Exercise 4.2.41: Add length directed cycle. Design a linearithmic algorithm to determine whether a digraph has an odd-length directed cycle

Solution:

- Digraph has odd-length cycle iff one or more of its SCC is nonbipartite.
- Recall bipartite is a graph whose vertices can be divided into 2 sets such that all edges connect a vertex in 1 set with a vertex in the other.
- We can run Kosaraju Sharirs SCC algorithm to create a SCC. This algorithm uses DFS once on reverse graph to calculate reverse post-order. It then uses this to determine which nodes to run DFS on for original graph to compute/create the SCC(Strongly connected components).
- If an edge $v \rightarrow w$ is pointing wrong direction, we can replace it with an odd-length path that is pointing in opposite direction.
- If path has odd length, then we replace edge $v \rightarrow w$ path. If even-length, then path combined with $v \rightarrow w$ for odd-cycle formation.