Miscellaneous Problems

Normal Distribution as limiting case of Binomial distribution

Normal distribution is limiting case of Binomial distribution when n is very large and neither p nor q is very small.

1) Find the probability of getting 3 heads to 6 heads in 10 tosses of a fair coin using
   (i) binomial distribution
   (ii) the normal approximation to the binomial distribution

Soln: $p = \frac{1}{2}$; $q = \frac{1}{2}$; $n = 10$

$$P(X = x) = n_{C_x}\, p^x q^{n-x}$$

$$P\begin{pmatrix}\text{Getting atleast} \\ \text{3 heads}\end{pmatrix} = P(3 \le X \le 6)$$

$$= P(3) + P(4) + P(5) + P(6)$$

$$\therefore P\begin{pmatrix}\text{Getting atleast} \\ \text{3 heads}\end{pmatrix} = 0.7734$$

If we take data as continuous, it follows that 3 to 6 heads can be considered as 2.5 to 6.5 heads.

Mean is $\boldsymbol{\mu = np = 10\left(\frac{1}{2}\right) = 5}$

Standard deviations is $\boldsymbol{\sigma = \sqrt{npq} = 1.58}$

If $\boldsymbol{X}$ is a normal variate, then standard normal variable is $\boldsymbol{Z = \frac{X-\mu}{\sigma}}$

When

$$\boldsymbol{X = 2.5,\ \ Z = \frac{2.5-5}{1.58} = -1.58}$$

$$\boldsymbol{X = 6.5,\ \ Z = \frac{6.5-5}{1.58} = 0.95}$$

Now, required probability is

$$\boldsymbol{P(2.5 \le X \le 6.5) = P(-1.58 < Z < 0.95)}$$

$$= \boldsymbol{P(-1.58} < Z < 0) + \boldsymbol{P(0} < Z < 0.95)$$

$$= \boldsymbol{P(0} < Z < \boldsymbol{1.58}) + \boldsymbol{P(0} < Z < 0.95)$$

$$\boldsymbol{= A(1.58) + A(0.95)}$$

$$\boldsymbol{= 0.4429 + 0.3289 = 0.7718}$$

Fitting of Binomial distribution

$$P(X = x) = n_{C_x} p^x q^{n-x}$$

Recurrence formula:

If X is a binomial variate, then

$$P(x + 1) = \left(\frac{n - x}{x + 1}\right)\left(\frac{p}{q}\right)P(x)$$

1. Five unbiased coins are tossed and numbers of heads are noted. The experiment is repeated 64 times and the following distribution is obtained.

No. of heads	0	1	2	3	4	5	Total
Frequencies	3	6	24	26	4	1	64

Soln: $p = \frac{1}{2}\ ; q = \frac{1}{2}\ ; n = 5; N = 64$

$$P(X = x) = n_{C_x} p^x q^{n-x}$$

Expected frequencies: $f(x) = NP(x)$

$$P(X = 0) = P(0) = n_{C_0} p^0 q^{n-0} = q^5 = \frac{1}{32}$$

Expected frequency is

$$f(0) = NP(0) = 64\left(\frac{1}{32}\right) = 2$$

Wkt

$$P(x+1) = \left(\frac{n-x}{x+1}\right)\left(\frac{p}{q}\right)P(x)$$

$$P(1) = \left(\frac{5-0}{0+1}\right)(1)P(0) = \frac{5}{32}$$

Expected frequency is

$$f(1) = NP(1) = 64\left(\frac{5}{32}\right) = 10$$

X	Probabilities $P(x)$	Expected frequencies $f(x) = NP(x)$	Observed Frequencies
0	1/32	2	3
1	5/32	10	6
2	10/32	20	24
3	10/32	20	26
4	5/32	10	4
5	1/32	2	1
Total		64	64

Note: Probabilities can be computed directly without using the recurrence relation