Exponential Distribution :

A continuous random variable x has the pdf $f(x) = \begin{cases} \lambda e^{-\lambda x} & , x \geq 0 \\ 0 & , \text{otherwise} \end{cases}$

is known as Exponential distribution provided $f(x) \geq 0$ and $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

Properties of Exponential Distribution:

Mean of E.D : We know the Mean $\mu = \int_{-\infty}^{\infty} x f(x)\,dx$

$$\therefore \quad \mu = \cancel{\int_{-\infty}^{0} x f(x)\,dx} + \int_{0}^{\infty} x f(x)\,dx$$

$$= \int_{0}^{\infty} x \lambda e^{-\lambda x}\,dx$$

$$= \lambda \left[x\left(\frac{e^{-\lambda x}}{-\lambda}\right) - 1\cdot\left(\frac{e^{-\lambda x}}{\lambda^2}\right) \right]_0^{\infty}$$

$$= \lambda \left[0 - \left(0 - \frac{1}{\lambda^2}\right) \right]$$

$$\boxed{\mu = \frac{1}{\lambda}}$$

Variance of E.D : We know that variance

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx = \int_{0}^{\infty} (x-\mu)^2 \lambda e^{-\lambda x}\,dx$$

$$= \lambda \left[(x-\mu)^2 \left(\frac{e^{-\lambda x}}{-\lambda}\right) - 2(x-\mu)\left(\frac{e^{-\lambda x}}{\lambda^2}\right) + 2\left(\frac{e^{-\lambda x}}{-\lambda^3}\right) \right]_0^{\infty}$$

$$= \frac{1}{\lambda^2}$$

$$\therefore \quad \boxed{\sigma = \frac{1}{\lambda}}$$

Problems on Exponential distribution:

① If X is an Exponential variate with mean '3' then find (i) $P(X>1)$ (ii) $P(X<3)$.

Sol:- The Pdf of E.D is $f(x) = \begin{cases} \lambda e^{-\lambda x} & , x \geq 0 \\ 0 & , \text{otherwise.} \end{cases}$

Mean of E.D $\mu = \frac{1}{\lambda}$

Given mean $=3$

$\therefore \frac{1}{\lambda} = 3 \Rightarrow \lambda = \frac{1}{3}$

$\therefore f(x) = \begin{cases} \frac{1}{3} e^{-x/3} & , x \geq 0 \\ 0 & , \text{otherwise.} \end{cases}$

(i) $P(X>1) = 1 - P(X \leq 1)$

$= 1 - \int_0^1 f(x)dx$

$= 1 - \int_0^1 \frac{1}{3} e^{-x/3} dx$

$= e^{-1/3} = 0.72.$

(ii) $P(X<3) = \int_0^3 f(x)dx$

$= \int_0^3 \frac{1}{3} e^{-x/3} dx$

$= \frac{1}{3} \left(\frac{e^{-x/3}}{-1/3} \right)_0^3$

$= \left(-e^{-x/3} \right)_0^3$

$= -(e^{-1} - 1) = 1 - \frac{1}{e} = 0.63.$

(2) The sales per day in a shop are exponentially distributed with average sale amounting to Rs. 100 and net profit is 8%. Find the probability that the net profit exceeds Rs 30 on two consecutive days.

25

Sol :- Let X = Sales in the shop.

The pdf of E.D is $f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise}. \end{cases}$

Given mean $\mu = 100$.

$\therefore \frac{1}{\lambda} = 100 \Rightarrow \lambda = 0.01$

$\therefore f(x) = \begin{cases} (0.01) e^{-(0.01)x}, & x \geq 0 \\ 0, & \text{otherwise}. \end{cases}$

Let A be the amount for which profit is 8%.
since the profit is Rs30,

$$A \cdot \frac{8}{100} = 30$$

$$\Rightarrow A = 375.$$

consider P(Net profit exceeds 30) = P(Sales > 375)

$$= P(X > 375)$$

$$= 1 - P(X \leq 375)$$

$$= 1 - \int_0^{375} (0.01) e^{-(0.01)x} \, dx$$

$$= e^{-3.75}$$

Prob.
$\therefore$ for 2 consecutive days $= e^{-3.75} \cdot e^{-3.75}$

$$= 0.0006.$$

③ The length of telephone conversation in a booth has been an exponential distribution and found that on an average to be 5 mins. Find the prob that a random call made from this booth (i) ends less than 5 mins
(ii) between 5 and 10 mins.

Sol: Let X = length of telephone conversation.

The pdf of E.D is $f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise.} \end{cases}$

Given mean $\mu = \frac{1}{\lambda} = 5$

$\Rightarrow \boxed{\lambda = \frac{1}{5}}$

$\therefore f(x) = \begin{cases} \frac{1}{5} e^{-x/5}, & x \geq 0 \\ 0, & \text{otherwise.} \end{cases}$

(i) $P(\text{less than 5 mins}) = P(X < 5)$

$$= \int_0^5 \frac{1}{5} e^{-x/5} dx$$

$$= \frac{1}{5} \left(\frac{e^{-x/5}}{-1/5} \right)_0^5$$

$$= 1 - \frac{1}{e} = 0.6321$$

(ii) $P(\text{between 5 and 10 mins}) = \int_5^{10} \frac{1}{5} e^{-x/5} dx$

$$= \left(-e^{-x/5} \right)_5^{10}$$

$$= -\left(e^{-2} - e^{-1} \right)$$

$$= \frac{1}{e} - \frac{1}{e^2} = 0.2325$$