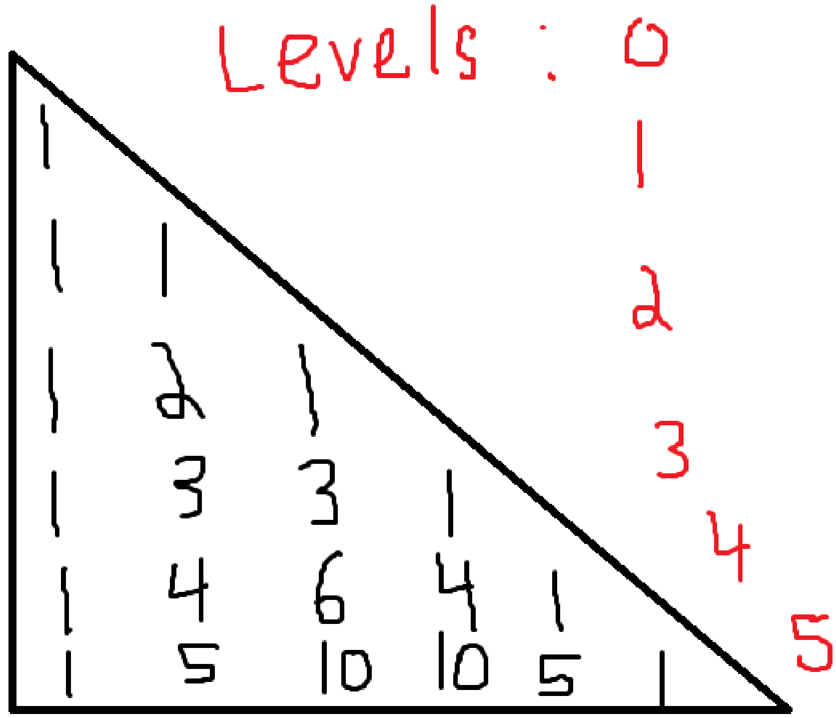
1.5.15: Binomial trees. Show that the number of nodes at each level in the worst-case trees for weighted quick-union are binomial coefficients. Compute the average depth of a node in a worst-case tree with N = 2n nodes.

Solution: Recall binomail coeffs have each successive row created by taking the sum of 2 points before it



For a tree with $k = 2^n$ nodes, there are $n + 1$ levels.
Notice that the sum of each level corresponds to 2^n where n represents the level. For example; $n = 0$ has a sum of 1, $n = 4$ has sum of 16 since 2^4. $n = 5$ has sum of 32 since 2^5. This is single depth.

By summing all levels until $n = 4$, we get 32 as total node depth. Recall single level nodes of $n = 4$ was 16 node depth.

Therefore, *Avg node depth*: $\frac{\textit{single node depth}}{\textit{total node depth}}$.
For example; $\frac{16}{32}$ results in $\frac{1}{2}$.

So total node depth increases by 2 each time.

We can also prove this by induction. *Avg node depth*: $\frac{n+1}{2}$