UNIT-5

Linear programming problems

Part of operations Research

* LPP deals with optimisation of a linear function subject to linear constraints.

Formulation:

A problem needs to be presented in a LPP form which requires defining the variables involved, establishing relation b/w the variables & formulating the objective function & constraints.

Graphical Method:

If a LPP contains only two decision variables, we can apply graphical method.

Working procedure:

① Formulate the given LPP, then convert constraints into equations and plot each equation in X-Y plane and determine the convex region formed by the equations & find the optimal values at each point of the convex polygon.

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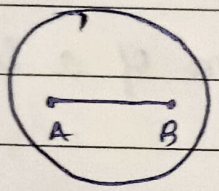
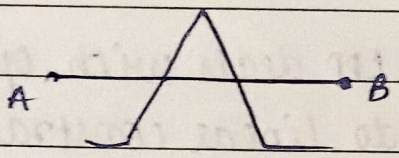
Convex region:

A region (convex) is a set of points that lies on a line which completely lies with in the region.



Convex region



Not a convex region.

* The XYZ company during the festival season combines two factors A and B to form a gift pack which must weigh 5 kg. At least 2 kg of A and not more than 4 kg of B should be used. The net profit contribution to the company is Rs. 5 per kg for A and Rs. 6 per kg for B. Formulate LPP model to find the optimal factor mix.

Let $A = x_1$ & $B = x_2$] → decision variables

$x_1 + x_2 = 5$

$x_1 \geq 2$ & $x_2 \leq 4$

$\Rightarrow 2 \leq x_1$

$x_2 \leq 4$ } constraints

Max Z (profit) $= 5x_1 + 6x_2$ ⇒ Maximization fn.

$x_1 \geq 0, x_2 \geq 0$ } non-negative restrictions

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(12)th Q.B. * A manufactures produces 2 types of models M1 & M2. Each model of the type M1 requires 4 hrs of grinding & 2 hrs of polishing; whereas each model of the type. M2 requires 2 hrs of grinding & 5 hrs of polishing. the manufactures has 2 grinders & 3 polishers. .....

	grinding	polishing
M1	4	2
M2	2	5

No. of grinders G = 2

No. of polishers P = 3

$M_1 = x_1$ } decision variables.
$M_2 = x_2$ }

1 Grinder = 40hrs 1polishers = 60 hrs

2 Griders = 80hrs 3 polishers = 180hrs.

Maximize profit. (z) = $3x_1 + 4x_2$.

$4x_1 + 2x_2 \leq 80$

$2x_1 + 5x_2 \leq 180$

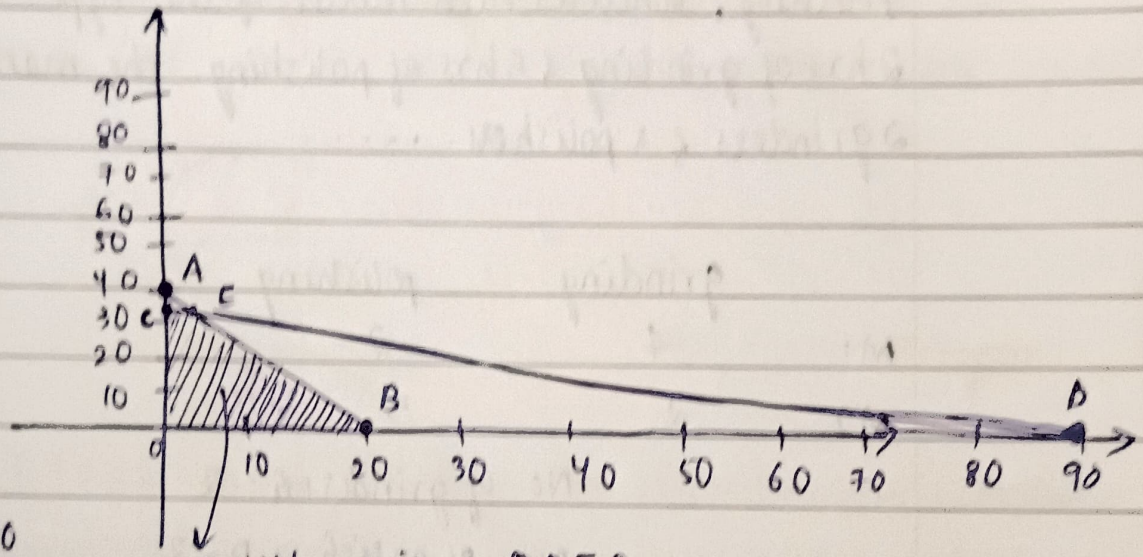
$x_1 \geq 0, x_2 \geq 0$ } non-negative constraints

Graphical method:

convert the inequalities to equalities

$4x_1 + 2x_2 = 80$ A(0,40), B(20,0)

$2x_1 + 5x_2 = 180$ C(0,36), D(90,0)



$4x_1 + 2x_2 = 80$

$-4x_1 - 10x_2 = -360$

feasible region: OBEC

$-8x_2 = -280$

$x_2 = 35$

$x_1 = \frac{5}{2} = 2.5$

put	$Z = 3x_1 + 4x_2$
O (0,0)	0
B (20,0)	60
E (2.5, 35)	147.5
C (0,36)	144

Max profit (z) $= 3x_1 + 4x_2 = 147.5$

$x_1 = 2.5$

$x_2 = 35$

32. I st column

iii)

Max $Z = 6x_1 + 11x_2$

$2x_1 + x_2 \leq 104$

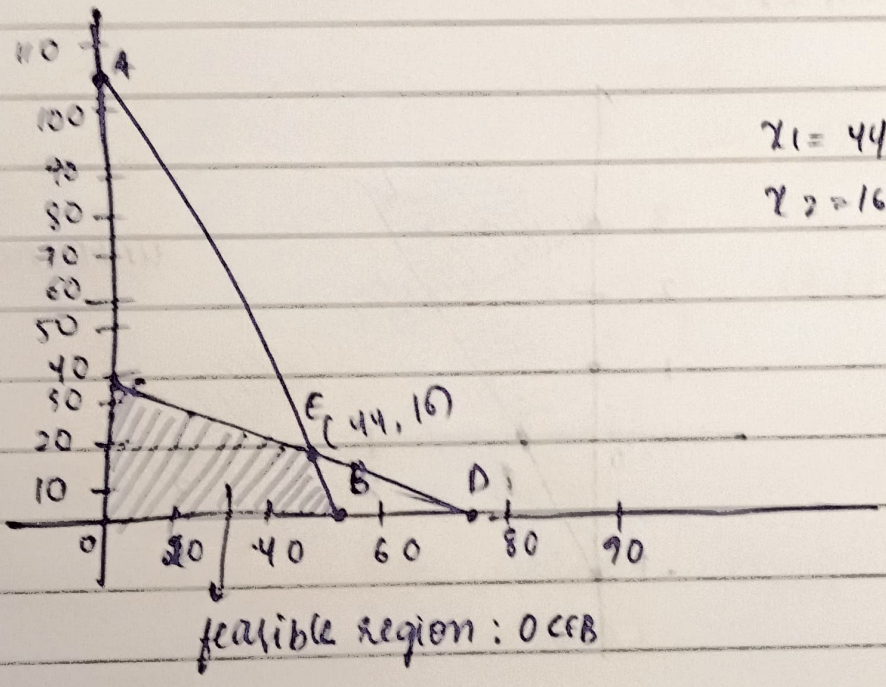
$x_1 + 2x_2 \leq 76$

$x_1, x_2 \geq 0$

Graphical method:

$2x_1 + x_2 = 104$, $x_1 + 2x_2 = 76$

A(0,104), B(52,0) C(0,38), D(76,0)



$x_1 = 44$

$x_2 = 16$

feasible region : OCEB

put	$Z = 6x_1 + 11x_2$
O(0,0)	0
C(0,38)	418
E(44,16)	440
B(52,0)	312

Max profit = 440

$x_1 = 44$

$x_2 = 16$

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* Max $Z = x_1 + 2x_2$

Subject to

$x_1 + x_2 \geq 3$

$x_1 - x_2 \leq 1$

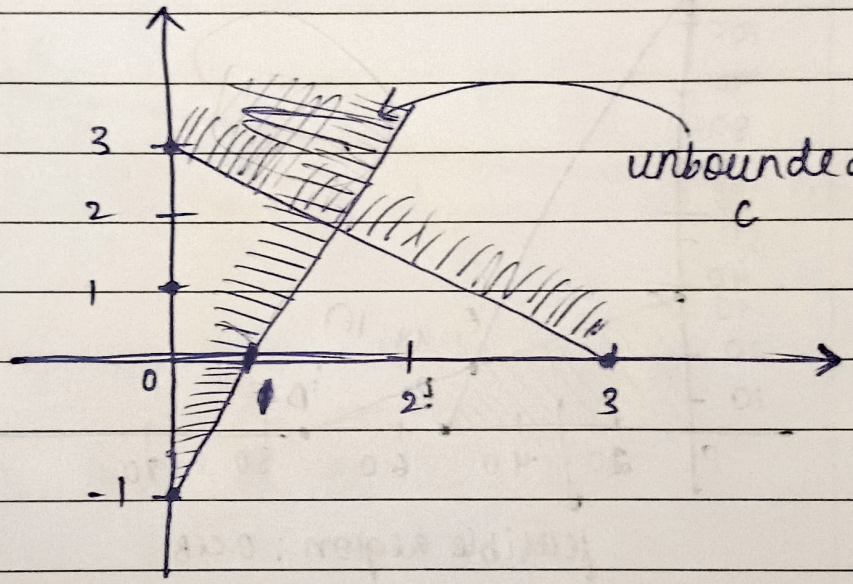
$x_1, x_2 \geq 0$

$x_1 + x_2 = 3$ —①

$x_1 - x_2 = 1$ —②

A(3,0), B(0,3)

C(1,0), D(0,−1)



* Solve graphically,

Max $Z = 2x_1 + 3x_2$

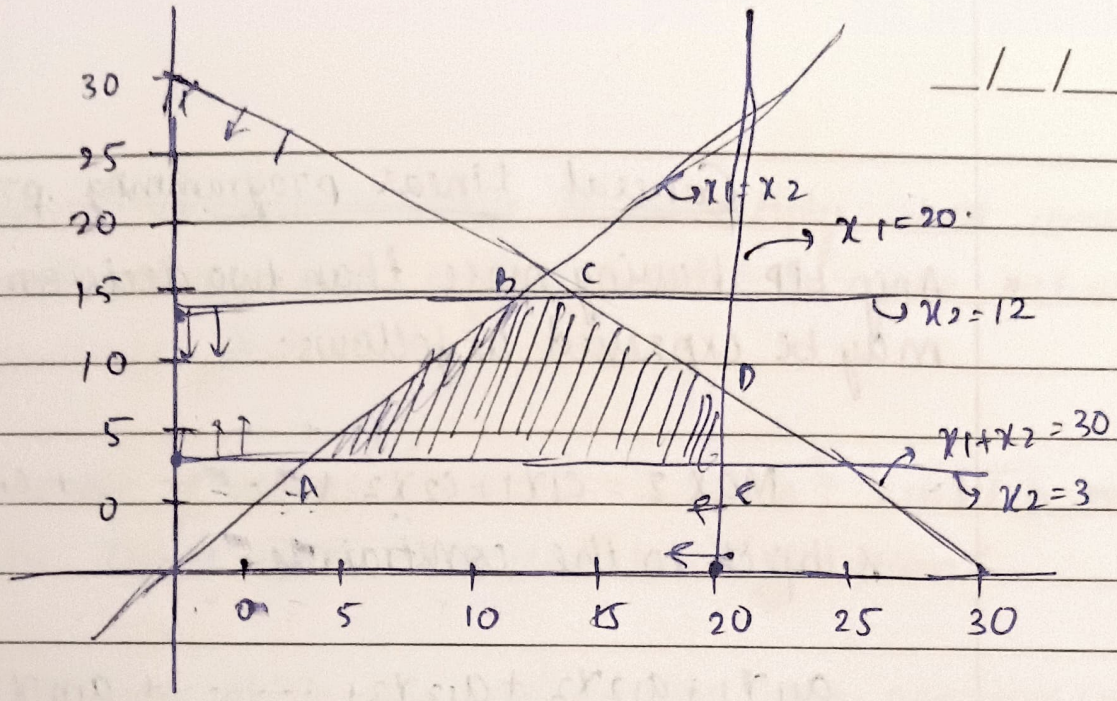
$x_1 + x_2 \leq 30$ —① (30,0)(0,30)

$x_1 - x_2 \geq 0$ —② (0,0) $\Rightarrow x_1 = x_2$

$x_2 \geq 3$ —③ (0,3)

$x_2 \leq 12$ —④ (0,12)

$x_1 \leq 20$ —⑤ (20,0)



A ⇒ $x_1 = x_2$, $x_2 = 3$ } (3,3)

B ⇒ (12, 12)

C ⇒ (18, 12)

D ⇒ (20, 10)

E ⇒ (20, 3)

put	$Z = 2x_1 + 3x_2$	Decision
A(3,3)	15	Min profit.
B(12,12)	60	
C(18,12)	(72)	Max profit
D(20,10)	70	
E(20,3)	49	