RAMAIAH
Institute of Technology

Department of Mathematics

CS/IS41

Engineering Mathematics – IV

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IV Semester B.E.

CSE/ISE

Term: 08.03.2021 to 26.06.2021

Ramaiah Institute of Technology

(Autonomous Institute, Affiliated to VTU)

Vidya Soudha, M.S.R. Nagar,M.S.R.I.T. Post, Bangalore-54

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Vision of the Institute

To be an Institution of International Eminence, renowned for imparting quality technical education, cutting edge research and innovation to meet global socio economic needs

Mission of the Institute

RIT shall meet the global socio-economic needs through

- Imparting quality technical education by nurturing a conducive learning environment through continuous improvement and customization
- Establishing research clusters in emerging areas in collaboration with globally reputed organizations
- Establishing innovative skills development, techno-entrepreneurial activities and consultancy for socio-economic needs

Quality Policy

We, at Ramaiah Institute of Technology, Bangalore strive to deliver Comprehensive, Continually enhanced, Global Quality Technical and Management Education through an established Quality Management System complemented by the Synergetic Interaction of the Stakeholders concerned.

Vision of the Department

To mould the students to have strong mathematical and analytical skills to meet the challenges open to them

Mission of the Department

To provide the students with a strong mathematical foundation through courses which cater to the needs of industry, research and higher education

Table of Contents

SYLLABUS

Engineering Mathematics-IV

Course Code:CS/IS41 **Course Credits:**3:1:0
Prerequisite: Calculus & Probability **Contact Hours:**42+14

➢ **Course Objectives:**

The students will

1. Learn the concepts of finite differences, interpolation and their applications.
2. Learn the concepts of Random variables and probability distributions.
3. Learn the concepts of joint probability distributions and stochastic processes.
4. Learn the concepts of Markov chain and queuing theory.
5. Determine whether there is enough statistical evidence in favour of the hypothesis about the population parameter.

Unit I

Finite Differences and Interpolation: Forward, Backward differences, Interpolation, Newton-Gregory Forward and Backward Interpolation formulae, Lagrange's interpolation formula and Newton's divided difference interpolation formula (no proof).

Numerical Differentiation and Numerical Integration: Derivatives using Newton-Gregory forward and backward interpolation formulae, Newton-Cotes quadrature formula, Trapezoidal rule, Simpson's 1/3rd rule, Simpson's 3/8th rule.

Unit II

Random Variables: Random Variables (Discrete and Continuous), Probability density function, Cumulative distribution function, Mean, Variance, Moment generating function.

Probability Distributions: Binomial distribution, Poisson distribution, Uniform distribution, Exponential distribution, Gamma distribution andNormal distribution.

Unit III

Joint probability distribution: Joint probability distribution (both discrete and continuous), Conditional probability, Conditional expectation, Simulation of random variable.

Stochastic Processes: Introduction, Classification of stochastic processes, Discrete time processes, Stationary, Ergodicity, Autocorrelation, Power spectral density.

Unit IV

Markov Chain: Probability Vectors, Stochastic matrices, Regular stochastic matrices, Markov chains, Higher transition probabilities, Stationary distribution of Regular Markov chains and absorbing states, Markov and Poisson processes.

Queuing theory: Introduction, Symbolic representation of a queuing model, Single server Poisson queuing model with infinite capacity (M/M/1 : ∞ /FIFO), when $\lambda_n = \lambda$ and $\mu_n = \mu \quad (\lambda < \mu)$, Performance measures of the model, Single server Poisson queuing model with finite capacity (M/M/S : N/FIFO), Performance measures of the model, Derivations of difference equations and expressions for L_s, L_q, W_s, W_q of M/M/1 queuing model with finite and infinite capacity, Multiple server Poisson queuing model with infinite capacity (M/M/S : ∞ /FIFO), when $\lambda_n = \lambda$ for all n, $(\lambda < S\mu)$, Multiple server Poisson queuing model with finite capacity (M/M/S : N/FIFO), Introduction to M/G/1 queuing model.

Unit-V

Sampling and Statistical Inference: Sampling distributions, Concepts of standard error and confidence interval, Central Limit Theorem, Type-1 and Type-2 errors, Level of significance, One tailed and two tailed tests, Z-test: for single mean, for single proportion, for difference between means, Student's t –test: for single mean, for difference between two means, F – test: for equality of two variances, Chi-square test: for goodness of fit, for independence of attributes.

Text Books:

1. R.E. Walpole, R. H. Myers, R. S. L. Myers and K. Ye – Probability and Statistics for Engineers and Scientists – Pearson Education – Delhi – 9th edition – 2012.
2. B.S.Grewal - Higher Engineering Mathematics - Khanna Publishers – 44th edition-2017.
3. T. Veerarajan- Probability, Statistics and Random processes – Tata McGraw-Hill Education – 3rd edition -2017.

Reference Books:

1. Erwin Kreyszig - Advanced Engineering Mathematics-Wiley-India publishers- 10th edition-2015.
2. Sheldon M. Ross – Probability models for Computer Science – Academic Press, Elsevier– 2009.
3. Murray R Spiegel, John Schiller & R. Alu Srinivasan – Probability and Statistics – Schaum's outlines -4nd edition-2012.
4. Kishore S. Trivedi – Probability & Statistics with Reliability, Queuing and Computer Science Applications – John Wiley & Sons – 2nd edition – 2008.

- **Course Outcomes**

At the end of the course, students will be able to

1. Find functional values, derivatives, areas and volumes numerically from a given data.(PO-1,2 & PSO-2)
2. Analyze the given random data and their probability distributions.(PO-1,2 & PSO-2)
3. Calculate the marginal and conditional distributions of bivariaterandom variables and determine the parameters of stationary random processes.(PO-1,2 & PSO-2)
4. Use Markov chain in prediction of future events and in queuing models.(PO-1,2 & PSO-2)
5. Choose an appropriate test of significance and make inference about the population from a sample. (PO-1,2 & PSO-2)

LESSON PLAN

Course Contents and Lecture Schedule:

Lesson No/ Session No	Topics	No. of hours
	Unit-I (09 Hours) **FINITE DIFFERENCES AND INTERPOLATION, NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION**	
1	Introduction, Forward and Backward differences, Interpolation	1 Hr
2	Newton-Gregory forward and backward interpolation	1 Hr
3	Problems continued on Lesson No.2	1 Hr
4	Lagrange's interpolation	1 Hr
5	Newton divided difference interpolation	1 Hr
6	Numerical differentiation using Newton's interpolation formula	1 Hr
7	Problems continued on Lesson No.6	1 Hr
8	Numerical integration, Newton-Cotes Quadrature, Trapezoidal Rule	1 Hr
9	Simpson's one third Rule, Simpson's three eighth Rule	1 Hr
	Unit-II (09 Hours) **RANDOM VARIABLES& PROBABILITY DISTRIBUTIONS**	
10	Introduction, discrete random variables, mean, variance	1 Hr
11	Continuous random variables, Probability density function, cumulative density function	1 Hr
12	Problems on Lesson 10 and 11	
13	Moment generating function	1 Hr
14	Binomial distribution	1 Hr
15	Poisson distribution	1 Hr
16	Uniform and Exponential distribution	1 Hr
17	Gamma distribution	1 Hr
18	Normal distribution	1 Hr
	Unit-III (08 Hours) **JOINT PROBABILITY AND STOCHASTIC PROCESS**	
19	Joint probability distributions – discrete	1 Hr
20	Joint probability distributions – continuous, conditional expectation	1 Hr
21	Problems on Lesson 19 and 20	
22	Simulation of Random variables and Problems	1 Hr
23	Stochastic Process: Introduction, Classification of stochastic processes, discrete time processes	1 Hr
24	Problems on L23	1 Hr
25	Stationary, Ergodicity, Autocorrelation, power spectral density	1 Hr
26	Problems on L25	1 Hr
	Unit-IV (08 Hours) **MARKOV CHAIN AND QUEUING THEORY**	
27	Introduction to Markov chain.	1 Hr
28	Higher transition probabilities, stationary distribution of regular Markov chains and absorbing states.	1 Hr

Lesson No/ Session No	Topics	No. of hours
29	Markov and Poisson processes	1 Hr
30	Introduction to Queuing Theory- M/M/1 – Single server with infinite capacity.	1 Hr
31	M/M/s – Multiple server with infinite capacity	1 Hr
32	M/M/1 – Single server with finite capacity	1 Hr
33	M/M/s – Multiple server with finite capacity	1 Hr
34	M/G/1 queuing system	1 Hr
Unit-V (08 Hours) **SAMPLING THEORY**		
35	Introduction to Sampling theory	1 Hr
36	Central limit theorem, Test of hypothesis of means, confidence limits for means	1 Hr
37	Z-test: for single mean, for single proportion, for difference between means	1 Hr
38	Problems Continued on Lesson No. 37	1 Hr
39	Student's t –test: for single mean, for difference between two means	1 Hr
40	F – test: for equality of two variances	1 Hr
41	Chi-square test: for goodness of fit, for independence of attributes	1 Hr
42	Problems continued on Lesson No.42	1 Hr

Tutorials Schedule:

Tutorial No.	Topics covered	Contact hours	Tutorial No.	Topics covered	Contact hours
1	Lesson No.01 to 03	2 hours	8	Lesson No.22 to 26	2 hours
2	Lesson No.04 to 07	2 hours	9	Lesson No.27 to 29	2 hours
3	Lesson No.08 to 09	2 hours	10	Lesson No.30 to 31	2 hours
4	Lesson No.10 to 13	2 hours	11	Lesson No.32 to 34	2 hours
5	Lesson No.14 to 15	2 hours	12	Lesson No.35 to 38	2 hours
6	Lesson No.16 to 18	2 hours	13	Lesson No.39 to 40	2 hours
7	Lesson No.19 to 21	2 hours	14	Lesson No.41 to 42	2 hours

Internal Assessment Details

T_1& T_2 – each of 30 marks, Test marks**= 30 marks (Avg. of T_1& T_2)**

Quiz Test = 10 **marks**

Assignment = 10 **marks**

CIE Total = 50 **Marks**

Syllabus for Tests

Test	Unit	Lesson No.
Test – 1	Unit I and Part of Unit II	Lesson 1 – Lesson 15
Test – 2	Part of Unit II &Unit III	Lesson 16 – Lesson 26

Unit – I

Finite Differences, Interpolation, Numerical Differentiation And Numerical Integration

Forward and Backward Differences

Recall/Comprehension (2/4 mark questions)

1. Define forward and backward difference operator.
2. Determine $\Delta^2 y_2$ and $\Delta^3 y_1$ from the following data

x	1	2	3	4	5
y	0.0567	1.2345	1.5678	1.0023	1.786

3. Construct the forward difference table of the function $y = \sin x + e^x$ over the interval $(0,1)$ with step length $h = 0.25$ and hence write the value of $\Delta^2 y_1$.
4. Estimate the missing term in the following table

x	0	1	2	3	4
$f(x)$	1	3	9	-	81

5. Find the missing terms from the following data

x	45	50	55	60	65
y	3.0	-	2.0	-	-2.4

Newton-Gregory Forward & Backward Interpolation Formula

Recall/Comprehension (2/4 mark questions)

6. Define interpolation and extrapolation.
7. Write the Newton's forward interpolation formula.
8. Write the Newton's backward interpolation formula.

Newton-Gregory Forward & Backward Interpolation Formula

Apply/Analyze/Evaluate/Create (7 marks questions)

9. Derive Newton Gregory forward interpolation formula.
10. Derive Newton Gregory backward interpolation formula.
11. Using the Newton's forward interpolation formula, find $f(1.4)$ from the following table

x	1	2	3	4	5
y	10	26	58	112	194

12. Using Newton's forward interpolation formula, find the interpolating polynomial for the function $y = f(x)$ given by $f(0) = 1,\ f(1) = 2,\ f(2) = 1,\ f(3) = 10$. Hence evaluate $f(0.75)$ and $f(-0.5)$.
13. Given $\sin 45^0 = 0.7071,\ \sin 50^0 = 0.7660,\ \sin 55^0 = 0.8192,\ \sin 60^0 = 0.8660$, find $\sin 48^0$ using Newton's forward formula.

14. The table gives the distances in nautical miles of the visible horizon for the given heights in feet above the earth's surface. Find the value of y when $x = 225$ft.

x =height	200	250	300	350	400
y = distance	15.04	16.81	18.42	19.90	21.27

15. From the following table, estimate the number of students who obtained marks between 75&80.

Marks	30 - 40	40 - 50	50 - 60	60 - 70	70 – 80
No. of students	33	24	26	40	38

16. Compute $f(17)$ from the following table by applying Newton's backward interpolation formula.

x	10	12	14	16	18
$y = f(x)$	0.240	0.281	0.318	0.352	0.384

17. Given that $\sqrt{12} = 3.464$, $\sqrt{14} = 3.742$, $\sqrt{16} = 4$, $\sqrt{18} = 4.243$, $\sqrt{20} = 4.472$,compute $\sqrt{19}$by using Newton's backward interpolation formula.

18. Using Newton's backward interpolation formula, find the interpolating polynomial for the function $y = f(x)$ given by $f(2) = 4$, $f(4) = 56$, $f(6) = 204$ and $f(8) = 496$. Hence find $f(7)$.

19. The areas y of circles for different diameters x are given below. Calculate the area when $x = 105$

x	80	85	90	95	100
y	5026	5674	6362	7088	7854

20. Extrapolate the value of for $y = f(x)$ for $x = 25.4$, given

x	19	20	21	22	23
y	91	100.25	110	120.25	131

21. The following table gives the population of a city during the last six censuses. Estimate the increase in the population during the period from 1985 to 1988.

Years (x)	1941	1951	1961	1971	1981	1991
Population in lakhs (y)	12	15	20	27	39	52

22. A survey conducted in a factory reveals the following information. Estimate the probable number of persons in the income group 20 to 25.

Income per hour (Rs.)	<10	10 - 20	20 - 30	30 - 40	40 – 50
No. of persons	20	45	115	210	115

Lagrange's Interpolation Formula

Recall/Comprehension (2/4 mark questions)

23. Define inverse interpolation.
24. Write Lagrange's interpolation formula.
25. Write Lagrange's inverse interpolation formula.

26. Use Lagrange's interpolation formula to find $f(4)$, given

x	0	2	3	6
$f(x)$	- 4	2	14	158

27. For the following data, find x as a polynomial in y using the inverse Lagrange's method and hence find x for $y=5$

x	2	10	17
y	1	3	4

Lagrange's Interpolation Formula

Apply/Analyze/Evaluate/Create (7 marks questions)

28. Using the Lagrange's Interpolation formula, calculate the profit in the year 2000 from the following data and also estimate the increase in profit during the period 2000 & 2001

Year (x)	1997	1999	2001	2003
Profit in lakhs (y)	43	65	159	248

29. Find the value of x when $y = 9$ from the following data

x	5	7	11	13	17
y	150	392	1452	2366	5202

30. For the following data, find x as a polynomial in y using the inverse Lagrange's method and hence find x for $y=5$.

x	2	10	17
y	1	3	4

31. If $y(0)=-3$, $y(3)=9$, $y(4)=30$ and $y(6)=132$ then find the Lagrange's interpolating polynomial that takes these values.

32. Given $\log_{10} 654 = 2.8156$, $\log_{10} 658 = 2.8182$, $\log_{10} 659 = 2.8189$, $\log_{10} 661 = 2.8202$, find the value of $\log_{10} 656$ using Lagrange's interpolation formula.

33. Apply Lagrange's formula to find a root of the equation $f(x)=0$, given that $f(30)=-30$, $f(34)=-13$, $f(38)=0$ and $f(42)=18$.

Newton Divided Difference Interpolation Formula

Recall/Comprehension (2/4 mark questions)

34. Write Newton's divided difference interpolation formula.
35. Find the first order divided difference of $x_0 = 1$ & $x_1 = 2$ if $y = x^3 + 4x + 3$.

Newton Divided Difference Interpolation Formula

Apply/Analyze/Evaluate/Create (7 marks questions)

36. Applying the method of divided differences for interpolation, find the value of y when $x=9$ given

x	5	7	11	13	17
$f(x)$	150	392	1452	2366	5202

37. Use Newton's divided difference formula to find $f(0.5)$ from the following data

x	0	2	3	5	6
$f(x)$	0	6	21	105	186

38. Using Newton's divided difference formula, evaluate $f(8)$ and $f(15)$ given

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

39. Find an interpolating polynomial representing the following data using Newton's divided difference method.

x	0	1	2	3
$f(x)$	2	3	12	147

40. Fit an interpolating polynomial for the following data using suitable interpolation formula.

x	-1	0	2	3
$f(x)$	-8	3	1	12

41. Using Newton's divided difference formula find an interpolating polynomial for the following data and hence find $f(0.3)$ & $f(1.6)$.

x	0	0.5	1	2
$f(x)$	0	0.57	1.46	5.05

42. Find the cubic polynomial which passes through the points $(2,4)$, $(4,56)$, $(9,711)$, $(10,980)$ using Newton's divided difference method and hence estimate the value of y when $x=1.5$.

Numerical Differentiation

Recall/Comprehension (2/4 mark questions)

43. What is numerical differentiation?
44. Write the formulae to find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ by using Newton-Gregory forward interpolation formula.
45. Write the formulae to find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ by using Newton-Gregory backward interpolation formula.

Numerical Differentiation

Apply/Analyze/Evaluate/Create (7 marks questions)

46. Evaluate $y'(1)$ from the following data

x	1	2	3	4
y	1	8	27	64

47. Derive an expression for $f''(x)$ using Newton's forward interpolation formula and hence find $f''(1)$ from the following data

x	1	1.2	1.4
$f(x)$	0	6	21

48. Derive an expression for $f''(x)$ using Newton's backward interpolation formula and hence find the value of $f''(1.8)$ from the following data

x	1.4	1.6	1.8
$f(x)$	9.88	11.68	13.72

49. Find $y'(0)$ and $y''(0)$ form the following table.

x	0	1	2	3	4	5
$f(x)$	4	8	15	7	6	2

50. Find $y'(0.5)$ and $y''(0.5)$ form the following table.

x	0	1	2	3
$f(x)$	1	3	7	13

51. The following table gives the temperature θ in degrees centigrade of a cooling body at different instant of time t in seconds. Find the rate of cooling at $t=8$ sec.

t	1	3	5	7	9
θ	85.3	74.5	67	60.5	54.3

52. Use an appropriate interpolation formula to find the radius of curvature at $x=3.0$ from the following data.

x	3	5	7	9	11
y	28.27	78.54	153.93	254.47	380.13

53. If θ is the observed temperature of vessel cooling water, t is the time in minutes from the beginning of observation, find the approximate rate of cooling when $t=3$ and $t=3.5$

t	1	3	5	7	9	11
θ^0 c	85.3	74.5	67.0	60.5	54.3	41.8

54. A slider in a machine moves along a fixed straight rod. Its distance x cm along the rod is given below for various value of the time t sec. Find the velocity of the slider and its acceleration when $t = 0.5$ sec

t	0	0.1	0.2	0.3	0.4	0.5
x	30.13	31.62	32.87	33.64	33.95	33.81

55. Find the maximum and minimum values of the function $y = f(x)$ from the following data.

x	1	3	5	7	9
y	9	11	13	63	209

56. Tabulate the value of the function $y = (\sin\sqrt{x} + 1)$ in [0,0.75] with h=0.25 to construct the difference table and use Newton Gregory forward interpolation formula to compute $\frac{\cos\sqrt{x}}{2\sqrt{x}}$. Also estimate the absolute error.

Numerical Integration

Recall/Comprehension (2/4 mark questions)

57. What is numerical integration?
58. Write the Trapezoidal rule for numerical integration.
59. Give the geometrical interpretation of trapezoidal rule.
60. Evaluate $\int_0^1 e^x dx$ approximately in steps of 0.2 by using trapezoidal rule.
61. Evaluate $\int_0^1 e^{x^2} dx$ using trapezoidal rule considering four sub intervals.
62. Write the Simpson's 1/3rd rule for numerical integration.
63. Write the Simpson's 3/8th rule for numerical integration.

Numerical Integration

Apply/Analyze/Evaluate/Create (7 marks questions)

64. Derive Newton cote's quadrature formula & hence arrive at trapezoidal rule.
65. Find an approximate value of $\log 2$ applying the Simpson's 1/3rd rule to the integral $\int_0^1 \frac{1}{1+x} dx$ using the eleven ordinates.
66. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's 3/8th rule taking four ordinates and hence find approximate value of π.
67. The velocity of a particle at distance from a point on its path is given by the table. Estimate the time taken to travel 60 ft by using Simpson's 1/3 rule.

s (ft)	0	10	20	30	40	50	60
v (ft/sec)	47	58	64	65	61	52	38

68. Write the Simpson's 3/8th rule for numerical integration.
69. Using Simpson's 3/8th rule to obtain the approximate value of $\int_0^1 (1+6x^3)^{1/2} dx$ by considering seven ordinates

70. Evaluate $\int_0^{\pi} \frac{dx}{2+\cos x}$ using Simpson's 3/8th rule taking 6 sub intervals.

71. Find the approximate value of $\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta$ by Simpson's rule dividing $\left(0, \frac{\pi}{2}\right)$ into 6 equal parts.

72. Find by Simpson's rule the value of the interval $I = \int_4^{5.2} \log_e x\, dx$ considering 7 ordinates.

73. Evaluate $\int_0^1 \frac{\sin x}{x} dx$ using simpson's rule by considering six sub intervals.

Unit – II

Random variables & Probability Distributions

Random variables

Recall/Comprehension (2/4 mark questions)

1. Define Mathematical expectation of the function $\phi(X)$ of the random variate X in both discrete and continuous cases.
2. Define discrete random variable with an example.
3. Define continuous random variable with an example.
4. Define probability density function.
5. Define probability mass function.
6. The probability distribution of a random variable X is given by the following table. Find k and evaluate the mean.

X	0	1	2	3	4	5
$P(X=x)$	k	5k	10k	10k	5k	k

7. A random variable X has the density function $f(x)=\dfrac{k}{1+x^2}$, $-\infty<x<\infty$, determine k and evaluate $P(X\geq 0)$.
8. Prove that $\sigma^2=E(X^2)-[E(X)]^2$.
9. A random variable X has the following distribution. Find the mean & variance of X

X	1	2	3	4
$P(X=x)$	2/6	3/6	0	1/6

10. Define Moment generating function for discrete and continuous random variables.
11. Determine the series expansion of mgf about $X=a$ in terms of moments about X about the origin.
12. Determine the series expansion of mgf about $X=\mu$ in terms of central moments.
13. Show that $M_{X=a}(t)=e^{-at}M_x(t)$
14. Show that $\mu_r'=\frac{d^r}{dt^r}\left(M_x(t)\right)$ at $t=0$

Random variables

Apply/Analyze/Evaluate/Create (7 marks questions)

15. A random variable X has the following probability function

X	1	2	3	4	5	6	7
$P(x)$	k	2k	2k	3k	k^2	$2k^2$	$7k^2$+k

Find (i) k (ii) $P(X\geq 6)$ (iii) $P(X<6)$ (iv) $P(1\leq X<5)$ (v) $E(X)$.

16. The probability density function of a variate X is

X	0	1	2	3	4	5	6
$P(x)$	k	3k	5k	7k	9k	11k	13k

Find (i) $P(X<4)$, $P(X \geq 5)$, $P(3<X<6)$ (ii) Mean and variance

17. The probability distribution of a finite random variate X is given by the following table

X	-2	-1	0	1	2	3
$P(X=x)$	0.1	K	0.2	2k	0.3	k

(i) Find the value of k, (ii) Find $P(x<1)$ (iii) $P(x>-1)$ (iv) $P(-1<x\leq 1)$
(v) calculate the mean and variance.

18. The pdf of a random variable X is given by $P(X=x)=\begin{cases} x & , \ 0\leq x\leq 1 \\ 2-x, & 1<x\leq 2 \\ 0 & , \ elsewhere \end{cases}$

Find (i) Cumulative distribution function $F(x)$ and (ii) $P(X\geq 1.5)$.

19. If a continuous random variable X has pdf $f(x)=\begin{cases} \frac{1}{4}, & -2<x<2 \\ 0, & elsewhere \end{cases}$. Obtain

(i) $P(X<1)$ (ii) $P(|X|>1)$ (iii) $P(2X+3>5)$.

20. A continuous random variable X has pdf given by $f(x)=\begin{cases} 2e^{-2x}, & 0\leq x<\infty \\ 0 & , \ x\leq 0 \end{cases}$

Evaluate (i) $E(X)$ and (ii) $E(X^2)$. Hence find the standard deviation.

21. Find $E(X)$, $E(X^2)$ and σ^2 for the probability function P(X) defined by the following table, Where k is an appropriate constant.

X_i	1	2	3	....	n
$P(X_i)$	k	2k	3k	....	nk

22. A random variable X has the density function $f(x)=\begin{cases} kx^2, & -3\leq x\leq 3 \\ 0, & otherwise \end{cases}$.

Find (i) value of k (ii) $p(1\leq x\leq 2)$ (iii) $p(x\leq 2)$ (iv) $p(x>1)$.

23. A continuous random variable X has the pdf $f(x)=\begin{cases} \frac{k}{1+x^2}, & -\infty<x<\infty \\ 0, & elsewhere \end{cases}$. Find k

and hence evaluate $p(x\geq 0)$ and $p(0<x<1)$.

24. A random variable X has the probability function $p(x)=\frac{1}{2^x}$, $x=1,2,3\ldots\ldots$
Find the mean and variance using the concept of moment generating function.

Binomial distribution

Recall/Comprehension (2/4 mark questions)

25. Write pdf of Binomial distribution.
26. Write the formula to find mean and variance of Binomial distribution.
27. In a Binomial distribution the mean is 20 &standard deviation is $\sqrt{15}$, then find p.
28. In a Binomial distribution mean and variance are $\frac{15}{4}$ and $\frac{15}{6}$. Find number of trials.
29. The mean and variance of a binomial variate X with parameters (n, p) are 16 and 8. Find the distribution of X .
30. The probability that a man aged 60 will live up to 70 is 0.65. What is the probability that out of 10 men, now aged 60, at least 7 will live up to 70.
31. The mean and variance of a random variable X having Binomial distribution are 4 and 2 respectively. Find $P(X > 6)$.
32. When a coin is tossed 4 times, find the probability of getting (i) exactly one head, (ii) at most 3 heads and (iii) at least two heads.

Binomial distribution

Apply/Analyze/Evaluate/Create (7 marks questions)

33. Find the mean and S.D of the binomial probability distribution.
34. Determine the probability of getting 9 exactly twice in 3 throws with a pair of fair dice.
35. Out of 800 families with 5 children each, how many would you expect to have (a) 3 boys (b) 5 girls (c) either 2 or 3 boys. Assume equal probabilities for boys and girls.
36. The probability that a pen manufactured by a company will be defective is 0.1. If 12 such pens are selected at random, find the probability that (i) Exactly two pens will be defective (ii) At most two pens will be defective (iii) At least two pens will be defective (iv) None will be defective.
37. The probability that a bomb dropped from a plane will strike the target is 1/5. If six Bombs are dropped, find the probability that i) exactly two will strike the target (ii) at least two will strike the target.
38. The number of telephone lines busy at an instant of time is a binomial variate with probability 0.2. If at an instant 10 lines are chosen at random, what is the probability that (i) 5 lines are busy? (ii) at most 2 lines are busy?
39. An airline knows that 5% of the people making reservations on a certain flight will not turn up. Consequently, their policy is to sell 52 tickets for a flight that can only hold 50 people. What is the probability that there will be a seat for every passenger who turns up?

Poisson distribution

Recall/Comprehension (2/4 mark questions)

40. Write pdf of Poisson distribution.

41. Write the formula to find mean and variance of poison distribution.

42. If a random variable X has a Poisson distribution such that $P(X=1)=P(X=2)$. Find the mean and variance.

43. A Poisson variate X is such that $P(X=2)=9P(X=4)+90P(X=6)$. Find the mean and variance.

44. The probabilities of a Poisson variate taking the values 3 and 4 are equal. Find the pmf of the variate.

Poisson distribution

Apply/Analyze/Evaluate/Create (7 marks questions)

45. Find the mean and Variance of the Poisson probability distribution

46. Stating the assumptions derive Poisson distribution as a limiting case of the binomial distribution. Also find mean and variance of the Poisson distribution.

47. The probabilities of a Poisson variate taking the values 3 and 4 are equal. Calculate the probabilities of the variate taking the values 0 and 1.

48. 2% of the fuses manufactured by a firm are found to be defective. Find the probability that a box containing 200 fuses contains (i) no defective fuses (ii) 3 or more defective.

49. In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10, use Poisson distribution to calculate the approximate number of packets containing no defective, one defective and two defective blades respectively in a consignment of 10,000 packets.

50. Alpha particles are emitted by a radioactive source at an average rate of 5 in 20 minutes interval. Using Poisson distribution, find the probability that there will be (i) two emissions (ii) at least two emissions in a particular 20 minute interval.

51. The probability that an individual suffers a bad reaction from a certain injection is 0.002. Determine the probability that out of 1000 individuals, (i) exactly 3 and (ii) more than 2 will suffer a bad reaction.

52. A communication channel receives independent pulses at the rate of 12 pulses per micro second. The probability of transmission error is 0.001 for each micro second. Compute the probability of (i) no error during a micro second (ii) one error per micro second (iii) at least one error per micro second. (iv) two errors (v) At most two errors.

53. The number of accidents in a year to taxi drivers in a city follows a poisson distribution with mean 3. Out of 1000 taxi drivers find approximately the number of the drivers with (i) no accident in a year (ii) more than 3 accidents in a year (iii) at most 3 accidents in a year (iv) at least accidents in a year.

Exponential distribution

Recall/Comprehension (2/4 mark questions)

54. Write pdf of Exponential distribution.
55. If X is an exponential variate with mean 3 find (i) $P(X > 1)$ (ii) $P(X < 3)$.
56. If X is an exponential variate with mean 5, evaluate (i) $P(0 < X < 1)$ (ii) $P(-\infty < X < 10)$ (iii) $P(X \leq 0 \; or \; X \geq 1)$.
57. If X is an exponential variate with mean 4 evaluate (i) $P(0 < X < 1)$ (ii) $P(X > 2)$.

Exponential distribution

Apply/Analyze/Evaluate/Create (7 marks questions)

58. Find the mean and S.D of the Exponential probability distribution

59. The sales per day in a shop is exponentially distributed with average sale amounting to Rs.100 and net profit is 8%. Find the probability that the net profit exceeds Rs.30 on two consecutive days.

60. The length of telephone conversation in a booth has been an exponential distribution and found on average to be 5 minutes. Find the probability that a random call made from this booth (i) ends less than 5 minutes (ii) between 5 and 10 minutes.

61. The mileage which car owner get with a certain kind of radial tire is a random variable having an exponential distribution with mean 40,000km. Find the probabilities that one of these tires will last (i) at least 20,000km (ii) at most 30,000km.

62. If x is an exponential variate with mean 5, evaluate (i) $P(0 < x < 1)$ (ii) $P(-\infty < x < 10)$ (iii) $P(x \leq 0 \; or \; x \geq 1)$
63. The increase in sales per day in a shop is exponentially distributed with mean Rs.600. The sales tax is to be levied at the rate of 9%. What is the probability that the sales tax will exceed Rs.81 per day?

Uniform distribution

Recall/Comprehension (2/4 mark questions)

64. Write pdf of Uniform distribution.
65. A random variable X is uniformly distributed over the interval (-3, 3). Find $P(X < 1)$ & $P\left(|X - 1| \geq \frac{1}{2}\right)$
66. A random variable X has a uniform distribution over $(-3, 3)$, find k for which $P(X > k) = \frac{1}{3}$

Uniform distribution

Apply/Analyze/Evaluate/Create (7 marks questions)

67. Find the mean and Variance of the Uniform probability distribution.

68. On a certain city transport route, buses ply every 30 minutes between 6 a.m. and 10 p.m. If a person reaches a bus stop on this route at a random time during this period, what is the probability that he will have to wait for at least twenty minutes?

69. Buses arrive at a specified stop at 15 min intervals starting at 7 A.M., that is, they arrive at 7, 7:15, 7:30, 7:45 and so on. If a passenger arrives at the stop at a random time that is uniformly distributed between 7 and 7:30 A.M., find the probability that he waits (a) Less than 5 min for a bus and (b) At least 12 min for a bus.

Normal distribution

Recall/Comprehension (2/4 mark questions)

70. Write pdf of Normal distribution.
71. For the standard normal distribution of a random variable Z, evaluate (i) $P(0 \le Z \le 1.45)$ (ii) $P(-2.60 \le Z \le 0)$ (iii) $P(-3.40 \le Z \le 2.65)$ (iv) $P(1.25 \le Z \le 2.1)$ (v) $P(Z \ge 1.7)$.

Normal distribution

Apply/Analyze/Evaluate/Create (7 marks questions)

72. In a normal distribution (Gaussian random variable), 7% are under 35 and 89% are under 63. Find the mean and the standard deviation, given that A(1.23) = 0.39 and A(1.48) = 0.43.

73. Steel rods are manufactured to be 3 cm in diameter but they are acceptable if they are inside the limits 2.99 cm and 3.01 cm. It is observed that 5% are rejected as oversized and 5% are rejected as undersized. Assuming that the diameters are normally distributed, find the standard deviation of the distribution.

74. The life of a certain type of electrical lamps is normally distributed with mean 2040 hours and standard deviation 60 hours. In a consignment of 3000 lamps, how many would be expected to burn for (i) more than 2150 hours (ii) less than 1950 hours and (iii) between 1920 and 2160 hours.

75. The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be (i) less than 65 (ii) more than 75 and (iii) between 65 and 75.

76. In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S D of 60 hours. Estimate the number of bulbs likely to burn for more than 2150 hours, (ii) less than 1950 hours and (iii) more than 1920 hours and but less than 2160 hour.

77. The marks obtained by a number of students in a certain subject are approximately normally distributed with mean 65 and standard deviation 5. If 3 students are selected at random from this group, what is the probability that at least one of them would have scored above 75.

Gamma distribution

Recall/Comprehension (2/4 mark questions)

78. Write pdf of Gamma distribution.

79. Find mean and standard deviation of Gamma distribution

80. The demand for a certain item is distributed as a Gamma distribution with mean 8 and variance 32. Find the probability that there will be a demand for at least 10 items.

Gamma distribution

Apply/Analyze/Evaluate/Create (7 marks questions)

81. The daily consumption of milk in a town, in excess of 30,000 liters is distributed as a Gamma distribution with parameters $\alpha = 2$ and $\beta = 10{,}000$. The town has a daily stock of 40,000 litres. Find the probability that the stock is adequate on a particular day.

82. After the appointment of a new sales manager in a shop, the increase in sales per day in the shop is found as gamma variate with parameters $\alpha = 2$ and $\beta = 2000$. What is the probability that the increase in sales tax returns on a randomly chosen day exceeds Rs.200, given that the sales tax is 5% of the sales.

83. The no. of accidents per day on a certain highway is a gamma variate with an average 6 and variance 18. Find the probability that there will be (i) more than 8 accidents (ii) Between 5 and 8 accidents.

84. The daily sales of a certain brand of bicycles in a city in excess of 1000 pieces is distributed as the Gamma distribution with parameters $\alpha = 2$ and $\beta = 500$. The city has a daily stock of 1500 pieces of the brand. Find the probability that the stock is insufficient on a particular day.

Unit – III

Joint Probability distribution & Stochastic Processes

Joint Probability Distribution

Recall/Comprehension (2/4 mark questions)

1. Define joint probability distribution of *X* and *Y*.
2. Define marginal probability distribution of *X* and *Y.*
3. Find marginal distribution of *X* and *Y*, given that:

X \ Y	-4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

4. Find the joint distribution of *X* and *Y*, which are independent random variables with the following respective distributions also find *E(X).*

x_i	1	2
$f(x_i)$	0.7	0.3

y_i	-2	5	8
$g(y_i)$	0.3	0.5	0.2

5. The joint probability mass function given by $p(x,y)=k(2x+3y)$, $x=0,1,2;\ y=0,1,2$. Find k and also find the joint probability distribution of *X* and *Y*.
6. The joint PDF of a two-dimensional random variable is given by $f(x,y)=xy^2+\frac{x^2}{8}, 0\le x\le 2,\quad 0\le y\le 1,$ Compute (i) $P(X>1)$ (ii) $P(Y<1/2)$.
7. Find the value of *k* from the joint probability density function of *X* and *Y* given by,
$$f(x,y)=\begin{cases} k(6-x-y) & 0<x<2,\ 2<y<4 \\ 0 & otherwise \end{cases}$$
8. From the following Joint density function, find the marginal density function of *X* & *Y*,
$$f(x,y)=\begin{cases} k(2x+y) & 2<x<6,\ 0<y<5 \\ 0 & elsewhere \end{cases}$$

Joint Probability Distribution

Apply/Analyze/Evaluate/Create (7 marks questions)

9. The joint distribution of two variables *X* and *Y* are given below

X \ Y	-4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

Find (i) Marginal distributions of *X* and *Y* (ii) σ_X *and* σ_Y (iii) $COV(X,Y)$ (iv) $\rho(X,Y)$.

10. Given the following bivariate probability distribution, obtain (i) marginal distributions of *X* and *Y* (ii) The conditional distribution of *Y* given *X* = 2 (iii) The conditional distribution of *X* given *Y* = -1.

X \ Y	-1	0	1
0	1\15	2\15	1\15
1	3\15	2\15	1\15
2	2\15	1\15	2\15

11. The joint distribution of two variables *X* and *Y* are given below:

X \ Y	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Evaluate the following (i) $E(X)$ and $E(Y)$ (ii) $E(XY)$ (iii) σ_x & σ_y (iv) $COV(X,Y)$ (v) $\rho(X,Y)$.

12. The joint probability distribution of two random variables X and Y is given by the following table:

X \ Y	1	3	9
2	1/8	1/24	1/12
4	1/4	¼	0
6	1/8	1/24	1/12

Find Marginal distribution of *X* and *Y* and evaluate $COV(X,Y)$ & $\rho(X,Y)$.

13. The joint distribution of two variables X and Y are given below:

X \ Y	0	1
0	0.1	0.2
1	0.4	0.2
2	0.1	0

Evaluate (i) Marginal distribution of *X* and *Y* (ii) $E(XY)$ (iii) σ_x & σ_y iv) $COV(X,Y)$ (v) *P(X + Y>1)* and (vi) Verify that *X* and *Y* are not independent.

14. A fair coin is tossed three times. Let *X* denote *0* or *1* according as the head or tail occurs on the first toss. Let *Y* denote the number of heads which occur. (i) Find the marginal distributions of *X* and *Y*, (ii) determine the joint distribution of *X* and *Y* and (iii) $COV(X,Y)$.

15. A bag contains 3 white, 2 red, 2 green bulbs. 3 bulbs are selected at random. If *X* and *Y* are discrete random variable denoting number of white and red bulbs respectively. Determine (i) joint distribution of *X* and *Y* (ii) Marginal distribution of *X*and*Y*&(iii) $COV(X,Y)$.

16. Two cards are selected at a random from a box which contains five cards numbered *1, 1, 2, 2* and *3*. Find the joint distribution of *X* and *Y,* where *X* denotes the sum and *Y* the maximum of the two numbers drawn. Also determine $COV(X,Y)$ and $\rho(X,Y)$.

17. Let *X* and *Y* be continuous random variables having the joint density function as follows. $f(x,y)=\begin{cases}4xy & 0<x<1, 0<y<1\\ 0 & otherwise\end{cases}$ Verify that (i) $E(X+Y)=E(X)+E(Y)$ (ii) $E(XY)=E(X)\,E(Y)$.

18. Suppose that the joint density function of two continuous random variables *X* and *Y* is $f(x,y)=\begin{cases}x^2+{xy}/{3}, & 0<x<1, 0<y<2\\ 0, & otherwise\end{cases}$ Find (i) $P\left(X>1/2\right)$ (ii) $P\left(Y/X\right)$.

19. If the probability function of the random variables *X* and *Y* is given by $f(x,y)=\begin{cases}\frac{1}{8}(6-x-y), & 0<x<2,\ 2<y<4\\ 0, & otherwise\end{cases}$ Find (i) $P(X<1, Y<3)$ (ii) $P(X+Y<3)$

20. If X and Y are continuous random variables having the joint density function as follows. Find (i) the value of c (ii) $P(1<X<2,\ 2<Y<3)$ (iii) $P(X\geq 3,\ Y\leq 2)$
$$f(x,y)=\begin{cases}c\left(x^2+y^2\right), & 0\leq x\leq 4, 0\leq y\leq 4\\ 0, & otherwise\end{cases}$$

Stochastic Processes

Recall/Comprehension (2/4 mark questions)

21. Define stochastic process.
22. Classify stochastic process.
23. Write the expressions for autocorrelation, auto covariance and correlation coefficient.
24. Define wide sense stationary (WSS) processes.
25. Define strict sense stationary (SSS) processes.

Stochastic Processes

Apply/Analyze/Evaluate/Create (7 marks questions)

26. A random process *X(t)* is represented by the ensemble*{-k, -2k, -3k, k, 2k, 3k}* (*k*> 0) corresponding to the outcomes of an event which are equally probable. Show that the random process is SSS but not WSS.

27. Find the autocorrelation and correlation coefficient of X(t) with the random process

Outcome	1	2	3	4	5	6
X(t)	-4	-3	1	2	-t	t

Compute the following probabilities (i) P[X(1) = -3] (ii) P[X(1) ≤ 0]

28. Show that $X(t) = A\cos\omega t + B\sin\omega t$ where A and B are uncorrelated random variables each with mean 0 and variance 1 and ω is a positive constant is a WSS.

29. Find the autocorrelation, auto covariance and correlation coefficient of *X(t)* from the following table.

Outcome	1	2	3	4	5	6
X(t)	-2	-1	1	2	-T	T

30. Consider a stochastic process defined on a finite sample space with three sample points. Its description is provided by the specifications of the three sample functions. $X(t, \lambda_1) = 3,\ X(t, \lambda_2) = 3\cos t,\ X(t, \lambda_3) = 3\sin t$, Also given the probability of the assignment $P(\lambda_1) = P(\lambda_2) = P(\lambda_3) = \frac{1}{3}$, compute $\mu(t)$ and $R(t_1, t_2)$. Decide whether the process is SSS or WSS.

31. Find the ACF of the stochastic process defined by X(t) = A cos(10t + θ) where A is a random variable with mean 0 and variance 1 and θ is uniformly distributed in the interval [$-\pi$, π]

32. A random process *X(t)* is described by the ensemble x_k(t), (k = 1 to 6) respectively by 5, 3, 1, -1, -3, -5. Assuming that the occurrences of the outcomes are equally probable show that the random process is SSS.

33. Find the autocorrelation, auto covariance and correlation coefficient *r(1, 2)* of *X(t)* in respect of the stochastic process.

No. of dots	1	2	3	4	5	6
X(t)	-3	-1	1	3	-t/3	t/3

34. A random process *X(t)* is described by the ensemble $\{1, \sin t, -\sin t, \cos t, -\cos t, -1\}$. Show that *X(t)* is WSS but not SSS.

35. Find the auto correlation $R(t_1, t_2)$ of the stochastic process defined by $X(t) = A\cos(\omega t + \alpha)$ where the random variables *A* and α are independent and α is uniform in the interval [$-\pi$, π].

36. A stochastic process with its ensemble functions is assumed to have equal probabilities and is given by: $x_1(t) = 3, x_2(t) = 3\sin t, x_3(t) = -3\sin t,\ x_4(t) = 3\cos t,$ $x_5(t) = -3\cos t, x_6(t) = -3$. Show that the process is WSS but not SSS.

Unit – IV

Markov Chain & Queuing Theory

Markov Chain

Recall/Comprehension (2/4 mark questions)

1. Define stochastic process
2. Define stochastic matrix
3. Define regular stochastic matrix
4. Define Periodic state
5. Define Absorbing state
6. Define Recurring state
7. Define Probability vector
8. Define Fixed probability vector of a stochastic matrix
9. Find the fixed probability vector of $\begin{bmatrix} 0.5 & 0.25 & 0.25 \\ 0 & 1 & 0 \\ 0.3 & 0.7 & 0 \end{bmatrix}$
10. Which of the stochastic matrices are regular? (i) $\begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix}$ (ii) $\begin{bmatrix} 0 & 0 & 1 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$
11. Find the fixed probability vector of the following:
(i) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & 0 \\ 1/2 & 1/2 \end{bmatrix}$ (iv) $\begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$

Markov Chain

Apply/Analyze/Evaluate/Create (7 marks questions)

12. A student's study habits are as follows. If he studies one night, he is 67% sure not to study the next night. On the other hand, if he does not study one night, he is 58% sure not to study the next night as well. In the long run, how often does he study?

13. A salesman's territory consists of three cities A, B & C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in city B. However, if he sells in either B or C, then the next day he is thrice as likely to sell in city A as in other city. In the long run how often does he sell in each of the cities?

14. Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C. But C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball, find the probabilities that (i) A has the ball, (ii) B has the ball and (iii) C has the ball, for the fourth throw.

15. There are 2 white marbles in bag A and 3 red marbles in bag B. At each step of the process a marble is selected at random from each bag and the two marbles selected are interchanged then find (i) transition probability matrix (ii) what is the probability that there are 2 red marbles in A after 3 steps (iii) In the long run what is the probability that there are 2 red marbles in A?

16. Consider repeated tosses of a fair die. Let X_n be the maximum of the numbers occurring in the first n trials. (i) Find transition matrix of the markov chain (ii) Is this matrix regular(iii) Find the probability distribution after first toss (iv) Find $P^{(2)}$ *and* $P^{(3)}$.

17. Two boys b_1 and b_2 and two girls g_1 and g_2 are throwing a ball from one to the other. Each boy throws the ball to the other with probability ½ and to each girl with probability ¼. On the other hand, each girl throws the ball to each boy with probability ½ and never to the girl. In the long run, how often does each receive the ball?

18. A man's smoking habits are as follows. If he smokes filter cigarettes one week, he switches to no filter cigarettes the next week with probability 0.2. On the other hand if he smokes no filter cigarettes one week, there is a probability of 0.7 that he will smoke no filter cigarettes the next week as well. In the long run how often does he smoke filter cigarettes.

19. A player has Rs.300. At each play of a game, he loses Rs.100 with probability ¾ but wins Rs.200 with probability ¼. He stops playing if he has lost his Rs.300 or he has won at least Rs.300. (i) Determine the transition probability matrix of the markov chain (ii) Find the probability that there are at least 4 plays to the game.

20. Every year, a man trades his car for a new car. If he has a Maruthi, he trades it for an Ambassador. If he has an Ambassador, he trades for a Santro. However, if he has a Santro, he is just as likely to trade it for a new Santro as to trade it for a Maruti or an Ambassador. In 2000 he bought his first car, which was a Santro. (i) Find the probability that he has (a) 2002 Santro (b) 2002 Maruti (c) 2003 Ambassador (d) 2003 Santro (ii) In the long run, how often will he have a Santro?

Queuing Theory

Recall/Comprehension (2/4 mark questions)

(M/M/1): (∞/FIFO) Single Server With Infinite Capacity

21. Derive the expression to determine the expected number of customers (L_s) in the system for (M/M/1): (∞/FIFO)
22. Derive the expression to determine the expected number of customers in the queue (L_q) for (M/M/1): (∞/FIFO)
23. Derive the expression to determine the expected number of customers in the nonempty queue (L_w) for (M/M/1): (∞/FIFO).
24. Derive the expression to determine the expected waiting time of a customer in the system (W_s) for (M/M/1): (∞/FIFO).
25. Derive the expression to determine the average waiting time of a customer in the queue (W_q) for (M/M/1): (∞/FIFO).
26. Derive Little's Formulae for (M/M/1): (∞/FIFO).
27. What is the probability that a customer has to wait more than 15 minutes to get his service completed in (M/M/1): (∞/FIFO) queue system if $\lambda = 6$ per hour and $\mu = 10$ per hour?
28. Find the probability that there are at least n customers in the system for (M/M/1): (∞/FIFO).
29. Find the probability of at least 10 customers in the system for (M/M/1): (∞/FIFO) model if $\lambda = 6$ & $\mu = 8$.
30. In the usual notation of an (M/M/1):(∞/FIFO) model, if $\rho = 0.6$, then what is the probability that the queue contains 5 or more customers?
31. Suppose that customers arrive at a Poisson rate of one per every 12 minutes and that the service time is exponential at a rate of one service per 8 minutes. What is
    a) The average number of customers in the system
    b) The average time a customer spends in the system

(M/M/s): (∞/FIFO) Multiple Server With Infinite Capacity

32. Write the expression for L_s , L_q, W_s & W_q for (M/M/s):(∞/FIFO) queuing system.
33. Find the average waiting time in the queue for (M/M/s): (∞/FIFO) model if $\lambda = 30$ & $\mu = 10$ & s=4.
34. Find the expected number of customers in the system for (M/M/s): (∞/FIFO) model if $\lambda = 12$ & $\mu = 5$ & s=3.
35. A petrol pump has 4 pumps. The service time follows an exponential distribution with a mean of 6 minutes and cars arrive for service in a Poisson process at the rate of 30 cars per hour. Find the average waiting time in the queue, average time spent in the system and the average number of cars in the system.
36. A bank has three teller counters. The customers to the bank choose a teller counter at random. It is given that the arrivals to the teller counters are Poisson-distributed at the average rate λ and the mean service rate at the teller counters is $\frac{\lambda}{2}$. Find the steady average queue at each counter.

(M/M/1): (k/FIFO) Single Server With Finite Capacity

37. Derive the expression to determine the expected number of customers (L_s) in the system for (M/M/1): (k/FIFO).
38. Derive the expression to determine the expected number of customers in the queue (L_q) for (M/M/1): (k/FIFO).
39. Derive the expression to determine the expected number of customers in the nonempty queue (L_w) for (M/M/1): (k/FIFO).
40. Derive the expression to determine the expected waiting time of a customer in the system (W_s) for (M/M/1): (k/FIFO).
41. Derive the expression to determine the average waiting time of a customer in the queue (W_q) for (M/M/1): (k/FIFO).
42. Derive the modified Little's Formulae for (M/M/1): (k/FIFO).
43. Find the expected number of customers in the system for (M/M/1): (k/FIFO) model if $\lambda = 4$ & $\mu = 4$ & maximum capacity is k=7.
44. Find the expected number of customers in the queue for (M/M/1): (k/FIFO) model if $\lambda = 18$ & $\mu = 6$ & maximum capacity is k=3.

(M/M/s): (k/FIFO) Multiple Server With Finite Capacity

45. Write the expression for L_s , L_q, W_s & W_q for (M/M/s):(k/FIFO) queuing system.
46. Write the expression for P_0 & P_n for (M/M/s):(k/FIFO) model.
47. Find the probability of no customer in the system for (M/M/s):(k/FIFO) model if $\lambda = 10$ $\mu = 8$, s=2 and k=5.

(M/G/1) Queuing System

48. Write the expression for L_s , L_q, W_s & W_q for (M/G/1): (∞/FIFO) model.

Queuing Theory

Apply/Analyze/Evaluate/Create (7 marks questions)

(M/M/1): (∞/FIFO) Single Server With Infinite Capacity

49. With usual notations, prove that $P_n = \rho^n(1-\rho)$ for (M/M/1): (∞/FIFO) model.

50. A supermarket has a single cashier. During peak hours, customers arrive at a rate of 20 per hour. The average number of customers that can be processed by the cashier is 24 per hour. Calculate
    a) The probability that the cashier is idle
    b) The average number of customers in the queuing system
    c) The average time a customer spends in the system
    d) The average number of customers in the queue
    e) The average time a customer spends in the queue waiting for service

51. Customers arrive at a sales counter manned by a single person according to a Poisson process with a mean rate of 20 per hour. The time required to serve a customer has an exponential distribution with a mean of 100 seconds. Find the average waiting time of a customer.

52. Customers arrive at a watch repair shop according to a Poisson process at a rate of one per every 10 minutes and the service time is exponential with mean 8 minutes. Find the average number of customers in the shop, average waiting time a customer spends in the shop and the average time a customer spends in the queue for service.

53. In a city airport, flights arrive at a rate of 24 flights per day. It is known that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average of 30 minutes. Find the following
    a) The probability that the system will be idle
    b) The mean queue size
    c) The average number of flights in the queue
    d) The probability that the size exceeds 7

54. A Xerox machine is maintained in an office and operated by a secretary who does other jobs also. The service rate is Poisson-distributed with a mean service rate of 10 jobs per hour. Generally, the requirements for use are random over the entire 8-hour working day but arrive at a rate of 5 jobs per hour. Several people have noted that a waiting line develops occasionally and have questioned the office policy of maintaining only one Xerox machine. If the time of the secretary is valued at Rs. 10 per hour, find the following:
    a) Utilization of the Xerox machine
    b) The probability that an arrival has to wait
    c) The mean number of jobs of the system
    d) The average waiting time of a job in the system
    e) The average cost per day due to waiting and operating the machine

(M/M/s): (∞/FIFO) Multiple Server With Infinite Capacity

55. A travel centre has three service counters to receive people who visit to book air tickets. The customers arrive in a Poisson distribution with the average arrival of 100 persons in a 10 hour service day. It has been estimated that the service time follows an exponential distribution. The average service time is 15 minutes. Find the
    a) Expected number of customers in the system
    b) Expected number of customers in the queue
    c) Expected time a customer spends in the system
    d) Expected waiting time for a customer in the queue
    e) Probability that a customer must wait before he gets active

56. A local hospital has 3 doctors for treating patients. The patients arrive at the hospital according to Poisson distribution with a an average of 8 per hour. On an average, a doctor takes about 15 minutes to treat each patient and the actual time taken is known to vary approximately exponentially around this average. Find
    a) Traffic intensity of the system
    b) Probability that a customer has to wait for service
    c) Average number of customers waiting in the queue
    d) Average number of customers in the system
    e) Average waiting time a customer spends in the queue
    f) Expected time a customer spends in the system

57. On every Sunday morning, a dental hospital renders free dental service to the patients. As per the hospital rules, 3 dentists who are equally qualified and experienced will be on duty then. It takes on an average 10 minutes for a patient to get treatment and the actual time taken is known to vary approximately exponentially around this average. The patients arrive according to the Poisson distribution with an average of 12 per hour. The hospital management wants to investigate the following:
    a) The expected number of patients waiting in the queue
    b) The average time that a patient spends at the hospital

(M/M/1): (k/FIFO) Single Server With Finite Capacity

58. With usual notations, prove that $P_n = \begin{cases} \frac{\rho^n(1-\rho)}{1-\rho^{k+1}} & if\ \rho \neq 1 \\ \frac{1}{k+1} & if\ \rho = 1 \end{cases}$ for (M/M/1): (k/FIFO)model

59. Patients arrive at a doctor's clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 9 patients. Examination time per patient is exponential with mean rate of 20 per hour. Find the:
    a) Probability that an arriving patient will not wait
    b) Effective arrival rate
    c) Average number of patients in the clinic
    d) Expected time a patient spends in the clinic
    e) Average number of patients in the queue
    f) Expected waiting time of a patient in the queue

60. A city has a one-person barber shop which can accommodate a maximum of 5 people at a time (4 waiting and 1 getting haircut). On average, customers arrive at the rate of 8 per hour and the barber takes 6 minutes for serving each customer. It is estimated that the arrival process is Poisson and the service time is an exponential random variable. Find:
    a) Percentage of time the barber is idle
    b) Fraction of potential customers who will be turned away
    c) Effective arrival rate of customers for the shop
    d) Expected number of customers in the barber shop
    e) Expected time a person spends in the barber shop
    f) Expected number of customers waiting for a hair-cut
    g) Expected waiting time of a customer in the queue

(M/M/s): (k/FIFO) Multiple Server With Finite Capacity

61. A dispensary has two doctors and four chairs in the waiting room. The patients who arrive at the dispensary leave when all four chairs in the waiting room of the dispensary are occupied. It is known that patients arrive at the average rate of 8 per hour and spend an average of 10 minutes for their check-up and medical consultation. The arrival process is Poisson and the service time is an exponential random variable. Find:

a) Probability that an arriving patient will not wait
b) Effective arrival rate at the dispensary
c) Expected number of patients at the queue
d) Expected waiting time of a patient at the queue
e) Expected number of patients at the dispensary
f) Expected time a patient spends at the dispensary

62. A beauty salon has 2 barbers and 6 chairs to accommodate waiting customers. Potential customers, who arrive when all the 6 chairs are full, leave the salon immediately. Customers arrive at the salon at the average rate of 10 per hour and spend an average of 10 minutes in the barber's chair. The arrival process is Poisson and the service time is an exponential random variable. Find:
a) Probability of no customer in the beauty salon
b) Effective arrival rate at the beauty salon
c) Expected number of customers in the queue
d) Expected waiting time of a customer in the queue
e) Expected number of customers at the beauty salon
f) Expected time a customer spends at the beauty salon

(M/G/1) Queuing System

63. Automatic car wash facility operates with only one bay. Cars arrive according to a Poisson distribution with a mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. If the service time for all cars is constant and equal to 10 minutes, then determine L_s , L_q, W_s & W_q.

64. A car manufacturing plant uses one big crane for loading cars into a truck. Cars arrive for loading by the crane according to a Poisson distribution with a mean of 5 cars per hour. Given that the service time for all cars is constant and equal to 6 minutes, determine L_s , L_q, W_s & W_q.

65. Customers arrive at a one-man barber shop in a remote village according to a Poisson distribution with the average arrival rate of 8 per hour. It is estimated that the service time follows a random distribution with the mean service time of 6 minutes and standard deviation equal to 15 minutes. Find the
a) Average number of customers in the queue
b) Average number of customers in the barber shop
c) Average waiting time of a customer at the queue
d) Average time a customer spends at the barber shop

Unit – V

Sampling and Statistical Inference

Sampling And Statistical Inference

Recall/Comprehension (2/4mark questions)

1. Define Type-I and Type – II errors.
2. Define Standard error.
3. Define confidence interval.
4. Define student's t- distribution.
5. Define F-distribution
6. Define Chi-square distribution.
7. Define sampling distribution of means.
8. Write formula for Chi-square test and explain the terms involved.
9. State the condition when F test is used
10. Define null hypothesis and alternate hypothesis.
11. Difference between student's t- distribution and F- distribution.
12. Write a short note on i). Type I and Type II errors ii). Testing of hypothesis
13. Write a short note on i). Level of significance ii). One tailed and two tailed tests
14. State central limit theorem.
15. The nine items of a sample have the following values:45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these differ significantly from the assumed mean of 47.5?
16. A machinist is making engine parts with axle diameter of 0.7 inch. A random sample of 10 parts shows mean diameter 0.742 inch with a SD of 0.04inch .On the basis of the sample, would you say that the work is inferior?
17. A certain stimulus administered to each of the 12 patients resulted in the following change in blood pressure.5,2, 8, -1,3,0,6,-2,1,5,0,4. Can it be concluded that the stimulus will increase the blood pressure?
18. Test the equality of standard deviations for the data given below at 5% level of significance: $n_1=10$; $n_2=14$; $s_1=1.5$; $s_2=1.2$.
19. The mean height and the SD height of 8 randomly chosen soldiers are 166.9 and 8.29 cm respectively. The corresponding values of 6 randomly chosen sailors are 170.3 and 8.50 cm respectively. Based on this data, can we conclude that soldiers are, in general, shorter than sailors?
20. A population consists of the four numbers 3, 7, 11, 15. Consider all possible samples of size 2 which can be drawn (i) with replacement and (ii) without replacement from this population. In both the cases find (a) the population mean (b) the population standard deviation (c) the mean of the sampling distribution of means (d) the standard error of means.

Sampling And Statistical Inference

Apply/Analyze/Evaluate/Create (7 marks questions)

Z test for single mean, for single proportion, for difference between means

21. A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased at 5% level of significance.
22. A sample of 12 measurements of the diameter of a metal ball gave the mean $\overline{X}$ = 7.38mm with S.D. S = 1.24mm. Find a) 95% and b) 99% confidence limits for the actual diameter.
23. A survey was conducted in a slum locality of 2000 families by selecting a sample of size 800. It was revealed that 180 families were illiterates. Find the probable limits of the illiterate families in the population of 2000.

24. A sample of 900 members is found to have a mean of 3.4cm. Can it be reasonably regarded as a truly random sample from a large population with mean 3.25cm and SD 1.61cm.

25. According to the norms established for a mechanical aptitude test, persons who are 18 years old have an average height of 73.2 with a standard deviation of 8.6. If 4 randomly selected persons of that age averaged 76.7, test the hypothesis $\mu = 73.2$ against the alternative hypothesis $\mu > 73.2$ at the 0.01 level of significance.

26. An ambulance service claims that it takes on the average less than 10 minutes to reach its destination in emergency calls. A sample of 36 calls has a mean of 11 minutes and the variance of 16 minutes. Test the significance at 0.05 level.

27. The means of two large samples of sizes 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the same population of S.D 2.5 inches?

28. The mean yield of wheat from a district A was 210 pounds with S.D. 10 pounds per acre from a sample of 100 plots. In another district the mean yield was 220 pounds with S.D. 12 pounds from a sample of 150 plots. Assuming that the S.D. of yield in the entire state was 11 pounds, test whether there is any significant difference between the mean yield of crops in the two districts.

29. Sample of students were drawn from two universities and from their weights in kilograms, mean and standard deviations are calculated and shown below. Make a large sample test to test the significance of the difference between the means.

	Mean	**S.D**	**Sample size**
University A	55	10	400
University B	57	15	100

30. A company claims that its bulbs are superior to those of its main competitor. If a study showed that a sample of 40 of its bulbs has a mean life time of 647 hrs of continuous use with a S.D of 27 hrs. While a sample of 40 bulbs made by its main competitor had a mean life time of 638 hrs of continuous use with a S.D of 31 hrs. Test the significance between the difference of two means at 5% level.

31. In a sample of 1000 people in Karnataka, 540 are rice eaters and the rest are wheat eaters. Can we assume that both rice and wheat are equally popular in this state at 1% level of significance?

32. In a big city, 325 men out of 600 men were found to be smokers. Does this information support the conclusion that the majority of men in this city are smokers?

33. A die was thrown 9000 times and of these 3220 yielded a 3 or 4. Is this consistent with the hypothesis that the die was unbiased? Use 1% level of significance.

34. A manufacture claims that only 4% of his products are defective. A random sample of 500 men was taken among which 100 were defective. Test the hypothesis at 0.05 level.

t- test:

35. The heights of 10 males of a given locality are found to be 175,168,155,170, 152, 170, 175, 160, 160 and 165 cms. Based on this sample, find the 95% confidence limits for the height of males in that locality.

36. The mean lifetime of a sample of 25 bulbs is found as 1550 hours with a SD of 120h. The company manufacturing the bulbs claims that the average life of their bulbs is 1600h. Is the claim acceptable at 5% LOS?

37. Two independent samples of sizes 8 and 7 contained the following values:

Sample 1	19	17	15	21	16	18	16	14
Sample 2	15	14	15	19	15	18	16	

Is the difference between the sample means significant?

38. Sample of two types of electric bulbs were tested for length of life and the following data were obtained

	Size	Mean	SD
Sample1	8	1234 h	36 h
Sample2	7	1036 h	40 h

Is the difference in the means sufficient to warrant that type 1 bulbs are superior to type 2 bulbs?

39. A sample of 6 persons in an office reveled an average daily smoking of 10,12,8,9,16,5 cigarettes. The average level of smoking in the whole office has to be estimated at 90% level of confidence. $t = 2.015$ for 5 degree of freedom.

40. A fertilizer mixing machine is set to give 12 kg of nitrate for quintal bag of fertilizer. Ten 100 kg bags are examined, the percentage of nitrate per bag are as follows: 11, 14, 13, 12, 13, 12, 13, 14, 11, 12. Are there any reasons to believe that the machine is defective? Value of t for 9 degree of freedom is 2.262.

41. A random sample of size 16 values from a normal population showed a mean of 53 and a sum of squares of deviation from the mean is equal to 150. Can this sample be regarded as taken from the population having 56 as mean? Obtain 95% and 99% confidence limits of the mean of the population.

42. In the past, a machine has produced washers having a thickness of 0.50mm. To determine whether the machine is in proper working condition, a sample of 10 washers is chosen for which the mean thickness is found as 0.53mm with standard deviation 0.03mm. Test the hypothesis that the machine is in proper working condition, using a level of significance of (i) 0.05 (ii) 0.01.

43. Eleven school boys were given as test in drawing. They were given a month's further tuition and a second Test at of equal difficulty was held at the end of it. Do the marks give evidence that the students have benefited by extra coaching?

Boys	1	2	3	4	5	6	7	8	9	10	11
Marks I test	23	20	19	21	18	20	18	17	23	16	19
Marks II test	24	19	22	18	20	22	20	20	23	20	17

44. The average breaking strength of steel rods is specified to be 18.5 thousand pounds. To test this a sample of 14 rods was tested. The mean and standard deviation obtained were 17.85 and 1.955 respectively. Is the result of the experiment significant with 95% confidence?

45. A group of 10 rats fed on a diet A and another group of 8 rats fed on a different diet B recorded the following increase in weights in gms.

Diet A	5	6	8	1	12	4	3	9	6	10
Diet B	2	3	6	8	1	10	2	8	-	-

Test whether the diet A is superior to diet B.

F-test:

46. A sample of size 13 gave an estimated population variance of 3.0, while another sample of size 15 gave an estimate of 2.5. could both samples be from populations with the same variance?

47. Two samples of sizes 9 and 8 gave the sums of squares of deviations from their respective means equal to 160 and 91 respectively. Can they be regarded as drawn from the same normal population?

48. The nicotine contents in two random samples of tobacco are given below.

Sample 1	21	24	25	26	27	
Sample 2	22	27	28	30	31	36

Can you say that the two samples came from the same population?

49. Two independent samples of sizes 7 and 6 have the following values:

Sample A	28	30	32	33	33	29	34
Sample B	29	30	30	24	27	29	-

Examine Whether the samples have been drawn from normal population having the same variance?

50. Two random samples drawn from 2 normal populations are given below. Test whether the 2 populations have the same variance.

Sample I	20	16	26	27	23	22	18	24	25	19	-	-	n=10
Sample II	17	23	32	25	22	24	28	6	31	33	20	27	n=12

51. Ten individuals are chosen at random from a population and their heights are found to be in inches 63, 63, 64, 666, 69, 69,70,70,71. Discus the suggestion that the mean height of universe is 65.

52. The mean life time of sample of 100 fluorescent light bulbs produced by a company is computed to be 1570 hours with a standard deviation of 120 hours. The company claims that the average lift of the bulbs produced by it is 1600 hours. Using the level of significance of 0.05, is the claim acceptable?

χ^2 test:

53. Fit a Poisson distribution for the following data and test the goodness of fit.

x:	0	1	2	3	4	5	6	Total
f:	273	70	30	7	7	2	1	390

54. Among 64 offspring of a certain cross between Guinea pigs 34 were red, 10 were black and 20 were white. According to the genetic model these numbers should be in the ratio 9:3:4. Are the data consistent with the model at 5% level?

55. The theory predicts that the proportion of beans available in four groups A, B,C,D should be 9:3:3:1. In an experiment with 1600 beans the numbers in the four groups were 882, 313, 287 and 118. Use χ^2- test to verify whether the experimental result supports the theory.

56. The following table shows the result of an experiment to investigate the effect of vaccination induced on the animals against a particular disease. Use χ^2- test to test the hypothesis that there is no difference between and vaccinated and unvaccinated groups i.e. vaccination and this disease are independent

	Got disease	Did not get disease
vaccinated	9	42
Not vaccinated	17	28

Model Question paper (CIE-01)

Sub Code:	CS/IS41	**Sub:**	Engineering Mathematics-IV	**Test:**	I
Semester:	IV	**Sec:**	CSE/ISE	**Marks:**	30

Note: Answer any TWO full questions. Each main question carries 15 marks

Q.No.		Questions	Blooms Level	CO's	Marks
1	(a)	The probabilities of a Poisson variate taking the values 3 and 4 are equal. Find the pmf of the variate.	L1	CO2	2
	(b)	Find the missing terms from the following data x: 45, 50, 55, 60, 65 y: 3.0, -, 2.0, -, -2.4	L2	CO1	3
	(c)	Find the approximate value of $\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta$ by Simpson's rule dividing $(0, \pi/2)$ into 6 equal parts.	L3	CO1	5
	(d)	Find the cubic polynomial which passes through the points $(2,4)$, $(4,56)$, $(9,711)$, $(10,980)$ using Newton's divided difference method and hence estimate the value of y when $x=1.5$.	L4	CO1	5
2	(a)	Write Lagrange's inverse interpolation formula..	L1	CO1	2
	(b)	Find the mean of the binomial distribution.	L2	CO2	3
	(c)	Use an appropriate interpolation formula to find the radius of curvature at $x=3.0$ from the following data. x: 3, 5, 7, 9, 11 y: 28.27, 78.54, 153.93, 254.47, 380.13	L4	CO1	5
	(d)	A random variable X has the probability function $p(x) = \frac{1}{2^x}$, $x = 1,2,3 \dots\dots$ Find the mean and variance using the concept of moment generating function.	L3	CO2	5
3	(a)	Find the first order divided difference of $x_0 = 1$ & $x_1 = 2$ if $y = x^3 + 4x + 3$.	L1	CO1	2
	(b)	Determine the series expansion of mgf about $X = \mu$ in terms of central moments.	L2	CO2	3
	(c)	The probability that an individual suffers a bad reaction from a certain injection is 0.002. Determine the probability that out of 1000 individuals, (i) exactly 3 and (ii) more than 2 will suffer a bad reaction.	L3	CO2	5
	(d)	A random variable X has the following probability function X: 1, 2, 3, 4, 5, 6, 7 P(X): k, 2k, 2k, 3k, k^2, $2k^2$, $7k^2+k$ Find (i) k (ii) $P(X \geq 6)$ (iii) $P(X < 6)$ (iv) $P(1 \leq X < 5)$ (v) E(X).	L5	CO2	5

Model Question paper (CIE-02)

Sub Code:	CS/IS41	Sub:	Engineering Mathematics-IV	Test:	II
Semester:	IV	Sec:	CSE/ISE	Marks:	30

Note: Answer any TWO full questions. Each main question carries 15 marks

Q.No.		Questions	Blooms Level	CO's	Marks
1	(a)	Write the expressions for autocorrelation & auto covariance.	L1	CO3	2
	(b)	A random variable X is uniformly distributed over the interval (-3, 3). Find $P(X<1)$ & $P\left(\lvert X-1\rvert \geq \frac{1}{2}\right)$	L2	CO2	3
	(c)	In a normal distribution (Gaussian random variable), 7% are under 35 and 89% are under 63. Find the mean and the standard deviation, given that A(1.23)=0.39 & A(1.48)=0.43.	L3	CO2	5
	(d)	The daily consumption of milk in a town, in excess of 30,000 litres is distributed as a Gamma distribution with parameters $\alpha=2$ and $\beta=10{,}000$. The town has a daily stock of 40,000 litres. Find the probability that the stock is adequate on a particular day.	L4	CO2	5
2	(a)	For the standard normal distribution of a random variable Z, evaluate $P(-3.40 \leq z \leq 2.65)$	L1	CO2	2
	(b)	The joint probability mass function given by $p(x,y)=k(2x+3y)$ $x=0,1,2;\ y=0,1,2.$ Find k.	L2	CO3	3
	(c)	Suppose that the joint density function of two continuous random variables X and Y is $f(x,y)=\begin{cases} x^2+\frac{xy}{3}, & 0<x<1,\ 0<y<2 \\ 0, & otherwise \end{cases}$ Find (i) $P\left(X>\frac{1}{2}\right)$ (ii) $P\left(\frac{Y}{X}\right)$	L4	CO3	5
	(d)	Find the auto correlation $R(t_1, t_2)$ of the stochastic process defined by $X(t)=A\cos(\omega t+\alpha)$ where the random variables A and α are independent and α is uniform in the interval $[-\pi, \pi]$.	L3	CO3	5
3	(a)	Define strict sense stationary (SSS) processes.	L1	CO3	2
	(b)	On a certain city transport route, buses ply every 30 minutes between 6 a.m. and 10 p.m. If a person reaches a bus stop on this route at a random time during this period, what is the probability that he will have to wait for at least twenty minutes?	L2	CO3	3
	(c)	The increase in sales per day in a shop is exponentially distributed with mean Rs.600. The sales tax is to be levied at the rate of 9%. What is the probability that the sales tax will exceed Rs.81 per day?	L3	CO2	5
	(d)	A bag contains 3 white, 2 red, 2 green bulbs. 3 bulbs are selected at random. If X and Y are discrete random variable denoting number of white and red bulbs respectively. Determine (i) joint distribution of X and Y (ii) Marginal distribution of X and Y & (iii) $COV(X,Y)$.	L5	CO3	5

RAMAIAH INSTITUTE OF TECHNOLOGY

(Autonomous Institute, Affiliated to VTU, Belgaum)

MODEL QUESTION PAPER-1

Course & Branch:	**B.E. (CS/IS)**	**Semester:**	**IV**
Subject:	**Engineering Mathematics – IV**	**Max. Marks:**	**100**
Subject Code:	**CS/IS 41**	**Duration:**	**3 Hrs**

Instructions to the candidates:

Answer ONE full question from each unit.

UNIT-I

1. a) Write the Trapezoidal rule, Simpson's (3\8) th rule for numerical integration. (02)

b) Find the polynomial $f(x)$ from Newton's divided difference interpolation formula. Given that $f(34) = 13$, $f(38) = 3$, $f(42) = 18$. (04)

c) Find $y'(0.5)$ and $y''(0.5)$ form the following table. (07)

x	0	1	2	3
y	1	3	7	13

d) Using the Lagrange's formula, find the interpolation polynomial that approximates to the function described the following table. Hence find $f(0.5)$ and $f(3.1)$ (07)

x	5	6	9	11
y	12	13	14	16

2. a) Write Newton-Gregory backward interpolation formula. (02)

b) Estimate the missing term in the following table (04)

x	0	1	2	3	4
$f(x)$	1	3	9	-	81

c) The following table gives the temperature θ in degrees centigrade of a cooling body at different instant of time t in seconds. (07)

t	1	3	5	7	9
θ	85.3	74.5	67	60.5	54.3

Find the rate of cooling at t= 8 sec.

d) A solid of revolution is formed by rotating about the x-axis, the area between the x-axis, the line $x = 0$ and $x = 1$ and a curve through the points with the following coordinates: (07)

x	0	0.25	0.5	0.75	1
y	1	0.9896	0.9589	0.9089	0.8415

Estimate the volume of the solid formed using Simpson's rule.

UNIT-II

3. a) In a Binomial distribution the mean is 20 and standard deviation is $\sqrt{15}$, then find p. (02)

b) A Poisson variate X is such that $P(X=2)=9P(X=4)+90P(X=6)$. Find the mean and variance. (04)

c) The mileage which car owner get with a certain kind of radial tire is a random variable having an exponential distribution with mean 40,000km. Find the probabilities that one of these tires will last (i) at least 20,000km (ii) at most 30,000km. (07)

d) If a continuous random variable X has pdf $f(x)=\begin{cases}\frac{1}{4}, & -2<x<2 \\ 0, & elsewhere\end{cases}$. Obtain (07)

(i) $P(X<1)$ (ii) $P(|X|>1)$ (iii) $P(2X+3>5)$.

4. a) Define Moment generating function for discrete and continuous random variables. (02)

b) For the normal distribution with mean 2 and standard deviation 4, evaluate (04)

(i) $P(x \geq 5)$ (ii) $P(|x|<4)$ (iii) $P(|x|>3)$ (iv) $P(-6<x<3)$.

c) The probability that a pen manufactured by a company will be defective is 0.1. If 12 such pens are selected at random, find the probability that (i) Exactly two pens will be defective (ii) At most two pens will be defective (iii) At least two pens will be defective (iv) None will be defective. (07)

d) On a certain city transport route, buses ply every 30 minutes between 6 a.m. and 10 p.m. If a person reaches a bus stop on this route at a random time during this period, what is the probability that he will have to wait for at least twenty minutes. (07)

UNIT-III

5. a) Classify stochastic process. (02)

b) Find marginal distribution of *X* and *Y*, given that: (04)

X \ Y	-4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

c) A random process *X(t)* is represented by the ensemble *{-k, -2k, -3k, k, 2k, 3k}* (*k*> 0) corresponding to the outcomes of an event which are equally probable. Show that the random process is SSS but not WSS. (07)

d) If the probability function of the random variables *X* and *Y* is given by $f(x,y)=\begin{cases}\frac{1}{8}(6-x-y) & 0<x<2,\ 2<y<4\\ 0 & otherwise\end{cases}$ Find (i) $P(X<1,\ Y<3)$ (ii) $P(X+Y<3)$ (07)

6. a) Define wide sense stationary (WSS) processes. (02)

b) Find the value of *k* from the joint probability density function of *X* and *Y* is given by,

$$f(x,y)=\begin{cases}k(6-x-y) & 0<x<2,\ 2<y<4\\ 0 & otherwise\end{cases}.$$ (04)

c) Find the autocorrelation, auto covariance and correlation coefficient of X*(t)* from the following table. (07)

Outcome	1	2	3	4	5	6
X(t)	-2	-1	1	2	-T	T

d) A fair coin is tossed three times. Let *X* denote *0* or *1* according as the head or tail occurs on the first toss. Let *Y* denote the number of heads which occur. (i) Find the marginal distributions of *X* and *Y*, (ii) determine the joint distribution of *X* and *Y* and (iii) $COV(X,Y)$. (07)

UNIT-IV

7. a) Define stochastic matrix (02)

b) Trains arrive at the yard every 15 minutes and the service time is 33 minutes. If the line capacity of the yard is limited to 5 trains, find the probability that the yard is empty and the average number of trains in the system. (04)

c) Two boys b_1 and b_2 and two girls g_1 and g_2 are throwing a ball from one to the other. Each boy throws the ball to the other with probability ½ and to each girl with probability ¼. On the other hand, each girl throws the ball to each boy with probability ½ and never to the girl. In the long run, how often does each receive the ball? (07)

d) A firm has a single mechanic in a repair shop. He works 8 hours a day and on an average four machines break each day. It takes on the average one hour to repair a machine. Find (i) the expected number of machines in the repair shop. (ii) The expected number of machines in the shop on which the mechanic has not started to work. (iii) The average downtime (waiting for repairs or undergoing repairs per) machine. (iv) The proportion of time a facility will be idle. (07)

8. a) Find the fixed probability vector of $\begin{bmatrix} 0.5 & 0.25 & 0.25 \\ 0 & 1 & 0 \\ 0.3 & 0.7 & 0 \end{bmatrix}$ (03)

b) With usual notations, prove that $P_n = \begin{cases} \frac{\rho^n (1-\rho)}{1-\rho^{k+1}} & if\ \rho \neq 1 \\ \frac{1}{k+1} & if\ \rho = 1 \end{cases}$ for (M/M/1): (k/FIFO) model (10)

c) A salesman's territory consists of three cities A, B & C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in city B. However, if he sells in either B or C, then the next day he is twice as likely to sell in city A as in other city. In the long run how often does he sell in each of the cities? (07)

UNIT-V

9. a) Define Type-I and Type – II errors. (02)

b) A manufacture claims that only 4% of his products are defective. A random sample of 500 men was taken among which 100 were defective. Test the hypothesis at 0.05 level. (04)

c) In the past, a machine has produced washers having a thickness of 0.50mm. To determine whether the machine is in proper working condition, a sample of 10 washers is chosen for which the mean thickness is found as 0.53mm with standard deviation 0.03mm. Test the hypothesis that the machine is in proper working condition, using a level of significance of (i) 0.05 (ii) 0.01. (07)

d) Among 64 offspring of a certain cross between Guinea pigs 34 were red, 10 were black and 20 were white. According to the genetic model these numbers should be in the ratio 9:3:4. Are the data consistent with the model at 5% level? (07)

10. a) Define null hypothesis and alternate hypothesis. (02)

b) Write a note on Level of significance & One tailed and two tailed tests. (04)

c) A population consists of the four numbers 3, 7, 11, 15. Consider all possible samples of size 2 which can be drawn (i) with replacement and (ii) without replacement from this population. In both the cases find (a) the population mean (b) the population standard deviation (c) the mean of the sampling distribution of means (d) the standard error of means. (07)

d) Two random samples drawn from 2 normal populations are given below. Test whether the 2 populations have the same variance. (07)

Sample I	20	16	26	27	23	22	18	24	25	19	-	-	n=10
Sample II	17	23	32	25	22	24	28	6	31	33	20	27	n=12

Note: **Students should not be under the impression that questions from model question paper will appear in SEE.**

RAMAIAH INSTITUTE OF TECHNOLOGY

(Autonomous Institute, Affiliated to VTU, Belgaum)

MODEL QUESTION PAPER-2

Course & Branch:	**B.E. (CS/IS)**	**Semester:**	**IV**
Subject:	**Engineering Mathematics – IV**	**Max. Marks:**	**100**
Subject Code:	**CS/IS 41**	**Duration:**	**3 Hrs**

Instructions to the candidates:

Answer ONE full question from each unit.

UNIT-I

1. a) Obtain the expression for $\Delta^2 y_n$ and $\nabla^2 y_n$ in terms of y values. (02)

b) Evaluate $\int_0^1 e^x dx$ approximately in steps of 0.2 by using trapezoidal rule. (04)

c) For the following data, find x as a polynomial in y using the inverse Lagrange's method and hence find x for $y = 5$. (07)

x	2	10	17
y	1	3	4

d) Find $y'(0)$ and $y''(0)$ form the following table. (07)

x	0	1	2	3	4	5
$f(x)$	4	8	15	7	6	2

2. a) Write the Newton's forward interpolation formula. (02)

b) Find the missing terms from the following data (04)

x	45	50	55	60	65
y	3.0	-	2.0	-	-2.4

c) Using Simpson's 3/8th rule to obtain the approximate value of $\int_0^{0.3} (1-8x^3)^{1/2} dx$ by considering seven ordinates (07)

d) Applying the method of divided differences for interpolation, find the value of y when $x = 9$ given (07)

x	5	7	11	13	17
$f(x)$	150	392	1452	2366	5202

UNIT-II

3. a) Define probability mass function. (02)

b) A random variable X is uniformly distributed over the interval (-2, 2). Find (i) $P(X < 1)$ (ii) $P(|X - 1| \geq 1/2)$ (04)

c) 2% of the fuses manufactured by a firm are found to be defective. Find the probability that a box containing 200 fuses contains (i) no defective fuses (ii) 3 or more defective. (07)

d) The probability distribution of a finite random variate X is given by the following table (07)

X	-2	-1	0	1	2	3
$P(X=x)$	0.1	K	0.2	2k	0.3	k

(i) Find the value of k, (ii) Find $P(x<1)$ (iii) $P(x>-1)$ (iv) $P(-1<x\le 1)$

(v) calculate the mean and variance.

4. a) Write the formula to find mean and variance of poison distribution. (02)

b) Show that $M_{X=a}(t) = e^{-at} M_x(t)$ (04)

c) The life of a certain type of electrical lamps is normally distributed with mean 2040 hours and standard deviation 60 hours. In a consignment of 3000 lamps, how many would be expected to burn for (i) more than 2150 hours (ii) less than 1950 hours and (iii) between 1920 and 2160 hours. (07)

d) Out of 800 families with 5 children each, how many would you expect to have (a) 3 boys (b) 5 girls (c) either 2 or 3 boys. Assume equal probabilities for boys and girls. (07)

UNIT-III

5. a) Define marginal probability distribution of *X* and *Y.* (02)

b) The joint PDF of a two-dimensional random variable is given by $f(x,y) = xy^2 + \frac{x^2}{8}, 0 \le x \le 2, \quad 0 \le y \le 1,$ Compute (i) $P(X>1)$ (ii) $P(Y<1/2)$. (04)

c) A fair coin is tossed three times. Let *X* denote *0* or *1* according as the head or tail occurs on the first toss. Let *Y* denote the number of heads which occur. (i) Find the marginal distributions of *X* and *Y*, (ii) determine the joint distribution of *X* and *Y* and (iii) $COV(X,Y)$. (07)

d) Show that $X(t) = A\cos\omega t + B\sin\omega t$ where A and B are uncorrelated random variables each with mean 0 and variance 1 and ω is a positive constant is a WSS. (07)

6. a) Write the expressions for autocorrelation, auto covariance and correlation coefficient. (02)

b) Two cards are selected at a random from a box which contains five cards numbered *1, 1, 2, 2* and *3*. Find the joint distribution of *X* and *Y,* where *X* denotes the sum and *Y* the maximum of the two numbers drawn. (04)

c) Suppose that the joint density function of two continuous random variables (07)

X and Y is $f(x,y)=\begin{cases} x^2+xy/3 & 0<x<1,\ 0<y<2 \\ 0, & otherwise \end{cases}$. Find (i) $P(X>1/2)$ (ii) $P(Y/X)$

d) A stochastic process with its ensemble functions are assumed to have equal probabilities are given by: $x_1(t)=3,\ x_2(t)=3\sin t, x_3(t)=-3\sin t$, $x_4(t)=3\cos t$, $x_5(t)=-3\cos t$, $x_6(t)=-3$. Show that the process is WSS but not SSS. (07)

UNIT-IV

7. a) Define Periodic state (02)

b) Trains arrive at the yard every 15 minutes and the service time is 33 minutes. If the line capacity of the yard is limited to 5 trains, find the probability that the yard is empty and the average number of trains in the system. (04)

c) A player has Rs.300. At each play of a game, he loses Rs.100 with probability ¾ but wins Rs.200 with probability ¼. He stops playing if he has lost his Rs.300 or he has won at least Rs.300. (i) Determine the transition probability matrix of the markov chain (ii) Find the probability that there are at least 4 plays to the game. (07)

d) Patients arrive at a doctor's clinic at mean intervals of 20 minutes and spend a mean of 15 minutes in consultation, both times exponentially distributed. The doctor wishes to have enough seats in the waiting room so that no more than about 1% of arriving patients will have to stand. How many seats should be provided? (07)

8. a) Define M/G/1 queuing system. (02)

b) Derive the expression to determine the expected number of customers in the queue (L_q) for (M/M/1): (∞/FIFO) (04)

c) A student's study habits are as follows. If he studies one night, he is 70% sure not to study the next night. On the other hand, if he does not study one night, he is 60% sure not to study the next night as well. In the long run, how often does he study? (07)

d) A self-service store employee one cashier at its counter. 8 customers arrive on an average every 5 minutes while the cashier can serve 10 customers in the same time. Assuming Poisson distribution for arrival and exponential distribution service rate, determine (i) Average number of customers in the system. (ii) Average number of customers in queue. (iii) Average time a customer spends in the system. (iv) Average time a customer waits before being served. (07)

UNIT-V

9. a) State central limit theorem. (02)

b) Test the equality of standard deviations for the data given below at 5% level of significance: $n_1=10$; $n_2=14$; $s_1=1.5$; $s_2=1.2$ (04)

c) The nicotine contents in two random samples of tobacco are given below. (07)

Sample 1	21	24	25	26	27	
Sample 2	22	27	28	30	31	36

Can you say that the two samples came from the same population?

d) A sample of 900 members is found to have a mean of 3.4cm. Can it be reasonably regarded as a truly random sample from a large population with mean 3.25cm and SD 1.61cm? (07)

10. a) Define sampling distribution of means. (02)

b) Write a short note on Type I and Type II errors (04)

c) The following table shows the result of an experiment to investigate the effect of vaccination induced on the animals against a particular disease. Use the χ^2 test to test the hypothesis that there is no difference between and vaccinated and unvaccinated groups i.e. vaccination and this disease are independent (07)

	Got disease	Did not get disease
vaccinated	9	42
Not vaccinated	17	28

d) Sample of two types of electric bulbs were tested for length of life and the following data were obtained (07)

	Size	Mean	SD
Sample1	8	1234 h	36 h
Sample2	7	1036 h	40 h

Is the difference in the means sufficient to warrant that type 1 bulbs are superior to type 2 bulbs?

Note: **Students should not be under the impression that questions from model question paper will appear in SEE.**

Assignment Problems

1. The following table gives the population of a city during the last six censuses. Estimate the increase in the population during the period from 1985 to 1988.

Years (x)	1941	1951	1961	1971	1981	1991
Population in lakhs (y)	12	15	20	27	39	52

2. Find the cubic polynomial which passes through the points $(2,4)$, $(4,56)$, $(9,711)$, $(10,980)$ using Newton's divided difference method and hence estimate the value of y when $x=1.5$.

3. Derive an expression for $f''(x)$ using Newton's forward interpolation formula and hence find $f''(1)$ from the following data

x	1	1.2	1.4
$f(x)$	0	6	21

4. Find the approximate value of $\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta$ by Simpson's rule dividing $\left(0, \frac{\pi}{2}\right)$ into 6 equal parts.

5. The velocity of a particle at distance from a point on its path is given by the table. Estimate the time taken to travel 60 ft by using Simpson's 1/3 rule.

s (ft)	0	10	20	30	40	50	60
v (ft/sec)	47	58	64	65	61	52	38

6. A random variable X has the probability function $p(x) = \frac{1}{2^x},\ x = 1,2,3\ldots\ldots$
   Find the mean and variance using the concept of moment generating function.

7. The mileage which car owner get with a certain kind of radial tire is a random variable having an exponential distribution with mean 40,000km. Find the probabilities that one of these tires will last (i) at least 20,000km (ii) at most 30,000km.

8. The probability that a bomb dropped from a plane will strike the target is 1/5. If six Bombs are dropped, find the probability that i) exactly two will strike the target (ii) at least two will strike the target.

9. Steel rods are manufactured to be 3 cm in diameter but they are acceptable if they are inside the limits 2.99 cm and 3.01 cm. It is observed that 5% are rejected as oversized and 5% are rejected as undersized. Assuming that the diameters are normally distributed, find the standard deviation of the distribution.

10. The daily sales of a certain brand of bicycles in a city in excess of 1000 pieces is distributed as the Gamma distribution with parameters $\alpha = 2$ and $\beta = 500$. The city has a daily stock of 1500 pieces of the brand. Find the probability that the stock is insufficient on a particular day.

11. A box contains 2 red, 3 white and4 black balls. 3 balls are drawn at random. If X and Y are discrete random variables denoting number of white and red balls in the draw respectively. Determine (i) joint distribution of X and Y (ii) Marginal distribution of $X$and$Y$&(iii) $COV(X,Y)$.

12. A random observation on a bivariate population (X, Y) can yield one of the following pairs of values with probabilities noted against them:

For each observation pair	Probability
(1, 1); (2, 1); (3, 3); (4, 3)	1/20
(3, 1); (4, 1); (1, 2); (2,2); (3,2); (4, 2); (1, 3); (2, 3)	1/10

Find the (i) Marginal distributions of X and Y (ii) Mean of X and Y
(iii) Probability that Y=2, given X = 4 (iv) Are X and Y independent?

13. Let (X, Y) be a two dimensional continuous random variable having joint pdf given by
$$f(x,y) = \begin{cases} kx^2y, & 0 < x < 1,\ 0 < y < 1 \\ 0, & otherwise \end{cases}$$

Find (i) the value of k (ii) P (0< X <3/4, 1/3 < Y <1) (iii) P (X+Y < 1)
(iv) P (X < 1/2 / Y < 1/3)

14. Let $X(t) = \{\sin t, -\sin t, \cos t, -\cos t\}$, each with equal probability. Determine if it defines a stationary process and in what sense.

15. Show that $X(t) = A\cos\omega t + B\sin\omega t$ where A and B are uncorrelated random variables each with mean 0 and variance 1 and ω is a positive constant is a WSS.

16. There are 3 white marbles in bag A and 4 red marbles in bag B. At each step of the process a marble is selected at random from each bag and the two marbles selected are interchanged then find (i) transition probability matrix (ii) what is the probability that there are 2 red marbles in A after 3 steps?

17. A player has Rs.300. At each play of a game, he loses Rs.100 with probability ¾ but wins Rs.200 with probability ¼. He stops playing if he has lost his Rs.300 or he has won at least Rs.300. (i) Determine the transition probability matrix of the markov chain (ii) Find the probability that there are at least 4 plays to the game.

18. Customers arrive at a bank counter manned by a single person, according to a Poisson process with a mean rate of 15 per hour. The time required to serve a customer has an exponential distribution with a mean of 5 minutes. Find (i) the average number in the system. (ii) the average waiting time of a customer in the queue. (iii) the probability that there would be 2 customers in them queue.

19. Patients arrive at a doctor's clinic at mean intervals of 24 minutes and spend a mean of 16 minutes in consultation, both times exponentially distributed. The doctor wishes to have enough seats in the waiting room so that no more than about 1% of arriving patients will have to stand. How many seats should be provided?

20. With usual notations, prove that $P_n = \begin{cases} \frac{\rho^n(1-\rho)}{1-\rho^{k+1}} & if\ \rho \neq 1 \\ \frac{1}{k+1} & if\ \rho = 1 \end{cases}$ for (M/M/1): (k/FIFO)model

21. A beauty salon has 2 barbers and 6 chairs to accommodate waiting customers. Potential customers, who arrive when all the 6 chairs are full, leave the salon immediately. Customers arrive at the salon at the average rate of 10 per hour and spend an average of 10 minutes in the barber's chair. The arrival process is Poisson and the service time is an exponential random variable. Find:
    a) Probability of no customer in the beauty salon

b) Effective arrival rate at the beauty salon
c) Expected number of customers in the queue
d) Expected waiting time of a customer in the queue
e) Expected number of customers at the beauty salon
f) Expected time a customer spends at the beauty salon

22. A car manufacturing plant uses one big crane for loading cars into a truck. Cars arrive for loading by the crane according to a Poisson distribution with a mean of 5 cars per hour. Given that the service time for all cars is constant and equal to 6 minutes, determine L_s , L_q, W_s & W_q.

23. Eleven school boys were given a test in drawing. They were given a month's further tuition and a second Test of equal difficulty was held at the end of it. Do the marks give evidence that the students have benefited by extra coaching?

Boys	1	2	3	4	5	6	7	8	9	10	11
Marks I test	22	22	18	20	23	22	18	17	24	21	19
Marks II test	24	19	20	22	20	22	19	21	23	20	18

24. The mean life time of sample of 100 fluorescent light bulbs produced by a company is computed to be 1670 hours with a standard deviation of 110 hours. The company claims that the average life of the bulbs produced by it is 1700 hours. Using the level of significance of 0.05, is the claim acceptable?

25. A coin was tossed 500 times and the head turned up 266 times. Test the hypothesis that the coin is unbiased at 5% level of significance.