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Unit - II

Continuous Probability Distributions.

→ Exponential Distribution:

$$P(\alpha, x) = \begin{cases} \alpha e^{-\alpha x}, & 0 \le x < x \\ 0, & \text{elsewhere} \end{cases}, \quad \alpha \in R^{+}$$

α is the parameter of the distribth

- Mean, $\mu = \frac{1}{\alpha}$
- S.D $\Rightarrow \sigma = \frac{1}{\alpha}$

$$P(a < x < b) = \int_a^b P(\alpha, x)\, dx$$

→ Questions:

1. The duration of a telephone convo follows exponential distributth with mean 3 mins. Find the prob that the convo may last
   i) more than 1 min.
   ii) less than 3 mins
   iii) b/w 2 and 4 mins.

→ $\mu = \frac{1}{\alpha} \Rightarrow \alpha = \frac{1}{\mu} = \frac{1}{3}$

$P(\alpha, x) = \begin{cases} \frac{1}{3} e^{-\frac{1}{3}x}, & x \ge 0 \\ 0, & \text{elsewhere} \end{cases}$

$$P(x > 1) = \frac{1}{3}\int_1^\infty e^{-\frac{1}{3}x}\, dx$$

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$$= (-1)\, e^{-\frac{1}{3}x}\Big|_1^\infty$$

$$= (-1)\{e^{-\infty} - e^{-1/3}\}$$

$$= (-1)(-0.7165)$$

$$= \underline{0.7165}$$

ii) $P(x < 3) = \frac{1}{3}\int_0^3 e^{-\frac{1}{3}x}\, dx$

$$= (-1)\, e^{-\frac{1}{3}x}\Big|_0^3$$

$$= (-1)\{e^{-1} - e^{0}\}$$

$$= (-1)\{0.3678 - 1\}$$

$$= \underline{0.6321}$$

iii) $P(2 < x < 4) = \frac{1}{3}\int_2^4 e^{-\frac{1}{3}x}\, dx$

$$= (-1)\, e^{-\frac{1}{3}x}\Big|_2^4$$

$$= (-1)\cdot\{e^{-4/3} - e^{-2/3}\}$$

$$= \underline{0.2498}$$

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2. The mileage that a car owner gets with a certain type is an exponential avariate with mean 40000 km. Find the prob that these tyres will last

i) At least 20k km

ii) At most 30k km.

$\mu = 40k.$

$\alpha = \frac{1}{40k}$

ii) $P(x \leq 30k)$.

$= \frac{1}{40k}\int_{0}^{30k} e^{-1/40k(x)}\, dx.$

$= (-1)\, e^{-1/40k\, x}\Big|_{0}^{30k}$ $k = 10^3$

$= (-1)\{e^{-30k/40k} - e^{0}\}$

$= (-1).\{0.7788 - 1\}$

~~0.2212~~

$= 0.3127.$

i) $P(x > 20k)$

$= \frac{1}{40k}\int_{20k}^{\infty} e^{-1/40k\, x}\, dx.$

$= (-1)\, e^{-1/40k}\Big|_{20k}^{\infty}$

$= (-1)\{e^{-\infty} - e^{-0.5}\}$

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$= 0.6065$

3. The average daily turn out in a store is ₹. 10,000 and the average profit is 8%. If the turn out is exp variant, find the prob. that the net profit will exceed ₹. 1000 on two consecutive days.

$\mu = 800$ $\alpha = \frac{1}{800}$

$P(x > 1000) = \frac{1}{800}\int_{1000}^{\infty} e^{-1/800\, x}\, dx.$

$= (-1)\, e^{-1/800\, x}\Big|_{1000}^{\infty}$

$= (-1)\{e^{-\infty} - 0.2865\}$

$= 0.2865$

The prob that $x > 1000$ on 2 consecutive days $= (0.2865)(0.2865) = 0.0821$

4) The life of a bulb is advertised to have a mean of 200 hrs. If it has exp. dist, find the prob that the bulb will last for

i) Less than 200 hrs

ii) b/w 100 & 300 hrs.

iii) More than 250 hrs.

$\mu = 200 hrs \qquad \alpha = \frac{1}{200}$

i) $P(x < 200)$

$$= \frac{1}{200}\int_0^{200} e^{-1/200 x}\, dx$$

$$= \frac{1}{200}\left\{ (-1)\, e^{-1/200 x} \Big|_0^{200} \right.$$

$$= (-1)\{e^{-1} - e^{0}\}$$

$$= (-1)\{0.3678 - 1\}$$

$$= 0.6321$$

ii) $P(100 < x < 300)$

$$= \frac{1}{200}\int_{100}^{300} e^{-1/200 x}\, dx$$

$$= (-1)\, e^{-1/200 x} \Big|_{100}^{300}$$

$$= (-1)\{e^{-1.5} - e^{-0.5}\}$$

$$= (-1)\{-0.3834\}$$

$$= 0.3834$$

iii) $P(X > 250)$

$$= \frac{1}{200}\int_{250}^{\infty} e^{-1/200 x}\, dx$$

$$= (-1)\, e^{-1/200} \Big|_{250}^{\infty}$$

$$= (-1)\{e^{-\infty} - 0.2865\}$$

$$= 0.2865$$

5) The length of time for a person to be served at a cafe is an exp. var with $\mu = 4$ mins. Find the prob that a person is served in less than 3 mins on at least 4 of the next 6 days.

$\mu = 4$ mins. $\qquad \alpha = \frac{1}{4}$

$$P(X < 3) = \frac{1}{4}\int_0^3 e^{-1/4 x}\, dx$$

$$= (-1)\, e^{-1/4 x} \Big|_0^3$$

$$= (-1)\{e^{-3/4} - e^{0}\}$$

$$= 0.5276$$

$$P(6, 0.5276, 4) = {}^6C_4 (0.5276)^4 (0.4724)^2$$

$$= \frac{6!}{4!(6-4)!}(0.0775)(0.2231)$$

$$= 0.259$$

→ Normal Distribution.

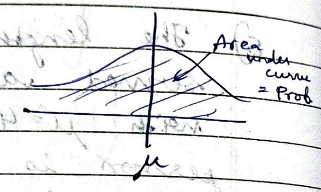
Any quantity whose variation depends on random causes is distributed according to the normal law. It is defined as

$$N(\mu, \sigma, x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}$$

This is also called Gaussian distribut^n.

* i) $N(\mu, \sigma, x) \geq 0$

ii) $\int_{-\infty}^{\infty} N(\mu, \sigma, x)\, dx = 1$

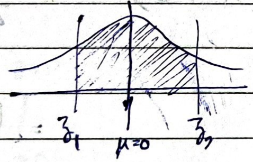


→ Standard Normal distribution.

The normal distribut^n for which $\mu = 0$ and $\sigma = 1$ is called SND.

In this case,

$$z = \frac{x-\mu}{\sigma}$$



$$P(z_1 \leq z \leq z_2) = P(0 \leq z \leq z_1) + P(0 \leq z \leq z_2)$$

Eg:- If x is normally distributed with mean 5 & variance 4, find the prob that x lies b/w 2 & 6.

$\mu = 5$, $\sigma^2 = 4 \Rightarrow \sigma = 2$.

If $x = 2$, $z = \frac{2-5}{2} = \frac{-3}{2} = -1.5$

If $x = 6$, $z = \frac{6-5}{2} = 0.5$

$P(2 \leq x \leq 6) = P(-1.5 \leq z \leq 0.5)$

$= \underbrace{P(0 \leq z \leq 1.5) + P(0 \leq z \leq 0.5)}_{\text{Area under curve is +ve.}}$

$P(2 \leq x \leq 6) = 0.4332 + 0.1915$

$= 0.6247$

→ Questions:-

1) In a city, the no. of power breakdowns per week is a normal variate with $\mu = 11.6$ & $\sigma = 3.3$. Find the prob. that there will be atleast 8 breakdowns in a week.

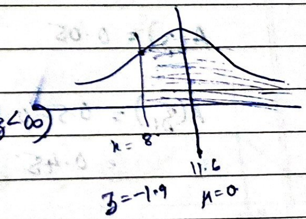
$z = \frac{8 - 11.6}{3.3} = \frac{-3.6}{3.3} = -1.09$.

$P(x \geq 8) = P(z > -1.09)$

$= P(0 \leq z < 1.09) + P(0 \leq z < \infty)$

$= 0.3621 + 0.5$

$= 0.8621$.



2. The average length of metal bars produced by a company is 68.22 cm with variance 10.8 cm. How many bars in a consignment of 1000 are expected to be over 72 cm.

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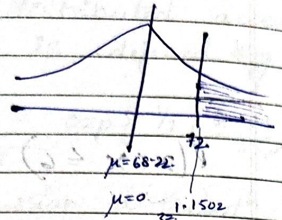
$\mu = 68.22$, $\sigma^2 = 10.8$ cm
$\sigma = 3.2863$

$z = \frac{72 - 68.22}{3.2863}$



$z = 1.1502$

$P(x > 72) = P(z > 1.1502)$

$= P(0<z<\infty) - P(0 \le z < 1.1502)$

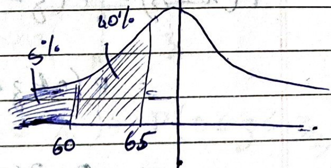
$= 0.5 - 0.3749$

$= 0.1251$

out of 1000 bars
$= 1000 \times 0.1251$
$= 125$ bars

3) In a normal distribut^n 5% of the items are under 60 & 40% are b/w 60 & 85. Find the mean & s.d of the dist.

$A(z) = 0.05$



$A(z_1) = 0.5 - 0.05$
$= 0.45$

$z_1 = -1.64$

$-1.64 = \frac{60 - \mu}{\sigma} \Rightarrow \mu - 1.64\sigma = 60$

$A(z) = 0.4$

$A(z_2) = 0.05 - 0.4$
$= 0.1$

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$z_2 = -0.27$

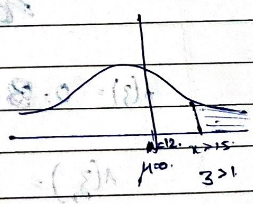
Q) A sample of 500 items have a mean life of 12 & s.d 3. If it follows normal dist, how many items are expected to have life
i) more than 15
ii) less than 6
iii) B/w 10 to 14

$\mu = 12$, $\sigma = 3$

i) $x = 15$, $z = \frac{x-\mu}{\sigma} = \frac{15-12}{3} = 1$.

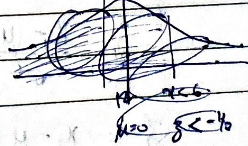
$P(x > 15) = P(z > 1)$

$= 0.5 - P(0 \le z \le 1)$



$= 0.5 - 0.3413$
$= 0.1587 \times 500$

ii) $x < 6$ $z = \frac{15-6}{3} = -1/3$.



$P(x<6) = P(z < 1/3)$

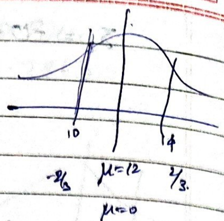
$= 0.5 - P(0 \le z < 1/3) = 0.3707 \times 500$

$= 0.5 - 0.1293$

$x_1 = 10$ $\quad z_1 = \frac{10-12}{3} = 2/3$

$x_2 = 14$

$z_2 = \frac{14-12}{3} = -2/3$

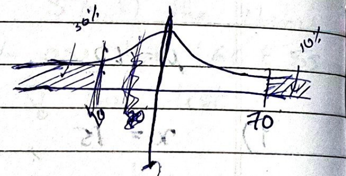


$P(10 \le x \le 14) = 2\{P(0 \le z \le 2/3)\}$

$= 2(0.2454)$

$= 0.4908 \times 500$

Q) In a N.D, 80% of items are under 40 & 10% are over 70 Find μ & σ.



$A(z) = 0.\cancel{4}$

$A(z_1) = 0.5 - 0.\cancel{1}$

$= 0.4\cancel{0}$ $\quad P(z_1 < 40)$

$\cancel{0.21879} = 0.1554$

$A(z) = 0.\cancel{3}$

$A(z_2) = 0.5 - 0.3$

$= 0.2$

$P(x_2 > 70) = 0.0793$

$z_1 = \frac{x_1 - \mu}{\sigma} \quad 0.1554\sigma + \mu = 40$

$z_2 = \frac{x_2 - \mu}{\sigma} = 0.0793\sigma + \mu = 70$

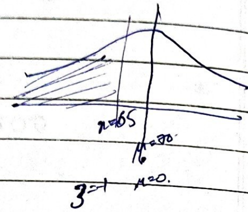
$\sigma = -394.2181$

$\mu = 101.2619$

6) The marks of ~~two~~ students in an exam follow ND with mean 70 & sd 5. Find the no. of students whose marks will be

i) less than 65
ii) more than 75
iii) b/w 64 & 73.

i) $x = 65$



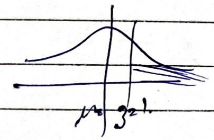
$z = \frac{65-70}{5} = -1$

$P(x<65) = 0.5 - P(0 \le z \le 1)$

$= 0.5 - 0.3413$
$= 0.1587$

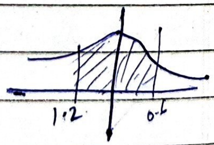
ii) $x = 75$

$z = \frac{75-70}{5} = 1$



$P(x > 75) = 0.5 - P(0 \le z \le 1)$
$= 0.5 - 0.3413$
$= 0.1587$

iii) b/w 64 to 73.



$x_1 = 64$

$z_1 = \frac{64-70}{5} = -1.2$

$x_2 = 73$

$z_1 = \frac{73-70}{5} = 0.6$

$P(64 \le x \le 73) = P(0 \le z \le 1.2) + P(0 \le z \le 0.6)$

$= 0.3849 + 0.2257$

$= 0.6106$

7) If 4 stds are randomly selected from this group, what is the prob that at least one of them would have scored above 75

For $x = 75$

$p = 0.1587, \quad q = 1 - 0.1587$

$B(4, 0.1587, 1) = {}^4C_0\, p^{n-x} q^n + (0.1587)^4$

$= {}^4C_1 (0.1587)^0 (0.8413)^4 + (0.1587)^7$

$= \frac{4!}{1! \times 3!} (0.01345)$

$= 0.5009$

$1 - P(X=0)$
$1 - 0.5009$
$= 0.4991$

8) For a N.D. with mean μ & σ (s.d)
i) $P(|x-\mu| \le \sigma)$
ii) $P(|x-\mu| \le 2\sigma)$
iii) $P(|x-\mu| \le 3\sigma)$

i) $z = \frac{x-\mu}{\sigma}$

$P(|x-\mu| \le \sigma) = P(-\sigma \le x-\mu \le \sigma)$

$= P(-1 \le z \le 1)$

$= 2\,P(0 \le z \le 1)$

$= 2(0.3413)$

$= 0.6826$

ii) $= 2P(0 \le z \le 2)$

$= 2(0.4772)$

$= 0.9544$

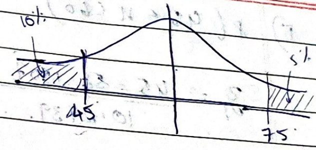
iii) $= 2P(0 \le z \le 3)$

$= 2(0.4987)$

$= 0.9974$

9) In an exam, a student is considered to have failed, secured 3rd class, 2nd class or 1st class according as he/she scores <45%, b/w 45% to 60%, b/w 60% to 75% & above 75% marks. If 10% of the students fail & 5% got 1st class, find the % of stds who got second & third class.

$A(z_1) = 0.1$



$A(z_1) = 0.5 - 0.1$

$= 0.4$

$z_1 = \cancel{0.3888}\ -1.28$

$A(z_2) = 0.05$

$= 0.5 - 0.05$

$= 0.45$

$z_2 = 1.64$

$z_1 = \frac{x_1 - \mu}{\sigma}$

$-1.28\sigma + \mu = 45$ — (1)

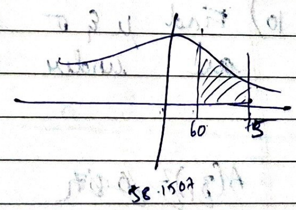
$z_2 = \frac{x_2 - \mu}{\sigma}$

$1.64\sigma + \mu = 75$

$\sigma = \cancel{88.88}\ 10.2739$

$\mu = 58.1507$

i) $P(60 \le x \le 75)$



$z_1 = \frac{60 - 58.1507}{10.2739} = 0.1799$

$z_2 = \frac{75 - 58.1507}{10.2739} = 1.685$

$P(0 \le z \le 1.685) - P(0 \le z \le 0.1799)$

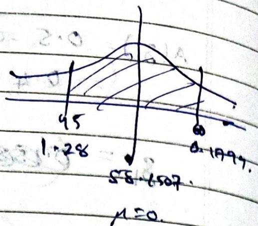
$= 0.4535 - 0.0675$

$= 0.386$

$= 38.6\ \%$

ii) $P(45 \le x \le 60)$.

$$z_1 = \frac{45 - 58.1507}{10.2739} = -1.28$$



$$z_2 = \frac{60 - 58.1507}{10.2739} = 0.1799$$

$P(1.28 \le z \le 0.1799)$

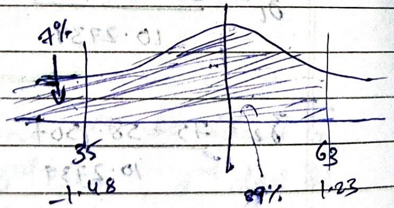
$= P(0 \le z \le 1.28) + P(0 \le z \le 0.1799)$

$= 0.3997 + 0.0714$

$= 0.4711$

$= 47.11\%$

10) Find μ & σ of a N.D if 7% of the items are under 35 & 89% are under 63.



$\cancel{A(z_1) = 0.07}$

$A(z_1) = 0.5 - 0.07$

$= 0.43$

$z_1 = -1.48$

$A(z_2) = 0.89 - 0.5$

$= 0.39$

$z_2 = 1.28$

$$z_1 = \frac{x_1 - \mu}{\sigma} \qquad z_2 = \frac{x_2 - \mu}{\sigma}$$

$\mu - 1.48\sigma = 35$ $\qquad$ $\mu + 1.23\sigma = 63$

$\therefore \mu = 50.2915$

$\sigma = 10.3321$

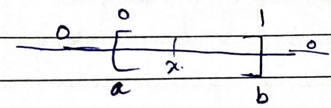
→ Uniform Distribution (for CRV)

A RV 'x' is said to be uniformly distributed if $a \le x \le b$ if $f(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & \text{otherwise} \end{cases}$

- Mean of UD, $\mu = \frac{1}{2}(a+b)$
- Variance of UD, $\sigma^2 = \frac{1}{12}(b-a)^2$

→ Cumulative Prob. function

$$F(x) = \begin{cases} 0 & -\infty < x < a \\ \frac{x-a}{b-a} & a \le x \le b \\ 1 & x > b \end{cases}$$



$$F(x) = \int_{-\infty}^{x} f(x)\,dx = \int_{a}^{x} \frac{1}{b-a}\,dx$$

$$= \frac{1}{b-a}[x]_a^x$$

$$= \frac{x-a}{b-a}.$$

→ Questions.

1. If x is uniformly distributed with mean 1 and variance $\frac{4}{3}$, find $P(X<0)$

→ $\mu = \frac{1}{2}(a+b)$ $\sigma^2 = \frac{1}{12}(b-a)^2$

$2 = a+b$ $\sqrt{\frac{4}{3}} = \sqrt{\frac{1}{12}(b-a)^2}$

$\frac{2}{\sqrt{3}} = \frac{1}{2\sqrt{3}}(b-a)$

$b-a = 4$

$b+a = 2$

$2b = 6$

$\boxed{b=3}$

$4 = b-a$

$\boxed{a=-1}$

$P(X<0)$.

$F(x) = \int_a^x \frac{1}{b-a}\,dx = \int_{-1}^{0} \frac{1}{3+1}\,dx$

$= \frac{1}{4}[x]_{-1}^{0}$

$= \frac{1}{4}(0-(-1))$

$= \frac{1}{4} = 0.25$

2. Buses arrive at a stop at 15 min interval starting at 7am. If a passenger arriving at the stop at a random time ~~that~~ is uniformly distributed b/w 7 & 7:30, find the prob that he waits,

a) Less than 5 mins

ii) Atleast 12 min

$\mu = \frac{1}{2}(7+7{:}30)$

$= 7{:}15$ mins.

$\sigma^2 = \frac{1}{12}(7{:}30-7)^2 = \frac{1}{12}(30)^2 = 75$ min.

i) $P(X<5)$.

7 7.15 7.30

$f(x) = \begin{cases} \frac{1}{30}, & 7 \le x \le 7.30 \\ 0, & \text{otherwise} \end{cases}$

$P(X<5) = \int_{7.10}^{7.15} \frac{1}{30}\,dx + \int_{7.25}^{7.30} \frac{1}{30}\,dx$

$= \frac{5}{30} + \frac{5}{30}$

$= \frac{1}{3}$

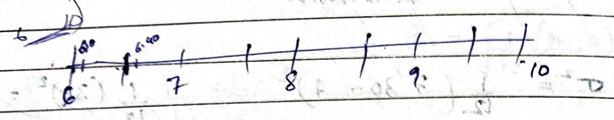
ii) $P(X \ge 12)$ $= \int_{7.00}^{7.03} \frac{1}{30}\,dx + \int_{7.15}^{7.18} \frac{1}{30}\,dx$

$= \frac{3}{30} + \frac{3}{30}$

$= \frac{6}{30}$

$= 0.2$

3. In a certain city transport route, buses ply every 30 mins b/w 6 am & 10 am. If a person reaches a bus stop at a random time during this period, what is the prob that he has to wait for at least 20 mins.



$$f(x) = \begin{cases} \frac{1}{240} \\ \end{cases}$$

6.10

$$P(x > 20) = 8\int_{6.0} \frac{1}{240}\, dx$$

$$= 8\left\{\frac{1}{\cancel{240}}(1\cancel{0})\right\}$$

3

$$= \frac{1}{3} = 0.33$$

4. A bus travels b/w cities A & B 100 miles apart. The dist X of the pt. of breakdown from A is a uniform variate. There are service garages in city A, city B & midway b/w A & B. If a breakdown occurs, a tow truck is sent from the garage closest to the pt. of breakdown. estimate the prob that the tow truck has to travel more than 10 miles to reach the bus.

0.4

$$4 \times \int \frac{(40)10}{100}$$

$$\frac{4}{10} = 0.4$$

$$4\int$$

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→ Gamma Distribution.

Gamma function:

$$\sqrt{n} = \int_0^\infty x^{n-1} e^{-x} dx \quad , x > 0$$

$\sqrt{n+1} = n\sqrt{n}$

If n is an integer, $\sqrt{n+1} = n!$

The CRV X has a gamma distributⁿ with parameters α and β if its density functⁿ is given by

$$f(x) = \begin{cases} \frac{1}{\beta^\alpha \sqrt{\alpha}} x^{\alpha-1} e^{-x/\beta} & , x > 0 \\ 0 & , \text{otherwise} \end{cases}$$

If $\alpha = 1$

$$f(x) = \begin{cases} \frac{1}{\beta} e^{-x/\beta} & , x > 0 \\ 0 & , \text{otherwise.} \end{cases}$$

for $\alpha = 1$, gamma distributⁿ reduces to exponential distributⁿ.

- Mean of gamma distributⁿ, $\mu = \alpha\beta$ $\left\{ \int_0^\infty x f(x) dx \right.$

Variance, $(\sigma^2) = \alpha\beta^2$

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→ Questions :-

i) A R.V X is g-dist'd with $\alpha = 3$, $\beta = 2$

Find i) $P(X \leq 1)$

ii) $P(1 \leq X \leq 2)$

i) $P(X \leq 1) = \int_0^1 \frac{1}{\beta^\alpha \sqrt{\alpha}} x^{\alpha-1} e^{-x/\beta} dx.$

$\sqrt{2+1} = 2! = 2$

$$= \int_0^1 \frac{1}{2^3 \sqrt{3}} x^2 e^{-x/2} dx$$

$$= \frac{1}{8} \int_0^1 \frac{1}{2} x^2 e^{-x/2} dx.$$

$$= \frac{1}{16} \int_0^1 x^2 e^{-x/2} dx.$$

$$= \frac{1}{16} \left\{ x^2 \frac{e^{-x/2}}{-1/2} - 2\int \frac{e^{-x/2}}{-1/2} (2x) dx \right\}$$

$$= \frac{1}{16} \left\{ -2x^2 e^{-x/2} + 8 \left\{ -x \frac{e^{-x/2}}{-1/2} - \frac{e^{-x/2}}{-1/2}(1) \right\} \right\}$$

$$= \frac{1}{16} \left\{ -2x^2 e^{-x/2} - 16 x e^{-x/2} - 16 e^{-x/2} \right\}_0^1$$

$$= e^{-x/2} \left\{ -\frac{x^2}{8} - x - 1 \right\}_0^1$$

$$= e^{-1/2} \left\{ -\frac{1}{8} - 1 - 1 \right\}$$

Ans: 0.0143

2) The demand for a certain item follows gamma dist. with mean 8 and variance 32. Find the prob that there will be demand for atleast 10 items.

$\alpha\beta = 8 \qquad \alpha\beta^2 = 32$

$\beta = \frac{32}{8} = 4. \qquad \therefore \alpha = 2.$

$$P(10 \le x < \infty) = \int_{10}^{\infty} \frac{1}{4^2 \sqrt{2}} x^1 e^{-x/4} \, dx.$$

$$= \int_{10}^{\infty} \frac{1}{16(1)} x e^{-x/4} \, dx.$$

$$= \frac{1}{16} \int_{10}^{\infty} x e^{-x/4} \, dx$$

$$= \frac{1}{16} \left\{ x \frac{e^{-x/4}}{-1/4} + 4\int e^{-x/4} (1) \, dx \right\}_{10}^{\infty}$$

$$= \frac{1}{16} \left\{ -4x e^{-x/4} - 16 e^{-x/4} \right\}_{10}^{\infty}$$

$$= \frac{1}{16} \left\{ +4(10) e^{-5/2} + 16 e^{-5/2} \right\}$$

$$= \frac{5}{2} e^{-5/2} + e^{-5/2}$$

$$= e^{-5/2} \left(\frac{5}{2} + 1 \right)$$

$$= e^{-5/2} (7/2) = 0.2873$$

3) The daily consumption of milk in a city, in excess of 20,000 litres, is distributed as a Gamma variate with $\alpha = 2$, $\beta = 10{,}000$. The city has a daily stock of 30,000 litres. What is the prob that the stock is insufficient on a given day.

$X \rightarrow$ daily consumption of milk in the city

$Y = X - 20{,}000 \rightarrow$ gamma variate.

$\alpha = 2, \quad \beta = 10{,}000$

$$P(X > 30{,}000) = P(Y > 10000) = \int_{10{,}000}^{\infty} \frac{1}{10000^2 \sqrt{2}} x^1 (e^{-x/10000}) \, dx$$

$$= \frac{1}{10000^2} \int_{10000}^{\infty} x \, e^{-x/10000} \, dx$$

$$= \frac{1}{10000^2} \left\{ x \frac{e^{-x/10000}}{-1/10000} - \int \frac{e^{-x/10000}}{-1/10000} (1) \, dx \right\}_{10000}^{\infty}$$

$$= \frac{1}{10000^2} \left\{ -10000 x e^{-x/10000} - 10000^2 e^{-x/10000} \right\}_{10000}^{\infty}$$

$$= \frac{1}{10000^2} \left\{ 10000^2 e^{-1} + 10000^2 e^{-1} \right\}$$

$$= 2e^{-1}$$

$$= 0.7357$$

4) The time required to repair an item at a repair shop follows gamma dist with mean 2 hrs and 1 hr² resp. Estimate the prob that the next time ~~to~~ arrive will require less than 1 hr to be repaired.

$\alpha\beta = 2 \qquad \alpha\beta^2 = 1$

$\boxed{\beta = \frac{1}{2}} \qquad \boxed{\alpha = 4}$

$$P(X \le 1) = \int_0^1 \frac{1}{(1/2)^4 \Gamma 4} x^3 e^{-2x}\,dx$$

$$= \frac{16}{6} \int_0^1 x^3 e^{-2x}\,dx$$

$$= \frac{8}{3}\left\{ \frac{x^3 e^{-2x}}{-2} - \int \frac{e^{-2x}}{-2} 3x^2\,dx \right\}_0^1$$

$$= -\frac{4}{3}\left\{ x^3 e^{-2x} - 3\int e^{-2x} x^2\,dx \right\}_0^1$$

$$= -\frac{4}{3}\left\{ x^3 e^{-2x} - 3\left\{ x^2 \frac{e^{-2x}}{-2} - \int \frac{e^{-2x}}{-2}(2x)\,dx \right\}\right\}_0^1$$

$$= -\frac{4}{3}\left\{ x^3 e^{-2x} + \frac{3}{2} x^2 e^{-2x} + \int e^{-2x} x\,dx \right\}_0^1$$

$$= -\frac{4}{3}\left\{ x^3 e^{-2x} + \frac{3}{2} x^2 e^{-2x} + x\frac{e^{-2x}}{-2} - \int \frac{e^{-2x}}{-2}\,dx \right\}_0^1$$

$$= -\frac{4}{3}\left\{ x^3 e^{-2x} + \frac{3}{2} x^2 e^{-2x} + \frac{x}{2} e^{-2x} - \frac{e^{-2x}}{4} \right\}_0^1$$

$$= -\frac{4}{3}\left\{ e^{-2} + \frac{3e^{-2}}{2} - \frac{e^{-2}}{2} - \frac{e^{-2}}{4} + \frac{1}{4} \right\}$$

$$= -\frac{4}{3}\left\{ e^{-2}\left\{1 + \frac{3}{2} - \frac{1}{2} - \frac{1}{4}\right\} + \frac{1}{4} \right\}$$

$$= -\frac{4}{3}\left\{ e^{-2}(1.75) + 0.25 \right\}$$

0.1426

→ Joint Probability distribution

A JPD is used to determine the likelihood of certain events happening, given certain other events happening.

If X & Y are two discrete R V's then the joint prob f^n of X and Y can be defined as

$P(X = x, Y = y) = h(x, y) =$ Prob that x & y occur together.

$h(x,y)$ satisfies 2 cond^n.

i) $h(x,y) \ge 0 \qquad$ ii) $\sum_y \sum_x h(x,y) = 1$

Let $X = \{x_1, x_2, \dots, x_n\}$ & $Y = \{y_1, y_2 \dots y_n\}$

Then, $P(X = x_i, Y = y_j) = h(x_i, y_j) = J_{ij}, \quad 1 \le j \le m, \; 1 \le j \le n$

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J_{ij} is called the joint prob dist of X & Y.

This is written in the form of a table.

X \ Y	y_1	y_2	...	y_n	$f(x)$
x_1	J_{11}	J_{12}		J_{1n}	$f(x_1)$
x_2	J_{21}	J_{22}		J_{2n}	$f(x_2)$
⋮	⋮			⋮	⋮
x_n	J_{n1}	J_{n2}	..	J_{nn}	$f(x_n)$
$g(y)$	$g(y_1)$	$g(y_2)$	...	$g(y_n)$	1

→ Questions:

1) A fair coin is tossed twice. Let X denote 0 or 1 ascending as head or tail appears first. Let Y denote the no. of heads obtained. Write the JPD of X & Y.

	HH	HT	TH	TT
X	0	0	1	1
Y	2	1	1	0

JPD

X \ Y	0	1	2	f(x)
0	0	1/4	1/4	1/2
1	1/4	1/4	0	1/2
g(y)	1/4	1/2	1/4	1

→ Marginal Prob. Dist.

If we add up the rows & columns of a joint prob dist, we get:

$$f(x_i) = J_{i1} + J_{i2} + \cdots + J_{in}$$

$$g(y_j) = J_{1j} + J_{2j} + \cdots + J_{mj}$$

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$$\sum_i f(x_i) = \sum_j g(y_j) = \sum_{j}\sum_{i} J_{ij} = 1$$

$f(x_i)$ & $g(y_j)$ are called the Marginal Prob dist of X & Y.

→ Independent RV's : X & Y are independent iff.

$P(X=x, Y=y) = P(X=x) \cdot P(Y=y)$

(or)

$P(X=x_i, Y=y_j) = P(X=x_i) \cdot P(Y=y_j)$

$\Rightarrow J_{ij} = f(x_i) \cdot g(y_j) \;\forall\, i,j$

→ Expectation, Variance, Co-variance & Correlation:

- $\mu_X = E(X) = \sum_{i=1}^{m} x_i f(x_i)$

  $\mu_Y = E(Y) = \sum_{j=1}^{n} y_j f(y_j)$

  $E(XY) = \sum_j \sum_i x_i y_j J_{ij}$

- Variance : $\sigma_X^2 = \sum_{i=1}^{m} (x_i - \mu_X)^2 f(x_i)$

  $\sigma_Y^2 = \sum_{j=1}^{n} (y_j - \mu_Y)^2 g(y_j)$

- Co-variance : $cov(XY) = E(XY) - E(X)E(Y)$

- Correlation : $\rho(X,Y) = \dfrac{cov(XY)}{\sigma_X \sigma_Y}$ (always b/w -1 to 1)

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(*) Note:

If X and Y are independent, then
$E(XY) = E(X)E(Y)$.
$\Rightarrow cov(XY) = 0$

$\Rightarrow \rho(XY) = 0$

→ Questions:-

1) The JPD of 2 RVS X & Y is given below

X \ Y	-2	-1	4	5	f(x)
1	0.1	0.2	0	0.3	0.6
2	0.2	0.1	0.1	0	0.4
g(y)	0.3	0.3	0.1	0.3	1

Find i) Marginal, ii) Expectectation of X, Y & XY
iii) S.D's of X & Y
iv) Cov(x, y)
v) cor (x, y)
vi) Are x & Y independent?

i) $f(x)$ & $g(y)$.

ii) $\mu_x = \sum_{i=1}^{2} x_i f(x_i) = 1(0.6) + 2(0.4)$
$= 0.6 + 0.8$
$= 1.4$

1.9
-0.9

$\mu_y = -2(0.3) - 1(0.3) + 4(0.1) + 5(0.3)$
$= -0.6 - 0.3 + 0.4 + 1.5$
$= 1$

$E(XY) = 1\left[(-2)(0.1) + (-1)(0.2) + 4(0) + 5(0.3)\right]$
$+ 2\left[-2(0.2) + (-1)(0.1) + 4(0.1) + 5(0)\right]$
$= 0.9$

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iii) $\sigma_x^2 = (1-1.4)^2\, 0.6 + (2-1.4)^2\, 0.4$
$= 0.096 + 0.144$
$= 0.24$

$\sigma_x = 0.4899$

$\sigma_y^2 = (-2-1)^2\, 0.3 + (-1-1)^2\, 0.3 + (4-1)^2\, 0.1 + (5-1)^2\, 0.3$
$= 2.7 + 1.2 + 0.9 + 4.8$
$= 9.6$

$\sigma_y = 3.0983$

iv) $= E(XY) - E(X)E(Y)$

$cov(XY) = 0.9 - 1(1.4)$
$= -0.5$

v) $cor(XY) = \dfrac{cov(XY)}{\sigma_x \sigma_y}$

$= \dfrac{-0.5}{0.4899 \times 3.0983}$ $= \dfrac{-0.5}{1.5179}$

$= -0.3294$

vi) X & Y are not independent.

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Q). Find the correltn b/w X & Y

Y \ X	-1	0	1	g(y)
0	1/15	2/15	1/15	4/15
1	3/15	2/15	1/15	6/15
2	2/15	1/15	2/15	5/15
f(x)	6/15	5/15	4/15	1

$$E(X) = \frac{-6}{15} + 0 + \frac{4}{15} = -2/15$$

$$E(Y) = 0 + \frac{6}{15} + \frac{10}{15} = \frac{16}{15}$$

$$E(XY) = -1\left\{0 + 3/15 + 4/15\right\} + 0 + 1\left\{0 + 1/15 + 4/15\right\}$$

$$= \frac{-7}{15} + \frac{5}{15} = \frac{-2}{15}$$

$$cov(XY) = \frac{-2}{15} - \left(\frac{-2}{15}\right)\left(\frac{16}{15}\right)$$

$$= \frac{-2}{15} + \frac{32}{225} = \frac{-30}{225} + \frac{32}{225}$$

$$= 2/225$$

$$\sigma_X^2 = \left(-1 + 2/15\right)^2 \frac{6}{15} + \left(0 + 2/15\right)^2 \left(\frac{5}{15}\right) + \left(1 + 2/15\right)^2 \frac{4}{15}$$

$$= \left(\frac{-13}{15}\right)^2 \frac{6}{15} + \frac{4}{225}\left(\frac{5}{15}\right) + \left(\frac{17}{15}\right)^2 \frac{4}{15}$$

$$= 0.3004 + 0.0059 + 0.3425$$

$$= 0.6488$$

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$$\sigma_X = 0.8054$$

$$\sigma_Y^2 = \left(0 - 16/15\right)^2 \frac{4}{15} + \left(1 - 16/15\right)^2 \frac{6}{15} + \left(2 - \frac{16}{15}\right)^2 \frac{5}{15}$$

$$= 0.3034 + 0.0017 + 0.2903$$

$$= 0.5954$$

$$\sigma_Y = 0.7716.$$

$$cor(XY) = \frac{2/225}{0.6488 \times 0.7716} = \frac{2}{225 \times 0.5006}$$

$$cor(XY) = 0.0143.$$

JPD for DRV

1) A fair coin is tossed 3 times & the Sample space S is {HHH HHT HTH THH HTT THT TTH TTT}. Let X be 0 or 1 according as a head or tail occurs in the first toss and Y be the total no. of heads that occurs. Find

i) joint dist. of X & Y. ii) cov(X, Y)
iii) If X & Y are indep iv) cor(X, Y)

X \ Y	0	1	2	3	f(x)
0	0	1/8	2/8	1/8	1/2
1	1/8	2/8	1/8	0	1/2
g(y)	1/8	3/8	3/8	1/8	1

X → 0 head.
→ 1 tail.

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$E(X) = 0(1/2) + 1(1/2)$

$= \frac{1}{2}$

$E(Y) = 0 + 1(3/8) + 2(3/8) + 3(1/8)$

$= \cancel{1/2 + 1 + 1} \frac{3}{8}$

$= \frac{12}{8} = \frac{3}{2}$

$E(XY) = 0 + 1\{0 + \frac{2}{8} + \frac{4}{8} + 0\}$

$= \frac{3}{8}$

$cov(XY) = E(XY) - E(X)E(Y)$

$= \frac{3}{8} - \frac{1}{2} \times \frac{3}{2}$

$= \frac{3}{8} - \frac{3}{4}$

$= -\frac{3}{8}$

X & Y are not independent.

$\sigma_X^2 = (0 - 1/2)^2\, 1/2 + (1 - 1/2)^2\, 1/2$

$= \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$

$\sigma_X = \frac{1}{2}$

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$\sigma_Y^2 = (0 - 3/2)^2 \frac{1}{8} + (1 - \frac{3}{2})^2 \frac{3}{8} + (2 - 3/2)^2 \frac{3}{8}$
$+ (3 - 3/2)^2 \frac{1}{8}$

$= \frac{9}{32} + \frac{3}{32} + \frac{3}{32} + \frac{9}{32}$

$= \frac{24}{32} = \frac{3}{4}$ 4×6

$\sigma_Y = \cancel{\frac{2\sqrt{3}}{6}} \cancel{\frac{2}{\sqrt{6}}}\ \frac{\sqrt{3}}{2}$

$cor(XY) = \frac{-3/8}{1/8 \times \sqrt{3}/2} = -2\sqrt{3}$

2. 2 balls are selected at random from a box containing 3 blue, 2 red and 3 green balls. If X is the no. of blue balls and Y is the no. of red balls selected, find

i) Joint prob of X & Y.

ii) $P[(XY) \in A]$ where A is the region $\{(x, y) / x + y \le 1\}$

iii) $cov(XY)$

→ 2 out of 8 balls can be picked in ${}^8C_2$ ways
$= \frac{7 \times 8}{2} = 28$ ways.

X \ Y	0	1	2	f(x)
0	3/28	6/28	1/28	10/28
1	9/28	6/28	0	15/28
2	3/28	0	0	3/28
g(y)	15/28	12/28	1/28	1

2 red 3 Gr.

$J_{11} = P(2G) = \frac{{}^3C_2}{{}^8C_2} = \frac{3}{28}$

$J_{12} = P(1R, 1G) = \frac{{}^2C_1 \times {}^3C_1}{{}^8C_2} = 6/28$

$J_{13} = P(2R) = \frac{{}^2C_2}{{}^8C_2} = 1/28$

$J_{21} = P(1B, 1G) = \frac{{}^3C_1 \times {}^3C_1}{{}^8C_2} = \frac{9}{28}$

$J_{22} = P(1B, 1R) = \frac{{}^3C_1 \times {}^2C_1}{{}^8C_2} = 6/28$

$J_{32} = P(2B) = \frac{{}^3C_2}{{}^8C_2} = \frac{3}{28}$

$E(X) = 0 + 1(15/28) + 2(3/28)$
$= \frac{21}{28}$.

$E(Y) = 0 + 12/28 + 2(1/28)$
$= \frac{14}{28} = 0.5$

$E(XY) = 0 + 1\{0 + 6/28 + 0\} + 2\{0\}$
$= \frac{6}{28}$//.

$cov(XY) = E(XY) - E(X)\,E(Y)$
$= \frac{6}{28} - \frac{21}{28} \times 0.5$

$= \frac{6 - 10.5}{28}$

$= \frac{-4.5}{28}$

$P[(XY) \in A \,/\, x+y \le 1\} = \frac{3}{28} + \frac{3}{14} + \frac{9}{28} = \frac{9}{14}$//

3. 2R, 3W, 4 Black, 3 balls chosen at random
Find cov(X Y).

X \ Y	0	1	2	3	f(x)
0					
1					
2					
3					
g(y)					

4) The JPD for 2 DRVs X & Y is given by $f(x,y) = c(2x+y)$ where X and Y can assume all integer values for $0 \le x \le 2$, $0 \le y \le 3$ & $f(x,y) = 0$ otherwise. Find

i) c

ii) $P(X \ge 1, Y \le 2)$

iii) Marginal dist of X & Y

x \ y	0	1	2	3	f(x)
0	0	c	2c	3c	6c
1	2c	3c	4c	5c	14c
2	4c	5c	6c	7c	22c
g(y)	6c	8c	12c	15c	1

i) $6c + 14c + 22c = 1$

$42c = 1$

$c = \frac{1}{42}$

ii) $P(X \ge 1, Y \le 2) = 2c + 3c + 4c + 4c + 5c + 6c$

$= 24c$

$= \frac{24}{42} = \frac{4}{7}$

iii) Marginal dis $X = \frac{6}{42}, \frac{14}{42}, \frac{22}{42}$

$Y = \frac{6}{42}, \frac{8}{42}, \frac{12}{42}, \frac{15}{42}$

→ Conditional Prob. Dist.

$h(y/x) = \frac{h(x,y)}{f(x)}$

$h(x/y) = \frac{h(x,y)}{g(y)}$

→ Questions:-

1) If X & Y are independent random v. with the dist given by.

X	1	2
f(x)	0.7	0.3

Y	-2	5	8
g(y)	0.3	0.5	0.2

Find the JPD of X & Y. ~~Also~~ find $P(Y=8 / X=1)$

x \ y	-2	5	8	f(x)
1	0.21	0.35	0.14	0.7
2	0.09	0.15	0.06	0.3
g(y)	0.3	0.5	0.2	1

$J_{ij} = f(x_i)\, g(y_j)$

$P(Y=8 / X=1) = \frac{0.14}{0.7} = \frac{1}{5}$

$P(X=1 / Y=8) = \frac{0.14}{0.2} = \frac{7}{10}$

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Q. Two cards are selected at random from a box which contains 5 cards numbered 1, 1, 2, 2, 3. If X denotes the sum and Y the max. of 2 numbers find the JPD of X and Y. Also find cov(X, Y) & cor(X, Y)

X \ Y	1	2	3	f(x)
2	0.1	0	0	0.1
3	0	0.4	0	0.4
4	0	0.1	0.2	0.3
5	0	0	0.2	0.2
g(y)	0.1	0.5	0.4	1

(1,1) (1,2) (1,2) (1,3)
(1,2) (1,2) (1,3)
(2,2) (2,3)
(2,3)

$J_{11} = \frac{{}^2C_2}{{}^5C_2} = \frac{1}{\frac{5\times4}{2\times1}} = \frac{1}{10}$

$J_{12} = \frac{4}{10}$

$E(X) = \Sigma x_i f(x_i)$
$= 2(0.1) + 3(0.4) + 4(0.3) + 5(0.2)$
$= 3.6$

$E(Y) = \Sigma y_i\, g(y_i)$
$= 1(0.1) + 2(0.5) + 3(0.4)$
$= 0.1 + 1.0 + 1.2$
$= 2.3$

$E(XY) = 2\{0.1\times0.1\} + 3\{0.4\times0.5\}$
$+ 4\{0.1\times0.5 + 0.2\times0.4\} + 5\{0.2\times0.4\}$

$= 0.02 + 0.6 + 0.52 + 0.4$
$= 8.8$

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$cov(X, Y) = E(XY) - E(X)\,E(Y)$
$= 8.8 - 3.6\times2.3$
$= 8.8 - 8.28$
$= 0.52$

$\sigma_X^2 = (2-3.6)^2\, 0.1 + (3-3.6)^2\, 0.4$
$+ (4-3.6)^2\, 0.3 + (5-3.6)^2\, 0.2$

$= 0.256 + 0.144 + 0.048 + 0.2$
$= 0.848$

$\sigma_X = 0.9165$

$\sigma_Y^2 = (1-2.3)^2\, 0.1 + (2-2.3)^2\, 0.5 + (3-2.3)^2\, 0.4$
$= 0.169 + 0.045 + 0.196$
$= 0.41$

$\sigma_Y = 0.6403$

$cor = \frac{0.52}{0.9165\times0.6403} = \frac{0.52}{0.5868}$
$= 0.8861$

→ JPD for CRV

If X & Y are 2 continuous R.V and $f(x,y)$ is a real valued f^n s.t $f(x,y)\geq 0$ & $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$, then $f(x,y)$ is called the JPD of X & Y

$P(a\leq x\leq b,\ c\leq y\leq d) = \int_c^d\int_a^b f(x,y)\,dx\,dy$

Marginal Distr.

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$$P(X \le x) = F_1(x) = \int_{-\infty}^{x}\int_{-\infty}^{\infty} f(x,y)\,dy\,dx$$ is the marginal dis of x

$$P(Y \le y) = F_2(y) = \int_{-\infty}^{y}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy$$

- Expectation, Variance and Covariance

$$\mu_x = E(X) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\, f(x,y)\,dx\,dy$$

$$\mu_y = E(Y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y\, f(x,y)\,dx\,dy$$

$$\sigma_x^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x-\mu_x)^2 f(x,y)\,dx\,dy = E\{(X-\mu_x)^2\}$$

$$\sigma_y^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (y-\mu_y)^2 f(x,y)\,dx\,dy = E\{(Y-\mu_y)^2\}$$

$$\operatorname{cov}(X,Y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x-\mu_x)(y-\mu_y) f(x,y)\,dx\,dy = E(XY) - E(X)E(Y)$$

If X & Y are independent, $\operatorname{cov}(X,Y) = 0$.

→ Questions

1) The JPD fn of X & Y is

$$f(x) = \begin{cases} cxy, & 0 < x < 4,\ 1 < y < 5 \\ 0, & \text{otherwise} \end{cases}$$ find

i) c ii) $P(1 < x < 2,\ 2 < y < 3)$
iii) Marginal distr fn of X & Y.

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$$\Rightarrow \int_1^5\int_0^4 cxy\,dx\,dy = 1$$

$$\Rightarrow c \left.\frac{x^2}{2}\right|_0^4 \left.\frac{y^2}{2}\right|_1^5 = 1$$

i) $$\Rightarrow c\left(\frac{16}{2}\right)\left\{\frac{25}{2} - \frac{1}{2}\right\} = 1$$

$$\Rightarrow c(8)(12) = 1$$

$$\Rightarrow 96c = 1$$

$$c = \frac{1}{96}$$

ii) $$\int_2^3\int_1^2 cxy\,dx\,dy$$

$$= \frac{1}{96} \times \left.\frac{x^2}{2}\right|_1^2 \left.\frac{y^2}{2}\right|_2^3$$

$$= \frac{1}{96} \times \left(\frac{4}{2} - \frac{1}{2}\right) \times \left(\frac{9}{2} - \frac{4}{2}\right)$$

$$= \frac{1}{\cancel{96}_{32}} \times \frac{\cancel{3}}{2} \times \frac{5}{2} = \frac{5}{128}$$

iii) $$F_1(x) = \int_0^x\int_1^5 cxy\,dy\,dx$$

$$= \int_0^x cx \left.\frac{y^2}{2}\right|_1^5 dx$$

$$= \int_0^x cx\left\{\frac{25}{2} - \frac{1}{2}\right\} dx$$

$$= \frac{12}{96}\int_0^x x\,dx$$

$$= \frac{12}{96} \left.\frac{x^2}{2}\right|_0^x$$

$$= \frac{x^2}{16}$$

$$F_1(x) = \begin{cases} 0 & , x < 0 \\ x^2/16 & , 0 < x < 4 \\ 1 & , x > 4 \end{cases}$$

$$F_2(y) = \int_1^6 \int_0^4 cxy \, dx \, dy.$$

$$= \int_1^y cy \left.\frac{x^2}{2}\right|_0^4 dy$$

$$= \int_1^y cy \, 16 \, dy.$$

$$= \frac{16}{96 \times 2} \int_1^y y \, dy.$$

$$= \frac{1}{12} \left.\frac{y^2}{2}\right|^y$$

$$= \frac{1}{12} \left\{ \frac{y^2}{2} - \frac{1}{2} \right\}.$$

$$= \frac{y^2 - 1}{24}$$

$$F_2(y) = \begin{cases} 0 & , y < 1 \\ y^2 - 1/24 & , 1 < y < 5 \\ 1 & , y > 5 \end{cases}$$

2) If X & Y are CRV's having JPD f^n

$$f(x,y) = \begin{cases} c(x^2+y^2) & , 0 \le x, y \le 1 \\ 0 & , \text{otherwise.} \end{cases}$$ Find

i) C.

ii) $P(X < \frac{1}{2}, Y > 1/2)$

iii) $P((1/4 < x < 3/4)$

iv) $P(Y < 1/2)$

i) $$\int_0^1 \int_0^1 c(x^2+y^2) \, dx \, dy. = 1$$

$cx^2 + cy^2$

$$\left.\frac{cx^3}{3}\right|_0 + \left.\frac{cy^3}{3}\right|_{-\infty} = 1$$

X Wrong Integration

$$c\left\{\frac{1}{3} + \frac{1}{3}\right\} = 1$$

$$c = \frac{3}{2}.$$

ii) $$\int_{1/2}^1 \int_0^{1/2} c(x^2+y^2). \, dx \, dy.$$

$$\cancel{\frac{3}{2}\left\{\left.\frac{x^3}{3}\right|_0^{1/2} + \left.\frac{y^3}{3}\right|_{1/2}^1\right\}}.$$

$$\cancel{= \frac{3}{2}\left\{\frac{1}{24} + \frac{1}{3} - \frac{1}{24}\right\}}.$$

$$= c \int_{1/2}^1 \left.\frac{x^3}{3} + xy^2\right|_0^{1/2} dy.$$

$$= c \int_{1/2} \frac{1}{24} + \frac{y^2}{2} \, dy.$$

$= c\left\{\frac{y}{24} + \frac{y^3}{6}\right|_{1/2}^{1}\Big\}$

$= c\left\{\frac{1}{24} + \frac{1}{6} - \frac{1}{48} - \frac{1}{48}\right\}$

$= c \times \frac{1}{6}$

$= \frac{3}{2} \times \frac{1}{6}$

$= \frac{1}{4}$

iii) $P(1/4 \le x \le 3/4)$

$= \frac{3}{2}\int_{y=0}^{1}\int_{x=1/4}^{3/4} x^2+y^2\,dx\,dy$

$= \frac{3}{2}\int_{0}^{1} \frac{x^3}{3}+xy^2\Big|_{1/4}^{3/4} dy$

$= \frac{3}{2}\int_{0}^{1} \frac{27}{192}+\frac{3y^2}{4} - \frac{1}{192} - \frac{1}{4}y^2\,dy$

$= \cancel{\frac{3}{2}\left\{\frac{27y}{192}+\frac{y^3}{4}\Big|_0^1\right\}} = \frac{3}{2}\int_0^1 \frac{26}{192}+\frac{1}{2}y^2\,dy$

$= \cancel{\frac{3}{2}\left\{\frac{27}{192}+\frac{1}{4}\right\}} = \frac{3}{2}\left[\frac{26y}{192}+\frac{y^3}{6}\right]_0^1$

$= \cancel{\frac{3}{2}\left\{\frac{9}{64}+\frac{16}{64}\right\}} = \frac{3}{2}\left\{\frac{26}{192}+\frac{1}{6}\right\}$

$= \frac{3}{2} \times \frac{?}{64}\left\{\frac{13}{96}+\frac{16}{96}\right\}$

$= \frac{3}{2} \times \frac{29}{96}$

$= \frac{29}{64}$

iv) $P(Y<1/2) = \frac{3}{2}\int_0^{1/2}\int_0^1 x^2+y^2\,dx\,dy$

$= \frac{3}{2}\int_0^{1/2} \frac{x^3}{3}+xy^2\Big|_0^1 dy$

$= \frac{3}{2}\int_0^{1/2} \frac{1}{3}+y^2\,dy$

$= \frac{3}{2}\left\{\frac{y}{3}+\frac{y^3}{3}\right\}_0^{1/2}$

$= \frac{3}{2}$

$= \cancel{\frac{3}{2}\left\{\frac{64}{3}\times\frac{1}{2}+\frac{1}{3}\times\frac{1}{8}\right\}}$

$= \frac{3}{2}\left\{\frac{32}{3}+\frac{1}{6}\right\}$

$= \frac{3}{2}\left\{\frac{64+1}{6}\right\}$

$= \frac{3}{2}\times\frac{65}{62}$

$= \frac{35}{4}$