

LALR(1) parser

(lookahead LR parser).

Algorithm: An easy, but space consuming LALR table construction.

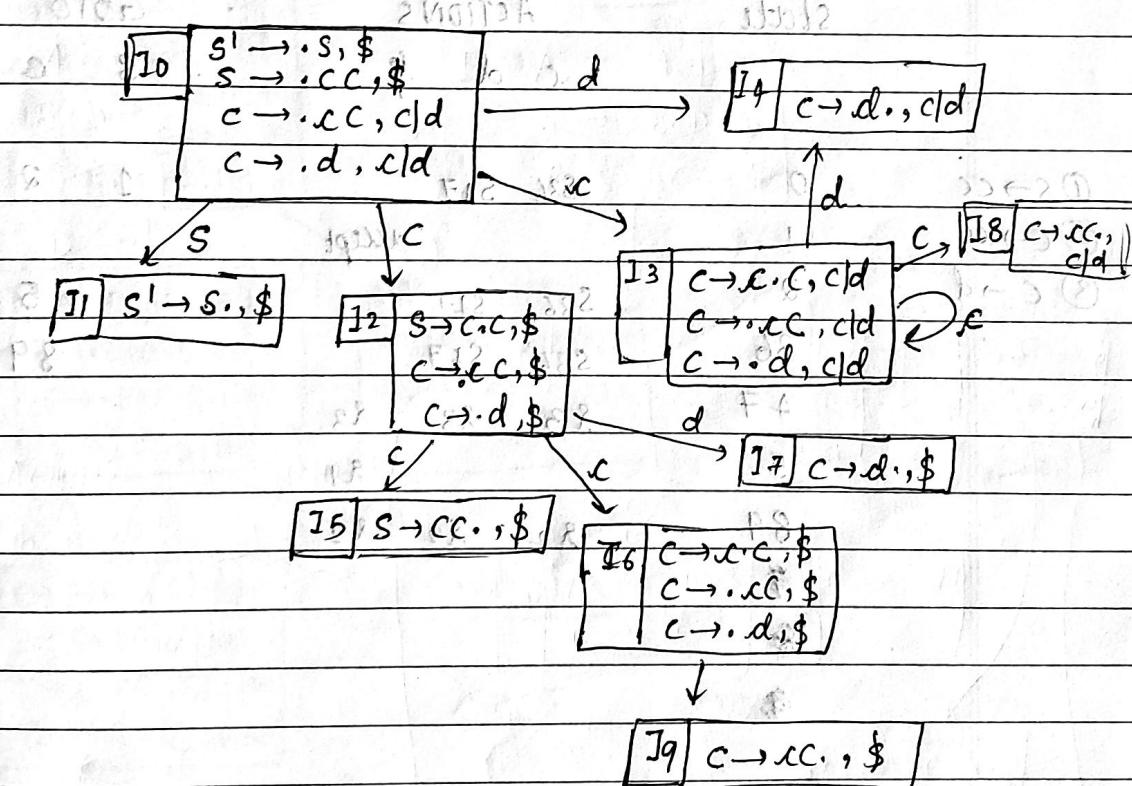
i)  $C = \{ J_0, J_1, \dots, J_n \}$ 

ii) For each core present in LR(1) set of items,

$$S \rightarrow CC$$

$$C \rightarrow CC/d$$

$J = I_1 \cap I_2 \cap \dots \cap I_n$ , merge the set of item whose first right senential form is same.

i) DO  $LR(1) \Rightarrow CLR$ 

Find two states whose first right senential form are same.

like:

J3 &amp; J6, J8 &amp; J9, J4 &amp; J7.

$I_3 \& I_6 \Rightarrow \text{Merge} \Rightarrow$	$I_36$
	$C \rightarrow C \cdot C, C d \$$
	$C \rightarrow \cdot C C, C d \$$
	$C \rightarrow \cdot d, C d \$$

$I_4 \& I_7 \Rightarrow \text{Merge} \Rightarrow$	$I_{47}$
	$C \rightarrow d, C d \$$

$I_8 \& I_9 \Rightarrow \text{Merge} \Rightarrow$	$I_{89}$
	$C \rightarrow CC, C d \$$

parse table.

	state	ACTIONS	GOTO
		$c \cdot d \cdot \$$	$\$ \cdot C$
① $S \rightarrow CC$	0	$S_{36} \quad S_{47}$	1 2
② $C \rightarrow CC$	1	accept	
③ $C \rightarrow d$	2	$S_{36} \quad S_{47}$	5
	36	$S_{36} \quad S_{47}$	89
	47	83 83 83	
	50	81	
	89	82 82 82	
	87		
	85		

\*  $E \rightarrow E+E \mid E * E \mid id.$

→ no common states, it is same so, as CLR

I0	$E' \rightarrow \cdot E, \$$ $E \rightarrow \cdot E+E, \$$ $E \rightarrow \cdot E * E, \$$ $E \rightarrow \cdot id, \$$ $E \rightarrow \cdot E+E, +$ $E \rightarrow \cdot E * E, *$ $E \rightarrow \cdot id, +$ $E \rightarrow \cdot E+E, *$ $E \rightarrow \cdot E * E, -$ $E \rightarrow \cdot id, -$
----	--

I0	$E' \rightarrow \cdot E, \$$ $E \rightarrow \cdot E+E, \$ \mid + \mid *$ $E \rightarrow \cdot E * E, \$ \mid + \mid *$ $E \rightarrow \cdot id, \$ \mid + \mid *$
----	--

can be written as

J1	$E' \rightarrow E \cdot \$$ $E \rightarrow E \cdot + E, \$$ $E \rightarrow E \cdot * E, \$$ $E \rightarrow E \cdot + E, +$ $E \rightarrow E \cdot * E, +$ $E \rightarrow E \cdot + E, *$
----	---

J2	$E \rightarrow id \cdot \$$ $E \rightarrow id \cdot +$ $E \rightarrow id \cdot *$
----	---

J2	$E \rightarrow id \cdot \$ \mid + \mid *$
----	---

J3	$E \cdot + E, \$ \mid + \mid *$ $E \rightarrow E+E, \$ \mid + \mid *$ $E \rightarrow E * E, \$ \mid + \mid *$ $E \rightarrow id, \$ \mid + \mid *$
----	---

J4	$E \cdot * E, \$ \mid + \mid *$ $E \rightarrow \cdot E+E, \$ \mid + \mid *$ $E \rightarrow \cdot E * E, \$ \mid + \mid *$ $E \rightarrow \cdot id, \$ \mid + \mid *$
----	---

J5	$E \rightarrow E * E \cdot \$ \mid + \mid *$ $E \rightarrow E \cdot + E, \$ \mid + \mid *$ $E \rightarrow E \cdot * E, \$ \mid + \mid *$
----	--

J6	$E \rightarrow E+E \cdot \$ \mid + \mid *$ $E \rightarrow E \cdot + E, \$ \mid + \mid *$ $E \rightarrow E \cdot * E, \$ \mid + \mid *$
----	--

J7	$E \rightarrow id \cdot \$ \mid + \mid *$
----	---

status      ACTIONS      GOTO

+ \* id \$

E

0            S2            1

1            S3 S4        accept

2            R3 R3        R3

3            S2            5

4            S2            6

5            S3/R1 S4/R1    R1

6            S3/R2 S4/R2    R2

Viable prefix: (valid prefix)

prefixes of right sentential form can appear on the stack of a SRP (Shift Reduce parser):

Ex:

Grammar:

$$E \rightarrow E + T \mid T$$

$$T \rightarrow T * F \mid F$$

$$F \rightarrow (E) \mid id$$

$$E \xrightarrow{hi} F * id \Rightarrow (E) * id$$

and

handle = (E)

except handle all the previous items are viable prefixes

except the handle

∴ Viable prefix = { (, (E) }

Viable prefix

NOTE: can't include \$

Q.T.P

INPUT

TRANS

\$ hi + id

ER

E

id

ER

E

id

ER

E

id

ER

E

id

Unit-1Introduction to AutomataDFA : Deterministic Finite Automata

\* consists of 5-tuple machine

$$(Q, \Sigma, \delta, q_0, F)$$

transition fn (q x Σ)

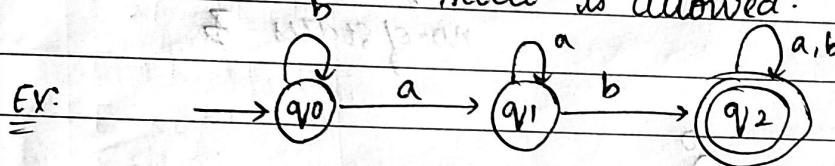
finite no. of states in building the machine

characters, symbol ip required to build the machine

initial state

Final state

\* only one deterministic path from one state to another state is allowed.



$$\Sigma = \{a, b\}$$

$$Q = \{q_0, q_1, q_2\}$$

$$\delta(q_0, a) = q_1$$

$$\delta(q_1, b) = q_2$$

$q_0 \rightarrow$  initial state / start state

$q_2 \rightarrow$  final state

Transition table

Present state	Next state for input a	Next state of input b
$\rightarrow q_0$	$q_1$	$q_0$
$q_1$	$q_1$	$q_2$
$* q_2$	$q_2$	$q_2$

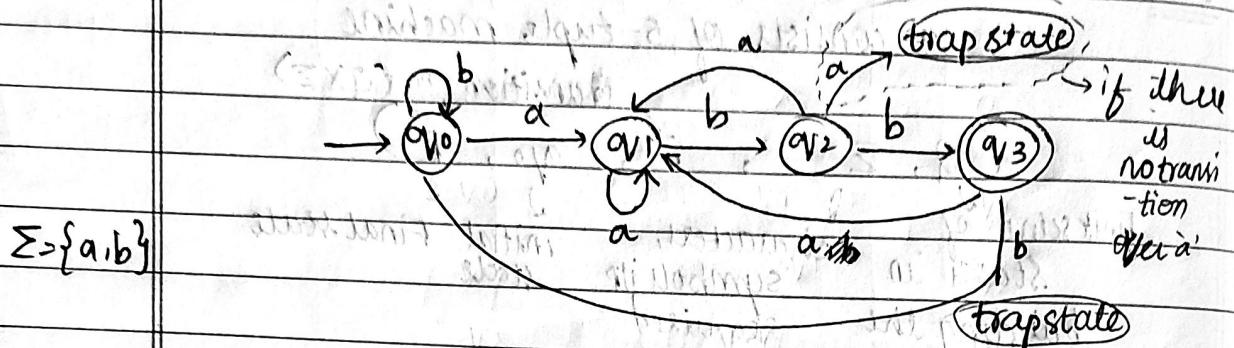
\* Regex:  $(a|b)^* abb = \{ abb, aabb, babb, \dots \}$

length = 3 (min)

so, atleast 4 states  
should exist

Condition: ends with "abb" or  
can be "abb" alone

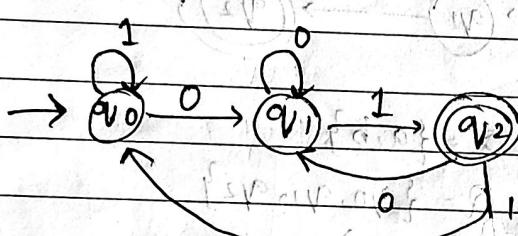
construct the DFA:



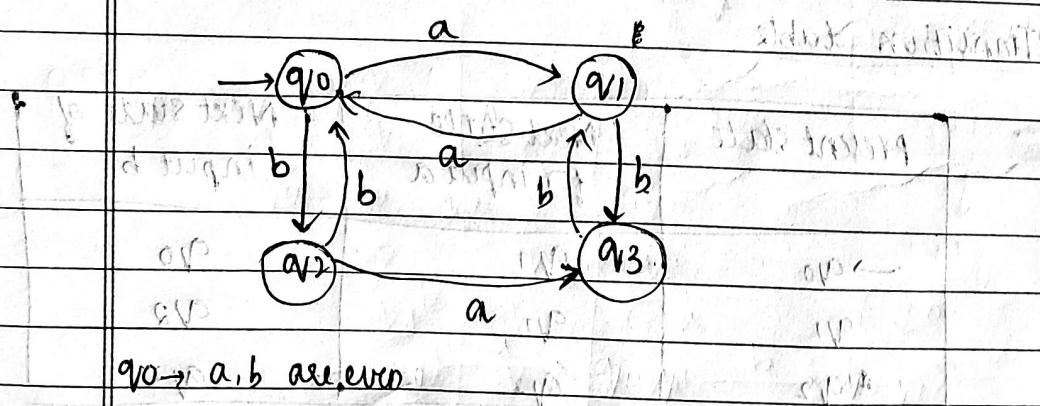
\* Regex:  $(0|1)^* 01 = \{ 001, 101, 0001, \dots \}$

min length = 2

no. of states = 3



\* Accept even a's & even b's including empty string.  
 $= \{ \text{empty}, aaabb, bbba, abba, baab, \dots \}$



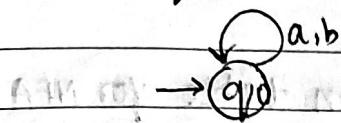
$q_0 \rightarrow a, b \text{ are even}$

$q_1 \rightarrow \text{odd } a's, \text{ even } b's$

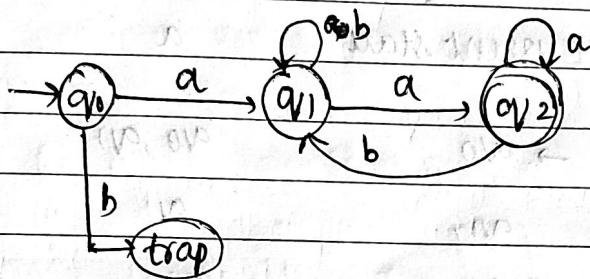
$q_2 \rightarrow \text{even } a's, \text{ odd } b's$

$q_3 \rightarrow \text{odd } a's, \text{ odd } b's$

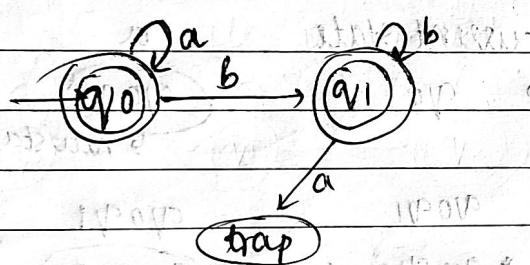
\* construct DFA for  $(a|b)^*$



\* construct DFA for  $a(a|b)^*a$



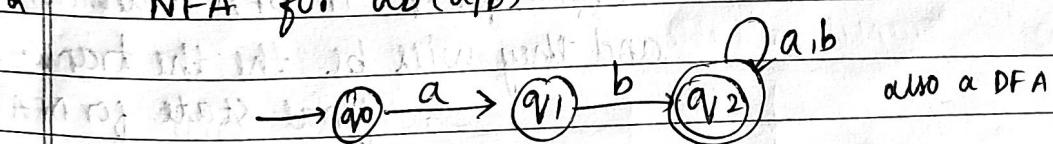
\* construct DFA for  $a^*b^* = \{ \epsilon, a, b, ab, bb, \dots \}$



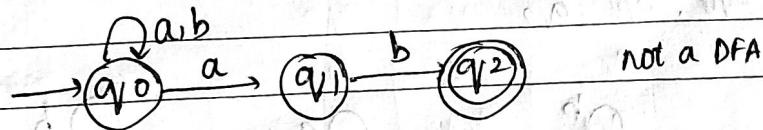
NFA (Non-Deterministic Finite Automata)

5 tuple  $(Q, \Sigma, \delta, q_0, F)$

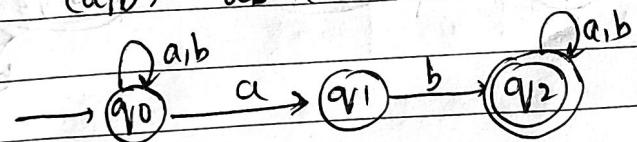
\* NFA for  $ab(a|b)^*$



\* NFA for  $(a|b)^*ab$



\* NFA for  $(a|b)^*ab(a|b)^*$



## \* Conversion of NFA to DFA

Step 2: Draw the transition table for NFA

For previous example:

current state	a	b
$\rightarrow q_0$	$q_0, q_1$	$q_0$
$q_1$	$q_1$	$q_2$
$* q_2$	$q_2$	$q_2$

Step 2: construct transition table for DFA

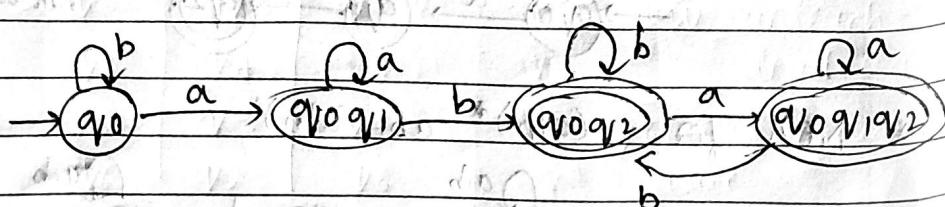
current state	a	b
$\rightarrow q_0$	$q_0, q_1$	$q_0$
$q_0, q_1$	$q_0, q_1$	$(q_0, q_2) \rightarrow$ new state so include
$* q_0, q_2$	$q_0, q_1, q_2$	$q_0, q_2$ new state, so include

check in NFA,  $q_2$  was final state

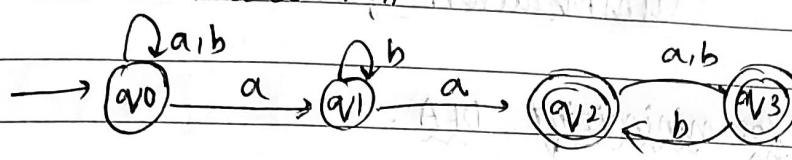
check for  $q_2$  in DFA transition table  
and they will be the final state for DFA

Step 3: construct DFA

$q_0, q_1, q_2$



\* Given NFA convert to DFA



NFA:

current state

$\rightarrow q_0$

$q_1$

\*  $q_3$

$q_0 q_1$

$q_2$

-

$q_0$

$q_1$

$q_2$

~~$q_3$~~

DFA:

current state

a      b

$q_0 q_1$

$q_0$

$q_0 q_1$

$q_0 q_1 q_2$

$q_0 q_1$

\*  $q_0 q_1 q_2$

$q_0 q_1 q_2 q_3$

$q_0 q_1 q_3$

\*  $q_0 q_1 q_2 q_3$

$q_0 q_1 q_2$

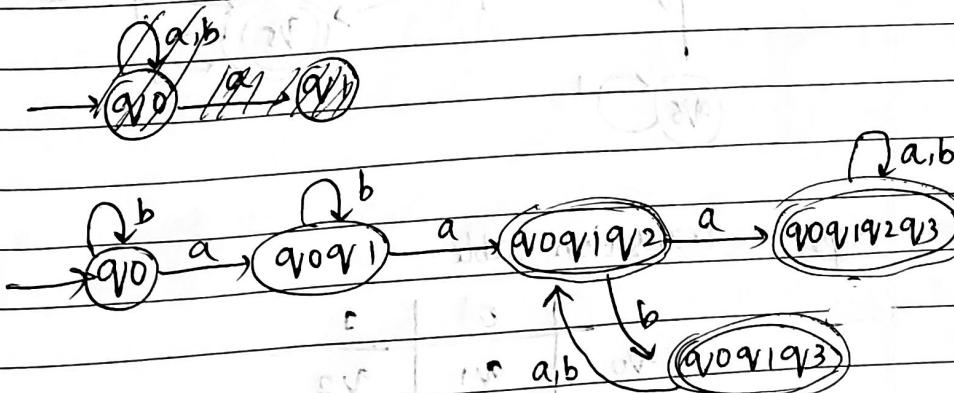
$q_0 q_1 q_3 q_2$

\*  $q_0 q_1 q_3$

$q_0 q_1 q_2$

$q_0 q_1 q_2$

DFA construction:

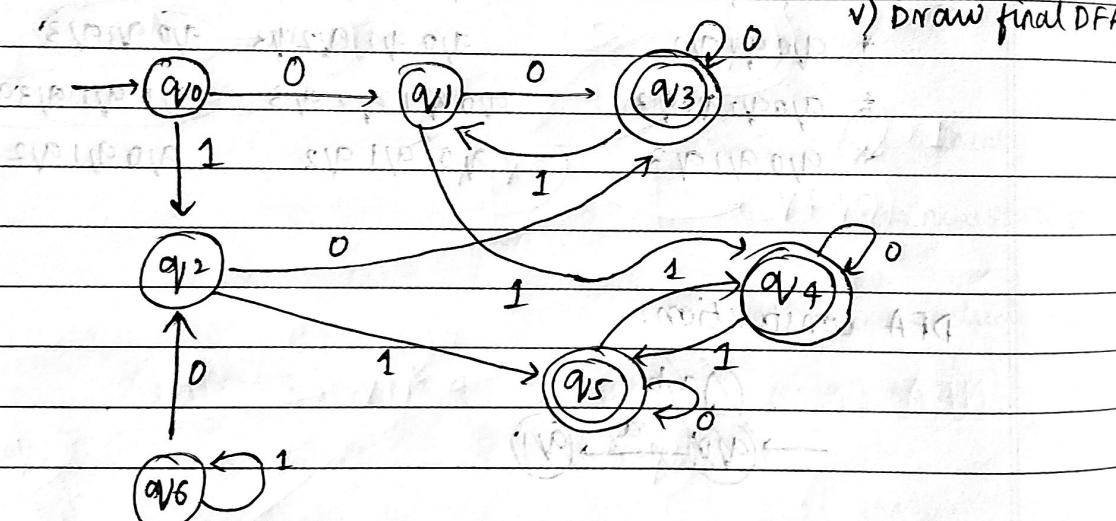


## Minimization of DFA

Steps for minimizing DFA:

- i) Remove the unreachable / dead state.
- ii) construction of the transition table for DFA with  $\Sigma$ , as set of alphabets.
- iii) Divide into equivalence classes i) states which are not final/accepting.  
ii) final/accepting states.
- iv) continue finding equivalence classes until no new ones are found.

Ex:



Step 2: Transition table

	0	1
$\rightarrow q_0$	$q_1$	$q_2$
$q_1$	$q_3$	$q_4$
$q_2$	$q_3$	$q_5$
$q_3$	$q_3$	$q_1$
$q_4$	$q_4$	$q_5$
$q_5$	$q_5$	$q_4$
* $q_6$	$q_2$	$q_6$

now, unreachable  
dead state.

$q_6 \rightarrow$  is unreachable  
state.

so, remove it.

Step 2: current transition table:

	0	1
$\rightarrow q_0$	$q_1$	$q_2$
$q_1$	$q_3$	$q_4$
$q_2$	$q_3$	$q_5$
$q_3$	$q_3$	$q_4$
$q_4$	$q_4$	$q_5$
$q_5$	$q_5$	$q_4$

Step 3:  $C_1$  (not final states)  $C_2$  (final states)

$$\{q_0, q_1, q_2\} \quad \{q_3, q_4, q_5\}$$

check for more equivalence classes,

check  $\{q_0, q_1, q_2\}$



i)  $\{q_0, q_1\}$  with 0 =  $q_1, q_3$

in  $C_1$  in  $C_2$

both in different classes

so discard

no, need to check with

$\{q_0, q_1\}$  with 1.

ii)  $\{q_0, q_2\}$  with 0 =  $q_1, q_3$

same problem so, discard

& no need to check  $\{q_0, q_2\}$  with 1.

iii)  $\{q_1, q_2\}$  with 0 =  $q_3, q_5$ , all belong to

$\{q_1, q_2\}$  with 1 =  $q_4, q_5$  same class  $C_2$

so, accept it.

if it can be written as  $\{q_0\} \{q_1, q_2\}$

$q_0$  as separate class bcz it didn't satisfy with 0 & 1.

Similarly for  $\{q_3, q_4, q_5\}$

Step 4:

$$\Rightarrow \{q_3\} \cup \{q_4, q_5\}$$

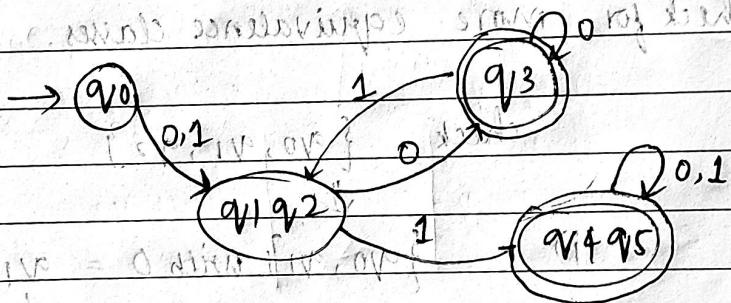
1st equivalence  $\Rightarrow \{q_0\} \{q_1, q_2\}, \{q_3\} \{q_4, q_5\}$

2nd equivalence  $\rightarrow$  further  $q_0, q_3$  can't be classified  
as they are isolated  
and remaining  $\{q_1, q_2\}$  &  $\{q_4, q_5\}$   
can't be further  
classified.

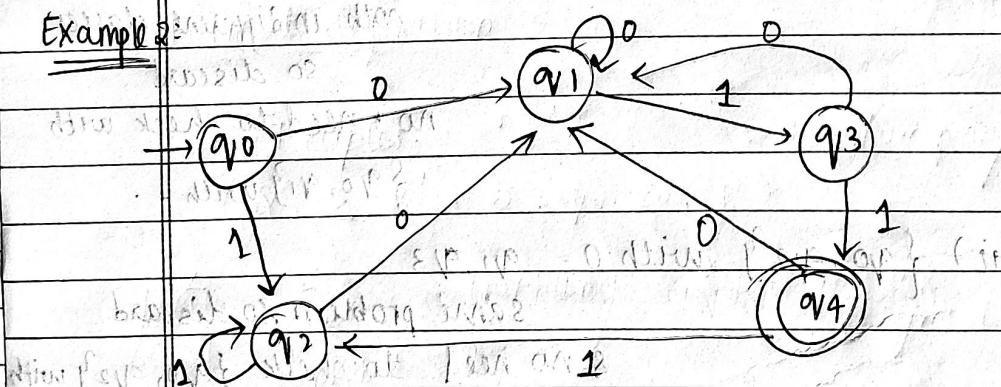
So, 3rd equivalence:  $\{q_0\}, \{q_1, q_2\}, \{q_3\}, \{q_4, q_5\}$

Step 5:

Draw finalized DFA



Example 2:



No dead/unreachable state

Step 4: Transition table:

		0	1
→ q0	q1	q2	
q1	q1	q3	q4
q2			
q3			
q4			

For initial form  $\{q_0\} \{q_1, q_2\} \{q_3\} \{q_4\}$ , no removal  
of any dead states

Step 3:

$$C_1 = \{q_0, q_1, q_2, q_3\}$$

$$C_2 = \{q_4\}$$

check  $q_0, q_3$  with 0  $\Rightarrow q_1, q_1$  } different classes  
 $q_0, q_3$  with 1  $\Rightarrow (q_2, q_4)$  so, discard

$q_1, q_3$  with 0  $\Rightarrow q_1, q_1$  } different classes  
 $q_1, q_3$  with 1  $\Rightarrow (q_3, q_4)$  so, discard.

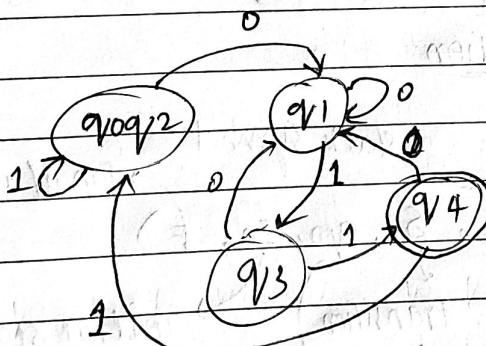
$q_2, q_3$  with 0 & 1  $\Rightarrow$  discard.

so,

1st equivalence :  $\{q_0, q_1, q_2\}, \{q_3\}, \{q_4\}$

2nd equivalence :  $q_0, q_1$  with 0  $= \{q_1, q_1\}$  } disc.  
 $q_0, q_1$  with 1  $= \{q_2, q_3\}$  -rd.

$q_1, q_2$  with 0  $= \{q_1, q_2\}$  } disc.  
 $q_1, q_2$  with 1  $= \{q_2, q_1\}$  -rd.  
 so, 2nd equivalence:  $\{q_0, q_2\}, \{q_1\}, \{q_3\}, \{q_4\}$



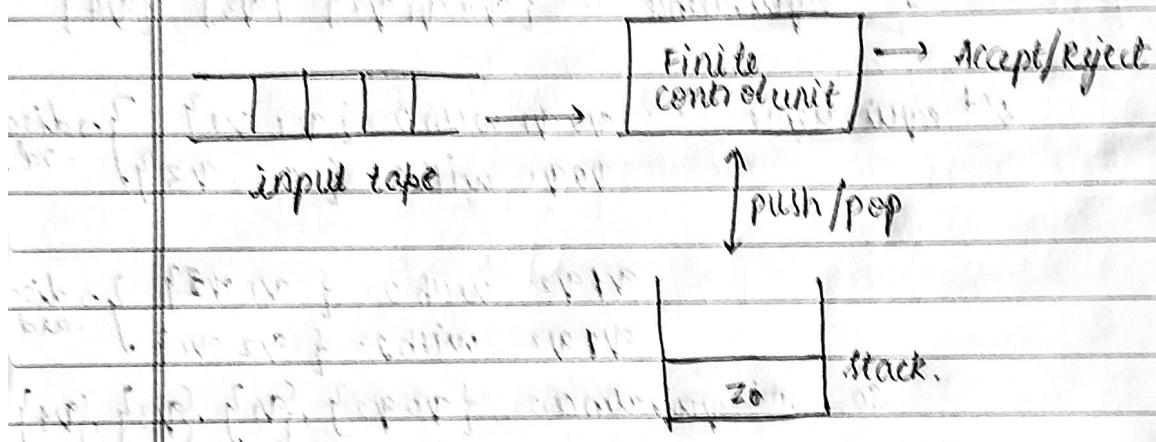
UNIT - 2push Down Automata (PDA):

- a way to implement a CFG.

→ PDA has 3 components.

- input tape
- control unit
- stack with infinite size.

PDA has to read the top of the stack in every transition

Formal definition:

7 tuples:

$(Q, \Sigma, \Gamma, S, q_0, z_0, F)$	$\begin{matrix} \nearrow \text{finite states} \\ \nearrow \text{set of input alphabets} \\ \searrow \text{transition} \\ \searrow \text{state/actual state} \end{matrix}$	$\begin{matrix} \nearrow \text{stack symbol} \\ \nearrow \text{initial stack top symbol.} \\ \searrow \text{final/accepting state} \end{matrix}$
---------------------------------------	---	--

\* Construct a PDA which will accept strings for language

$$L = \{ 0^n 1^{2n} \mid n \geq 1 \}$$

no. of times it occurs.

The no. of 1's is the twice the no. of 0s

push phase:

- For each 0 encountered in the input, push a symbol (e.g., X) onto the stack
- This keeps track of the number of 0s

pop phase:

- When encountering the 1's, pop one symbol from the stack for every two 1's
- This ensures there are exactly twice as many 1's as 0s.

Acceptance:

After processing the entire input, the stack should be empty, and the automation will accept the string.

If string: 001111ε₀

$$\delta(q_0, 0, z_0) = (q_0, 0z_0)$$

$$\delta(q_0, 0, 0) = (q_0, 0 \cdot 0)$$

$$\delta(q_0, 1, 0) = (q_1, 0)$$

$$\delta(q_1, 1, 0) = (q_2, \epsilon_0)$$

Pops operation (when 2nd 1 is encountered)

$$\delta(q_2, 1, 0) = (q_1, 0)$$

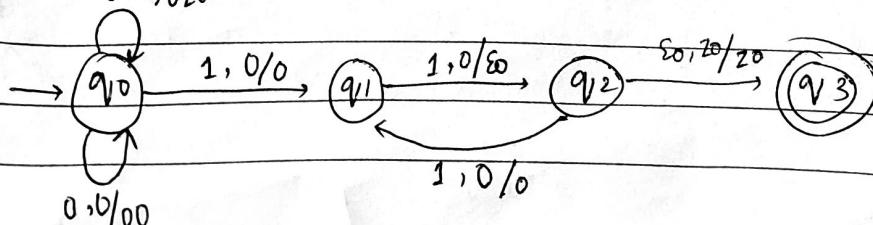
$$\delta(q_1, 1, 0) = (q_2, \epsilon_0)$$

$$\delta(q_2, \epsilon_0, z_0) = (q_3, z_0)$$

p →	0
p →	0
→	z₀

Move to new state & then pop

0, z₀/z₀



UNIT-IVSynthesized attribute:

↳ value is computed from children or node itself.

SDD → S attributed definition  
 ↳ synthesized

Inherited attribute:

↳ value is computed from parent/siblings /itself.

Top Down parser stack (use of inherited attribute)

$D \rightarrow DB | B$  it has left recursion

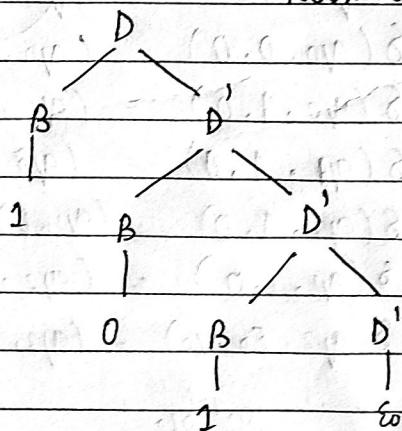
$B \rightarrow 0 | 1$  so eliminate it

↓

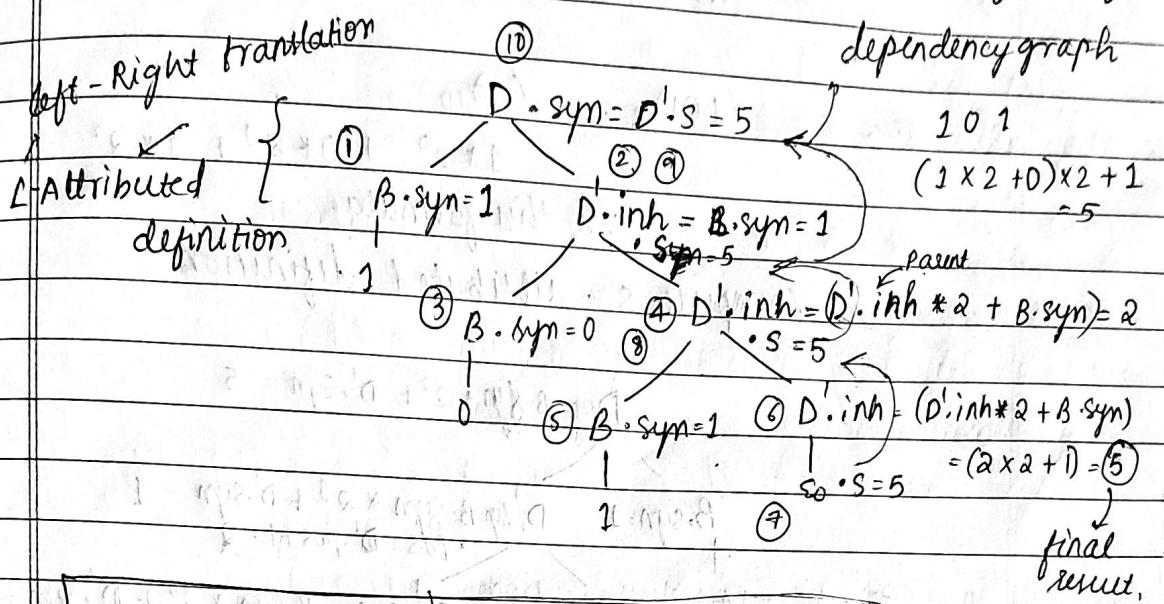
$D \rightarrow BD'$  ↳ suitable for  
 $D' \rightarrow BD' | \epsilon_0$  top-down  
 $B \rightarrow 0 | 1$  parser

I/P :- 101

parse tree



→ convert parse tree to Annotated parse tree  
(post order traversal) so, start from left



production	Semantic Rule	now pass this final result to root to print the result.
$D \rightarrow BD'$	$D' \cdot \text{inh} > B \cdot \text{sym}$ $D \cdot \text{syn} = D' \cdot s$	
$D' \rightarrow B D_1'$	$D_1' \cdot \text{inh} = D' \cdot \text{inh} * 2 + B \cdot \text{sym}$	
↳ did	$D_1' \cdot s = D_1 \cdot s$	so, create a new synthesized attribute(s)
$D' \rightarrow \epsilon_0$	$D' \cdot s = D' \cdot \text{inh}$	to pass the result from leaf to root
$B \rightarrow 0$	$B \cdot \text{syn} = 0$	
$B \rightarrow 1$	$B \cdot \text{syn} = 1$	

then at root, write it as  $D \cdot \text{syn} = D' \cdot s = 5$

because it will hold n print

the result.

### Dependency Graph:

[DAG] to find the

order in which the parser executes.

An edge is made b/w two consecutive nodes that are performed one after another.

$i \rightarrow j$  then 'i' should be executed before 'j'.

then write the topological sort.

Example:

$$D \rightarrow BD'$$

$$D' \rightarrow BD' \mid \epsilon_0$$

$$B \rightarrow 0 \mid 1$$

$$(101)_2 = (5)_{10}$$

$$= 1 * 2^0 + 0 * 2^1 + 1 * 2^2$$

use this formula:

compute  $s$  - attributed definition.

$$D.\text{sym} = B.\text{sym} + 2^2 + D'.\text{sym} = 5$$

$$\begin{array}{ll} B.\text{sym} = 1 & D'.\text{sym} = B.\text{sym} * 2^1 + D'.\text{sym} = 1 \\ | & | \\ 1 & \cdot \text{pos} = D'.\text{pos} + 1 = 2 \end{array}$$

$$\begin{array}{ll} 2 & B.\text{sym} = 0 \quad D'.\text{sym} = B.\text{sym} * 2^0 + D'.\text{sym} = 1 \\ | & | \\ 0 & \cdot \text{pos} = 1 \end{array}$$

$$\begin{array}{ll} 0 & B.\text{sym} = 1 \quad D'.\text{pos} = 0 \\ | & | \\ 1 & \cdot \text{sym} = 0 \end{array}$$

Application of SDDSDD for array type declaration

production

$$T \rightarrow B \ id \ C$$

$$B \rightarrow \text{int}$$

$$B \rightarrow \text{float}$$

$$C \rightarrow [\text{num}] C$$

$$C \rightarrow S_0$$

semantic rules

$$c.\text{inh} = B.\text{type} = \text{int}$$

$$T.\text{type} = c.\text{type} \& \text{addType(id.entry, } T.\text{type)}$$

$$B.\text{type} = \text{int}$$

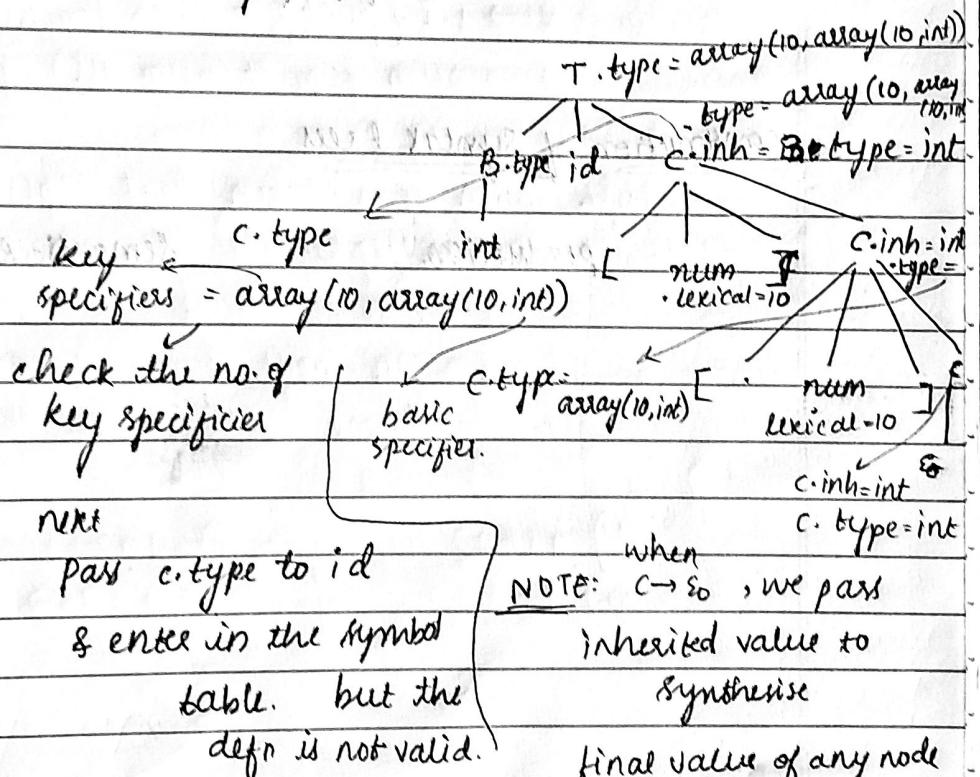
$$B.\text{type} = \text{float}$$

$$c_1.\text{inh} = c.\text{inh} \& c.\text{type} = \text{array}(\text{num.lexical, } c_1.\text{type})$$

$$c.\text{type} = \text{int}$$

Parse tree.

type = syn. i/p: int a[10][10]



Ref

LV

type

1

a

int [10][10]

so, addType(id.entry, T.type)

## SDT for array type declaration.

Translated  
SDP to SDT

by placing  
all synthesis  
at the end  
& inherited  
at the  
beginning  
of the  
body  
of production

## production

$$T \rightarrow B \text{ id } C$$

$$B \rightarrow \text{int}$$

$$B \rightarrow \text{float}$$

$$c \rightarrow [\text{num}] c_1$$

$$\{ c_1.\text{inh} = c.\text{inh}; \} \quad c_1 \{ c.\text{type} = \text{array}(\text{num}, c_1.\text{type}); \}$$

$$c \rightarrow \text{so} \quad \{ c.\text{type} = c.\text{inh}; \}$$

## Semantic Rules

## translation

$$\rightarrow B \text{ id } \{ c.\text{inh} = B.\text{type}; \} \quad c \{ ST.\text{type} = B.\text{type}; \}$$

$$\{ B.\text{type} = \text{int}; \}$$

$$\{ B.\text{type} = \text{float}; \}$$

Construction of syntax trees

## production

## semantic rules

## Eliminating Left recursion from SDTs

SDT for postfix SDT  
Syntax Directed Translation

$$A \rightarrow A_1 Y \{ A.a = g(A_1.a, Y.y) \}$$

$$A \rightarrow X \{ A.a = f(X.x) \}$$

$X, Y \rightarrow$  tokens / terminals

(non-terminal) non-terminal with attributes

$A.a \rightarrow$  synthesized attribute of left-recurcive nonterminal  $A$  &  $x, Y$  are single

grammar symbols

$f, g$  (functions) with synthesized attributes

$X.x$  &  $Y.y$

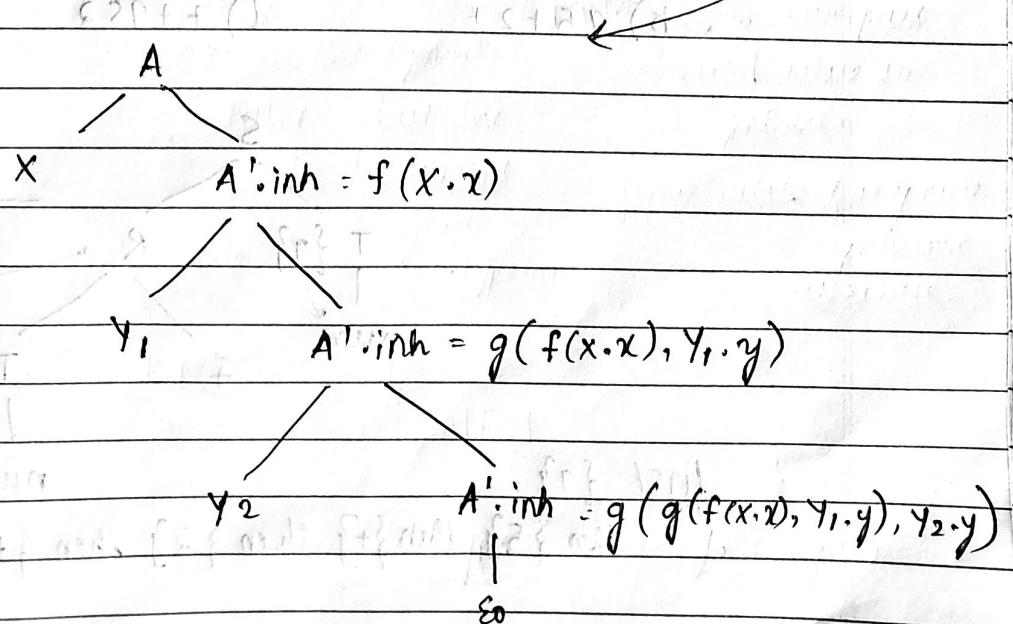
Actions are  $g()$  → group computation.

written in the end of body

of production.

$$\begin{array}{l} A \rightarrow X \\ A \rightarrow X A' \\ A' \rightarrow Y A' \epsilon_0 \end{array}$$

Annotated parse tree



SDT for 1-attributed Defn:

$$\begin{aligned} A &\rightarrow X \{ A'.\text{inh} = f(x, x) \} \cdot A' \\ A' &\rightarrow y \quad A' \\ A' &\rightarrow s_0 \end{aligned}$$

\* J/P:  $q + 5 + 2$

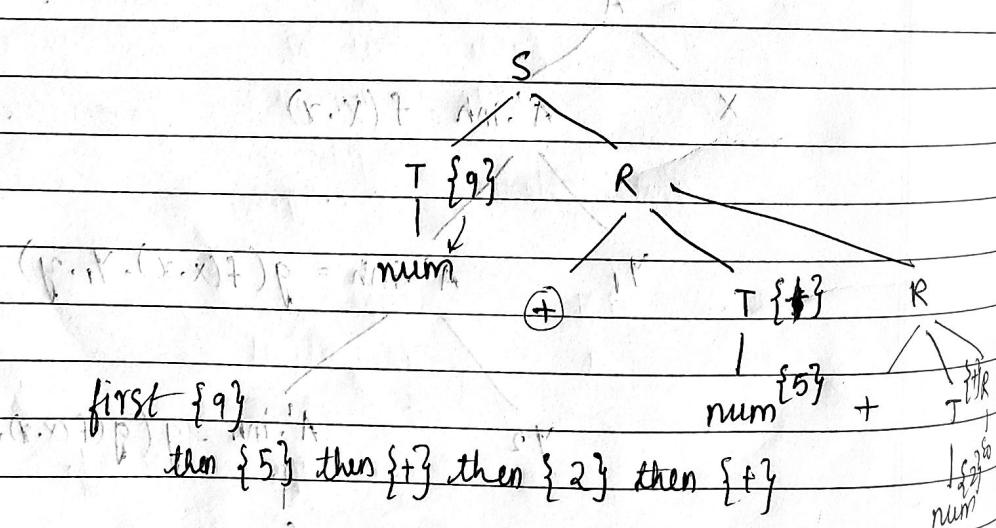
Consider the translation scheme shown

$$\begin{aligned} S &\rightarrow TR \xrightarrow{\text{actions}} \\ R &\rightarrow + T \{ \text{print}('+''); \} R | s_0 \\ T &\rightarrow \text{num} \{ \text{print}(\text{num}. \text{val}); \} \end{aligned}$$

Here num is a token that represents an integer and num.val represents the corresponding integer value.

For an input string '9+5+2', this translation scheme will print:

- a)  $9+5+2$
- b)  $95+2+$
- c)  $9.52++$
- d)  $++952$



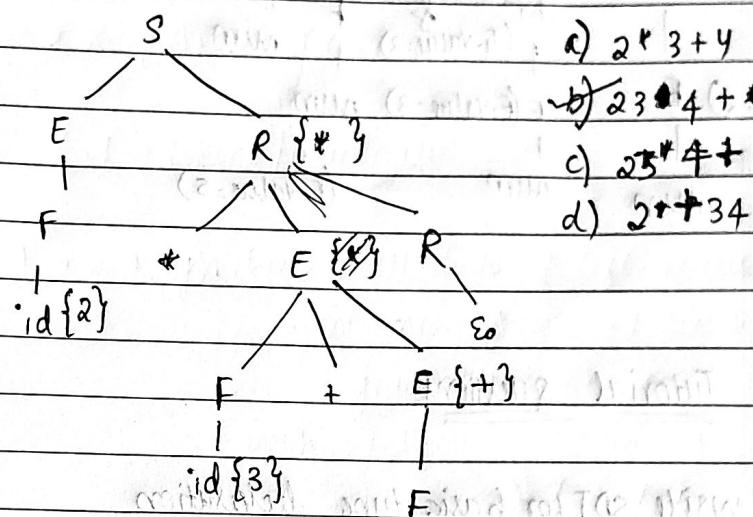
\*  $S \rightarrow ER$

$R \rightarrow *E \{ \text{print}('*'); \} R \mid \epsilon_0$

$E \rightarrow F+E \{ \text{print}('+') \} \mid F$

$F \rightarrow (S) \mid \text{id} \{ \text{print(id.value)}; \}$

i/p: '2\*3+4'



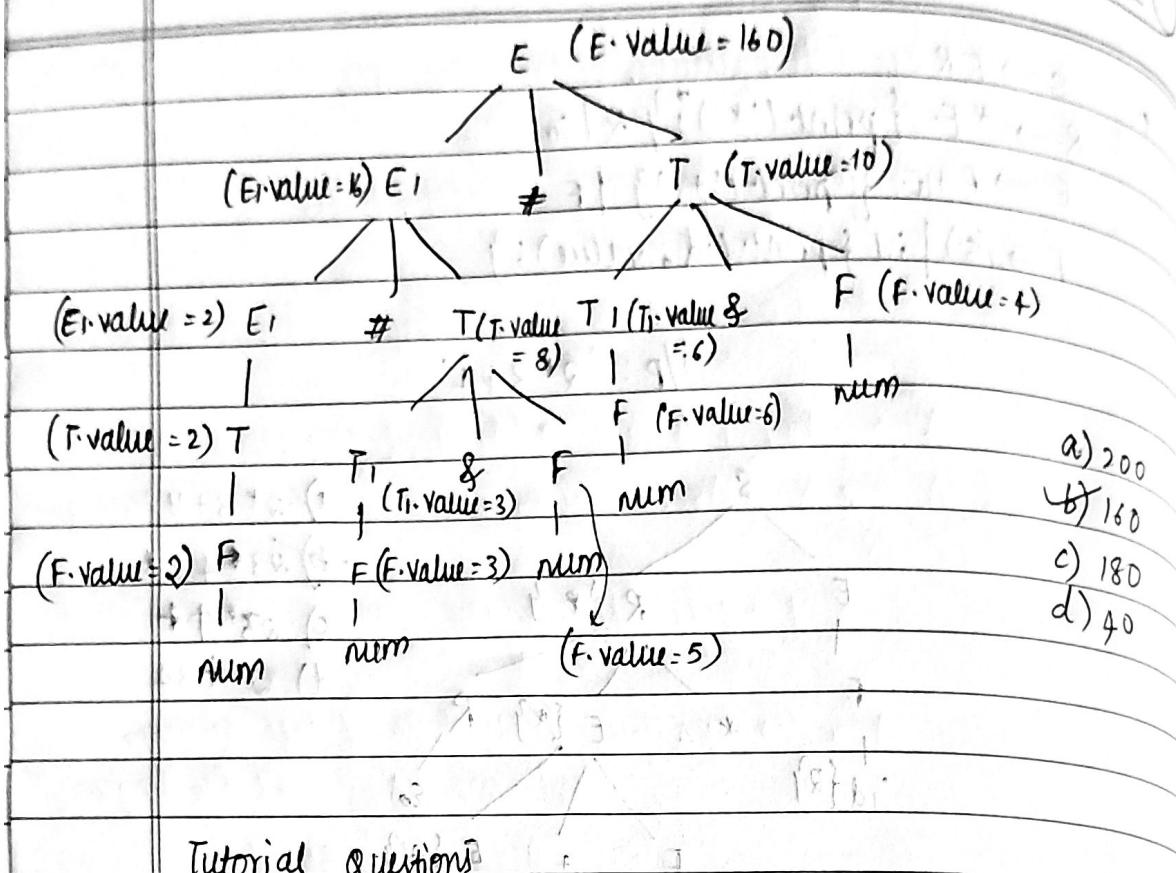
\* consider the grammar with the following translation rules and E as the start symbol.

$E \rightarrow E_1 \# T : \{ E \cdot \text{value} = (E_1 \cdot \text{value} * T \cdot \text{value}) \} \mid T$   
 $\{ E \cdot \text{value} = T \cdot \text{value} \}$

$T \rightarrow T_1 \& F : \{ T \cdot \text{value} = T_1 \cdot \text{value} + F \cdot \text{value} \} \mid f$   
 $\{ T \cdot \text{value} = F \cdot \text{value} \}$

$F \rightarrow \text{num} : \{ F \cdot \text{value} = \text{num} \cdot \text{value} \}$

compute E.value for the root of the parse tree  
for the expression: 2#3&5#6&4

Tutorial Questions

① Write SDT for basic type declaration

② Draw APT & DG for

i) float a, b, c, d

ii) int a[2]

iii) int a[2][3][2]

for i=0 to 2

    for j=0 to 3

        for k=0 to 2

            cout << a[i][j][k] << endl;

            a[i][j][k] = rand() % 100;

        }