RAMAIAH
Institute of Technology

APPLIED GRAPH THEORY

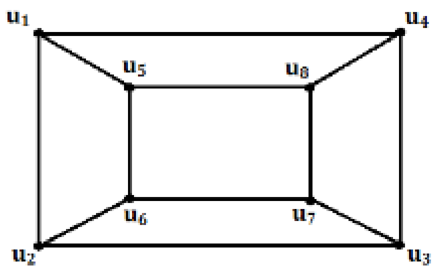
Unit-I
Introduction to Graph Theory
Basic concepts of graphs, standard definitions, types of graphs, graph isomorphism, connected & disconnected graphs, Operations on Graphs, Euler graphs, Hamiltonian Paths and Circuits. Trees, properties of trees, Rooted & Binary Trees.

I. Two/Four marks questions :

1. Define a graph with an example.
2. Define a directed graph with an example.
3. Define a weighted graph with an example.
4. Define finite and infinite graphs.
5. Define degree of a vertex with an example.
6. Define isolated and pendent vertices.
7. Define simple graph and multi graph.
8. Define regular graph with an example.
9. Define complete graph with an example.
10. State hand shaking property.
11. Define a sub graph with an example.
12. Define edge disjoint sub graphs with an example.
13. Define vertex disjoint sub graphs with an example.
14. Define union of two graphs with an example.
15. Define intersection of two graphs with an example.
16. Define ring sum of two graphs with an example.
17. Define fusion of two vertices with an example.
18. Define decomposition of a graph with an example.
19. Define complement of a graph.
20. Define walk with an example.
21. Define path with an example.
22. Define circuit with an example.
23. Give an example of a closed walk which is not a circuit.
24. Give an example of open walk which is not a path.
25. Define connected and disconnected graphs.
26. Define component of a graph.
27. Define Euler graph with an example.
28. Define Hamiltonian graph with an example.
29. Define arbitrarily traceable graph with an example
30. Write the condition for a graph to be arbitrarily traceable from a vertex.
31. Define Unicursal graph with an example.
32. Define minimally connected graph with an example.

33. Define a tree.
34. Define center of a tree.
35. Define a rooted tree.
36. Define a binary tree.
37. Prove that every complete graph is a regular graph. Is converse true? Justify.
38. Prove that in every graph, the number of vertices of odd degree is even.
39. Show that in a complete graph of n vertices (K_n) the degree of every vertex is (n-1) and that the total number of edges is $\frac{n(n-1)}{2}$
40. If k is odd, then show that the number of vertices in a k-regular graph is even.
41. Prove that a path with n vertices is of length n.
42. If a circuit has n vertices then prove that it has n edges.
43. Prove that degree of every vertex in a circuit is two.
44. If a graph has exactly two vertices of odd degree then prove that there must be a path connecting these vertices.
45. Show that the following graphs are Hamiltonian but not Eulerian

a)



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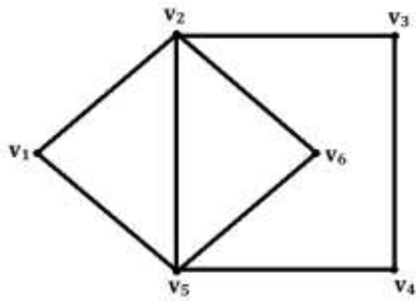
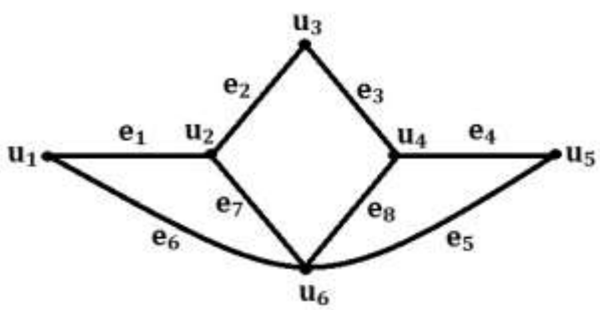
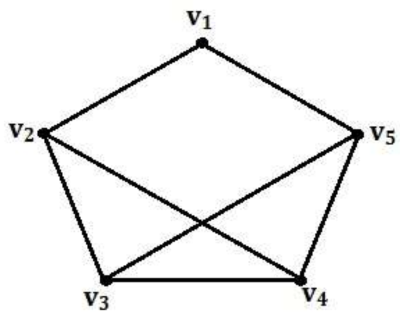


46. Find the complement of the following graphs:

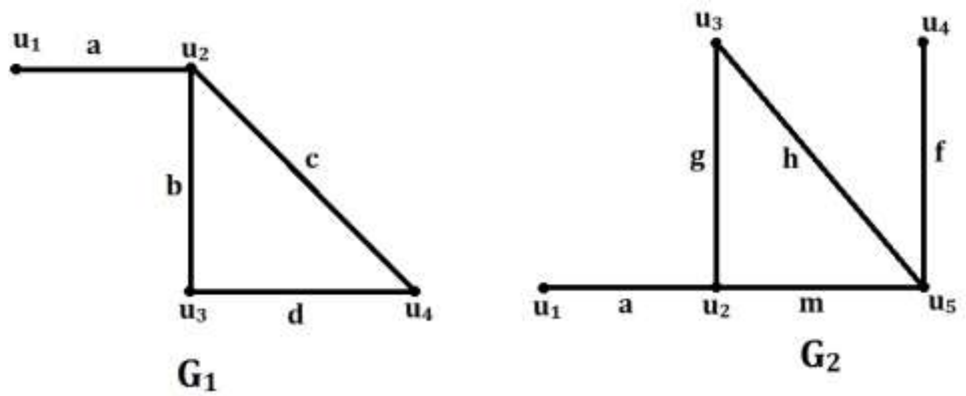
a)



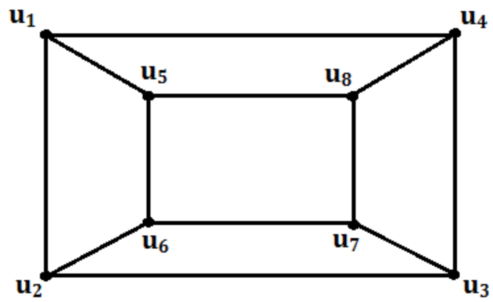
b)



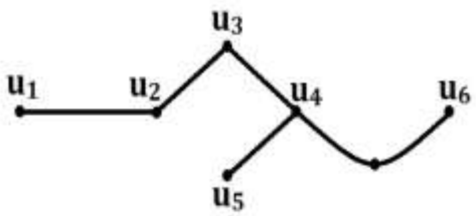
47. Find the ring sum of the graphs G_1 and G_2



48. Find the fusion of the vertices u_1 and u_2



49. Define Bipartite graph with an example.
50. Define Complete bipartite graph.
51. Prove that in a tree, there is one and only path between every pair of vertices.
52. If in a graph G, there is one and only one path between every pair of vertices then prove that G is a tree.
53. Find eccentricity of all the vertices and hence center of G

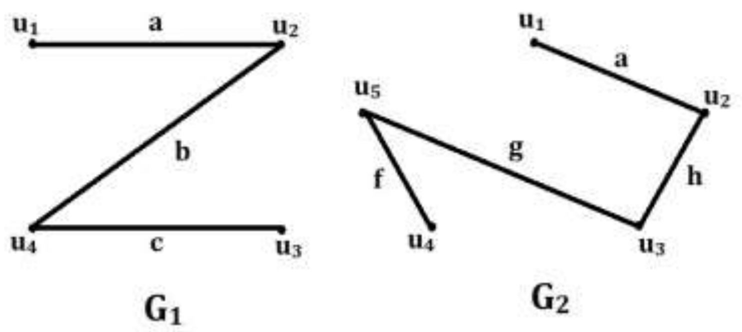


54. Define Isomorphic graphs with an example.

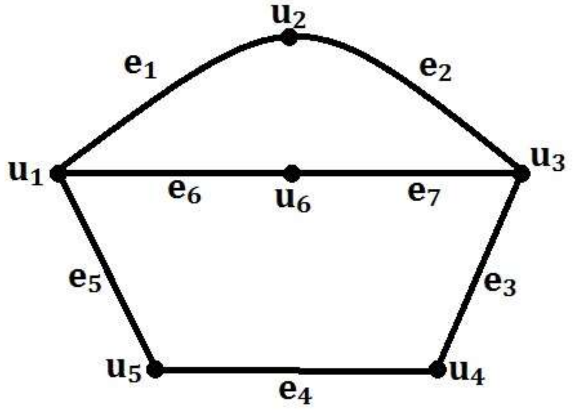
II. Five/Seven Marks questions:

1. Define union, intersection and ring sum of two graphs with an example. Also define decomposition of a graph.
2. Prove that a simple graph with n vertices and k components can have at most have (n-k).(n-k+1)/2 edges
3. Prove that a given connected graph G is an Euler graph if and only if all vertices of G are of even degree.

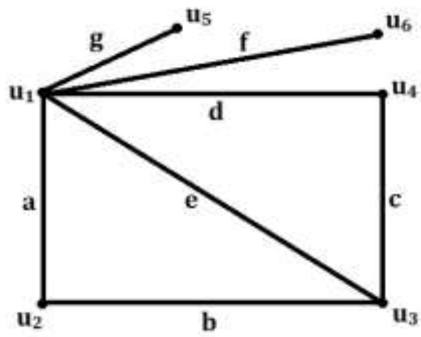
4. Write a note on Konigsberg bridge problem.
5. Write an note on Travelling Salesman problem.
6. Write a note on Seating arrangement problem.
7. Prove that a tree with n vertices has n-1 edges.
8. Prove that any connected graph with n vertices and n-1 edges is a tree.
9. Prove that a graph G is a tree if and only if it is minimally connected.
10. Prove that
    i) In a binary tree, the number of vertices is always odd
    ii) In a binary tree with n vertices, the number of pendent vertices is (n+1)/2
11. Prove that a connected graph G is a tree if and only if adding an edge between any two vertices in G creates exactly one circuit.
12. i) Draw a 4-regular graph which is not a complete graph and also find its complement.
    ii) Show that it is not possible to have a set of seven persons such that each person in the set knows exactly three other persons in the set.
13. Find union, intersection & ring sum of G_1 and G_2.



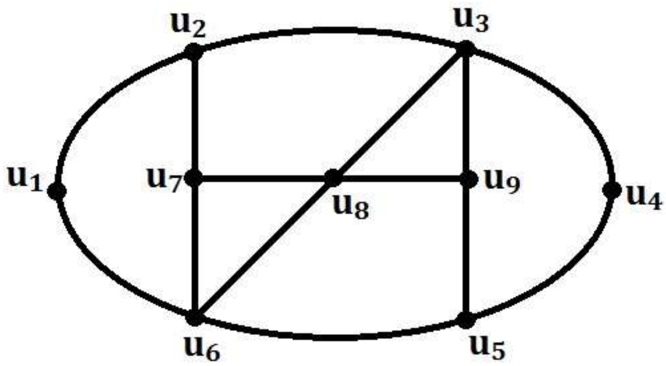
14. Find (i) Fusion of u_1 and u_2 (ii) $G - e_1$ (iii) $G - v_1$ (iv) Edge disjoint sub graphs of G and (v) decomposition of G.



15. Find (i) Fusion of u_1 and u_3 (ii) $G - a$ (iii) $G - c$ (iv) Edge disjoint sub graphs of G and (v) decomposition of G.



16. From the graph G given below, find the following if exists.
    (i) Hamiltonian cycle
    (ii) Euler line
    (iii) A subgraph with at least 5 vertices which is a binary tree
    (iv) Vertex disjoint sub graphs of G.



17. Draw a graph which is
    i) Both Hamiltonian and Eulerian
    ii) Hamiltonian but not Eulerian
    iii) Eulerian but not Hamiltonian
    iv) Neither Eulerian nor Hamiltonian.