Question Bank.

Important Questions

Engineering Mathematics – I (BAS103)

(Year: 2022-23)

SECTION A:

1. Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{bmatrix}$, a, b, c are being real.
2. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & -1 \\ 3 & 4 & 5 \end{bmatrix}$, find the eigen values of $A^2 - 3A - 2I$.
3. Find the sum and product of the eigen values of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$.
4. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$, find the eigen values of $A^2 + 2A - 3I$.
5. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ 1 & 2 & 2 \end{bmatrix}$, find the eigen values of $A^2 - 5A - 3A^{-1}$.
6. Show that the matrix iA is Skew-Hermitian where $A = \begin{bmatrix} 3 & 5+2i & -3 \\ 5-2i & 7 & 4i \\ -3 & -4i & 5 \end{bmatrix}$.
7. Prove that the matrix $A = \frac{1}{2}\begin{bmatrix} 1+i & -1+i \\ 1+i & 1-i \end{bmatrix}$ is unitary.
8. Prove that the matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary.
9. Prove that the matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$ is unitary. Where ω is the cube root of unity.
10. Show that the vectors $X_1 = (1,-1,1)$, $X_2 = (2,1,1)$ and $X_2 = (3,0,2)$ are linearly dependent.
11. Examine the linear dependence of the vectors $X_1 = (1,1,-1,1)$, $X_2 = (1,-1,-2,-1)$ and $X_3 = (3,1,0,1)$.
12. Find the n[th] derivatives of $y = \frac{1}{x^2+a^2}$.
13. Find the n[th] derivatives of $y = x^2 \sin 2x$.
14. Find the nth derivative of sin^3x.
15. Find the nth derivative of $x^2 sin2x$ at x = 0 by using Leibnitz theorem.
16. If $u = \log\left(\frac{x^4+y^4}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$ by using Euler's theorem.
17. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = sin2u$ by using Euler's theorem.
18. If $u = \sin^{-1}\left(\frac{x+2y+3z}{\sqrt{x^8+y^8+z^8}}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} + 3tanu = 0$ by using Euler's theorem.
19. If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, prove that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$.
20. If $u = f(2x-3y, 3y-4z, 4z-2x)$, prove that $\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z} = 0$.
21. If $u = f(y-z, z-x, x-y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.
22. If $u = \log(\tan x + \tan y)$, show that $sin2x\frac{\partial u}{\partial x} + \sin 2y\frac{\partial u}{\partial y} = 2$.
23. If $u = e^{xyz}$, then find $\frac{\partial^3 u}{\partial x \partial y \partial z}$.
24. If $z = \tan(y+ax) - (y+ax)^{3/2}$, find the value of $\frac{\partial^2 z}{\partial x^2}$.
25. What is the maximum value of $f(x,y) = 1 - x^2 - y^2$.

26. What is the minimum value of $f(x,y) = x^2 + y^2$.
27. Find the extreme point for the function $x^3y^2(1-x-y)$.
28. Find the extreme point of $x^2 + y^2 - 6x + 10$.
29. Find the stationary point of $f(x,y) = 5x^2 + 10y^2 + 12xy - 4x - 6x + 1$.
30. If $u = \frac{yz}{x}, v = \frac{xz}{y}$ and $w = \frac{xy}{z}$, find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$.
31. If $y_1 = 1 - x_1, y_2 = x_1(1-x_2)$ and $y_3 = x_1x_2(1-x_3)$, find $\frac{\partial(y_1,y_2,y_3)}{\partial(x_1,x_2,x_3)}$.
32. Use the Jacobian to prove that the functions $u = x+y+z, v = x^2+y^2+z^2$ and $w = xy+yz+zx$ are not independent.
33. Use the Jacobian to prove that the functions $u = x+y-z, v = x-y+z$ and $w = x^2+y^2+z^2-2yz$, are not independent.
34. Use the Jacobian to prove that the functions $u = x+2y+z, v = x-2y+3z$ and $w = 2xy - xz + 4yz - 2z^2$ are not independent.
35. If $x = e^v secu, y = e^v tanu$, find $\frac{\partial(x,y)}{\partial(u,v)}$.
36. Find the possible percentage error in computing the parallel resistance r of three resistances r_1, r_2, r_3. if r_1, r_2, r_3 are each in error by +1.2%.
37. Find the possible percentage error in computing the series resistance r of three resistances r_1, r_2, r_3. if r_1, r_2, r_3 are each in error by +1%.
38. Prove that $div(curl\vec{V}) = \nabla.(\nabla \times \vec{V}) = 0$.
39. Prove that $curl(grad\emptyset) = \nabla \times \nabla\emptyset = \vec{0}$.

SECTION B:

1. Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.
2. Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.
3. Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.
4. Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.
5. Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix}$.
6. If $y = (sin^{-1}x)^2$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$. Also find $y_n(0)$.
7. If $y = sin(asin^{-1}x)$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 - a^2)\,y_n = 0$. Also find $y_n(0)$.
8. If $y = \left[\log\left(x + \sqrt{1+x^2}\right)\right]^2$ then find $y_n(0)$.
9. If $y = \left[x + \sqrt{1+x^2}\right]^m$, find $(y_n)_0$.
10. If $y = e^{m\sin^{-1}x}$ then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$ Also find $y_n(0)$.
11. If u, v, w *are the roots of the cubic* $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ in λ. Find the value of $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.
12. If $u^3 + v + w = x + y^2 + z^2, u + v^3 + w = x^2 + y + z^2$ and $u + v + w^3 = x^2 + y^2 + z$. Show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{1-4(xy+yz+zx)+16xyz}{2-3(u^2+v^2+w^2)+27u^2v^2w^2}$.
13. If $u^3 + v^3 + w^3 = x + y + z, u^2 + v^2 + w^2 = x^3 + y^3 + z^3$ and $u + v + w = x^2 + y^2 + z^2$. Show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{(x-y)(y-z)(z-x)}{(u-v)(v-w)(w-u)}$.
14. Find the dimensions of a rectangular box of maximum capacity whose surface area is given when (i) Box is open at the top (ii) Box is closed

15. Divide 24 into three parts such that the continued product of the first, square of the second and cube of the third may be maximum.
16. Find the constant a, b, c so that $\vec{F} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is irrotational. If $\vec{F} = grad\emptyset$, Find the velocity potential $\emptyset$.
17. Show that $\vec{A} = (6xy + z^3)\hat{\imath} + (3x^2 - z)\hat{\jmath} + (3xz^2 - y)\hat{k}$ is irrotational. Find the velocity potential $\emptyset$ such that $\vec{A} = \nabla\emptyset$.

SECTION C:

1. Verify Cayley-Hamilton Theorem for $A = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$. Also find A^{-1}.
2. Verify Cayley-Hamilton Theorem for $A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$. Also find A^{-1}.
3. By using Cayley-Hamilton Theorem find A^{-1} of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$.
4. If $A = \begin{bmatrix} 7 & -1 & 3 \\ 6 & 1 & 4 \\ 2 & 4 & 8 \end{bmatrix}$, then find A^{-1} by using Cayley-Hamilton theorem.
5. If $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$, then find A^{-1} by using Cayley-Hamilton theorem.
6. Evaluate the expression $A + 5I + A^{-1}$. If $A = \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$
7. Express $2A^5 - 3A^4 + A^2 - 4I$ as a linear polynomial in A, where $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$.
8. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 3 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix}$ by reducing it to normal form.
9. Find the rank of the matrix $A = \begin{bmatrix} 0 & 1 & 2 & -1 \\ 1 & 2 & 1 & 1 \\ 3 & 1 & 0 & 2 \end{bmatrix}$ by reducing it to normal form.
10. Find the rank of the matrix $A = \begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 3 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}$ by reducing it to normal form.
11. Find the rank of $A = \begin{bmatrix} -2 & -1 & 3 & -1 \\ 1 & 2 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$, by reducing into its normal form.
12. Find the rank of $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$, by reducing into its normal form.
13. Investigate, for what values of λ and μ do the system of equations $2x - 5y + 2z = 8$, $2x + 4y + 6z = 5$, $x + 2y + \lambda z = \mu$ have (i) no solution (ii) unique solution (iii) infinite solutions?
14. Investigate, for what values of λ and μ do the system of equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have (i) no solution (ii) unique solution (iii) infinite solutions?
15. Show that the matrix $A = \begin{bmatrix} \alpha + i\gamma & -\beta + i\delta \\ \beta + i\delta & \alpha - i\gamma \end{bmatrix}$ is unitary if $\alpha^2+\beta^2+\gamma^2+\delta^2 = 1$.
16. Find the value of n so that the equation $V = r^n(3\cos^2\theta - 1)$ satisfy the relation $\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) = 0$.
17. If $u = \log(x^3+y^3+z^3 - 3xyz)$, prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$.
18. If $u = f(r)$, $r^2 = x^2 + y^2$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r}f'(r)$.

19. If $x^x y^y z^z = c$, show that at $x = y = z$, $\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$.

20. If $u = \log(x^3 + y^3 - x^2 y - xy^2)$, then show that $u_{xx} + 2u_{xy} + u_{yy} = -\frac{4}{(x+y)^2}$.

21. If $y = (x^2 - 1)^n$, prove that $(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$.

22. If $x = \tan(\log y)$, prove that $(1 + x^2)y_{n+2} + (2(n+1)x - 1)y_{n+1} + n(n+1)y_n = 0$

23. If $u = cosec^{-1}\left(\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}\right)^{1/2}$, then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{144}(13 + \tan^2 u)$.

24. If $u = sin^{-1}\left(\frac{x^{1/4}+y^{1/4}}{x^{1/6}+y^{1/6}}\right)$, then evaluate $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$.

25. If $u = cosec^{-1}\left(\frac{x^{1/4}+y^{1/4}}{x^{1/6}+y^{1/6}}\right)^2$, then evaluate $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$.

26. If $u = sec^{-1}\left(\frac{x^3-y^3}{x+y}\right)$, find the value of $x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy}$ by using Euler's theorem.

27. If $u = sin^{-1}\left(\frac{x^3+y^3}{\sqrt{x}+\sqrt{y}}\right)$, find the value of $x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy}$ by using Euler's theorem.

28. Find the nth derivative of $tan^{-1}\left(\frac{1+x}{1-x}\right)$.

29. Find the nth derivative of $tan^{-1}\left(\frac{a+x}{a-x}\right)$.

30. Find the nth derivative of $tan^{-1}\left(\frac{2x}{1-x^2}\right)$.

31. Expand $f(x, y) = e^x \log(1 + y)$ in powers of x and y up to third degree terms.

32. Expand $f(x, y) = e^x tan^{-1} y$ in powers of $(x - 1)$ and $(y - 1)$ up to two degree terms.

33. Express the function $f(x, y) = x^2 + 3y^2 - 9x - 9y + 26$ as Taylor's series expansion about the point $(1, 2)$.

34. Expand $x^2 y + \sin y + e^x$ in power of $(x - 1)$ and $(y - \pi)$ up to the third degree terms by using Taylor's theorem.

35. Expand $e^x cosy$ at $(1, \pi/4)$ by using Taylor's theorem.

36. If $y_1 = \frac{x_2 x_3}{x_1}$, $y_2 = \frac{x_1 x_3}{x_2}$, $y_3 = \frac{x_2 x_1}{x_3}$, then find $\frac{\partial(y_1, y_2, y_3)}{\partial(x_1, x_2, x_3)}$.

37. If $x + y + z = u$, $y + z = u^2 v$, $z = u^3 w$ then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

38. Examine for extreme values of $x^3 + y^3 - 3axy$.

39. Examine for extreme values of $\cos x + \cos y + \cos(x + y)$.

40. A rectangular box which is open at the top has a capacity of 32 cc. Determine, using Lagrange's method of multipliers, and the dimension of the box such that the least material is required for the construction of the box.

41. Find the maximum and minimum distances of the point $(3, 4, 12)$ from the sphere $x^2 + y^2 + z^2 = 1$

42. Find the shortest and longest distances from the point $(1, 2, -1)$ to the sphere $x^2 + y^2 + z^2 = 24$.

43. A balloon in form of right circular cylinder of radius 1.5 m and length 4 m is surmounted by hemispherical ends. If the radius is increased by 0.01 m. Find the percentage change in volume of the balloon.

44. Find the approximate value of $[(0.98)^2 + (2.01)^2 + (1.94)^2]^{1/2}$.

45. In estimating the cost of a pile of bricks measured as 6m x 50m x 4m, the tape is stretched 1% beyond the standard length. If the count is 12 bricks in 1 m^3 and bricks cost Rs. 100 per 1000. Find the approximate error in the cost.

46. Show that the vector $\vec{F} = \frac{\vec{r}}{r^3}$ is solenoidal.

47. Find the directional derivative of $\emptyset(x, y, z) = xy^2 + yz^3$ at point (2, -1, 1) in the direction of the normal to the surface $x \log z - y^2 + 4$ at (2, -1, 1).

48. Calculate the angle between the normal to the surface $xy = z^2$ at the point (4, 1, 2) and (1, 0, 2).

49. Prove that $div(r^n \vec{r}) = (n + 3)r^n$, further show that $r^n \vec{r}$ is solenoidal only if $n = -3$.

50. Find the directional derivative of $\phi = (x^2 + y^2 + z^2)^{-1/2}$ at the point P(3, 1, 2) in the direction of the vector $yz\hat{i} + zx\hat{j} + xy\hat{k}$.

51. If $u = x + y + z, v = x^2 + y^2 + z^2, w = xy + yz + zx$ prove that *grad u*, *grad v* and *grad w* are coplanar.