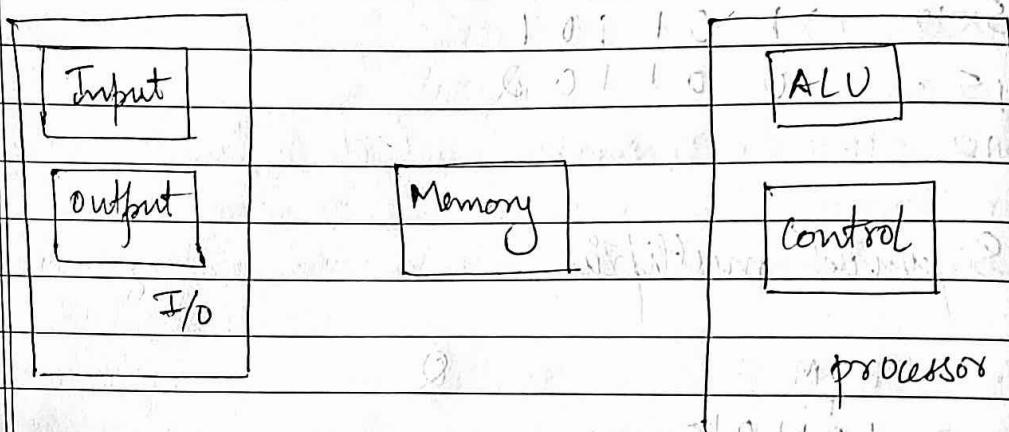


D	D	M	M	Y	Y	Y	Y
---	---	---	---	---	---	---	---

Computer Organisation & Architecture

Functional units:

Input, ALU, Memory, Control, I/O



Computer Organisation is concerned with structural and behaviours of the computer system as seen by the user.

Computer Architecture is concerned with the way hardware components are connected together to form a computer system. It acts as an interface between the hardware & software.

Computer Organisation is a low level design whereas Computer Architecture is a high level design.



mollisco



Clock: defines regular time intervals called clock cycles.

$$\text{Exe}(2) + (\text{Exe}(3)) + (\text{Exe}(4)) = 2 \times 4 = 16$$

P: length of one clock cycle.



\rightarrow P one clock cycle.

Clock Rate: - The no. of clock cycles per second.

$$R = \frac{1}{P}$$

CPV, speed / clock rate, is measured in Hertz.
↓
clock

Basic performance equation:-

$$T = N \times S$$

↓
Clock Rate

T: process time required to execute a program.

N: Number of instructions.

R: Clock rate

S: Number of basic instructions required to perform 1 instruction

- Q) A program contain 1000 instruction of out of that 25% instruction requires 4 clock cycles. 40% instruction require 5 clock cycle. for execution. Find the total time required to execute the program running in a 1 GHz machine.

Solⁿ

$S = 25\% \text{ of } N = 250 \text{ instructions requires } 4 \text{ clock cycles}$

$40\% \text{ of } N = 400 \text{ instructions requires } 5 \text{ clock cycles.}$

$35\% \text{ of } N = 350 \text{ instructions require } 3 \text{ clock cycles.}$



$$R = 1 \text{ GHz} = 10^9 \text{ Hz}$$

$$T = \frac{N \times S}{R} = \frac{(250 \times 4) + (400 \times 5) + (350 \times 3)}{10^9}$$

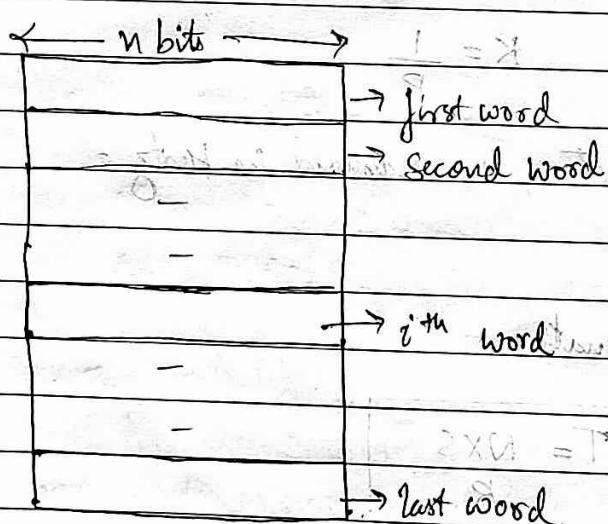
$$= \frac{4050}{10^9} = 4.05 \times 10^{-6}$$

$$\therefore = 4.05 \mu\text{s}$$

Memory location and Addresses

Group of n bits is word

n is the word length



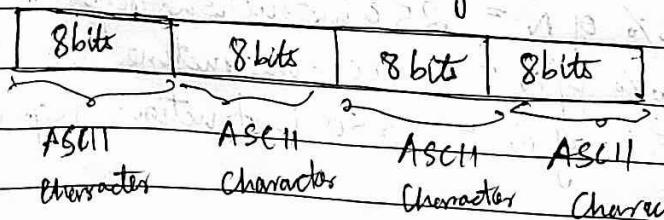
Memory words

If the word length is 32 bits, single word can store 32 bits or 4 bytes ASCII characters.



32 bits

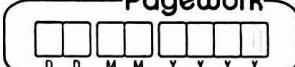
sign bit : $b_{31} = 0$ for +ve numbers
 $b_{31} = 1$ for -ve numbers



Bus

- 1) Address Bus
- 2) Data Bus
- 3) Control Bus

PageWork



For storing 12 34 56 78 :-

LSB

→ Big - Endian

12 34 56 78

→ Little - Endian

78 56 34 12

0	0	1	2	3
4	4	5	6	7
3	3	2	1	0
2	2	1	0	1

2^{k-4}

Memory address

0 1 2 3

7 6 5 4

2^{k-4}

Big - Endian

assignment

Little - Endian

assignment.

Memory Operations

- 1) Load (or Read or Fetch)
- 2) Store (or Write)

Array of cells

MAR

MDR

Memory Address

Memory Data

register

Register

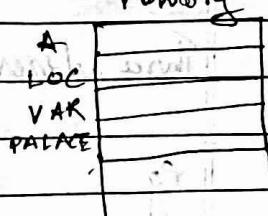
Instruction and Instruction Sequencing

Data Transfer :-

Memory location is given symbolic name.

→ Name for memory address location.

A, LOC, VAR, PLACE



→ Name for Processor register names.

R1, R2, R3, ...

→ Name for I/O register names.
DATIN, DATOUT

- Register Transfer Notation (RTN)
- Assembly Language Notation.

RTN :-

$$\begin{array}{l} R_3 \leftarrow [R_1] + [R_2] \\ R_1 \leftarrow [LOC] \end{array}$$

Processor

$$R_1 = 10$$

$$R_2 = 20$$

$$R_3 = 30$$

Assembly Language Notation :-

Move LOC, R1

Add R1, R2, R3

Basic Instruction types.

Three address instruction format : operation Source1, Source2, Destination
 two address instruction format
 one address instruction format
 zero address instruction format.

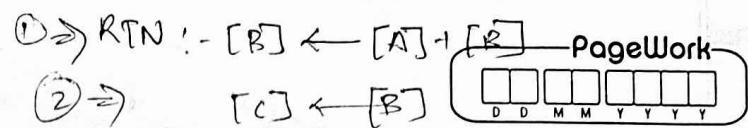
Label : operation operands.

Three address :- operation Source1, Source2, Destination.

e.g:- Add A, B, C

Add the contents of A and B then store the result in C.

Add A, B



two addresses :- Operation, Source, Destination

Eg :- 1) Add A, B loses its original value.

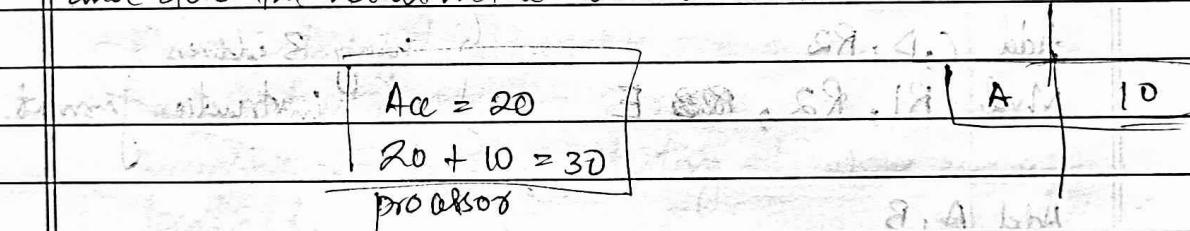
add the contents at A and B then store the result in B.

2) Add A, B retains its original value.
Move B, C
Add A, C

one address :- Operation, Operand.

→ can be source or destination depending on the type of instruction

Eg:- 1) Add A $(A+3) \rightarrow (8+A)$ Second operand is taken from the accumulator and store the result in the accumulator.



2)

Load A → source operand

Store A → destination operand

Examples:- Add R_i, R_j or

Add R_i, R_j, R_k

→ directly using the processor register

Move Same, Destination

Eg:- Move R1, R2

Move A, R1 } Both are same instruction.
Load A, R1 }

Move R1, A } Both are same.
Store R1, A }

e) evaluate $(A+B)*(C+D)$

Add A, B, R1

Add C, D, R2

Mul R1, R2, E } using 3 address instruction format.

Add A, B

Move B, R1

Add C, D

Move D, R1

Mul R1, R2, E

Move R2, E

using 2 address instruction format.



Instruction and Instruction Sequencing :-

Straight line Sequencing :- To begin executing a program, the address of the first instruction will be placed in program counter, then processor control unit will use the information in PC to fetch and execute the instruction one at a time. In the order of increasing addresses. This is called straight line sequencing.

Q) Write a program for $C \leftarrow [A] + [B]$:

Address	Contents
i	Move A, RD
i+4	ADD B, RD
i+8	Move RD, C

Assume.

Memory word length is 32 bits

Memory is byte addressable

Three instructions are stored in

Successive word locations starting at location i.

Each instruction is 4 bytes long, i+4, i+8

Instruction Execution:-

Executing a given instruction is a two phase procedure

- 1) Instruction Fetch
- 2) Instruction Execute

CPSR \Rightarrow Current Program Status Register.

31 30 29 28

CPSR

N - Negative

Z - Zero

C - Carry

V - Overflow



: Set of to 1 if the result is -ve; else cleared to 0

: Set to 1 if the result is 0

:

:

Condition Code flags

Addressing modes :-

Different ways in which the location of an operand is specified is called addressing modes.

different addressing modes :-

- 1) Register mode
 - 2) Absolute mode
 - 3) Immediate mode
 - 4) Indirect mode
 - 5) Index mode
 - 6) Base with Index mode
 - 7) Base with Index and Offset mode
 - 8) Relative Mode
 - 9) Auto increment mode
 - 10) Autodecrement mode
- Two Addressing modes to access variables
To deal with lists and arrays

1) Two Register mode:-

The operand is the content of a processor register, the name (address) of a register is given in the instruction.

eg: Move LOC, R1 → was processor register as operand.
Move R1, R2 → was processor register as operand.

2) Absolute mode (Direct mode) :-

The operand is in memory location, the address of memory location is given explicitly in the instruction.

eg: Move A, B...
→ Address of memory location.
Add R1, A

3) Immediate mode :- Addressing mode to represent constants.

Add # 100, R2 : Add instruction, adds 100 to R2's content.
→ constant value

If a value does not have # sign in front of it then it is the address of a memory location.

The # sign in front of the value indicates the immediate value to be operated.

Store R2, 100H

→ it does not have # sign hence store will consider it as address and store the contents of R2 into the memory location 100H.

8)

$$A = B + 6$$

Move B, RI :- Register addressing mode

Add #6, RI :- Immediate

Move RI, A :- Absolute or direct

9)

Indirect mode

Add (RI), RO

Main memory :-

Add (RI), RO

Register Address : A

= A. 12 44

-

Register Address : B

Operand

Register RI

B

Add (A), RO

A

B

Add (A), RO

B

Operand

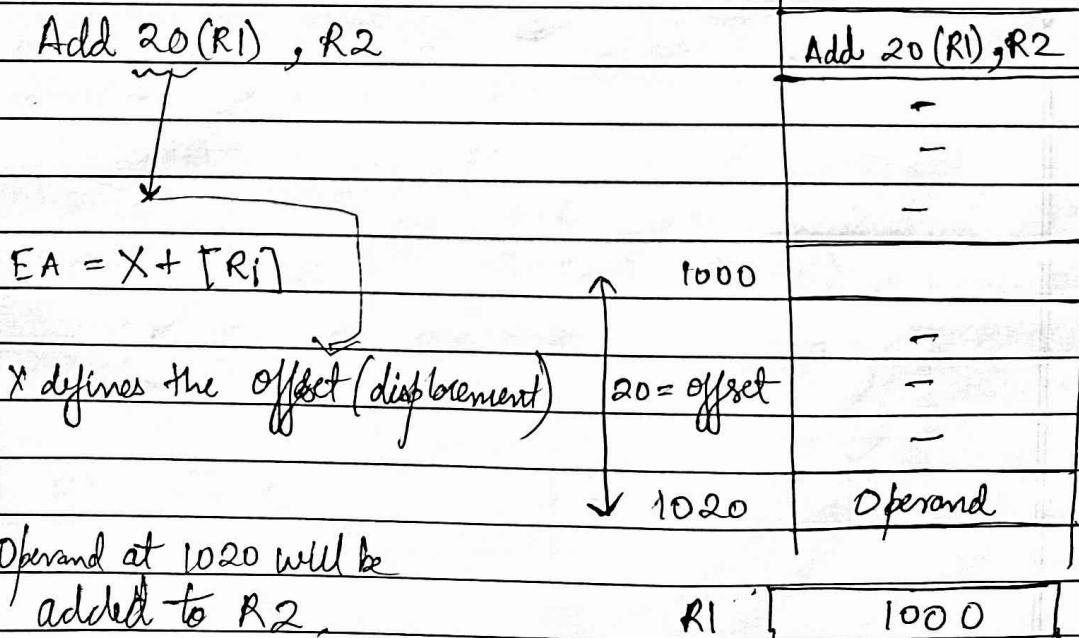
Q) Adding a List of Numbers:-

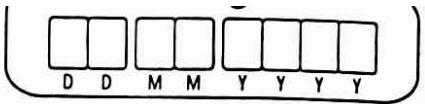
Address	Contents	
	Move N, R1	
	Move #NUM1, R2	} initialization
	Clear R0	
→ LOOP	Add (R2), R0	
	Add #4, R2	
	Decrement R1	
	Branch > 0 LOOP	
	Move R0, SUM	

⇒ Addressing modes to deal with lists and arrays

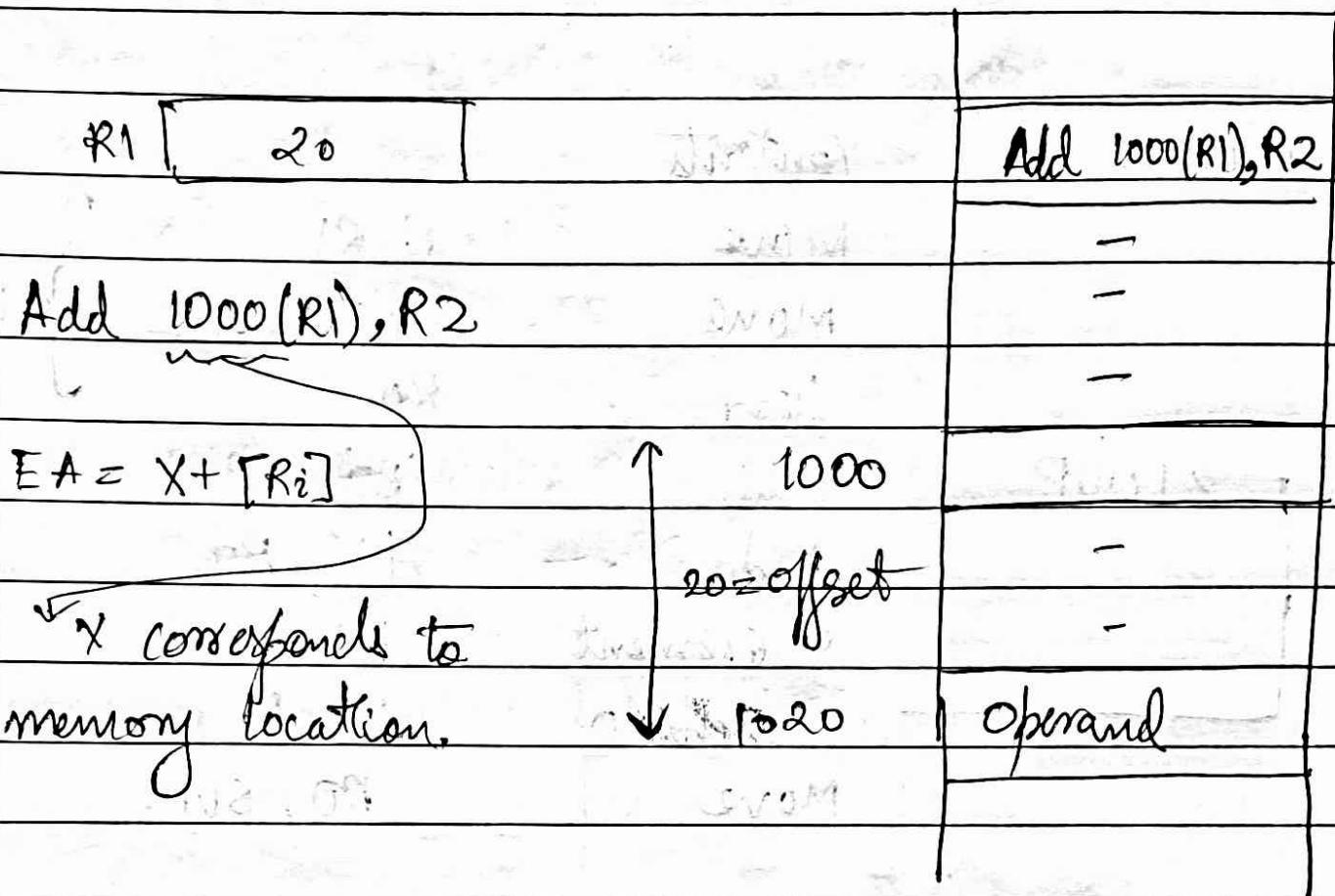
- 5) Index mode
- 6) Base with Index mode
- 7) Base with Index and offset mode.

5) Index Mode.





Offset is in index register



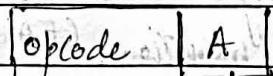
Instruction word that will look at address register A

B

start point (2)

8) Relative mode.

Common use to specify target address in Branch instruction.



PC

Operands

A. 1000

1000

1001

1002

1003

1004

1005

1006

1007

1008

1009

1010

1011

1012

1013

1014

1015

1016

1017

1018

1019

101A

101B

101C

101D

101E

101F

101G

101H

101I

101J

101K

101L

101M

101N

101O

101P

101Q

101R

101S

101T

101U

101V

101W

101X

101Y

101Z

101A

101B

101C

101D

101E

101F

101G

101H

101I

101J

101K

101L

101M

101N

101O

101P

101Q

101R

101S

101T

101U

101V

101W

101X

101Y

101Z

101A

101B

101C

101D

101E

101F

101G

101H

101I

101J

101K

101L

101M

101N

101O

101P

101Q

101R

101S

101T

101U

101V

101W

101X

101Y

101Z

101A

101B

101C

101D

101E

101F

101G

101H

101I

101J

101K

101L

101M

101N

101O

101P

101Q

101R

101S

101T

101U

101V

101W

101X

101Y

101Z

101A

101B

101C

101D

101E

101F

101G

101H

101I

101J

101K

101L

101M

101N

101O

101P

101Q

101R

101S

101T

101U

101V

101W

101X

101Y

101Z

101A

101B

101C

101D

101E

101F

101G

101H

101I

101J

101K

101L

101M

101N

101O

101P

101Q

101R

101S

101T

101U

101V

101W

101X

101Y

101Z

101A

101B

101C

101D

101E

101F

101G

101H

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101K

101L

101M

101N

101O

101P

101Q

101R

101S

101T

101U

101V

101W

101X

101Y

101Z

101A

101B

101C

101D

101E

101F

101G

101H

101I

101J

101K

101L

101M

101N

101O

101P

101Q

101R

101S

101T

101U

101V

101W

101X

101Y

101Z

101A

101B

101C

101D

101E

101F

101G

101H

101I

101J

101K

101L

101M

101N

101O

101P

101Q

101R

101S

101T

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101L

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101N

101O

101P

101Q

101R

101S

101T

101U

101V

101W

101X

101Y

101Z

101A

101B

101C

101D

Subroutines

Memory location Calling Program

Memory location Subroutine SUB

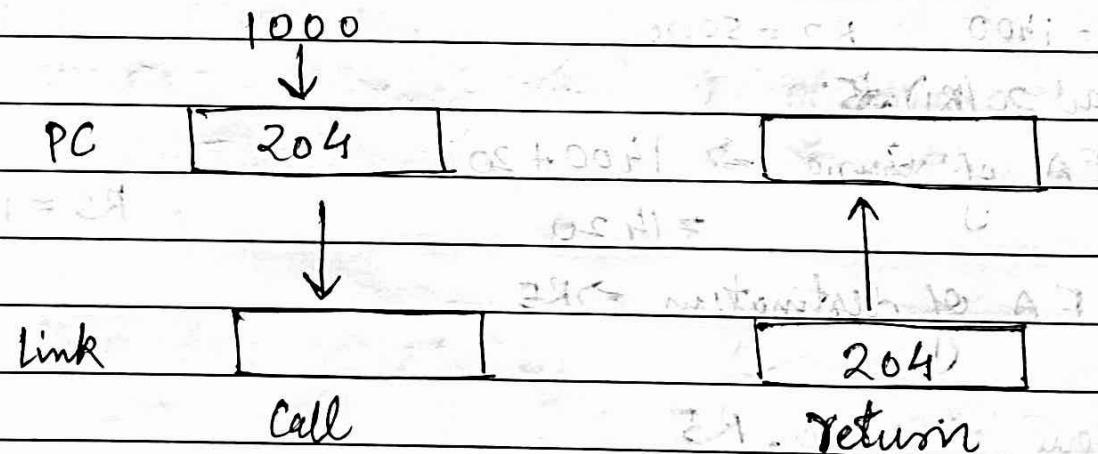
200 call SUB → 1000 first instruction

204 next instruction

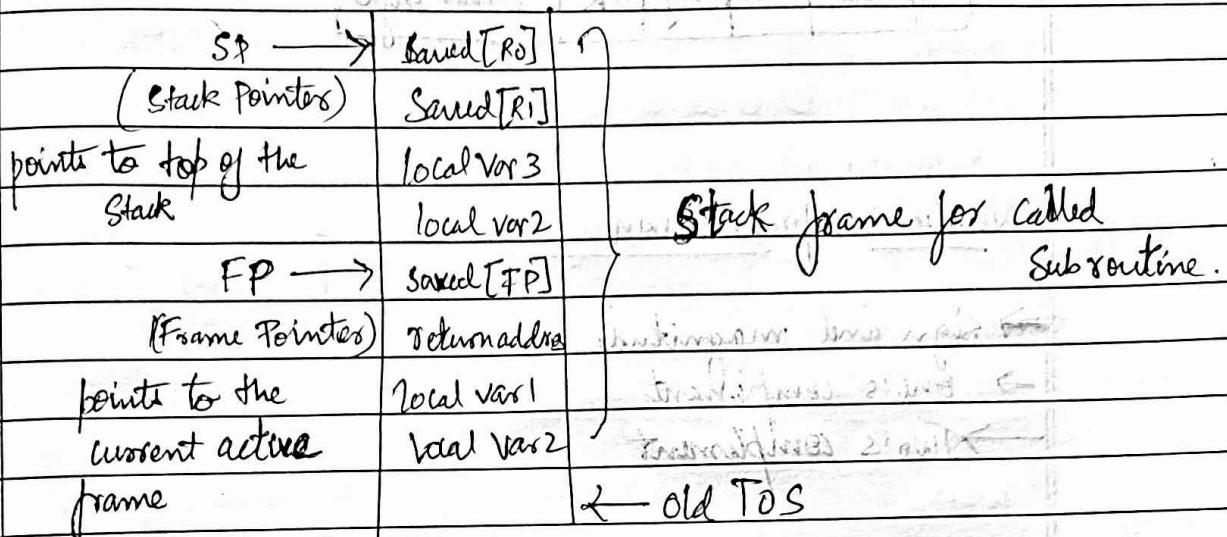
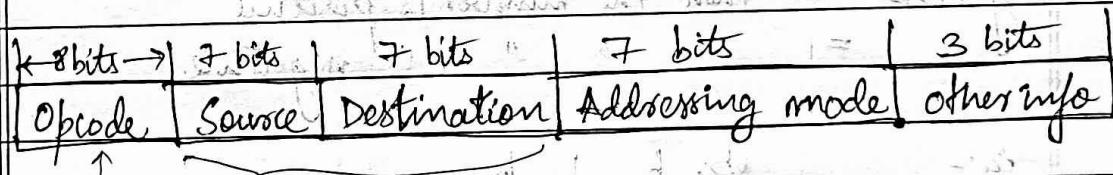
return

The way in which a computer makes it possible to call and return from subroutines is referred to as its Subroutine Linkage method.

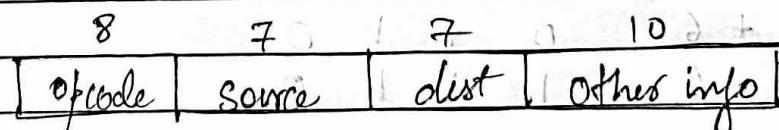
Link register: saves the return address in a specific location



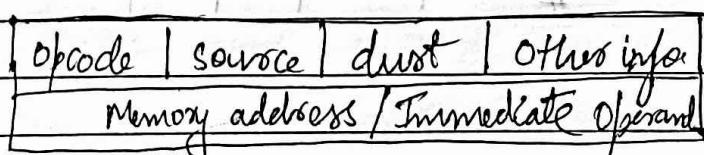
Nested Subroutine uses Stack

Stack Frame :-Encoding of Machine instructionencoding instruction into 32 bit word :-

a) One word instruction: → RISC



b) Two word instruction: → RISC



0	0	M	M	Y	Y	Y
---	---	---	---	---	---	---

3) Three operand instruction :-

opcode	Ri	Rj	Rk	. Other info
--------	----	----	----	--------------

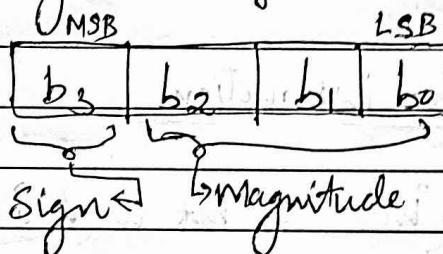
Number representation

→ Sign and magnitude

→ one's complement

→ Two's complement

Sign and magnitude system



If MSB = 0 then the number is positive

If MSB = 1 then the number is negative

e.g:-

$$\begin{array}{r}
 & b_3 & b_2 & b_1 & b_0 \\
 +0 & 0 & 0 & 0 & 0 \\
 -0 & 1 & 0 & 0 & 0
 \end{array}
 \quad \text{2 representation of zero}$$

$$\begin{array}{r}
 +6 & 0 & 1 & 1 & 0 \\
 -6 & 1 & 0 & 1 & 0
 \end{array}$$

$$\begin{array}{r}
 +7 & 0 & 1 & 1 & 1 \\
 -7 & 1 & 0 & 1 & 1
 \end{array}$$

I's Complement

	b_3	b_2	b_1	b_0				
+7	0	1	1	1				
-7	1	0	0	0	0	0	0	0

Here also we have

+ 0 0 . 0 0 0 } 2 different representation for zero
- 0 1 1 1 }

$$\begin{array}{r|rrrr} + & 0 & 0 & 0 & | \text{ Separation boundary } 2 \\ -1 & 1 & 1 & 1 & 0 \end{array}$$

2's Complement System:

$$2^{\text{'s}} \text{ Complement} = 1^{\text{'s}} \text{ complement} + 1$$

$$+2 \rightarrow 0010$$

1's 110

3300' west wall of trail

$$2^{\text{st}} \text{ side} \xrightarrow{\text{cancel } 110} -2$$

11. *Chloris* *viridissima*

$$+5 \longrightarrow 0101$$

$$\begin{array}{r} 0.015 \text{ (0.15)} \\ \times 0.01 \text{ (0.1)} \\ \hline 0.0015 \text{ (0.0015)} \end{array}$$

$$+7 \rightarrow 0111$$

monitors that take advantage of the fact that most of the data

King is about 1000 feet above sea level: ($K \rightarrow$)

IX 1st-iii b/w

21S 1001 → -7

$$10 \rightarrow 0000$$

1's 1111

1

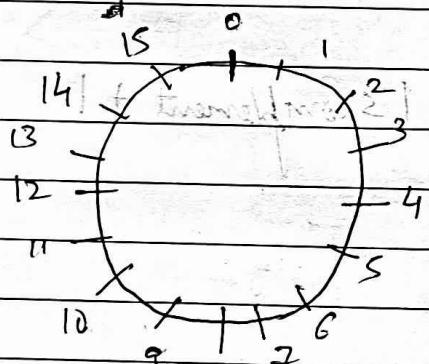
2's 0000

$\rightarrow 00111100$

So, here zero has only one unique representation.

2's complement arithmetic

N=16



$$(a+b) \bmod N$$

$$(7+4) \bmod 16$$

$$= 11010000 - 80$$

locate 7 then move 4 units in
the clockwise direction to arrive at
the answer 11.

Q)

C	1	*
+2	0010	
+3	0011	
<u>+5</u>	0101	

1	0	1	1	0	0	1	*
+4	0	1	0	0	0	0	\rightarrow written in 2's
-6	1	0	1	0	0	0	\rightarrow " complement
-2	0	1	1	0	0	0	"

Subtraction Rule:- Two numbers X and Y (that is to perform $X-Y$) ; take the 2's complement of Y and add it to X.

eg:

$$\begin{array}{r} X - 3 \\ Y - 7 \end{array}$$

2's complement of $-7 \Rightarrow 1001$

$$0110$$

$$-3 \Rightarrow 1101$$

$$2's \text{ complement of } -7 \Rightarrow 0111$$

$$X - Y \Rightarrow \underline{0100} \rightarrow +4$$

$$\begin{array}{r} X + 6 \\ Y + 3 \end{array}$$

$$2's \text{ complement of } +3 \Rightarrow \begin{array}{r} 0011 \\ 1100 \\ \hline 1101 \end{array}$$

$$(X - Y) \Rightarrow \begin{array}{r} 0110 \\ 1101 \\ \hline 0011 \end{array}$$

$$0011 \rightarrow +3$$

Overflow in integer arithmetic

$$(+7) + (+4)$$

$$\begin{array}{r} X + 7 \\ Y + 4 \\ \hline S \end{array}$$

$\underline{1011} \rightarrow -5$ overflow has occurred.

If sum of two -ve numbers yield a +ve number then

Sum has overflowed.

$$(-5) + (-6)$$

$$\begin{array}{r}
 X \quad -5 \quad 1 \ 0 \ 1 \ 1 \\
 Y \quad -6 \quad 1 \ 0 \ 1 0 \\
 \hline
 S \quad 0 \ 1 \ 1 \ 0 1
 \end{array}$$

$\rightarrow +5$ overflow has occurred.

$$(+7) + (-4)$$

$$\begin{array}{r}
 X \quad +7 \quad 0 \ 1 \ 1 \ 1 \\
 Y \quad -4 \quad 1 \ 1 \ 0 0 \\
 \hline
 S \quad 0 \ 0 \ 1 0 1
 \end{array}$$

$\rightarrow +3$ + No overflow.

UNIT-2

Arithmetic Unit

Multiplication of unsigned numbers :-

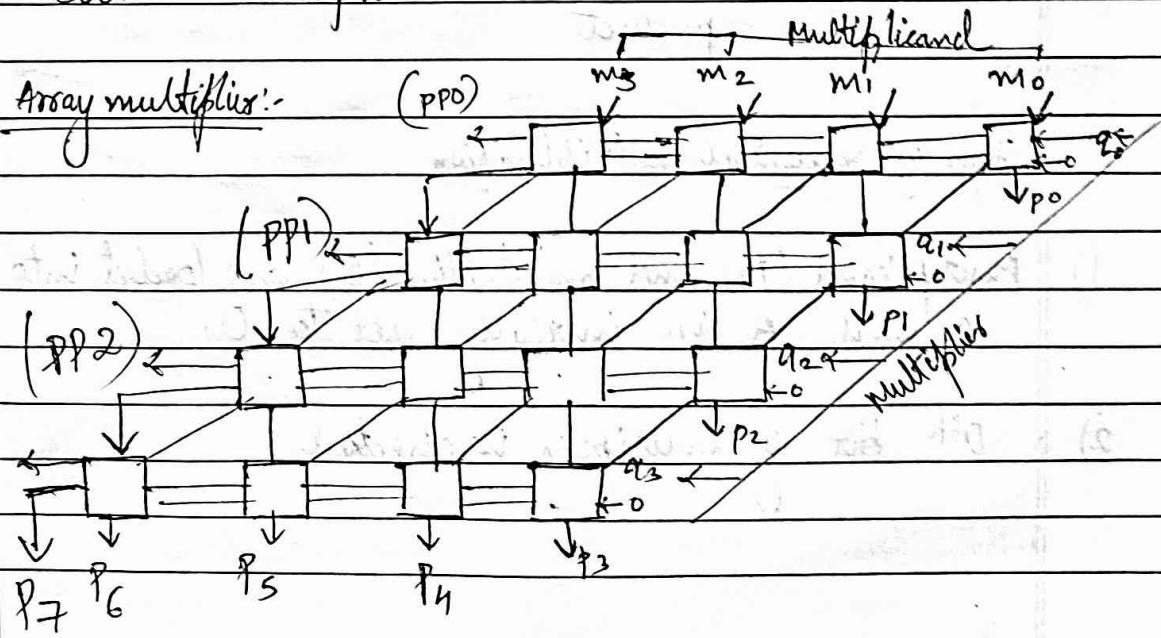
- e) Find the product multiplicand 13 and multiplier 11
(M) (Q)

$$\underline{M \times Q = ?}$$

M	1	1	0	1	0	(13) Multipliand	2	6
Q	1	0	1	1		(11) Multipliisor.	2	3
	1	1	0	1			1	1
	1	0	0	1			1	1
	0	0	0	0			0	0
	1	0	1	1			1	0
	0	0	0	1			0	0

Implementation of Multiplication

- 1) Array multiplier
 - 2) Sequential multiplier
 - 3) Booth multiplier.



9)

Find the product of multiplicand 13 and multiplier 11 using sequential multiplication.

1101 M (13)
1011 Q (11)

1011 Q (11)

11-201 00:0.1 1151 P (143)

(a) $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ (b) $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

M

$$\left. \begin{array}{c} \text{Initial} \\ \text{configuration} \end{array} \right\} \begin{array}{c} \boxed{1 \ 1 \ 0 \ 1} \\ \quad \quad \quad Q \\ \boxed{0 \ 0 \ 0 \ 0} \quad \quad \quad \boxed{1 \ 0 \ 1 \ 0 \ 1} \end{array}$$

Initial state: 0011
After 1 clock cycle: 1100
Carry bit: shift

0	0 0 1 1	1 1 0 1	Add } 2nd cycle
0	1 0 0 1 (shift)	1 1 1 0	Shift } 01

0 1 0 0 | 1 1 1 0 Shift No add 3rd cycle
0 1 0 0 | 1 1 1 1 Shift

1	0 0 0 1	1 1 1 1 0	Add } 4th cycle. shift } <hr style="border-top: 1px solid black; margin-bottom: 5px;"/>
0	1 0 0 0	1 1 1 1 1	

Steps of sequential multiplication

- 1) multiplicand (m) and multiplier (α) are loaded into registers C and A are initially set to 0.
 - 2) 0th Bit of multiplier is checked

3) If bit 0 = 1

Multiplicand and partial product is added and all the bits of Q, A_r and C registers are shifted to right one bit.

4)

If bit 0

No addition is performed, only shift operation is carried out.

Step 2,3,4 are repeated till the answer is reached.

Signed Multiplication

case i) Multiplicand is (-ve) and Multiplier is (+ve).

e.g.

$$1 \ 0 \ 0 \ 1 - 1 \ 0 (-13)$$

~~0 1 0 1 1 (+11)~~

1 1 1 1 1 1 0 0 1 1

Sign

الآن نحن في مرحلة الانتهاء من المهمة

0 0 0 0 0 0 0

written in

1 3 1 1 1 0 0 1 1

black

~~00 00 00 00 - (21+) - 10/11 and 15/15~~

~~T 10181 - 00010 (+143)~~

Booth Algorithm

Booth multiplier recoding table :-

Multiples		Version of multiplicand selected by bit $i-1$
Bit i	Bit $i-1$	$0 \times M$
0	0	$+1 \times M$
1	0	$-1 \times M$
1	1	$0 \times M$

e.g:- 1) Multiplier is scanned from right to left

$$\begin{array}{r} 1 \ 1 \ 0 \ 1 \ 0 \ 0 \\ \hline 0 \ -1 \ +1 \ -1 \ 0 \end{array}$$

(+ve) is multiplicand (0ve) is multiplicand (0)

2) recode $\begin{array}{r} 1 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \\ \hline -1 \ 0 \ 1 \ -1 \ 0 \ 0 \ 1 \end{array}$

$$\begin{array}{r} 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0 \\ \hline 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0 \end{array}$$

Q)

Booth multiplication with negative numbers.

$$(+13) \times (-6)$$

2's complement for (-6)

$$1101 (+13) \rightarrow 11010$$

$$01101 (-6) \rightarrow 10101$$

$$0110$$

Add sign bit \Rightarrow

$$01001$$

$$\begin{array}{r} 01101 (+13) \\ 11010 (-6) \\ \hline 10100 \end{array}$$

$$\underline{10100}$$

recode the multiplier.

$$11010 [0$$

$$\cdot 0-1+1-10$$

$$\begin{aligned}
 1+1 &= 10 \\
 1+1+1 &= 11 \\
 1+1+1+1 &= 100 \\
 1+1+1+1+1 &= 101 \\
 1+1+1+1+1+1 &= 110
 \end{aligned}$$

whenever we are

PageWork



multiplying

with
a negative
number

2's complement of
multiplicand.

$$\Rightarrow 0\ 11\ 01$$

$$, , , 0-11-10$$

$$00\ 00\ 00\ 00\ 00$$

$$11\ 11\ 10\ 10\ 11$$

$$00\ 00\ 11\ 01$$

$$11\ 10\ 01\ 10\ 01$$

$$00\ 00\ 00\ 00\ 10$$

$$1) 1110110010 (-78)$$

ignore

$$\Theta) M = -13$$

$$Q = -20$$

2's complement

$$01101 (-13) \rightarrow 10011 (-13)$$

$$10100 (-20) \rightarrow 100100 (-20)$$

$$\begin{array}{r}
 01101 \quad 10100 \\
 +10010 \quad 01011 \\
 \hline
 10011 \quad 01100
 \end{array}$$

$$\begin{array}{r}
 01101 \quad 10100 \\
 +10010 \quad 01011 \\
 \hline
 10011 \quad 01100
 \end{array}$$

$$\text{Add sign bit to both } 01110011 (-13)$$

$$(2^7 000) \quad (2^7 000) \quad 0111001100 (-13)$$

$$(2^7 000) \quad (2^7 000) \quad 0111001100 (-13)$$

$$\text{scale the multiplier (Q): } \frac{10110010}{+10101001}$$

$$\begin{array}{r}
 111110010011 \\
 +111101100100 \\
 \hline
 000000000000
 \end{array}$$

$$\begin{array}{r}
 000000000000 \\
 +000001101001 \\
 \hline
 000000000001
 \end{array}$$

$$\begin{array}{r}
 000000000001 \\
 +111100110000 \\
 \hline
 000000000000
 \end{array}$$

$$\begin{array}{r}
 000000000000 \\
 +000110110000 \\
 \hline
 000100000000
 \end{array}$$

2's complement of
multiplicand

of m. 110011

001100

001101

(260)

$$\theta) M = -13$$

$$Q = +9$$

$$1101 \text{ (}-13\text{)} \rightarrow 0011 \text{ (}-13\text{)}$$

$$1001 \text{ (}9\text{)} \rightarrow 110010 \text{ (}9\text{)}$$

2's complement

$$1101$$

Adding sign bit

$$0010$$

$$10011 \text{ (}-13\text{)}$$

$$010010 \text{ (}9\text{)}$$

$$\begin{array}{r} 1 \\ - \\ 0011 \\ \hline 0111 \end{array}$$

Decoding the multiplier

$$\begin{array}{r} 010010 \\ +1-10+1-1 \\ \hline \end{array}$$

$$10011$$

$$1-10+1-1$$

$$0000001101$$

$$111110011$$

$$00000000$$

$$0001101$$

$$110011$$

$$11100010+11 \text{ (}-117\text{)}$$

$$11111001101$$

2's complement of

multiplicand

$$10011$$

$$000101100$$

$$110100101101$$

$$001100101101$$

$$110100101101$$

2's complement

$$\theta)$$

$$M = +14 \quad 11011110 \text{ (}14\text{)}$$

$$Q = -8 \quad 00001100 \text{ (-}8\text{)}$$

$$1110 \text{ (}14\text{)}$$

$$1000 \text{ (-}8\text{)}$$

$$1000 \text{ (}0\text{) Adding sign bit}$$

$$0111 \text{ (}0\text{) Adding sign bit}$$

$$\begin{array}{r} 1 \\ - \\ 1000 \\ \hline 1000 \end{array}$$

Decoding the multiplier

$$110010$$

$$00100$$

$$\begin{array}{r} 1 \\ - \\ 00100 \\ \hline 1000 \end{array}$$

2's complement of multiplicand

$$01110$$

$$10001$$

$$\begin{array}{r} 1 \\ - \\ 10001 \\ \hline 10010 \end{array}$$

$$11000011010$$

$$0011000-11000$$

$$000000000000$$

$$000000000000$$

$$000000000000$$

$$111001000000$$

$$000000000000$$

$$111001000000$$

$$111001000000$$

$$(-112)$$

Binary format of dividend & divisor (MSB first)

+2 -1 -1

$$\begin{array}{r}
 0000000.0^{\circ} 1^{\circ} 0^{\circ} 1^{\circ} 1 \\
 0000001^{\circ} 0^{\circ} 1^{\circ} 1^{\circ} 1^{\circ} 1 \\
 \hline
 0110101^{\circ} 0^{\circ} 1^{\circ} 0^{\circ} 1^{\circ} 1 \\
 \hline
 1101101^{\circ} 0^{\circ} 1^{\circ} 1^{\circ} 1^{\circ} 1 (-297)
 \end{array}$$

Binary format

2's complement

Q) $M = +23$

~~$10111^{\circ} 0^{\circ} 1^{\circ} 1^{\circ} 1^{\circ} 1 (23)$~~

$Q = -20$

~~$10100^{\circ} 0^{\circ} 1^{\circ} 0^{\circ} 0^{\circ} 0 (-20)$~~

~~10100~~

~~10111~~

~~01111~~

~~01000~~

~~01000~~

Add the sign bit

~~01011~~

~~01100~~

~~10100 101100~~

Decoding the multiplier

~~10100 10100~~

~~101100~~

~~+ -2 +1~~

~~⇒ 01011~~

~~-1 -2 +1~~

~~2's complement of
multiplier~~

~~00000010111~~

~~11110000100~~

~~11101001~~

~~01011~~

~~101000~~

~~101001~~

~~x 20~~

~~0000000~~

~~101001~~

~~1010010~~

~~00000000000~~

~~1111101001~~

~~11101001~~

~~111000110100. (-460)~~

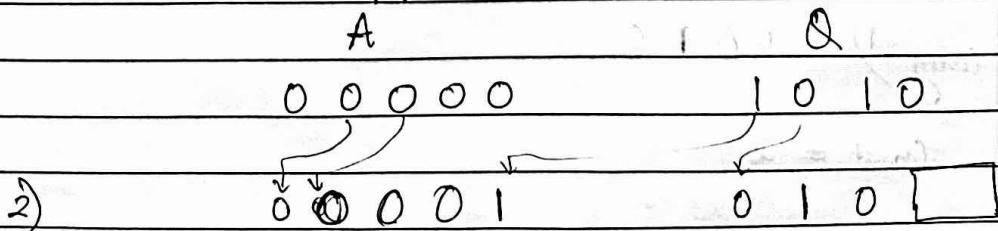


Integer Division

Algorithm for restoring division:-

- i) $A \leftarrow 0$, $M \leftarrow \text{Divisor}$, $Q \leftarrow \text{Dividend}$, $\text{Count} \leftarrow 0$
- 2) Shift $A \& Q$ left one binary position.
- 3) $A \leftarrow A - M$
- 4) if A is 1 \rightarrow sign bit value of A
 - Yes \rightarrow Assign $Q_0 \leftarrow 0$
 $A \leftarrow A + M$ (Restore)
 - No \rightarrow $Q_0 \leftarrow 1$
- 5) $\text{Count} \leftarrow \text{Count} - 1$
- 6) Repeat 2, 3, 4, 5 n times.
- 7) Dividend : $Q \rightarrow 1010$ \rightarrow 4 bit long $\Rightarrow n = 4$
Divisor : $M \rightarrow 0011$

D) \Rightarrow ^{Registers} A should be assigned with $(n+1)$ zeroes
Count = 4.



3) $A \leftarrow A - M$ $M \rightarrow 00011$

11100

00001

11101

11101

A $\leftarrow 11110$ 4) $Q_0 \leftarrow 0$

010 [0]

 $A \leftarrow A + M$

11110

00011

1) 00001

(carry ignore)

5) Count = 4 - 1 =

Count = 3

~~2nd cycle~~

Repeat Step 2, 3, 4, 5

A

Q

00001

00010

0100

01100

11101

1100 [] Adjunct

01111

1100 [] Adjunct

00011

100 [0]

00010

Count = 2

3rd cycle

$$\begin{array}{r} A \quad 0011 \\ \times 1000 \\ \hline 00010100 \end{array}$$

shift:

$$\begin{array}{r} 00101 \\ 000 \square \end{array}$$

$$\begin{array}{r} 011101 \end{array}$$

$$\begin{array}{r} \text{ignore } 1) 00010 \\ \text{From } 10000 \end{array}$$

$$\begin{array}{r} 10111 \end{array}$$

$$\begin{array}{r} 000111 \end{array}$$

$$\begin{array}{r} 11000 \end{array}$$

$$\begin{array}{r} 10000 \square \end{array}$$

Count = 1

$$\begin{array}{r} 11000 \end{array}$$

$$\begin{array}{r} 11000 \square \end{array}$$

4th cycle

$$\begin{array}{r} A \end{array}$$

$$\begin{array}{r} 00000 \end{array}$$

shift

$$\begin{array}{r} 1100010 \\ 1010100 \end{array}$$

$$\begin{array}{r} 0001 \end{array}$$

$$\begin{array}{r} 1110 \end{array}$$

$$\begin{array}{r} 10111 \end{array}$$

$$\begin{array}{r} 1) 0001 \\ 10100 \end{array}$$

$$\begin{array}{r} 001100 \end{array}$$

$$\begin{array}{r} 001000 \end{array}$$

$$\begin{array}{r} 000100 \end{array}$$

$$A = 00001$$

$$Q = 0011$$

(Remainder)

$$\begin{array}{r} 000100 \end{array}$$

(Quotient)

$$\begin{array}{r} 1011 \end{array}$$

Count = 0

$$\begin{array}{r} 00000 \end{array}$$

$$\begin{array}{r} 1011 \end{array}$$

$$\begin{array}{r} 00000 \end{array}$$

$$\begin{array}{r} 000000 \end{array}$$

(i) Q = 0 - overflow
 (ii) R = 1 - remainder



Q)

Dividend Q \rightarrow 1100 $n=4$ Divisor M \rightarrow 0011

10100

A

1st cycle

00000

00001

11101

~~①~~ 11110

00011

100001

2nd cycle

00011

11101

~~②~~ 00000

000000

11101

~~③~~ 11101

00011

100000

3rd cycle

000000

11101

~~④~~ 11101

00011

100000

A

00000

Q

0100

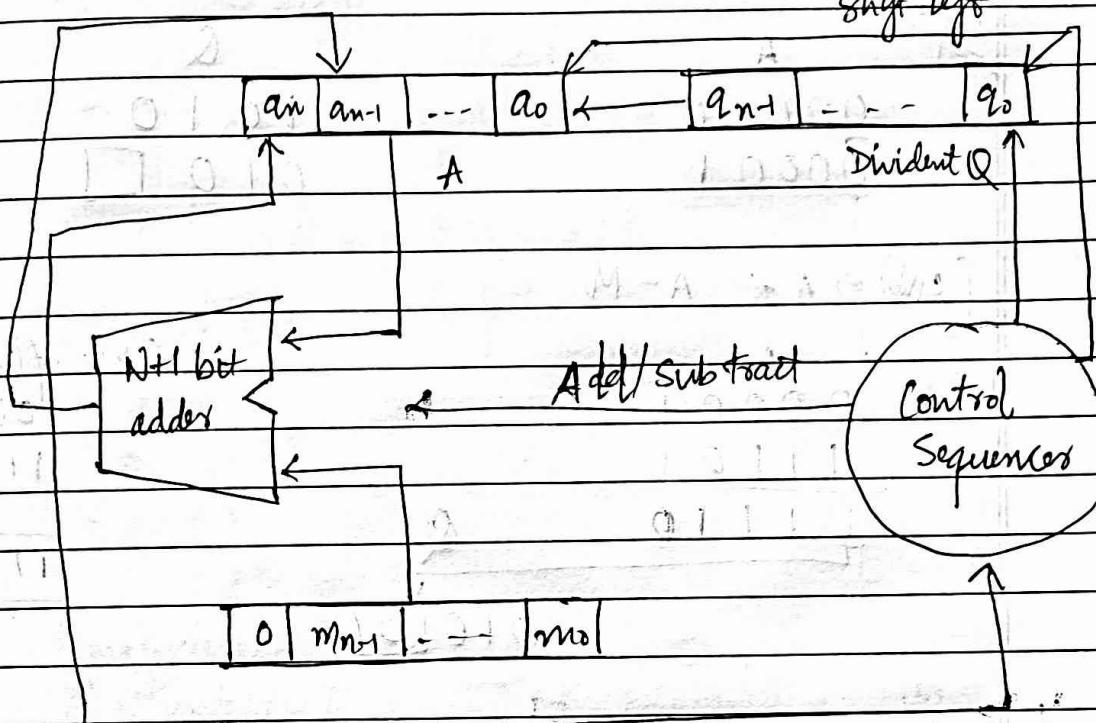
 \Rightarrow Remainder = 0. (0)

Quotient = 0100 (4)

Circuit Arrangement

Restoring division

Shift left

Non-Restoring Division - Steps

- 1) $A \leftarrow 0, M \leftarrow \text{Divisor}, Q \leftarrow \text{Divident}, \text{count} \leftarrow n$
- 2) Shift left $A \& Q$
- 3) If sign of A is 0
 - Yes, $A \leftarrow A - M$ (sub) $\rightarrow \begin{cases} A \text{ is } 1, Q_0 = 0 \\ A \text{ is } 0, Q_0 = 1 \end{cases}$
 - No, $A \leftarrow A + M$ (Add) $\rightarrow \begin{cases} A \text{ is } 1, Q_0 = 0 \\ A \text{ is } 0, Q_0 = 1 \end{cases}$
- 4) Repeat 2 & 3 for n times
- 5) If the sign of A is 1, add divisor to A

Q) Divisor M : 0011

Dividend Q : 1010

cycle 1

A

Q

00000 → 0011 | 1010

00001

A

010□

(Sub) $\rightarrow A \leftarrow A - M$

00001

2's complement M

11101

00011

01110

Q

11100

11101

01010 (or 10)

2nd cycle

A

Q

11110 → 0100

01100

100□

(Add) $\rightarrow A \leftarrow A + M$

A

11100

00011

01111

A is A + M (or 10)

01000

3rd cycle

A

Q

01111 → 000□

(Add) $\rightarrow A \leftarrow A + M$

11111

00011

00010

000□

000□

4th cycle

$$\begin{array}{r}
 A \\
 00010 \\
 \textcircled{D} 0100 \\
 \end{array}
 \quad
 \begin{array}{r}
 Q \\
 0001 \\
 11001 \square
 \end{array}$$

(Sub)

$$A \leftarrow A - M$$

$$00100111$$

$$111010110$$

$$\begin{array}{r}
 \textcircled{D} 0001 \\
 \end{array}
 \quad
 \begin{array}{r}
 Q \\
 00111
 \end{array}$$

A

$$00001$$

Q

$$0011$$

Remainder $\Rightarrow 1$ Quotient $\Rightarrow 011$

e)

$$Q : 1011$$

$$M - A \rightarrow A$$

$$M : 0101$$

cycle 1

$$101100$$

A

$$11011$$

Q

$$1011$$

$$\textcircled{D} 0001$$

$$011\square$$

(Sub)

$$A \leftarrow A - M$$

division: first step of remainder division

25 M

$$00001$$

$$11011$$

$$\textcircled{D} 1100$$

$$011\square$$

$$00101$$

$$11010$$

I

$$11011$$

2nd Cycle

A

Q

$$01000$$

$$110\square$$

(Add)

$$11000$$

$$00101$$

→

$$110\square 0$$

$$\textcircled{D} 1101$$

3rd cycle

A

$$\textcircled{1} \ 011 \quad 100 \boxed{1}$$

(Add)

$$A \leftarrow A + M - A \quad (\text{Add})$$

$$1101 \boxed{0} 0000$$

$$00101 \ 01111 \ 100 \boxed{1}$$

$$\textcircled{1} \ \textcircled{0} 000 \ 000000$$

11010

4th cycle

A

$$\textcircled{0} 000 \ 1$$

1 = Quotient

$$11000 \boxed{1} 000$$

(Sub)

$$A \leftarrow A - M$$

$$1101 : 0 \quad 10$$

$$1010 - M$$

$$0000 \ 1$$

$$1101 \ 1$$

$$\underline{\textcircled{1} \ 1100 \ 0000 \ 0010}$$

11010

$$A : \textcircled{1} 1100$$

\rightarrow Add divisor to A to get remainder.

$$11100$$

$$00101$$

$$\underline{00001}$$

Quotient : 0.010

IEEE Standard for Floating point numbers

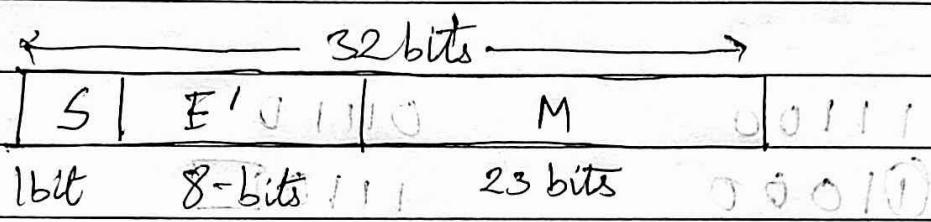
Floating Point Numbers has 3 parts | 0100 = M

M: Mantissa $\pm M \times B^E$

B: Base

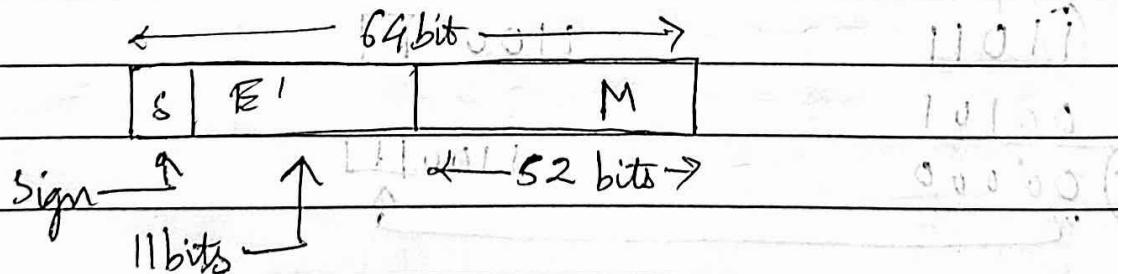
E : Exponent

Single Precision Format



$$\text{Normalize} : - \pm 1. M \times 2^{E+127}$$

Double Precision Format



Normalize :-

$$N = 1. M' \times 10^{23}$$

Q) $(12.59, 125)_{10}$ in ~~fix~~ single and double precision IEEE floating point formats.

Step 1) Convert decimal to binary form.

$$(1259)_{10} = 10011101011$$

$(0.125)_{10}$	\Rightarrow	$0.125 \times 2 = 0.25$	0	001
		$0.25 \times 2 = 0.50$	0	
		$0.5 \times 2 = 1.0$	1	

$$(1259.125)_{10} = (10011101011.001)_2$$

Step 2) Normalize the number.

$$10011101011.001_2 = (11)$$

$$\rightarrow 1.0011101011001 \times 2^{10}$$

Step 3) Represent in Single Precision form

$$\pm 1.M \times 2^{E-127} \rightarrow 1.0011101011001 \times 2^{10}$$

$$\Rightarrow 2^{E-127} = 2^{10}$$

$$E' - 127 = 10$$

$$E' = 137 \rightarrow \text{Binary form } 10001001$$

0	10001001	00111010110010000000000
S-1 bit	E' - 8 bits	M - 23 bits

Step 4)

Represent in Double precision form

$$1.0011101011001 \times 2^{10}$$

$$\pm 1.M \times 2^{E'-1023}$$

$$\Rightarrow 1.02^{E'-1023} = 2^{10} \quad (\text{E281})$$

$$E' - 1023 = 10$$

 $E' = 1033 \rightarrow$ Binary form

10000001001

0	1	10000001001	0011101011001000...00
S-1 bit	$E' - 11$ bits		M-52 bits

$$100.1101011001 = (251.825)$$

Q2) Compute the single precision IEEE 754 binary representation of the number $(-118.625)_{10}$

Ans:-

Step 1:-

$$(118)_{10} = 1110110.1101011001$$

$$(0.625)_{10} = 0.625 \times 2 = 1.25$$

$$0.25 \times 2 = 0.50$$

$$0.50 \times 2 = 1.00$$

$$(-118.625)_{10} = (1110110.101)_2$$

Step 2:

Normalize the number

$$1110110.101$$

$$\Rightarrow 1.110110101 \times 2^6$$

3:

$$\pm 1.M \times 2^{E'-127} \Rightarrow 1.110110101 \times 2^6$$

$$2^{E'-127} \rightarrow 2^6$$

$$E' = 127 + 6 = 133 \rightarrow 10000101$$

1	10000101	110110101.0000000000000000
S-1 bit	E-8 bits	23 bits

83)

$$(85.125)_{10}$$

Step 1)

$$(85)_{10} = 1010101$$

$(0.125)_{10}$	$0.125 \times 2 = 0.25$	0	1001
0	$0.25 \times 2 = 0.50$	0	0101
1	$0.50 \times 2 = 1$	1	

$$(85.125)_{10} = (1010101.001)_{10}$$

2)

$$1.010101001 \times 2^6$$

3)

$$\pm 1.M \times 2^{E' - 127} = 1.010101001 \times 2^6$$

$$2^{E' - 127}$$

$$= 2^6$$

$$E' = 133 \rightarrow 10000101$$

0	10000101	01010100100000000000000000000000
S-1bit	$E' - 8$ bits	M - 23 bits

4)

$$1.0000 \pm 1.M \times 2^{E' - 1023} = 1.010101001 \times 2^6$$

$$2^{E' - 1023} = 2^6$$

$$E' = 1029 \rightarrow 10000000101$$

0	10000000101	01010100100...000
S-1bit	$E' - 11$ bits	M - 52-bits

6

(263.3)₁₀

ok31.2j

22

(263) 100000111

(0.3) 10	$0.3 \times 2 = 0.6$	0
	$0.6 \times 2 = 1.2$	1
	$0.2 \times 2 = 0.4$	0
	$0.4 \times 2 = 0.8$	0
	$0.8 \times 2 = 1.6$	1
	$0.6 \times 2 = 1.2$	1
	$0.2 \times 2 = 0.4$	0

$$(263, 3)_{10} = 1000000111_2 = 0100\ 1100\ 11_2 \text{, i.e. }$$

Single Precision format :-

$$\pm 1.M \times 2^{E'-127} \rightarrow 1.00000111010011 \times 2^8$$

$$E' - 127 = 8$$

$$E' = 8 + 127 = 135$$

0	1 0000111	$\Rightarrow 10000111_1$
S-1 bit	E-8 bit	M-23 bits

1000000000	1000000000	1000000000
1000000000	1000000000	1000000000