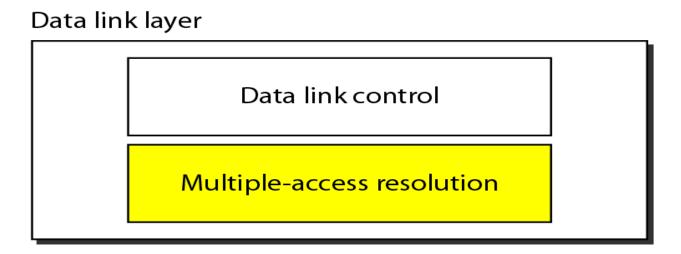




Module 4 Multiple Access Protocols

Figure 12.1 Data link layer divided into two functionality-oriented sublayers



The data link layer has 2 sub layer:

Logical link layer (LLC): responsible for data link control ie flow control, framing and error control

Media Access Control: Multiple Access resolution responsible for resolving access to the shared media

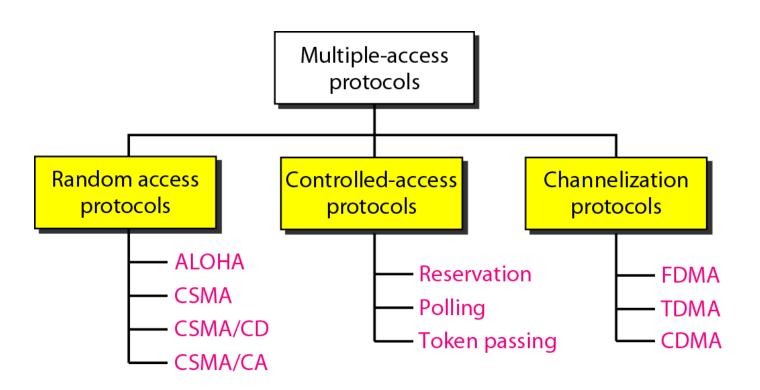
Why Multiple Access Protocols?

If there is a dedicated link between the sender and the receiver then data link control layer is sufficient. However, if there is no dedicated link present, then multiple stations can access the channel simultaneously. Here multiple access protocol are required to decrease collision and avoid cross talk

Multiple Access Protocols are checking

- When a station access the medium?
- What can the station do if medium is busy
- How can station determine the success or failure of transmission?
- What can station do if there is access conflicts?

Figure 12.2 Taxonomy of multiple-access protocols



1 RANDOM ACCESS PROTOCOL

In random access or contention methods, no station is superior to another station and none is assigned the control over another. No station permits, or does not permit, another station to send. At each instance, a station that has data to send uses a procedure defined by the protocol to make a decision on whether or not to send.

There is no scheduled time for a station to transmit. Transmission is random among the stations. That's why these method are called random access

4 CATEGORIES:

ALOHA

Carrier Sense Multiple Access with Collision Detection
Carrier Sense Multiple Access with Collision Avoidance

1.1 ALOHA

- Earliest random access method.
- Designed for radio wireless LAN, but used on any shared medium
- The medium is shared between station. When a station sends a data, another station may attempt to do at the same time. the data from 2 stations are collide and become garbled.

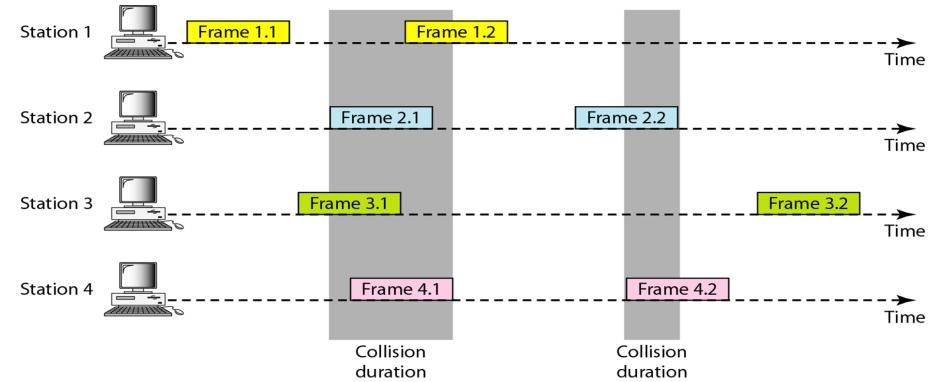
2 CATEGORIES:

Pure ALOHA Slotted ALOHA

1.1.1 Pure ALOHA

- Original ALOHA, simple elegant protocol
- The idea is that each station sends a frame whenever it has a frame to send. However, since there is only one channel to share there is possibility of collision between frames from different stations

Figure 12.3 Frames in a pure ALOHA network



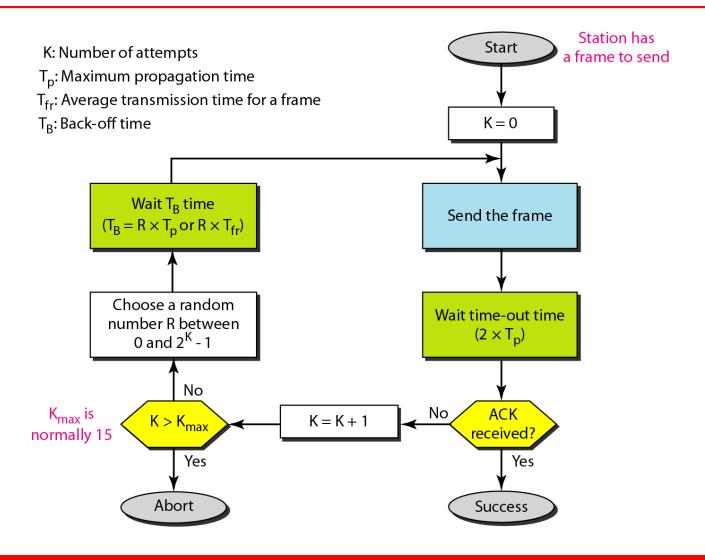
1.1.1 Pure ALOHA

- According to Figure 12.3, only 2 frame survives Frame 1.1 from Station 1 and Frame 3.2 from Station 3.
- We have to resend the frames that have been destroyed during transmission.
- Pure ALOHA protocol relies on acknowledgment from receiver. when a station sends a frame, it expects the receiver to send an acknowledgement. If acknowledgement does not arrive after a time-out period, the station assumes that the frame has been destroyed and resends the frame.

1.1.1 Pure ALOHA

- A collision involves 2 or more stations. If all these stations try to resend their frames after the time-out period, frame will collide. So ALOHA when time-out period passes, each station waits a random amount of time before resending its frame. The randomness will help to avoid more collision. This time is called back-off Time T_B
- Pure ALOHA has a second method to **prevent congestion** of channel with retransmission of frames. After a maximum number of retransmission attempts k_{max} , station give up.

Figure 12.4 Procedure for pure ALOHA protocol



Example 12.1

The stations on a wireless ALOHA network are a maximum of 600 km apart. If we assume that signals propagate at 3×10^8 m/s, we find

$$T_p = (600 \times 10^5) / (3 \times 10^8) = 2 \text{ ms.}$$

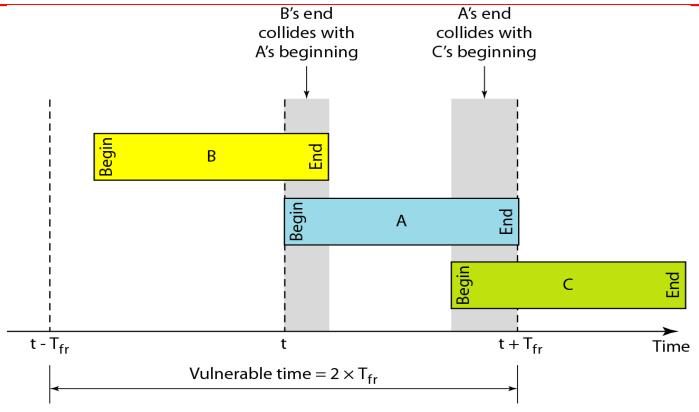
Now we can find the value of T_B for different values of K.

a. For K = 1, the range is $\{0, 1\}$. The station needs to generate a random number with a value of 0 or 1. This means that T_B is either 0 ms (0×2) or 2 ms (1×2) , based on the outcome of the random variable.

Example 12.1 (continued)

- b. For K = 2, the range is $\{0, 1, 2, 3\}$. This means that T_B can be 0, 2, 4, or 6 ms, based on the outcome of the random variable.
- c. For K = 3, the range is $\{0, 1, 2, 3, 4, 5, 6, 7\}$. This means that T_B can be $0, 2, 4, \ldots, 14$ ms, based on the outcome of the random variable.
- d. We need to mention that if K > 10, it is normally set to 10.

Figure 12.5 Vulnerable time for pure ALOHA protocol



Since no rule that define when station can send its frame. So the station can send soon after another station has started or soon before another station has finished.

Vulnerable time of Pure ALOHA is 2*Tfr.

Example 12.2

A pure ALOHA network transmits 200-bit frames on a shared channel of 200 kbps. What is the requirement to make this frame collision-free?

Solution

Average frame transmission time T_{fr} is 200 bits/200 kbps or 1 ms. The vulnerable time is 2×1 ms = 2 ms. This means no station should send later than 1 ms before this station starts transmission and no station should start sending during the one 1-ms period that this station is sending.

Note

The throughput for pure ALOHA is $S = G \times e^{-2G}$. The maximum throughput $S_{max} = 0.184$ when G = (1/2).

Example 12.3

A pure ALOHA network transmits 200-bit frames on a shared channel of 200 kbps. What is the throughput if the system (all stations together) produces

- a. 1000 frames per second b. 500 frames per second
- c. 250 frames per second.

Solution

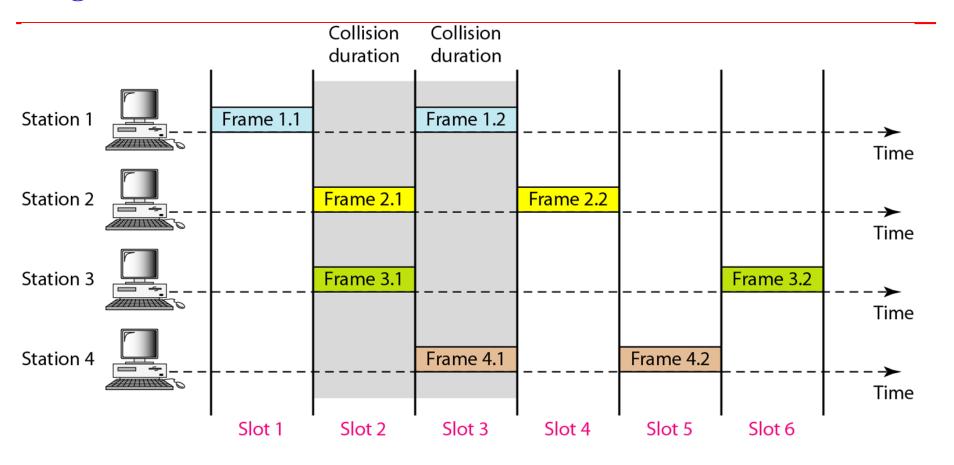
The frame transmission time is 200/200 kbps or 1 ms.

a. If the system creates 1000 frames per second, this is 1 frame per millisecond. The load is 1. In this case $S = G \times e^{-2 G}$ or S = 0.135 (13.5 percent). This means that the throughput is $1000 \times 0.135 = 135$ frames. Only 135 frames out of 1000 will probably survive.

Example 12.3 (continued)

- b. If the system creates 500 frames per second, this is (1/2) frame per millisecond. The load is (1/2). In this case $S = G \times e^{-2G}$ or S = 0.184 (18.4 percent). This means that the throughput is $500 \times 0.184 = 92$ and that only 92 frames out of 500 will probably survive. Note that this is the maximum throughput case, percentagewise.
- c. If the system creates 250 frames per second, this is (1/4) frame per millisecond. The load is (1/4). In this case $S = G \times e^{-2G}$ or S = 0.152 (15.2 percent). This means that the throughput is $250 \times 0.152 = 38$. Only 38 frames out of 250 will probably survive.

Figure 12.6 Frames in a slotted ALOHA network



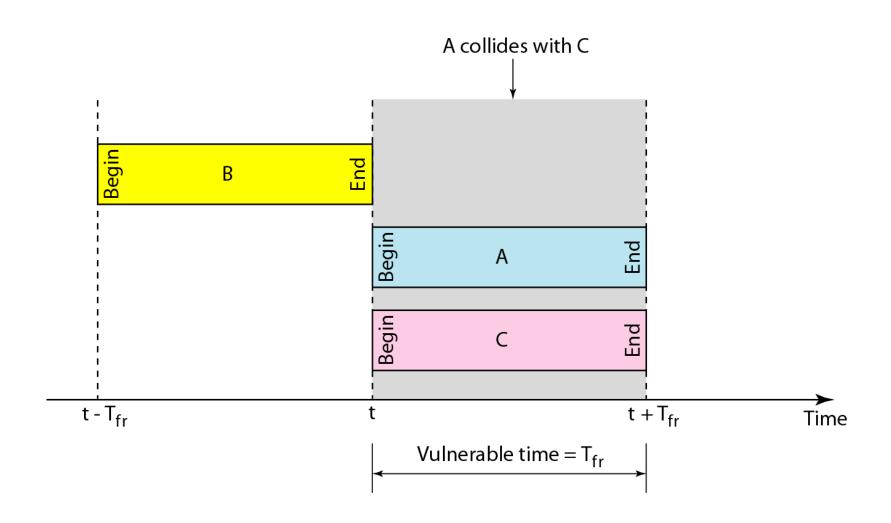
Here Frame 2.1 of Station 2 and Frame 3.1 of Station 3 are causing collision in Slot 2. Similarly the Frame 1.2 of Station 1 and Frame 4.1 of Station 4 are causing collision in Slot 3

Throughput

Note

The throughput for slotted ALOHA is $S = G \times e^{-G}$. The maximum throughput $S_{max} = 0.368$ when G = 1.

Figure 12.7 Vulnerable time for slotted ALOHA protocol



Example 12.4

A slotted ALOHA network transmits 200-bit frames on a shared channel of 200 kbps. What is the throughput if the system (all stations together) produces

- a. 1000 frames per second b. 500 frames per second
- c. 250 frames per second.

Solution

The frame transmission time is 200/200 kbps or 1 ms.

a. If the system creates 1000 frames per second, this is 1 frame per millisecond. The load is 1. In this case $S = G \times e^{-G}$ or S = 0.368 (36.8 percent). This means that the throughput is $1000 \times 0.0368 = 368$ frames. Only 386 frames out of 1000 will probably survive.

Example 12.4 (continued)

- b. If the system creates 500 frames per second, this is (1/2) frame per millisecond. The load is (1/2). In this case $S = G \times e^{-G}$ or S = 0.303 (30.3 percent). This means that the throughput is $500 \times 0.0303 = 151$. Only 151 frames out of 500 will probably survive.
- c. If the system creates 250 frames per second, this is (1/4) frame per millisecond. The load is (1/4). In this case $S = G \times e^{-G}$ or S = 0.195 (19.5 percent). This means that the throughput is $250 \times 0.195 = 49$. Only 49 frames out of 250 will probably survive.

1.1.2 CSMA (Carrier Sense Multiple Access)

- CSMA protocol was developed to minimize the chances of collision, so as to improve the performance.
- CSMA is based on the principle of carrier sense.
- The station sensed the carrier /medium (channel) before transmitting a frame.
- If a channel is idle, the station can send data to the channel. Otherwise it must wait until the channel become idle.
- It does not sense the entire transmission medium, rather it sense a region of transmission medium before transmitting the frame. So it cannot avoid collision totally.

Persistence Methods

1. 1-Persistent CSMA:

- In this method station that wants to transmit data continuously senses the channel to check whether the channel is idle or busy. If channel is busy, the station wait until it become idle.
- When the station detects an idle-channel, it immediately transmit the frames with probability 1. Hence it is called 1-persistent CSMA.
- This method has the highest chances of collision because two or more stations may find channel to be idle at the same time and transmit their frames immediately.

2. Non- Persistent CSMA:

- A station that has a frame to send senses the channel. If the station wants to transmit a frame and it finds that the channel is busy, then it will wait for fixed interval of time. After this time, it again checks the status of the channel and if it is free, it will transmit.
- It reduces the chances of collision

Persistence Methods

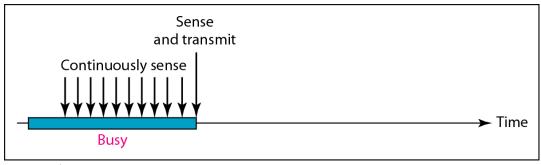
1. P-Persistent CSMA:

- It is the combination of 1-persistent and non-persistent modes.
- The p-persistent mode defines that each node senses the channel and if channel is inactive, it sends a frame with a 'p' probability.
- If the medium is busy, the station continues to listen until the channel is idle and repeats the same procedure when the medium is idle.
- It reduce the chances of collision and improves efficiency.

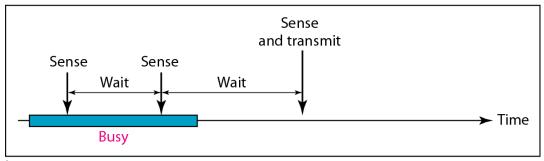
Steps for p-persistent

- a. With the probability p, the station send its frame
- b. With the probability q=1-p, the station wait for the beginning of the next time slot, and check the channel again
- a. If the channel is idle, it goes to step 1.
- b. If the line is busy, it acts as though a collision has occurred and uses back-off procedure to resend. It depends on k (No: of retransmission).

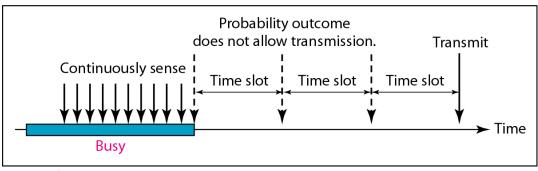
Figure 12.10 Behavior of three persistence methods



a. 1-persistent

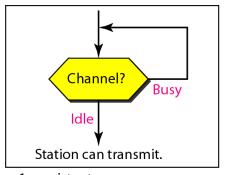


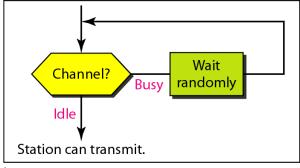
b. Nonpersistent



c. p-persistent

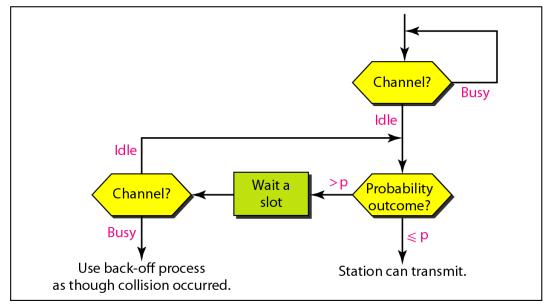
Figure 12.11 Flow diagram for three persistence methods





a. 1-persistent

b. Nonpersistent



c. p-persistent

Figure 12.8 Space/time model of the collision in CSMA

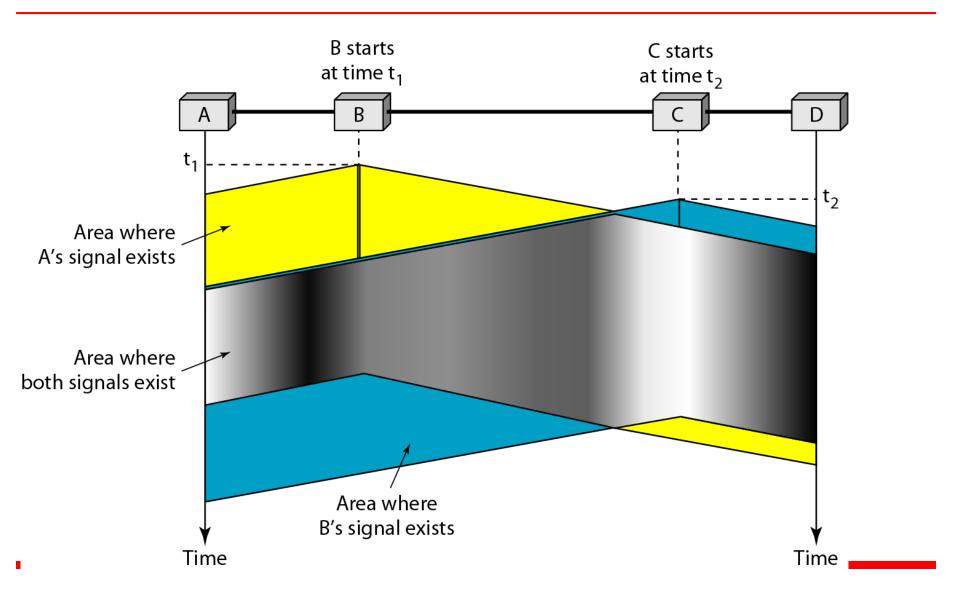
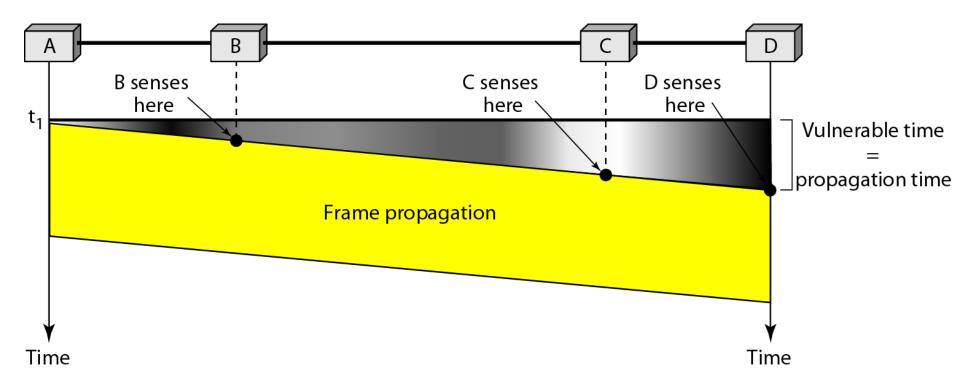


Figure 12.9 Vulnerable time in CSMA



Vulnerable time for CSMA is the propagation time Tp. This is the time needed for a signal to propagate from one medium to other

1.1.3 **CSMA/CD**

In CSMA/CD monitors the medium after it sends a frame to see if the transmission was successful. If so, the transmission is finished. If however, there is a collision, the frame is sent again.

Figure 12.12 Collision of the first bit in CSMA/CD

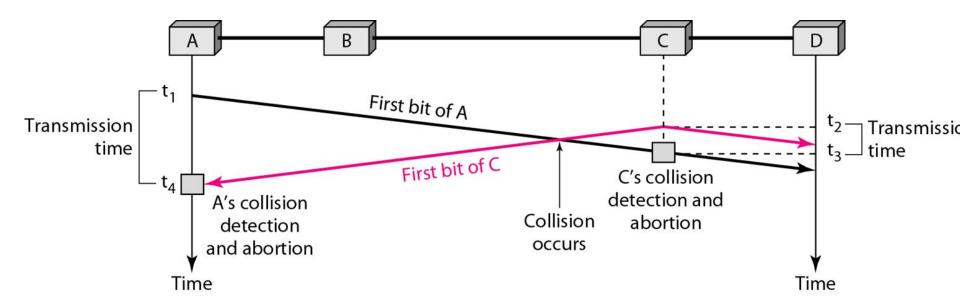
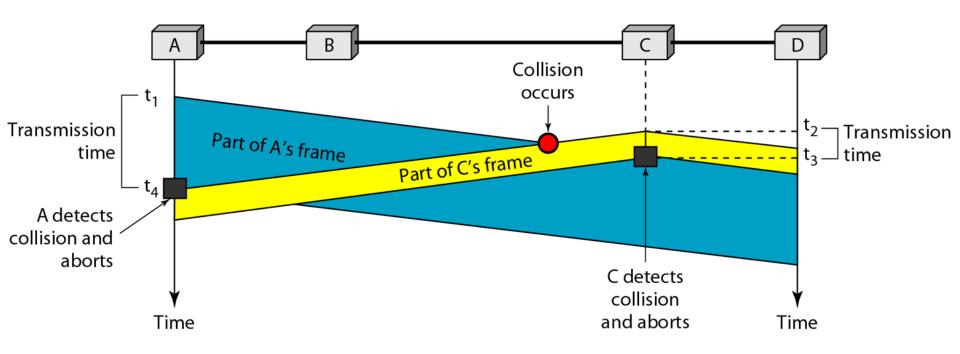


Figure 12.12 Collision of the first bit in CSMA/CD

- At the time t1, Station A has executed its persistence procedure and starts sending the bits of its frame.
- At time t2, Station C has not yet sensed the first bit sent by A.
- Station C executes its persistence procedure and starts sending the bit in its frame, which propagate both to left and to right.
- The collision occurs sometime after time t2. Station C executes its persistence procedure and starts sending the bit in its frame, which propagate both to left and to right.
- The collision occurs sometime after time t2. Station C detects a collision at time t3 when it receives the first bit of A's frame.
- Station C immediately aborts transmission. Similarly Station A detects collision at time t4 when it receives the first bit of C's frame, it also immediately abort the transmission.

Figure 12.13 Collision and abortion in CSMA/CD



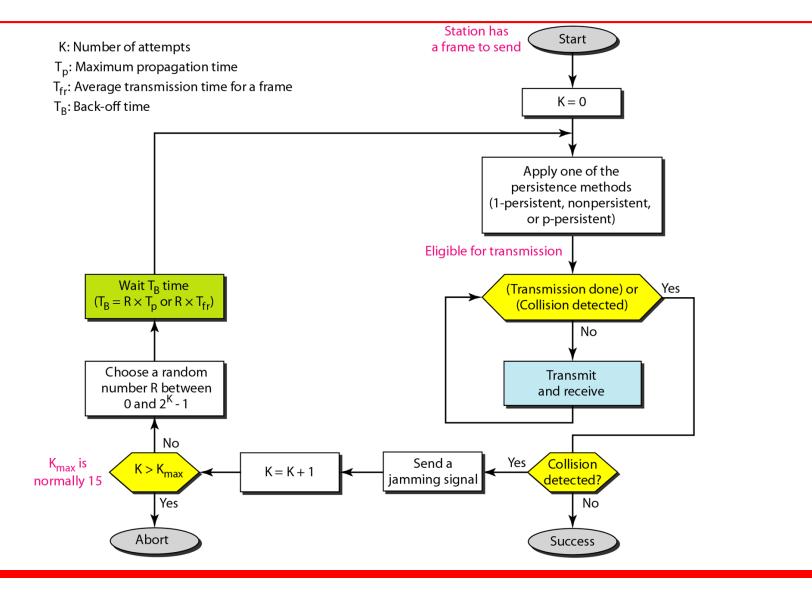
Example 12.5

A network using CSMA/CD has a bandwidth of 10 Mbps. If the maximum propagation time (including the delays in the devices and ignoring the time needed to send a jamming signal, as we see later) is 25.6 µs, what is the minimum size of the frame?

Solution

The frame transmission time is $T_{fr} = 2 \times T_p = 51.2 \ \mu s$. This means, in the worst case, a station needs to transmit for a period of 51.2 μs to detect the collision. The minimum size of the frame is 10 Mbps \times 51.2 $\mu s = 512$ bits or 64 bytes. This is actually the minimum size of the frame for Standard Ethernet.

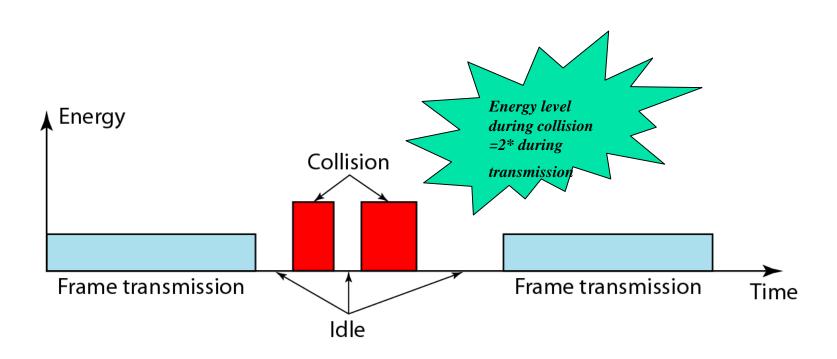
Figure 12.14 Flow diagram for the CSMA/CD



Energy level during transmission, idleness, or collision

- The level of energy in a channel can have 3 values: zero, normal and abnormal.
- At the zero level, the channel is idle
- At the **normal level**, a station can successfully captured the channel and sending its frame.
- At the **abnormal level**, there is a collision and the level of the energy is twice the normal level.
- A station that has a frame to send or is sending a frame needs to monitor the energy level to determine if the channel is idle, busy or in collision mode

Figure 12.15 Energy level during transmission, idleness, or collision



Throughput

- The throughput of CSMA/CD is greater than that of pure and slotted ALOHA.
- For the 1 persistent method, the maximum throughput is around 50% when G=1, where G is the number of frames generated by a station at a time.
- For the non-persistent method, the max throughput can go upto 90% when G is between 3 and 8.

1.1.4 **CSMA/CA**

- In wireless network, much of the sent energy is lost in transmission. The received signal has very little energy. The collision can add hardly 5 to 10%
 - of additional energy. This is not useful for effective collision detection.
- Collisions on wireless network cannot be detected. So we need to avoid
 - collisions on wireless networks.
- Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) was invented for wireless networks.
- Collisions are avoided through the use of CSMS/CA's 3 strategies
 - The interframe space(IFS)
 - The contention window
 - Acknowledgments

Interframe space (IFS)

- First, collision can be avoided by deferring transmission, even if channel is found idle.
- When an idle channel is found, the station does not send immediately. It waits for a period of time called the interframe space or IFS.
- Even though the channel may appear idle when it is sensed, a distant station may have already started transmitting.
- The distant station's signal has not yet reached this station. The IFS time allows the front of the transmitted signal by distant station to reach this station.
- If after the IFS time the channel is still idle, the station can send, but it still needs to wait a time equal to the contention time.

Note

In CSMA/CA, the IFS can also be used to define the priority of a station or a frame. The IFS time will be set by the administrator.

Contention Window

- The contention window is an amount of time divided into slots.
- A station that is ready to send chooses the random number of slots as its waiting time. The number of slots in the window changes according to the binary exponential backoff strategy. This means that it is set to one slot time and then doubles each time.
- This is very similar to p-persistent method except that a random outcome defines the number of slots taken by waiting station.
- In Contention window strategy, the station need to sense the channel after each time slot.
- However, if the station finds the channel busy, it does not restart the process. It just stop the timer and restarts it when the channel is sensed idle. This gives priority to the station with the longest waiting time

Note

In CSMA/CA, if the station finds the channel busy, it does not restart the timer of the contention window; it stops the timer and restarts it when the channel becomes idle.

Acknowledgment

- With all these precautions, there still may be a collision resulting destroyed data
- In addition, the data may be corrupted during transmission
- The positive acknowledgment and time-out timer can help guarantee that the receiver received the frame.

Figure 12.16 Timing in CSMA/CA

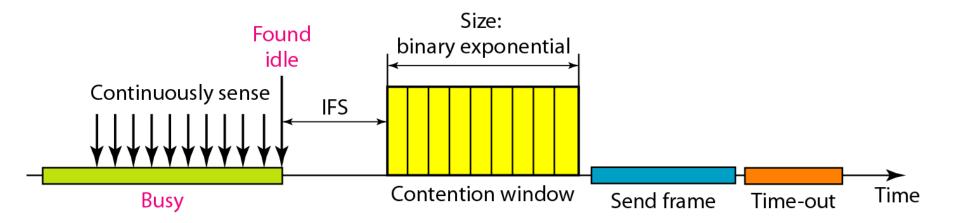
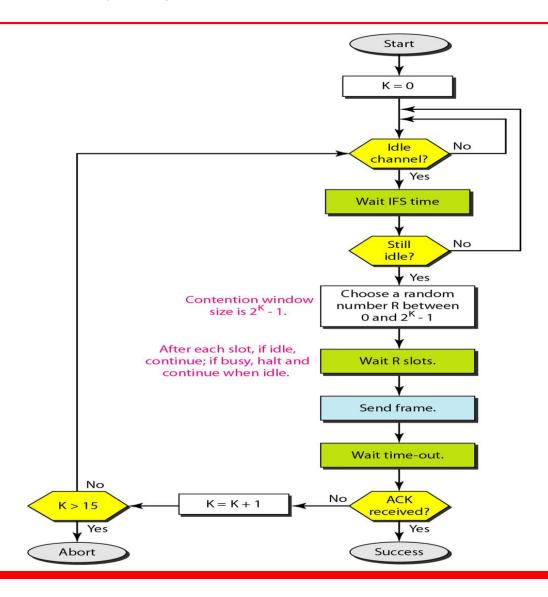


Figure 12.17 Flow diagram for CSMA/CA



1.2 CONTROLLED ACCESS

In controlled access, the stations consult one another to find which station has the right to send. A station cannot send unless it has been authorized by other stations. There are three popular controlled-access methods.

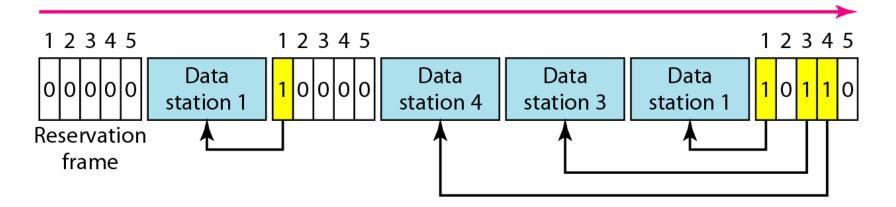
Controlled Access Methods:

Reservation
Polling
Token Passing

1.2.1 Reservation

- In reservation method, a station needs to make a reservation before sending data.
- Time is divided into intervals and each interval, a reservation frame precedes the data frames sent in that interval.
- If there are N stations in the system, there are exactly N reservation minislots in the reservation frame.
- Each minislot belongs to a station
- When a station need to send a data frame, it make a reservation in its own minislot.
- The station that have made reservation can send their data frame after the reservation frame.

Figure 12.18 Reservation access method

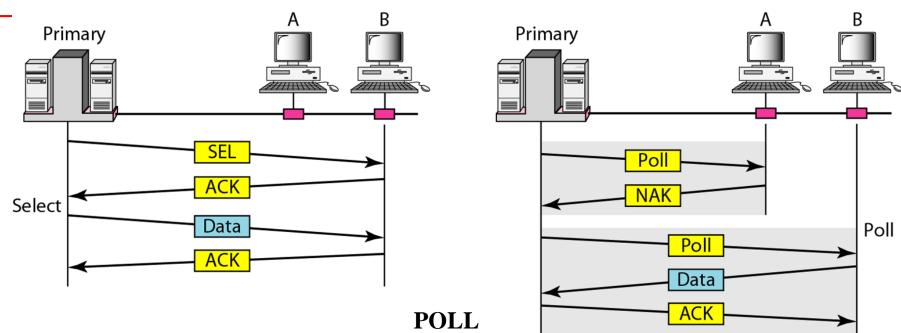


- Figure shows a situation with 5 station and 5 minislot reservation frame.
- In the first interval, only Station1,3, and 4 have made reservation. In the second interval, only station1 has made a reservation.

1.2.2 Polling

- Polling works with topologies in which one device is designated as a primary station and the other devices are secondary stations.
- All data exchanges must be made through the primary devices even when the ultimate destination is a secondary device.
- The primary devices controls the link, the secondary devices follow its instructions.
- It is up to the primary device to determine which device is allowed to use the channel at a given time
- The primary device is always the initiator of a session.
- This method uses poll and select function to prevent collision. However, the drawback is if the primary station fails, the system goes down.

Figure 12.19 Select and poll functions in polling access method



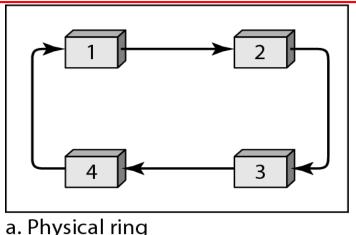
SELECT

- The select function is used whenever primary device has something to send.
- Before sending data, the primary creates and transmit a select (SEL) frame, one field of which includes the addresses of the intended secondary
- The poll function is used by the primary device to solicit transmission from the secondary.
- When the primary is ready to receive data, it must ack (poll) each device in turn it has anything to send.
- If the response is negative (NAK frame), the primary polls the nest secondary in the same manner until it finds one with data to send.
- If the response is positive(data frame), the primary reads the frame and returns an acknowledgment (ACK frame), verifying its receipt.

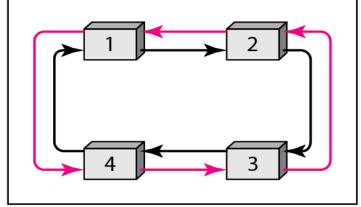
1.2.3 Token Passing

- In token ring passing method, the station in a network are organized in a logical ring; i.e., for each station, there is a successor and a predecessor.
- In this method, a special packet called token circulates through the ring. The possession of token gives the station the right to access the channel and send its data.
- When a station has some data to send, it waits until it receives the token from its predecessor. It holds token and send its data. When the station has no more data to send, it releases the token and passing it to the next logical station in the ring.
- The station cannot send data until it receives the taken again in the next round.
- Token management is needed for his access method. The taken must be monitored to ensure it has not been lost or destroyed.
- Token management assign priorities to the stations and to the type of data being transmitted.
- Token management is needed to make low priority stations release the token to high priority stations

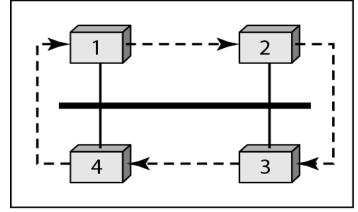
Figure 12.20 Logical ring and physical topology in token-passing access method



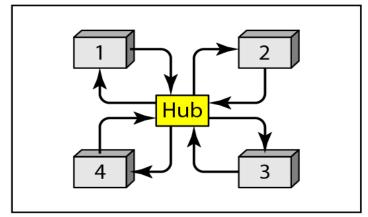
a. Physical ring



b. Dual ring



c. Bus ring



d. Star ring

Not physically connected but the ring can be a logical one.

* Please refer textbook for detailing

12-3 CHANNELIZATION

Channelization is a multiple-access method in which the available bandwidth of a link is shared in time, frequency, or through code, between different stations. In this section, we discuss three channelization protocols.

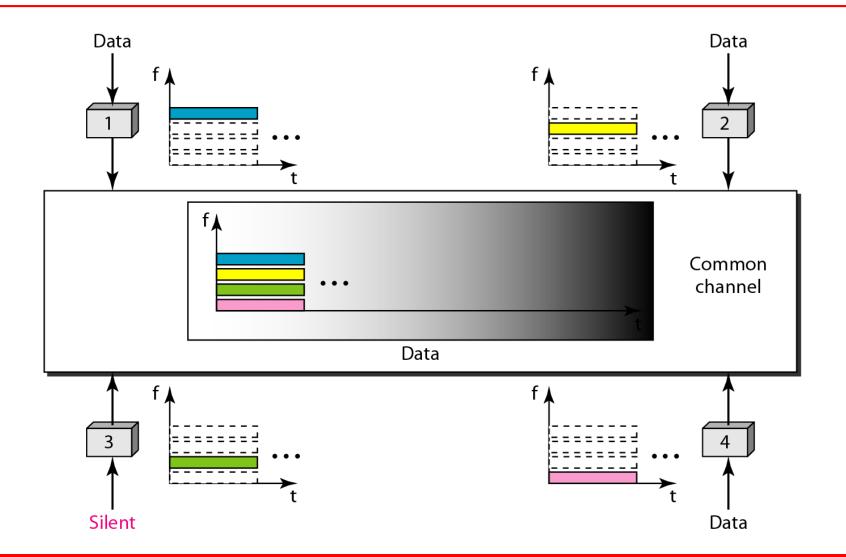
Topics discussed in this section:

Frequency-Division Multiple Access (FDMA)
Time-Division Multiple Access (TDMA)
Code-Division Multiple Access (CDMA)

1.3.1 FDMA(Frequency Division Multiple Access)

- The available bandwidth is divided into frequency bands. Each station is allocated a band to send its data.
- Each band is reserved for a specific station, and it belong to the station all the time.
- Each station also use a band pass filter to confine the transmitter frequencies.
- To prevent station interference, the allocated bands are separated from one another by small guard band.
- In FDMA, the available bandwidth of the common channel is divided into bands that are separated by guard bands.
- FDMA specifies a predetermined frequency band for entire period of communication.
- Stream data(continuous flow of data that may not be packetized) can easily be used with FDMA.
- This feature can be used in cellular telephone system.

Figure 12.21 Frequency-division multiple access (FDMA)



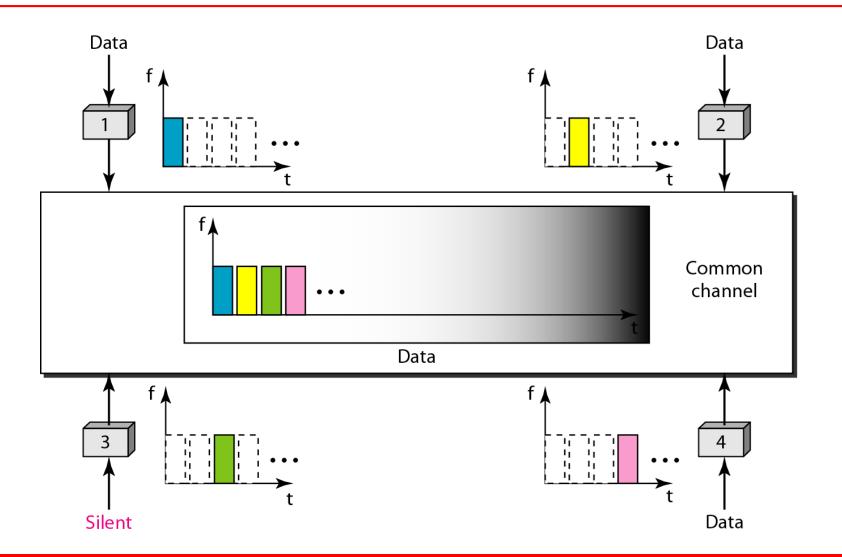
Note

In FDMA, the available bandwidth of the common channel is divided into bands that are separated by guard bands.

1.3.2 TDMA(Time Division Multiple Access)

- The station share the bandwidth of the channel in time.
- Each station is allocated a time slot during which it can send data.
- Each station transmit its data in its assigned time slot.
- The main problem with TDMA lies in achieving synchronization between the different stations.
- Each station need to know the beginning of its slot and the location of its slot. This may be difficult because of the propagation delays introduced in the system, if the station are spread over a large area.
- To compensate for delays, we can insert guard time. Synchronization is normally accomplished by having some synchronization bits (preamble bits) at the beginning of each slot.
- In TDMA, bandwidth is just one channel i.e; time shared between different stations.
- TDMA is an access method in data link layer. The data link layer in each station tell its physical layer to use the allocated time slot.

Figure 12.22 Time-division multiple access (TDMA)



Note

In TDMA, the bandwidth is just one channel that is timeshared between different stations.

1.3.3 CDMA (Code Division Multiple Access)

- CDMA differs the FDMA because only one channel occupies the entire bandwidth of the link.
- It differs from TDMA because all stations can send data simultaneously; there is no timesharing.
- In CDMA one channel carries all transmission simultaneously.
- CDMA means communication with different codes.

Note

In CDMA, one channel carries all transmissions simultaneously.

Figure 12.23 Simple idea of communication with code

- Using the same channel, it allows communication between several couples using different codes.
- Let us assume we have 4 stations 1, 2,3 and 4 connected to same channel. The data from Station 1 are d1, and those from Station 2 are d2 and so on. The code assigned to first Station is c1, to the second c2 and so on.
- The assigned code have 2 properties
- 1. If we multiply each code by another, we get 0.
- 2. If we multiply each code with itself, we get 4(i.e., number of stations)

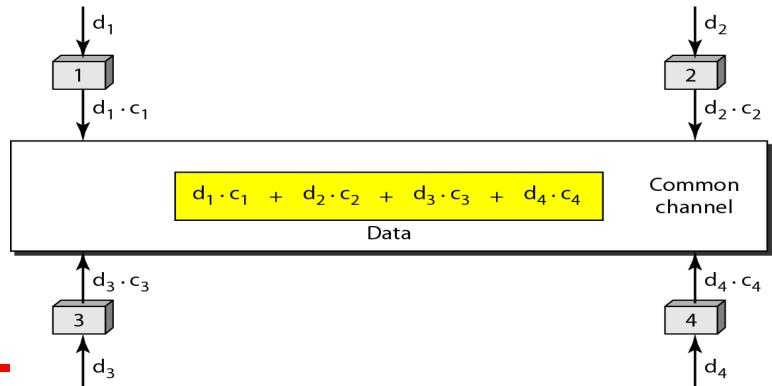


Figure 12.24 Chip sequences

Code should be selected very carefully. They are called orthogonal sequence, having properties.

- 1. Each sequence is made of N elements, where N is the number of stations.
- 2. If we multiply a sequence by a number, every element in the sequence is multiplied by that element. This is called multiplication of sequence by a scalar.

$$2.[+1 + 1 - 1 - 1] = [+2 + 2 - 2 - 2]$$

If we multiply 2 equal sequence, element by element and add the results, we get 'N', where 'N' is the number of elements in the each sequence.

$$[+1 + 1 - 1 - 1]$$
. $[+1 + 1 - 1 - 1] = 4$

3. If we multiply two different sequence, element by element and add the results, we get 0. This is the inner product of 2 different sequence.

$$[+1 + 1 - 1 - 1]$$
. $[+1 + 1 + 1 + 1] = 0$

Adding 2 sequence means adding the corresponding elements.

$$[+1 + 1 - 1 - 1] + [+1 + 1 + 1 + 1] = [+2 + 2 \ 0 \ 0]$$

C.

$$[+1 +1 +1 +1]$$

 C_{j}

 C_3

 $\mathsf{C}_{\mathtt{\Delta}}$

Figure 12.25 Data representation in CDMA

If a station needs to send 0 bit, it encode as -1. If it needs to send a 1 bit, it encodes as +1. When station is idle, it send no signal and encodes as 0. This is the data representation in CDMA

Data bit 1───+1

Silence → 0

Figure 12.26 Sharing channel in CDMA

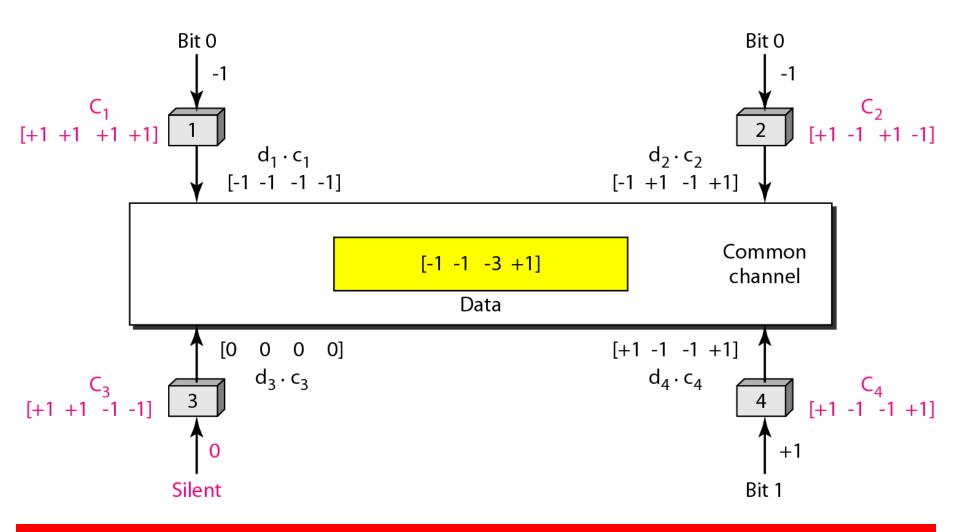


Figure 12.27 Digital signal created by four stations in CDMA

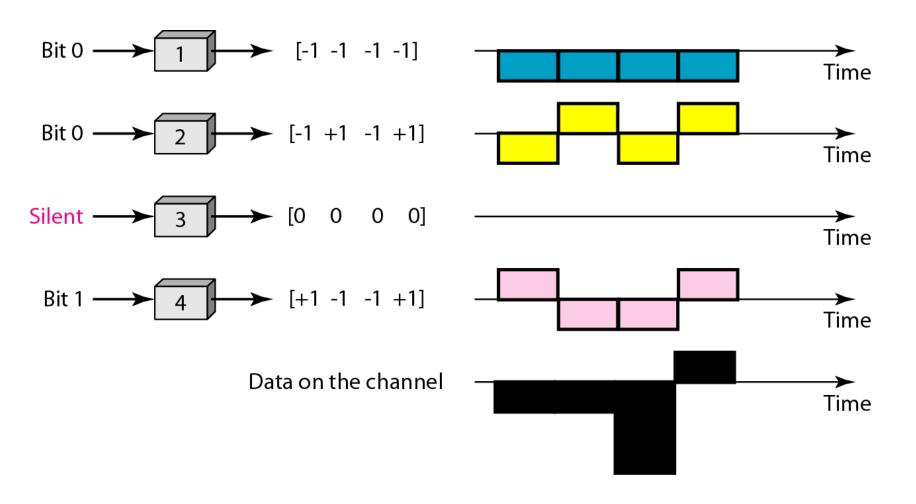


Figure 12.28 Decoding of the composite signal for one in CDMA

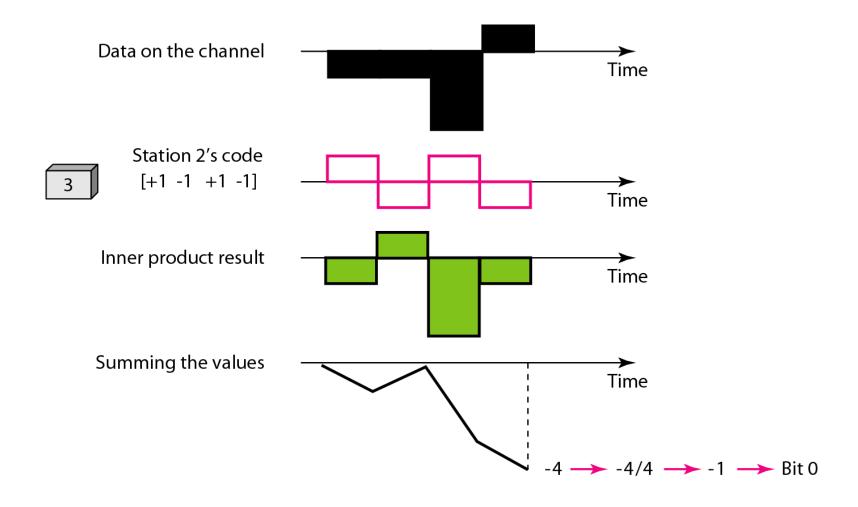


Figure 12.29 General rule and examples of creating Walsh tables

$$W_1 = \begin{bmatrix} +1 \end{bmatrix} \qquad W_{2N} = \begin{bmatrix} W_N & W_N \\ W_N & \overline{W}_N \end{bmatrix}$$

a. Two basic rules

$$W_{1} = \begin{bmatrix} +1 \\ +1 \end{bmatrix}$$

$$W_{2} = \begin{bmatrix} +1 \\ +1 \\ +1 \end{bmatrix}$$

$$W_{4} = \begin{bmatrix} +1 \\ +1 \\ +1 \end{bmatrix}$$

$$W_{4} = \begin{bmatrix} +1 \\ +1 \\ +1 \end{bmatrix}$$

$$W_{1} = \begin{bmatrix} +1 \\ +1 \\ +1 \end{bmatrix}$$

$$W_{2} = \begin{bmatrix} +1 \\ +1 \\ +1 \end{bmatrix}$$

$$W_{3} = \begin{bmatrix} +1 \\ +1 \\ +1 \end{bmatrix}$$

b. Generation of W_1 , W_2 , and W_4

Note

The number of sequences in a Walsh table needs to be $N = 2^{m}$.

Example 12.6

Find the chips for a network with a. Two stations b. Four stations

Solution

We can use the rows of W_2 and W_4 in Figure 12.29: a. For a two-station network, we have [+1 +1] and [+1 -1].

b. For a four-station network we have [+1 +1 +1 +1], [+1 -1 +1 -1], [+1 +1 -1], [+1 +1 -1 -1], and [+1 -1 -1 +1].

Example 12.7

What is the number of sequences if we have 90 stations in our network?

Solution

The number of sequences needs to be 2^m . We need to choose m = 7 and $N = 2^7$ or 128. We can then use 90 of the sequences as the chips.

Example 12.8

Prove that a receiving station can get the data sent by a specific sender if it multiplies the entire data on the channel by the sender's chip code and then divides it by the number of stations.

Solution

Let us prove this for the first station, using our previous four-station example. We can say that the data on the channel

 $D = (d_1 \cdot c_1 + d_2 \cdot c_2 + d_3 \cdot c_3 + d_4 \cdot c_4).$ The receiver which wants to get the data sent by station 1 multiplies these data by c_1 .

Example 12.8 (continued)

$$\begin{aligned} D \cdot c_1 &= (d_1 \cdot c_1 + d_2 \cdot c_2 + d_3 \cdot c_3 + d_4 \cdot c_4) \cdot c_1 \\ &= d_1 \cdot c_1 \cdot c_1 + d_2 \cdot c_2 \cdot c_1 + d_3 \cdot c_3 \cdot c_1 + d_4 \cdot c_4 \cdot c_1 \\ &= d_1 \times N + d_2 \times 0 + d_3 \times 0 + d_4 \times 0 \\ &= d_1 \times N \end{aligned}$$

When we divide the result by N, we get d_1 .