algebric structure $(B, +, \cdot)$ is called a Boolean algebra if the elements of set B obeys and follows the following laws.

Unit-2
Boolean Algebra

1) Closure law -

$$a + b \in B$$
$$a \cdot b \in B, \quad \forall\, a, b \in B$$

2) Commutative law -

$$a + b = b + a$$
$$a \cdot b = b \cdot a$$

3) distributive law -

$$a \cdot (b + c) = a \cdot b + a \cdot c$$
$$a + (b \cdot c) = (a + b) \cdot (a + c)$$

4) Identity law -

0 is the identity element w.r.t operation '+' and 1 is the identity element w.r.t operation '·'.

$a + 0 = a$	$a + 1 = 1$
$a \cdot 1 = a$	$a \cdot 0 = 0$

5) Compliment law or law of inverse -

let $a' \in B$ is the compliment or inverse of $a \in B$.

$$a + a' = 1$$
$$a \cdot a' = 0$$

=> Above these five laws are the postulates of Boolean Algebra.

6) Associative law -

$(a+b)+c = a+(b+c)$

$(a\cdot b)\cdot c = a\cdot(b\cdot c)$

7) Demorgan's law-

$(a+b)' = a'\cdot b'$

$(a\cdot b)' = a'+b'$

8) Idempotent law-

$a+a = a$

$a\cdot a = a$

Ques proof the following boolean expressions.

(i) $A + AB = A$

(ii) $A + \bar{A}B = A + B$

(iii) $(A+B)\cdot(A+C) = A+BC$

Sol^n (i) $A + A\cdot B = A$

$LHS = A + A\cdot B$

$= A\cdot 1 + A\cdot B$ (By identity law)

$= A\cdot(1+B)$ (distributive law)

$= A\cdot 1$ [Identity law]

$= A$ (Identity law)

$= RHS$

$\Rightarrow$

(ii) LHS $= A + (\bar{A} \cdot B)$
$= (A + \bar{A}) \cdot (A + B)$ (Distributive law)
$= 1 \cdot (A + B)$ (complement law)
$= A + B$ (Identity law)

(iii) LHS $= (A+B) \cdot (A+C)$
$= ((A+B) \cdot A) + (A+B) \cdot C$ (Distributive law)
$= (A \cdot A + B \cdot A) + (A \cdot C + B \cdot C)$ (Distributive)
$= A + AB + A \cdot C + B \cdot C$ (absorption law)
$= A + AC + BC$ (absorption law)
$= A + BC$ (Idempotent law)

Ques simplify the following Boolean expression.

(i) $Y = \overline{\overline{AB} + \bar{A} + AB}$.
$= (\overline{\overline{AB}}) \cdot \bar{\bar{A}} \cdot \overline{AB}$
$= (AB) \cdot A \cdot (\bar{A} + \bar{B})$
$= A(BA) \cdot (\bar{A} + \bar{B})$
$= A(AB) \cdot (\bar{A} + \bar{B})$
$= (A \cdot A) \cdot B \cdot (\bar{A} + \bar{B})$
$= A \cdot B \cdot (\bar{A} + \bar{B})$
$= 0$.

(Or)
$= (\overline{\overline{AB}}) \cdot \bar{\bar{A}} \cdot \overline{AB}$
$= A \cdot B \cdot A \cdot \overline{AB}$
$= A \cdot A \cdot B \cdot \overline{AB}$
$= A \cdot B \cdot \overline{AB}$
$= 0$

(ii) $Y = (A+C) \cdot (A+D) \cdot (B+C) \cdot (B+D)$.
$= (A+C) \cdot (A+D) \cdot (B+C) \cdot (B+D)$ {distributive)

$= (A+CD)\cdot(B+CD)$ (Distributive law)

$= AB+CD.$ (distributive law).

(iii) $Y = (B+BC)\cdot(B+\bar{B}C)\cdot(B+D)$

$= (B+BC)\cdot(B+\bar{B}C)\cdot(B+D)$

$= ((B+BC)\cdot B + (B+BC)\cdot \bar{B}C)\cdot(B+D)$

$= BB + BBC + \bar{B}BC + \bar{B}BCC)\cdot(B+D)$

$= (B + BC + 0 + 0)\cdot(B+D)$

$= BB + BBC + BD + BCD$

$= B + BC + BD + BCD$

$= B(1+D) + BC(1+D)$

$= B\cdot 1 + BC\cdot 1$

$= B(1+C).$

$= B.$

Boolean Functions -

Boolean functions can be represented by two ways.

- SOP form - sum of product.
- POS form - Product of sum.

***Literals -**

single variable or its complement is known as literals.

* min term and max term -

1) Three variables -

2)

H.W (4) variable.

$\Rightarrow$

A	B	C	Decimal equivalence	Minterm Designation	Minterm expression (Prod. form)	Maxterm Designation	Maxterm expression (sum form)
0	0	0	0	m_0	$\bar{A}\bar{B}\bar{C}$	M_0	$A+B+C$
0	0	1	1	m_1	$\bar{A}\bar{B}C$	M_1	$A+B+\bar{C}$
0	1	0	2	m_2	$\bar{A}B\bar{C}$	M_2	$A+\bar{B}+C$
0	1	1	3	m_3	$\bar{A}BC$	M_3	$A+\bar{B}+\bar{C}$
1	0	0	4	m_4	$A\bar{B}\bar{C}$	M_4	$\bar{A}+B+C$
1	0	1	5	m_5	$A\bar{B}C$	M_5	$\bar{A}+B+\bar{C}$
1	1	0	6	m_6	$AB\bar{C}$	M_6	$\bar{A}+\bar{B}+C$
1	1	1	7	m_7	ABC	M_7	$\bar{A}+\bar{B}+\bar{C}$

minterms are the compliments of maxterm & and vice versa.

2) Four variables -

A	B	C	D	Decimal equivalence	Minterm Designation	Minterm expression	Maxterm Designation	Maxterm expression
0	0	0	0	0	m_0	$\bar{A}\bar{B}\bar{C}\bar{D}$	M_0	$A+B+C+D$
0	0	0	1	1	m_1	$\bar{A}\bar{B}\bar{C}D$	M_1	$A+B+C+\bar{D}$
0	0	1	0	2	m_2	$\bar{A}\bar{B}C\bar{D}$	M_2	$A+B+\bar{C}+D$
0	0	1	1	3	m_3	$\bar{A}\bar{B}CD$	M_3	$A+B+\bar{C}+\bar{D}$
0	1	0	0	4	m_4	$\bar{A}\cdot B\bar{C}\bar{D}$	M_4	$A+\bar{B}+C+D$
0	1	0	1	5	m_5	$\bar{A}B\bar{C}D$	M_5	$A+\bar{B}+C+\bar{D}$
0	1	1	0	6	m_6	$\bar{A}BC\bar{D}$	M_6	$A+\bar{B}+\bar{C}+D$
0	1	1	1	7	m_7	$\bar{A}BCD$	M_7	$A+\bar{B}+\bar{C}+\bar{D}$
1	0	0	0	8	m_8	$A\bar{B}\bar{C}\bar{D}$	M_8	$\bar{A}+B+C+D$
1	0	0	1	9	m_9	$A\bar{B}\bar{C}D$	M_9	$\bar{A}+B+C+\bar{D}$
1	0	1	0	10	m_{10}	$A\bar{B}C\bar{D}$	M_{10}	$\bar{A}+B+\bar{C}+D$
1	0	1	1	11	m_{11}	$\bar{A}\bar{B}CD$	M_{11}	$\bar{A}+B+\bar{C}+\bar{D}$
1	1	0	0	12	m_{12}	$AB\bar{C}\bar{D}$	M_{12}	$\bar{A}+\bar{B}+C+D$
1	1	0	1	13	m_{13}	$AB\bar{C}D$	M_{13}	$\bar{A}+\bar{B}+C+\bar{D}$
1	1	1	0	14	m_{14}	$ABC\bar{D}$	M_{14}	$\bar{A}+\bar{B}+\bar{C}+D$
1	1	1	1	15	m_{15}	$ABCD$	M_{15}	$\bar{A}+\bar{B}+\bar{C}+\bar{D}$

Ques Simplify the following three variables expression?

* $Y = \Sigma m(2,4,6)$.

$\Rightarrow$ $Y = \Sigma m(2,4,6)$

$= m_2 + m_4 + m_6$

$= \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$

$= \bar{A}B\bar{C} + AB\bar{C} + A\bar{B}\bar{C}$ (commutative)

$= (\bar{A}+A)\cdot B\bar{C} + A\bar{B}\bar{C}$ (Distributive)

$= 1\cdot B\bar{C} + A\bar{B}\bar{C}$ (compliment)

$= B\bar{C} + A\bar{B}\bar{C}$ (identity)

$= (B + A\bar{B})\cdot\bar{C}$ (distributive)

$= (A+B)\cdot\bar{C}$

$= A\bar{C} + B\bar{C}$

* $Y = \pi\ m(1,3,5)$

$= M_1 \cdot M_3 \cdot M_5$.

$= (A+B+\bar{C})\cdot(A+\bar{B}+\bar{C})\cdot(\bar{A}+B+\bar{C})$

$= (A+B+\bar{C})\cdot(\bar{A}+B+\bar{C})\cdot(A+\bar{B}+\bar{C})$ (distributive)

$= (A\cdot\bar{A}) + (B+\bar{C})\cdot(A+\bar{B}+\bar{C})$ (compliment)

$= 0 + (B+\bar{C})\cdot(A+\bar{B}+\bar{C})$ (identity)

$= (B+\bar{C})\cdot(A+\bar{B}+\bar{C})$

$= [B\cdot(A+\bar{B})] + \bar{C}$ (distributive)

$= (AB + B\bar{B}) + \bar{C}$ (Compliment)

$= (AB + 0) + \bar{C}$ (identity)

$= AB + \bar{C}$

* Obtain Canonical (or) standard (or) complete SOP form -

* $F(A,B,C) = AB + AC + BC$.

$\Rightarrow F(A,B,C) = AB + AC + BC$

$= AB\cdot 1 + AC\cdot 1 + BC\cdot 1$ [identity]

$= AB(C+\bar{C}) + AC(B+\bar{B}) + BC(A+\bar{A})$ $\because \boxed{a+a'=1}$

$= ABC + AB\bar{C} + ABC + A\bar{B}C + ABC + \bar{A}BC$

$= ABC + AB\bar{C} + A\bar{B}C + \bar{A}BC$.

* Obtain canonical in POS form - $\boxed{a\cdot a'=0}$

* $F(A,B,C) = (A+B)\cdot(A+\bar{C})$

$= (A+B+0)\cdot(A+\bar{C}+0)$ [identity]

$= (A+B+C\cdot\bar{C})\cdot(A+\bar{C}+B\cdot\bar{B})$ [distribu]

$= (A+B+C)(A+B+\bar{C})\cdot(A+\bar{B}+\bar{C})\cdot A+B+\bar{C}$

$= (A+B+C)(A+B+\bar{C})\cdot(A+\bar{B}+\bar{C})$

K-MAP [Karnaugh map] -

* 2 variable - $2^n =$ $n=2$

$= 4$.

A \ B	$\bar{B}$	B
$\bar{A}$	0	1
A	2	3

$\boxed{F(A,B) = \bar{B}}$

* 3 variable - $2^n =$ $n=3$

$= 8$

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0	1	3	2
A	4	5	7	6

* 4 variable -

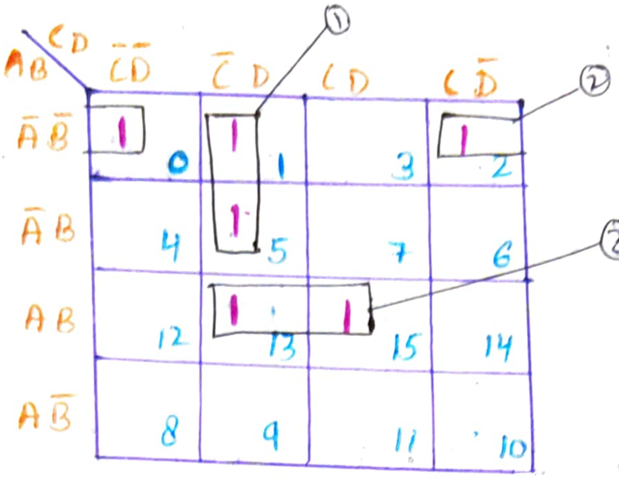
AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	3	2
$\bar{A}B$	4	5	7	6
AB	12	13	15	14
$A\bar{B}$	8	9	11	10

Ques minimise following Boolean function using K-map?

* $F(A,B,C,D) = \Sigma m(0,1,2,5,13,15)$

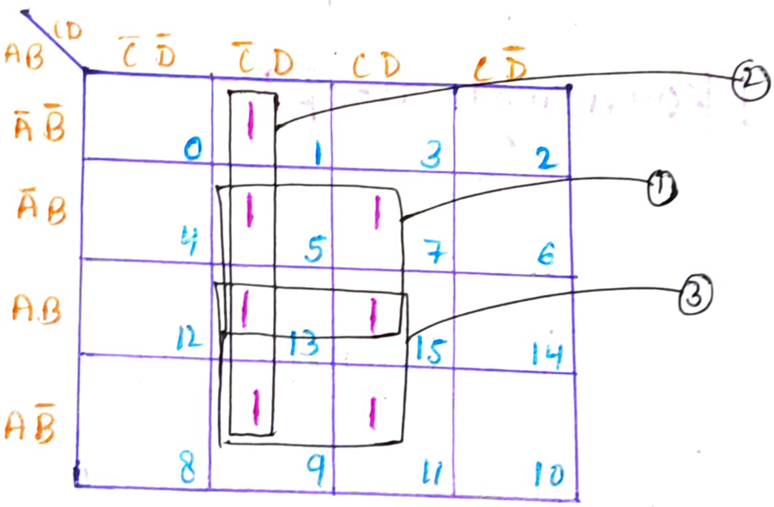
$\Rightarrow$

[SOP - DNF
POS - CNF]



$\Rightarrow \bar{A}\bar{B}\bar{D} + \bar{A}\bar{C}D + ABD.$

* $F(A,B,C,D) = \Sigma m(1,5,7,9,11,13,15)$



$\Rightarrow \bar{C}D + BD + AD.$

Home work

Ques $Y = \Sigma m(1,3,5,9,11,13)$

Ques $Y = \Sigma m(1,3,4,5,7,9,11,13,15)$

Ques $Y = \Sigma m(1,2,9,10,11,14,15)$

Ques $Y = \Sigma m(0,2,5,6,7,8,10,13,15)$

Answers

solution ① $Y = \Sigma m(1,3,5,9,11,13)$

$AB \backslash CD$	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1 1	1 3	2
$\bar{A}B$	4	1 5	7	6
AB	12	1 13	15	14
$A\bar{B}$	8	1 9	1 11	10

$\Rightarrow \bar{C}D + \bar{B}D$

<u>solution</u> ②

$AB \backslash CD$	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1 1	1 3	2
$\bar{A}B$	1 4	1 5	1 7	6
AB	12	1 13	1 15	14
$A\bar{B}$	8	1 9	1 11	10

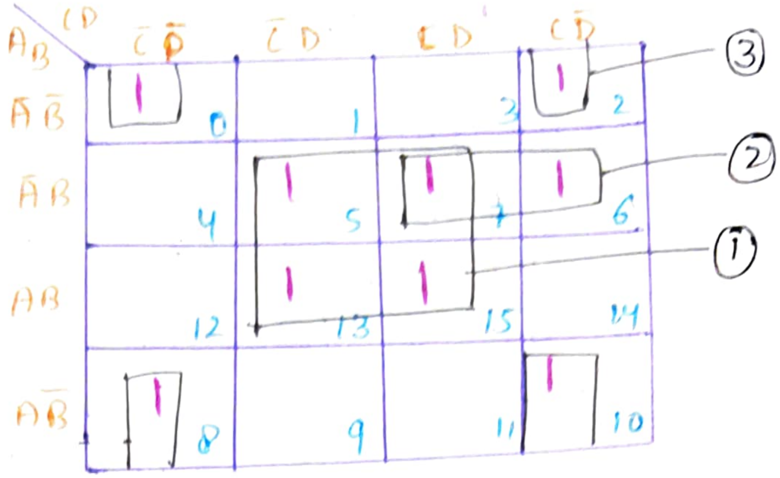
$\Rightarrow D + \bar{A}B\bar{C}$

<u>solution</u> ③

$AB \backslash CD$	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1 1	3	1 2
$\bar{A}B$	4	5	7	6
AB	12	13	1 15	1 14
$A\bar{B}$	8	1 9	1 11	1 10

$\Rightarrow AC + \bar{B}\bar{C}D + \bar{B}C\bar{D}$

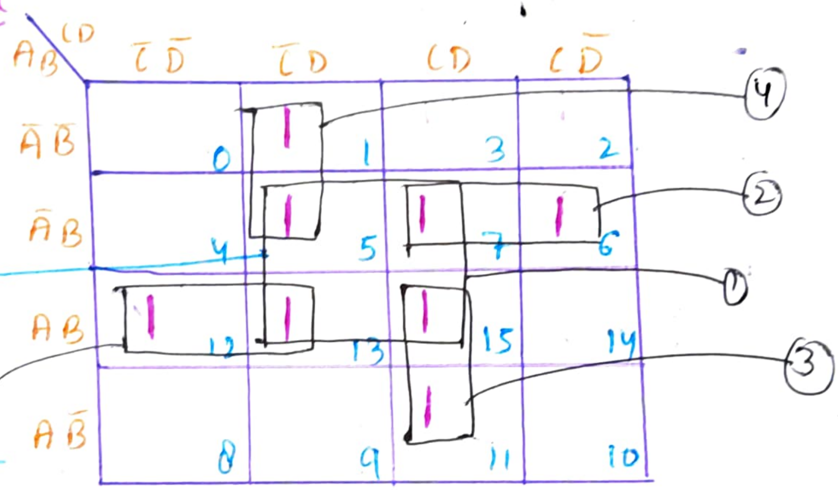
Solution (4)



$\Rightarrow BD + \bar{A}BC + \bar{B}\bar{D}$

* $Y = \Sigma m(1, 5, 6, 7, 11, 12, 13, 15)$

exceptional case



This group will be remove (5)

$\Rightarrow AB\bar{C} + \bar{A}\bar{C}D + \bar{A}BC + ACD$

* **Redandant group -**

If all the ones in a group are already involve with some another group. then that group is called Redandant grp. The Redandant group has to be eliminated because it increase no. of logic gates required.

* **Don't care condition -**

We involve element also can not be use. (X)

* $Y = \sum m(1, 3, 7, 11, 15) + d(0, 2, 5)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	X (0)	1 (1)	1 (3)	X (2)
$\bar{A}B$	(4)	X (5)	1 (7)	(6)
AB	(12)	(13)	1 (15)	(14)
$A\bar{B}$	(8)	(9)	1 (11)	(10)

$\Rightarrow \bar{A}D + CD.$

* $Y = \sum m(0, 1, 5, 9, 13, 14, 15) + d(3, 4, 7, 10, 11)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1 (0)	1 (1)	X (3)	(2)
$\bar{A}B$	X (4)	1 (5)	X (7)	(6)
AB	(12)	1 (13)	1 (15)	1 (14)
$A\bar{B}$	(8)	1 (9)	X (11)	X (10)

$\Rightarrow D + AC + \bar{A}\bar{C}$

* $Y = \pi M(1, 3, 7, 11, 15) + d(0, 2, 5)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	X (0)	0 (1)	0 (3)	X (2)
$\bar{A}B$	(4)	X (5)	0 (7)	(6)
AB	(12)	(13)	0 (15)	(14)
$A\bar{B}$	(8)	(9)	0 (11)	(10)

$\Rightarrow (A+\bar{D}) \cdot (\bar{C}+\bar{D}).$