SEMESTER END EXAMINATIONS – AUGUST 2024

Program	:	**B.E. – Common to CSE / ISE / CSE(CY) / CSE (AI & ML)**	**Semester**	:	**IV**
Course Name	:	**Statistics, Probability and Linear Programming**	**Max. Marks**	:	**100**
Course Code	:	**CS / IS / CY / CI41**	**Duration**	:	**3 Hrs**

Instructions to the Candidates:

- Answer one full question from each unit.
- Statistical Table will be provided.

UNIT - I

1. a) Write the formula for regression coefficients of the regression lines of y on x and x on y. CO1 (02)

b) The probability distribution of a random variable X is given by the following table: CO1 (04)

X	0	1	2	3	4	5
$P(X=x)$	k	5k	10k	10k	5k	k

Find 'k' and evaluate the mean.

c) In some determinations of the volume V of Carbon dioxide dissolved in a given volume of water at different temperatures θ the following pairs of values were obtained. Find a relation of the form $V=a+b\theta$ which best fits to these observations. CO1 (07)

θ	0	5	10	15
V	1.80	1.45	1.18	1.00

d) Out of 600 families with 5 children each, how many would you expect to have (i) 3 boys (ii) 5 girls (iii) either 2 or 3 boys. Assume equal probabilities for boys and girls. CO1 (07)

2. a) Write the normal equations using the method of least squares to fit the curve of the form $z=a+bx_1+cx_2$ for the given data. CO1 (02)

b) A random variable X has the density function $f(x)=\dfrac{c}{x^2+1}$, where $-\infty<x<\infty$. CO1 (04)

(i) Find the value of the constant c

(ii) Find the probability that X lies between 1/3 and 1.

c) In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10, use Poisson distribution to calculate the approximate number of packets containing no defective, one defective and two defective blades respectively in a consignment of 10,000 packets. CO1 (07)

d) Obtain the coefficient of correlation and the equations of the lines of regression for the data: CO1 (07)

x	0	1	2	3	4
y	14	13	11	9	8

UNIT - II

3. a) Write the mean and variance of exponential distribution. CO2 (02)

b) A random variable X has a uniform distribution over $(-3, 3)$, find k for which $P(X > k) = \frac{1}{2}$. CO2 (04)

c) If SAT scores (which are measured to nearest integer) are normally distributed with mean 465 and standard deviation 100. What is the probability that a randomly chosen SAT score is (i) over 700? (ii) at least 500 but not more than 600? (iii) 64% of sat scores are over what number?. CO2 (07)

d) Two cards are selected at a random from a box which contains four cards numbered *1, 2, 2* and *3*. Find the joint distribution of *X* and *Y*, where *X* denotes the sum and *Y* the maximum of the two numbers drawn. Also determine $COV(X,Y)$ and $\rho(X,Y)$. CO2 (07)

4. a) For the standard normal distribution of a random variable Z, evaluate $P(Z > 1.5)$. CO2 (02)

b) The mileage which car owner get with a certain kind of radial tire is a random variable having an exponential distribution with mean 40,000km. Find the probabilities that one of these tires will last at least 20,000km. CO2 (04)

c) Let *X* and *Y* be continuous random variables having the joint density function as follows. $f(x,y) = \begin{cases} 4xy & 0 < x < 1, 0 < y < 1 \\ 0 & otherwise \end{cases}$. Verify that CO2 (07)

(i) $E(X+Y) = E(X) + E(Y)$ (ii) $E(XY) = E(X)\ E(Y)$.

d) The daily consumption of milk in a town, in excess of 30,000 liters is distributed as a Gamma distribution with parameters $\alpha = 2$ and $\beta = 10000$. The town has a daily stock of 40,000 liters. Find the probability that the stock is adequate on a particular day. CO2 (07)

UNIT - III

5. a) Find the average waiting time in the queue for (M/M/s): (∞/FIFO) model if $\lambda = 30$ & $\mu = 10$ and s=4. CO3 (02)

b) Show that the stochastic matrix $\begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix}$ is regular. CO3 (04)

c) A person's playing habits are as follows. If he plays one day, he is 70% sure not to play the next day. On the other hand, if he does not play one day, he is 60% sure not to play the next day as well. In the long run, how often does he play? CO3 (07)

d) A supermarket has a single cashier. During peak hours, customers arrive at a rate of 20 per hour. The average number of customers that can be processed by the cashier is 24 per hour. Calculate: CO3 (07)

(i) The probability that the cashier is idle.
(ii) The average number of customers in the queuing system.
(iii) The average time a customer spends in the system.
(iv) The average number of customers in the queue.
(v) The average time a customer spends in the queue waiting for service.

6.	a)	Write the expression to determine the expected number of customers (L_s) in the system for (M/M/1): (k/FIFO).	CO3	(02)
	b)	Find the fixed probability vector of the stochastic matrix: $\begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$	CO3	(04)
	c)	Automatic car wash facility operates with only one bay. Cars arrive according to a Poisson distribution with a mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. If the service time for all cars is constant and equal to 10 minutes, then determine L_s, L_q, W_s & W_q.	CO3	(07)
	d)	There are 2 white marbles in bag A and 3 red marbles in bag B. At each step of the process a marble is selected at random from each bag and the 2 marbles selected are interchanged, then find (i) transition probability matrix (ii) what is the probability that there are 2 red marbles in A after three steps (iii) In a long run, what is the probability that there are 2 red marbles in A?	CO3	(07)

UNIT- IV

7\. a) Define One tailed and two tailed tests. CO4 (02)

b) The mean lifetime of a sample of 25 bulbs is found as 1550 hours with a SD of 120hours. The company manufacturing the bulbs claims that the average life of their bulbs is 1600hours. Is the claim acceptable at 5% Level of Significance ? CO4 (04)

c) Five Coins are tossed 320 times. The number of heads observed is given below. Examine whether the coin is unbiased. CO4 (07)

No. of heads	0	1	2	3	4	5	Total
Frequency	15	45	85	95	60	20	320

d) A fertilizer mixing machine is set to give 12 kg of nitrate for quintal bag of fertilizer. Ten 100 kg bags are examined, the percentage of nitrate per bag are as follows: 11, 14, 13, 12, 13, 12, 13, 14, 11, 12. Are there any reasons to believe that the machine is defective? CO4 (07)

8\. a) Define the null and the alternate hypothesis. CO4 (02)

b) A sample of 900 members is found to have a mean of 3.4cm. Can it be reasonably regarded as a truly random sample from a large population with mean 3.25cm and SD 1.61cm. Use 5% level of significance. CO4 (04)

c) Two independent samples of sizes 7 and 6 have the following values: CO4 (07)

Sample A	28	30	32	33	33	29	34
Sample B	29	30	30	24	27	29	-

Examine Whether the samples have been drawn from normal population having the same variance?

d) A population consists of the four numbers 3, 7, 11, 15. Consider all possible samples of size 2 which can be drawn without replacement from this population. Find (i) the population mean (ii) the population standard deviation (iii) the mean of the sampling distribution of means (iii) the standard error of means. CO4 (07)

UNIT - V

9. a) Write the dual of the LPP to minimize $z = 2x_1 + 5x_2 + 6x_3$, subject to the constraints $5x_1 + 6x_2 - x_3 \le 3, -2x_1 + x_2 + 4x_3 \le 4, x_1 - 5x_2 + 3x_3 \le 1$, $-3x_1 - 3x_2 + 7x_3 \le 6$, $x_1 \ge 0$, $x_2 \ge 0$ and $x_3 \ge 0$. CO5 (04)

b) Solve the following LLP by Graphical method: CO5 (08)

$$\begin{aligned} &Max\, Z = 5x_1 + 7x_2 \\ &s/t \quad x_1 + x_2 \le 4 \\ &\quad 3x_1 + 8x_2 \le 24 \\ &\quad 10x_1 + 7x_2 \le 35 \\ &\quad x_1, x_2 \ge 0 \end{aligned}$$

c) Solve the following LLP by Simplex method: CO5 (08)

$$\begin{aligned} &Max\, Z = 7x_1 + 5x_2 \\ &s/t \quad x_1 + x_2 \le 6 \\ &\quad 4x_1 + 3x_2 \le 12 \\ &\quad x_1, x_2 \ge 0 \end{aligned}$$

10. a) Solve the following LLP by Big M method: CO5 (10)

$$\begin{aligned} &Min\, Z = 2x_1 + 3x_2 \\ &s/t \quad x_1 + x_2 \ge 5 \\ &\quad x_1 + 2x_2 \ge 6 \\ &\quad x_1, x_2 \ge 0 \end{aligned}$$

b) Solve the following LLP by Two phase method: CO5 (10)

$$\begin{aligned} &Min\, Z = x_1 + x_2 \\ &s/t \quad 2x_1 + x_2 \ge 4 \\ &\quad x_1 + 7x_2 \ge 7 \\ &\quad x_1, x_2 \ge 0 \end{aligned}$$

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