Subject Code: KAS103

PAPER ID-411938

Roll No:

BTECH
(SEM I) THEORY EXAMINATION 2021-22
MATHEMATICS-I

Time: 3 Hours ***Total Marks: 100***

Notes:
- *Attempt all Sections and Assume any missing data.*
- *Appropriate marks are allotted to each question, answer accordingly.*

SECTION-A	Attempt **All** of the following Questions in brief Marks**(10X2=20)**	CO
Q1(a)	Find the eigen value of A^3 where A= $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.	1
Q1(b)	Show that the system of vectors $X_1 = (1, -1, 1), X_2 = (2, 1, 1), and$ $X_3 = (3, 0, 2)$ are linearly dependent or linearly independent.	1
Q1(c)	If $y = A\,sin\,nx + B\cos nx$, prove that $y_2 + n^2 y = 0$.	2
Q1(d)	Find the asymptotes parallel to y-axis of the curve $\frac{a^2}{x} + \frac{b^2}{y} = 1$.	2
Q1(e)	If $x = rcos\theta, y = rsin\theta, find \ \frac{\partial(r,\theta)}{\partial(x,y)}$.	3
Q1(f)	An error of 2% is made in measuring length and breadth then find the percentage error in the area of the rectangle.	3
Q1(g)	Evaluate $\int_0^1 \int_0^{x^2} e^{\frac{y}{x}}\, dy\, dx$.	4
Q1(h)	Find the volume common to the cylinders $x^2 + y^2 = a^2\ and\ x^2 + z^2 = a^2$.	4
Q1(i)	Find p such that $\vec{A} = (px + 4y^2 z)i + (x^3 sinz - 3y)j - (e^x + 4\cos x^2 y)k$ is solenoidal.	5
Q1(j)	State Green's theorem for a plane region.	5

SECTION-B	Attempt **ANY THREE** of the following Questions Marks**(3X10=30)**	CO
Q2(a)	Find the eigen values and corresponding eigen vectors of $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.	1
Q2(b)	Verify Rolle's theorem for the function $f(x) = \sqrt{4 - x^2}\ \ in\ [-2, 2]$.	2
Q2(c)	Find the first six terms of the expansions of the function $e^x \log(1 + y)$ in a Taylor series in the neighborhood of the point $(0, 0)$.	3
Q2(d)	Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} xydydx$ and hence evaluate the same.	4
Q2(e)	If a vector field is given by $\vec{F} = (x^2 - y^2 + x)i - (2xy + y)j$ Is this field irrotational? If so, find its scalar potential.	5

SECTION-C	Attempt **ANY ONE** following Question Marks **(1X10=10)**	CO
Q3(a)	Find for what values of $\lambda\ and\ \mu$ the system of linear inequation: $x + y + z = 6$, $x + 2y + 5z = 10$, $2x + 3y + \lambda z = \mu$ has(i) a unique solution, (ii) no solution, (iii) infinite solution. Also find the solution for $\lambda = 2\ and\ \mu = 8$.	1
Q3(b)	Find the rank of matrix reducing it to normal form $A = \begin{bmatrix} 1 & 3 & 4 & 2 \\ 2 & -1 & 3 & 2 \\ 3 & -5 & 2 & 2 \\ 6 & -3 & 8 & 6 \end{bmatrix}$	1

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SECTION-C	Attempt **ANY ONE** following Question	Marks **(1X10=10)**	**CO**
Q4(a)	If $y = (sin^{-1} x)^2$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$ and calculate $y_n(0)$.		2
Q4(b)	Verify mean value theorem for the function $f(x) = x(x-1)(x-2)\ in\ \left[0, \frac{1}{2}\right]$.		2

SECTION-C	Attempt **ANY ONE** following Question	Marks **(1X10=10)**	**CO**
Q5(a)	A rectangular box which is open at the top having capacity 32c.c.Find the dimension of the box such that the least material is required for its constructions.		3
Q5(b)	If u, v and w are the roots of $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$, cubic in λ, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.		3

SECTION-C	Attempt **ANY ONE** following Question	Marks **(1X10=10)**	**CO**
Q6(a)	Find by double integration the area enclosed by the pair of curves $y = 2 - x \quad and \quad y^2 = 2(2-x)$.		4
Q6(b)	Find C.G. of the area in the positive quadrant of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.		4

SECTION-C	Attempt **ANY ONE** following Question	Marks **(1X10=10)**	**CO**
Q7(a)	Find the directional derivative of $f(x,y,z) = xyz$ at the point $P(1,-1,2)$ in the direction of the vector $(2i - 2j + 2k)$.		5
Q7(b)	Verify Stoke's Theorem for $\vec{F} = (y - z + 2)i + (yz + 4)j - (xz)k$ over the surface of cube $x = 0, y = 0, z = 0, x = 2, y = 2, z = 2$ above the XOY plane.		5

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BTECH
(SEM I) THEORY EXAMINATION 2021-22
ENGINEERING MATHEMATICS-I

Time: 3 Hours ***Total Marks: 100***

Notes:

- *Attempt all Sections and Assume any missing data.*
- *Appropriate marks are allotted to each question, answer accordingly.*

SECTION-A	Attempt **All** of the following Questions in brief — Marks **(10X2=20)**	**CO**
Q1(a)	If the matrix A= $\begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$, then find the eigen value of $A^3+5A+8I$.	1
Q1(b)	Reduce the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ into normal form and find its rank.	1
Q1(c)	Find the envelope of the family of straight line $y=mx+\frac{a}{m}$, where m is a parameter.	2
Q1(d)	Can mean value theorem be applied to $f(x)=\tan x$ in $[0, \pi]$.	2
Q1(e)	State Euler's Theorem on homogeneous function.	3
Q1(f)	Find the critical points of the function $f(x,y)=x^3+y^3-3axy$.	3
Q1(g)	Find the area bounded by curve $y^2=x \ and \ x^2=y$.	4
Q1(h)	Find the value of $\int_0^1 \int_0^x \int_0^{x+y} dx\, dy\, dz$.	4
Q1(i)	Find a unit normal vector to the surface $z^2=x^2+y^2$ at the point $(1,0,-1)$.	5
Q1(j)	State Stoke's Theorem.	5

SECTION-B	Attempt **ANY THREE**of the following Questions — Marks **(3X10=30)**	**CO**
Q2(a)	Find the characteristic equation of the matrix A= $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$, compute A^{-1} and prove that $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I=\begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$.	1
Q2(b)	State Rolle's theorem and verify Rolle's theorem for the function $f(x)=\frac{sinx}{e^x} \ in \ [0,\pi]$.	2
Q2(c)	If u, v and w are the roots of $(\lambda-x)^3+(\lambda-y)^3+(\lambda-z)^3=0$, cubic in λ, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.	3
Q2(d)	Find the volume bounded by the cylinder $x^2+y^2=4$ and the plane $y+z=4$ and z = 0.	4
Q2(e)	Apply Green's theorem to evaluate $\int_C [(2x^2-y^2)dx+(x^2+y^2)dy]$,where C is th boundary of the area enclosed by the x-axis and the upper half of the circle $x^2+y^2=a^2$.	5

SECTION-C	Attempt **ANY ONE** following Question — Marks **(1X10=10)**	**CO**
Q3(a)	Find the value of k for which the system of eqations $(3k-8)x+3y+3z=0$, $3x+(3k-8)y+3z=0$, $3x+3y+(3k-8)z=0$ has a non-trivial solution.	1
Q3(b)	Find the eigen values and eigen vectors of matrix A= $\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$.	1

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ENGINEERING MATHEMATICS-I

SECTION-C	Attempt **ANY ONE** following Question	Marks **(1X10=10)**	**CO**
Q4(a)	If $f(x) = \frac{x}{1+e^{\frac{1}{x}}}$; $x \neq 0$ and $f(0) = 0$, then show that the function is continuous but not differentiable at $x = 0$.		2
Q4(b)	If $y = \left(x + \sqrt{1+x^2}\right)^m$, find $y_n(0)$.		2

SECTION-C	Attempt **ANY ONE** following Question	Marks **(1X10=10)**	**CO**
Q5(a)	Expand x^y in powers of $(x-1)$ and $(y-1)$ up to the third-degree terms and hence evaluate $(1.1)^{1.02}$.		3
Q5(b)	A rectangular box which is open at the top having capacity 32c.c. Find the dimension of the box such that the least material is required for its constructions.		3

SECTION-C	Attempt **ANY ONE** following Question	Marks **(1X10=10)**	**CO**
Q6(a)	Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$ and hence evaluate the same.		4
Q6(b)	Find the position of the C.G. of a semicircular lamina of radius, a if its density varies as the square of the distance from the diameter.		4

SECTION-C	Attempt **ANY ONE** following Question	Marks **(1X10=10)**	**CO**
Q7(a)	Find the directional derivative of $\nabla(\nabla f)$ at the point $(1, -2, 1)$ in the direction of the normal to the surface $xy^2z = 3x + z^2$ where $f = 2x^3y^2z^4$.		5
Q7(b)	Find the constants a, b, c so that $\vec{F} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is irrotational and hence find function $\emptyset$ such that $\vec{F} = \nabla\emptyset$.		5