

DEPARTMENT OF MATHEMATICS

Sub Code: CS41

Sub: ENGINEERING MATHEMATICS IV

Test: II

Time: 9.30 to 10.30 am

Term: 8.03.2021 to 26.06.2021

Marks: 30

Date: 13.07.2021

Semester: IV

Section: CSE

Note: Answer any TWO full questions. Each main question carries 15 marks

Q. No.	Questions	Bloom's Level	CO's	Marks
1. (a)	Define (i) Stochastic Process (ii) Ergodic process	L1	CO3	2
(b)	Find the fixed probability vector of $\begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 1/4 & 1/4 \\ 3/4 & 1/4 & 0 \end{bmatrix}$	L2	CO4	3
(c)	A bag contains 3 white, 3 red and 2 green bulbs. 2 bulbs are selected at random. If X and Y are discrete random variables denoting the number of white and red bulbs respectively, determine (i) joint distribution of X and Y (ii) Marginal distribution of X and Y & (iii) $COV(X, Y)$.	L3	CO3	5
(d)	A supermarket has two billing counters. The customers arrive in a Poisson fashion at the rate of 12 per hour. The service time for each customer is exponential with mean 6 minutes. Find (i) the probability that an arriving customer has to wait for the service (ii) the average number of customers in the system (iii) the average time spent by a customer in the supermarket	L4	CO4	5
2. (a)	Define regular stochastic matrix. Is the matrix $A = \begin{bmatrix} 1/4 & 3/4 \\ 0 & 1 \end{bmatrix}$ regular?	L1	CO4	2
(b)	A random process $X(t)$ is represented by the ensemble $\{1, 5, -3.5, -4, 1.5\}$ corresponding to the outcomes of an event which are equally likely. Check if the process is SSS.	L2	CO3	3
(c)	Find the value of k from the joint probability density function of X and Y given by $f(x, y) = \begin{cases} k \left(x^2 + \frac{xy}{2} \right) & 0 < x < 1, 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}$. Also find (i) $P(x > 0.5)$ (ii) $P(y > 1 x < 0.5)$	L4	CO3	5
(d)	The mean and standard deviation of a sample of size 50 are 1000 and 100 respectively. Can this be assumed to come from a population with mean 1050? Also find 95% confidence interval for the population mean.	L3	CO5	5
3. (a)	Define (i) Null hypothesis (ii) One tailed test	L1	CO5	2
(b)	Trains arrive in a yard in a Poisson fashion with a mean of one every 20 minutes but the service time is exponential with mean 30 minutes. Assuming that there is only one service station and the line capacity of the yard is limited to 4 trains, find the probability that the yard is empty.	L2	CO4	3
(c)	In a cascade of binary communication channels, the symbol 1 and 0 are transmitted in successive stages. At any stage, the probability that a transmitted 1 is received as 1 is 0.8 and the probability that 0 is received as 0 is 0.75. Find the probability that (i) 1 transmitted in the first stage is received correctly at third stage (ii) 0 transmitted in the first stage is received correctly at third stage	L4	CO4	5
(d)	Find the autocorrelation function of the stochastic process defined by $X(t) = 5 \sin(\omega t + \alpha)$ where α is uniform in the interval $[-\pi, \pi]$. Hence verify if the process is WSS.	L3	CO3	5