



SEMESTER END EXAMINATIONS – AUGUST 2024

Program	: B.E. – CSE (Cyber Security) / CSE (Artificial Intelligence and Machine Learning)	Semester	: IV
Course Name	: Design and Analysis of Algorithms	Max. Marks	: 100
Course Code	: CY43 / CI43	Duration	: 3 Hrs

Instructions to the Candidates:

- Answer one full question from each unit.

UNIT - I

1. a) *Alternating disks* You have a row of $2n$ disks of two colors, n dark and n light. They alternate: dark, light, dark, light, and so on. You want to get all the dark disks to the right-hand end, and all the light disks to the left-hand end. The only moves you are allowed to make are those that interchange the positions of two neighbouring disks. CO1 (06)



Design an algorithm for solving this puzzle and determine the number of moves it takes.

- b) Solve the following: CO1 (08)
- Consider the following $f(n)$ and $g(n)$
 $f(n) = 3n + 5$
 $g(n) = n$
 - Prove that $f(n) \in O(n)$. Compute c and n_0 .
 - Consider the following $f(n)$ and $g(n)$
 $f(n) = 2n^2 + 24n + 188$
 $g(n) = n^2$

Prove that $f(n) \in O(n^2)$. Compute c and n_0 .

- c) **ALGORITHM** $S(n)$ CO1 (06)
- ```

//Input: A positive integer n
//Output: The sum of the first n cubes
if n = 1 return 1
else return S(n - 1) + n * n * n

```

For the algorithm given,

- Set up a recurrence relation for the number of times the algorithm's basic operation is executed.
  - Solve the recurrence relation using substitution method.
2. a) Define the asymptotic bounds for algorithms with an example. Explain two kinds of analyzing the efficiency of an algorithm. CO1 (06)
- b) Given the following asymptotic functions prove that it belongs to  $O(n^2)$ ,  $O(n^3)$  CO1 (06)
- $f(n) = 3n^2 + 2n + 2$   
 $f(n) = 8n^3 + 6n + 3$
- c) Given a set of preferences among hospitals and med-school students. Input. A set of  $n$  hospitals  $H$  and a set of  $n$  students  $S$ . CO3 (08)

|         | favorite | least favorite  |                 |                 | favorite | least favorite  |                 |                 |
|---------|----------|-----------------|-----------------|-----------------|----------|-----------------|-----------------|-----------------|
|         | ↓        | 1 <sup>st</sup> | 2 <sup>nd</sup> | 3 <sup>rd</sup> | ↓        | 1 <sup>st</sup> | 2 <sup>nd</sup> | 3 <sup>rd</sup> |
| Atlanta |          | Xavier          | Yolanda         | Zeus            | Xavier   | Boston          | Atlanta         | Chicago         |
| Boston  |          | Yolanda         | Xavier          | Zeus            | Yolanda  | Atlanta         | Boston          | Chicago         |
| Chicago |          | Xavier          | Yolanda         | Zeus            | Zeus     | Atlanta         | Boston          | Chicago         |

### hospitals' preference lists

|         |         |         |      |
|---------|---------|---------|------|
| Atlanta | Xavier  | Yolanda | Zeus |
| Boston  | Yolanda | Xavier  | Zeus |
| Chicago | Xavier  | Yolanda | Zeus |

### students' preference lists

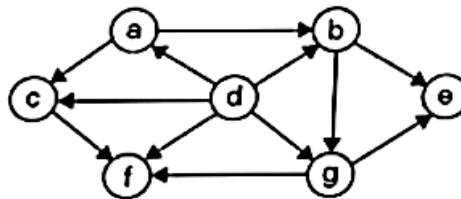
|         |         |         |         |
|---------|---------|---------|---------|
| Xavier  | Boston  | Atlanta | Chicago |
| Yolanda | Atlanta | Boston  | Chicago |
| Zeus    | Atlanta | Boston  | Chicago |

Obtain the following:

- Perfect Matching
- Stable Matching
- Hospital Optimal.

## UNIT - II

- Apply merge sort algorithm to the following elements {20, 10, 5, 15, 25, 30, 50, 35} using divide and conquer method. CO2 (08)
  - Explain how BFS can be used to check connectness of a graph and also to find the number of components in a graph. CO2 (08)
  - Distinguish between BFS and DFS. CO2 (04)
- Discuss how Quicksort works to sort the list {M,E,R,G,E,S,O,R,T} in alphabetical order. Also derive its best, worst and average case time complexity. CO2 (07)
  - Apply the DFS-based algorithm to solve the topological sorting problem for the following digraph, considering source vertex as "a". CO2 (07)



- Write the recurrence relation for merge-sort algorithm, Analyze the running time of merge-sort using Masters theorem. CO2 (06)

## UNIT - III

- Construct a heap for the list 1, 8, 6, 5, 3, 7, 4 by the bottom-up algorithm. Specify the property of building Max Heap. Maximum Key Deletion from a heap. CO3 (06)
  - Compute the shortest path from source node to all other nodes for the following graph using Dijkstra's algorithm. Describe the algorithm by demonstrating them relaxation procedure. CO2 (07)
  - Consider the Knapsack instance  $n=7$ ,  $m=15$ , and  $(P_1, P_2, \dots, P_7) = (10, 5, 15, 7, 6, 18, 3)$  and  $(W_1, W_2, \dots, W_7) = (2, 3, 5, 7, 1, 4, 1)$ . Find an optimal solution to the Fractional knapsack problem making use of greedy approach. CO4 (07)
- Apply Kruskal's algorithm to the graph given in Fig 6(a). Include in the priority queue all the vertices not already in the tree. CO4 (07)

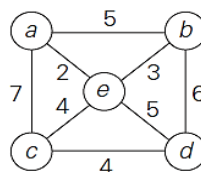


Fig. 6(a) Graph

- b) Construct a Huffman code for the data given in Table 6b.

CO4 (07)

Table 6b: Data Table

| Character | Frequency |
|-----------|-----------|
| a         | 5         |
| b         | 9         |
| c         | 12        |
| d         | 13        |
| e         | 16        |
| f         | 45        |

- Construct a Huffman Tree and compute the codeword for all the characters.
  - Encode "adadcabdcdaefabcde" using the codeword computed.
- c) Solve the following instances of the single-source shortest-paths problem with vertex *a* as the source, for the graph given in Fig 6(c).

CO4 (06)

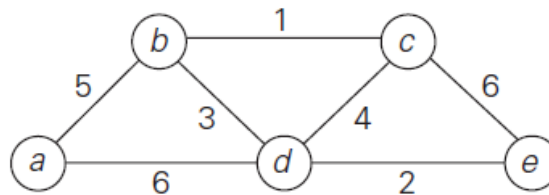


Fig. 6(c) Graph

## UNIT- IV

7. a) For the set of items given in Table 7a and knapsack capacity = 5 kg, find the optimal solution for the 0/1 knapsack problem making use of dynamic programming approach.

CO4 (08)

Table 7a: Items and values

| Item | Weight | Value |
|------|--------|-------|
| 1    | 2      | 3     |
| 2    | 3      | 4     |
| 3    | 4      | 5     |
| 4    | 5      | 6     |

- b) Solve the all-pairs shortest-path problem using Floyd's algorithm for the digraph with the weight matrix given in Fig 7(b).

CO4 (06)

$$\begin{bmatrix} 0 & 2 & \infty & 1 & 8 \\ 6 & 0 & 3 & 2 & \infty \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & \infty & \infty & \infty & 0 \end{bmatrix}$$

Fig. 7(b) Weight Matrix

- c) Describe the memorization Approach used in Dynamic Programming by illustrating the recursive Fibonacci function. Construct the control stack and illustrate the process of Memorization table.

CO4 (06)

```
int fibonacci(int n) {
 if(n == 0)
 return 0;
 else if(n == 1)
 return 1;
 else
 return (fibonacci(n-1) + fibonacci(n-2));
}
```

8. a) Differentiate between Dynamic Programming and Divide and conquer techniques. CO4 (06)
- b) Apply the shortest-augmenting path algorithm to find a maximum flow and a minimum cut in the network given in Fig. 8(b). Compute the Maximum Flow. CO4 (07)

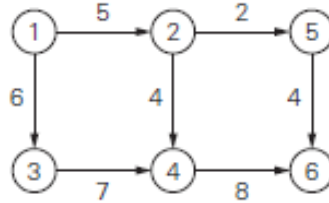


Fig 8(b) Flow Network

- c) Identify and compute the optimal set of intervals with maximum value for the set of intervals and weights given in Table 8c. CO4 (07)

Table 8c Weighted Interval

| Interval | Start time | Finish time | Weight |
|----------|------------|-------------|--------|
| 1        | 1          | 6           | 10     |
| 2        | 2          | 4           | 3      |
| 3        | 5          | 9           | 4      |
| 4        | 7          | 14          | 20     |
| 5        | 11         | 18          | 2      |

## UNIT - V

9. a) Explain the approximation algorithms for NP-hard problems. CO5 (07)
- b) Apply the branch-and-bound algorithm to solve the traveling salesman problem for the graph given in Fig. 9(b) considering 'a' source vertex: CO5 (07)

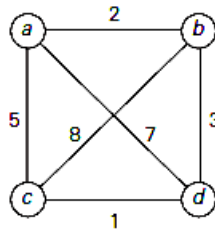


Fig. 9(b)

- c) What is the central principle of backtracking? Taking 4-queens problem as an example, explain the solution process. CO5 (06)
10. a) Explain P, NP and NP-Complete problems. CO5 (10)
- b) Describe Hamiltonian cycle. Find a Hamiltonian cycle in the graph given in Fig. 10(b). CO5 (10)

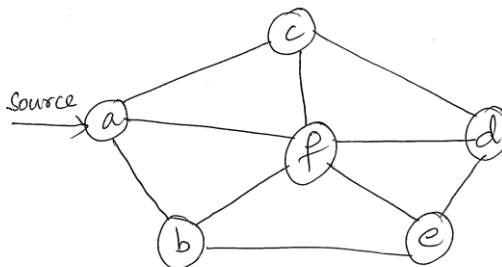


Fig 10(b)

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