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UNIT-V

Linear Programming Problems

In this it deals with optimization of linear function subject to linear constraints.

Formulation A problem is to be presented in a linear programming problem form which requires establishing relation b/w variables & formulating objective function & constraints.

Graphical Method

If a LPP contains only 2 decision variables we can apply graphical method

Working Procedure

I) Formulate the given LPP 2) convert constraints into equations & plot each equation in the XY plane and determine the convex region formed by the equations & find the optimal values at each point of convex polygon

Convex Region

It is a set of points that on a line which completely lies within the region.

Problems

II) The XYZ company during the festival season combines 2 factors A and B to form a gift pack which must weigh 5 kg. At least 2 kg of A & not more than 4 kg of B should be used. The net profit contribution to the company is Rs 5 per kg for A & Rs 6 per kg for B. Forming Formulate Linear Programming model to find the optimal factor mix.

A

A B

Let $\left.\begin{array}{l} A = x_1 \\ B = x_2 \end{array}\right\}$ which are decision variables.

$x_1 + x_2 = 5$

$x_1 \geq 2$

$x_2 \leq 4$

Max $Z = 5x_1 + 6x_2$

$x_1 \geq 0$, $x_2 \geq 0$ - non negative restrictions.

12) A manufacturer produces 2 types of models M1 & M2. Each model of the type M1 requires 4 hours of grinding and 2 hours of polishing; whereas each model of the type M2 requires 2 hours of grinding and 5 hours of polishing. The manufacturer has 2 grinders & 3 polishers. Each grinder works 40 hours a week and each polisher works for 60 hrs a week. Profit on M1 model is 3.00 & on model M2 is 4.00. Whatever is produced in a week is sold in the market. How should the manufacturer allocate his production capacity to the 2 types of models so that he may make max profit in a week?

A

	grinding	polishing
M_1	4	2
M_2	2	5
	80	180

Let $M_1 = x_1$, $M_2 = x_2$
which are decision variables
Max $Z = 3x_1 + 4x_2$
Subject to the constraints

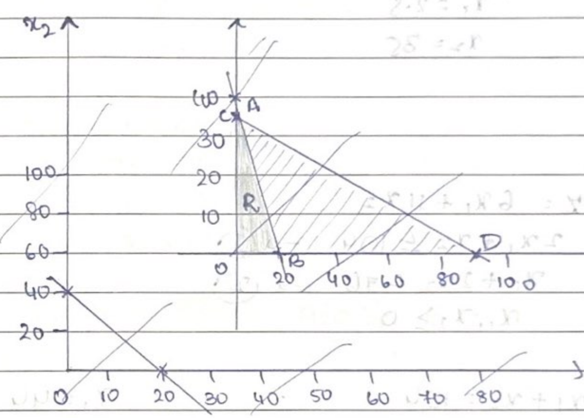
grinding
$4x_1 + 2x_2 \le 80$
$2x_1 + 5x_2 \le 180$ → polishers

$x_1 \ge 0$, $x_2 \ge 0$

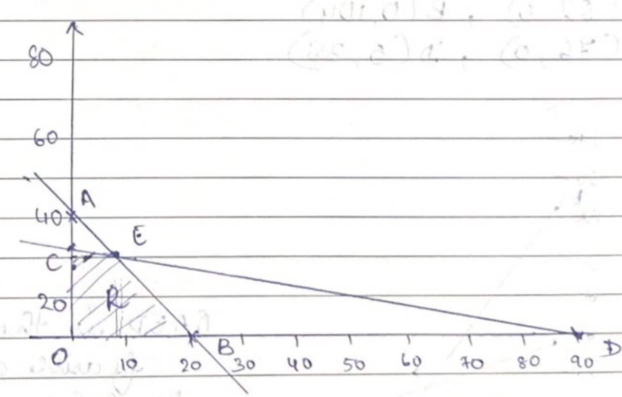
Graphical methode

$4x_1 + 2x_2 = 80$ — ①
$2x_1 + 5x_2 = 180$ — ②

A(0, 40), B(20, 0)
C(0, 36), D(90, 0)



Point of intersection for AB and CD



(2.5, 35)

At	$z = 3x_1 + 4x_2$
O (0,0)	0
B (20,0)	60
C (0,36)	144
D (2.5,35)	147.5

Max profit = 147.5

$x_1 = 2.5$

$x_2 = 35$

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32) Solve

ii) Max $z = 6x_1 + 11x_2$

S/t $2x_1 + x_2 \le 104 \rightarrow$ (1)

$x_1 + 2x_2 \le 76 \rightarrow$ (2)

$x_1, x_2 \ge 0$

$2x_1 + x_2 = 104$

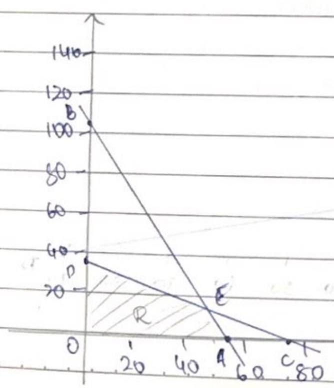
$x_1 + 2x_2 = 76$

$x_1 = 44$ find using calci

$y = 16$

A(52,0), B(0,104)

C(76,0), D(0,38)



OAED is the feasible region

Pt of intersection (44,16)

At	$z: 6x_1 + 11x_2$
O (0,0)	0
A (52,0)	312
E (44,16)	264 + 176 = 440
D (0,38)	418

Max profit = 440

$x_1 = 44$

$x_2 = 16$

32 1) Max $z = x_1 + 2x_2$

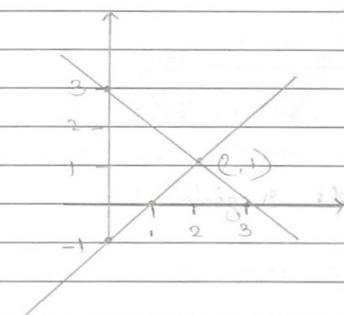
$x_1 + x_2 \ge 3$

$x_1 - x_2 \le 1$

$x_1, x_2 \ge 0$

$x_1 = 2$ A(0,3) B(3,0)

$x_2 = 1$ C(1,0) D(0,-1)



The solution is unbounded

2) Solve graphically

Max $z = 2x_1 + 3x_2$

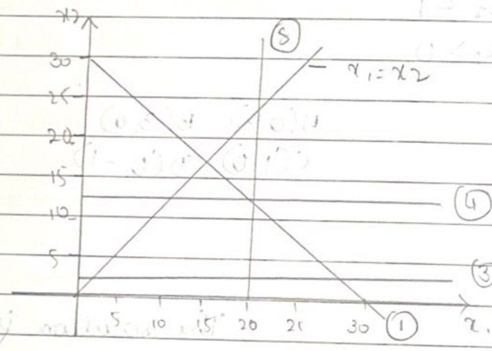
$x_1 + x_2 \le 30$ - (1) $x_2 \ge 3$ - (3)

$x_1 - x_2 \ge 0$ - (2) $x_2 \le 12$ - (4)

$x_1 \le 20$ — (5)
$x_1, x_2 \ge 0$

Pt	$Z = 2x_1 + 3x_2$
A(3,3)	15
B(20,3)	49
C(20,10)	70
D(18,12)	72 → Max
E(12,12)	60

Max $z = 72$
at $x_1 = 18$, $x_2 = 12$



ABCDE is the feasible region

The pts A (3,3)
B (20,3)
C is the intersection eqn (1) & (4)
Solving C(20,10)
D is the intersection of eqn (4) & (1)
Solving D (18,12)
E is the intersection of (4) & (2)
E (12,12)

General linear programming problem

Any linear programming problem having more than 2 decision variables may be expressed as follows :-

Maximize $Z = c_1x_1 + c_2x_2 + \dots + c_nx_n$
subject to the constraints

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \le b_1$$
$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \le b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \le b_m$$

With non negative ~~sectio~~ restrictions
$x_1, x_2 \dots x_n \ge 0$
May be of with $\ge$ type or with $=$ type.

In the above LPP programming the objective func may be of min type & the constraints may be of > or equal = type or equal type and sum of the decision variables may be unrestricted variables

$c_1, c_2 \dots c_n$ are called cost co-efficients.

In LPP all b values should be positive. If there is any negative b, multiply that constraint with a negative sign in order to make that b value positive.

Solution :- The set of values $\{x_1, x_2 \dots x_n\}$ which satisfies the constraints is called basic solution. The basic solution which also ~~which~~ satisfies non negative restrictions is called the feasible solution.

A feasible soln which optimize the function z is called optimal solution.

Slack and ~~Samp~~ Surplus variables

If the constraints are $\le$ type then we add some basic variables to the constraints in order to make inequalities to equal sign.

Suppose $\sum_{j=1}^{n} a_{ij} x_j \le b_i$

then we add S_i such that

$$\sum_{j=1}^{n} a_{ij} x_j + S_i = b_i \quad \text{for } i = 1, 2 \dots k$$

We subtract some basic variables for the constraints with $\ge$ type, i.e, these variables are called surplus variables

$$\sum_{j=1}^{n} a_{ij} x_j \ge b_i$$

$$\Rightarrow \sum_{j=1}^{n} a_{ij} x_i - S_i = b_i$$

Surplus variables

$\frac{S^8 P_0}{8!(1-\frac{S}{3})}$

9/1/24 Standard form of LPP

The general LPP can also put in the following form

1) Objective function is of max type
2) All constraints are expressed as equations by adding slack variables or subtracting surplus variables
3) RHS of each constraint is non negative
4) All variables are non negative

Working Procedure for simplex method

Assuming the existance of an initial basic feasible solution, an optimal soln to any LPP by simplex method is formed is as follows:-

Step 1:- Check whether the objective func is maximisation type. If it is minimisation problem, then convert

$$\boxed{\text{Min } z = \text{Max}(-z)}$$

2) Check all the b_i's are positive. If any of the b's are -ve, ~~negative~~ multiple both sides by -ve so as to make its RHS positive

Step 2:- Express the problem in the standard form

Convert all inequalities of constraints into equations by introducing slack or surplus variables in the constraints giving eqns of the form

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \sigma s_1 + \sigma s_2 + \sigma s_3 = b_1$$

Step 3:- Find initial basic feasible soln i.e by assigning each $x = 0$, then we have initial basic feasible soln for s_i
If all $s_i \geq 0$ this basic soln is feasible & non ~~negative~~ degenerate.

If one or more s_i's are zero, the solution is degenerate. Then form the initial simplex table as follows:-

	C_j	c_1	c_2	0	0	0	
C_B	Basis	x_1	x_2	s_1	s_2	s_3	b
0	s_1	a_{11}	a_{12}	1	0	0	b_1
0	s_2	a_{21}	a_{22}	0	1	0	b_2
0	s_3	a_{31}	a_{32}	0	0	1	b_3

Here s_1, s_2 & s_3 .... are called basic variables

x_1, x_2 & x_3 are called non basic variables
Here basis refers to the basic variables s_1, s_2 & s_3 and C_j = co-efficients of x_1, x_2.. in the objective function & C_B = co-efficients of s_1, s_2, s_3 in the objective func. All b values are a constants and all a values are the co-efficients in the equations.

Step 4 :- Compute $\Delta j = C_j - Z_j$
where $Z_j = C_B a_{ij}$

If all Δj values are negative, the optimal solution is obtained.
If one of the Δj value is positive, the solution is not optimal.

Step 5 :- Identify the incoming & outgoing variables. Choose the most positive of Δj. The variable corresponding to this column of Δj is Incoming variable. For outgoing variable find the 'θ' column.
The elements are obtained by diving the b column elements with the pivotal column elements.
In the theta column choose the least positive value, the variable corresponding to this least positive value is the outgoing variable & this row is called pivotal row.
The intersection of pivotal column with the pivotal gives the pivot element. Make this pivot element 1 & the other elements in the pivotal column 0 by using the row transformation. Again find Δj. Follow the above procedure until all the values of Δj are negative & the basic feasible soln is along the b column corresponding to the basic variables.

Duality

Every LPP is associated with another LPP called the dual of LPP. The original problem is called primal of LPP. Let the primal of LPP is defined as follows.

$$\text{Max } Z_P = C_1x_1 + C_2x_2 + \dots C_nx_n$$

s.t. constraints

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \le b_1$$
$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \le b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{22}x_2 \dots\dots + a_{mn}x_n \le b_m$$

Here $x_1, x_2 \dots x_n$ = primal variables. This primal is in std form. The dual of this LPP is given by

$$\text{Min } Z_D = b_1y_1 + b_2y_2 + \dots\dots + b_my_m$$

s.t constraints =

$$a_{11}y_1 + a_{21}y_2 + a_{31}y_3 + \dots\dots + a_{m1}y_m \ge C_1$$
$$a_{12}y_1 + a_{22}y_2 + a_{32}y_3 + \dots\dots + a_{m2}y_m \ge C_2$$
$$\vdots$$
$$a_{1n}y_1 + a_{2n}y_2 + a_{3n}y_3 + \dots\dots + a_{mn}y_m \ge C_n$$
$$y_1, y_2 \dots y_m \ge 0$$

The primal can be changed to dual by transposing rows & column

Note :- If ith variable in the primal is unrestricted inside then i'th constraint in the dual is an equality. If the ith constraint in the primal is an equality then i'th variable in the dual is unrestricted ~~inside~~ in sign.

Type the dual of following LPP

Min $Z = 3x_1 + x_2$

s.t $2x_1 + 3x_2 \geq 2$

$x_1 + x_2 \geq 1$

$x_1, x_2 \geq 0$

$$A = \begin{bmatrix} 2 & 3 & 2 \\ 1 & 1 & 1 \\ 3 & 1 & * \end{bmatrix}$$

$$A^T = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 1 \\ 2 & 1 & * \end{bmatrix}$$

Dual problem

Max $Z_D = 2y_1 + y_2$

St constraints

$2y_1 + y_2 \leq 3$

$3y_1 + y_2 \leq 1$

$y_1, y_2 \geq 0$

~~1+4+1=12~~

Construct the dual of LPP

i) Max $Z = x_1 - x_2 + 3x_3$

s.t constraints

$x_1 + x_2 + x_3 \leq 10$

$2x_1 - x_3 \leq 2$

$2x_1 - 2x_2 + 3x_3 \leq 6$

$x_1, x_2, x_3 \geq 0$

$$A = \begin{bmatrix} 1 & 1 & 1 & 10 \\ 2 & 0 & -1 & 2 \\ 2 & -2 & 3 & 6 \\ 1 & -1 & 3 & * \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 2 & 2 & 1 \\ 1 & 0 & -2 & -1 \\ 1 & -1 & 3 & 3 \\ 10 & 2 & 6 & * \end{bmatrix}$$

Min $Z_D = 10y_1 + 2y_2 + 6y_3$

s.t constraints

$y_1 + 2y_2 + 2y_3 \geq 1$

$y_1 - 2y_3 \geq -1$

$y_1 - y_2 + 3y_3 \geq 3$

$y_1, y_2, y_3 \geq 0$

Min $Z = 5x_1 + 3x_2 - x_3$

s.t $x_1 + x_2 + x_3 = 1$

$x_1 - 2x_2 + x_3 \geq 3$

$x_1, x_2, x_3 \geq 0$

2) Min $z = 5x_1 + 3x_2 - x_3$

s.t $x_1 + x_2 + x_3 = 1$

$x_1 - 2x_2 + x_3 \geq 3$

$x_1, x_2, x_3 \geq 0$

$x_1 + x_2 + x_3 = 1$

can be expressed as

$x_1 + x_2 + x_3 \geq 1$

and $x_1 + x_2 + x_3 \leq 1$

The std form

Min $z = 5x_1 + 3x_2 + x_3$

s.t $x_1 + x_2 + x_3 \geq 1$

$-x_1 - x_2 - x_3 \geq -1$

$x_1 - 2x_2 + x_3 \geq 3$

$x_1, x_2, x_3 \geq 0$

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 \\ 1 & -2 & 1 & 3 \\ 5 & 3 & -1 & * \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & -1 & 1 & 5 \\ 1 & -1 & -2 & 3 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & 3 & * \end{bmatrix}$$

Max $z = y_1 - y_2 + 3y_3$

st. constraints

$y_1 - y_2 + y_3 \leq 5$

$y_1 - y_2 - 2y_3 \leq 3$

$y_1 - y_2 - y_3 \leq -1$

$y_2, y_3 \geq 0$

y_1 is unrestricted variable

10/7/24 Fundamental theorem of duality

If the primal problem has a final optimal soln then the dual problem also have the same optimal soln

Problems

3). Write the dual of the LPP to minimize

Min $z = 3x_1 - 2x_2 + 4x_3$

st $3x_1 + 5x_2 + 4x_3 \geq 7$

$6x_1 + x_2 + 3x_3 \geq 4$

$7x_1 - 2x_2 - x_3 \leq 10$

$x_1 - 2x_2 + 5x_3 \geq 3$

$4x_1 + 7x_2 - 2x_3 \geq 2$ $\quad x_1, x_2, x_3 \geq 0$

$$A = \begin{bmatrix} 3 & 5 & 4 & 7 \\ 6 & 1 & 3 & 4 \\ -7 & +2 & +1 & -10 \\ 1 & -2 & 5 & 3 \\ 4 & 7 & -2 & 2 \\ 3 & -2 & 4 & * \end{bmatrix} \quad A^T = \begin{bmatrix} 3 & 6 & -7 & 1 & 4 & 3 \\ 5 & 1 & 2 & -2 & 7 & -2 \\ 4 & 3 & 1 & 5 & -2 & 4 \\ 7 & 4 & -10 & 3 & 2 & * \end{bmatrix}$$

Dual problem

Max $z = 7y_1 + 4y_2 - 10y_3 + 3y_4 + 2y_5$

st constraints

$3y_1 + 6y_2 - 7y_3 + y_4 + y_5 \leq 3$

$5y_1 + y_2 + 2y_3 - 2y_4 + 7y_5 \leq -2$

$4y_1 + 3y_2 + y_3 + 5y_4 - 2y_5 \leq 4$

$y_1, y_2, y_3, y_4, y_5 \geq 0$

30) Write the dual of the LPP Max $Z = 2x_1 + 3x_2 + x_3$

st $4x_1 + 3x_2 + x_3 = 6$

$x_1 + 2x_2 + 5x_3 = 4$

$x_1, x_2, x_3 \geq 0$

$$A = \begin{bmatrix} 4 & 3 & 1 & 6 \\ 1 & 2 & 5 & 4 \\ 2 & 3 & 1 & * \end{bmatrix} \qquad A^T = \begin{bmatrix} 4 & 1 & 2 \\ 3 & 2 & 3 \\ 1 & 5 & 1 \\ 6 & 4 & * \end{bmatrix}$$

Min $Z = 6y_1 + 4y_2$

s.t constraints

$4y_1 + y_2 \geq 2$

$3y_1 + 2y_2 \geq 3$

$y_1 + 5y_2 \geq 1$

when = sign is there

min = $\geq$

max = $\leq$

y_1, y_2 are unrestricted.

31) $Z = x_1 + x_2 + x_3$

$x_1 - 3x_2 + 4x_3 = 5$

$x_1 - 2x_2 \leq 3$

$2x_2 - x_3 \geq 4$, where x_1, x_2, x_3 are unrestricted

$$A \begin{bmatrix} 1 & -3 & 4 & 5 \\ -1 & 2 & * & -3 \\ * & 2 & -1 & 4 \\ 1 & 1 & 1 & * \end{bmatrix} = A^T = \begin{bmatrix} 1 & -1 & * & 1 \\ -3 & 2 & 2 & 1 \\ 4 & * & -1 & 1 \\ 5 & -3 & 4 & * \end{bmatrix}$$

Min $Z = 5y_1 - 3y_2 + 4y_3$

$y_1 - y_2 \geq 1$

$-3y_1 + 2y_2 + 2y_3 \leq 1$

$4y_1 - y_3 \leq 1$

~~$5y_1 - 3y_2 + 4y_1$~~

Simplex Method problems

Use simplex method to form the following LPP Max $Z = 5x_1 + 3x_2$

s.t $5x_1 + 2x_2 \leq 10$, $x_1 + x_2 \leq 2$

$3x_1 + 8x_2 \leq 12$

$x_1, x_2 \geq 0$

Convert to its standard form

$x_1 + x_2 + s_1 = 2$

$5x_1 + 2x_2 + s_2 = 10$

$3x_1 + 8x_2 + s_3 = 12$

$x_1, x_2, s_1, s_2, s_3 \geq 0$

Max $Z = 5x_1 + 3x_2 + 0s_1 + 0s_2 + 0s_3$

put $x_1 = 0$, $x_2 = 0$

$s_1 = 2$, $s_2 = 10$, $s_3 = 12$

Initial basic feasible soln

Initial Simplex table

	$C_j =$	5	3	0	0	0		
CB	Basis	x_1	x_2	s_1	s_2	s_3	b	θ
0	s_1	1	1	1	0	0	2	2
0	s_2	5	2	0	1	0	10	2
0	s_3	3	8	0	0	1	12	4
$Z_j = \Sigma C_B a_{ij} =$		0	0	0	0	0		
$\Delta = C_j - Z_j$		5-0	3-0	0	0	0		

Since all Δ values are non negative so the solution is not optimal soln

To find the incoming variable choose the most positive value of Δ. It occurs in the s_1 column. ~~It occurs~~ s_1 is the incoming variable.

To find the outgoing variable form the elements in θ columns by dividing the elements in b column with the corresponding elements of pivotal column.

In the θ column, choose min +ve quantity. The min +ve quantity occurs in S_1 row & s_2 row, we choose arboritarily s_1 as the outgoing variable.

Forming the 1st simplex table

	C_j	5	3	0	0	0	
Replace s_1 with x_1	~~x_1~~	~~1~~	~~1~~	~~1~~	~~0~~	~~0~~	
C_B	Basis	x_1	x_2	s_1	s_2	s_3	b
5	x_1	1	1	1	0	0	2
0	s_2	0	-3	-5	1	0	0
0	s_3	0	5	-3	0	1	6

In the pivotal column make other elements 0

$R_2 \to R_2 - 5(\text{New } R_1)$
$R_3 \to R_3 - 3(\text{New } R_1)$

	C_j	5	3	0	0	0	
C_B	Basis	x_1	x_2	s_1	s_2	s_3	b
5	x_1	1	1	1	0	0	2
0	s_2	0	-3	-5	1	0	0
0	s_3	0	5	-3	0	1	6
	$Z_j = \Sigma C_B a_{ij}$	5	5	5	0	0	
	$\Delta = C_j - Z_j$	0	-2	-5	0	0	

All Δ values are -ve so optimal solution

Solution :-

$(x_1 = b)$ $(x_2 = b)$

$x_1 = 2, \; x_2 = 0$

$\text{Max } z = 5 \times 2 + 0 \times 3$
$= 10$

11/7/24 Max $z = 7x_1 + 5x_2$

s.t $x_1 + x_2 \le 6$

$4x_1 + 3x_2 \le 12$

$x_1, x_2 \ge 0$

Convert to standard form Max $z = 7x_1 + 5x_2 + 0s_1 + 0s_2$

$x_1 + x_2 + s_1 = 6$

$4x_1 + 3x_2 + s_2 = 12$

I BFS $s_1 = 6, s_2 = 12$

Initial simplex table

	C_j	7	5	0	0		
C_B	Basis	x_1	x_2	s_1	s_2	b	θ
0	s_1	1	1	1	0	6	6
0	s_2	4	3	0	1	12	3 →
Z_j =	0	0	0	0	0		
$\Delta = C_j - Z_j$ =		7	5	0	0		

First simplex

	C_j	7	5	0	0	
C_B	Basis	x_1	x_2	s_1	s_2	b
0	s_1	0	1/4	1	-1/4	3
7	x_1	1	3/4	0	1/4	3
Z_j		7	21/4	0	7/4	
Δ		0	-1/4	0	-7/4	

All Δ values are -ve. Optimal soln

$x_1 = 3 \quad x_2 = 0$

Max $z = 7 \times 3 = 21$

Big M Method or (Charne's Penalty method)

This method is when the constraints are ≥ or = type. In this method, the constraints with greater than or equal to type we have artificial variables for the constraints with = sign we had artificial variable. The cost of the artificial variable we use -M and eliminate artificial variables as in the simplex Method. Here the value of M is assumed to be very large.

1) Min $z = 2x_1 + 3x_2$

s.t $x_1 + x_2 \ge 5$

$x_1 + 2x_2 \ge 6$

$x_1, x_2 \ge 0$

Solve by Big M method

Max $z = -2x_1 - 3x_2 + 0s_1 + 0s_2 - MA_1 - MA_2$

$x_1 + x_2 - s_1 + A_1 = 5$

$x_1 + 2x_2 - s_2 + A_2 = 6$

IBFS (Initial Basic Feasible soln)

$A_1 = 5, A_2 = 6$

Initial Simplex table

	C_j	2	3	0	0	-M	-M		
CB	Basis	x_1	x_2	s_1	s_2	A_1	A_2	b	θ
-M	A_1	1	1	-1	0	1	0	5	5
-M	A_2	1	2	0	-1	0	1	6	3 → outgoing
$Z_j = \sum C_B a_{ij}$		-2M	-3M	M	M	-M	-M		

$\Delta = C_j - Z_j$ $2+2M$ $3+3M$ $-M$ $-M$ 0   0

To find outgoing elements

$5/1 = 5$
$6/2 = 3$

First simplex table

	C_j	2	3	0 (s_1)	0 (s_2)	-M	b	θ
-M	A_1	1/2	0	-1	1/2	1	2	$2 \div 1/2$ → outgoing
3	x_2	1/2	1	0	-1/2	0	3	-ve
Z_j:		$-\frac{M}{2}+\frac{3}{2}$	3	M	$-\frac{M}{2}-\frac{3}{2}$	-M		

$\boxed{R_1 \Rightarrow R_1 - \text{New}(R_2) : \text{transformation}}$ Note

$\Delta = \frac{M}{2}+\frac{1}{2}$ 0 $-M$ $\frac{M}{2}+\frac{3}{2}$ 0

most positive, so its an incoming variable

All Δ are non negative so not an optimal solution

	C_j	2	3	0	0	
CB	Basis	x_1	x_2	s_1	s_2	b
0	s_2	1	0	-2	1	4
3	x_2	1	1	-1	0	4
Z_j		3	3	3	0	
Δ		-1	0	-3	0	

$~~R_2 \to R_2 + \frac{(\text{New}R)}{2}~~$

$R_2 = 2R_2 + R_1$

$2R_2 + R_1$

$-\frac{1}{2} + \frac{1}{2}$

$R_2 = R_2 + \frac{\text{New}(R_1)}{2}$

All Δ's are not +ve so its a optimal solution

12/7/24

1) Max $Z = 3x_1 + 2x_2$

s.t. $2x_1 + x_2 \le 2$
$3x_1 + 4x_2 \ge 12$
$x_1, x_2 \ge 0$

Max $Z = 3x_1 + 2x_2 + 0s_1 + 0s_2 - MA_1$
$2x_1 + x_2 + s_1 = 2$
$3x_1 + 4x_2 - s_2 + A_1 = 12$
$x_1, x_2, s_1, s_2, A_1 \ge 0$

IBFS $s_1 = 2$ $A_1 = 12$

Initial simplex table

	C_j	3	2	0	0	-M	b	
CB		x_1	x_2	s_1	s_2	A_1		
0	s_1	2	1	1	0	0	2/1	2
-M	A_1	3	4	-1	0	1	12/4	3

z_j : $-3M$ $-4M$ M 0 0 $-M$
Δ $3+3M$ $2+4M$ $-M$ 0 0 0
x_2 is incoming
s_1 is outgoing pivot 1

1st simplex table

	Cj	3	2	0	0	-M	
CB		x_1	x_2	S_1	S_2	A_1	b
2	x_2	2	1	1	0	0	2
-M	A_1	-5	0	-5	0	1	4
z_j		4+5M	2	2+5M	0	-M	
Δ		-1-5M	0	-2-10M	0	0	

Even though optimality is occured, Artificial variable is not removed ∴ Soln is redundant

2) Max $z = 12x_1 + 20x_2$
s.t $6x_1 + 8x_2 \le 100$
$7x_1 + 12x_2 \ge 120$
$x_1, x_2 \ge 0$
convert to std problem

Max $(-z)$: $-12x_1 - 20x_2$
$6x_1 + 8x_2 + s_1 = 100$
$7x_1 + 12x_2 - s_2 + A_1 = 120$
$x_1, x_2, s_1, s_2, A_1 \ge 0$

1 BFS $s_1 = 100$ $A_1 = 120$

	Cj	-12	-20	0	0	-M		
CB	Basis	x_1	x_2	s_1	s_2	A_1	b	θ
0	s_1	6	8	1	0	0	100	100/8 = 12.5
-M	A_1	7	12	0	-1	1	120	120/12 = 10 →
z_j		-7M	-12M	0	M	-M		
Δ		-12+7M	-20+12M	0	-M	0		

$R_1 \to R_1 - 8$ (New R_2)
$8 - 8(1)$
$6 - 8(7/12)$
$6 - 42/12$

	Cj	-12	-20	0	0	
	Basis	x_1	x_2	s_1	s_2	b
0	s_1	4/3	0	1	8/12	20
-20	x_2	7/12	1	0	-1/12	10
	z_j	-140/12	-20	0	20/12	

Δ

16/7/24 Two phase Simplex Method

Here we choose cost of artificial variables is -1
& " " " other variables is 0
& eliminate the artificial variables.

After eliminating the artificial variables in the basis by choosing the original cost to the variables & testing the soln for optimality by making all $\Delta \le 0$

1) Min $z = x_1 + x_2$ Solve by using 2 phase Simplex Method.
s.t $2x_1 + x_2 \ge 4$
$x_1 + 7x_2 \ge 7$
$x_1, x_2 \ge 0$

A Convert into std form

$Max(-z) = -x_1 - x_2 + 0s_1 + 0s_2 - A_1 - A_2$
$2x_1 + x_2 - s_1 + A_1 = 4$
$x_1 + 7x_2 - s_2 + A_2 = 7$

Initial basic feasible soln : $A_1 = 4$, $A_2 = 7$

Phase 1: Initial simplex table

	C_j	0	0	0	0	-1	-1		
	Basis	x_1	x_2	s_1	s_2	A_1	A_2	b	θ
-1	A_1	2	1	-1	0	1	0	4	4
-1	A_2	1	7	0	-1	0	1	7	1 → A_2
	Z_j	-3	-8	1	1	-1	-1		
	Δ $C_j - Z_j$	3	8	-1	-1	0	0		

All Δ are non negative, most positive quantity is along x_2 column.
A_2 is Outgoing

Make 7 as 1.

First simplex table

Divide Make 2 as 1

	C_j	0	0	0	0	-1		
	Basis	x_1	x_2	s_1	s_2	A_1	b	θ
-1	A_1	13/7	0	-1	1/7	1	3	21/13 → Min
0	x_2	1/7	1	0	-1/7	0	1	7
	Z_j →	-13/7	0	1	-1/7	-1		
	Δ	13/7	0	-1	1/7	0		

$R_1 = R_1 - \text{New } R_2$

x_1 is most positive
Min is A_1

	C_j	0	0	0	0	
	Basis	x_1	x_2	s_1	s_2	b
0	x_1	1	0	-7/13	1/13	21/3
0	x_2	0	1	1/73	~~-14/91~~ -2/13	10/3
	Z_j	0	0	0	0	
	Δ	0	0	0	0	

$R_2 = R_2 - \frac{1 (\text{New } R_1)}{7}$

this is end of ~~same~~ phase 1.

$\frac{1}{7}$ $\frac{1}{7}$
$0 - \left(-\frac{1}{7}\right)$
$-\frac{1}{7} - \frac{1}{7}$
$0 + \frac{7}{13}$
$-\frac{1}{7} \cdot \frac{7}{13}$
$-\frac{1}{7}\left(1 + \frac{1}{13}\right)$
$\frac{-1}{7} \frac{14}{13}$
$\frac{-14}{91} = \frac{-2}{13}$
$\frac{1}{7}\left(\frac{21}{13}\right)$

Instead of x_1 & x_2, we take original cost in G

In phase 2

C_j		-1	-1	0	0	
	Basis	x_1	x_2	s_1	s_2	b
-1	x_1	1	0	-7/13	1/3	21/13
-1	x_2	0	1	1/13	-2/13	10/13
	Z_j	-1	-1	6/13	1/13	
	Δ	0	0	-6/13	-1/13	

consider the phase 1 last matrix.

All Δ values are -ve so optimal soln

$x_1 = \frac{21}{13}$, $x_2 = \frac{10}{13}$

$\text{Max}(-z) = -\frac{21}{13} - \frac{10}{13} = +\frac{31}{13}$, $\text{Min}(z) = \frac{31}{13}$

2) Solve using 2 phase Simplex Method

$\text{Max } z = -12x_1 - 20x_2$

st $6x_1 + 8x_2 \geq 100$

$7x_1 + 12x_2 \geq 120$

$x_1, x_2 \geq 0$

Convert to the standard form

$\text{Max}(z) = -12x_1 - 20x_2 + 0s_1 + 0s_2 - A_1 - A_2$

st C $6x_1 + 8x_2 \geq 100 \quad s_1 + A_1$

$7x_1 + 12x_2 \geq 120 \quad s_2 + A_2$

$x_1, x_2 \geq 0$

$A_1, A_2, s_1, s_2, x_1, x_2 \geq 0$

Phase 1

IBFS : $A_1 = 100$, $A_2 = 120$

Initial simplex method

C_j		0	0	0	0	-1	-1		
	Basis	x_1	x_2	s_1	s_2	A_1	A_2	b	θ
-1	A_1	6	8	-1	0	1	0	100	100/8 12.5
-1	A_2	7	12	0	-1	0	1	120	120/12 = 10 →
	Z_j	-13	-20	1	1	-1	-1		
	Δ	+13	20 ↑	-1	-1	0	0		

All Δ's are non negative

Consider x_2 as most positive

A_2 is outgoing

First simplex table

C_j		0	0	0	0	-1		
	Basis	x_1	x_2	s_1	s_2	A_1	b	θ
-1	A_1	4/3	0	-1	2/3	1	20	15 →
0	x_2	7/12	1	0	-1/12	0	10	120/7
	Z_j	-4/3	0	1	-2/3	-1		
	Δ	4/3 ↑	0	-1	2/3	0		

All Δ's are non negative, so not an optimal soln

$R_1 = R_1 - 8(\text{New } R_2)$

$-1 - 8(0)$

$6 - \frac{8}{12} \times 7 = \frac{2}{3}$

$-1 - \frac{8}{12}\frac{7}{24}$

x_1 is incoming

A_1 is outgoing

$\frac{1}{12} - \frac{7}{12}\left(\frac{-1}{12}\right)$

$-\frac{2}{3} - \frac{7}{12}\left(\frac{1}{2}\right)$

$6 - \frac{7 \times 8}{12}$

$6 - \frac{14}{3}$

$\frac{18-14}{3} = \frac{4}{3}$

$0 - \frac{7}{12}$

$\frac{2}{3} - \frac{7}{24}$

$\frac{-16-7}{24} = \frac{-9}{24}$

2nd Simplex table.

C_j		0	0	0	0		
	Basis	x_1	x_2	s_1	s_2	b	θ
0	x_1	1	0	$-3/4$	$1/2$	15	
0	x_2	0	1	$7/16$	$-3/8$	$235/28$	
	Z_j	0	0	0	0	0	
	Δ	0	0	0	0	0	

end of phase 1

$R_2 = R_2 - \frac{7}{12}(\overset{New}{R_1})$

$0 - \frac{7}{12}\left(\frac{-3}{4}\right) = \frac{7}{16}$

$35 \times 7 = 245$

$\frac{120 - ...}{7}$

$\frac{480 - 2...}{28}$

480
245
235
24

Phase 2

C_j		-12	-20	0	0	
	Basis	x_1	x_2	s_1	s_2	b
-12	x_1	1	0	$-3/4$	$1/2$	15
-20	x_2	0	1	$7/16$	$-3/8$	$235/8$
	Z_j	-12	-20	$1/4$	$3/2$	
	Δ	0	0	$-1/4$	$-3/2$	

$\frac{36}{4}$

$-6 + ...$

$\frac{-12+15}{2}$

All value are negative
so soln is optimal

$$\boxed{Max\ Z = -12 \times 15 - 20 \times \frac{235}{28}}$$