By Anando Zaman

Exercise 4.2.10:: What is the maximum number of edges in a digraph with V vertices and no parallel edges? What is the minimum number of edges in a digraph with V vertices, none of which are isolated?

Solution:

Yes, reverse Post-Order DAG(Topological Sort). Basically DFS but push output onto stack instead of print.

- There are 3 cases, Proposition F:
- dfs(w) has already been called and has returned (w is marked).
- dfs(w) has not yet been called (w is unmarked), so $v \to w$ will cause dfs(w) to be called (and return), either directly or indirectly, before dfs(v) returns.
- dfs(w) has been called and has not yet returned when dfs(v) is called. The key to the proof is that this case is impossible in a DAG, because the recursive call chain implies a path from w to v and $v \to w$ would complete a directed cycle.

The third case has a cycle becayse we see dfs(w) but has not yet been returned, meaning w is still in queue/stack as it was not yet popped/marked, meaning a cycle can occur as not put in marked list yet.