Dijkstra's Algorithm

The Problem

Often when we traverse from one point to another or from one place to another place, we want to know the shortest path.

Given a path $G = (V, E)$ which models the shortest path problem, we may want to determine -

a) Given nodes u, v what is the shortest u-v path.

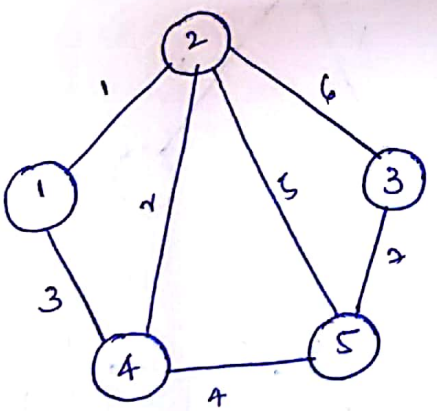
b) Given a start node s, what is the shortest path from s to each other node.

Set-up or Input Instance Setup

The edges in the graph has a length $l_e \geq 0$.

$l_e \rightarrow$ indicate time it takes to traverse e

$\rightarrow$ usually associated as cost to travel.



Take $u = 1$, $V = 2$ $l_e = 1$

$u = 1$ $V = 5$

$l_e = \{1, 4\} + \{4, 5\} = 3 + 4$

$= 7$.

$= \{1, 2\} + \{2, 5\} = 1 + 5 = 6$

$= \{1, 2\} + \{2, 4\} + \{4, 5\}$

$= 1 + 2 + 4 = 7$.

Algorithm :

Dijkstra's Algorithm (G, l)

Let S be the set of explored nodes.

for each $u \in S$, we store distance d(u)

Initially $S = \{s\}$ $d(s) = 0$.

While $S \neq V$

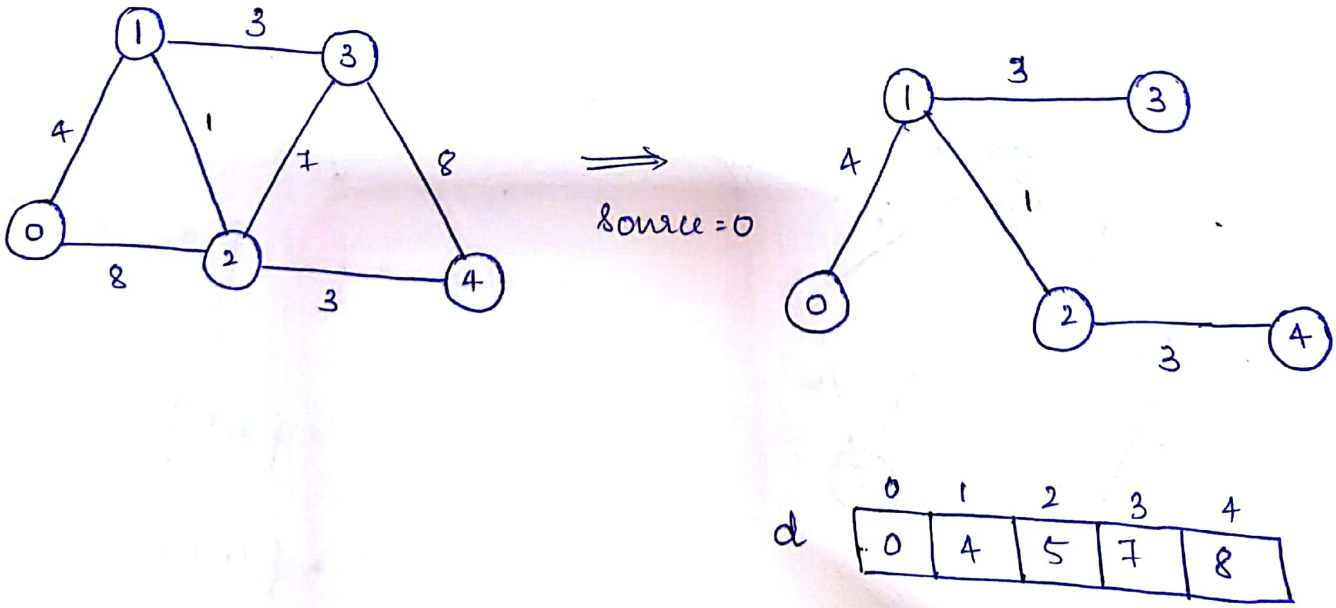
Select a node $v \notin S$ with atleast one edge from S for which

$$d'(v) = \min_{e=(u,v):\, u \in S} d(u) + l_e$$

add v to S and define $d(v) = d'(v)$

endwhile

Example :



	0	1	2	3	4
d	0	4	5	7	8

S

distance[ ]

d[ ]

u	v

$\min(d[u] + l_e)$

$e = (u, v) : u \in S$

$\{0\}$

d.

0	1	2	3	4
0	∞	∞	∞	∞

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$u = 0$, $v = 1$ or $u = 0$, $v = 2$

$\{0, 1\}$

d.

0	1
0	4

$\min(0 + 4) = 4$

~~$u = 0$, $v = 2$~~

$\{0, 1, 2\}$

d

0	1	2
0	4	5

$u = 0$, $v = 2$ | $u = 1$, $v = 2$

$\min(0 + 8) = 8$

$\min(4 + 1) = 5$ ✓

$\{0, 1, 2, 3\}$.

d

0	1	2	3
0	4	5	~~7~~

$u = 1$, $v = 3$ | $u = 2$, $v = 3$ | $u =$

$\min(~~4~~ + 3) = ~~7~~$ ✓

$\min(4 + 7) = 11$

$\{0, 1, 2, 3, 4\}$.

d

0	1	2	3	4
0	4	5	7	8

$u = 3$, $v = 4$ | $u = 2$, $v = 4$

$\min(3 + 8) = 11$

$\min(5 + 3) = 8$.

$S = V$

STOP