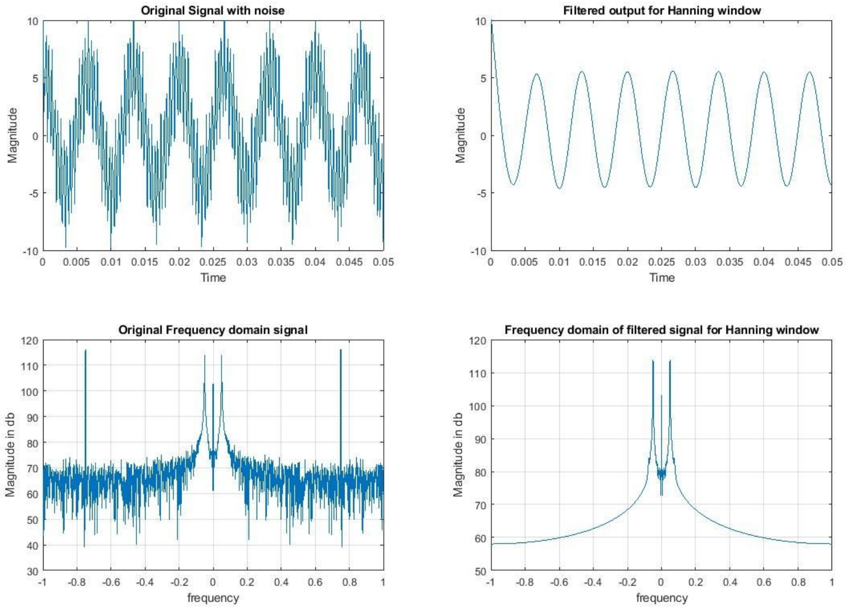
c) **<u>Using Hanning Window for different N</u>**
<u>N=8</u>



N=64



N=512

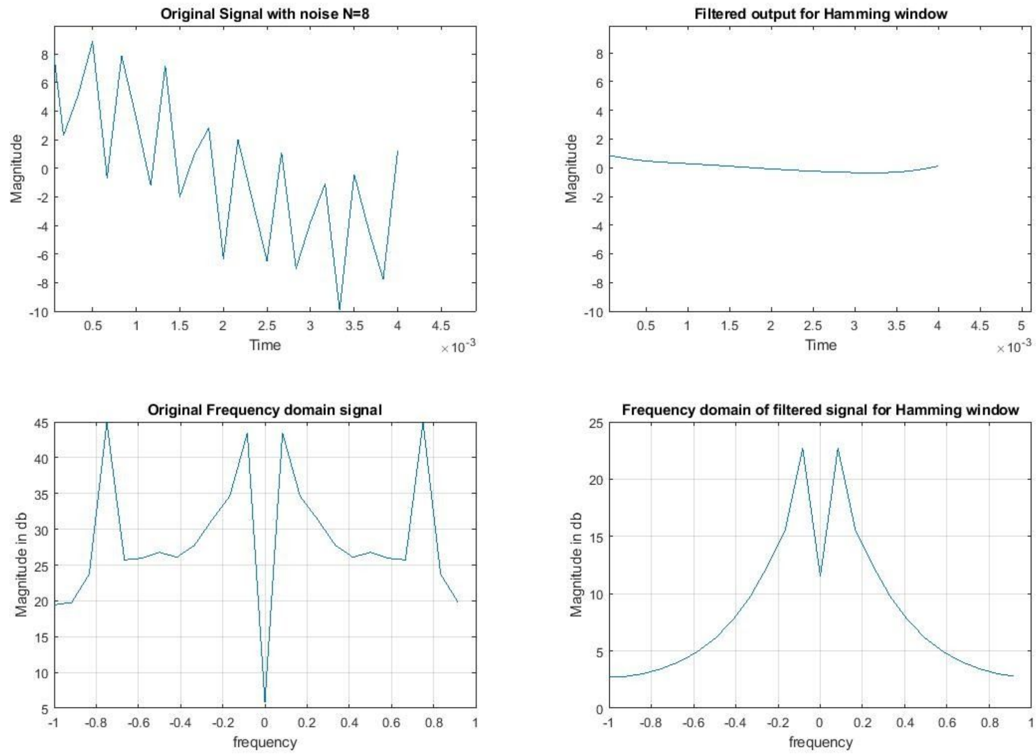


Observation Table:

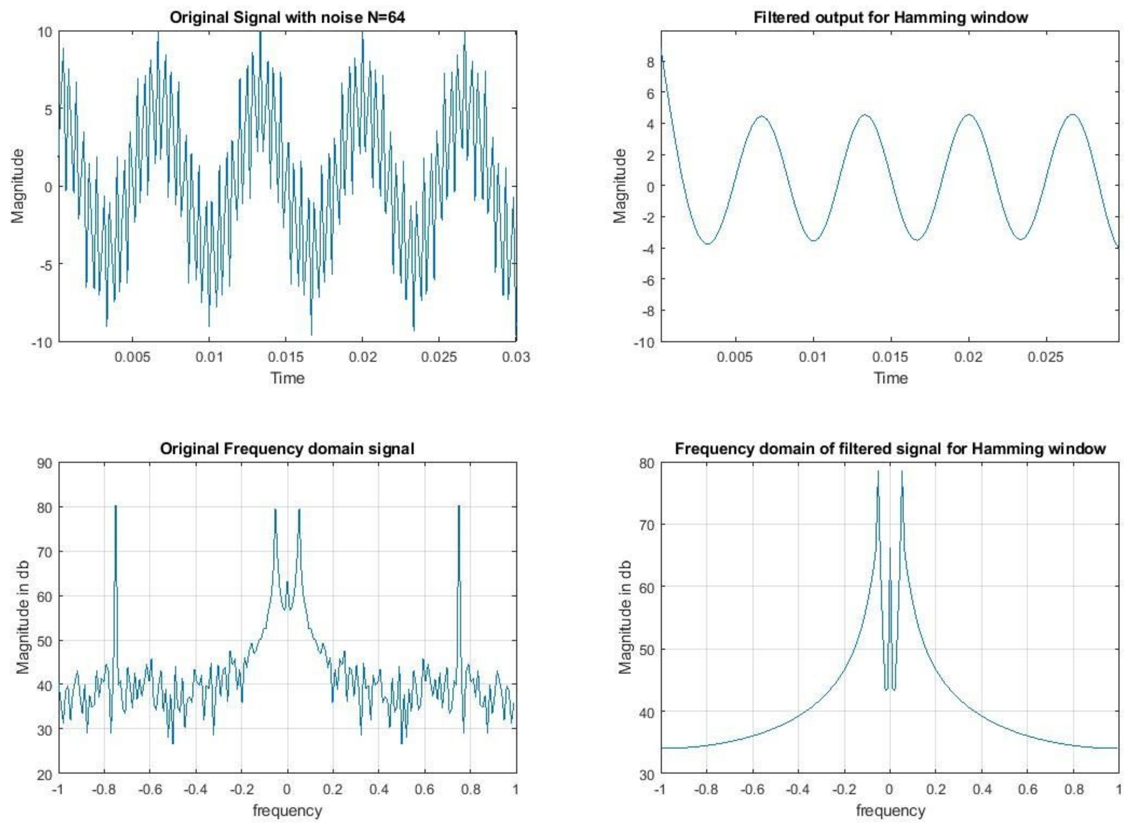
Hanning Window			
N	**Input SNR**	**Output SNR**	**SNR Gain (Output - Input)**
8	2.99	5.885	2.895
64	2.01	4.666	2.656
512	1.32	4.890	3.570

d) Using Hamming Window for different N

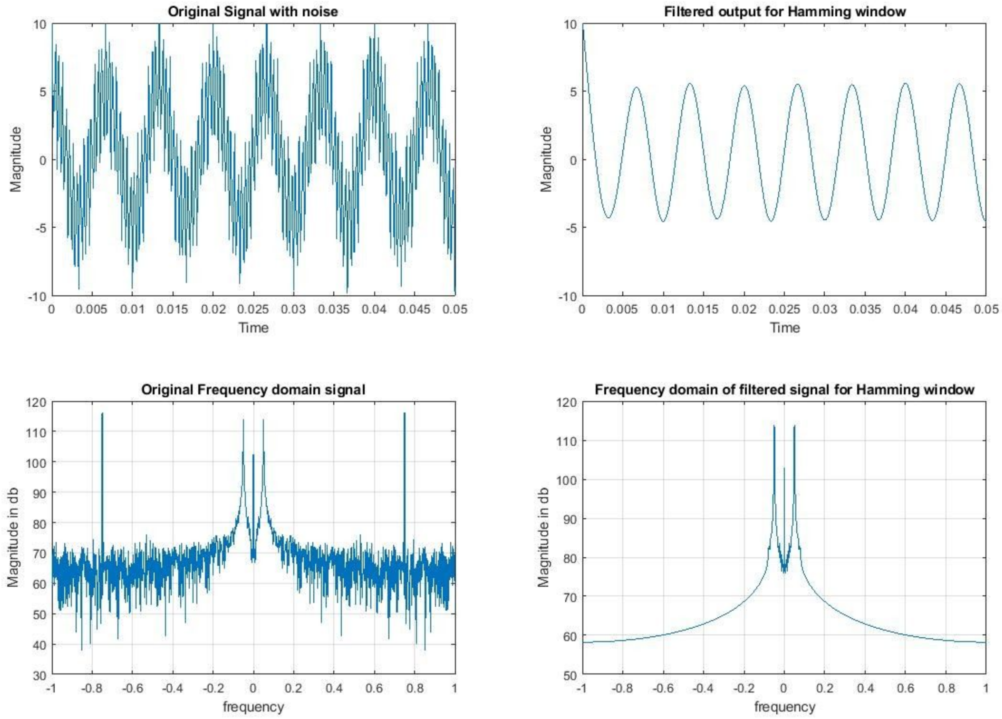
N=8



N=64



N=512

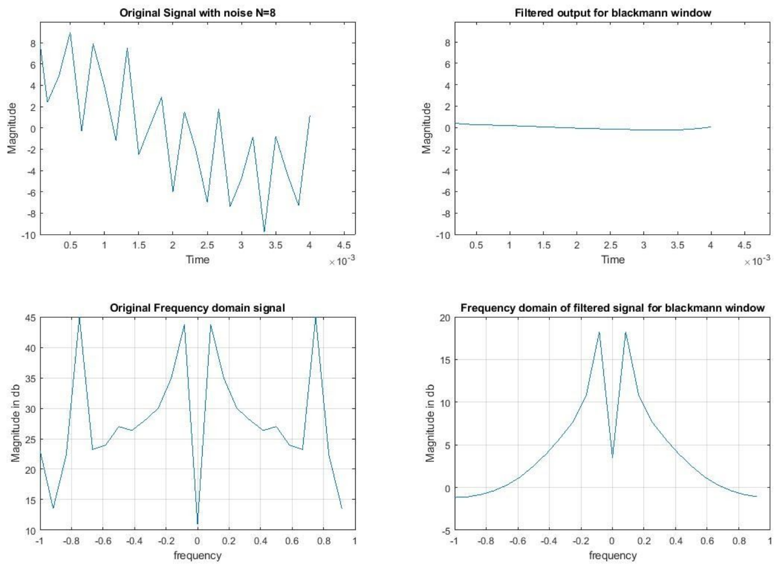


Observation Table:

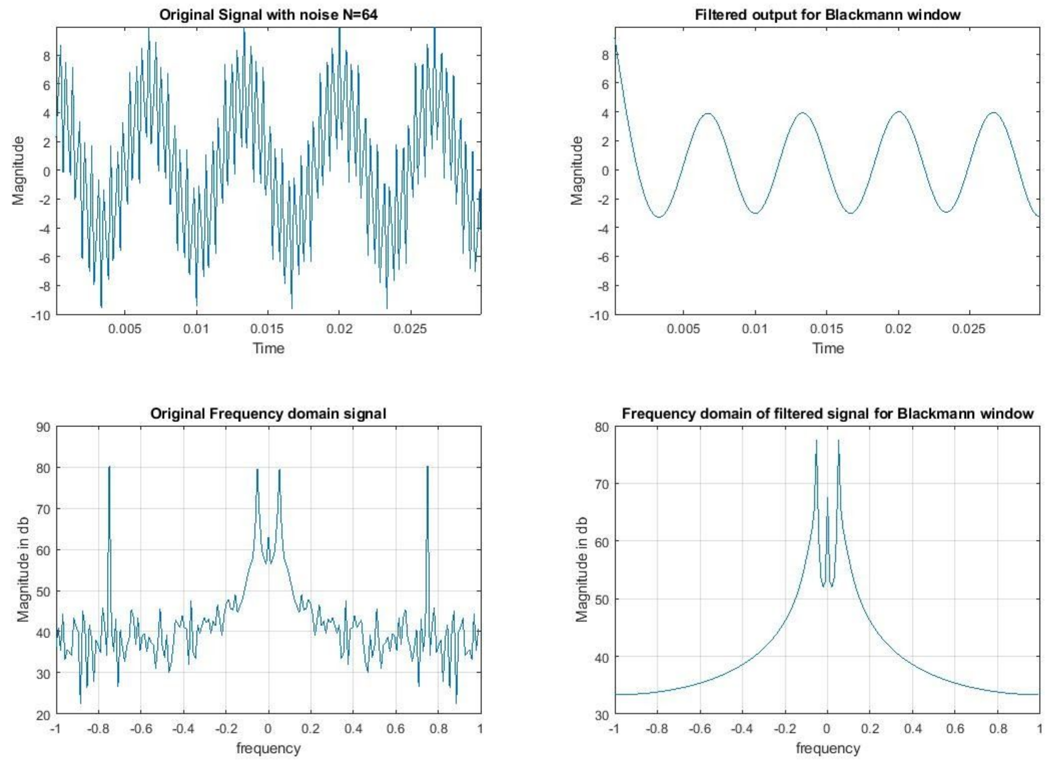
Hamming Window			
N	**Input SNR**	**Output SNR**	**SNR Gain (output - input)**
8	2.99	5.875	2.885
64	2.01	4.665	2.655
512	1.32	4.888	3.568

e) Using Blackman Window for different N

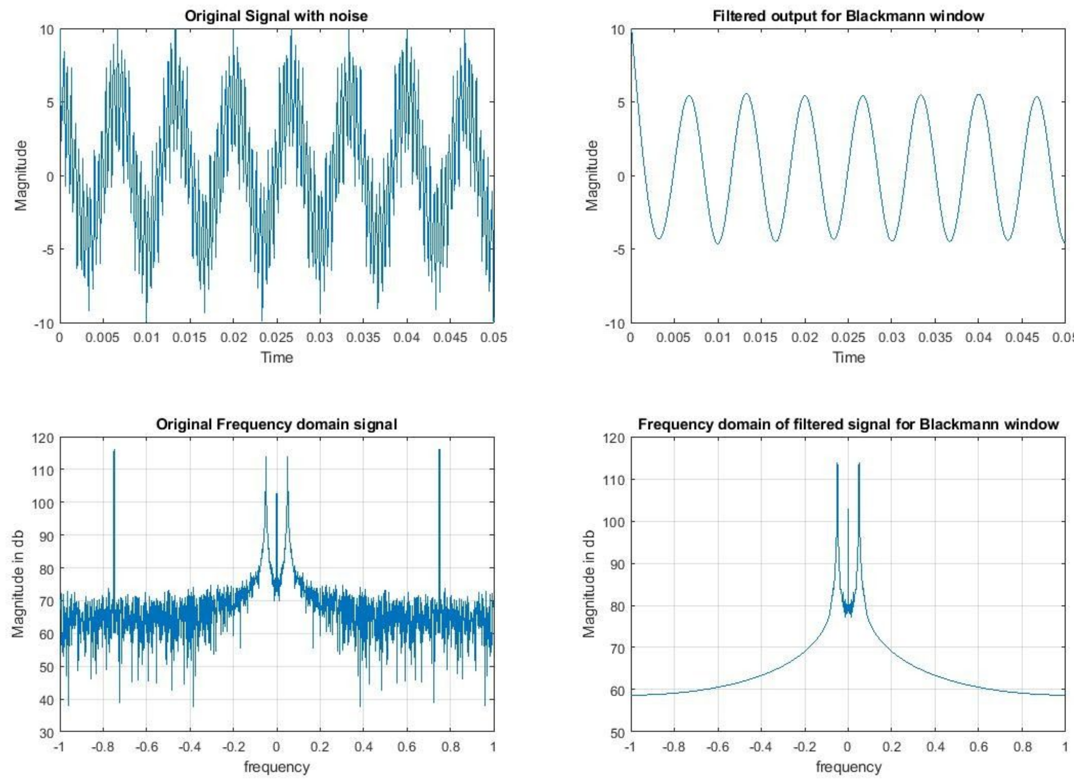
N=8



N=64


Original Signal with noise N=64
Magnitude
Time
Filtered output for Blackmann window
Magnitude
Time
Original Frequency domain signal
Magnitude in db
frequency
Frequency domain of filtered signal for Blackmann window
Magnitude in db
frequency

N=512


Original Signal with noise
Magnitude
Time
Filtered output for Blackmann window
Magnitude
Time
Original Frequency domain signal
Magnitude in db
frequency
Frequency domain of filtered signal for Blackmann window
Magnitude in db
frequency

Observation Table:

Blackman Window			
N	**Input SNR**	**Output SNR**	**SNR gain (output - input)**
8	2.99	5.889	2.899
64	2.01	4.510	2.500
512	1.32	4.899	3.579

Discussions:

- The noise is added using rand () function, which gives uniformly distributed values between 0 and 1, with mean 0.5. Thus to observe the effect of noise in the passband, it was normalized by adding A*(rand () – 0.5) to the signal, A being the noise amplitude.

- SNR is observed to increase for the filtered output as compared to the input signal, because noise removal capacity of different filters are different.

- The gain in SNR (Output – Input) is always highest for N=512, as with larger N, the filter's sharpness improves (see partA), so the noise filtering action improves.

- From the plots, I observed that the random spikes appearing in the input signal due to noise are removed effectively in the output, as output signal was more smooth and thus designed filter have high noise rejection capacity.

This link contains the original Matlab code for the experiment:
https://drive.google.com/drive/folders/1991oUyEI_mlqwBgSeDIaHZq1H2arPRZW?usp=sharing

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