MODEL QUESTION PAPER

Course & Branch	**: BE (OPEN ELECTIVE)**	**Semester:**	**V**
Subject	**: Applied Graph Theory**	**Max. Marks:**	**100**
Subject Code	**: MA0E04**	**Duration:**	**3Hrs**

Instructions to the candidates:

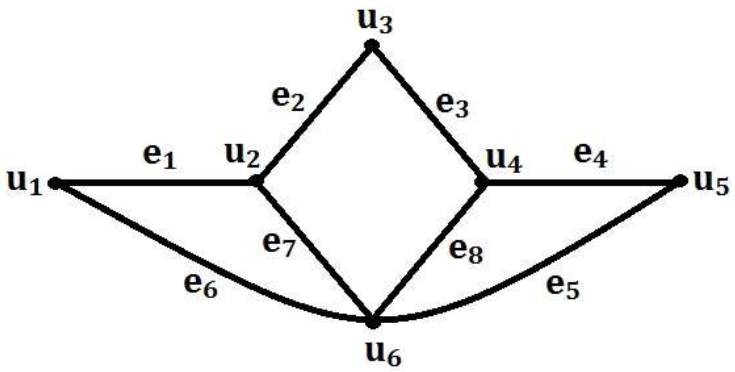
Answer ONE full question from each unit.

UNIT-I

1. a) Define a minimally connected graph with an example. CO1 (02)

   b) 11 Members of a new club meet each day for lunch at a round table. They decide to sit such that every member has different neighbors at each lunch. How many days can this arrangement last? Justify. CO1 (04)

   c) From the given graph G(**V,E**), find CO1 (07)
      i) Fusion of u_1 and u_5
      ii) G – u_6
      iii) G – e_8
      iv) Edge disjoint sub graphs of G
      v) Decomposition of G
      vi) Complement of G
      vii) Bipartites of **V** if exists



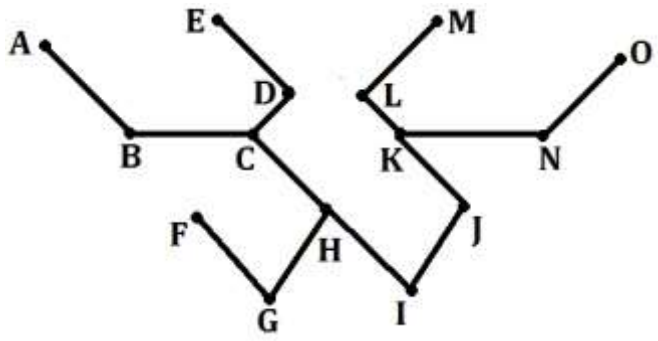
   d) Prove that CO1 (07)
      i) In a binary tree, the number of vertices is always odd.
      ii) In a binary tree with n vertices, the number of pendent vertices is (n+1)/2

2. a) Show that it is not possible to have a set of seven persons such that each person in the set knows exactly three other persons in the set. CO1 (02)

   b) Define graph isomorphism with an example. CO1 (04)

   c) With proper justification, draw CO1 (07)
      i) A connected graph that becomes disconnected when any edge is removed from it
      ii) A graph in which Euler line is also a Hamiltonian circuit
      iii) A graph that has a Hamiltonian path but no Hamiltonian circuit
      iv) A 4-regular graph which is not a complete graph

d) Find eccentricity of all vertices of tree T and hence find its centre, radius and diameter. CO1 (07)



UNIT-II

3. a) Define fundamental circuit with an example. CO2 (02)

b) Define planar graph and show that the complete graph K_5 is non-planar. CO2 (04)

c) Eight cities A, B, C, D, E, F, G, H are required to be connected by a new railway network. The possible tracks and the cost involved to lay them (in crores of rupees) are given below. Determine a railway network of minimal cost that connects all these cities. CO2 (07)

Track between	Cost		Track between	Cost
A and B	158		D and F	99
A and D	146		E and F	150
A and G	120		F and G	135
B and C	146		F and H	150
C and D	150		G and H	175
C and E	96		-	-

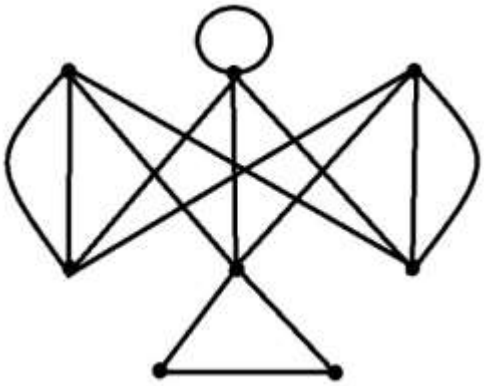
d) Show with an example that ring sum of any two cut-sets in a graph is either a third cut-set or an edge disjoint union of cut-sets. CO2 (07)

4. a) Let G be a 4-Regular connected planar graph having 16 edges. Find the number of regions of G. CO2 (02)

b) Define rank and nullity of a graph with an example. CO2 (04)

c) Write a note on (i) Edge connectivity (ii) Vertex connectivity (iii) Separable graphs & (iii) Euler's fundamental theorem on Planar graphs CO2 (07)

d) Explain the procedure of elementary reduction in the detection of Planarity and hence show that the following graph is non-planar. CO2 (07)

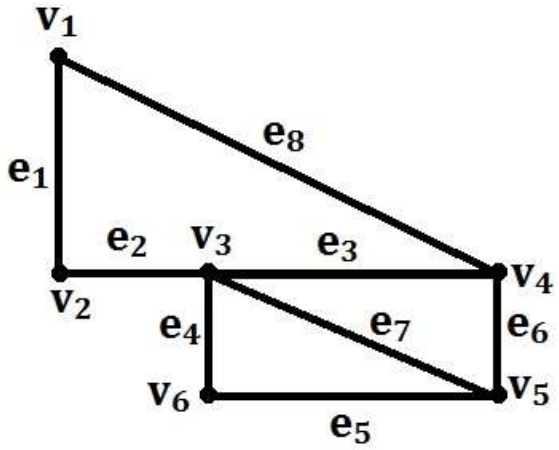


UNIT-III

5. a) Define Rank of Incidence matrix. CO3 (02)

b) Define cut-set matrix with an example. CO3 (04)

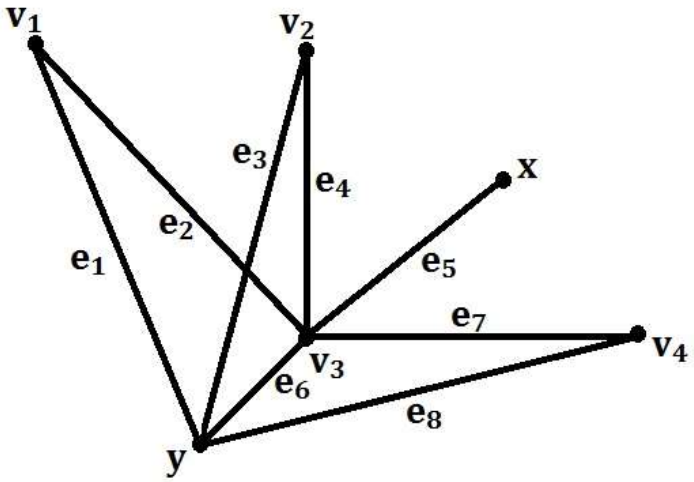
c) Write down the fundamental circuit matrix B_f for the following graph. Also express $B_f = [I_\mu : B_t]$ by taking any spanning tree T of G. CO3 (07)



d) Construct a graph whose adjacency matrix is given below. CO3 (07)

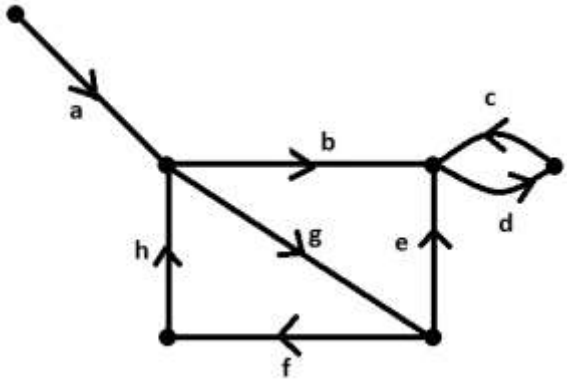
$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

6.	a)	Find the adjacency matrix of complete graph K_4	CO3	(04)
	b)	List the two properties associated with the circuit matrix with an example	CO3	(06)
	c)	If the edges of the following graph are arranged in the same order for the columns of the incidence matrix A and the path matrix $P(x,y)$ then show that $A \cdot P^T(x,y) = M$ in the product (mod 2) where the matrix M has 1's in two rows x & y and the rest rows are all 0's.	CO3	(10)



UNIT-IV

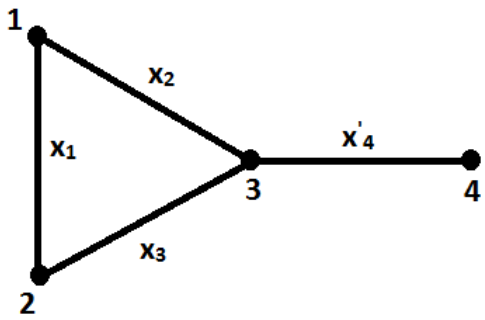
7.	a)	With an example, show that the sum of all out-degrees is equal to sum of all in-degrees in a digraph.	CO4	(02)
	b)	Define transitive directed graph with an example.	CO4	(04)
	c)	Define each of the following with an example i) Strongly connected digraph ii) Fundamental circuits in digraph iii) Arborescence	CO4	(07)
	d)	i) Define incidence matrix of a digraph with an example ii) Show with an example that ring sum of two directed circuits is not necessarily another directed circuit	CO4	(07)
8.	a)	Define complete symmetric digraph with an example.	CO4	(02)
	b)	Write a note on Euler digraphs.	CO4	(04)
	c)	Construct fundamental circuit matrix for the following digraph.	CO4	(07)



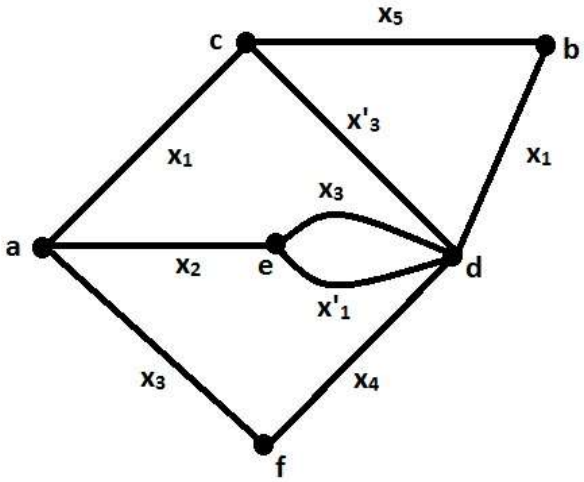
d) Prove that a digraph is an Euler digraph if and only if G is connected and is balanced. CO4 (07)

UNIT-V

9. a) i) Write the transmission matrix for the contact network CO5 (10)



ii) Define primitive connection of an n-vertex contact network and construct the same for the following contact network.



b) The transition matrix of a three state Markov Process is given below. CO5 (12)

$$P = \begin{array}{c} \\ \text{S1} \\ \text{S2} \\ \text{S3} \end{array} \begin{array}{c} \begin{array}{ccc} \text{S1} & \text{S2} & \text{S3} \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0.4 & 0.6 & 0 \\ 0.5 & 0.5 & 0 \end{bmatrix} \end{array}$$

i) Determine the transition digraph
ii) Find the probability of traversing from S3 to S2 in all possible cases with the directed edge sequences of length four and also show that the sum of these probabilities is equal to the entry in the (3,2) position of P^4

10. a) Write a note on the applications of graphs in coding theory. CO5 (10)
b) Explain how graph theory can be used to analyse the electrical networks. CO5 (10)