AKTU

B. Tech I-Year

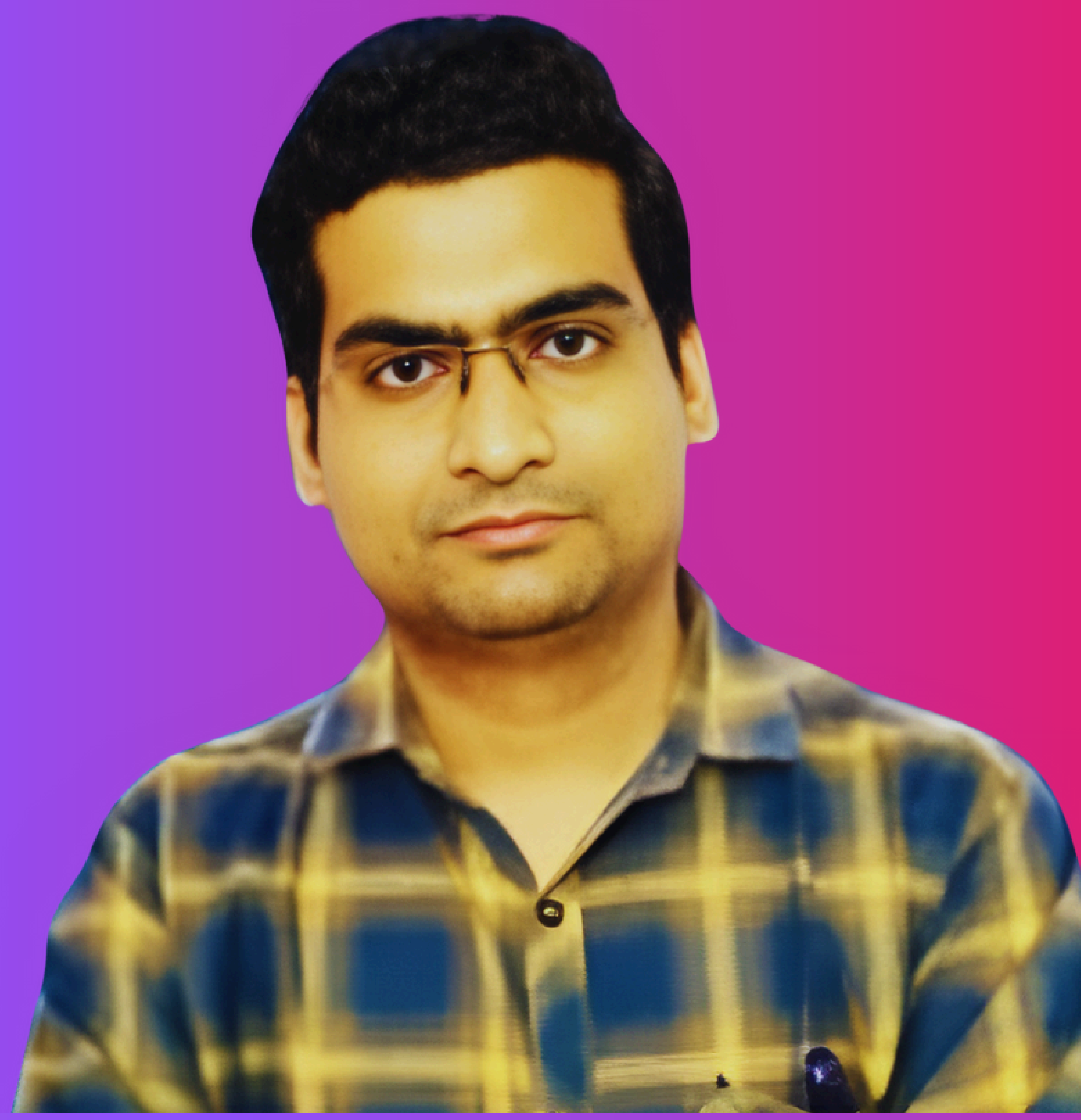
# Gateway Classes

Semester -I & II

Common to All Branches

## BEE101/201: FUNDAMENTALS OF ELECTRICAL ENGG.

## UNIT-2 ONE SHOT : Steady State Analysis

## Gateway Series for Engineering

- Topic Wise Entire Syllabus
- Long - Short Questions Covered
- AKTU PYQs Covered
- DPP
- Result Oriented Content

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BEE101 / BEE201: FUNDAMENTALS OF ELECTRICAL ENGINEERING

# Unit-2

# Introduction to Steady State Analysis

## Syallbus

Representation of Sinusoidal waveforms – Average and effective values, Form and peak factors. Analysis of single phase AC Circuits consisting R-L-C combination (Series and Parallel) Apparent, active & reactive power, Power factor. Concept of Resonance in series & parallel circuits, bandwidth and quality factor. Three phase balanced circuits, voltage and current relations in star and delta connections.


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Electrical Engg
One Shot
UNIT - II

# Syllabus BEE-201/101

- **Unit-2: (Steady State Analysis of Single Phase AC Circuits)**

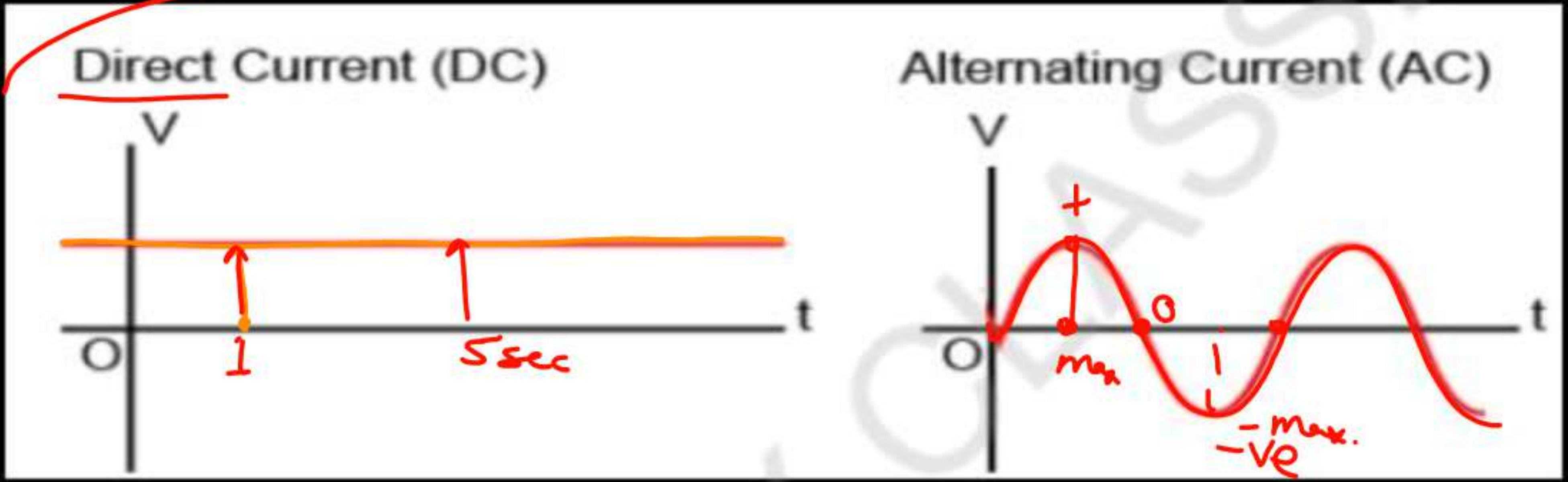
- Representation of Sinusoidal waveforms – Average and effective values, Form and peak factors.

- Analysis of single phase AC Circuits consisting R-L-C combination (Series and Parallel) Apparent, active & reactive power, Power factor. Concept of Resonance in series & parallel circuits, bandwidth and quality factor.

- Three phase balanced circuits, voltage and current relations in star and delta connections.

# Types of Electrical Supply

**Direct Current (D.C.) Supply:-** A DC power supply is one that provides a constant magnitude and direction with respect to time.




**Alternating Current (A.C.) Supply:-** Alternating supply is one whose magnitude and direction changes periodically with time.

*Advantage of AC over DC*

1. *Varying voltage level of AC by using transformer.*
2. *More Economical then DC*
3. *Generation of high voltage AC is much easier & Cheaper than DC*
4. *AC can be easily converted to DC.*
5. *AC machine are more simple in construction and requires less maintenance*

# Why We should use Pure Sine Wave ?

2 marks

- Easy to write equation for pure sine wave.
- Any other wave form can ne obtained by a series of sine or cosine wave form.
- Sine or cosine wave can pass through linear circuit without any distortion.
- Analysis of electrical networks with sine inputs is very easy.

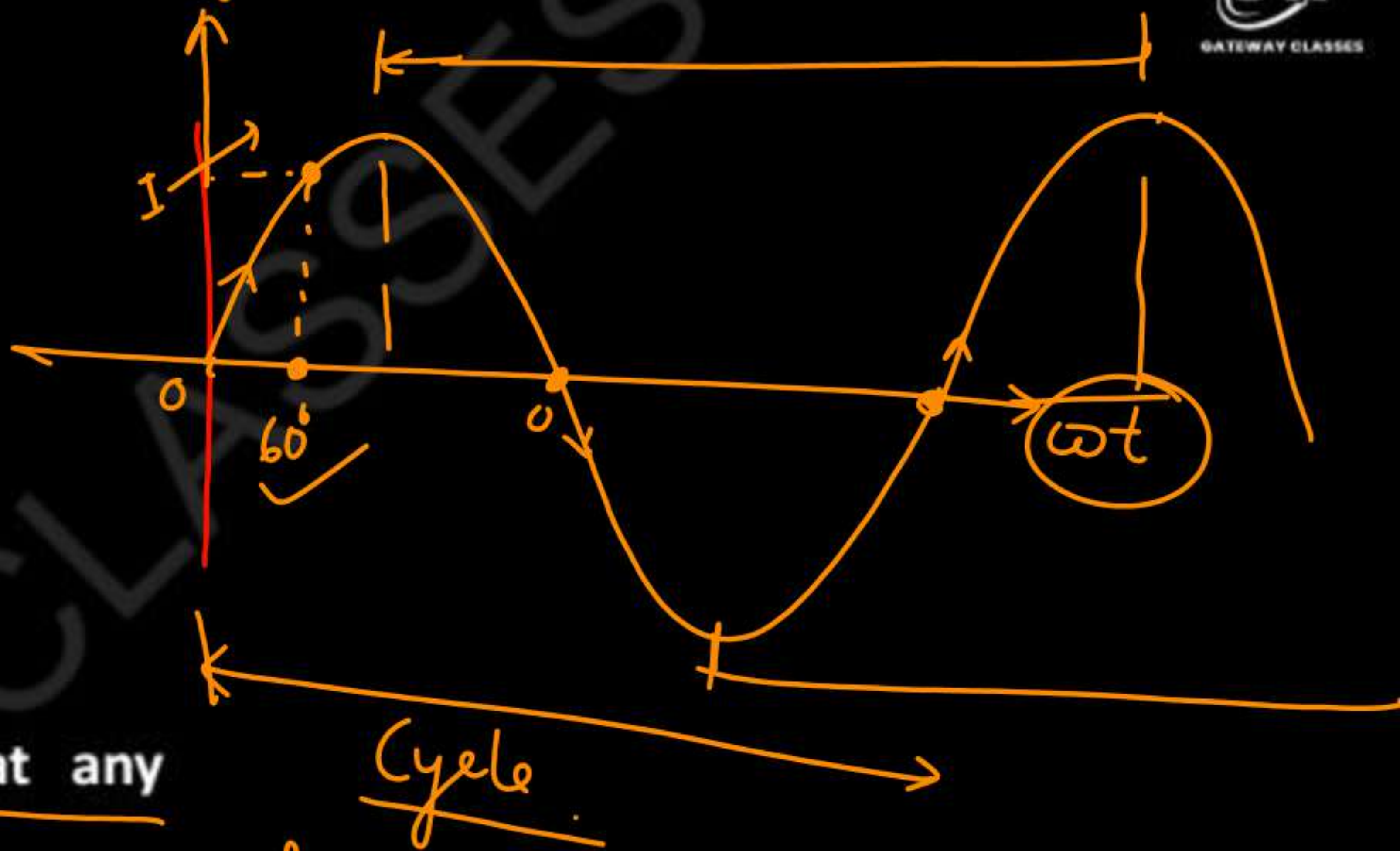
## Equation of Alternating Quantity (AKTU-2002-03):-

For a pure sine wave

$I = I_m \sin \omega t$ → instaneous Equation

$V = V_m \sin \omega t$

Max. Value    angular freq.




*Important Terms related to Alternating Quantity -*

**Instantaneous Value:-** Value of alternating quantity at any instant of time.

**Waveform Cycle:-** Each repetition of positive and negative instantaneous values of the AC Quantity.

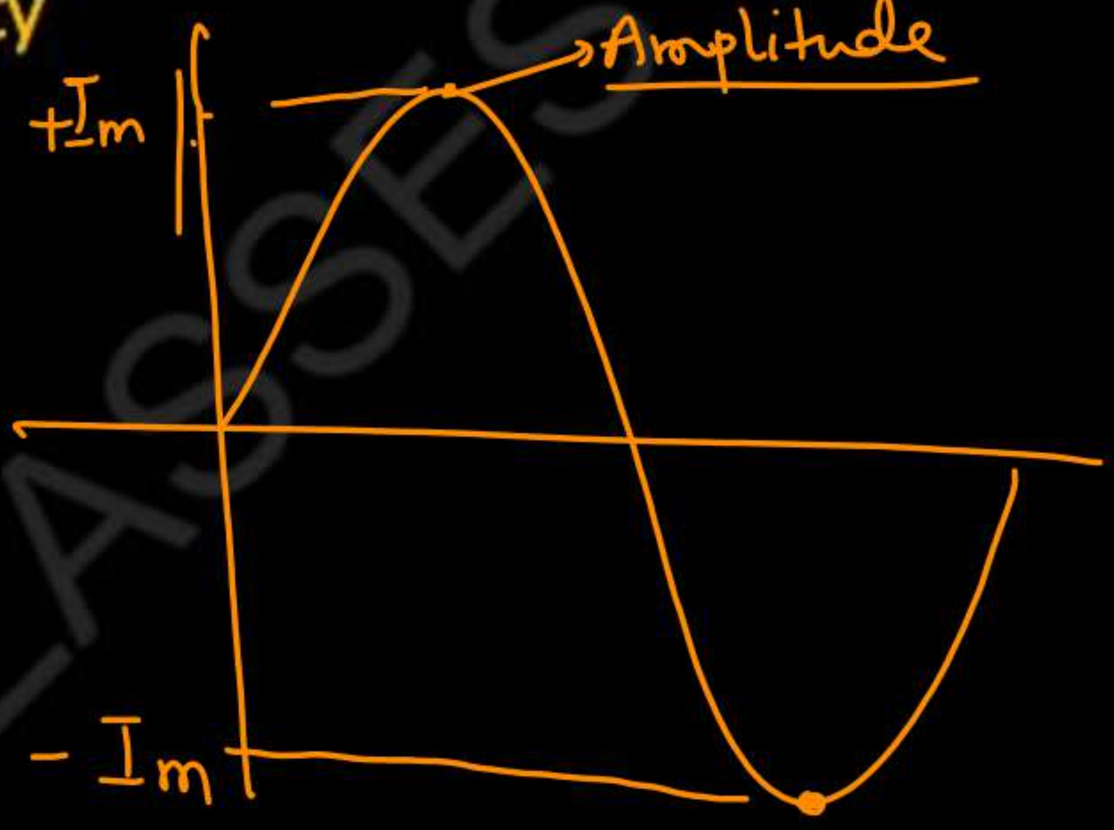
frequny = No. of Cycle/sec

**Time period:-** Time taken to complete one cycle.

**Frequency:-** Reciprocal of Time period or no of cycle in a second.

# Alternating Quantity

- **Amplitude:-** Maximum value attained by AC quantity during positive and negative cycle.




$P.P = 2I_m$

Amplite $I_m = \frac{P-P}{2}$

- **Angular Frequency:-** Frequency expressed in radian /sec.

$\omega = 2\pi f$

- **Peak to Peak Value:-** Value of AC quantity from positive maximum value to negative maximum Value.

  Amplitude = Peak to Peak Value /2

# Examples of Alternating Quantity

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**Ex-1** A sinusoidally varying alternating current of frequency 60 Hz has a maximum value of 15 amp.

(i) Write equation for instantaneous value, Find the value of current after 1/200 second

(ii) Find the time to reach 10 amperes for the first time? (AKTU-2002-03)

Soln $f = 60\,Hz$, $I_m = 15$ amp

$\omega = 2\pi f = 2\pi \times 60 = 120\pi$

(i) $i = I_m \sin \omega t$

$$\boxed{i = 15 \sin(120\pi) t}$$

Put $t = 1/200$ S

$i = 15 \sin 120\pi \cdot \frac{1}{200}$

$= 14.26$ amp.

(ii) $i = 10$ amp, $t = ?$

$10 = 15 \sin 120\pi \cdot t$

$\left(\frac{10}{15}\right) = \sin 120\pi t$

$120\pi t = \sin^{-1}\left(\frac{10}{15}\right)$

$t = \frac{\sin^{-1}\left(\frac{10}{15}\right)}{120\pi} = 1.93\,ms$ Ans

# Examples of Alternating Quantity

2 marks

**Ex-2 The equation of an alternating current is i= 42.42 sin 628t Determine Maximum Value, Frequency, rms value, average value, form factor (AKTU-2017-18, similar question in 2002-03,2003-4,2005-06,2007-08)**

**Sol.-**

$i = 42.42 \sin 628 t$

$i = I_m \sin \omega t$

$I_m = 42.42$ amp

$\omega = 628 = 2\pi f$

$f = \frac{628}{3.14 \times 2} = 100 Hz$

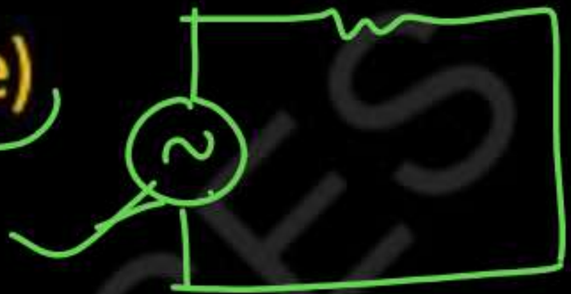
$i_{rms} = \frac{I_m}{\sqrt{2}} = \frac{42.42}{\sqrt{2}} =$

Pending

H.W

$I_{av} = \frac{2 I_m}{\pi} =$

# Average Value (Mean Value)

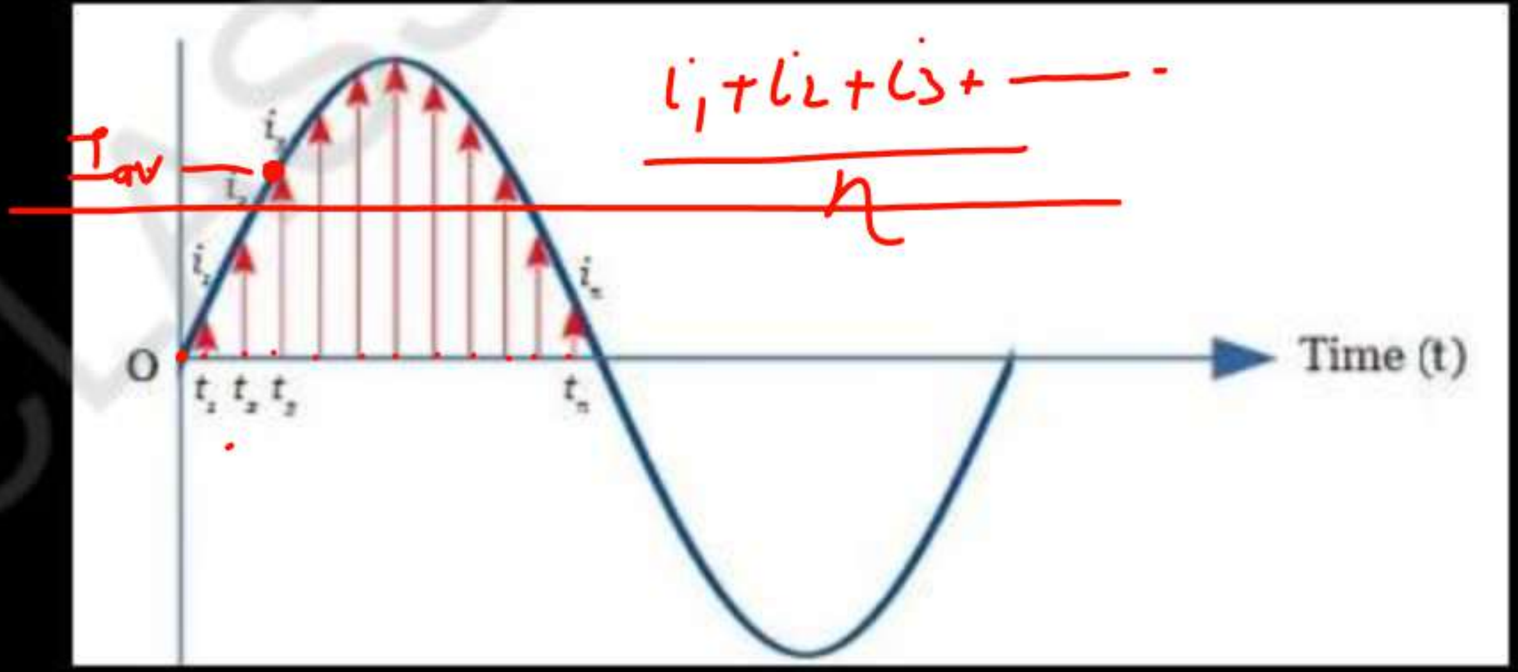


q charge

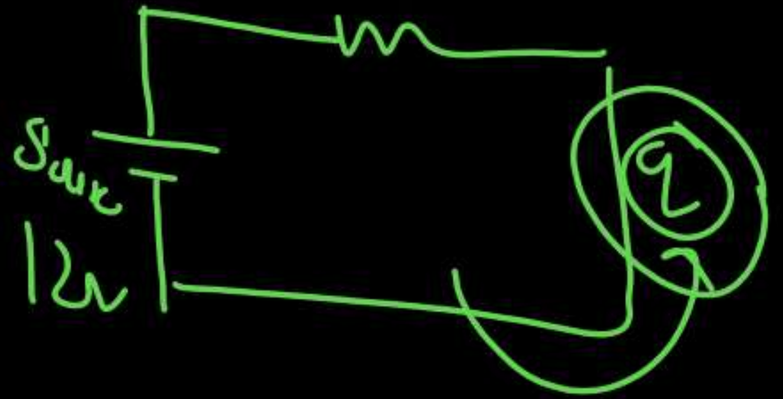
The average of all the instantaneous values of an alternating voltage and currents over one complete cycle is called **Average Value.**

**Or**

The Average Value (also known as Mean Value) of an Alternating Current (AC) is expressed by that Direct Current (DC) which transfers across any circuit the same amount of charge as is transferred by that Alternating Current (AC) during the same time




**Average Value by Graphical Method:-**




$$Average\ value = \frac{Sum\ of\ all\ instantaneous\ values\ over\ one\ cycle}{Number\ of\ instants}$$

$$I_{AV} = \frac{i_1 + i_2 + i_3 + \text{----} + i_n}{n}$$

# Average Value (Mean Value)

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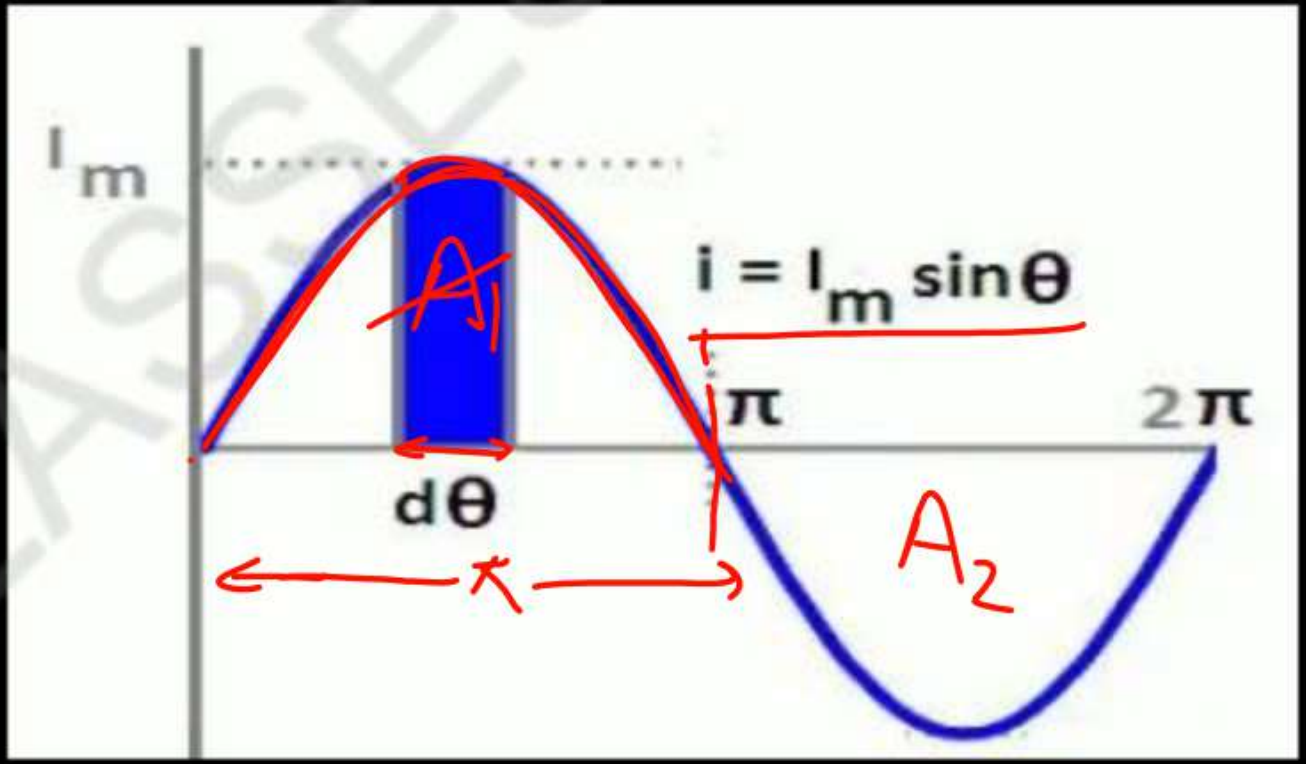
## Average Value by Analytical Method:-

$$I_{av} = \frac{\textit{Arrea under the curve for half cycle}}{\textit{Length of base over half cycle}}$$




$i = I_m \sin\theta$

$$I_{av} = \frac{\int_0^{\pi} i \, d\theta}{\pi} = \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta \, d\theta$$

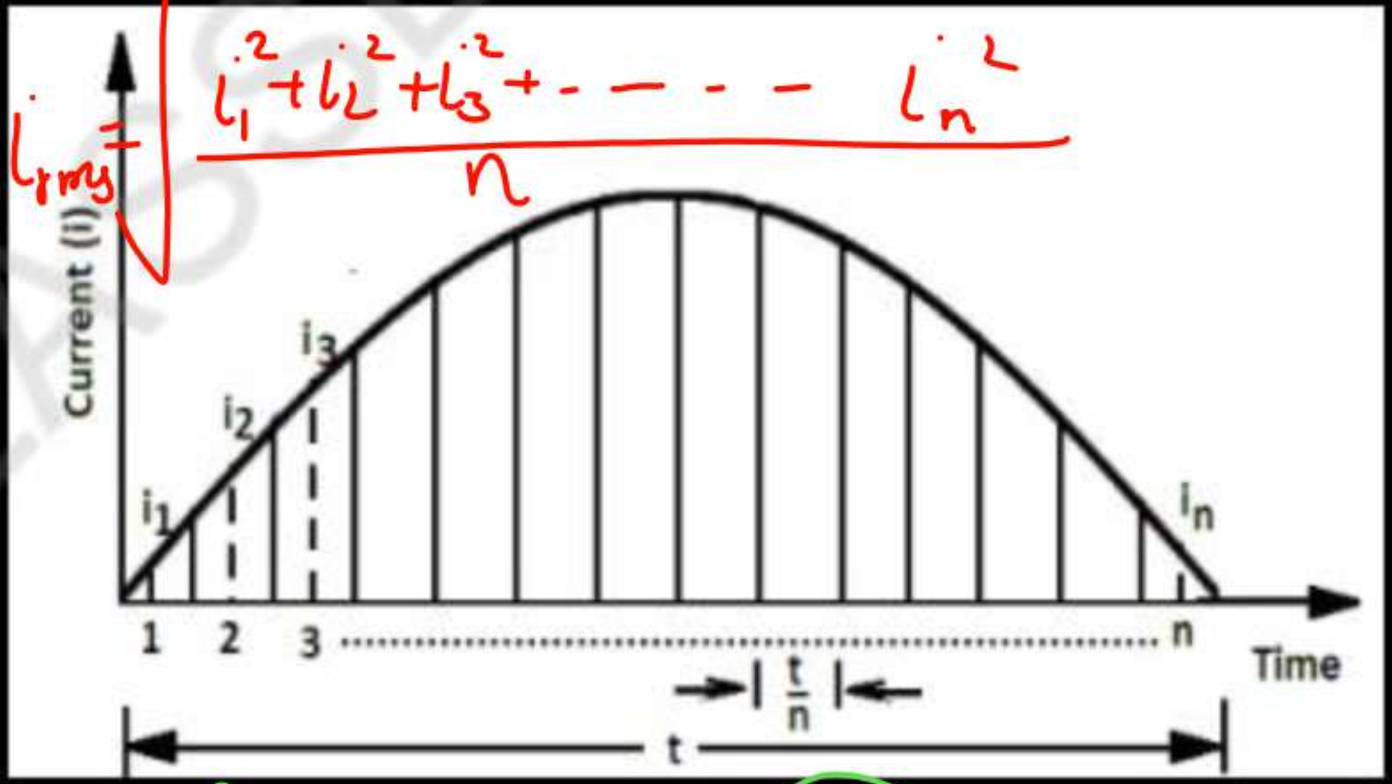
$$= \frac{I_m}{\pi}\int_0^{\pi} \sin\theta \, d\theta = \frac{I_m}{\pi}\left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{I_m}{\pi}\left[-\underbrace{\cos\pi}_{-1} + \underbrace{\cos 0}_{1}\right] = \boxed{\frac{2I_m}{\pi} = I_{av}}$$

# Root Mean Square (R.M.S.) Value/ Effective Value

**Definition:** The RMS value of AC current is equal to that amount of DC current which produces the same heating effect flowing through the same resistance for the same time.

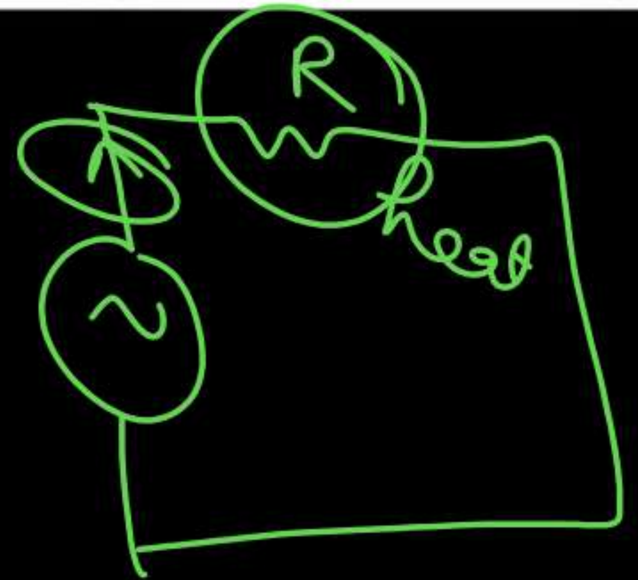
$$i_{rms} = \sqrt{\frac{i_1^2 + i_2^2 + i_3^2 + \dots\ i_n^2}{n}}$$




For example, if 3A (RMS) AC current is flowing through a circuit, And it will produce the same amount of heat (energy) as will be produced by 3A DC current.

$$RMS\ value\ of\ alternating\ current\ (I) = \sqrt{mean\ value\ of\ i^2}$$

$$= \sqrt{\frac{i_1^2 + i_2^2 + i_3^2 + \ldots + i_n^2}{n}}$$

# Root Mean Square (R.M.S.) Value/ Effective Value

## Analytical Method:-

$$I_{RMS} = \sqrt{\frac{\text{Area of half cycle squared wave}}{\text{base length of half cycle}}}$$

$$i = I_m \sin\theta$$

$$i^2 = I_m^2 \sin^2\theta$$

$$i_{rms} = \sqrt{\frac{\int_0^{\pi} I_m^2 \sin^2\theta \, d\theta}{\pi}}$$

$$i_{rms}^2 = \frac{I_m^2}{\pi}\int_0^{\pi} \sin^2\theta \, d\theta = \frac{I_m^2}{\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2}\, d\theta$$

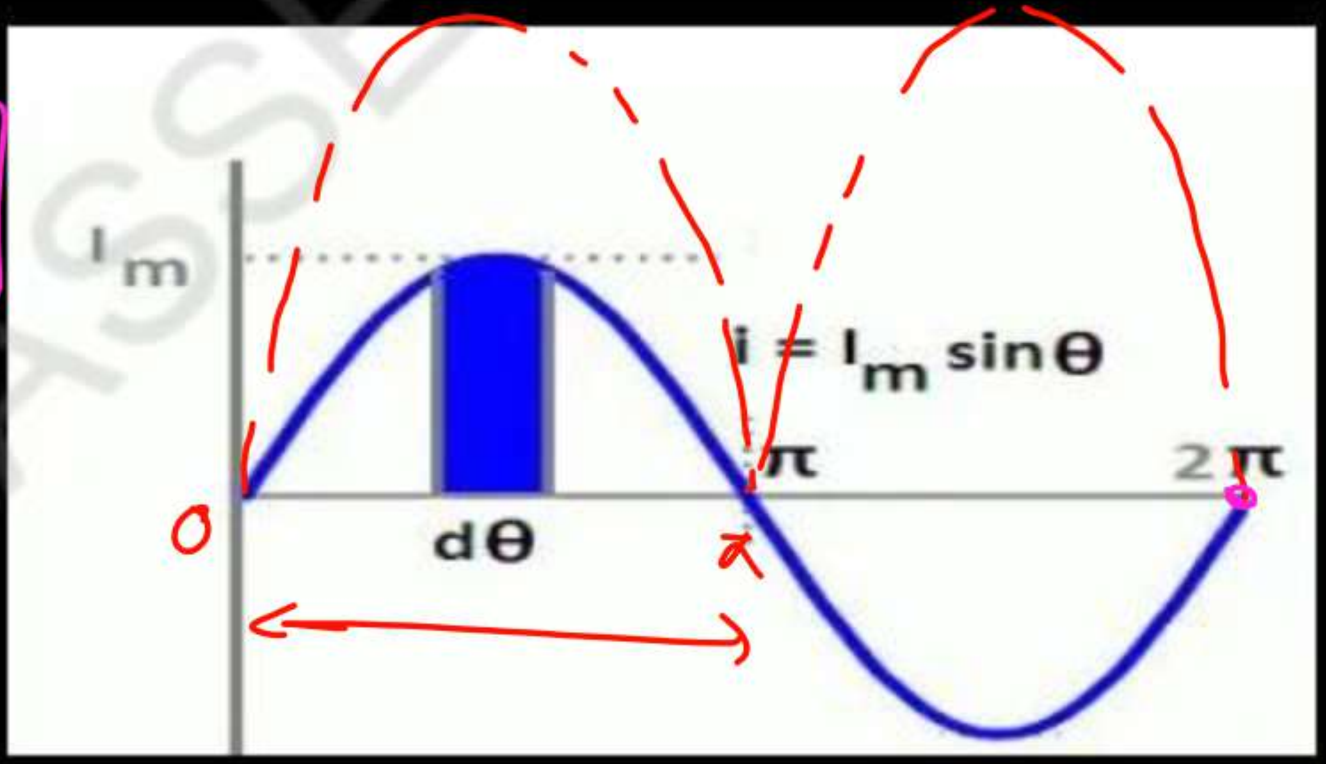
$$= \frac{I_m^2}{2\pi}\int_0^{\pi} (1-\cos 2\theta)\, d\theta$$

$$i_{rms}^2 = \frac{I_m^2}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{I_m^2}{2\pi}\left[\pi - \cancel{\frac{\sin 2\pi}{2}} - \cancel{0} + \cancel{\frac{\sin 0}{2}}\right]$$

$$I_{rms}^2 = \frac{I_m^2}{2\cancel{\pi}} \cdot \cancel{\pi} = \frac{I_m^2}{2}$$

$$\boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$$




$$I_{rms} = 0.707\, I_m$$
$$= 70.7\%\, I_m$$

- **Form Factor:-**The ratio of the root mean square value to the average value of an alternating quantity (current or voltage) is called **Form Factor.**

$$\text{Form Factor} = \frac{I_{r.m.s}}{I_{av}} \text{ or } \frac{E_{r.m.s}}{E_{av}}$$

For a sine wave,

$$\text{form factor} = \frac{\cancel{I_m}/\sqrt{2}}{2\cancel{I_m}/\pi} = 1.11$$

**Peak Factor/ Crest Factor :- Peak Factor** is defined as the ratio of maximum value to the R.M.S value of an alternating quantity.

**$K_p$ = Peak value / RMS value**

For a sine Wave

$$\text{Peak factor} = \frac{\cancel{I_m}}{\cancel{I_m}/\sqrt{2}} = \sqrt{2} \text{ Ans}$$

# Numerical

**Ex-3 Determine Form Factor and Peak Factor for Full wave rectified Signal (Vm = 10volt) ? (AKTU- 2002-03)**

$v = V_m \sin\theta$

$v = 10 \sin\theta \qquad 0 \longleftrightarrow \pi$

$V_{av} = \frac{1}{\pi}\int_0^{\pi} 10 \sin\theta \, d\theta$

$= \frac{10}{\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{10}{\pi}\left[-\cos\pi + \cos 0\right]$

$= \frac{10}{\pi}\cdot 2 = \boxed{\frac{20}{\pi} = V_{av}}$

$V_{rms}^2 = \frac{1}{\pi}\int_0^{\pi} 10^2 \sin^2\theta \cdot d\theta$

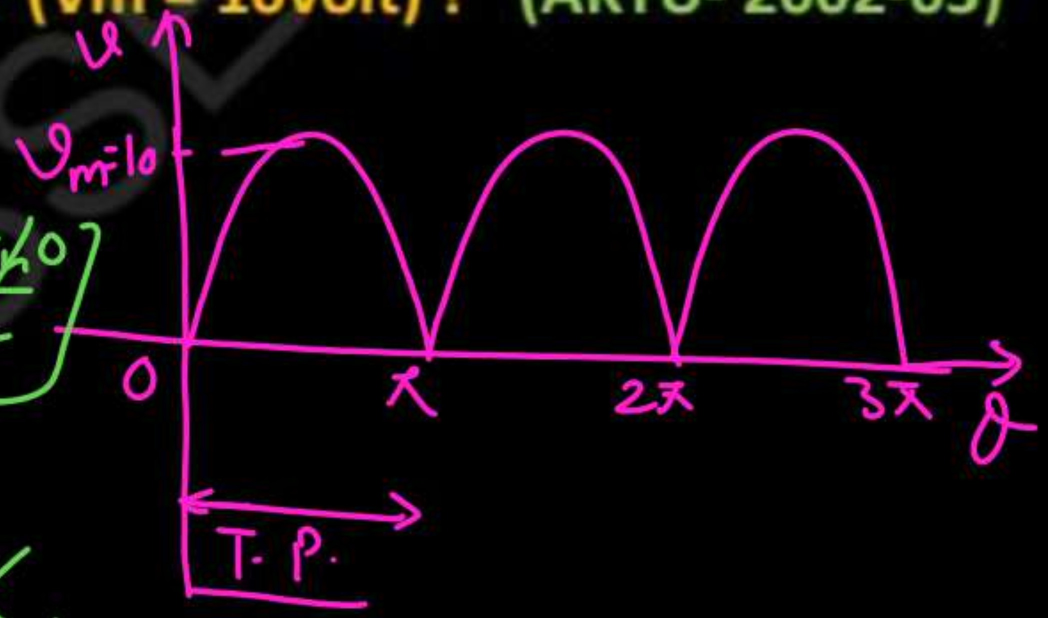
$= \frac{100}{2\pi}\int_0^{\pi}(1-\cos 2\theta)\, d\theta = \frac{100}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$

$V_{rms}^2 = \frac{50}{\pi}\left[\pi - \cancel{\frac{\sin 2\pi}{2}} - \cancel{0} + \cancel{\frac{\sin 0}{2}}\right]$

$V_{rms}^2 = \frac{50}{\cancel{\pi}} \times \cancel{\pi}$

$V_{rms}^2 = 50$

$\boxed{V_{rms} = \sqrt{50}}$




$F.F = \frac{V_{av}}{V_{rms}} = \frac{20/\pi}{\sqrt{50}} = 1.11$

$P.F = \frac{V_m}{V_{rms}}$

# Numerical (Try Yourself)

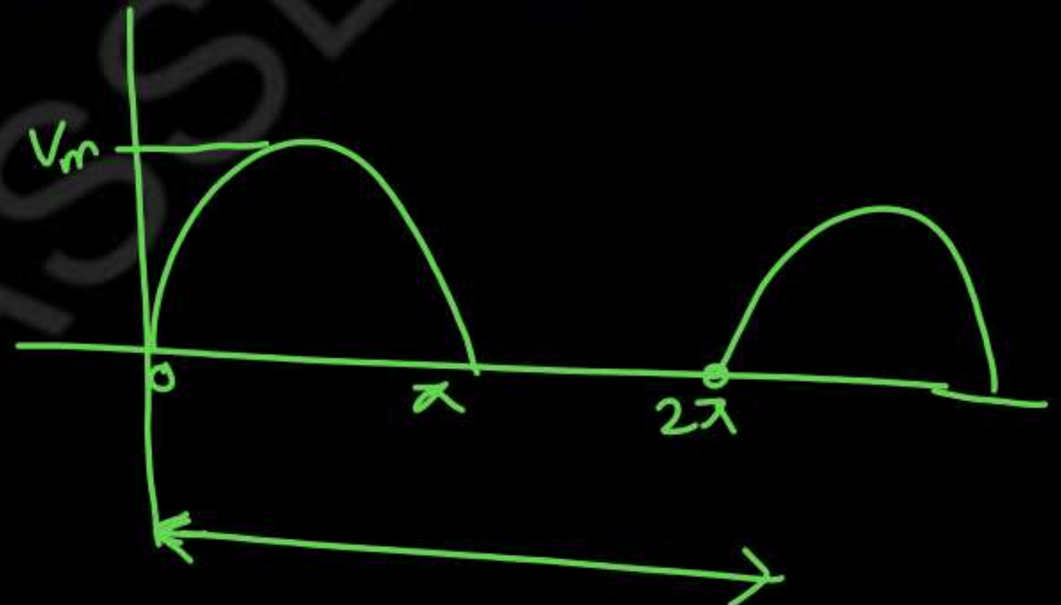
**Ex-4 Determine Form Factor and Peak Factor for Half wave rectified Signal ( Vm = 10volt) ? (AKTU- 2003-04, 2020-21)**

$v = V_m \sin\theta \qquad 0 - \pi$

$v = 0 \qquad \pi - 2\pi$

$$V_{av} = \int_0^{2\pi} i \, d\theta$$

$$= \frac{\left[\int_0^{\pi} V_m \sin\theta \, d\theta + \int_{\pi}^{2\pi} 0 \, d\theta\right]}{2\pi}$$

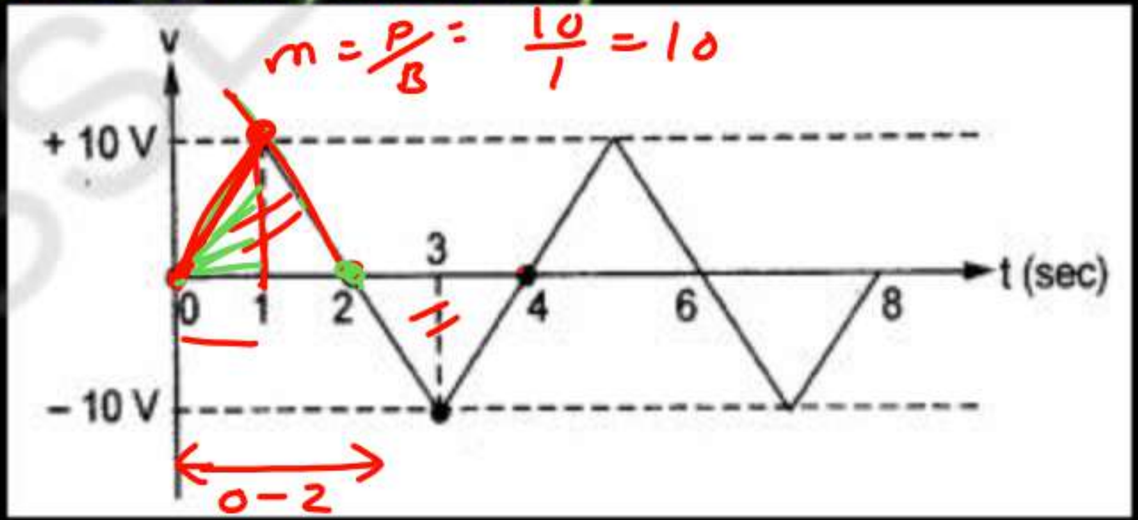



$f.f = 1.57$

$P.f = 2$

# Numerical

**Ex-5 Determine Form Factor and Peak Factor for the given Signal ?** (AKTU- 2004-05, 2017-18)




Sol^n:

$y = mx + c$

$v = 10t + 0$

$v = 10t \quad 1 > t > 0$

for (1-2 sec)

$v = -10t + c$

at $t = 2$, $v = 0$

$0 = -10 \times 2 + c$

$\boxed{c = 20}$

$\boxed{v = -10t + 20}$

$1 < t < 2$

$V_{av} = \frac{1}{2}\int_0^2 v \cdot dt$

$= \frac{1}{2}\left[\int_0^1 10t \cdot dt + \int_1^2 (-10t + 20)\,dt\right]$

$= \frac{1}{2}\left[\underbrace{\frac{1}{\cancel{2}} \times \cancel{10}^{5} \times 1}_{A\ \text{of}\ \Delta} + \frac{1}{\cancel{2}} \times \cancel{10}^{5} \times 1\right] = 5 \text{ volt}.$

$V_{rms}^2 = \frac{1}{2}\left[\int_0^1 (10t)^2 dt + \int_1^2 (-10t + 20)^2 dt\right]$

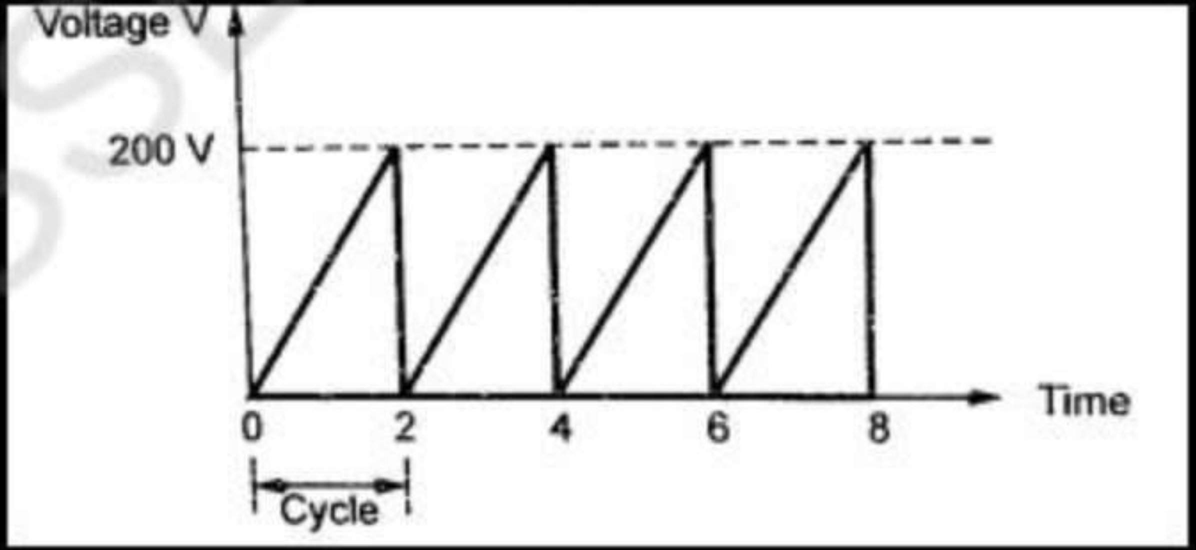
$V_{rms}^2 = \frac{1}{2}\left[100\left[\frac{t^3}{3}\right]_0^1 + \int_1^2 (100t^2 + 400 - 400t)\,dt\right]$

$V_{rms}^2 = \frac{1}{2}\left[100\left[\frac{1}{3}\right] + \left[100\frac{t^3}{3} + 400t - 400\frac{t^2}{2}\right]_1^2\right]$

$V_{rms} = \sqrt{\phantom{x}}$

$\boxed{V_{rms} = 5.7 \text{ volt}}$

**Ex-6 Determine Form Factor and Peak Factor for the given Signal ?** (AKTU- 2004-05, 2017-18)




$m$

0-2

$v = \dfrac{200}{2} t$

$v = 100t$ → $0 - 2$

# Concept of Phase and Phase Difference

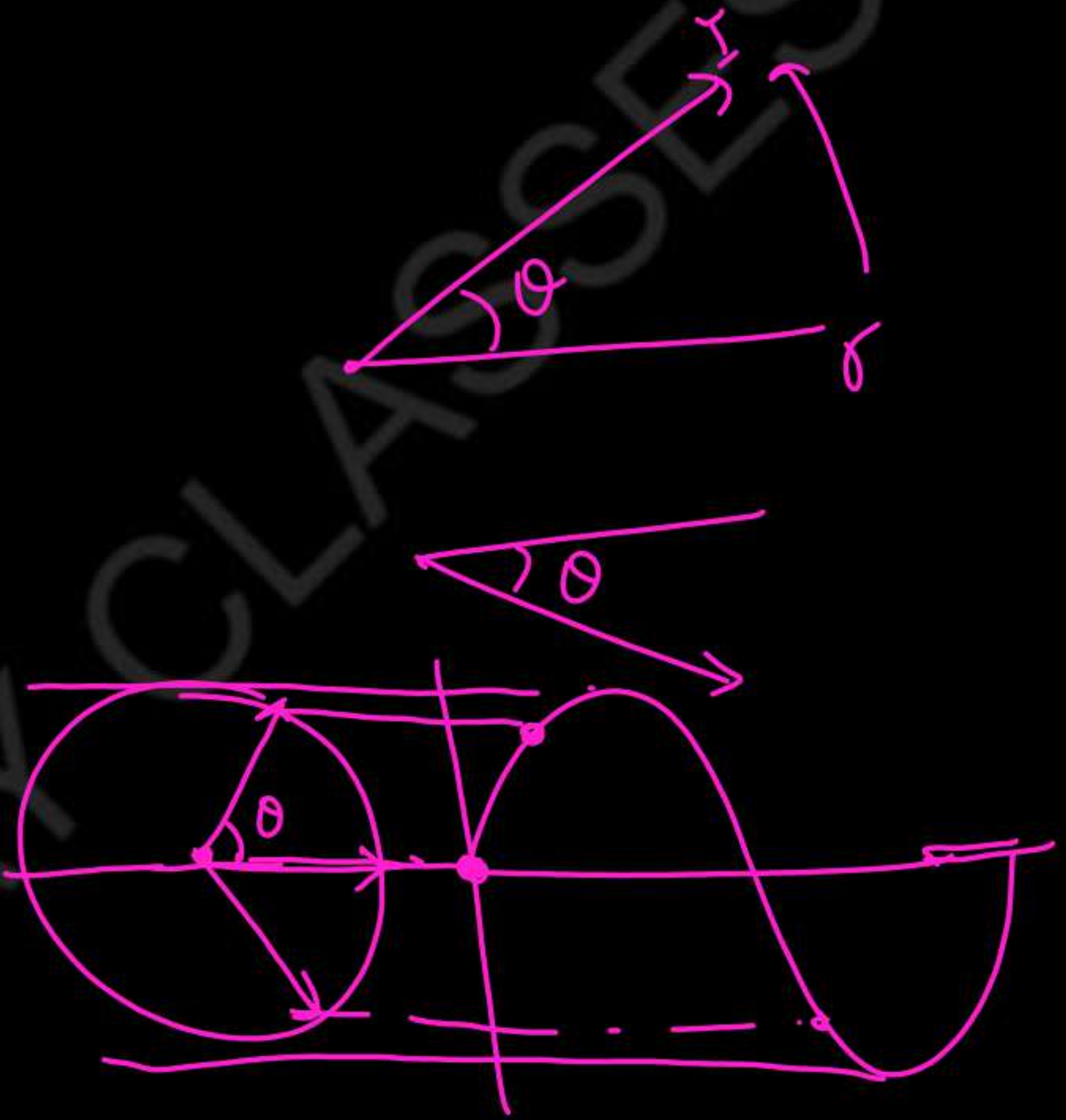
- **Phase :-** The Phase of an AC quantity is the angle travelled by phasor representing that alternating quantity up to the instant of consideration , measured from reference.

$$i = I_m \sin(\omega t + \phi)$$

- **when $\phi = 0$:** At t=0 AC quantity has Zero instantaneous Value.

- **When $\phi$ is positive :** At t=0 AC quantity has some positive instantaneous Value.

- **When $\phi$ is negative :** At t=0 AC quantity has some negative instantaneous Value.



# Concept of Phase and Phase Difference

- **Phase difference :-** The difference between the phase of two Alternating Quantities of same frequency is known as phase difference.

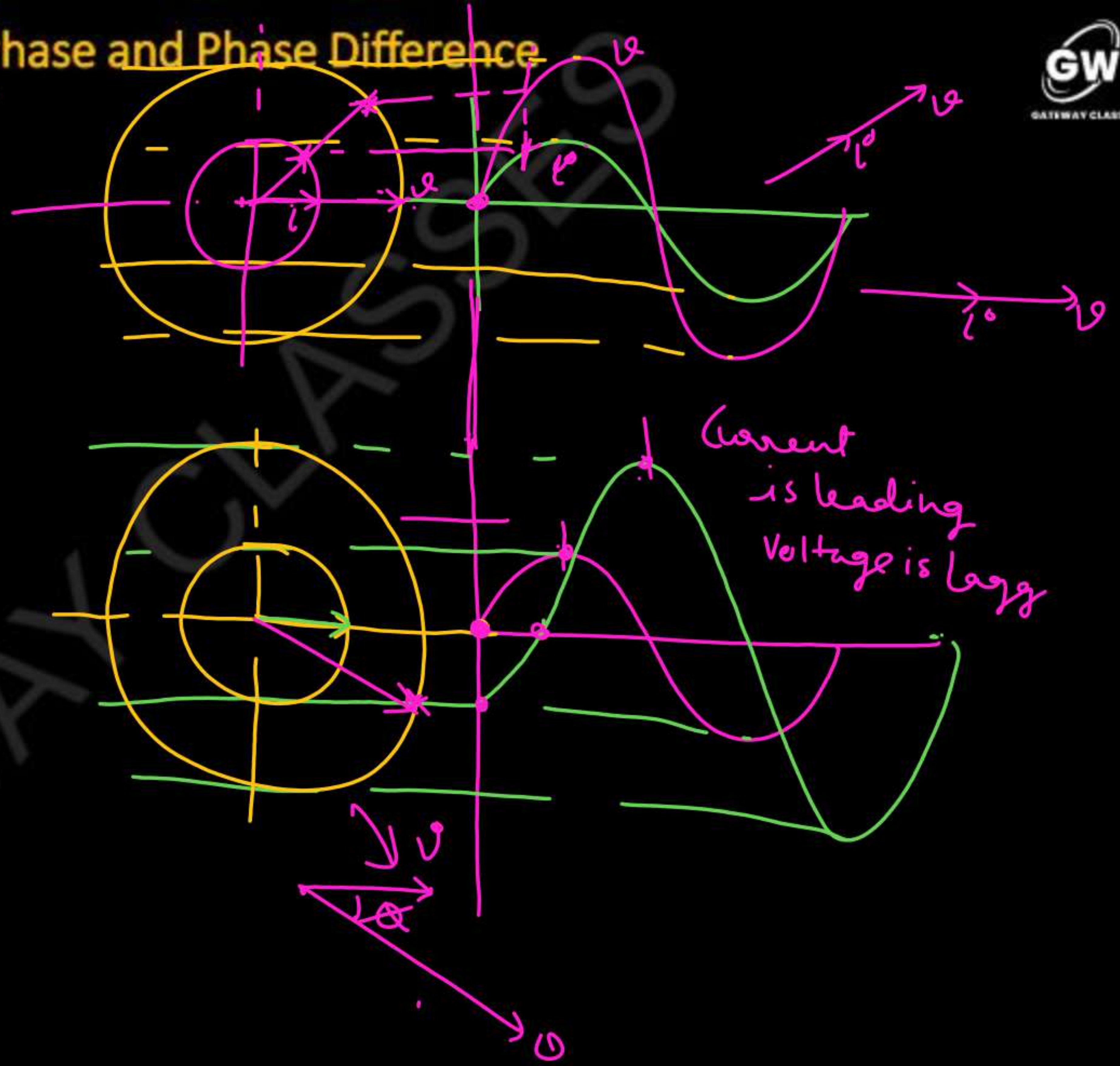
$v = ?$
$i = ?$

- **Zero Phase difference :-**

$v = V_m \sin \omega t$
$i = I_m \sin \omega t$

- **Leading Phase Difference :-**

- **Lagging Phase Difference :-**

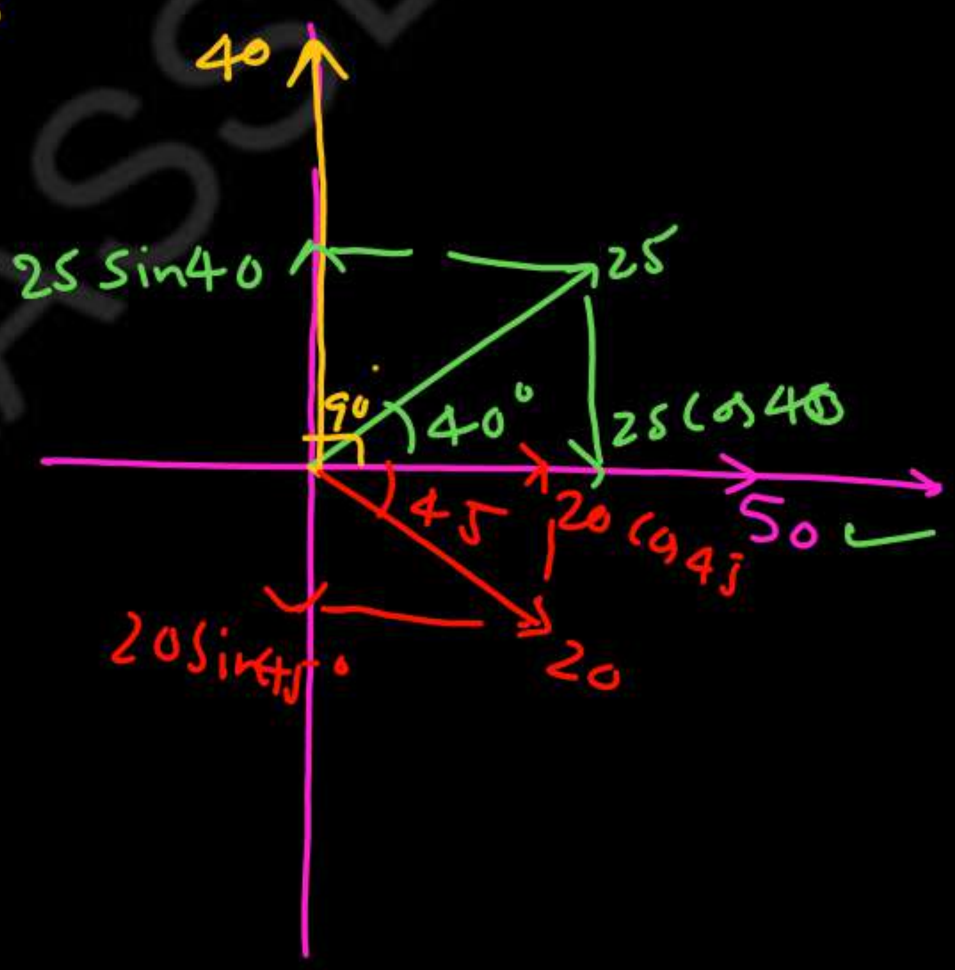


# Numerical

- Ex-7 $V_1 = 50 \sin \omega t$ , $V_2 = 25 \sin(\omega t + 40)$, $V_3 = 40 \cos \omega t$ $V_4 = \sin(\omega t - 45)$. Find Resultant voltage and phase angle.

20

$$v_1 = 50 \sin \omega t$$
$$v_2 = 25 \sin(\omega t + 40)$$
$$v_3 = 40 \cos \omega t = 40 \sin(\omega t + 90)$$
$$V_4 = 20 \sin(\omega t - 45^\circ)$$




$$V_x = 50 + 25\cos 40 + 20 \cos 45 = ?$$
$$V_y = 40 + 25 \sin 40 - 20 \sin 45 = ?$$
$$V_R = \sqrt{V_x^2 + V_y^2} = ? \quad , \theta = \tan^{-1} \frac{V_y}{V_x}$$

# Concept of Polar & Rectangular System

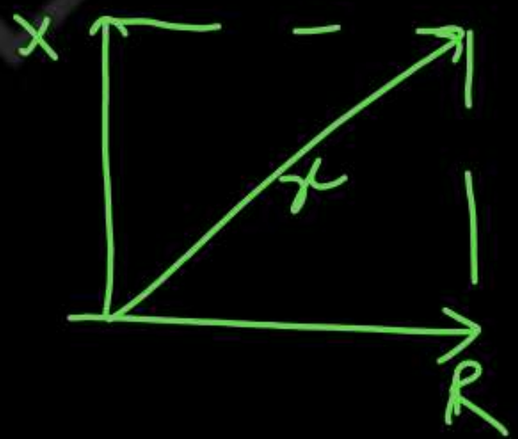
Complex

## Concept of Polar System:-

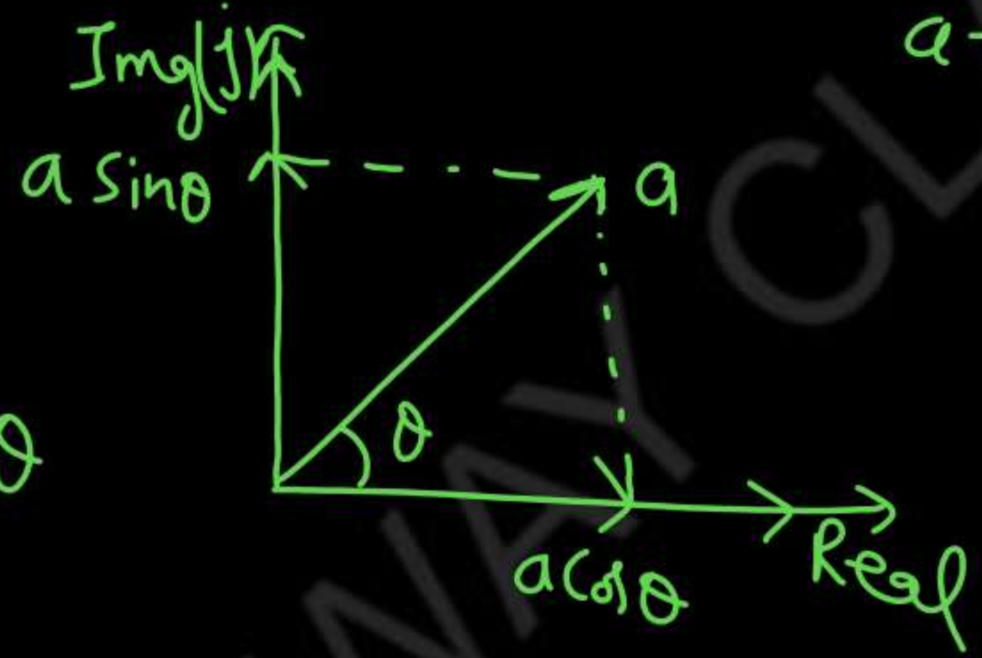
Polar — $r\angle\theta$

Rectangular — $a+ib$

$x = R + jX$




Polar $= a\angle\theta$




$\underbrace{a\cos\theta}_{b} + j\,\underbrace{a\sin\theta}_{c}$

$\boxed{b + jc}$

**Concept of Rectangular System:-** $R\angle\theta$ , $\underline{a+ib}$

Mode → Complx.

Type $R\angle\theta$ $\qquad (10\angle 30)$

$R$ → directly type

$\angle$ → (—) $\angle$

$\theta$ → Directly typ

$a + jb$

$a$ → directly

$j$ → $\boxed{ENG}$ → $i$

$b$ → directly

$\boxed{(2+3i)}$

# Polar to Rectangular conversion & Vice versa

$r\angle\theta$ , $a+ib$

Shift =    Shift =    $a+ib$ → $\boxed{\text{Shift}}\ \boxed{\ }$

## Polar to Rectangular conversion

Type

$(2\angle 60) \rightarrow \boxed{\text{Shift}} \rightarrow \boxed{-} \rightarrow \boxed{=}$

a

$\boxed{\text{Shift}}$

$\boxed{=}$

b i

$(2\angle 60)\ \boxed{=}$

(a)

↓ $\boxed{\text{Shift}}\ \boxed{=}$

(b) 17.08 i

$(2\angle 60^\circ) \times (3+7i) = \text{Ans}$

$\dfrac{(3+7j)}{(2\angle 60^\circ)}$    $3+7i/2\angle 60$

## Rectangular to Polar conversion

$(3+7i) \rightarrow \boxed{\text{Shift}} \rightarrow \boxed{+} \rightarrow \boxed{=} - \boxed{\text{Shift}} - \boxed{=}$

$r\angle\theta$    $r = 7.6$    $\angle\ \theta = 66.8$

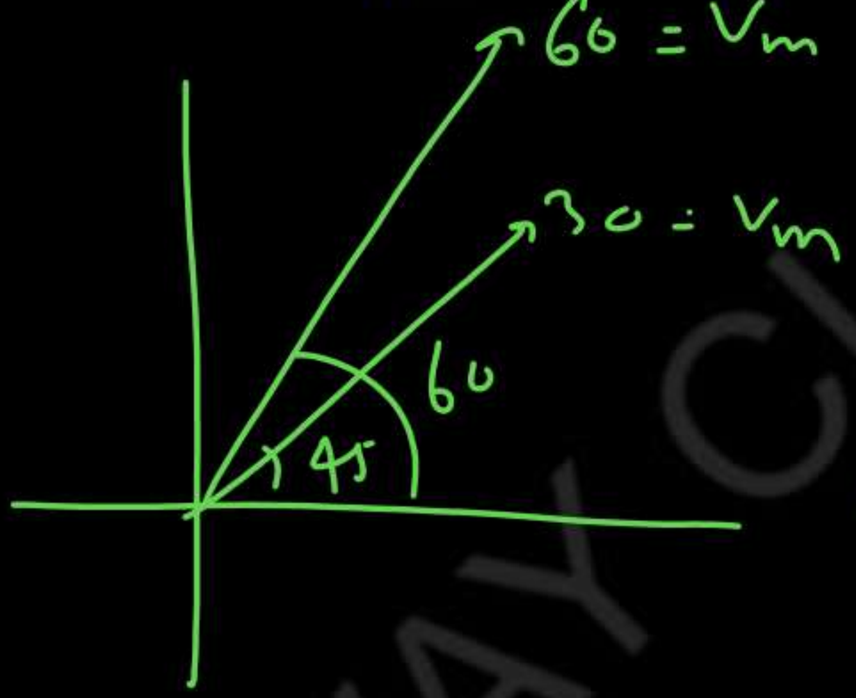
$(3+7i)\ r\angle\theta$

# Mathematics of Polar and Rectangular System:-

**Ex-8** Two a.c. voltages are represented bu $V_1(t) = 30 \sin(314t + 45^\circ)$ and $V_2(t) = 60 \sin(314t + 60^\circ)$. Calculate the resultant Voltage v(t) and express in the form $v(t) = V_m \sin(314t + \phi)$. (AKTU-2003-04)

$R = ? = V_m$

$\theta = ? = \tan^{-1} \frac{V_y}{V_x}$




H.W

$v(t) = R \sin(314t + \theta)$

# A.C. through Pure Resistive Circuit

Current & Voltage Equations

Reactance and Impedance

Phase Relationship

Power in AC Circuits

Waveform Analysis

# A.C. Through Pure Resistive Circuit

$v = V_m \sin\omega t \quad \text{—} (1)$

$i = \dfrac{v}{R} = \dfrac{V_m \sin\omega t}{R}$ } Compare $i = I_m \sin\omega t$

$i = I_m \sin\omega t \quad \text{—} (2)$

$$\boxed{I_m = \frac{V_m}{R}}$$

So Voltage & Current are in same phase.

$P = v . i$

$= V_m \sin\omega t . I_m \sin\omega t$

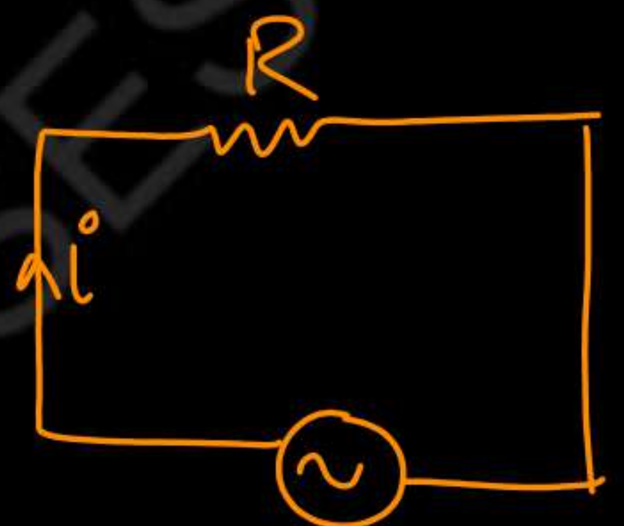
$= V_m I_m \sin^2\omega t$

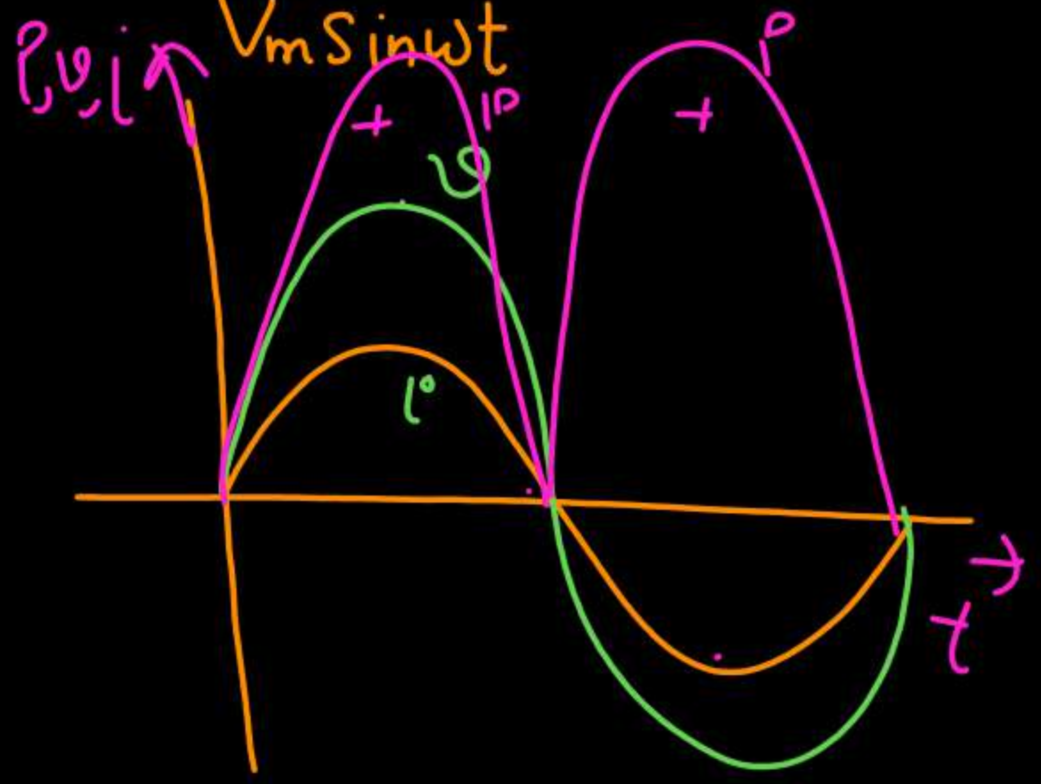
$= \dfrac{V_m I_m}{2}(1 - \cos 2\omega t)$ (over a complete Period its average value is zero)

$P = \dfrac{V_m I_m}{2}$

$P = \dfrac{V_m}{\sqrt{2}} . \dfrac{I_m}{\sqrt{2}}$

$$\boxed{P = V_{rms} . I_{rms}}$$



# A.C. Through Pure Inductive Circuit

$$\boxed{v = V_m \sin \omega t}$$

$$e = -L\frac{di}{dt}$$

$$v_L = -e = L\frac{di}{dt}$$

$$\frac{di}{dt} = \frac{v}{L} = \frac{V_m \sin \omega t}{L}$$

$$i = \frac{V_m}{L}\int \sin \omega t$$

$$i = -\frac{V_m}{L}\frac{\cos \omega t}{\omega}$$

$$i = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\boxed{i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)}$$

$$I_m = \frac{V_m}{\omega L}$$

$$\frac{V_m}{I_m} = \omega L$$

$$\boxed{X_L = \omega L}$$

This is known as Inductive Reactance

$$P = v.i$$

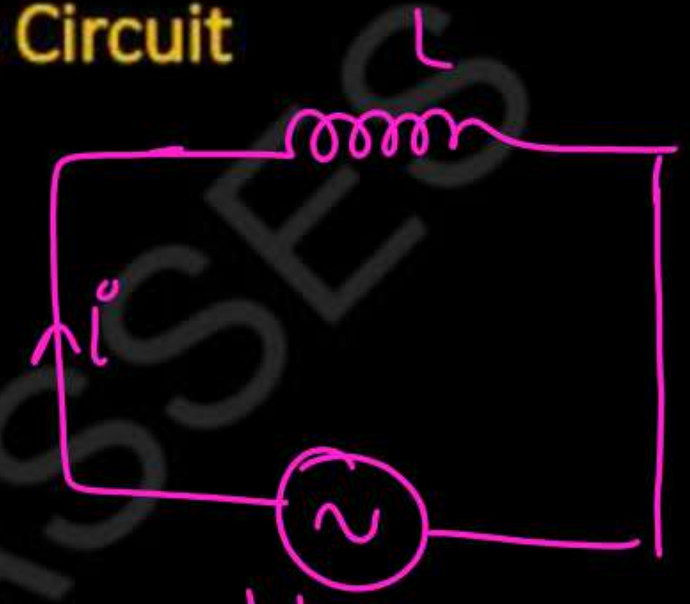
$$= -V_m \sin \omega t \, I_m \cos \omega t$$

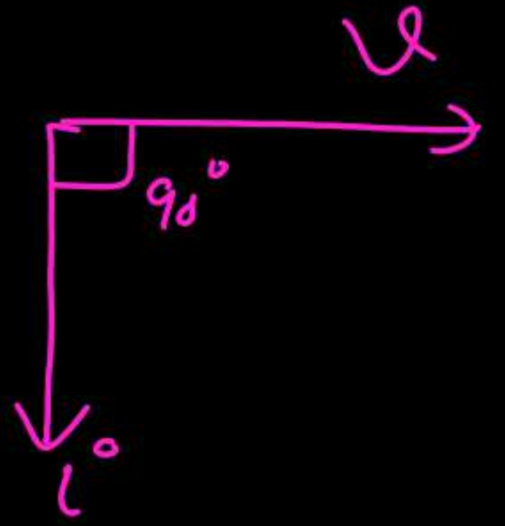
$$P = -\frac{V_m I_m}{2}\, 2\sin \omega t . \cos \omega t$$

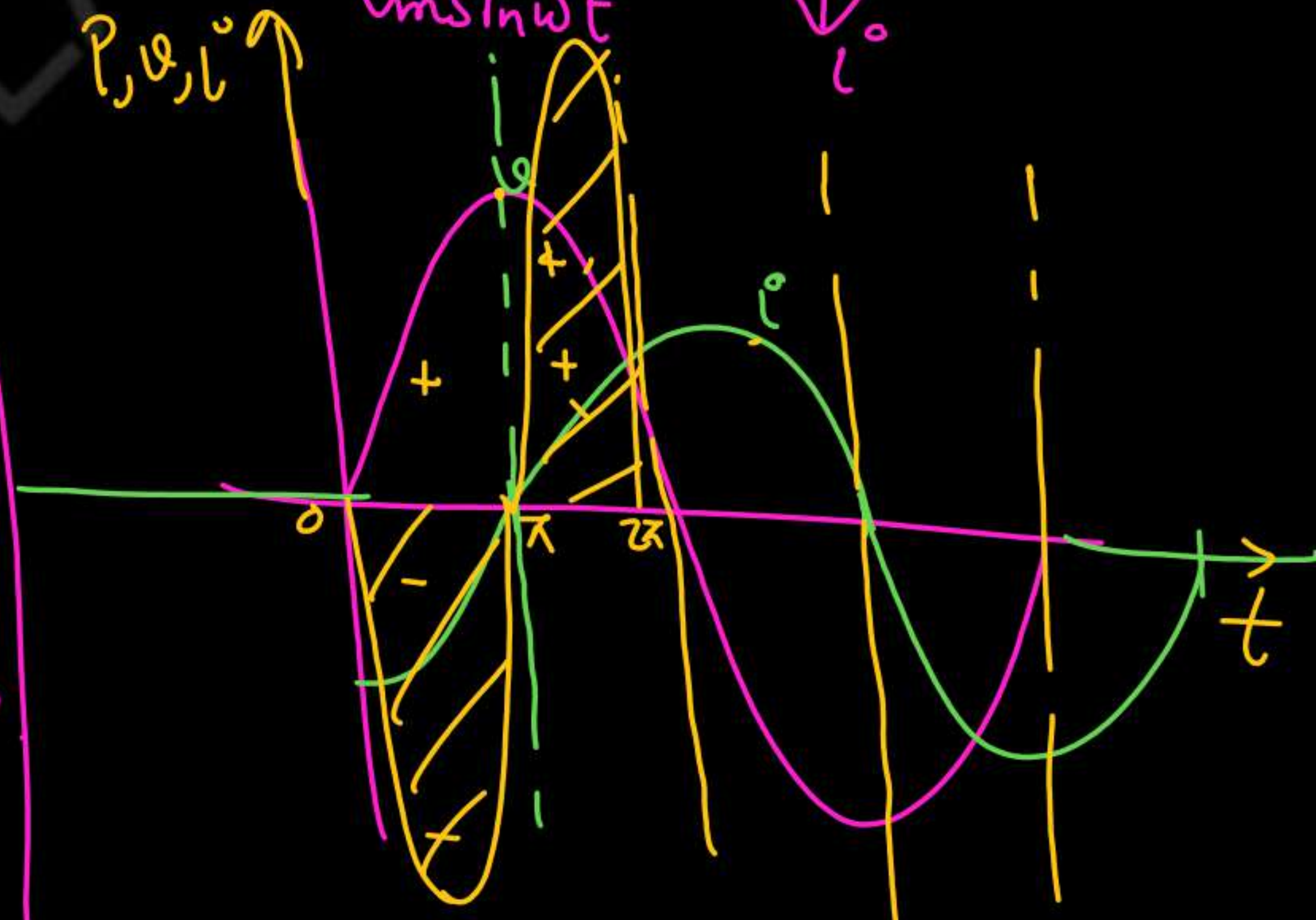
$$P = -\frac{V_m I_m}{2}\sin 2\omega t \rightarrow 0$$

over a Comp. cycle

$$\boxed{P_{avg} = 0}$$ Imp



# A.C. Through Pure Capacitive Circuit

$v = V_m \sin\omega t$ —— (1)

$q = Cv = CV_m \sin\omega t$

$i = \frac{dq}{dt} = CV_m(+\cos\omega t)\cdot\omega$

$i = +\omega C V_m \cos\omega t$

$i = \omega C V_m \sin(\omega t + \frac{\pi}{2})$

$i = I_m \sin(\omega t + \frac{\pi}{2})$ —— (2)

$I_m = \omega C V_m$

$\frac{V_m}{I_m} = \boxed{\frac{1}{\omega C} = X_C}$

Capacitive Reactance

Power

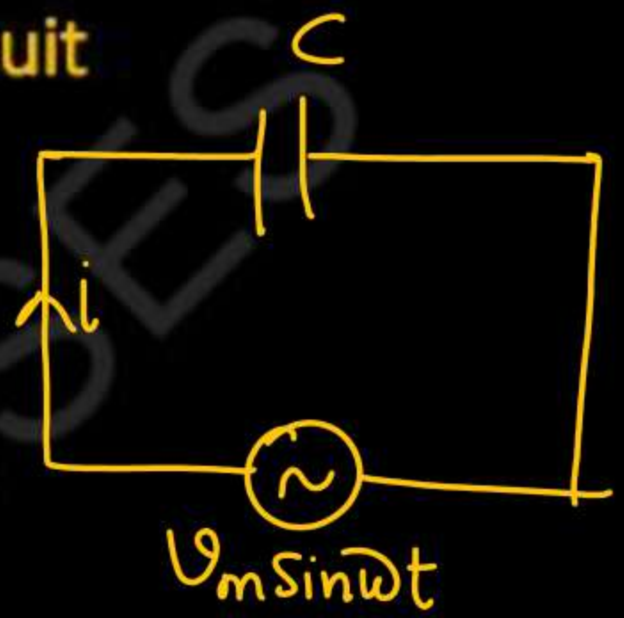
$P = Vi$

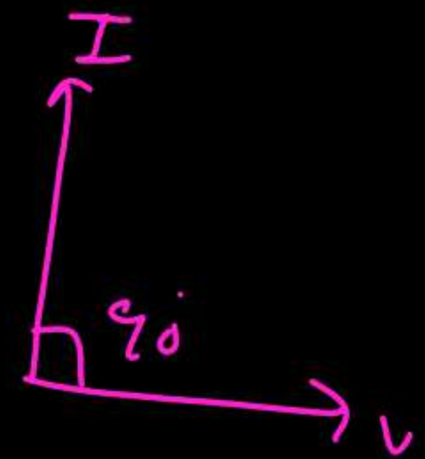
$= V_m \sin\omega t \, I_m \cos\omega t$

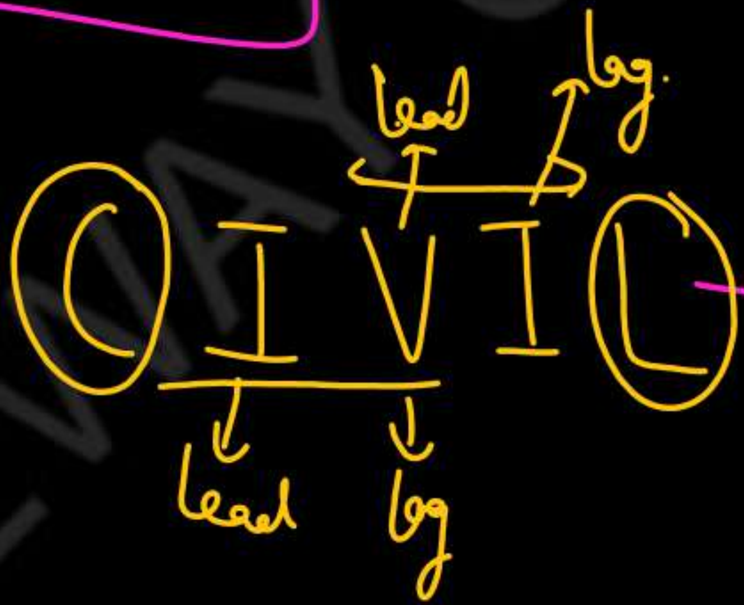
$P = \frac{V_m I_m}{2} 2\sin\omega t \cos\omega t$

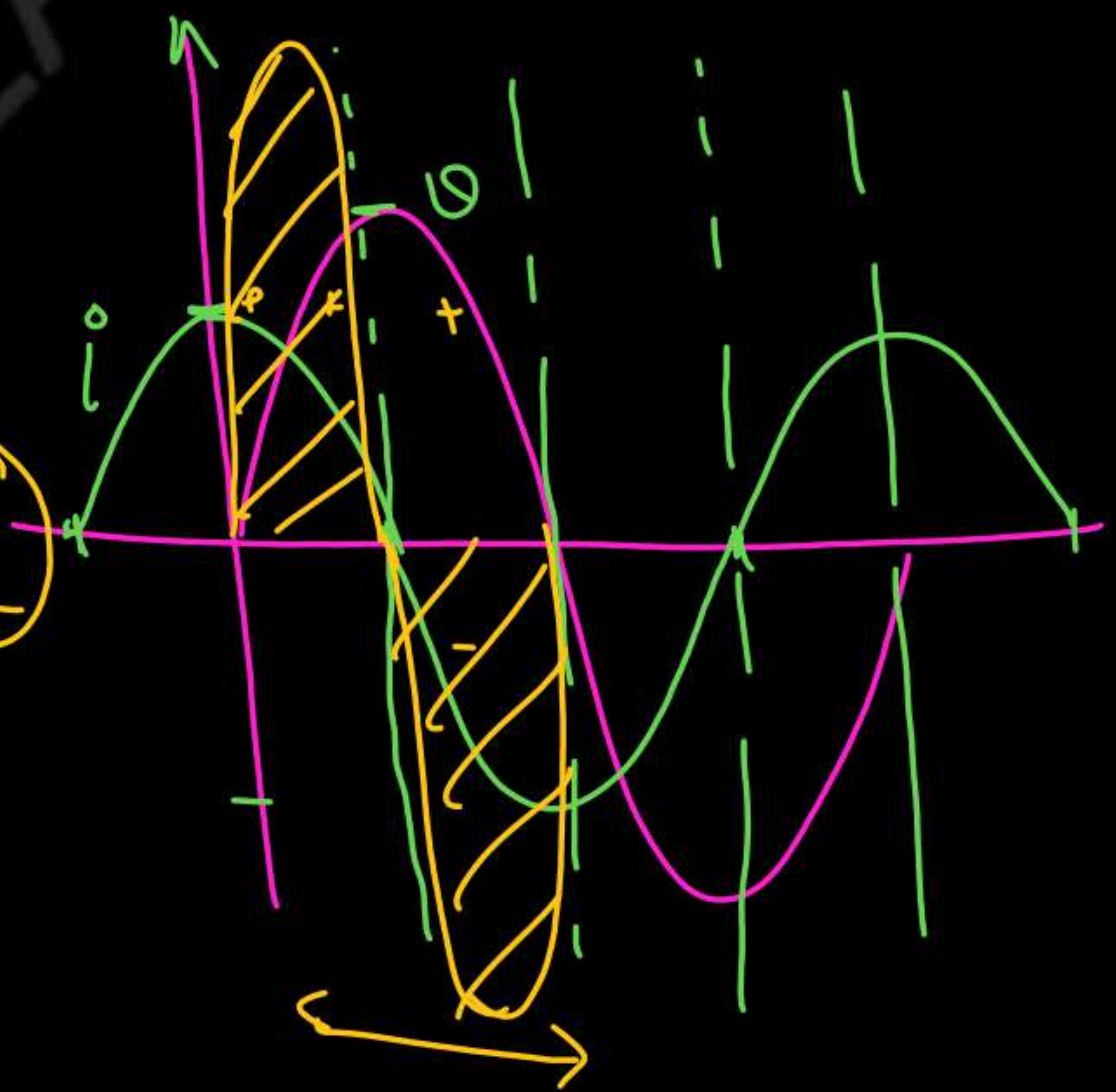
$P = \frac{V_m I_m}{2} \cancel{\sin 2\omega t} \rightarrow 0$

$\boxed{P_{avg} = 0}$



# Concept of Impedance

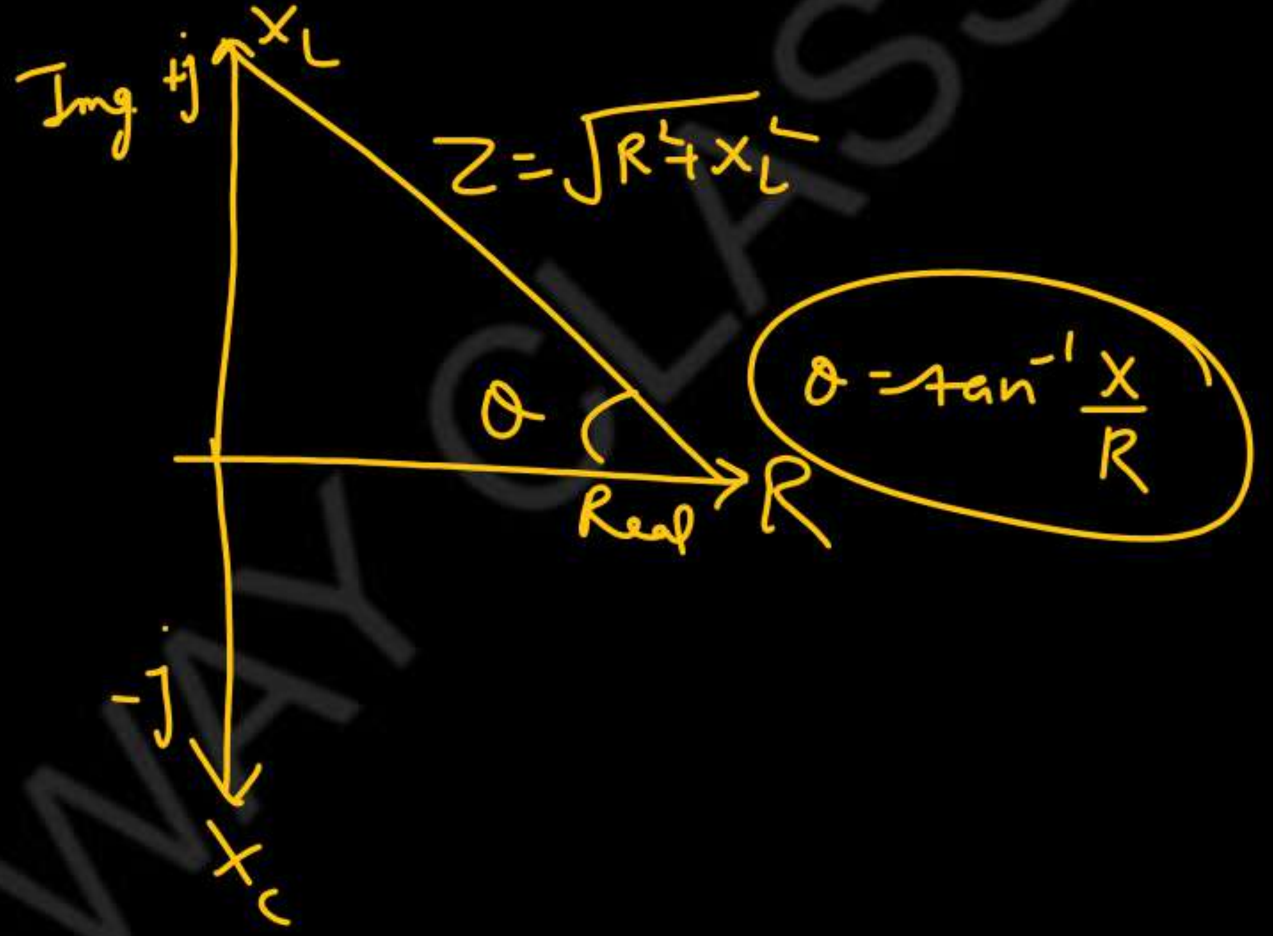
$$\text{Impedance} = \frac{\text{Resistance } \& \text{ Reactance}}{\text{Combination}}$$

R-L

$$Z = R + jX_L$$

R-C

$$Z = R - jX_C$$



# A.C. Through Series R-L Circuit

$\vec{v} = \vec{v_R} + \vec{v_L}$

$I\vec{Z} = \vec{IR} + \vec{IX_L}$

$IZ = \sqrt{(IR)^2 + (IX_L)^2}$

Imp. $\boxed{Z = \sqrt{R^2 + X_L^2}}$

$\phi = \tan^{-1}\frac{X_L}{R}$

$v = v_m \sin\omega t$

$i = I_m \sin(\omega t - \phi)$

$P = vi$

$= v_m \sin\omega t \cdot I_m \sin(\omega t - \phi)$

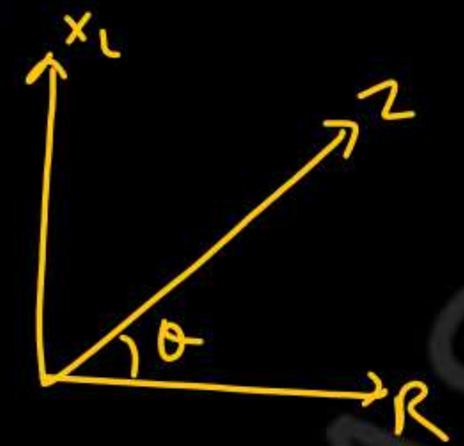
$= \frac{v_m I_m}{2}\left[2\sin\omega t \cdot \sin(\omega t - \phi)\right]$

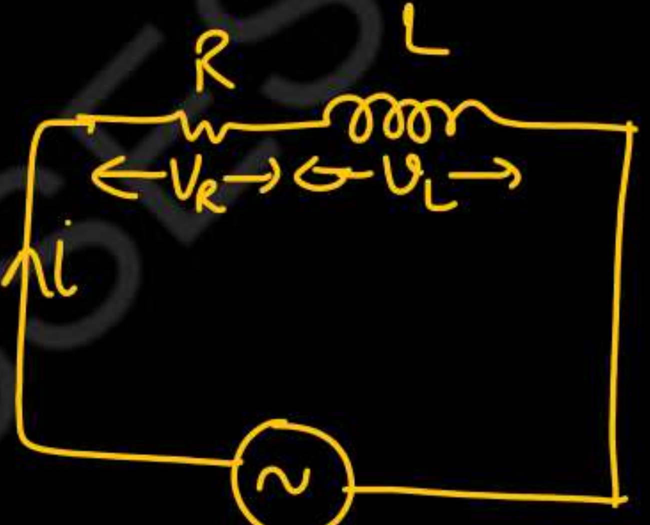
$= \frac{V_m I_m}{2}\left[\cos(\cancel{\omega t} - \cancel{\omega t} + \phi) - \cos(\omega t + \omega t - \phi)\right]$

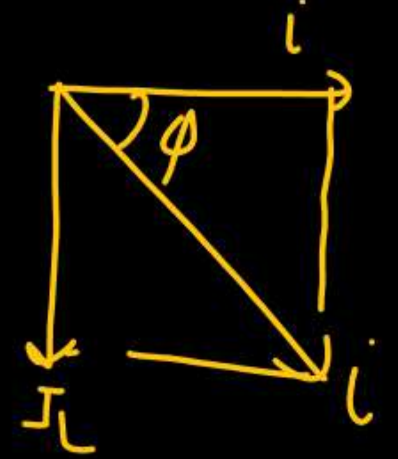
$= \frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t - \phi)\right]$

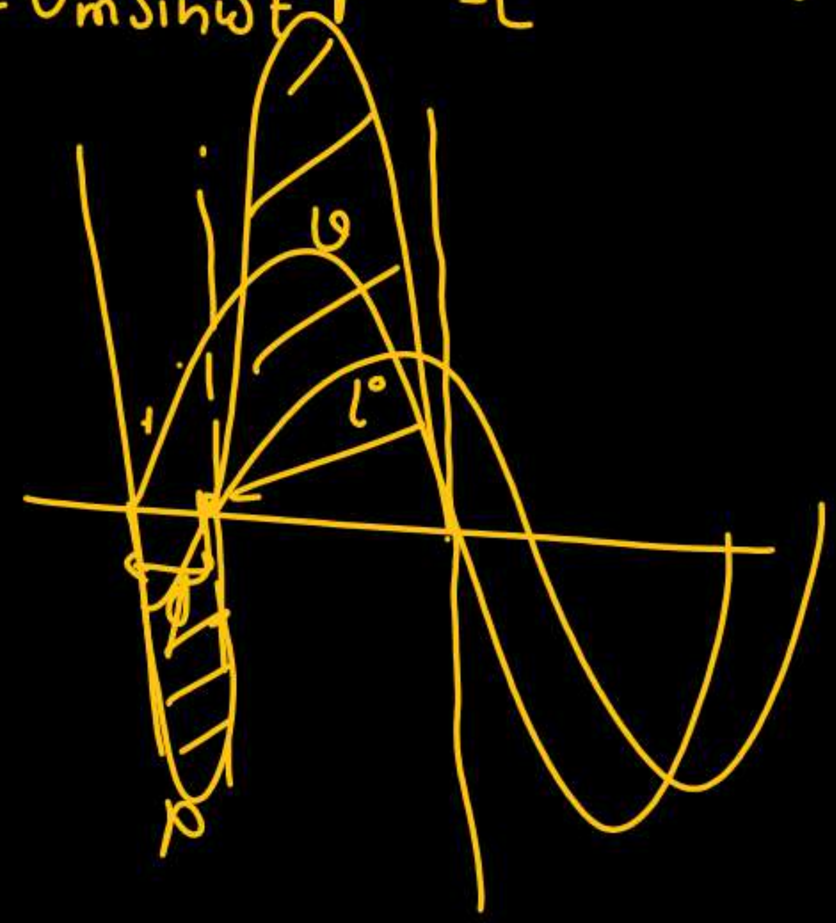
Avg = 0

$= \frac{V_m I_m}{2}\cos\theta = VI\cos\phi$



# A.C. Through Series R-C Circuit

$$\vec{v} = \vec{iR} + \vec{iX_C}$$

$$iZ = \sqrt{(iR)^2 + (iX_C)^2}$$

Imp. $\boxed{Z = \sqrt{R^2 + X_C^2}}$

$$\phi = \tan^{-1}\left(\frac{-X_C}{R}\right)$$

$$v = V_m \sin\omega t$$

$$i = I_m \sin(\omega t + \phi)$$

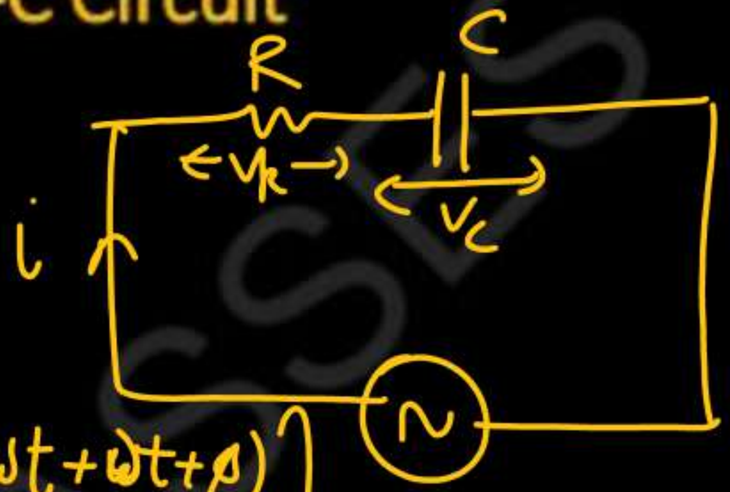
$$P = v.i$$

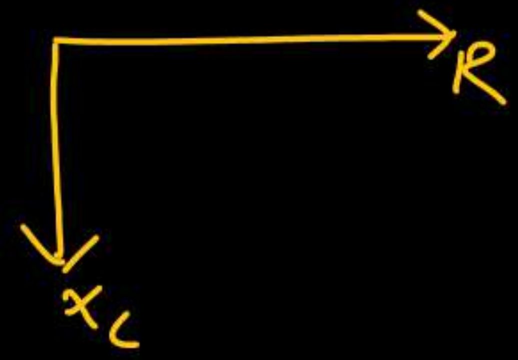
$$= \frac{V_m I_m}{2}\, 2\sin\omega t . \sin(\omega t+\phi)$$

$$= \frac{V_m I_m}{2}\left[\cos(\omega t - \omega t - \phi) - \cos(\omega t + \omega t + \phi)\right]$$

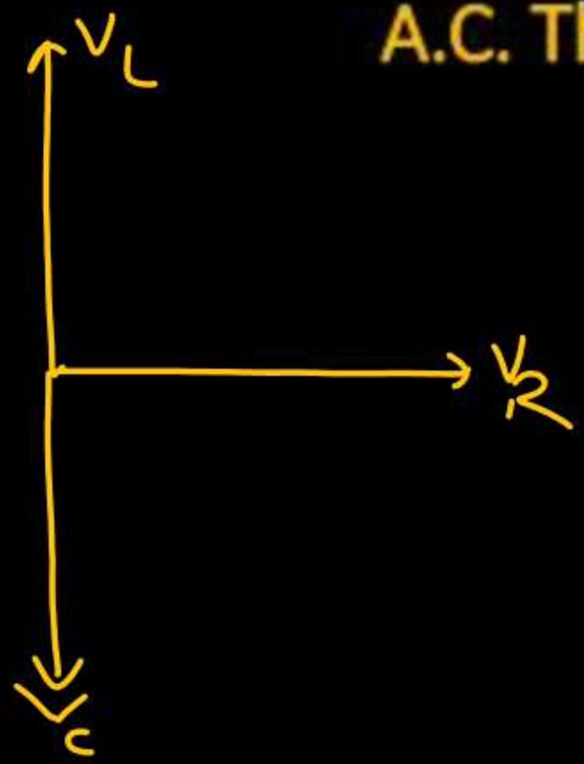
$$= \frac{V_m I_m}{2}\left[\cos(-\phi) - \underbrace{\cancel{\cos(2\omega t + \phi)}}_{0}\right]$$

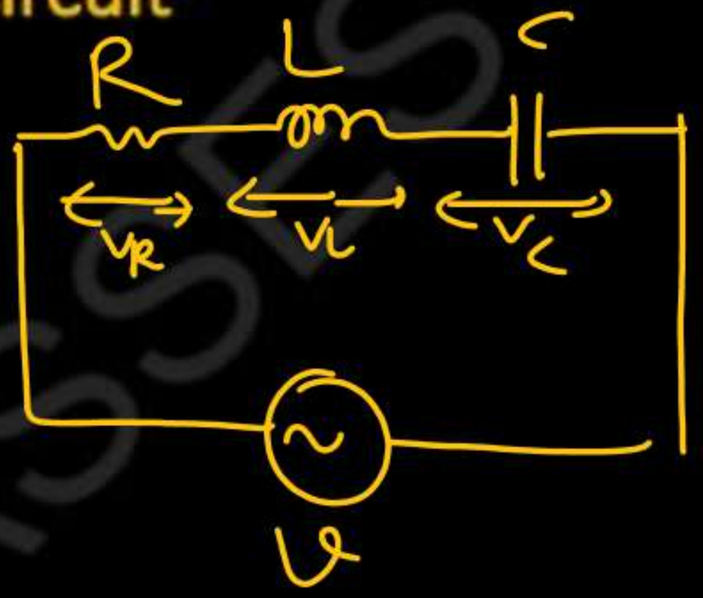
$$\boxed{P = VI\cos\phi}$$



# A.C. Through Series R-L-C Circuit




① $V_L > V_C \longrightarrow$ Ind. (R-L)

② $V_C > V_L \longrightarrow$ Capac. (R-C)

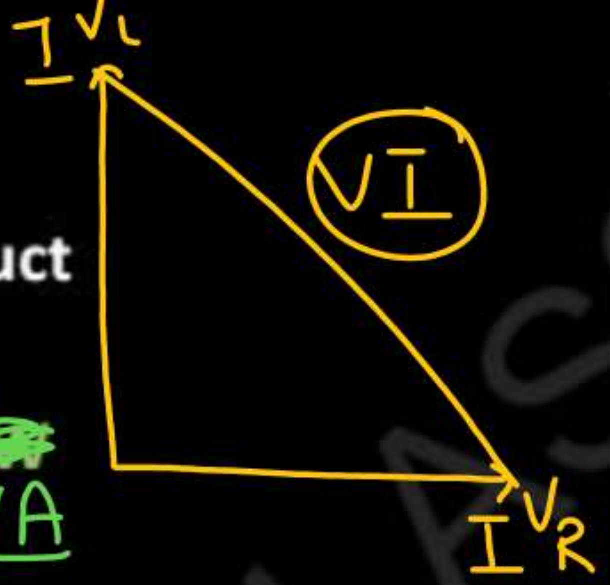
③ $V_L = V_C \longrightarrow$ Resistive (Pure)

## Concept of Power:-

- **Apparent Power:-** Apparent Power (S) is Product of rms value of Voltage (V) and current (I) .

  Unit- ~~KW~~ KVA

  $$\cancel{P}\, S = VI$$




- **Active/True Power :-** True Power (P ) is Product of applied Voltage (V) and active component of current (I).

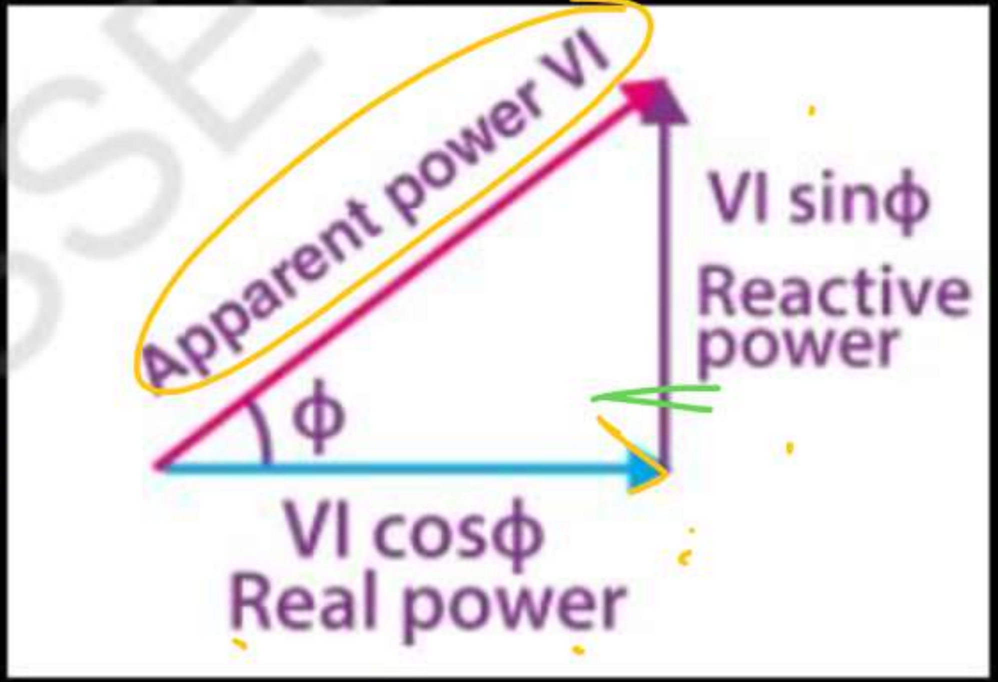
  Unit- ~~KW~~ KW

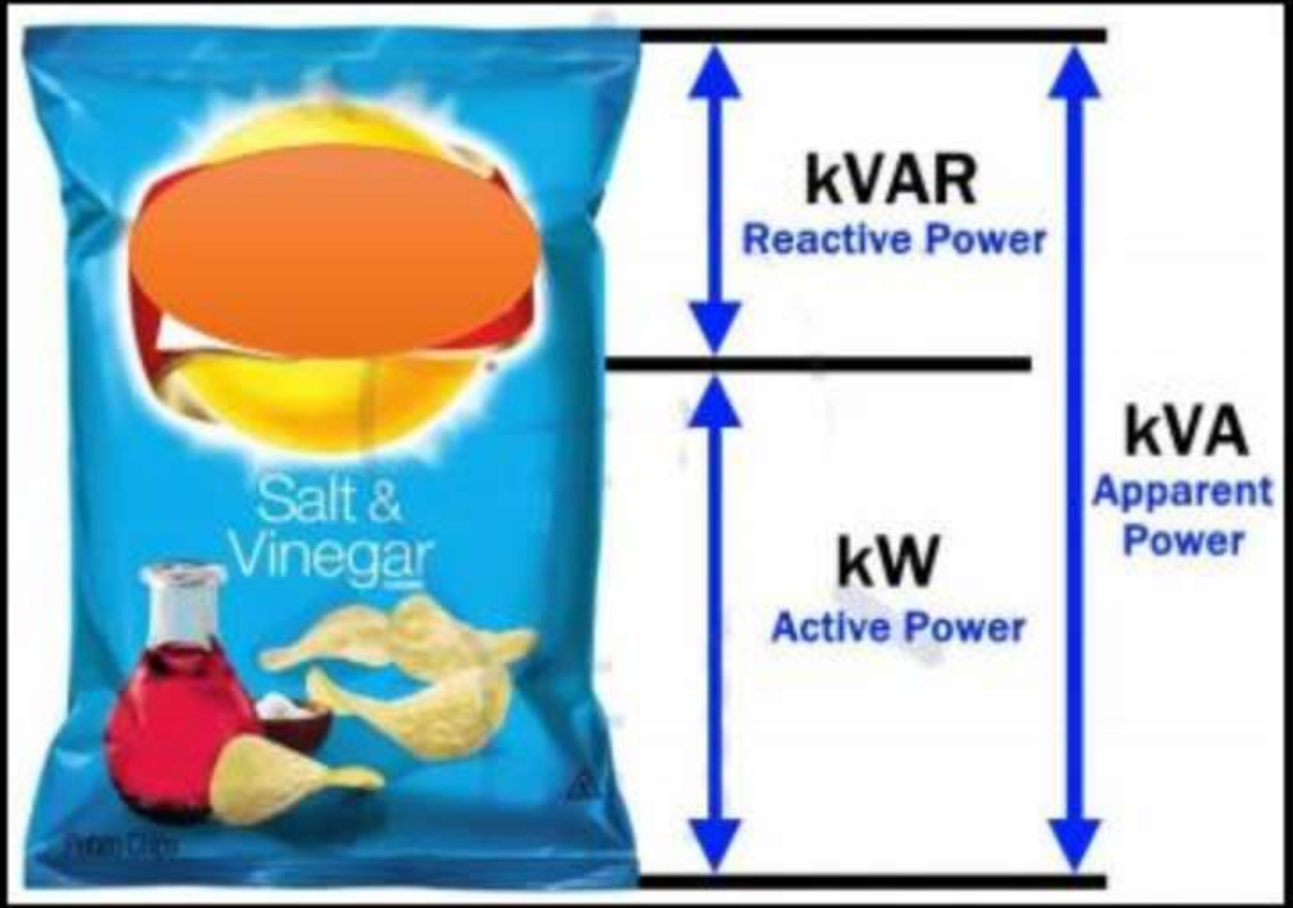
  $$\boxed{P = VI\cos\phi = S\cos\phi}$$

- **Reactive Power :-** Reactive power (Q ) is Product of applied Voltage (V) and reactive component of current (I).

  Unit- KVAR

  $$P = VI\sin\phi = S\sin\phi$$



# A.C. Through Pure Inductive Circuit



$0.9$ 90% $0.6$ — 60% use 40% waste

***Power factor is the***

- Ratio between true power and apparent power is called the for this circuit.
- Ratio of resistance to impedance.
- Cosine of phase angle between voltage and current.

$$\frac{VI\cos\phi}{VI} = \cos\phi = P.f$$

$$P.f = \frac{R}{Z}$$

**Cause of Low Power Factor.**

- All AC motor ~~transformers~~ has low power factor.
- Most of the loads has low power factors.
- Industrial heating furnace , induction furnace has low power factor.

**Improvement of Power Factor.**

- Using Phase advancers
- Using Capacitor Bank, Synchronous Condenser

**Effects of low power factor**

- a) Large Copper Losses b) Large kVA rating c) Poor Voltage Regulation.

# Numerical Examples

R

- Ex.-9 An A.C. circuit having a pure resistance of 10 ohm and is connected to an a.c. supply of 230 volt and 50Hz. Calculate (a) Current (b) Power Consume (c) Equation of voltage and current.

$V = 230$ volt , $V_m = V_{rms} \cdot \sqrt{2}$ , $\omega = 2\pi f$

$= 2\pi \times 50$

$= 100\pi$

$I = \frac{V}{R} = \frac{230}{10} = 2.3$ amp , $I_m = 2.3\sqrt{2}$

$P = I^2 R = (2.3)^2 \times 10$

$$\boxed{\begin{aligned} v &= V_m \sin \omega t = 230\sqrt{2} \sin 100\pi t \\ i &= 2.3\sqrt{2} \sin(100\pi \cdot t) \end{aligned}}$$

# Numerical Examples

- Ex.-10. A Pure inductive coil allow a current of 10A to flow from an a.c. supply of 230 volt and 50Hz. Calculate

(a) Inductive reactance (b) Inductance of coil (c) Power Consumed (c) Equation of voltage and current.

$i = 10A$ , $i_m = 10\sqrt{2}$

$v = 230V$ , $v_m = 230\sqrt{2}$

$f = 50$ , $\omega = 100\pi$

$$X_L = \frac{V}{I} = \frac{230}{10} = 23\,\Omega$$

$$X_L = \omega L$$

$$L = \frac{X_L}{\omega} = \frac{23}{100\pi} =$$

$0$

$v = v_m \sin \omega t$

$= 230\sqrt{2} \sin 100\pi t$

$i = 10\sqrt{2} \sin(100\pi t - 90^\circ)$

Ans

**Ex.-11.** A 318 microfarad capacitor is connected across a supply of 230 volt and 50Hz. Calculate (a) Capacitive reactance (b) RMS value of Current (c) Equation of voltage and current.

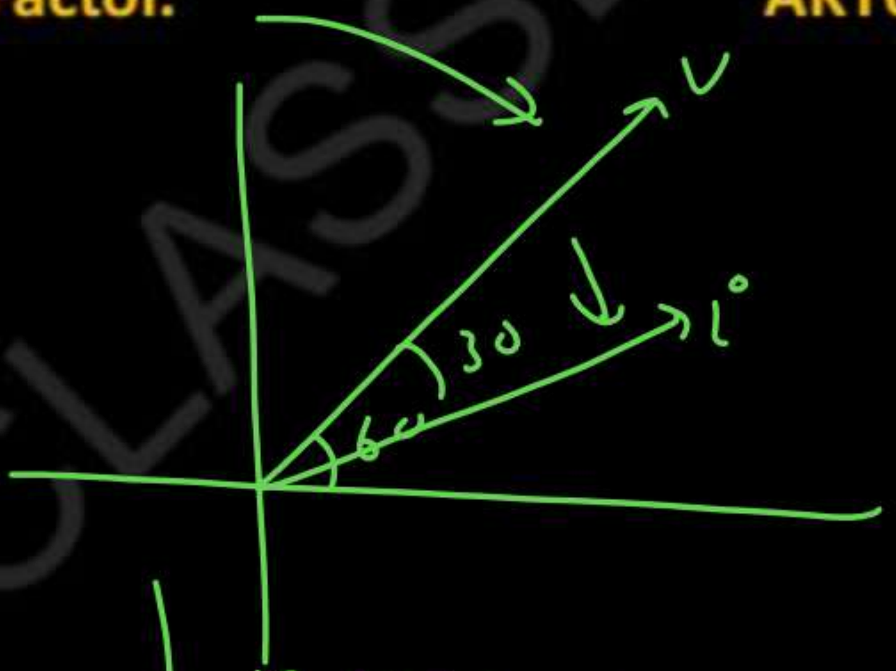
H.W

# Numerical Examples

**Ex.-12. An Electrical circuit connected to v=100 sin(628t+ 60) it draw current i=15 sin(628t+ 30), identify the circuit , also find the value of Resistance, Reactance, Power ,Power Factor.** **AKTU- 2020-21**

$v = 100 \sin(628t + 60)$

$i = 15 \sin(628t + 30)$




Current is lagging by 30°

Circuit is inductive.

$V_m = 100$

$I_m = 15$

$Z = \frac{V}{I} = \cancel{\frac{100}{15}} = 6.67$

$\phi = 30^\circ$

(−)

$Z = 6.67\angle 30^\circ$

$= \underbrace{5.77}_{R} + \underbrace{3.33}_{X_L} j$

$P = VI\cos\phi$

$= I^2R$

$= \left(\frac{10}{\sqrt{2}}\right)^2 \cdot 5.77 = ?$

$P.f = \cos 30$

$\theta$

# Numerical Examples

**Ex.-13.** Given $v = 200 \sin 377t$ volt and $i = 8 \sin (377t - 30^\circ)$ amp for an a.c. circuit. Determine:- a.) The power factor b.) True Power, c.) Apparent Power d.) Reactive Power. AKTU- 2008-09

Circuit inductive

$V_m = 100$ , $I_m = 8$

$V = \frac{100}{\sqrt{2}}$ , $I_{rms} = \frac{8}{\sqrt{2}}$

$P = VI \cos\phi$

$= \frac{V_m I_m}{2} \cos\phi$

$= \frac{100 \times 8}{2} \cos 30^\circ$

$= 400 \cos 30^\circ$

$= 346.416$

$\phi = 30^\circ$

$Q = VI \sin\phi$

$= 400 \sin 30^\circ$

$Q = 200$ VAR

$S = VI$

$= 400$ VA

# Numerical Examples

**Ex.-14. A 120 V, 60 Watt Lamp is to be operated on 220 V , 50 Hz supply mains, In order that lamp should operate on correct voltage Calculate value of (i) Non inductive resistance (ii) Pure Inductance (iii) Pure Capacitance**

**AKTU- 2005-06**

$V = V_R + V_{Lamp}$

$220 = V_R + 120$

$V_R = 220 - 120$

$V_R = 100$

$I = \frac{P}{V} = \frac{\cancel{60}}{\cancel{120}_2} = 0.5 \text{ amp}$

$R = \frac{V}{I} = \frac{100}{0.5} = 200\Omega$

$\vec{V} = \vec{V}_{Lamp} + \vec{V}_L$

$220 = \vec{120} + \vec{V}_L$

$V = \sqrt{V_L^2 + V_{Lamp}^2}$

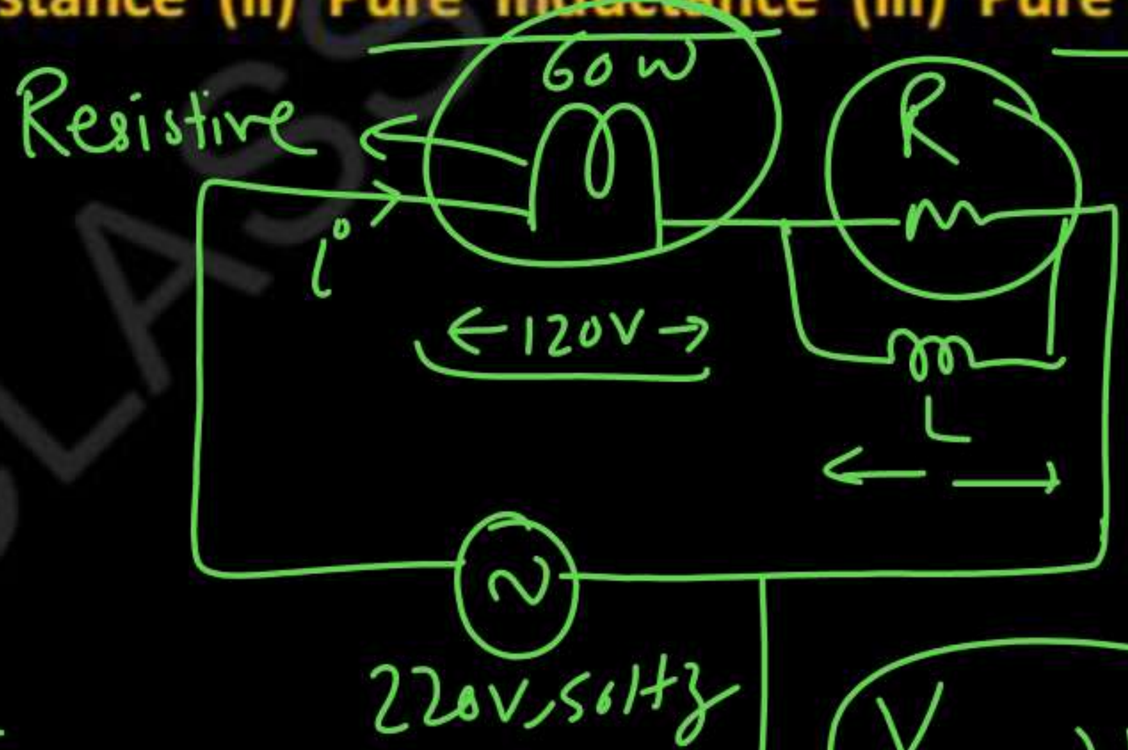
$V_L^2 = V^2 - V_{Lamp}^2$

$V_L^2 = 220^2 - 120^2$

$V_L = \sqrt{220^2 - 120^2} = 184.39 \text{ volt}$

$X_L = \frac{V_L}{I} = \frac{184.39}{0.5} = 368.78$

$L = \frac{X_L}{\omega} = ?$




$X_L \rightarrow$ H.W

Self

# Numerical Examples (Try Yourself)

- **Ex.-15. A resistance of 120 ohms and a capacitive reactance of 250 ohms are connected in series across a A.C. voltage source. If a current of 0.9 A is flowing in the circuit find out (i) power factor (ii) supply voltage (iii) voltage across resistance and capacitance (iv) Active and reactive power.**

$V_R = IR = ?$

$V_{X_C} = I X_C = ?$

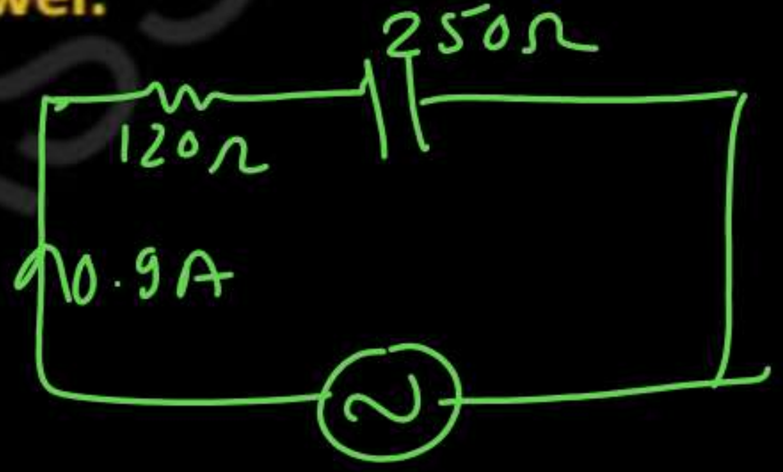
$V = \sqrt{V_R^2 + V_{X_C}^2}$

$\theta = \tan^{-1} \frac{V_{X_C}}{V_R}$

$P.f = \cos\theta$

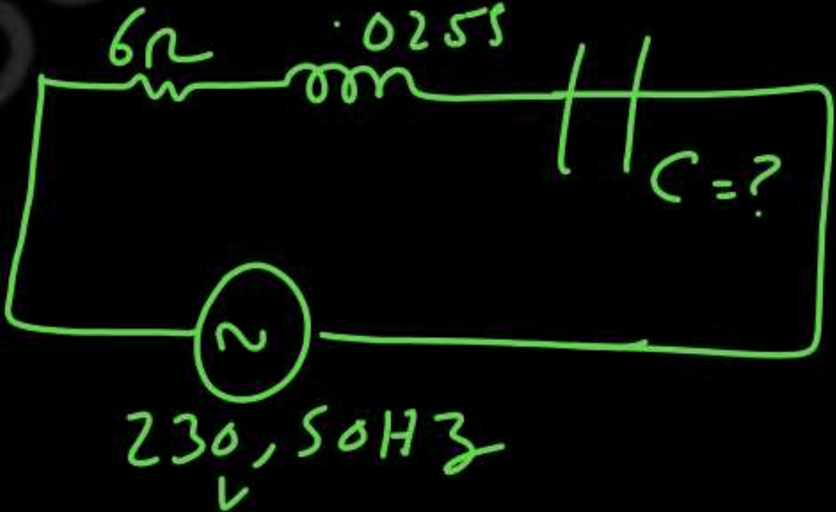
$P = I^2 R = VI\cos\phi$

$R = VI\sin\theta$



# Numerical Examples

- Ex.-16. A coil having a resistance of 6 ohm and an inductance of 0.0255 H is connected across a 230v, 50 Hz a.c. supply. Calculate (i) Current (ii) power factor (iii) Active power (iv) Reactive Power (v) Apparent Power (vi) It is desired to improve power factor to 0.8 .What value of capacitance to be connected in series R and what is reduction in reactive power. AKTU- 2009-10




$X_L = 2\pi f L$

$= 100\pi \times 0.0255 = 8\Omega$

$\boxed{Z = 6 + 8j}$

$i = \frac{230}{(6+8j)} = 23\angle -53.18^\circ$

$P.F = \cos 53.18 = 0.599$

$= 0.6$

$P = VI\cos\phi$

$Q = VI\sin\phi$

$S = VI$

$Z = R + j(X_L - X_C)$

$\cos\phi = \frac{R}{Z} = \frac{6}{Z} = 0.8$

$Z = \frac{6}{0.8} = 7.5$

$\boxed{7.5 = \sqrt{6^2 + (8 - X_C)^2}}$

$X_C = 3.5\Omega$

$X_C = \frac{1}{\omega C} \Rightarrow \boxed{C = \frac{1}{\omega X_C}}$

# Numerical Examples

- Ex.-17. A series RLC circuit is composed of 10 ohm resistance , 0.1H inductance and 50 microfarad capacitance. A voltage v(t) = 141.4 Cos(100π t) V is impressed upon the circuit . Determine (i) The expression for instantaneous current, (ii) The voltage drops VR,VL,VC across resistor capacitor and inductor (iii) Draw the phasor diagram using all the voltage relation. AKTU- 2014-15

$X_L = 100\pi \cdot 0.1 = 31.4\Omega$

$X_C = \frac{1}{100\pi \times 50\mu} = \underline{63.66\Omega}$

$X_C - X_L = 63.66 - 31.44 = 32.2$

$Z = R - j(X_C - X_L)$

$\boxed{Z = 10 - 32.26j} \rightarrow R\angle\theta$

$\theta = ?$

$i = \frac{V}{Z} = \frac{141.4/\sqrt{2}}{(10-32.26j)} = ?$

$I_m = \sqrt{2}\, I_{rms}.$

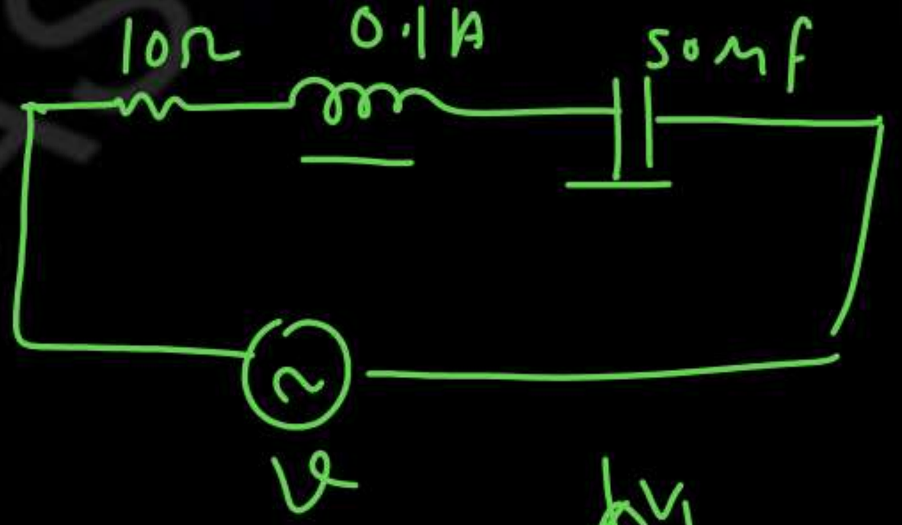
$\omega = \cancel{100\pi} = \cancel{2\pi} f$

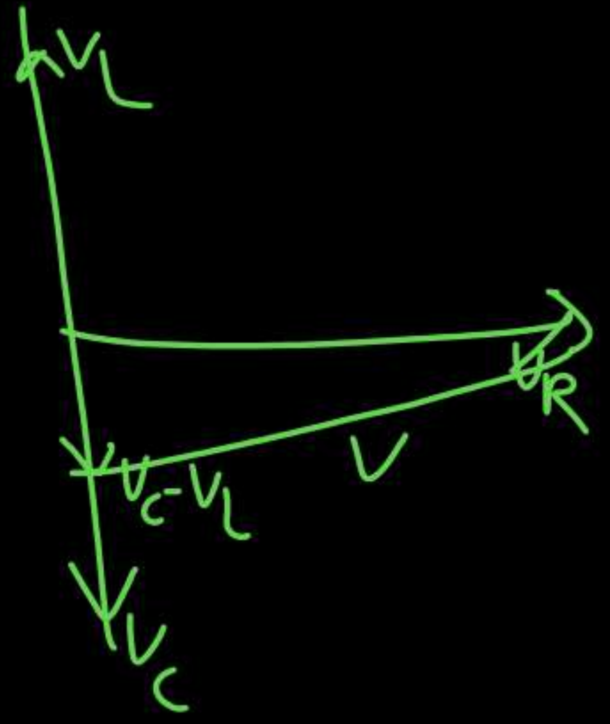
$f = 50$

$V_m = 141.4$

$V_{rms} = \frac{141.4}{\sqrt{2}}$

$i = I_m \sin(100\pi t + \theta)$



# Resonance in Series RLC circuit

$V_L = Q.V$

**Condition of Resonance in Series RLC circuit:-**

- Power Factor is unity — $\cos\phi = 1$ Purely resistive.
- Voltatge and current are in same phase
- A series resonant circuit has the capability to draw heavy current and power from the mains; it is also called **acceptor circuit**. 2 marks
- **Parameters at Resonance Frequency:**
- **(i) Resonance Frequency** Imp

$Z$ min $\quad I = \frac{V}{Z} \uparrow$ Max.

$(X_L - X_C) = 0$

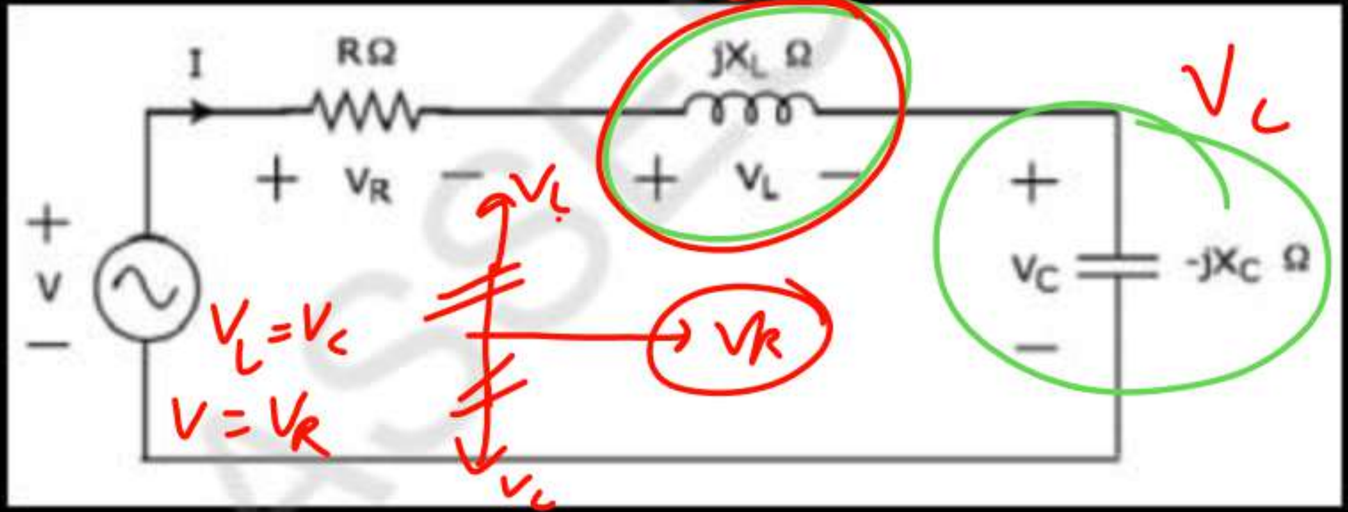
$X_L = X_C$

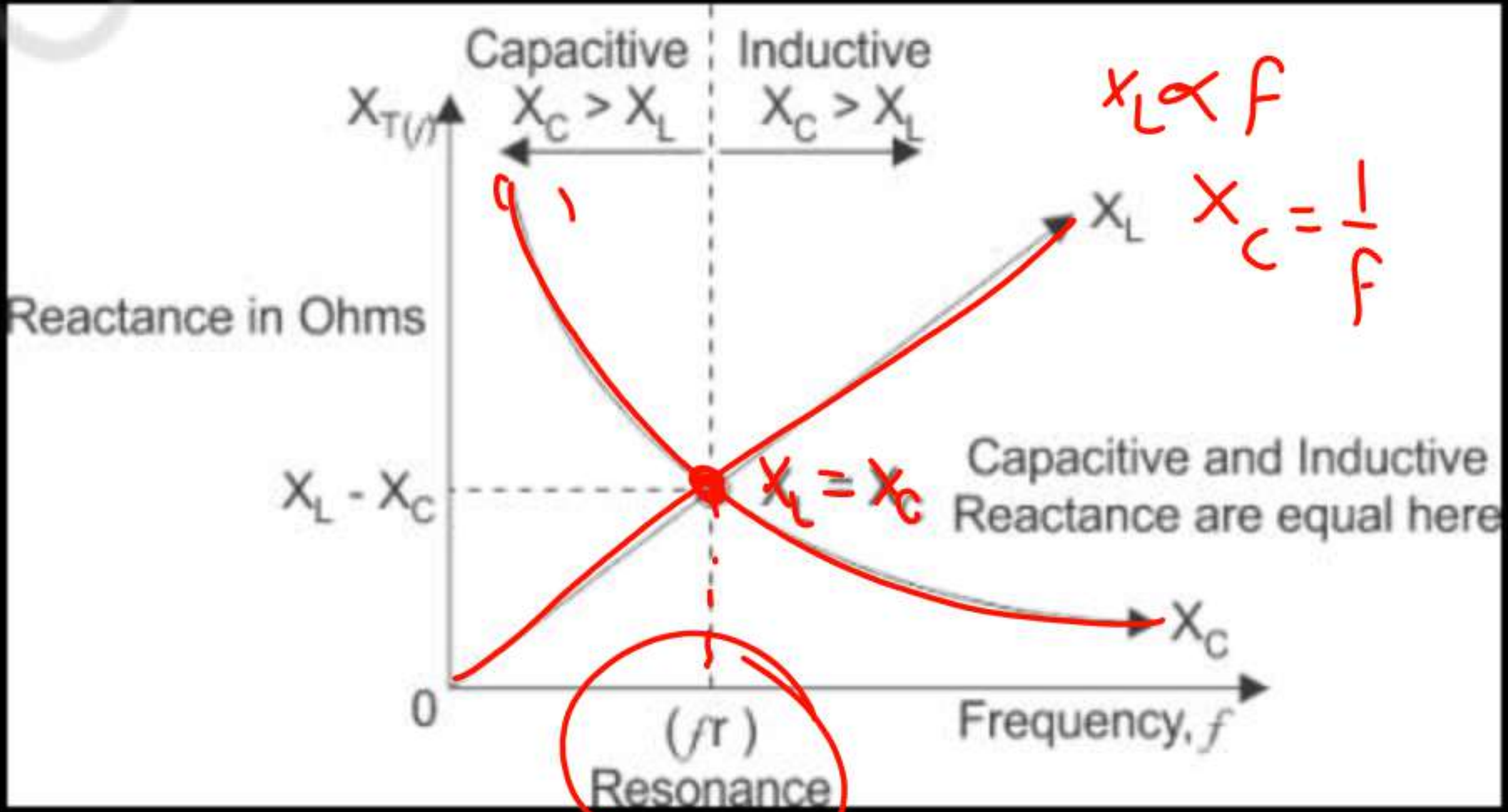
$2\pi f L = \frac{1}{2\pi f C}$

$4\pi^2 f_r^2 = \frac{1}{LC}$

$f_r^2 = \frac{1}{4\pi^2 LC}$

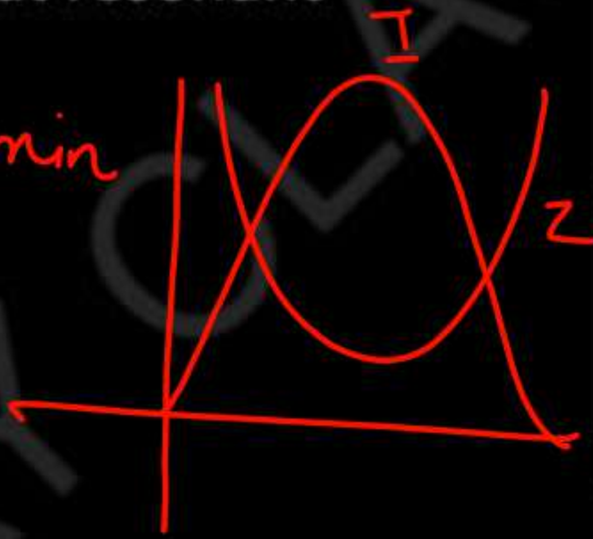
$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$



# Resonance in Series RLC circuit

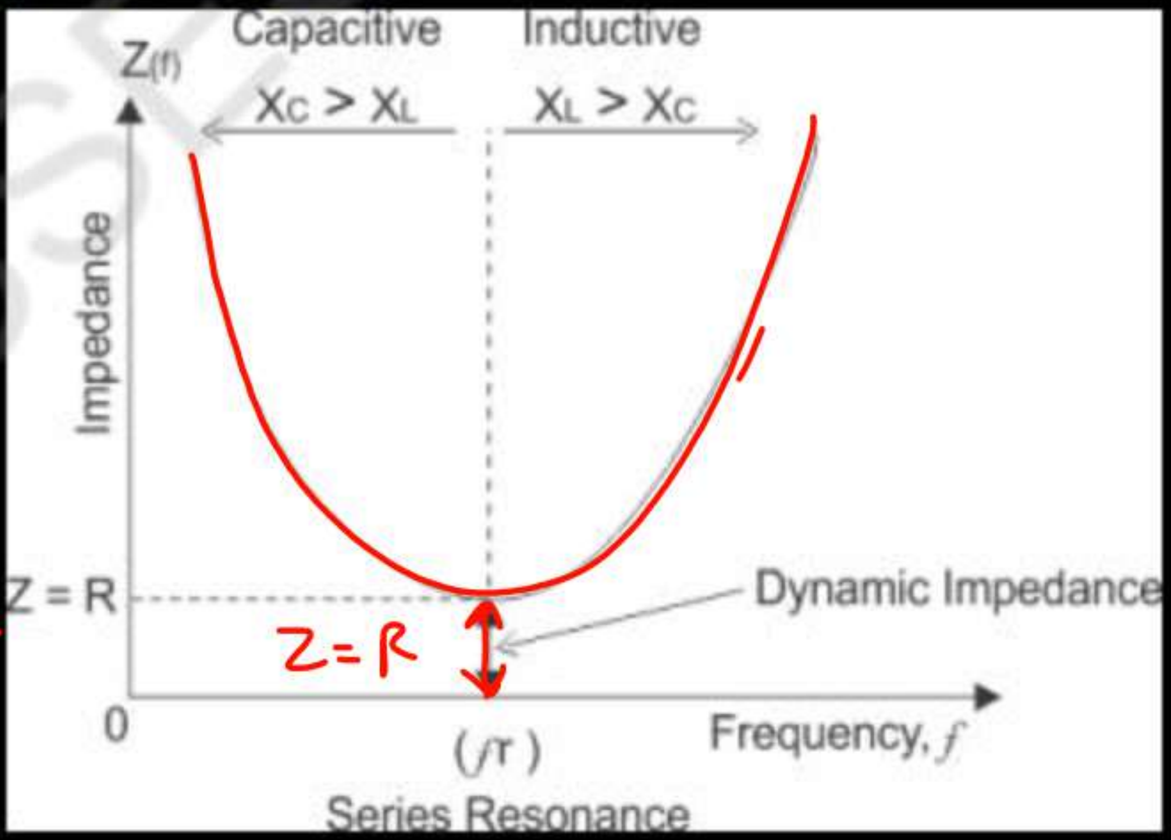
- **Parameters at Resonance Frequency:**
- **(ii) Impedance at Resonance:** At resonance, the total impedance of series RLC circuit is equal to resistance i.e ***Z = R***, impedance has only real part but no imaginary part and this impedance at resonant frequency is called dynamic impedance. $\boxed{Z=R}$ ↓min

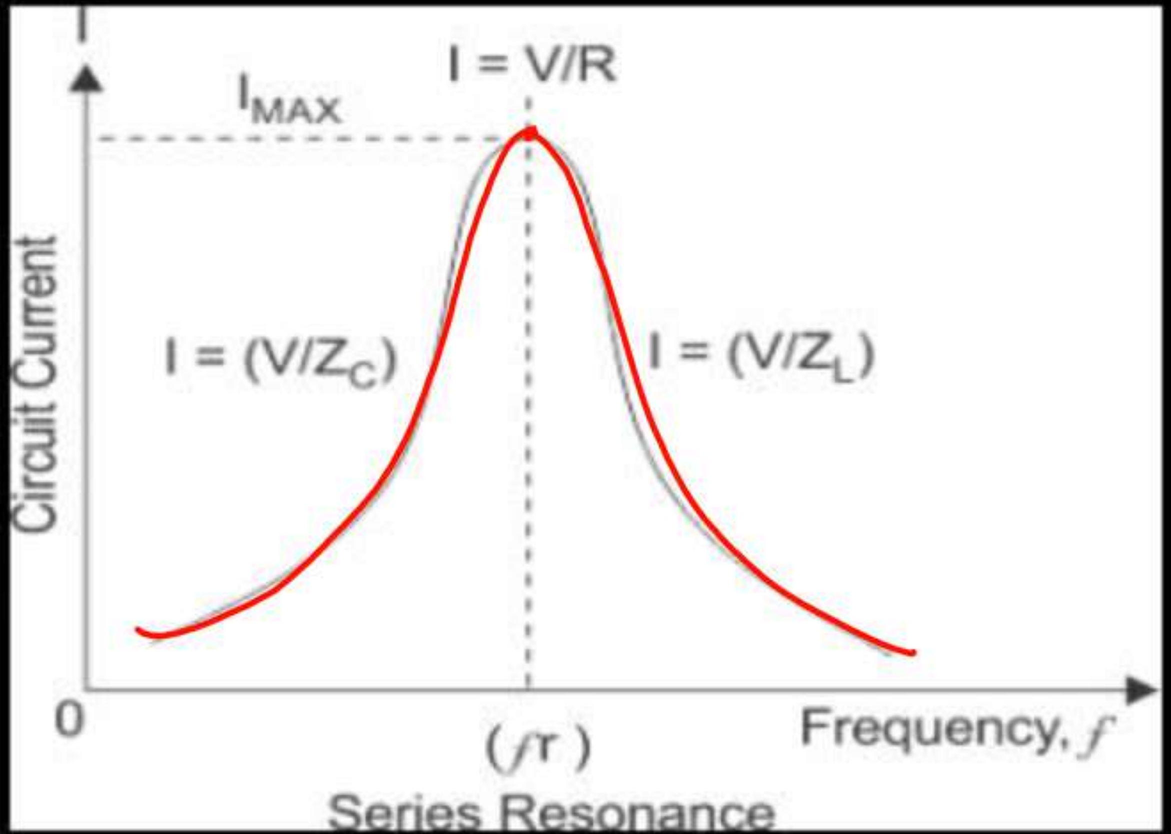



- **(iii) Current at Resonance:** I max. at Reso.
- **(iv) Quality Factor:-** The ratio of a voltage developed across the inductance or Capacitance at resonance to the applied voltage

$$Q = \frac{V_L \text{ or } V_C}{V} = \frac{\cancel{I} X_L}{\cancel{I} R} = \boxed{\frac{\omega L}{R} = Q}$$

**Resonance Curve:**



# Resonance in Series RLC circuit

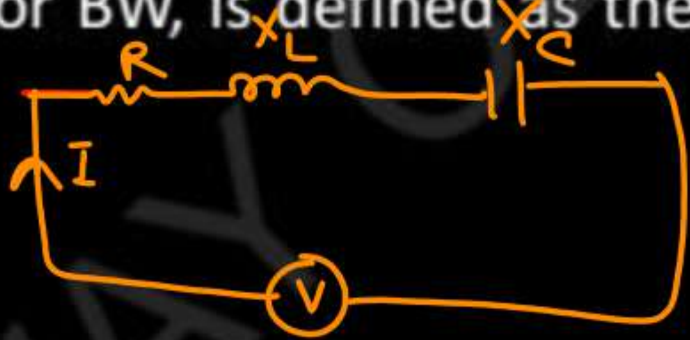
- Series resonance RLC circuit is called as **voltage magnification** circuit, because the magnitude of voltage across the inductor and the capacitor is equal to $Q$ times the input sinusoidal voltage $V$.

- **Application of Series RLC Resonant Circuit**
- Since **resonance in series RLC circuit** occurs at particular frequency, so it is used for filtering and tuning purpose as it does not allow unwanted oscillations that would otherwise cause signal distortion, noise and damage to circuit to pass through it.

# Bandwidth of Series RLC Circuit at resonance

The bandwidth of any system is the range of frequencies for which the current or output voltage is equal to 70.7% of its value at the resonant frequency, and it is denoted by BW.

Frequency $f_1$ is the frequency at which the current is 0.707 times the current at resonant value, and it is called the **lower cut-off frequency**. The frequency $f_2$ is the frequency at which the current is 0.707 times the current at resonant value (i.e. maximum value), and is called the **upper cut-off frequency**. The bandwidth, or BW, is defined as the frequency difference between f2 and f1.




$$\boxed{I = \frac{V}{Z} = \frac{V}{\sqrt{R^2+(X_L-X_C)^2}}}$$

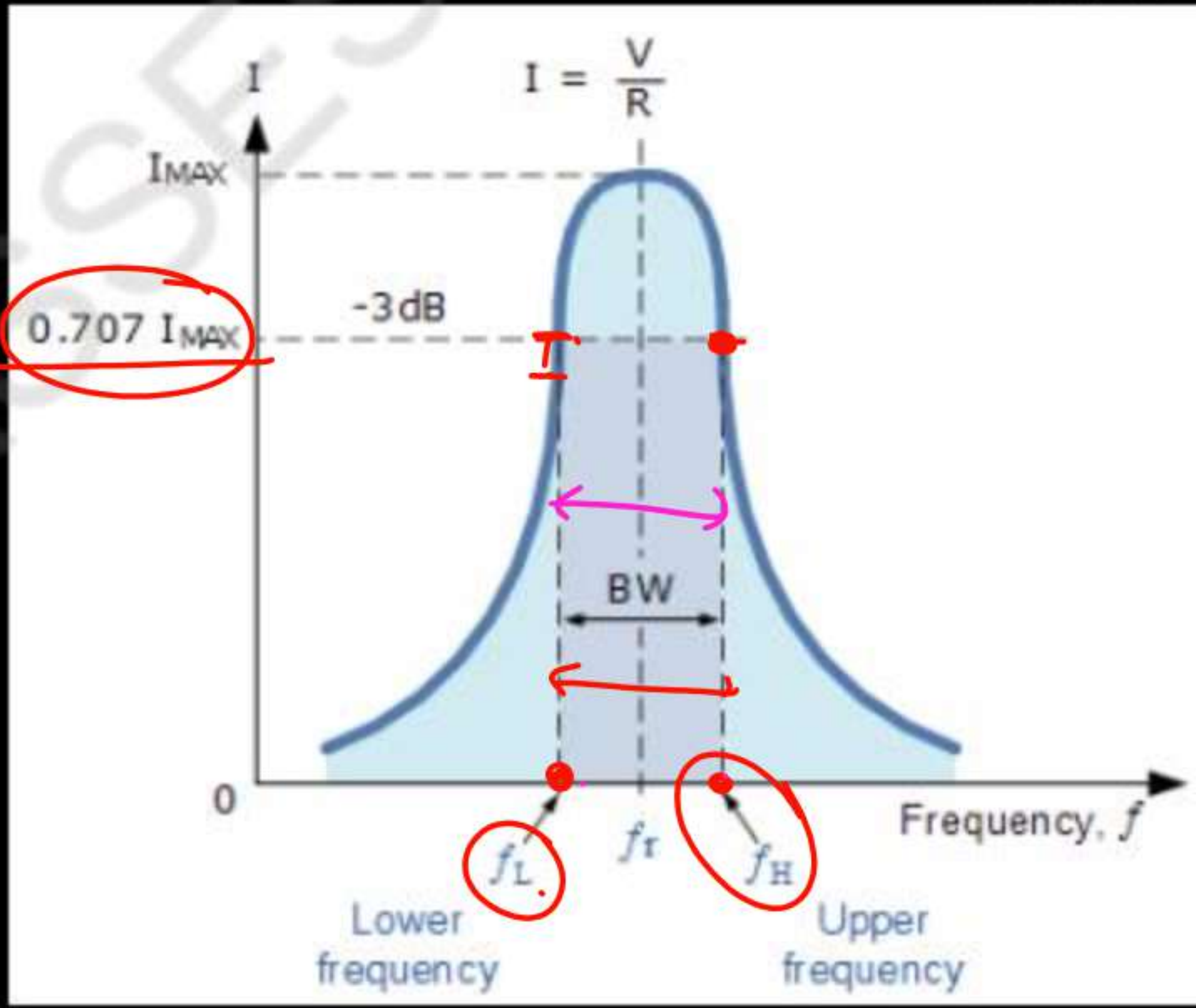
also

$$I = \frac{I_m}{\sqrt{2}}$$

at Resonance $Z = R$

$$I_m = \frac{V}{R}$$

$$\boxed{I = \frac{V}{R\sqrt{2}}}$$



# Bandwidth of Series RLC Circuit at resonance

$$\frac{\cancel{V}}{R\sqrt{2}} = \frac{\cancel{V}}{\sqrt{R^2+(X_L-X_C)^2}}$$

$$\sqrt{R^2+(X_L-X_C)^2} = R\sqrt{2}$$

$$R^2+(X_L-X_C)^2 = 2R^2$$

$$(X_L-X_C)^2 = R^2$$

$$X_L - X_C = \pm R$$

$$\omega L - \frac{1}{\omega C} = \pm R$$

So

$$\omega_1 L - \frac{1}{\omega_1 C} = +R \quad \text{— (1)}$$

$$\omega_2 L - \frac{1}{\omega_2 C} = -R \quad \text{— (2)}$$

Add (1) & 2

$$L(\omega_1+\omega_2) - \frac{1}{C}\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right) = 0$$

$$L(\omega_1+\omega_2) - \frac{1}{C}\left(\frac{\omega_2+\omega_1}{\omega_1\omega_2}\right) = 0$$

$$L - \frac{1}{C\omega_1\omega_2} = 0$$

$$L = \frac{1}{C\omega_1\omega_2}$$

$$\boxed{\omega_1\omega_2 = \frac{1}{LC}}$$

$$4\pi^2 f_1 f_2 = \frac{1}{LC}$$

$$f_1 f_2 = \frac{1}{4\pi^2 LC} = f_r^2$$

$$\boxed{f_r = \sqrt{f_1 f_2} = \frac{1}{2\pi\sqrt{LC}}}$$

Subtract (1) & (2)

$$L(\omega_1-\omega_2) - \frac{1}{C}\left(\frac{1}{\omega_1}-\frac{1}{\omega_2}\right) = 2R$$

$$L(\omega_1-\omega_2) - \frac{1}{C}\left(\frac{\omega_2-\omega_1}{\omega_1\omega_2}\right) = 2R$$

$$L + \frac{1}{C\omega_1\omega_2} = \frac{2R}{\omega_2-\omega_1}$$

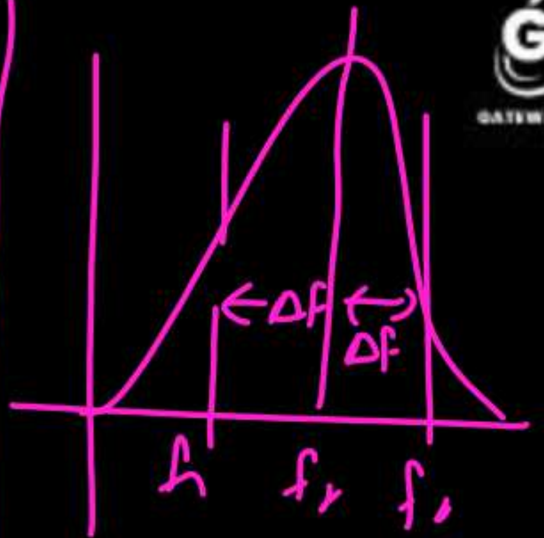
$$L + \frac{1}{\cancel{C}\times\frac{1}{L\cancel{C}}} = \frac{2R}{\omega_2-\omega_1}$$

$$2L = \frac{2R}{\omega_2-\omega_1}$$

$$\omega_2-\omega_1 = \frac{R}{L}$$

$$\boxed{f_2 - f_1 = \frac{R}{2\pi L}}$$

$$\boxed{B.W = \frac{R}{2\pi L}}$$




$$f_1 = f_r - \Delta f = f_r - \frac{R}{4\pi L}$$

$$f_2 = f_r + \Delta f = f_r + \frac{R}{4\pi L}$$

$$\Delta f = \frac{R}{4\pi L}$$

# Resonance in Parallel RLC circuit

## Condition of Resonance in Parallel RLC circuit:-

- **Parallel Resonance** means when the circuit current is in phase with the applied voltage of an AC circuit containing an inductor and a capacitor connected together in parallel.
- **Parameters at Resonance Frequency:**
- **(i) Resonance Frequency**

$\frac{L}{C} = Z_L^2 = R_L^2 + X_L^2$

$Z_L^2 = \left(\sqrt{R_L^2 + X_L^2}\right)^2$

$X_L^2 = \left(\frac{L}{C} - R^2\right) \Rightarrow X_L = \sqrt{\frac{L}{C} - R}$

$$I_C = I_L \sin\varphi_L$$

$$I_L = \frac{V}{Z_L}$$

$$\sin\varphi_L = \frac{X_L}{Z_L} \text{ and } I_C = \frac{V}{X_C}$$

$$\frac{V}{X_C} = \frac{V}{Z_L} \times \frac{X_L}{Z_L} \text{ or } X_L X_C = Z_L^2 \text{ or}$$

$$\frac{\omega L}{\omega C} = Z_L^2 = (R^2 + X_L^2) \text{ or}$$

$$\frac{L}{C} = R^2 + (2\pi f_r L)^2 \text{ or}$$

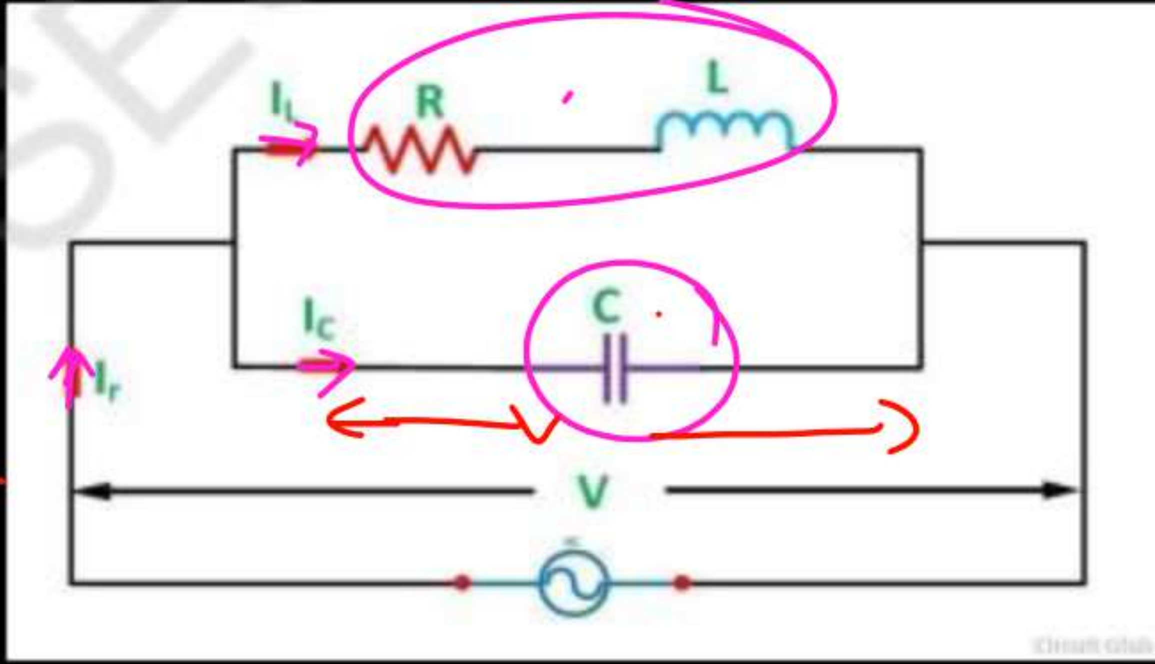
$$2\pi f_r L = \sqrt{\frac{L}{C} - R^2} \text{ or}$$

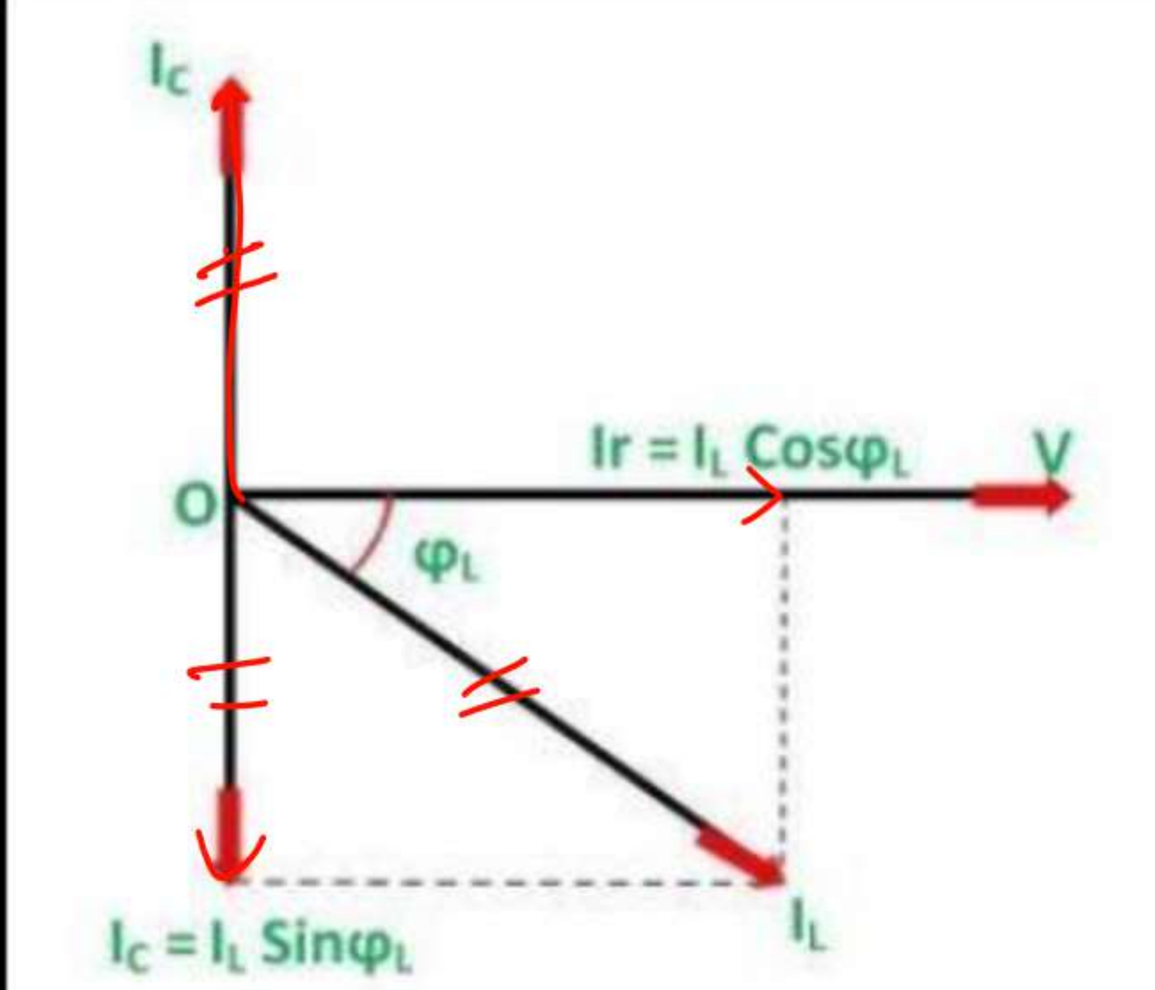
$$f_r = \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R^2} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

neglect

If R is very small as compared to L, then

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$



# Resonance in Series RLC circuit

- **(ii) Impedance at Parallel Resonance:**

$I_L \cos\phi = I_r$

- At parallel resonance line current $I_r = I_L \cos\varphi$ or

$$\frac{V}{Z_r} = \frac{V}{Z_l} \times \frac{R}{Z_l} \quad \text{or} \quad \frac{1}{Z_r} = \frac{R}{Z_L^2} \quad \text{or}$$

$$\frac{1}{Z_r} = \frac{R}{L/C} = \frac{CR}{L} \quad \left(\text{as } Z_L^2 = \frac{L}{C}\right)$$

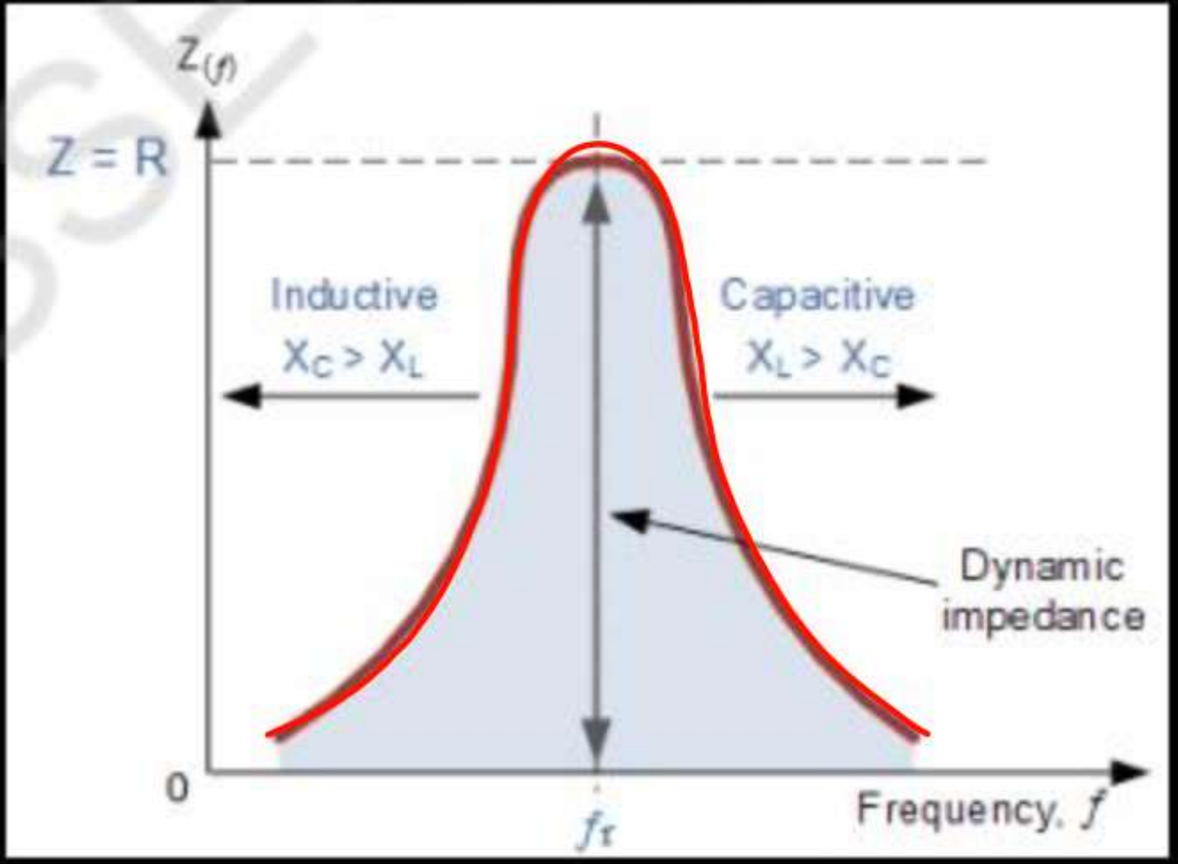
$$Z_r = \frac{L}{CR}$$

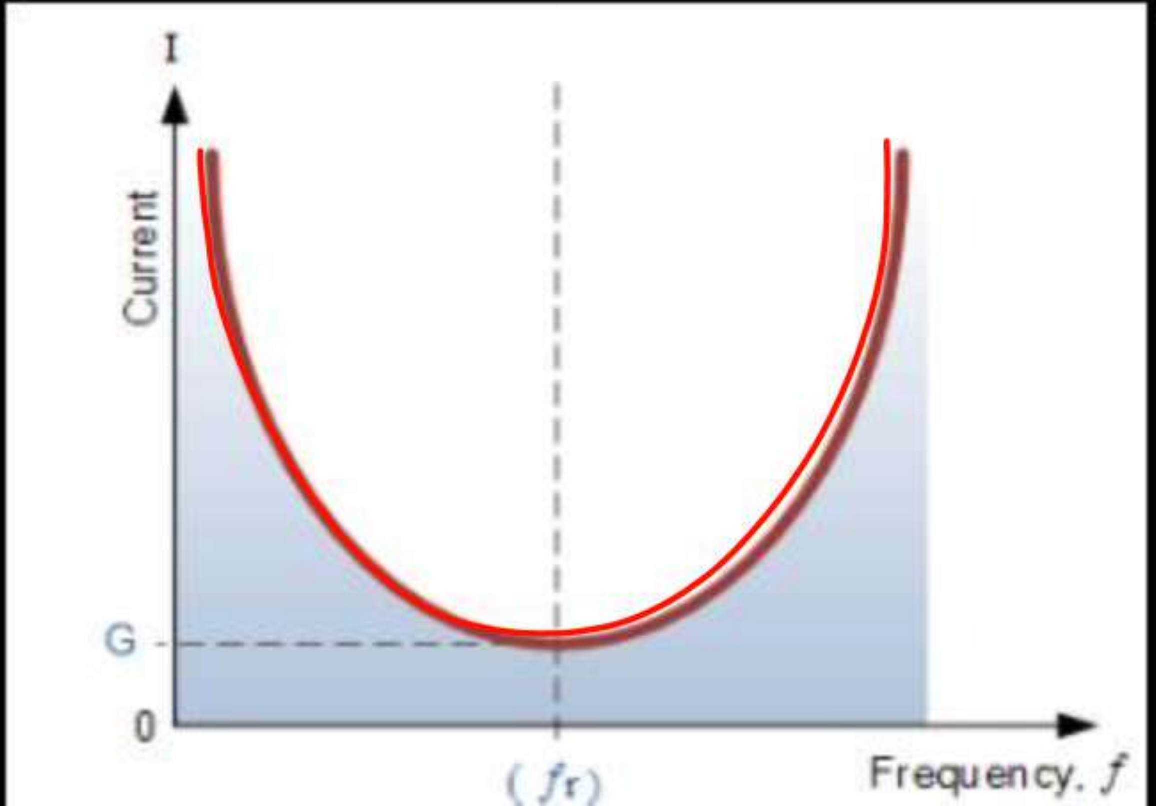
Imp = max.

Current = min

- The circuit impedance is purely resistive because there is no frequency term present in it. Circuit impedance $Z_r$ will be in Ohms.
- The value of **Zr will be very high** because the ratio L/C is very large at parallel resonance.
- The value of ***circuit current, Ir = V/Zr is very small*** because the value of Zr is very high.
- parallel resonant circuit can draw a very small current and power from the mains, therefore, it is also called as ***Rejector Circuit.***

## Resonance Curve:



# Quality Factor at Parallel Resonance

- The ratio of a Current across the inductance or Capacitance at resonance to the Current at resonance.
- Parallel resonance RLC circuit is called as **Current magnification** circuit.

$$Q = \frac{I_{L-C}}{I} = \frac{\cancel{V}/X_L}{\cancel{V}/Z_r} = \frac{Z_r}{X_L} = \frac{\cancel{L}/RC}{\omega \cancel{L}} = \boxed{\frac{1}{\omega RC} = Q}$$

$$= \frac{\cancel{V}/X_C}{\cancel{V}/Z_r} = \frac{Z_r}{X_C} = \frac{L/R\cancel{C}}{1/\omega \cancel{C}} = \boxed{\frac{\omega L}{R} = Q}$$

# Numerical Examples

- Ex.-18 A Series RLC circuit consisting a resistance of 20 ohm , inductance of 0.2 H and Capacitance of 150μF is connected across 230V, 50 Hz source. Calculate (i) Impedance (ii) Current (III) Power Factor (iv) Frequency Required to Make power factor Unity (v) Quality Factor AKTU- 2012-13

Resonance

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

$$= \frac{1}{2\pi\sqrt{0.2 \times 150 \times 10^{-6}}} = ?$$

$$Q = \frac{\omega L}{R} = \frac{2\pi f_r \cdot 0.2}{20}$$ Ans

# Numerical Examples (Try Yourself)

- Ex.-19. A 20 ohm resistor is connected in series with an inductor and a capacitor across a variable frequency 25 V supply. When the Frequency is 400 Hz , the current is at the maximum value of 0.5 A and the potential difference across the capacitor is 150 V Calculate the resistance and inductance of the inductor. AKTU- 2001-02

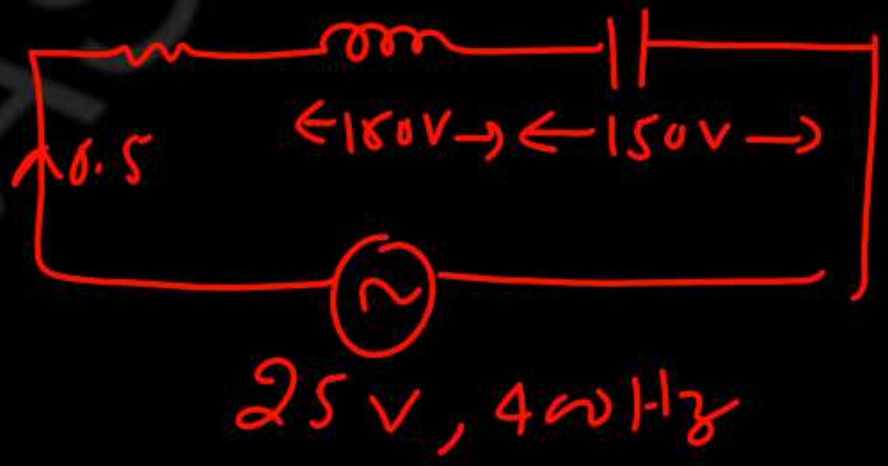
$I_m = 0.5$

$f_r = 400\,Hz$ , $V = 25$ volt

$Z = R = \frac{V}{I} = \frac{25}{0.5} = 50\,\Omega$

$X_L = \frac{V_L}{I} = \frac{150}{0.5} = 300\,\Omega = X_C$

$X_L \downarrow L$ $\quad$ $X_C \downarrow C$



# Numerical Examples

- **Ex.-20.** A series R – L – C circuit consists of R = 1000 Ohm, L = 100 mH and C = 10 μF. The applied voltage across the circuit is 100 V.
  (i) Find the resonant frequency of the circuit.
  (ii) Find the quality factor of the circuit at the resonant frequency.
  (iii) At what angular frequencies do the half power points occur?
  (iv) Calculate the bandwidth of the circuit.

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$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$Q = \frac{\omega L}{R}$$

$$f_1 = f_r + \Delta f = f_r + \frac{R}{4\pi L}$$

$$f_2 = f_r - \Delta f = f_r - \frac{R}{4\pi L}$$

$$B.W = f_2 - f_1$$

# Numerical Examples

- Ex.-21. A Series circuit consists of a resistance of 10 ohm and inductance of 50mH and a variable capacitance in series across a 100V , 50 Hz supply. Calculate (i) Value of capacitance to produce resonance (ii) Voltage across the capacitance (iii) Q- Factor. AKTU 2018-19

$X_L = 2\pi \times 50 \times 50\times10^{-3} = 15.70\Omega$

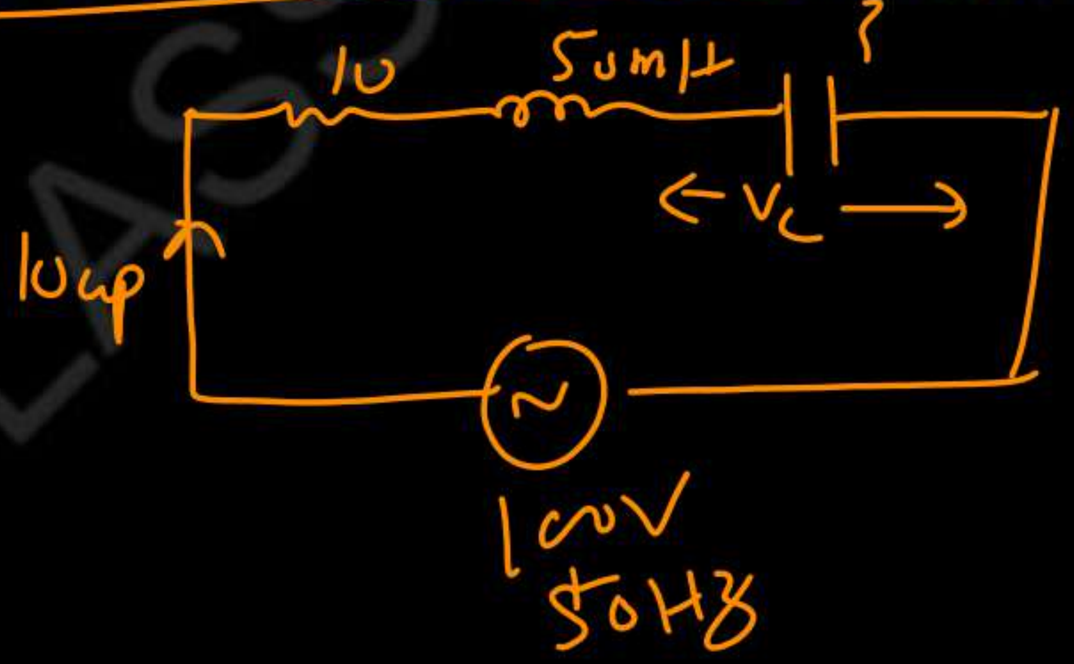
At Resonance.

$Z = R = 10\Omega$

$I_m = \frac{V}{Z} = \frac{100}{10} = 10$ amp

$V_L = IX_L = V_C$

$X_C = \frac{V_C}{I} = ?$



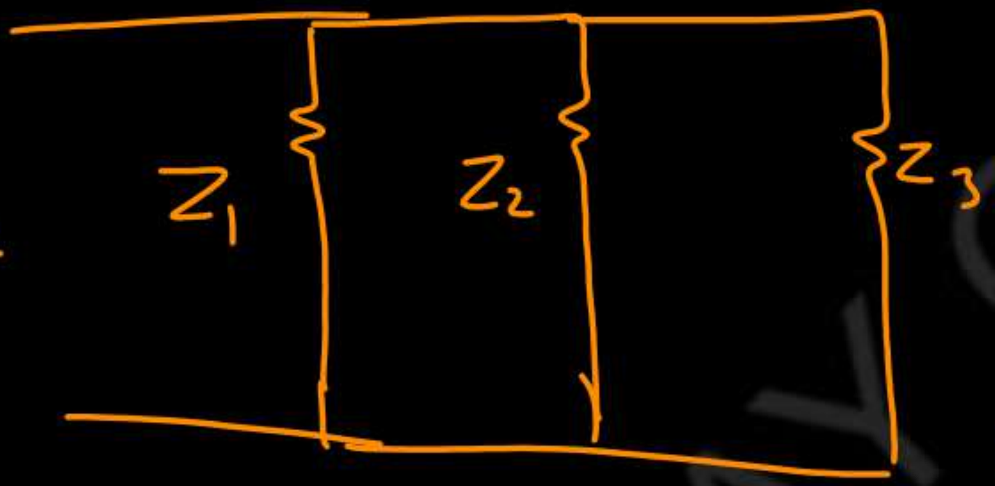
# Concept of Admittance

$\Lambda^{-1}$ $x^{-1}$

- **Admittance:-**

$$Y = \frac{1}{Z} = \frac{1}{\sqrt{R^2+X_L^2}}$$

$Z_{eq} =$





$$(2+3j)+(5+6j)+(7+8j)$$

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}$$

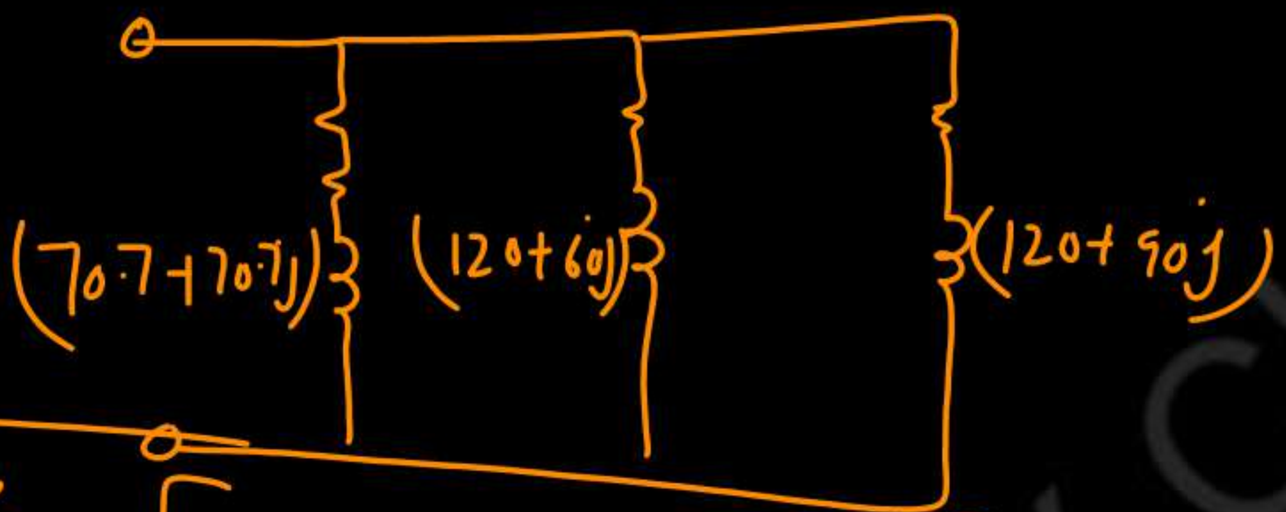
$$= Z_1^{-1}+Z_2^{-1}+Z_3^{-1}+\cdots$$

$$\frac{1}{Z_{eq}} = \left[(2+3j)^{-1}+(5+6j)^{-1}+(7+8j)^{-1}\right]$$

$$\boxed{Z_{eq} = [\text{Ans}]^{-1}}$$

# Numerical Examples

- Ex.-23. Three impedances of (70.7+j70.7)ohm, (120+j160)ohm and (120+j90) ohm are connected in parallel across a 250 V supply. Determine (i) admittance of the circuit (ii supply Current and (iii) power factor AKTU-2021-22




$$\boxed{Y_{eq} = \frac{1}{Z_{eq}} = \left[Z_1^{-1} + Z_2^{-1} + Z_3^{-1}\right]} =$$

$$\boxed{Z_{eq} = Any^{-1}}$$

$$I = \frac{V}{Z} = V \cdot Y_{eq}$$

$$I = r\angle\theta$$

$$\cos\varphi = p.f.$$

# Numerical Examples

- **Ex.-24. Two coils having resistance 5 Ω and 10 Ω and inductances 0.04 H and 0.05 H respectively are connected in parallel across a 200 V, 50 Hz supply. Calculate: i. Conductance, susceptance and admittance of each coil. ii. Total current drawn by the circuit and its power factor. iii. Power absorbed by the circuit.**

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$G = \frac{1}{R}$ , $S = \frac{1}{X}$ , $Y = \frac{1}{Z}$

$Z_{eq} = R + jX_L$

$X_{L_1} = 2\pi \times 50 \times 0.04 = 12.56\Omega$

$X_L = 2\pi \times 50 \times 0.05 = 15.70\Omega$

$Z_1 = 5 + 12.56j$

$Z_2 = 10 + 15.76j$

$Y_{eq} = Z_1^{-1} + Z_2^{-1}$

$= (5 + 12.56j)^{-1} + (10 + 15.76j)^{-1}$

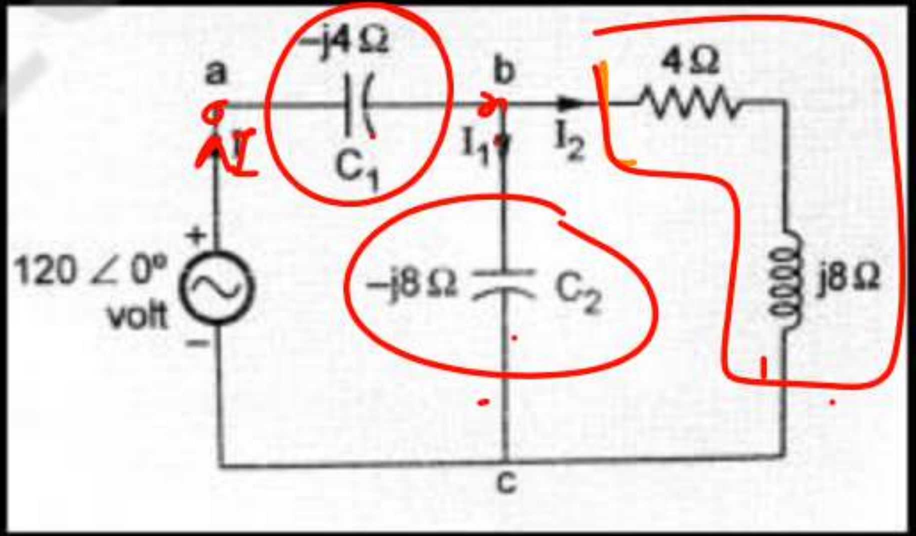
$= (\underbrace{0.027}_{B} - \underbrace{0.068}_{S} j)$

# Numerical Examples (Try Yourself)

- Ex.-25 In series parallel circuit A and B are in parallel and in series with C . The impedances are $Z_A$ = 4+3j ohm, $Z_B$= 4-j5 ohm and $Z_c$ = 2+j8 ohm . If the current Ic= 25+j0 amp Calculate (i) Branch voltage (ii) Branch Currents (iii) Total Power (iv) Phasor Diagram AKTU 2015-16

# Numerical Examples

- **Ex.-26. In the circuit shown in figure find (i) Current in each branch (ii) Voltage $V_{ab}$ and $V_{bc}$ (iii) Power Loss in the circuit.** AKTU- 2001-02




$Z_1 = 4 + 8j$ , $Z_2 = -8j$ , $Z_3 = -4j$

$Z_{eq} = (Z_1 \| Z_2) + Z_3$

$=$

$\boxed{Z_{eq} = 16 - 12j}$

$I = \frac{V}{Z_{eq}} = \frac{120}{16-12j}$

$V_{ab} = (-j4)(6+3j)$

Current distribution

$I_1 = I \times \frac{(4+8j)}{(4+8j-j8)}$

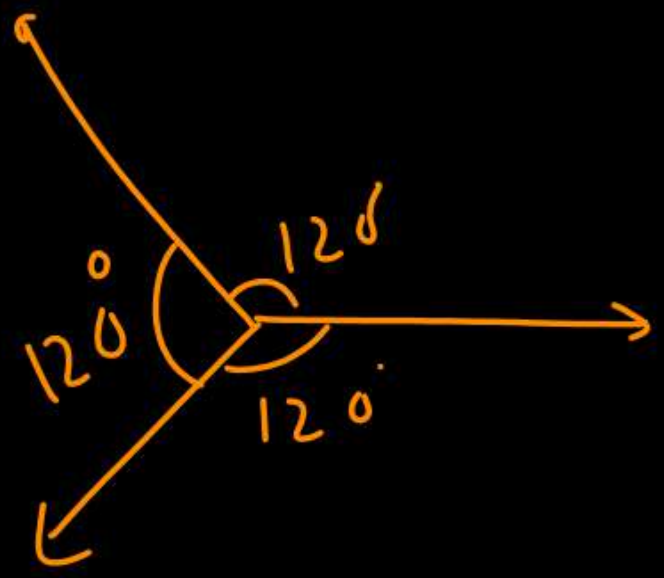
$I_1 = (16-12j)(1+2j)$

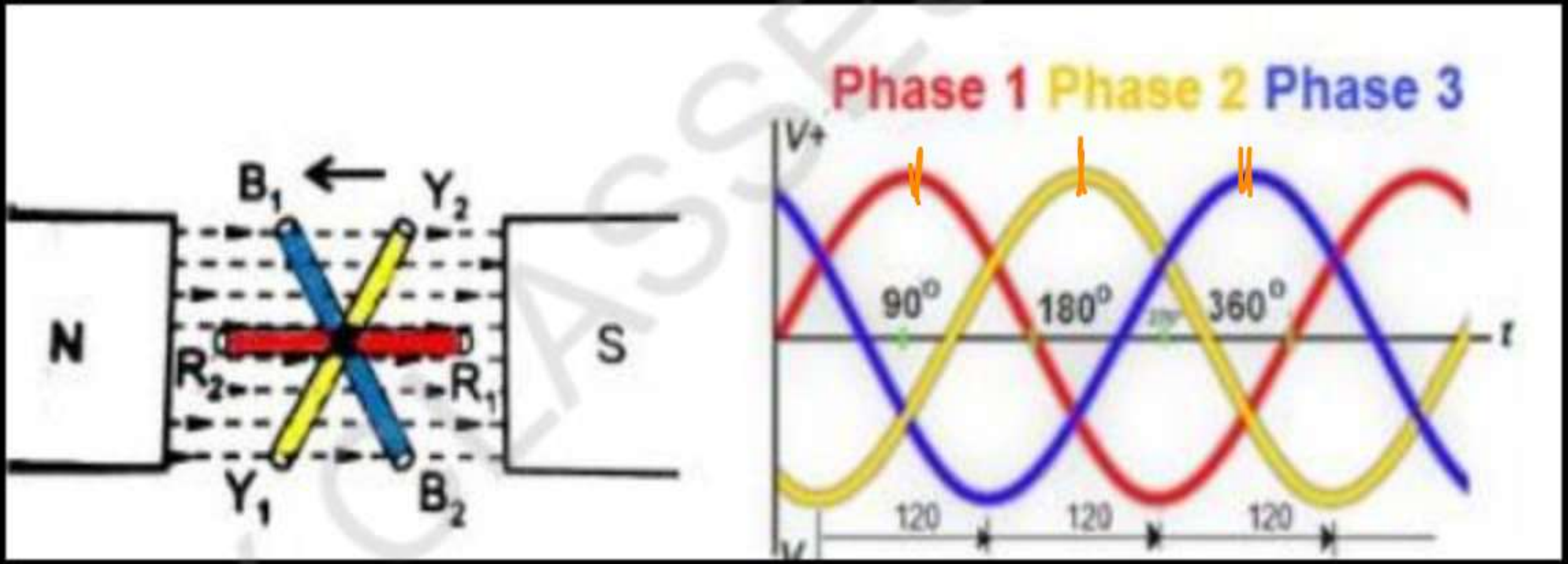
$V_{bc} = I_1 \times (-8j)$

$P = I^2 R$

$= I^2 \times 16$

# Generation of Three Phase Supply



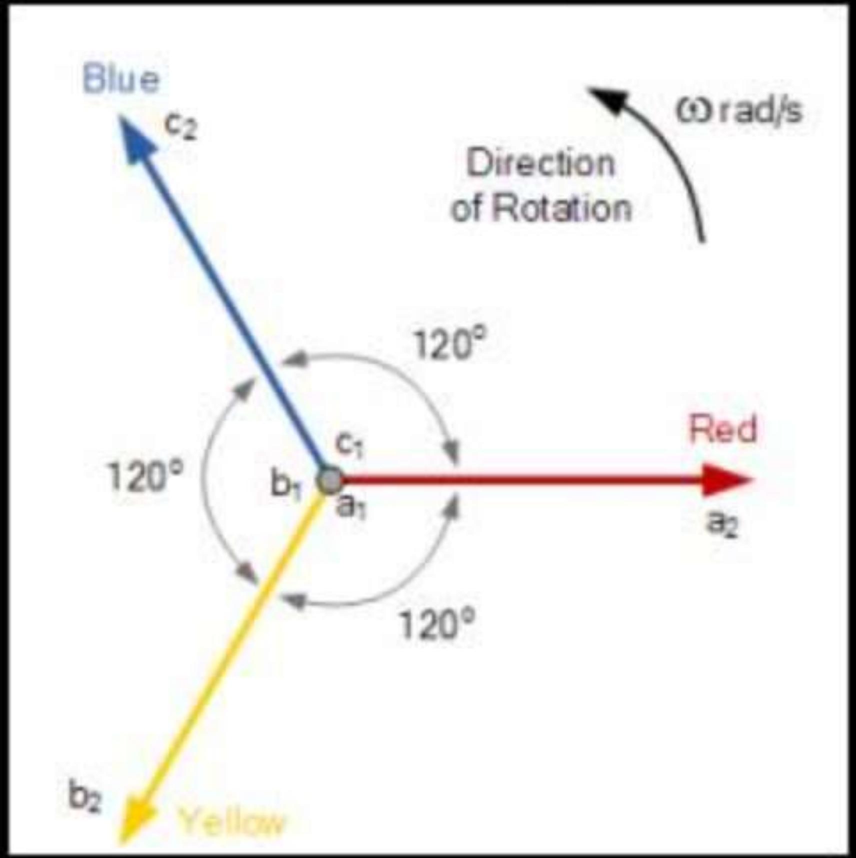




$E_R = E_m \sin\omega t$

$E_Y = E_m \sin(\omega t - 120°)$

$E_B = E_m \sin(\omega t - 240°)$

Sum of all three phasors at any instant is always zero



# Important Definitions

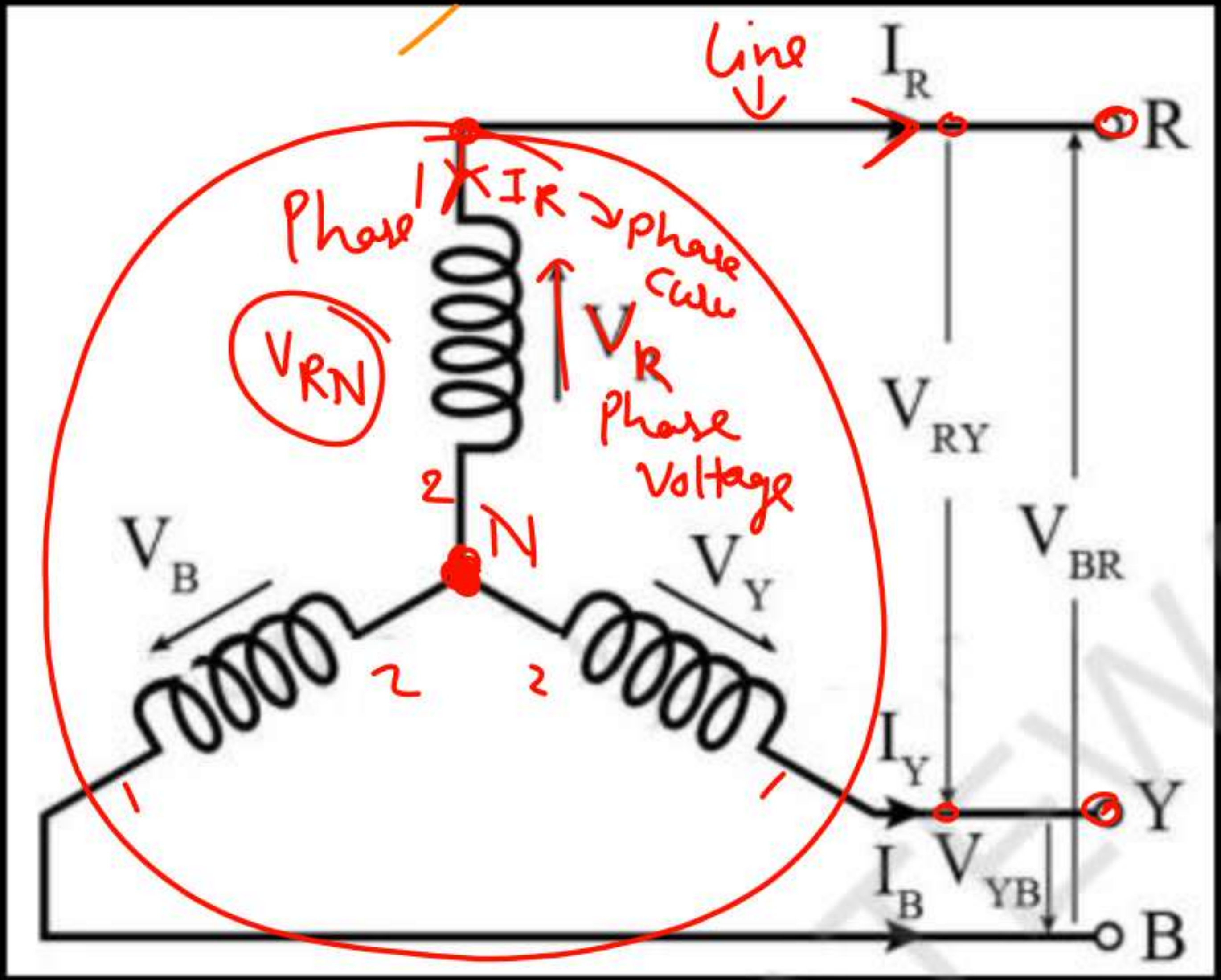
R-Y-B

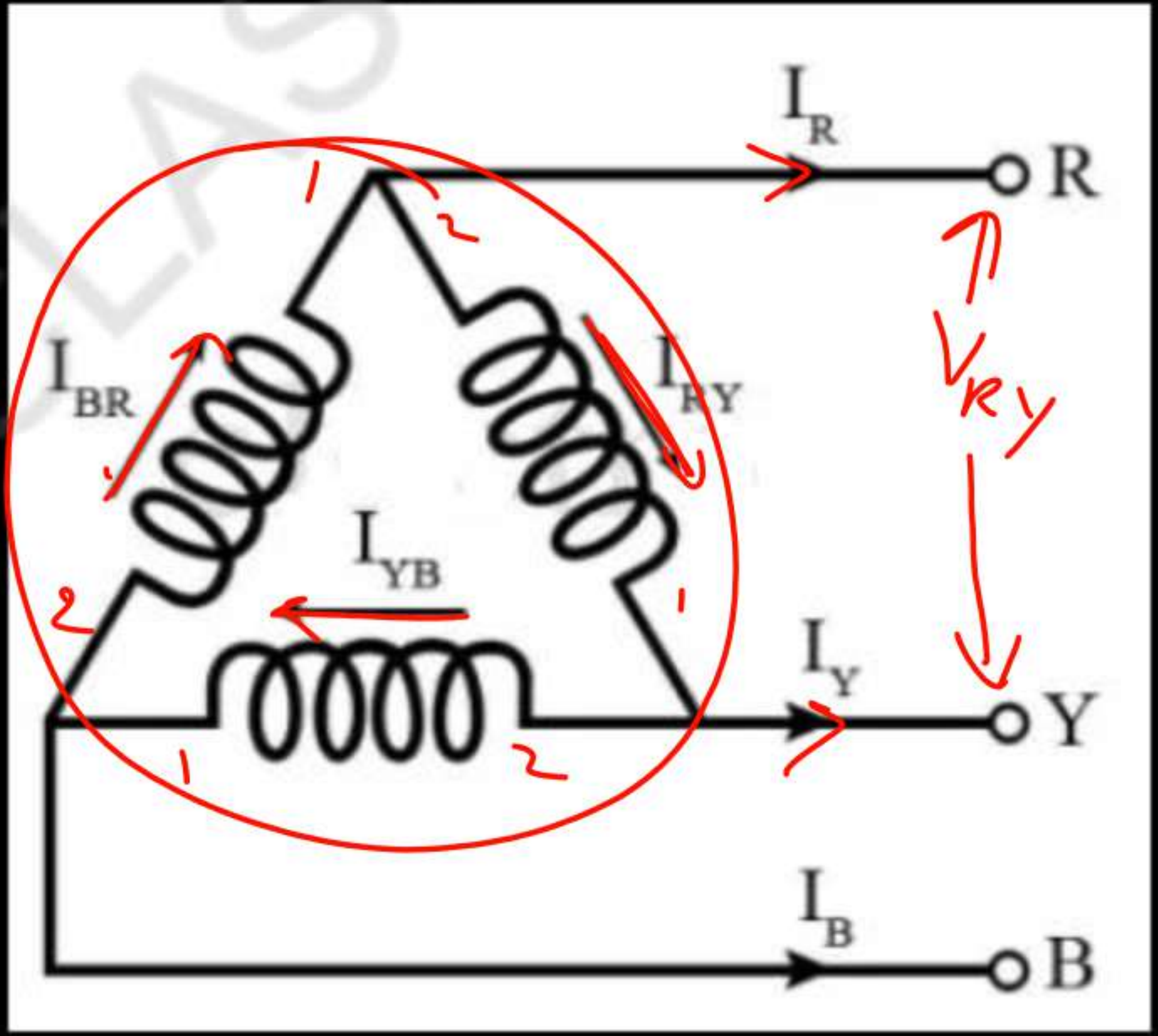
- **Phase Sequence** – The *phase sequence* is defined as the order in which the emf in three phase or coils of an alternator attains the positive maximum value. It is determined by the direction of rotation of alternator.
- **Naming the Phases** – The name of the three phases are given by the natural colors viz. Red (R), Yellow (Y) and Blue (B). Thus, the phase sequence being *RYB*.
- **Balanced Three-Phase Supply System** – In a balanced three-phase supply system, the three phase voltages are equal in magnitude and frequency.

## Advantages of Three Phase Supply

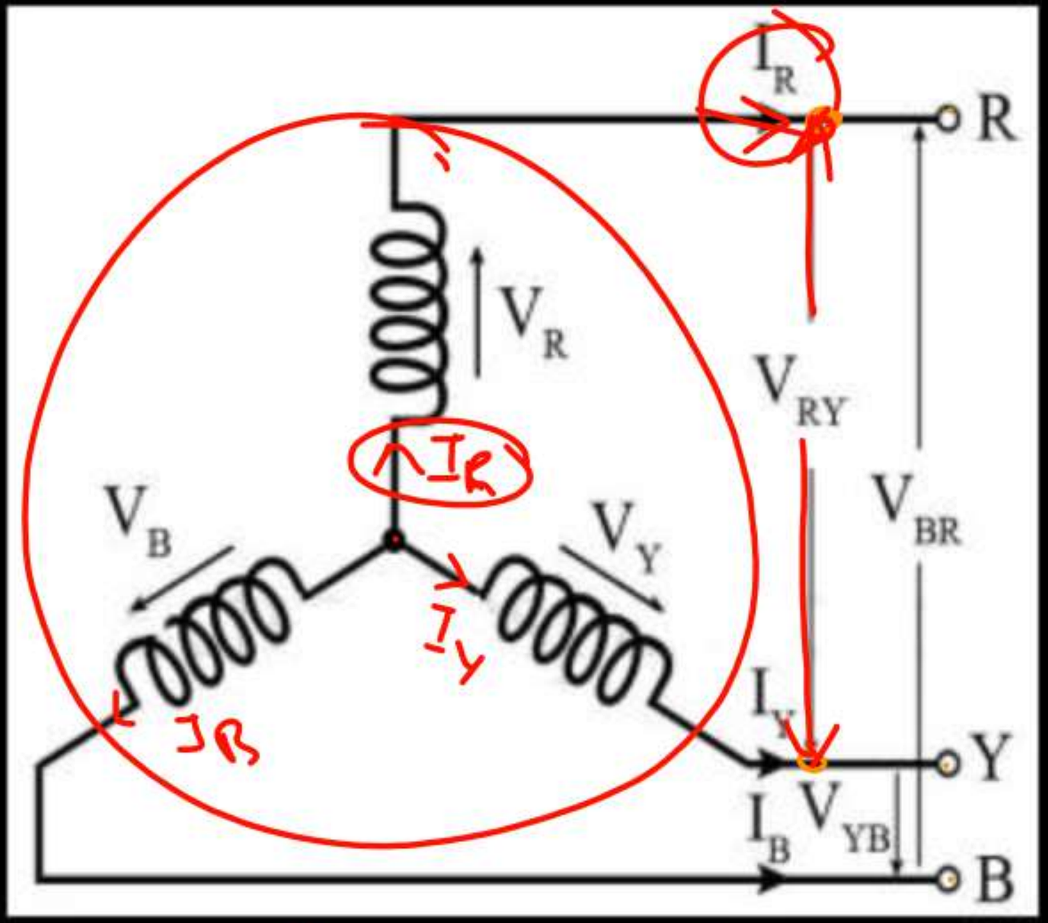
- The three phase power has a constant magnitude while the power in single phase system is the function of supply frequency i.e. pulsates from zero to maximum value at twice the supply frequency.
- In the 3-phase system, a rotating magnetic field can be created in stationary windings.
- For the same rating, three phase machines are smaller and simpler in construction
- To transmit the same amount of power over certain distance at a given voltage, the 3-ϕ system requires less (or 3/4) of the weight of copper than that required by the 1-ϕ system.
- The voltage regulation of three phase transmission lines is better than that of 1-phase line.
- A single phase load can by supplied by a three phase system but, the converse is not true.

# Types of Connections (Line and Phase Values)



# Relation Between Line and phase Values in Star Connection




In Case of Y Connect

$$\boxed{I_{ph} = I_L}$$

Line voltage $V_L = V_{RY} = V_{YB} = V_{BR}$

$V_{ph} = V_{RN} = V_{YN} = V_{BN}$

$V_{RY} = V_{RN} + V_{NY}$

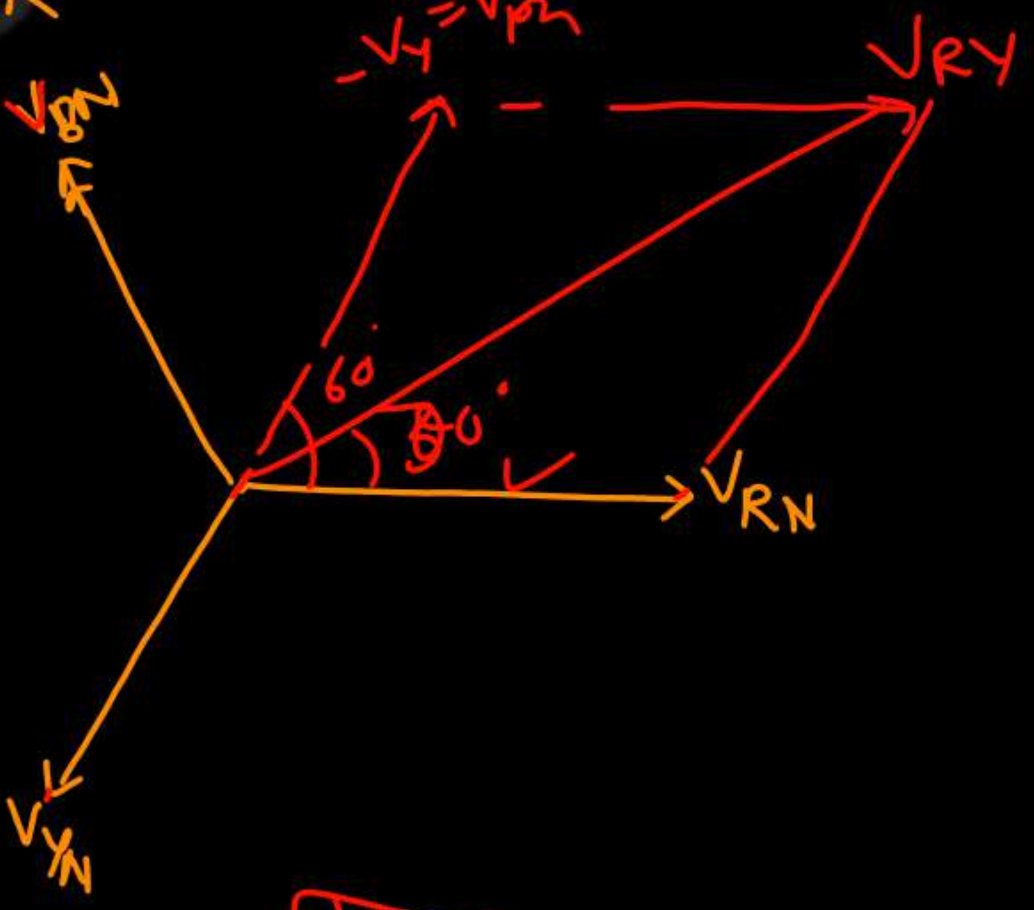
$= V_{RN} - V_{YN}$

$= V_R - V_Y$

$$V_{RY} = \sqrt{|V_R|^2 + |V_Y|^2 + 2|V_R||V_Y|\cos 60}$$

$$= \sqrt{V_{ph}^2 + V_{ph}^2 + 2V_{ph}V_{ph}\cos 60}$$

$$V_L = \sqrt{V_{ph}^2 + V_{ph}^2 + 2V_{ph}^2 \times \frac{1}{2}} = \sqrt{3}\, V_{ph}$$




$$\boxed{V_L = \sqrt{3}\, V_{ph}}$$

Power $= 3 V_{ph} I_{ph} \cos\phi$

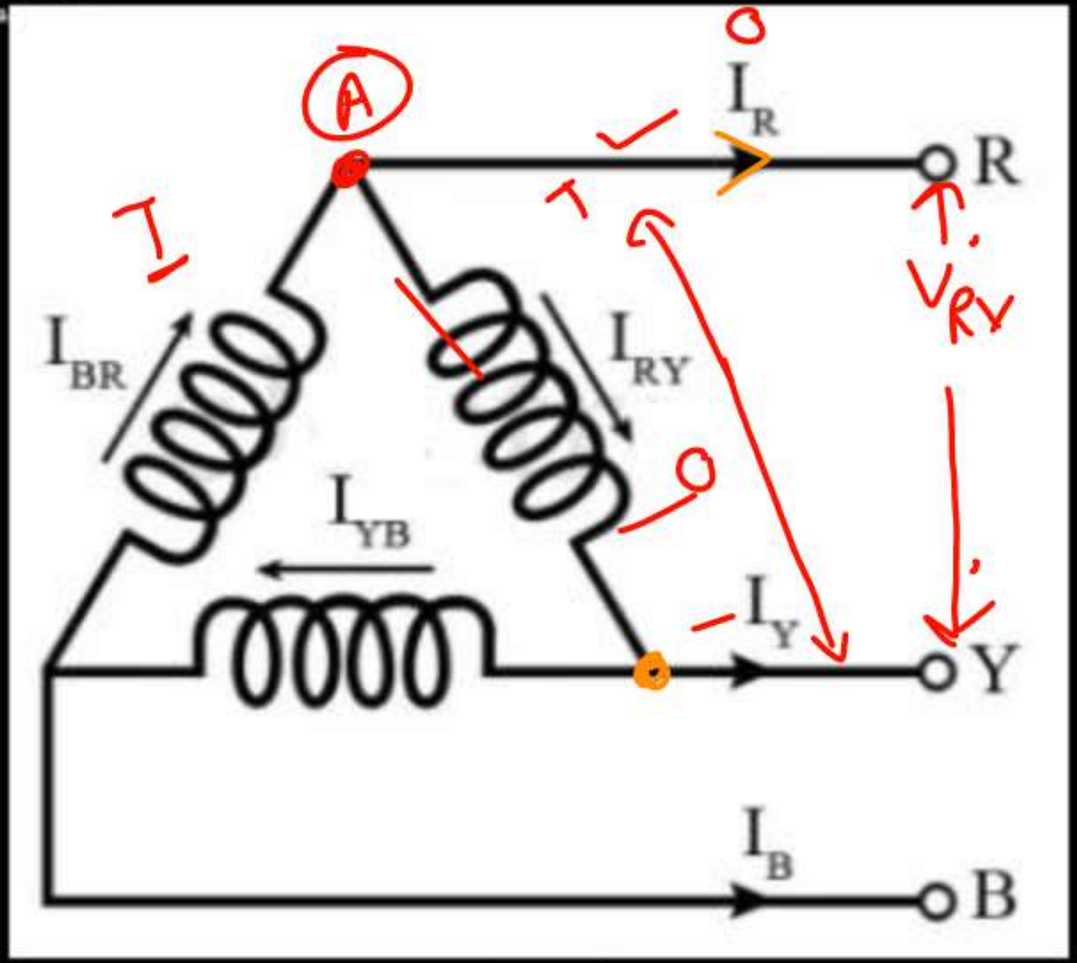
$= 3 \frac{V_L}{\sqrt{3}} I_L \cos\phi$

$$\boxed{P = \sqrt{3} V_L I_L \cos\phi}$$

$S = \sqrt{3} V_L I_L$

$Q = \sqrt{3} V_L I_L \sin\phi$

# Relation Between Line and phase Values in Delta Connection

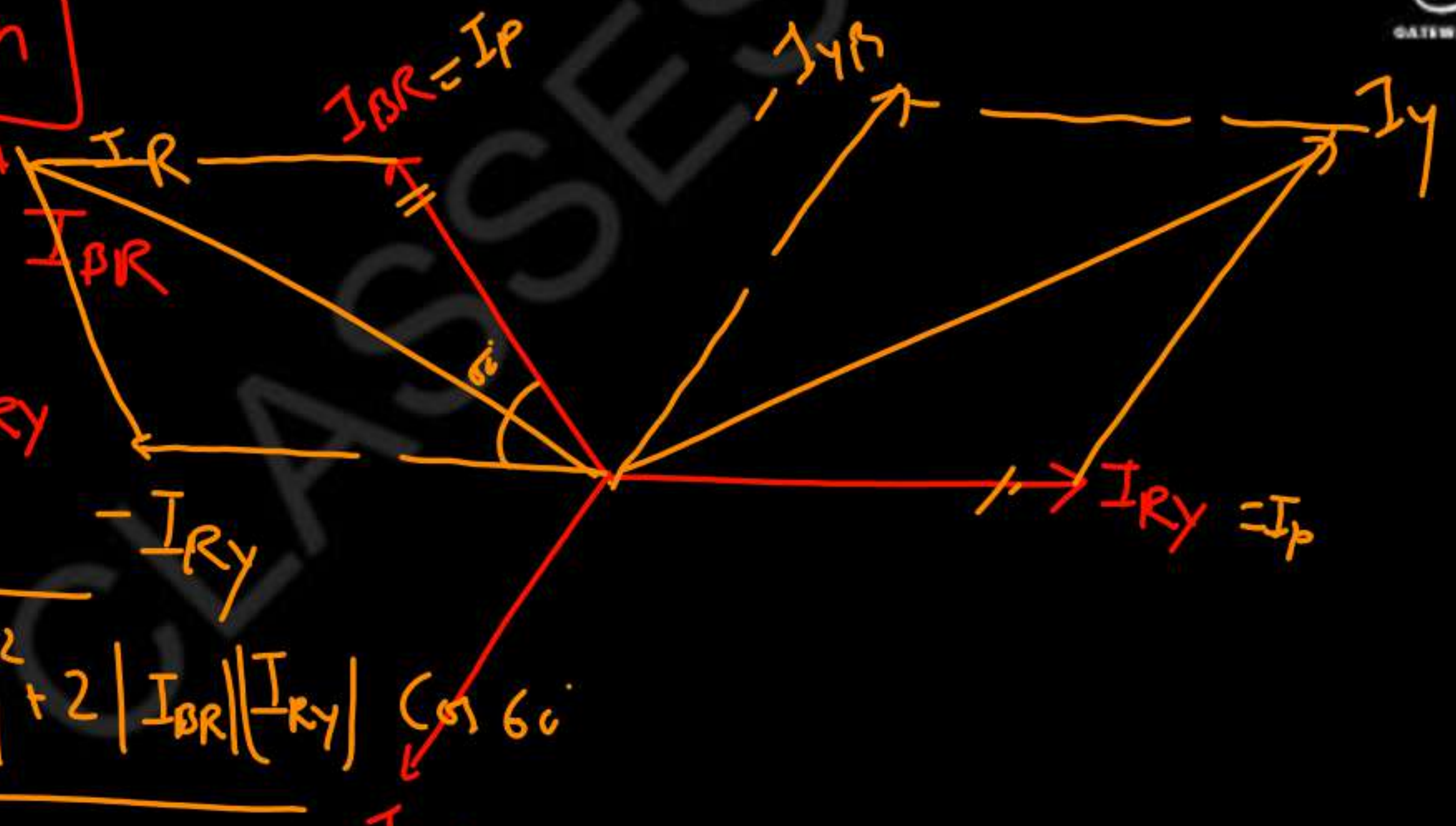



$$\boxed{V_L = V_{ph}}$$

KCL at Node A

$$I_R + I_{RY} = I_{BR}$$

$$I_R = I_{BR} - I_{RY}$$




$$I_R = \sqrt{|I_{BR}|^2 + |I_{RY}|^2 + 2|I_{BR}||I_{RY}| \cos 60°}$$

$$I_L = \sqrt{I_P^2 + I_P^2 + \cancel{2} I_P^2 \times \frac{1}{\cancel{2}}}$$

$$\boxed{I_L = \sqrt{3}\, I_{ph}}$$

$$I_Y = I_{RY} - I_{YB}$$

$$P = 3 V_P I_P \cos\phi$$

$$P = 3 V_L \frac{I_L}{\sqrt{3}} \cos\phi$$

$$= \sqrt{3} V_L I_L \cos\phi$$

# Power Relations in Star and Delta connection

$P = \sqrt{3} V_L I_L \cos\phi$
$= 3 V_{ph} I_{ph} \cos\phi$ KW

$Q = \sqrt{3} V_L I_L \sin\phi$
$= 3 V_{ph} I_{ph} \sin\phi$ KVAR

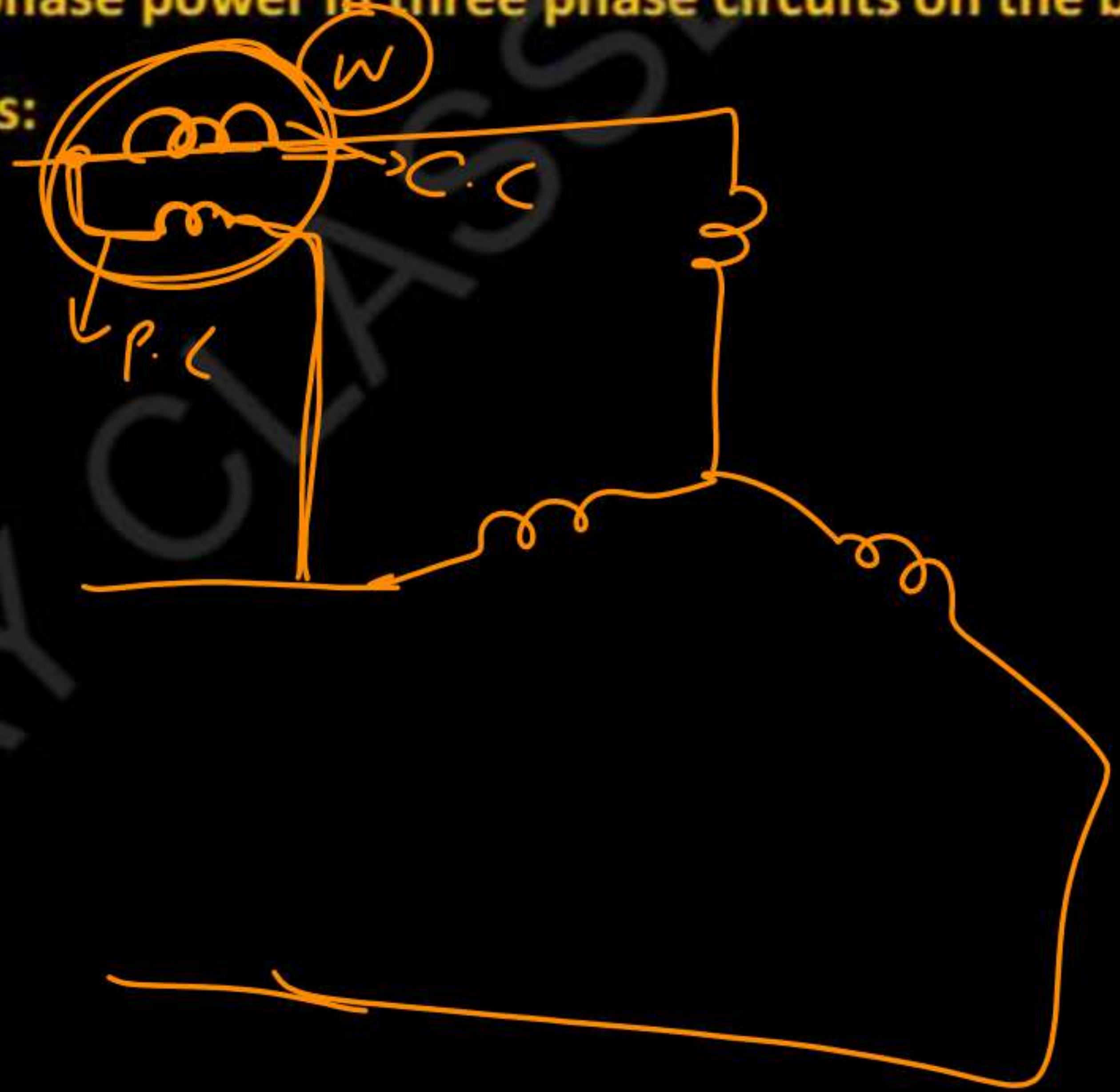
$S = \sqrt{3} V_L I_L$
$= 3 V_{ph} I_{ph}$ KVA

# Power Measurement by wattmeter

Various methods are used for measurement of three phase power in three phase circuits on the basis of number of wattmeters used. We have three methods to discuss:

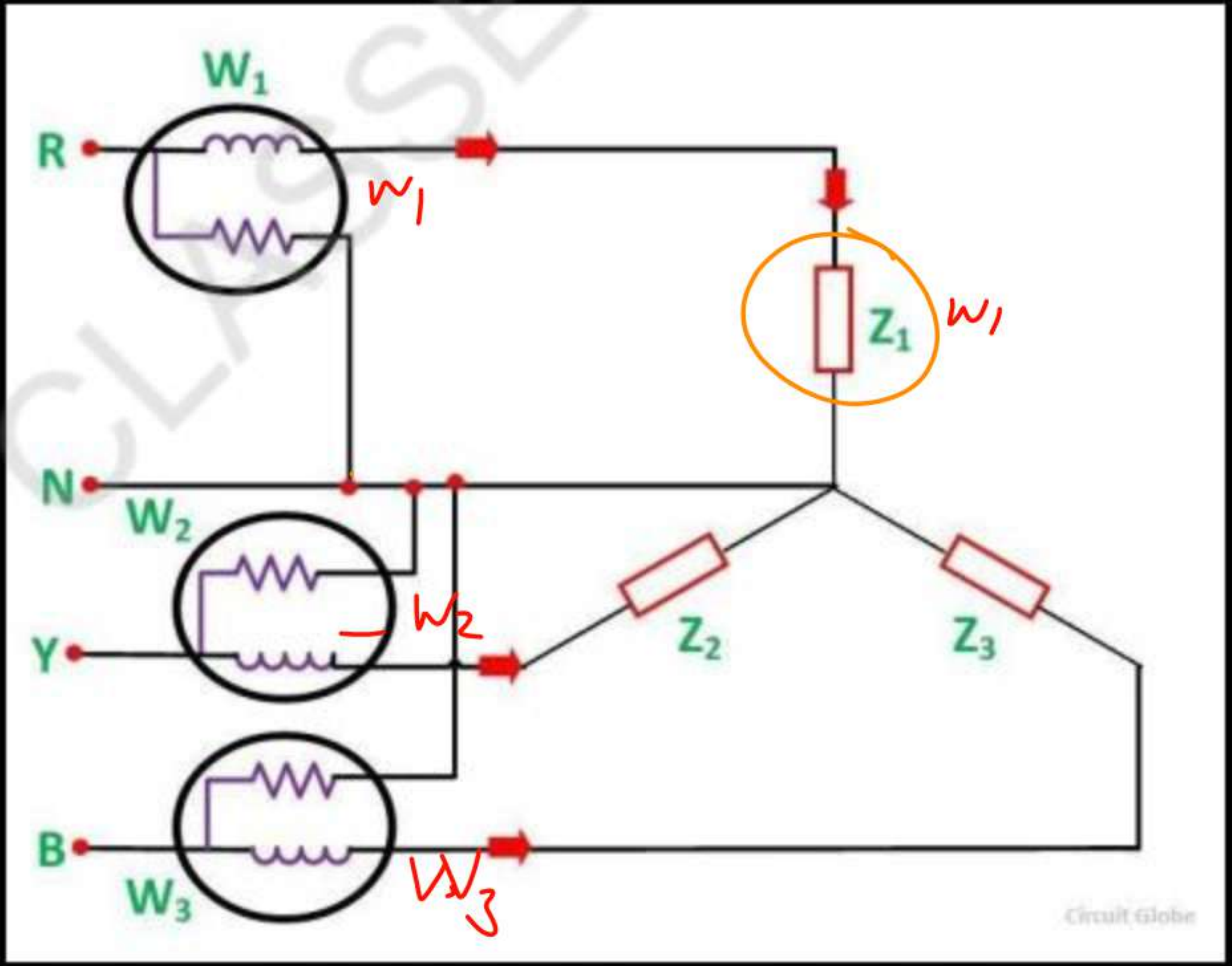
- Single wattmeter method.
- Two wattmeters method
- Three wattmeters method

$$P = 3W_1$$

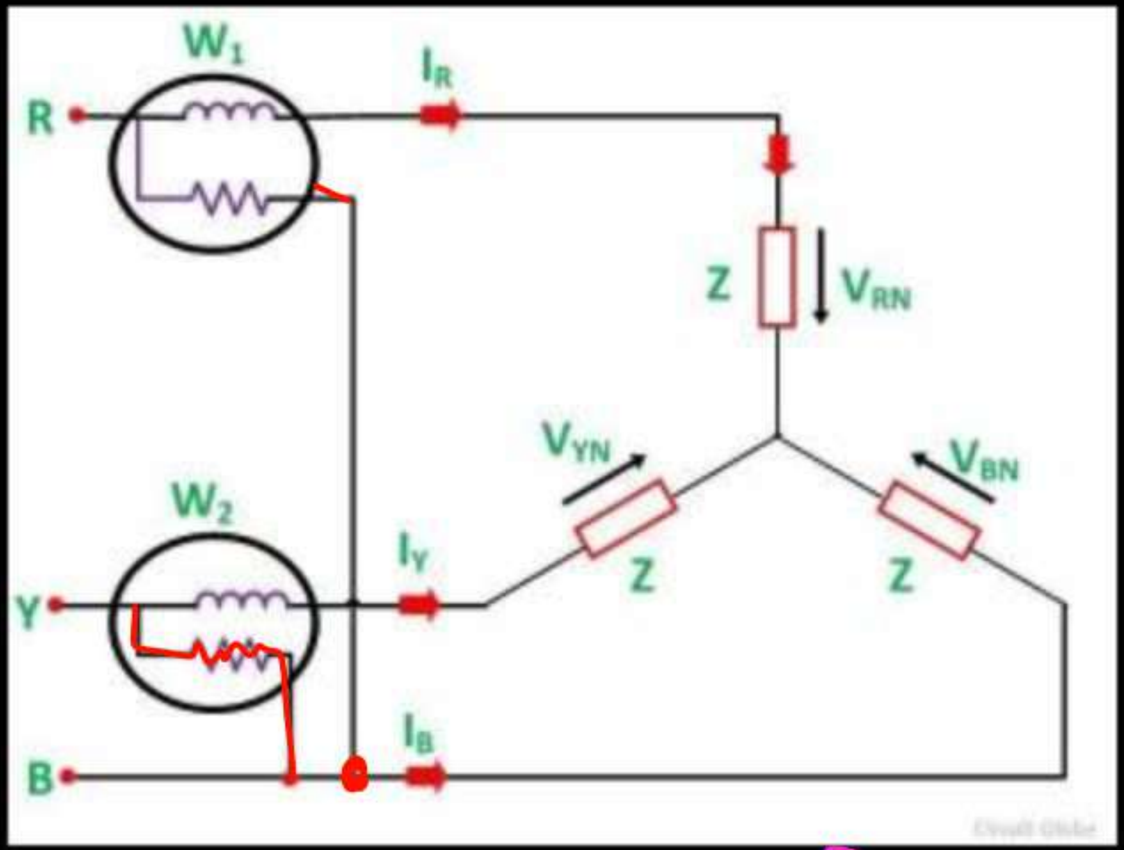


# Three wattmeter method

$$P = W_1 + W_2 + W_3$$



# Two Wattmeter Method (Star Connection )




$W_1 \rightarrow \underline{I_R, V_{RB}}, \ \theta = 30 - \phi$

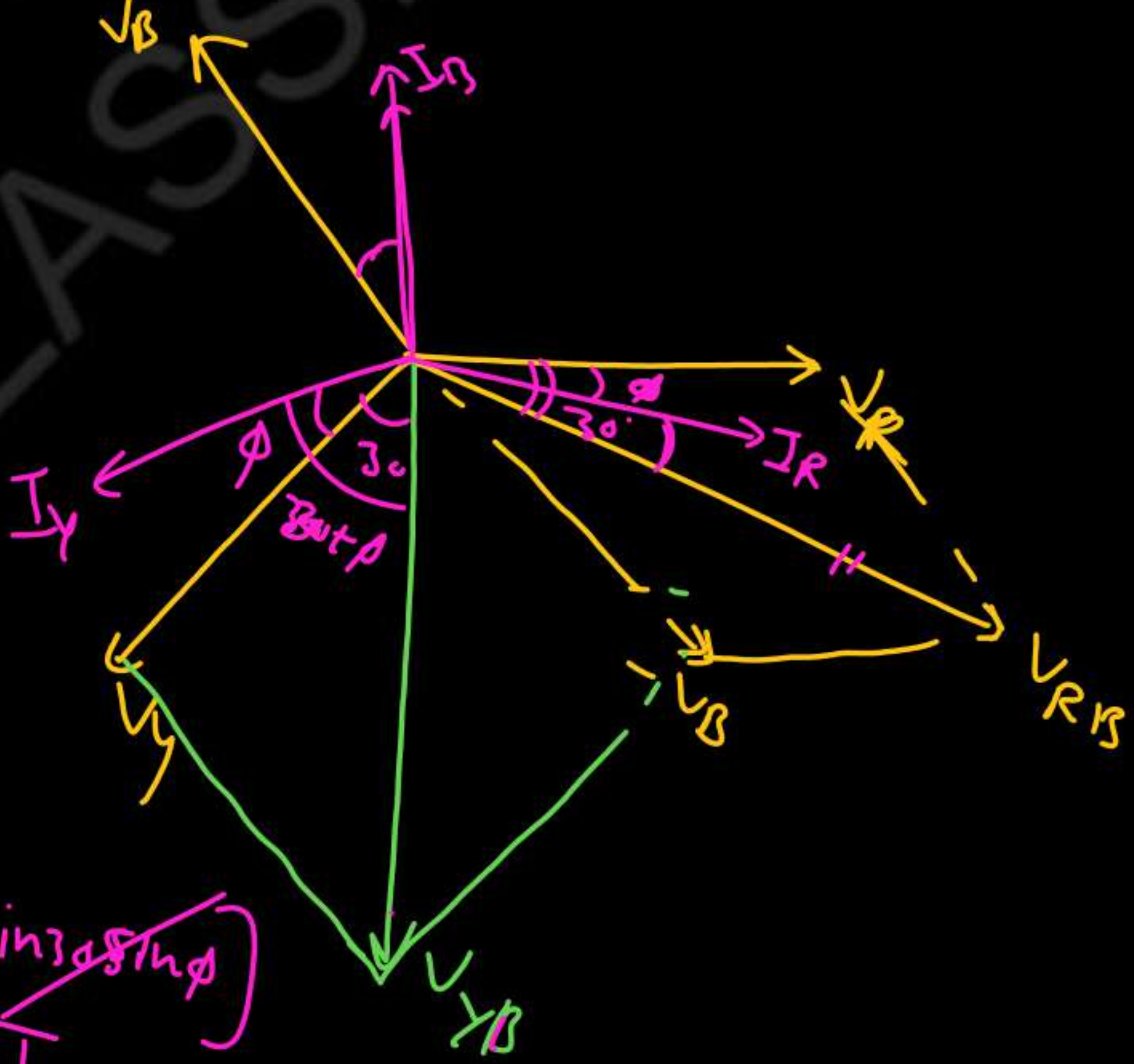
$W_2 \rightarrow \underline{I_Y}, \underline{V_{YB}}, \ \theta = 30 + \phi$

$V_{RB} = V_R - V_B$

$V_{YB} = V_Y - V_B$

$W_1 = \underline{V_{RB}}\, \underline{I_R} \cos(30 - \phi)$

$W_2 = V_{YB} I_Y \cos(30 + \phi)$




$$W_1 + W_2 = V_L I_L \left[\cos(30 - \phi) + \cos(30 + \phi)\right]$$

$$= V_L I_L \left[\cos 30 \cos\phi + \cancel{\sin 30 \sin\phi} + \cos 30 \cos\phi - \cancel{\sin 30 \sin\phi}\right]$$

$$= V_L I_L \left[2 \cos 30 \cos\phi\right] = \boxed{\sqrt{3}\, V_L I_L \cos\phi = P_{3-\phi}}$$

# Two Wattmeter Method (Delta Connection )

Self

# Power Factor calculation by Two Wattmeter Method

$$W_1 = V_L I_L \cos(30-\phi)$$

$$W_2 = V_L I_L \cos(30+\phi)$$

$$W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi$$

$$W_1 - W_2 = V_L I_L [\cos(30-\phi) - \cos(30+\phi)]$$

↓ (previous formula)

$$= V_L I_L [2 \underbrace{\sin 30}_{\frac{1}{2}} \sin\phi]$$

$$\boxed{W_1 - W_2 = V_L I_L \sin\phi}$$

$$\frac{W_1 + W_2}{W_1 - W_2} = \frac{\sqrt{3} \cancel{V_L I_L} \cos\phi}{\cancel{V_L I_L} \sin}$$

$$\tan\phi = \sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$$

$$\boxed{\phi = \tan^{-1}\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)}$$

# Numerical Examples

- Ex.-27 A balanced star connected load of (8 + j6) ohm per phase is connected to a balanced 3- phase, 400 V supply. Find the line current, phase current, power factor, power and total volt-amperes. AKTU- 2021-22

$V_L = 400V$ , $Z_{ph} = (8 + 6j)$ / phase

$I_L$ $V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231$

$I_L = \frac{V_{ph}}{Z_{ph}} = \frac{400}{\sqrt{3} \times (8+6j)} = 23.09 \angle -36.86 = I_{ph}$

$P.f = \cos\phi = \cos 36.86 = 0.8$

$S = 3 V_{ph} I_{ph} = 3 \times 231 \times 23.09$

# Numerical Examples

- Ex.-28. A balanced 3-Phase star connected load takes 30KW at a leading current of 48A from a 3-φ source of 500V, 50Hz. Find the circuit parameters per phase. AKTU- 2021-22

Capa.

$R - jX_C$

$V_L = 500\,V$ $\quad P = 30\,KW$

$V_{ph} = \frac{500}{\sqrt{3}} = 288.67\,V$ $\quad I_L = 48A$

$Z_{ph} = \frac{V_{ph}}{I_p} = \frac{288.67}{48} = 6.01\Omega$

$P = \sqrt{3} V_L I_L \cos\phi$

$\cos\phi = \frac{P}{\sqrt{3} \times 500 \times 48} = 0.7$

$\phi = 45.57$

$Z = 6.01 \angle -45.57$

$Z = \underbrace{4.33}_{R} - \underbrace{4.165}_{X_C} j$

# Numerical Examples

- Ex.-29. A balanced delta connected load of (12 + j 9) Ω/ phase is connected to 3- phase 400 V supply. Calculate line current, power factor and power drawn by it. AKTU- 2014-15

Self

# Numerical Examples

- Ex.-30. A 3-φ, 500 V motor has a power factor of 0.4 (lagging). Two wattmeters are connected to measure the input and show the total input power to be 30 kW. Find out the readings of each wattmeter AKTU- 2021-22

$\cos\phi = 0.4$, $V_L = 500$

$P = 30\,kW = W_1 + W_2$ — ①

$\tan\phi = \sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$

$W_1 - W_2 = \frac{\tan\phi\,(W_1 + W_2)}{\sqrt{3}}$ — ②

Solve Eqn ① & ②

$W_1 = ?$

$W_2 = ?$

# Numerical Examples

- Ex.-31. A Star Connected three phase load has a resistance of 8 ohm and an inductive reactance of 6 ohm . It is fed from 400 V 3 phase supply. Determine line current , power factor active and reactive power. Draw phasor diagram showing phase and line voltages and currents. If power is measured by two wattmeter method what will be the reading of both wattmeters. AKTU- 2004-05

$Z = 8 + 6j$ , $V_L = 400V \Rightarrow$

$V_{ph} = \frac{400}{\sqrt{3}} = 231V$

$I_L = I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{231}{(8+6j)} = 23.09\angle -36.86$

$W_1 = V_L I_L \cos(30 - \phi)$

$W_2 = V_L I_L \cos(30 + \phi)$

# Numerical Examples



- **Ex.-32. A balanced star connected inductive load is connected to a 400V, 50Hz supply. Two wattmeters are used to measure power indicate 8000W and 4000W respectively. Determine (i) Line Current (ii) Impedance of each phase(iii) power factor (iv) Resistance and inductance of each phase** AKTU- 2009-10

$W_1 + W_2 = 8000 + 4000 = 12000$

$W_1 - W_2 = 4000$

$\tan\phi = \sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$

$= \sqrt{3}\left(\frac{4000}{12000}\right)$

$\tan\phi = 0.577$

$\cos\phi = \cos(\tan^{-1}(0.577))$

$= 0.86$

$P = \sqrt{3} V_L I_L \cos\phi$, $V_L = 400$

$V_{ph} = 231$

$I_L = \frac{P}{\sqrt{3} V_L \cos\phi} = I_{ph}$

$Z_{ph} = \frac{V_{ph}}{I_{ph}} = Z\angle\theta$

$= R + jX$

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# Thank You