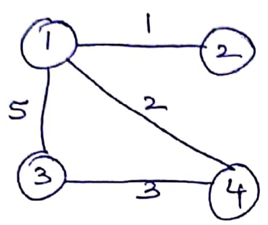
Prim's algorithm :- Used to obtain minimum spanning tree.

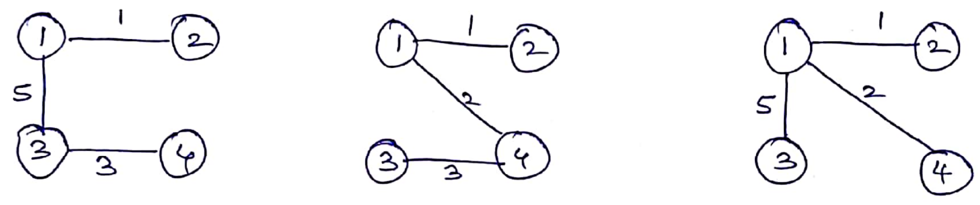
- A minimum spanning tree of a given graph is the minimum acyclic sub-graph of given graph.

Ex. :-



$V' = V$

Spanning trees :-



$C = 9$. $C = 6$ $C = 8$.

Algorithm :-

// Minimum Spanning tree = T
// Set of explored nodes = U

$T = \phi$

$U = \{r\}$ where r is any arbitrary vertex.

while $(U \neq V)$

let (u, v) be the lowest cost edge.

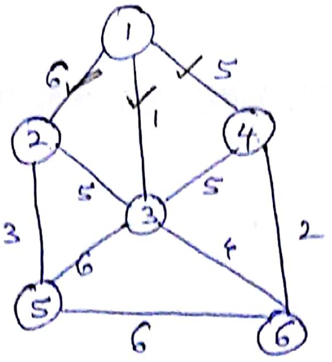
such that $u \in U$ and $v \in V - U$

$T = T \cup \{\{u, v\}\}$

$U = U \cup \{v\}$

end while.

Example :-



Consider the arbitrary vertex $r = 1$.

visited

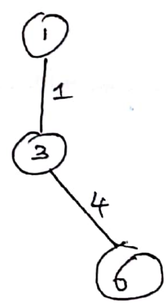
1	2	3	4	5	6
1	0	1	0	0	~~0~~ 1

Step	U	V − U	Cost	
1	1	2	6.	
	1	3	1	min cost.
	1	4	5	Add: $v = 3$ to U.
	1	5	∞	
	1	6	∞	

2) $U = \{1, 3\}$ $V - U = \{2, 4, 5, 6\}$

U	V − U	Cost	
1	2.	6.	
1	4.	5	
3.	2.	5	
3	4.	5	
3	5	6	
3	6	4.	min cost.

Add $v = 6$ to U.

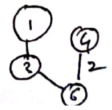


3) $U = \{1, 3, 6\}$ $V - U = \{2, 4, 5\}$

U	V − U	Cost
1	2.	6.
1	4	5.
1	5	∞
3	2	5.
3.	4.	5.
3	5	6

6	2	∞
6	4	2
6	5	6

min. Add 4 to U.



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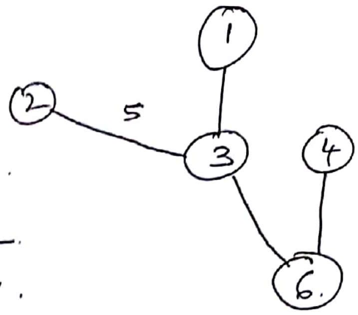
4) $U = \{1, 3, 6, 4\}$ $V - U = \{2, 5\}$

U	V − U	Cost
1	2	6
1	5	∞
3	2	5
3	5	6
6	2	∞
6	5	6

min cost
Add 2 to U.

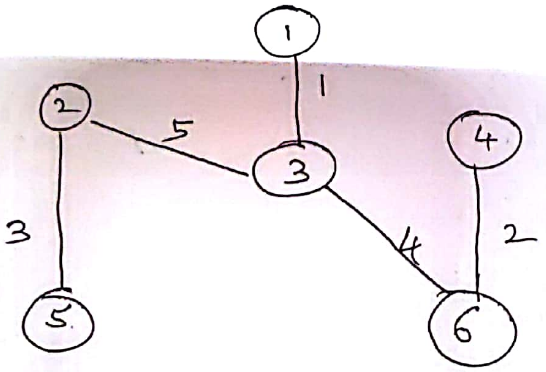


5) $U = \{1, 3, 6, 4, 2\}$ $V - U = \{5\}$

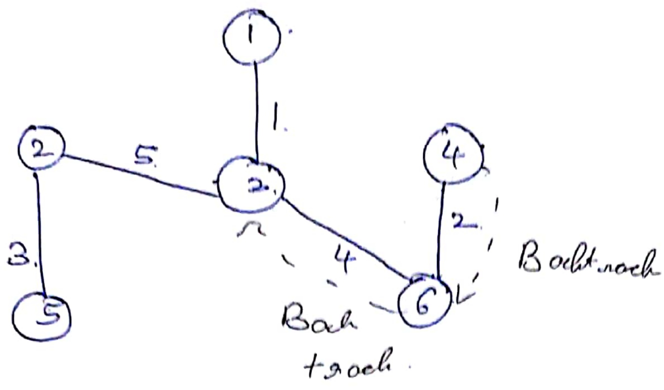
U	V − U	Cost
1	5	∞
3	5	~~6~~
6	5	6
4	5	∞
2	5	3

Min cost
Add 5 to U.

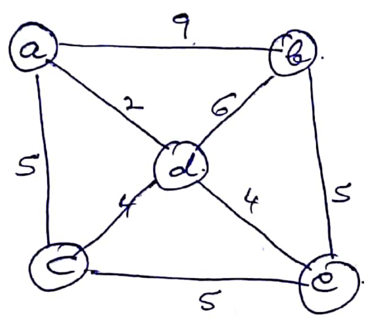
<u>Minimum Spanning tree is</u>



Shortcut



Example 2

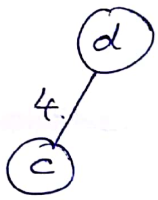


1)

$V - U = \{a, b, c, e\}$

U	V − U	Cost
d	a	2
d	b	6
d	**c**	**4**
d	e	4

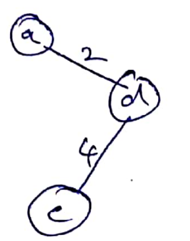
Add c to U.



2) $U = \{d, c\}$ $V - U = \{a, b, e\}$

U	V − U	Cost
d	**a**	**2**
d	b	6
d	c	4
c	a	5
c	b	∞
c	e	5

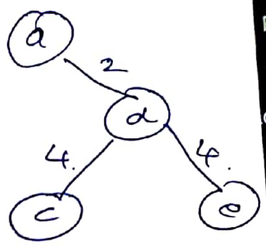
Add a to U.



3) $U = \{d, c, a\}$ $V-U = \{b, e\}$

U	V-U	Cost
d	b	6
d	e	4
c	b	∞
c	e	5
a	b	9
a	e	∞

Add e to U.



4) $U = \{d, c, a, e\}$ $V-U = \{b\}$

U	V-U	Cost
d	b	6
c	b	∞
a	b	9
e	b	5

Add b to U.

Min cost spanning tree