

Exercise 1.3.46: Forbidden triple for stack generability. Prove that a permutation can be generated by a stack (as in the previous question) if and only if it has no forbidden triple (a, b, c) such that $a \prec b \prec c$ with c first, a second, and b third (possibly with other intervening integers between c and a and between a and b).

solution: Suppose that there is a forbidden triple (a, b, c) . Item c is popped before a and b , but a and b are pushed before c . Thus, when c is pushed, both a and b are on the stack. Therefore, a cannot be popped before b .

- Numbers are pushed onto stack in ascending order.
- stacks are LIFO (Last-in-first-out), so items must be popped in descending order.
- If $a < b$, " a " cannot be above " b " on the stack.
- Therefore, a permutation would not exist when a forbidden triple exists as it contradicts the structure of a stack.

The point c is needed so that after a pop, you can verify if order of a and b are correct (ascending) for a stack. Otherwise, it's hard to tell.

