AKTU

B. Tech I-Year

Gateway Classes

Semester -I & II

Common to All Branches

BAS101 / BAS201: ENGINEERING PHYSICS

UNIT-2 ONE SHOT : Electromagnetic Field Theory

Gateway Series for Engineering

- Topic Wise Entire Syllabus
- Long - Short Questions Covered
- AKTU PYQs Covered
- DPP
- Result Oriented Content

Download App

For Full Courses including Video Lectures

AKTU

B. Tech I-Year

Gateway Classes

BAS101 / BAS201: ENGINEERING PHYSICS

Unit-2 ONE SHOT

Introduction to Electromagnetic Field Theory

Syallbus

Basic concept of Stoke's theorem and Divergence theorem, Basic laws of electricity and magnetism, Continuity equation for current density, Displacement current, Maxwell equations in integral and differential form, Maxwell equations in vacuum and in conducting medium, Poynting vector and Poynting theorem, Plane electromagnetic waves in vacuum and their transverse nature. Relation between electric and magnetic fields of an electromagnetic wave, Plane electromagnetic waves in conducting medium, Skin depth.

Engg. Physics

B. Tech 1st Year

One Shot Revision

Unit-2 : EMFT

- Notes
- PYQs
- DPP

By. Gulshan sir

B.Tech First Year : All Subjects

Maths-I / Maths-II	Video Lectures + Pdf Notes + PYQs + DPP
Physics / Chemistry	
Electrical Engg. / Electronics Engg.	
Basic Computer Engg. (PPS) / Mechanical Engg.	
Soft Skills / Environment	
B.Tech I-Year COMBO	

Courses Link in Description

Helpline No. 7819 0058 53

B.Tech Second Year : All Subjects

AKTU University

RGPV University

AKTU : SYLLABUS

- *Basic concepts of Stoke's Theorem and Divergence Theorem*
- *Basic laws of Electricity and Magnetism*
- *Continuity equation for current density*
- *Displacement current*
- *Maxwell's equations in integral and differential form*
- *Maxwell's equations in vacuum and in conducting medium*
- *Poynting vector and Poynting theorem*
- *Plane electromagnetic waves in vacuum and their transverse nature.*
- *Relation between Electric and Magnetic fields of an electromagnetic wave*
- *Plane electromagnetic waves in Conducting medium*
- *Skin Depth.*

B.TECH : Engineering Physics

UNIT : Electromagnetic Field Theory (EMFT)

Lecture-1

Today's target

- Stoke's Theorem and Gauss Divergence Theorem
- Basic laws of Electricity and Magnetism
- DPP
- PYQ

Electromagnetic Field Theory (EMFT)

Charge

Rest — Electric field

Accelerated Motion — Electric field and Magnetic field

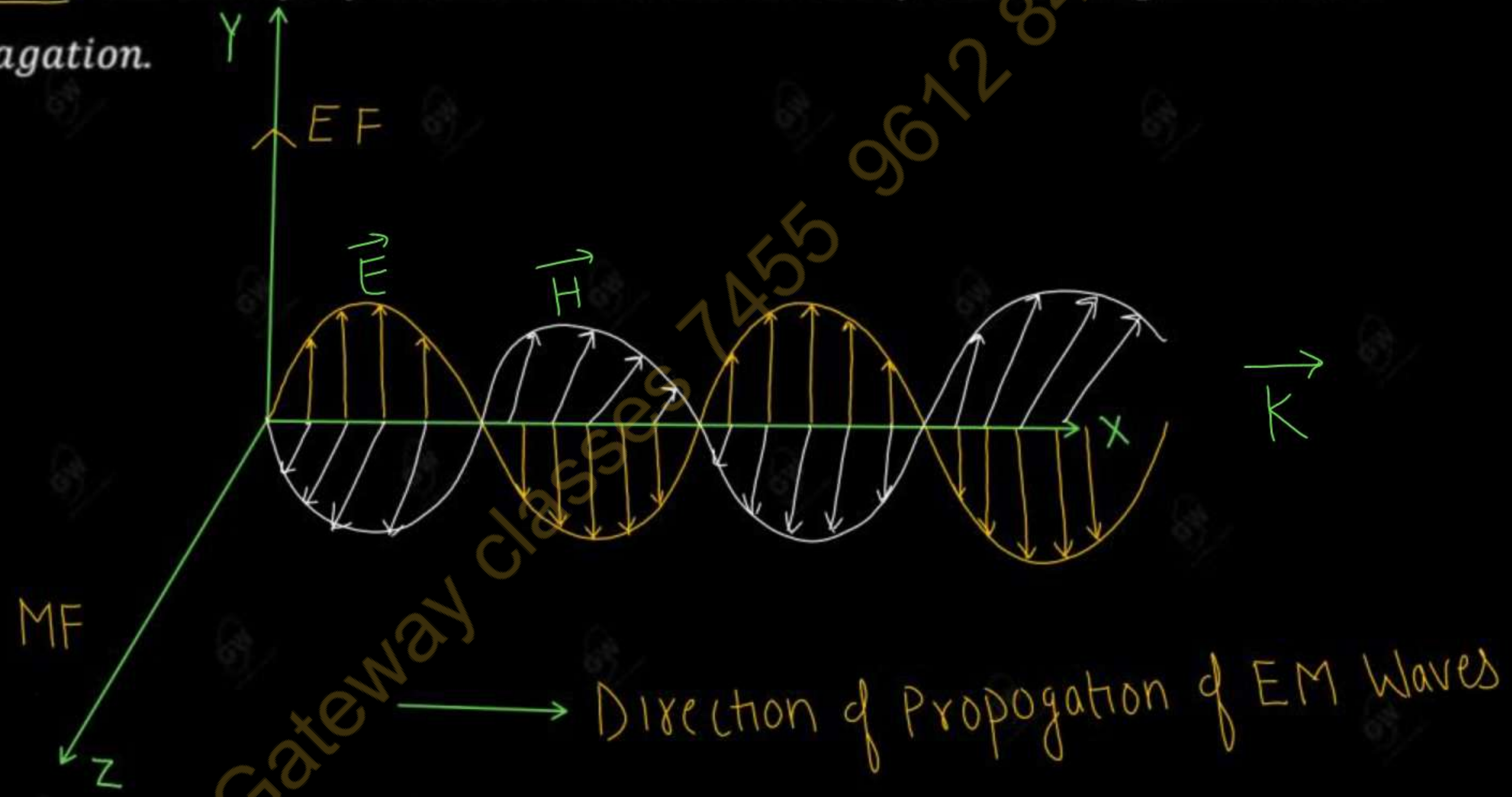
- *The mutual interaction of electric and magnetic fields produces an Eleclromagnetic Field.*

OR

The Electromagnetic Field is the combination of an electric field and magnetic field.

- *An electromagnetic field propogate in the form of wave.*
- *In 1864, Maxwell combined both electric and magnetic field and show that an accelerated charge particle generates electromagnetic waves.*

Note : *The electric and magnetic field of electromagnetic wave are perpendicular to each other and also perpendicular to the direction of Electromagnetic waves propagation.*



Mathematical Tools

(I) Dell Operator $(\vec{\nabla})$

$$(\vec{\nabla}) = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

Scalar Function (ϕ)

$\vec{\nabla}\phi$

Vector Function $(\vec{A})$

(i) $\vec{\nabla} . \vec{A}$

(ii) $\vec{\nabla} \times \vec{A}$

Where

(a) $\vec{\nabla}\phi$ = Gradient of ϕ

(b) $\vec{\nabla} . \vec{A}$ = Divergence of $\vec{A}$

(c) $\vec{\nabla} \times \vec{A}$ = Curl of $\vec{A}$

(2) Laplacian Operature ($\vec{\nabla}^2$)

$$\vec{\nabla}.\vec{\nabla} = \left(\frac{\partial}{\partial x}\,\hat{\imath} + \frac{\partial}{\partial y}\,\hat{\jmath} + \frac{\partial}{\partial z}\,\hat{k}\right).\left(\frac{\partial}{\partial x}\,\hat{\imath} + \frac{\partial}{\partial y}\,\hat{\jmath} + \frac{\partial}{\partial z}\,\hat{k}\right)$$

$$\boxed{\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}}$$

Stoke → (I) Line Integral → $\oint$

(II) Surface Integral → $\iint$ OR $\int_S$

Gauss → (III) Volume Integral → $\iiint$ OR $\int_V$

(1) Stoke's Theorem

Statement : *It state that "The Line integral of a vector $\vec{A}$ over a closed loop **C** is equal to surface integral of curl of the same vector $\vec{A}$ over the surface area enclosed by the loop **C**"*

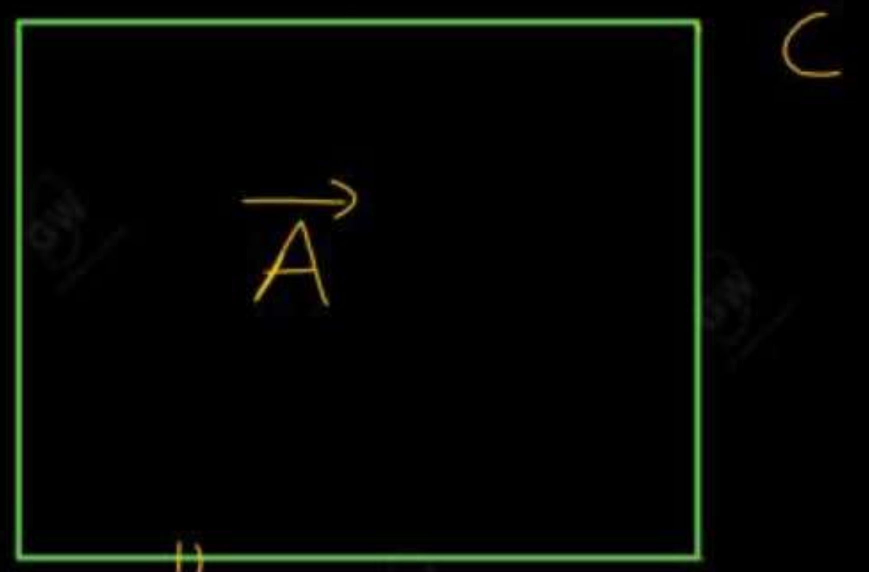
Mathmatically

$$\oint_c \vec{A}.\overrightarrow{dl} = \int\int (curl\ \vec{A}).d\vec{s}$$

$$= \int\int (\nabla \times \vec{A}).d\vec{s}$$

$$= \int_s (\nabla \times \vec{A}).d\vec{s}$$

$\nabla \times \vec{A}$



Note : *This theorem is used to convert a line integral in to a surface integral and vice – versa*

$$\boxed{\oint_c \vec{A}\cdot\vec{dl} = \int_s (\nabla \times \vec{A})\cdot d\vec{s}}$$

(2) Gauss Divergence Theorem

Statement : *It state that "The Surface integral of a vector $\vec{A}$ over a closed surface **S** is equal to volume integral of divergence of same vector $\vec{A}$ over the volume enclosed by that surface **S**"*

Mathmatically

$\nabla . \vec{A}$

$$\int\int \vec{A}.d\vec{s} = \int\int\int (div\ \vec{A})\ dv$$

$$= \int\int\int (\nabla . \vec{A})\ dv$$

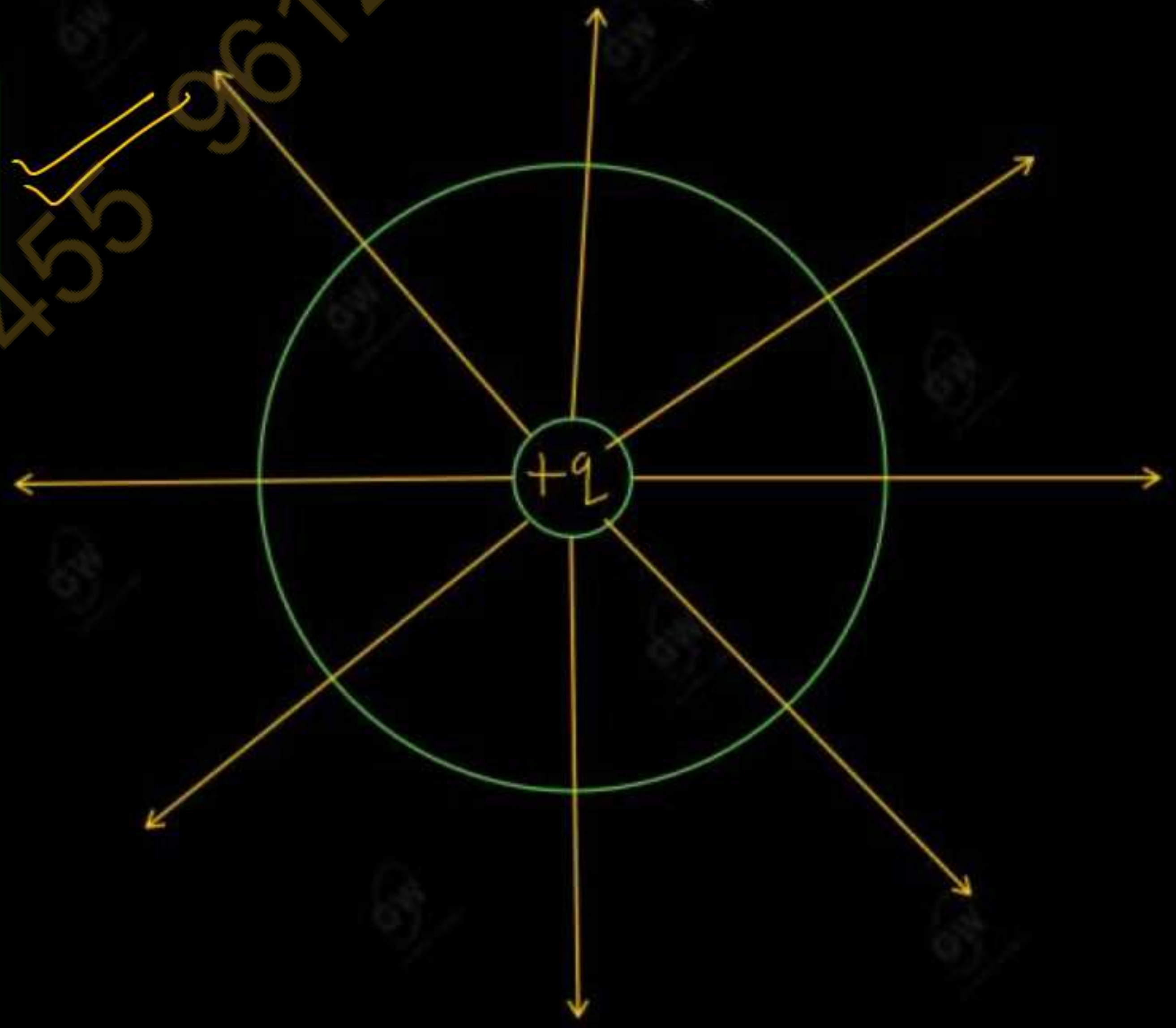
$$\boxed{\int_s \vec{A}.d\vec{s} = \int_v (\nabla . \vec{A})\ dv}$$

Note : *This theorem is used to convert a surface integral in to a volume integral and vice – versa*

(1) Gauss law of Electrostatics

It state that "the net electric flux through a closed surface $(3-D)$ is $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface"

$$\boxed{\phi_E = \frac{q_{total}}{\varepsilon_0}}$$



Also

$$\phi_E = \oint \vec{E}\,.\,\overrightarrow{ds} = \frac{q_{total}}{\varepsilon_0}$$

Where

- ϕ_E = Electric flux
- q_{total} = total charge enclosed by the surface
- ε_0 is Permittivity of free space

$(\varepsilon_0 = 8.85 \times 10^{-12} c^2 N^{-1} m^{-2})$

(2) Gauss law of Magnetism (Magnetostatics)

According to Guass law of magnetism the net magnetic flux (ϕ) linked through any closed surface is always zero

$$\boxed{\phi = \oint_s \vec{B}.\overrightarrow{ds} = 0}$$

$\oint_s \vec{B}.\overrightarrow{ds} \longrightarrow$ *Magnetic flux passing through any closed surface*

Reasons

(i) *Magnetic field lines are always form a closed curve*

(ii) *Magnetic monopoles do not exist*

(3) Faraday law of Electromagnetic induction

(i) Faraday's first law

Whenever the magnetic flux linked with a circuit is changed, an EMF is induced in the circuit.

(ii) Faraday second law

The magnitude of induced EMF is directly proportional to "Rate of change in magnetic flux"

$$e \propto \left(\frac{-d\,\phi_B}{dt}\right)$$

$$\boxed{e = \frac{-d\,\phi_B}{dt}} \quad \text{or} \quad \oint \vec{E}.\overrightarrow{dl} = \frac{d\,\phi_B}{dt}$$

Where

e = induced emf

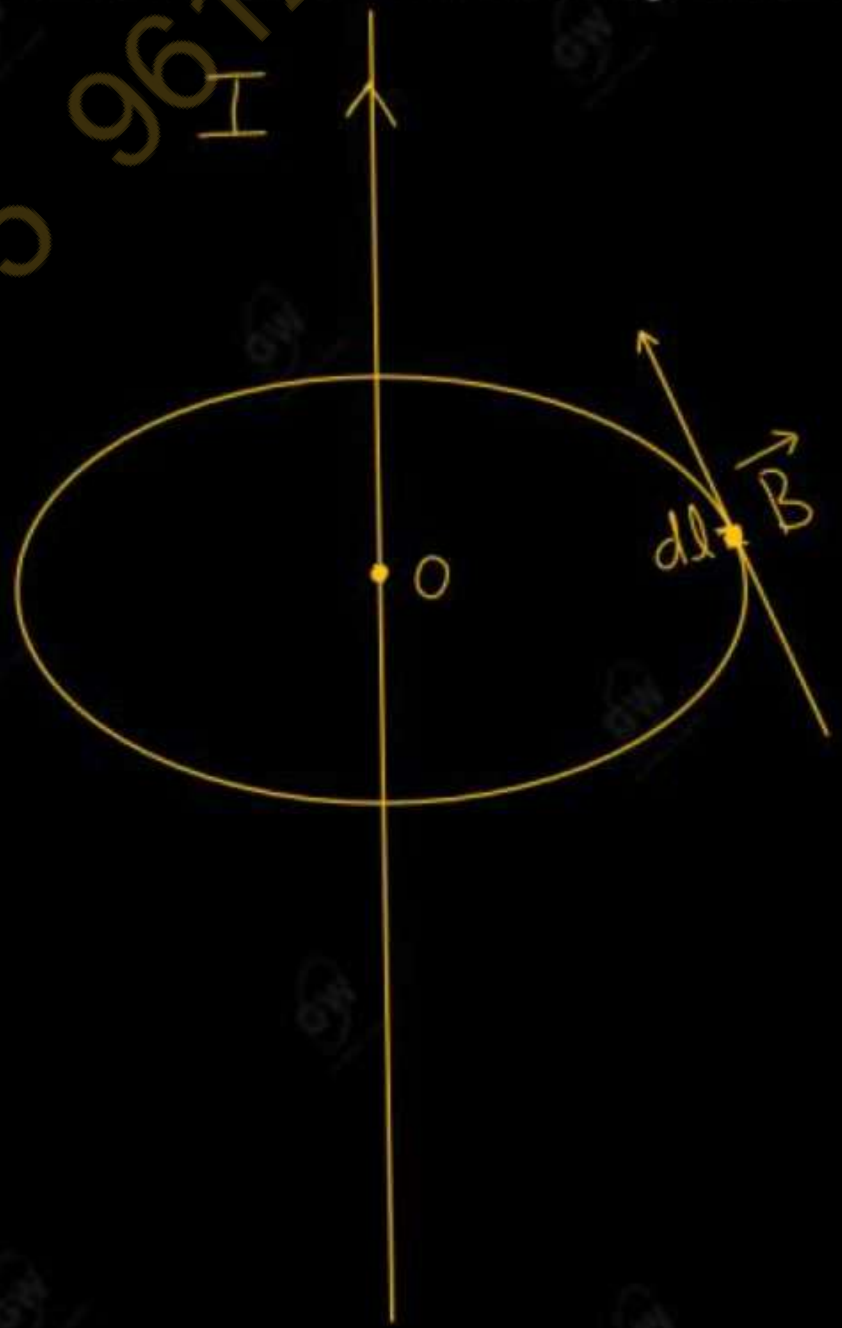
ϕ_B = Magnetic flux linked with circuit

$\vec{E}$ = electric field

(4) Ampere's Circuit law

According to Ampere's law the line integral of magnetic field B along a closed curve is equal to μ_0 times the net current through the area bounded by the curve.

$$\boxed{\oint \vec{B}\cdot\vec{dl} = \mu_0 I}$$



Where μ_0 = Permiability of free space.

DPP-1

UNIT : Electromagnetic Field Theory

Q.1 State and explain Stoke's theorem and Gauss divergence theorem.

Q.2 State and explain basic laws of electricity and magnetism

B.Tech First Year : All Subjects

<table>
<tr><td>Maths-I / Maths-II</td><td rowspan="6">Video Lectures
+
Pdf Notes
+
PYQs
+
DPP</td></tr>
<tr><td>Physics / Chemistry</td></tr>
<tr><td>Electrical Engg. / Electronics Engg.</td></tr>
<tr><td>Basic Computer Engg. (PPS) / Mechanical Engg.</td></tr>
<tr><td>Soft Skills / Environment</td></tr>
<tr><td>B.Tech I-Year COMBO</td></tr>
<tr><td colspan="2">Courses Link in Description Helpline No. 7819 0058 53</td></tr>
</table>

B.TECH : Engineering Physics

UNIT : Electromagnetic Field Theory (EMFT)

Lecture-2

Today's target

- *Continuity Equation*
- *Displacement current*
- DPP
- PYQ

Continuity Equation

It state that "The divergence of current density (J) is equal to the negative rate of change of volume charge density (ρ)"

$$\vec{J} = \frac{I}{A}$$

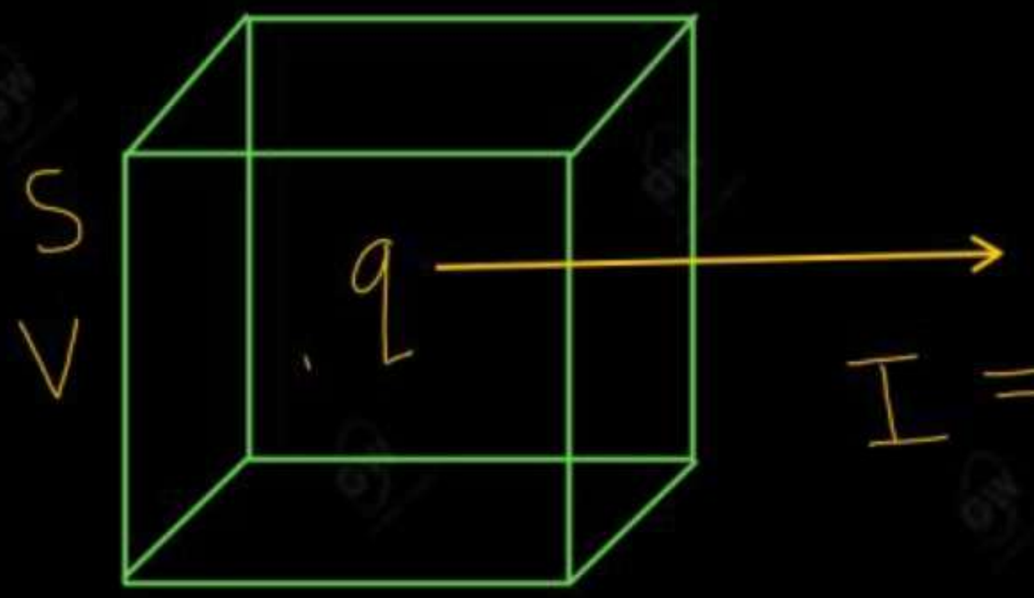
$$\vec{\nabla}.\vec{J} = \frac{-\partial\rho}{\partial t}$$

$$\rho = \frac{q}{V}$$

$$\boxed{\vec{\nabla}.\vec{J} + \frac{\partial\rho}{\partial t} = 0}$$

Derive the expression for continuity equation

- *Consider a closed surface (s) of volume (V) enclosing a charge (q) as shown in figure.*



$$I = \frac{dq}{dt}$$

- *As the current flows, there will be decrease in charges present inside the cube wrt to time*

$$I = -\frac{dq}{dt} \quad \text{——} \; (1)$$

- *Electric current through the surface is also given by*

$$I = \oint \vec{J} \cdot d\vec{s} \quad \text{——} \; (2)$$

- *By the law of consevation of charge*

$$\oint_S \vec{J} \cdot d\vec{s} = -\frac{dq}{dt} \quad \text{——} \; (3)$$

Since

$$q = \int_V \rho \, dV$$

Put q in (3)

$$\oint \vec{J} \cdot d\vec{s} = -\frac{d}{dt}\int_V \rho \, dV$$

$$\oint \vec{J} \cdot d\vec{s} = -\int_V \frac{\partial \rho}{\partial t} dV$$

Apply Gauss Divergence theorem

$$\int_V (\vec{\nabla} \cdot \vec{J}) dV = -\int_V \frac{\partial \rho}{\partial t} dV$$

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

$$\boxed{\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0}$$

For steady state

$$\frac{\partial \rho}{\partial t} = 0$$

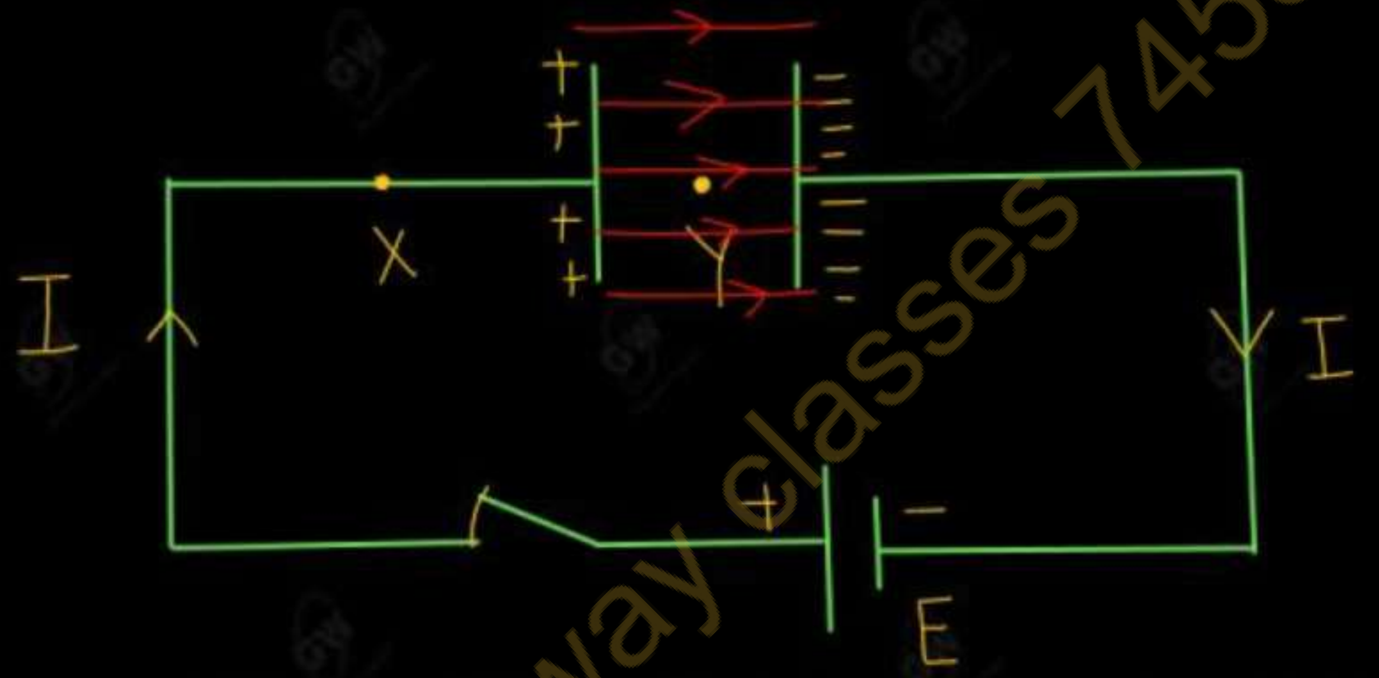
$$\boxed{\vec{\nabla} \cdot \vec{J} = 0}$$

Physical Significance : *Law of conservation of charge*

It states that the total current flowing out of some volume must be equal to rate of decrease of charge within volume assuming that charge can neither be created nor be destroyed.

Inconsistency of Ampere's cercuital law
OR
Why Ampere's law requrie modification

- *The idea of modification in Ampere's law arises in connection with capacitors with no medium between them.*
- *Consider an electrical circuit in which a capaciter is charged by a battery of emf E.*



A parallel plate capaciter being charged by a battery

- *Now consider two point X and Y as shown in above figure.*

- Apply Ampere's law at point X

$$\boxed{\oint \vec{B}.\overrightarrow{dl} = \mu_0 I}$$

- Apply Ampere's law at point Y

$$\oint \vec{B}.\overrightarrow{dl} = 0$$ {since current inside the capacitor is zero

- But Maxwell observed that the magnetic field exist between the plate of capacitor.
- So a need for modifying ampere's law was felt by Maxwell.
- Maxwell introduced the concept of displacement current and modified Ampere's law

Displacement current /Maxwell Displacement current

- *The concept of displacement current was introduced to resolve the inconsistency of Ampere's law*
- According to Maxwell

 (ii) *Changing Electric field Produces displacement current*

 (iii) *Displacement current Produces Magnetic field.*

 Conclusion : *Changing Electric field Produces Magnetic field.*

- *It means that changing electric field is equivalent to a current which flows as long as the electric field is changing and this equivalent current in vacuum or dielectric produces same magnetic field as the conduction current in conductor and is known as displacement current*

$$\boxed{I_d = \varepsilon_0 \frac{d\phi_E}{dt}}$$

Modification of Ampere's law

Ampere's law

$$\oint \vec{B}.\overrightarrow{dl} = \mu_0 I$$

Modified Ampere's law

I_d

Maxwell indroduced a new factor $\varepsilon_0 \frac{d\phi_E}{dt}$ to add in above equation.

This term $\varepsilon_0 \frac{d\phi}{dt}$ is known as displacement current (I_d)

$$\oint \vec{B}.\overrightarrow{dl} = \mu_0 \left(I + \varepsilon_0 \frac{d\phi_E}{dt}\right)$$

$$\boxed{\oint \vec{B}.\overrightarrow{dl} = \mu_0 (I + I_d)}$$

Difference between conduction current and displacement current

Conduction Current	Displacement current
➢ The electric current carried by conductors due to flow of charges is called conduction current. ➢ It exist even if flow of electron is at uniform rate. ➢ $I_c = \frac{V}{R}$	➢ The electic current due to changing electric field is called displacement current. ➢ It does not exist under steady condition ➢ $I_d = \varepsilon_0 \frac{d\phi_E}{dt}$

Q.1 *For a conducting medium,* $\sigma = 5.8 \times 10^6$ *siemens/m and* $\epsilon_r = 1$. *Find out conduction and displacement current densities if magnitude of electric field intensity E is given by* $E = 150\sin(10^{10}t)$ *volt/m.*

Given

$$\sigma = 5.8 \times 10^6 \ S/m$$

$$\varepsilon_r = 1$$

$$E = 150\sin(10^{10}t) \ V/m$$

Conduction Current Density ($\vec{J_c}$)

$$\boxed{\vec{J_c} = \sigma \vec{E}}$$

$$= 5.8 \times 10^6 \times 150\sin(10^{10}t)$$

$$\boxed{\vec{J_c} = 8.7 \times 10^8 \sin(10^{10}t) \ A/m^2}$$

$$\boxed{D = \varepsilon \vec{E}}$$

Displacement Current density ($\vec{J_d}$)

$$\boxed{\vec{J_d} = \frac{\partial \vec{D}}{\partial t} = \varepsilon \frac{\partial \vec{E}}{\partial t}}$$

$$\vec{J_d} = \varepsilon_r \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \left\{ \because \varepsilon_r = \frac{\varepsilon}{\varepsilon_0} \right\}$$

$$\vec{J_d} = \varepsilon_r \varepsilon_0 \frac{\partial}{\partial t}\left(150\sin(10^{10}t)\right)$$

$$\vec{J}_D = 1 \times 8.85 \times 10^{-12} \times 150 \cos(10^{10} t) \times 10^{10}$$

$$\boxed{J_D = 13.28 \cos(10^{10} t)}$$

DPP-2

UNIT : Electromagnetic Field Theory

Q.1 *Derive a suitable expression for continuity equation. Give its physical significance.*

OR

What is the equation of contiuity? Obtain the required expression for it. Also give its physical significance.

Q.2 *What is displacement current?*

Q.3 *What is the difference between conduction current and displacement current?*

Q.4 *Explain the concept of displacement current and show how it leads to modification of Ampere law.*

Q.5 *Why Maxwell proposed that Ampere law require modification?*

B.TECH : Engineering Physics

UNIT : Electromagnetic Field Theory (EMFT)

Lecture-3

Today's target

- Maxwell's Equations in differential form
  - (i) Maxwell's Equations in vacuum
  - (ii) Maxwell's Equations in conducting medium
- Maxwell's Equations in integral form
- Physical Significance of Maxwell's Equations
- DPP
- PYQ

Stoke's Theorem and Gauss Divergence Theorem

(1) Stoke's Theorem

This theorem is used to convert a line integral in to a surface integral or vice – versa

$$\oint_c \vec{A}.\overrightarrow{dl} = \int\int (\nabla \times \vec{A}).d\vec{s}$$

$$= \int_s (\nabla \times \vec{A}).d\vec{s}$$

(2) Gauss Divergence Theorem

This theorem is used to convert a surface integral in to a volume integral or vice – versa

$$\int\int \vec{A}.d\vec{s} = \int\int\int (\nabla.\vec{A})\, dv$$

$$\int_s \vec{A}.d\vec{s} = \int_v (\nabla.\vec{A})\, dv$$

Basic laws of Electromagnetism (or Electricity and Magnetism)

(1) Gauss law of Electrostatics

The net electric flux (ϕ) through a closed surface (3 − D) is $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface

$$\boxed{\phi = \oint_S \vec{E}\,.\,\overrightarrow{ds} = \frac{q_{total}}{\varepsilon_0}}$$

Where

$\varepsilon_0 =$ Permitivity of free space

(2) Gauss law of Magnetostatics

The net magnetic flux (ϕ_B) linked through any closed surface is always zero

$$\boxed{\phi_B = \oint_S \vec{B}.\overrightarrow{ds} = 0}$$

(3) Faraday law of Electromagnetic induction

Whenever the magnetic flux linked with a circuit is changed, an EMF is induced in the circuit

$$\boxed{e = \frac{-d\,\phi_B}{dt}} \quad \text{or} \quad \boxed{\oint \vec{E}\,.\,\vec{dl} = \frac{d\,\phi_B}{dt}}$$

Where

e = induced emf

(4) Ampere's Circuital law

According to Ampere's law the line integral of magnetic field B along a closed curve is equal to μ_0 times the net current through the area bounded by the curve.

$$\boxed{\oint \vec{B}\,.\,\vec{dl} = \mu_0\, I}$$

Where μ_0 = Permiability of free space.

(5). Modified Ampere's law (By Maxwell')

$$\oint \vec{B}.\overrightarrow{dl} = \mu_0 \left(I + \varepsilon_0 \frac{d\phi_E}{dt}\right)$$

$$\boxed{\oint \vec{B}.\overrightarrow{dl} = \mu_0 (I + I_d)}$$

Continuity Equation

The divergence of current density (J)is equal to the negative rate of change of volume charge density (ρ)

$$\vec{\nabla}.\vec{J} = \frac{-\partial\rho}{\partial t}$$

$$\boxed{\vec{\nabla}.\vec{J} + \frac{\partial\rho}{\partial t} = 0}$$

Maxwell's Equations

- *All the basic principles of electricity and magnetism can be explained in terms of four fundamental equations called Maxwell equations*
- *Maxwell did not discovered four equations, but he worked on them and stated that these four fundamental equations define complete electricity and magnetism.*

Maxwell's equations Differential form

$$\boxed{\begin{array}{l} (i)\ \vec{\nabla}.\vec{D} = \rho \\ (ii)\ \vec{\nabla}.\vec{B} = 0 \\ (iii)\ \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ (iv)\ \vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \end{array}}$$

$\vec{D} = \varepsilon_0 \vec{E}$

$\rho = \frac{q}{V}$

$\vec{B} = \mu_0 \vec{H}$

$\vec{H}$ = intensity of Magnetic field

$\vec{J} = \frac{I}{A}$

conduction current density

$\frac{\partial \vec{D}}{\partial t} \longrightarrow$ Displacement current density

Derivation of Maxwell's Equations in differential form

(i)Maxwell first equation

$$\boxed{\vec{\nabla}.\vec{D} = \rho}$$

By Gauss law of electrostatics

$$\boxed{\oint_s \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}}$$

$$\oint_s \varepsilon_0 \vec{E}.d\vec{s} = q$$

$$\oint_s \vec{D}.d\vec{s} = q$$

But $\boxed{q = \int_v \rho dv}$

$$\oint_s \vec{D}.d\vec{s} = \int_v \rho dv$$

Apply Gauss Divergence theorem in LHS

$$\boxed{\oint_s \vec{D}.d\vec{s} = \int_v (\nabla.\vec{D})dv}$$

$$\Rightarrow \int_v (\nabla.\vec{D})dv = \int_v \rho dv$$

$$\boxed{\nabla.\vec{D} = \rho}$$

Maxwell First equation

(ii) Maxwell's second equation

$$\boxed{\vec{\nabla}.\vec{B} = 0}$$

By Gauss law of Magnetostatics

$$\oint_S \vec{B}.d\vec{S} = 0$$

Apply Gauss Divergence theorem in LHS

$$\boxed{\oint_S \vec{B}.d\vec{S} = \int_V (\nabla.\vec{B})\,dv}$$

$$\Rightarrow \int_V (\nabla.\vec{B})\,dv = 0$$

$$\boxed{\nabla.\vec{B} = 0}$$

Maxwell second equation

(iii) Maxwell's Third equation

$$\boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

By Faraday law of EMI

$$e = -\frac{d\phi_B}{dt} \quad \text{—— (1)}$$

Also

$$e = \oint_c \vec{E}\cdot d\vec{l} \quad \text{—— (2)}$$

From (1) and (2)

$$\oint_c \vec{E}\cdot d\vec{l} = -\frac{d\phi_B}{dt} \quad \text{—— (3)}$$

Where

$$\phi_B = \oint_s \vec{B}\cdot d\vec{s}$$

Put ϕ_B in (3)

$$\oint_c \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\oint_s \vec{B}\cdot d\vec{s}$$

$$\oint_c \vec{E}\cdot d\vec{l} = \oint_s \left(-\frac{\partial \vec{B}}{\partial t}\right)\cdot d\vec{s}$$

Apply stoke's theorem in LHS

$$\boxed{\oint_c \vec{E}\cdot d\vec{l} = \oint_s (\nabla \times \vec{E})\cdot d\vec{s}}$$

$$\oint_s (\nabla \times \vec{E})\cdot d\vec{s} = \oint_s \left(\frac{-\partial \vec{B}}{\partial t}\right)\cdot d\vec{s}$$

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

Maxwell third Equation

(iv) Maxwell's Fourth equation

$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

By Ampere's law

$$\oint_c \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$\oint_c \cancel{\mu_0} \vec{H} \cdot d\vec{l} = \cancel{\mu_0} I$$

$$\oint_c \vec{H} \cdot d\vec{l} = I$$

But $I = \oint_s \vec{J} \cdot d\vec{s}$

$$\oint_c \vec{H} \cdot d\vec{l} = \oint_s \vec{J} \cdot d\vec{s}$$

Apply Stokes theorem in LHS

$$\oint_c \vec{H} \cdot d\vec{l} = \oint_s (\nabla \times \vec{H}) \cdot d\vec{s}$$

$$\Rightarrow \oint_s (\nabla \times \vec{H}) \cdot d\vec{s} = \oint_s \vec{J} \cdot d\vec{s}$$

$$\nabla \times \vec{H} = J \quad \text{——— (1)}$$

Taking Divergence both side

$$\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot \vec{J}$$

$$\boxed{\text{But } \nabla \cdot (\nabla \times \vec{H}) = 0}$$

$$0 = \nabla \cdot \vec{J}$$

$$\nabla \cdot \vec{J} = 0 \quad \text{——— (2)}$$

By continuity Equation

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t} \quad \text{——— (3)}$$

From (2) and (3)

$$-\frac{\partial \rho}{\partial t} = 0$$

$\Rightarrow \rho = \text{constant}$

→ Equation (1) is valid only for time independent field

→ For time dependent field, equation (1) required some Modification

→ Maxwell suggested that we must add $\vec{J}'$ in equation (1)

$$\nabla \times \vec{H} = \vec{J} + \vec{J}' \quad \text{—— (4)}$$

Taking Divergence both side

$$\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot (\vec{J} + \vec{J}')$$

$$0 = \nabla \cdot \vec{J} + \nabla \cdot \vec{J}'$$

$$\nabla \cdot \vec{J}' = -\nabla \cdot \vec{J}$$

Using equation (3)

$$\nabla \cdot \vec{J}' = -\left(-\frac{\partial \rho}{\partial t}\right)$$

$$\nabla \cdot \vec{J}' = \frac{\partial \rho}{\partial t} \quad \text{—— (5)}$$

From Maxwell First equation

$$\nabla \cdot \vec{D} = \rho \quad \text{—— (6)}$$

From (5) and (6)

$$\nabla \cdot \vec{J'} = \frac{\partial}{\partial t}(\nabla \cdot \vec{D})$$

$$\nabla \cdot \vec{J'} = \nabla \cdot \frac{\partial \vec{D}}{\partial t}$$

$$\vec{J'} = \frac{\partial \vec{D}}{\partial t}$$

Put $\vec{J'}$ in (4)

$$\boxed{\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

Maxwell equations

(i) $\vec{\nabla} \cdot \vec{D} = \rho$

(ii) $\vec{\nabla} \cdot \vec{B} = 0$

(iii) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

(iv) $\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

For Free Space or Vacuum

No conduction current

$\sigma = 0$

$J = \sigma E = 0$

$\rho = 0$

$\vec{D} = \varepsilon_o \vec{E}$

$\vec{B} = \mu_o \vec{H}$

Maxwell's equations in Free Space (or Vacuum)

(i) $\vec{\nabla} \cdot \vec{E} = 0$

(ii) $\vec{\nabla} \cdot \vec{H} = 0$

(iii) $\vec{\nabla} \times \vec{E} = -\mu_o \frac{\partial \vec{H}}{\partial t}$

(iv) $\vec{\nabla} \times \vec{H} = \varepsilon_o \frac{\partial \vec{E}}{\partial t}$

Where

$$\vec{\nabla} = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$\nabla = \frac{\partial}{\partial r}$$

Maxwell equations in differential form

(i) $\vec{\nabla} \cdot \vec{D} = \rho$

(ii) $\vec{\nabla} \cdot \vec{B} = 0$

(iii) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

(iv) $\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

For Conducting medium

Conduction current exist

$\sigma \neq 0$

$J = \sigma E \neq 0$

$\rho = 0$ (there is no net charge within a conductor because charge resides on the surface of conductor)

$\vec{D} = \varepsilon \vec{E}$

$\vec{B} = \mu \vec{H}$

Maxwell's equations in CONDUCTING MEDIUM

(i) $\vec{\nabla} \cdot \vec{E} = 0$

(ii) $\vec{\nabla} \cdot \vec{H} = 0$

(iii) $\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$

(iv) $\vec{\nabla} \times \vec{H} = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t}$

Where

$$\vec{\nabla} = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

Maxwell's equations in Integral Form

(i) $\oint \vec{E}.d\vec{s} = \frac{1}{\varepsilon_0}\int_v \rho dv$

(ii) $\oint \vec{B}.d\vec{s} = 0$

(iii) $\oint \vec{E}.d\vec{l} = -\frac{d}{dt}\int \vec{B}.d\vec{s}$

(iv) $\oint \vec{H}.d\vec{l} = I + \frac{d}{dt}\int \vec{D}.d\vec{s}$

Derivation of Maxwell's Equations in Integral form

(i) Maxwell's first equation

$$\boxed{\oint \vec{E}.d\vec{s} = \frac{1}{\varepsilon_0}\int_v \rho dv}$$

By First Maxwell Equation in Differential Form

$$\nabla.\vec{D} = \rho$$

Taking volume integral both side

$$\int_V (\nabla.\vec{D}) dv = \int_V \rho dv$$

Apply Gauss Divergence in LHS

$$\boxed{\oint_S \vec{D}.d\vec{s} = \int_V (\nabla.\vec{D}) dV}$$

$$\oint_S \vec{D}.d\vec{s} = \int_V \rho dV$$

$$\oint_S \varepsilon_0 \vec{E}.d\vec{s} = \int_V \rho dv$$

$$\boxed{\oint_S \vec{E}.d\vec{s} = \frac{1}{\varepsilon_0}\int_V \rho dV}$$

Maxwell First eqn in integral form

(ii) Maxwell's Second equation

$$\boxed{(ii)\ \oint \vec{B}.\vec{ds} = 0}$$

By Maxwell second eq^n in differential Form

$$\nabla . \vec{B} = 0$$

Taking volume integral both side

$$\int_V (\nabla . \vec{B}) dv = 0$$

Apply Gauss Divergence theorem in LHS

$$\oint_S \vec{B} . d\vec{s} = \int_V (\nabla . \vec{B}) dv$$

$$\boxed{\oint_S \vec{B} . d\vec{s} = 0}$$

Maxwell second eq^n in integral form

(iii) Maxwell's Third equation

$$\boxed{\oint \vec{E}.d\vec{l} = -\frac{d}{dt}\int \vec{B}.d\vec{s}}$$

By third Maxwell eqn in Differential form

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Taking surface integral both side

$$\oint_s (\nabla \times \vec{E}).d\vec{s} = \oint_s \left(-\frac{\partial \vec{B}}{\partial t}\right).d\vec{s}$$

Apply Stokes theorem

$$\boxed{\oint_c \vec{E}.d\vec{l} = \oint_s (\nabla \times \vec{E}).d\vec{s}}$$

$$\oint_c \vec{E}.d\vec{l} = \oint_s \left(-\frac{\partial \vec{B}}{\partial t}\right).d\vec{s}$$

$$\boxed{\oint_c \vec{E}.d\vec{l} = -\frac{d}{dt}\oint_s \vec{B}.d\vec{s}}$$

Maxwell third equation in integral form

(iv) Maxwell's Fourth equation

$$\boxed{\oint \vec{H}.d\vec{l} = I + \frac{d}{dt}\int \vec{D}.d\vec{s}}$$

By Fourth Maxwell equation in Differential Form

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Taking surface integral both side

$$\oint_s (\nabla \times \vec{H}).d\vec{s} = \oint_s \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right).d\vec{s}$$

$$\oint_s (\nabla \times H).d\vec{s} = \oint_s \vec{J}.d\vec{s} + \oint_s \left(\frac{\partial \vec{D}}{\partial t}\right).d\vec{s}$$

$$I$$

Apply stoke's theorm in LHS

$$\oint_c \vec{H}.d\vec{l} = \oint_s (\nabla \times \vec{H}).d\vec{s}$$

$$\boxed{\oint_c \vec{H}.d\vec{l} = I + \frac{d}{dt}\int \vec{D}.d\vec{s}}$$

Maxwell Fourth eqn in integral form

Physical Significance of Maxwell's Equations

1. Maxwell's First Equation

Maxwell's First Equation represent Gauss law in electrostatics in differential form.

OR

It is Gauss law in electrostatics. It states that the surface integral of electric field over any closed surface area is equal to $\frac{1}{\varepsilon_0}$ times of net charge enclosed by that surface.

2. Maxwell's Second Equation

Maxwell's Second Equation represent Gauss law of magnetostatics.

OR

It is Gauss law in magnetostatics. It states that there is no existence of magnetic monopole or the net magnetic flux through any closed surface area is zero. It also signifies that magnetic lines are closed curves.

3. Maxwell's Third Equation

Maxwell's Third Equation represent Faraday's law of electromagnetic induction

OR

It is Faraday's law in electromagnetic induction. It states that induced emf around any closed path is equal to the negative rate of change of magnetic flux bounded by the surface w.r.t time. (i.e. any changing manetic field produces a electric field.)

4. Maxwell's Fourth Equation

Maxwell's Fourth Equation represent modified Ampere's law

OR

It is modified Ampere's law. It states that any current carrying conductor as well as time varying electric field produces a magnetic field.

DPP-3

UNIT : Electromagnetic Field Theory

Q .1 Deduce four Maxwell's equation in free space.

Q.2 Write Maxwell's equation in differential form and in integral form and explain their physical significance with their proof.

B.TECH : Engineering Physics

UNIT : Electromagnetic Field Theory (EMFT)

Lecture-4

Today's target

- *Poynting Theorem and Poynting Vector*
- DPP
- PYQ

Maxwell's equations Differential form

(i) $\vec{\nabla}.\vec{D} = \rho$

(ii) $\vec{\nabla}.\vec{B} = 0$

$$\boxed{\begin{aligned} &(iii)\ \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ &(iv)\ \vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \end{aligned}}$$

Poynting Theorem

OR

Work Energy Theorem for the flow of energy in an electromagnetic field

- *Poynting Theorem is used to describe the flow of energy or power in an electromagnetic field during the propagation of uniform plane wave.*

$$\boxed{-\oint_S (\vec{E} \times \vec{H}).d\vec{s} = \int_v (\vec{E}.\vec{J})\,dv + \frac{d}{dt}\int_v \frac{1}{2}(\varepsilon E^2 + \mu H^2)\,dv}$$

$\oint(\vec{E} \times \vec{H}).d\vec{s} \longrightarrow$ *Total outgoing Electromagnetic energy from entire volume*

$\int_v (\vec{E}.\vec{J})\,dv \longrightarrow$ *Rate of transfer of Electromagnetic energy due to the motion of charge*

(Power loss)

$\frac{d}{dt}\int_v \frac{1}{2}(\varepsilon E^2 + \mu H^2)\,dv \longrightarrow$ *Rate of transfer of Electromagnetic energy in the form of electromagnetic field*

- *Poynting Theorem represent consevation of energy in an electromagnetic field*

Derivation

By Maxwell's third and Fourth equations

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{—— (1)}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \text{—— (2)}$$

Taking Dot product of equation (1) with $\vec{H}$ and equation (2) with $\vec{E}$

$$\vec{H}\cdot(\nabla \times \vec{E}) = -\vec{H}\cdot\frac{\partial \vec{B}}{\partial t} \quad \text{—— (3)}$$

$$\vec{E}\cdot(\nabla \times \vec{H}) = \vec{E}\cdot\vec{J} + \vec{E}\cdot\frac{\partial \vec{D}}{\partial t} \quad \text{—— (4)}$$

Subtract eqn (4) From eqn (3)

$$\vec{H}\cdot(\nabla \times \vec{E}) - \vec{E}\cdot(\nabla \times \vec{H}) = -\vec{H}\cdot\frac{\partial \vec{B}}{\partial t} - \vec{E}\cdot\vec{J} - \vec{E}\cdot\frac{\partial \vec{D}}{\partial t}$$

Using vector identity in above equation

$$\boxed{\nabla\cdot(\vec{E}\times\vec{H}) = \vec{H}\cdot(\nabla\times\vec{E}) - \vec{E}\cdot(\nabla\times\vec{H})}$$

$$\nabla\cdot(\vec{E}\times\vec{H}) = -\vec{H}\cdot\frac{\partial \vec{B}}{\partial t} - \vec{E}\cdot\vec{J} - \vec{E}\cdot\frac{\partial \vec{D}}{\partial t}$$

$$-\nabla\cdot(\vec{E}\times\vec{H}) = \vec{E}\cdot\vec{J} + \vec{E}\cdot\frac{\partial\vec{D}}{\partial t} + \vec{H}\cdot\frac{\partial\vec{B}}{\partial t}$$

Put $\vec{D} = \varepsilon\vec{E}$, $\vec{B} = \mu\vec{H}$

$$-\nabla\cdot(\vec{E}\times\vec{H}) = \vec{E}\cdot\vec{J} + \vec{E}\cdot\frac{\partial\varepsilon\vec{E}}{\partial t} + \vec{H}\cdot\frac{\partial\mu\vec{H}}{\partial t}$$

$$-\nabla\cdot(\vec{E}\times\vec{H}) = \vec{E}\cdot\vec{J} + \varepsilon\,\vec{E}\cdot\frac{\partial\vec{E}}{\partial t} + \mu\,\vec{H}\cdot\frac{\partial\vec{H}}{\partial t} \quad \text{--- (5)}$$

We know that

$$\frac{\partial}{\partial t}(\vec{E}\cdot\vec{E}) = \vec{E}\cdot\frac{\partial\vec{E}}{\partial t} + \vec{E}\cdot\frac{\partial\vec{E}}{\partial t}$$

$$\frac{\partial E^2}{\partial t} = 2\vec{E}\cdot\frac{\partial\vec{E}}{\partial t} \qquad \boxed{\because\ \vec{a}\cdot\vec{a} = a^2}$$

$$\vec{E}\cdot\frac{\partial\vec{E}}{\partial t} = \frac{1}{2}\frac{\partial E^2}{\partial t}$$

Similarly

$$\vec{H}\cdot\frac{\partial\vec{H}}{\partial t} = \frac{1}{2}\frac{\partial H^2}{\partial t}$$

Put these values in (5)

$$-\nabla\cdot(\vec{E}\times\vec{H}) = \vec{E}\cdot\vec{J} + \frac{1}{2}\varepsilon\frac{\partial E^2}{\partial t} + \frac{1}{2}\mu\frac{\partial H^2}{\partial t}$$

$$-\nabla\cdot(\vec{E}\times\vec{H}) = \vec{E}\cdot\vec{J} + \frac{1}{2}\left(\varepsilon\frac{\partial E^2}{\partial t} + \mu\frac{\partial H^2}{\partial t}\right)$$

Taking volume integral both side

$$-\int_V \nabla\cdot(\vec{E}\times\vec{H})\,dv = \int_V (\vec{E}\cdot\vec{J})\,dv + \int_V \frac{1}{2}\left(\varepsilon\frac{\partial E^2}{\partial t} + \mu\frac{\partial H^2}{\partial t}\right)dv$$

$$-\int_V \nabla\cdot(\vec{E}\times\vec{H}) = \int_V (\vec{E}\cdot\vec{J})\,dv + \int_V \frac{1}{2}\left(\frac{\partial \varepsilon E^2}{\partial t} + \frac{\partial \mu H^2}{\partial t}\right)dv$$

$$-\int_V \nabla\cdot(\vec{E}\times\vec{H}) = \int_V (\vec{E}\cdot\vec{J})\,dv + \frac{d}{dt}\int_V \frac{1}{2}(\varepsilon E^2 + \mu H^2)\,dv$$

By Gauss divergence theorem

$$\oint_S (\vec{E}\times\vec{H})\cdot d\vec{s} = \int_V \nabla\cdot(\vec{E}\times\vec{H})\,dv$$

$$\boxed{-\oint_S (\vec{E}\times\vec{H})\cdot d\vec{s} = \int_V (\vec{E}\cdot\vec{J})\,dv + \frac{d}{dt}\int_V \frac{1}{2}(\varepsilon E^2 + \mu H^2)\,dv}$$

Where

$\vec{E}\times\vec{H}$ is known as Poynting vector

Poynting Vector

- *Poynting vector gives the time rate of flow of electromagnetic wave energy per unit area of medium.*

OR

The cross product of electric field vector $\vec{E}$ and the magnetic field vector $\vec{H}$ is called Poynting Vector. It is denoted by $\vec{S}$

$$\vec{S} = \vec{E} \times \vec{H}$$

- *Magnitude of $\vec{S}$ is given by,* $|\vec{S}| = |\vec{E} \times \vec{H}| = |\vec{E}||\vec{H}| \sin 90 \; |\hat{n}| = EH$

$$\boxed{S = \frac{Energy}{Area \times Time} = \frac{Power\ Radiated}{Area}}$$

- *Direction of propagation of $\vec{S}$ is perpendicular to both $\vec{E}$ and $\vec{H}$ or in the direction of Propagation of wave electromagnetic wave*
- *Unit of Poynting vector is J/(m^2.sec)*
- *Dimension of Poynting vector is* MT^{-3}

Q.1 *Calculate the magnitude of poynting vector at surface of the Sun. Given that power radiated by Sun is 5.4×10^{28} w and its radius is $7 \times 10^{8} m$.*

Given

$P = 5.4 \times 10^{28} W$

$r = 7 \times 10^{8} m$

We know that

$$S = \frac{\text{Power radiated}}{\text{Area}}$$

$$S = \frac{P}{4\pi r^2}$$

$$S = \frac{5.4 \times 10^{28}}{4 \times 3.14 \times (7 \times 10^{8})^2}$$

$$\boxed{S = 8.77 \times 10^{9}\ W/m^2}$$

DPP-4

UNIT : Electromagnetic Field Theory

Q.1 What is Poynting theorem ?

Q.2 What is Poynting vector?

Q.3 Discuss the physical significance of Poynting theorem.

Q.4 State and deduce Poynting theorem for the flow of energy in an electromagnetic field.

OR

Discuss the work-energy theorem for the flow of energy in an electromagnetic field.

B.TECH : Engineering Physics

UNIT : Electromagnetic Field Theory (EMFT)

Lecture-5

Today's target

- Plane Electromagnetic Waves in free space (or Vacuum)
- Electromagnetic Waves are transverse in nature
- Relation between Electric and Magnetic field of an Electromagnetic Waves
- DPP
- PYQ

Maxwell's equation in space

$$\nabla \cdot \vec{E} = 0 \quad \text{--- (1)}$$

$$\nabla \cdot \vec{H} = 0 \quad \text{--- (2)}$$

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \quad \text{--- (3)}$$

$$\nabla \times \vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \text{--- (4)}$$

Taking curl of eq^n (3)

$$\nabla \times (\nabla \times \vec{E}) = -\nabla \times \left(\mu_0 \frac{\partial \vec{H}}{\partial t}\right)$$

$$\nabla \times (\nabla \times \vec{E}) = -\mu_0 \left(\nabla \times \frac{\partial \vec{H}}{\partial t}\right)$$

$$\nabla \times (\nabla \times \vec{E}) = -\mu_0 \frac{\partial}{\partial t}(\nabla \times \vec{H}) \quad \text{--- (5)}$$

Using eq^n (4) in eq^n (5)

$$\nabla \times (\nabla \times \vec{E}) = -\mu_0 \frac{\partial}{\partial t}\left(\varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$

$$\nabla \times (\nabla \times \vec{E}) = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{--- (6)}$$

Using vector identity

$$\boxed{\nabla \times (\nabla \times \vec{E}) = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E}}$$

$$\nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E} = -\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$$

$$0 - \nabla^2\vec{E} = -\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$$

$$+\nabla^2\vec{E} = +\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\boxed{\nabla^2\vec{E} = \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}} \longleftarrow (7)$$

Similarly

$$\boxed{\nabla^2\vec{H} = \mu_0\varepsilon_0\frac{\partial^2\vec{H}}{\partial t^2}} \longleftarrow (8)$$

Equation (7) and (8) represent plane electromagnetic waves in free space

General wave equation

$$\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \quad \text{--- (9)}$$

ψ = Wave function propagation with velocity v

on comparing eq^n 7 and 8 with eq^n 9 we get

$$\mu_0 \varepsilon_0 = \frac{1}{V^2}$$

$$V^2 = \frac{1}{\varepsilon_0 \mu_0}$$

$$\boxed{V = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}} \quad \text{—— (10)}$$

Where, $\varepsilon_0 = 8.85 \times 10^{-12}$

$\mu_0 = 4\pi \times 10^{-7}$

$$V = \frac{1}{\sqrt{8.85 \times 10^{-12} \times 4\pi \times 10^{-7}}}$$

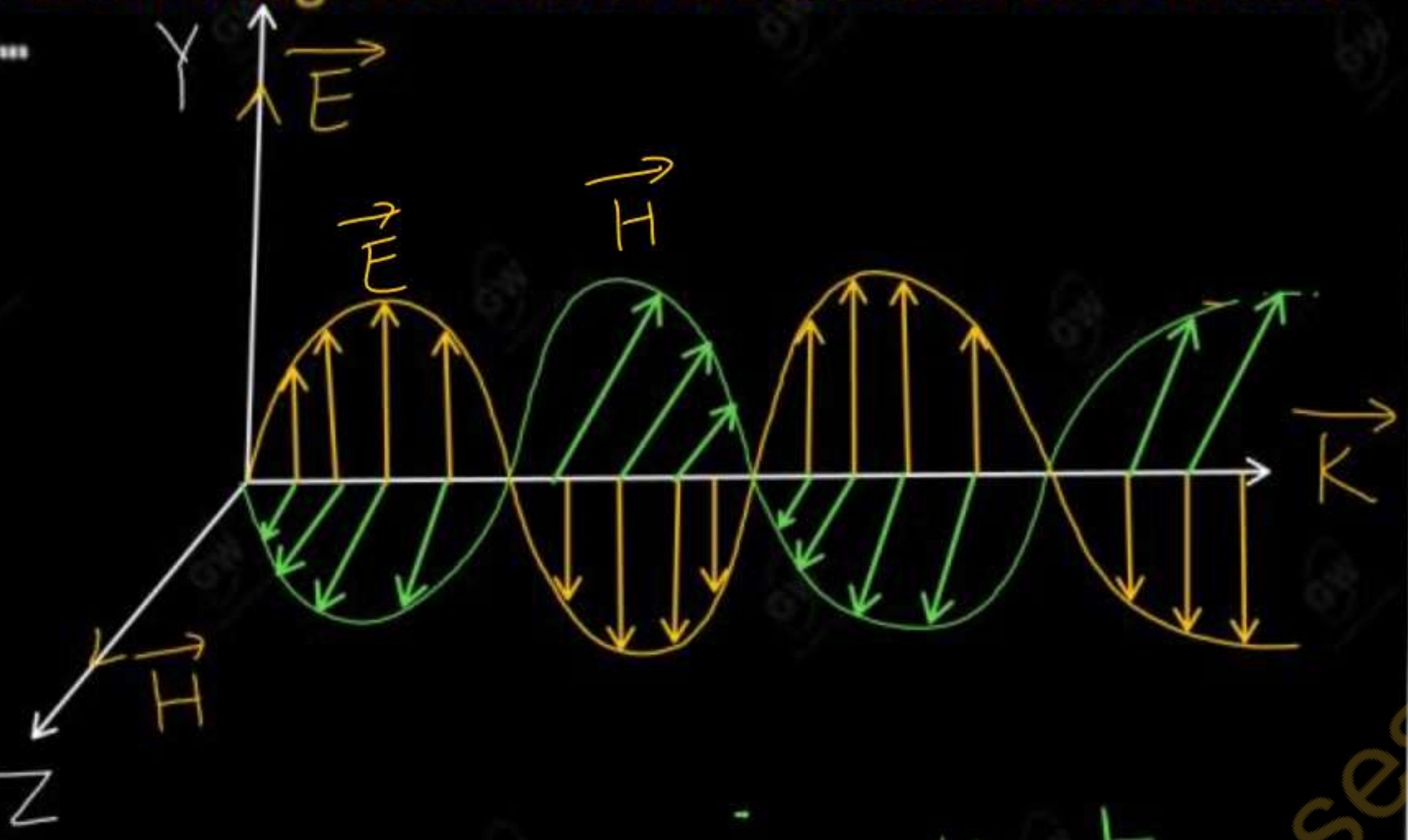
$$V = 2.99 \times 10^{8}\ m/s$$

$$\approx 3 \times 10^{8}\ m/s$$

Thus, Electromagnetic waves propagate with the velocity light (i.e $V = C$)

$$\boxed{\begin{aligned} \nabla^2 \vec{E} &= \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} \\ \nabla^2 \vec{H} &= \frac{1}{c^2} \frac{\partial^2 \vec{H}}{\partial t^2} \end{aligned}}$$

Electromagnetic Waves are transverse in nature



We have to prove that

(i) $\vec{K} \perp \vec{E}$

(ii) $\vec{K} \perp \vec{H}$

(iii) $\vec{E} \perp \vec{H}$

Wave equation in free space

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{--- (1)}$$

$$\nabla^2 \vec{H} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2} \quad \text{--- (2)}$$

General solution

$$\vec{E}(r,t) = E_0 e^{i(\vec{K}\cdot\vec{r} - wt)} \quad \text{--- (3)}$$

$$\vec{H}(r,t) = H_0 e^{i(\vec{K}\cdot\vec{r} - wt)} \quad \text{--- (4)}$$

Where

(i) E_0 and H_0 are the amplitudes

(ii) $\vec{K}$ is a propagation vector

(iii) $\vec{r}$ is a position vector

Partially Diff. eq^n (3) wrt r

$$\frac{\partial \vec{E}}{\partial r} = E_0 e^{i(\vec{K}.\vec{r}-\omega t)} \times i\vec{K}$$

$$\frac{\partial \vec{E}}{\partial r} = \vec{E} \times i\vec{K}$$

$$\frac{\partial \vec{E}}{\partial r} = i\vec{K}\vec{E}$$

$$\frac{\partial}{\partial r} = i\vec{K}$$

$$\nabla = i\vec{K} \quad \left\{ \because \nabla = \frac{\partial}{\partial r} \right\}$$

Using Maxwell's First equations

$$\nabla . \vec{E} = 0$$

$$i\vec{K}.\vec{E} = 0$$

$$i(\vec{K}.\vec{E}) = 0$$

$$\vec{K}.\vec{E} = 0$$

By the Definition of Dot product

$\therefore \quad \vec{K} \perp \vec{E}$

Using Maxwell's Second equation

$$\nabla \cdot \vec{H} = 0$$

$$i\vec{K} \cdot \vec{H} = 0$$

$$i(\vec{K} \cdot \vec{H}) = 0$$

$$\vec{K} \cdot \vec{H} = 0$$

By the definition of Dot product

$\vec{K} \perp \vec{H}$

Using Maxwell's third equation

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$i\vec{K} \times \vec{E} = -\mu_0 \frac{\partial}{\partial t} H_0 e^{i(\vec{K}\cdot\vec{r} - \omega t)}$$

$$i(\vec{K} \times \vec{E}) = -\mu_0 H_0 \frac{\partial}{\partial t} e^{i(\vec{K}\cdot\vec{r} - \omega t)}$$

$$i(\vec{K} \times \vec{E}) = -\mu_0 H_0 \times e^{i(\vec{K}\cdot\vec{r} - \omega t)} \times (-i\omega)$$

$$i(\vec{K} \times \vec{E}) = -\mu_0 \vec{H}(-i\omega)$$

$\cancel{i}(\vec{K} \times \vec{E}) = \cancel{i} \mu_0 \omega \vec{H}$

$$\boxed{(\vec{K} \times \vec{E}) = \mu_0 \omega \vec{H}}$$

By the Definition of cross product

$\vec{H} \perp \vec{K}$

$\vec{H} \perp \vec{E}$

Hence

(i) $\vec{K} \perp \vec{E}$

(ii) $\vec{K} \perp \vec{H}$

(iii) $\vec{E} \perp \vec{H}$

Thus, electromagnetic waves are transverse in nature

Relation between Electric and Magnetic field of an Electromagnetic Waves

We know that

$$\vec{K} \times \vec{E} = \mu_0 \omega \vec{H}$$

Taking Modulus both side

$$|\vec{K} \times \vec{E}| = \mu_0 \omega |\vec{H}|$$

$$K E \sin 90 = \mu_0 \omega H$$

$$K E = \mu_0 \omega H$$

$$E = \frac{\mu_0 \omega}{K} H$$

$$E = \mu_0 \frac{\omega}{K} H$$

$$E = \mu_0 c H \quad \left\{ \because \frac{\omega}{K} = c \right\} \quad \text{— (1)}$$

$$\frac{E}{H} = \mu_0 c$$

$$\frac{E}{H} = \mu_0 \times \sqrt{\frac{1}{\mu_0 \varepsilon_0}}$$

$$\frac{E}{H} = \sqrt{\frac{\mu_0^2}{\cancel{\mu_0} \varepsilon_0}}$$

V. imp

$$\boxed{\frac{E}{H} = \sqrt{\frac{\mu_0}{\varepsilon_0}}}$$

For Free space

From (1)

$$E = \mu_0 c H$$

$$E = \cancel{\mu_0} \times c \times \frac{B}{\cancel{\mu_0}}$$

$$\boxed{\frac{E}{B} = c} \quad \text{Free space}$$

Characteristic Impedance or Wave impedance of free space

- The ratio of magnitude of electric field $\vec{E}$ to the magnitude of magnetic field $\vec{H}$ is known as characteristic impedance.
- It is denoted by Z_o

$$Z_o = \frac{|\vec{E}|}{|\vec{H}|} = \frac{E}{H}$$

$$Z_o = \frac{E}{H} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$\frac{E}{H} = \sqrt{\frac{4\pi \times 10^{-7}}{8.85 \times 10^{-12}}}$$

$$\frac{E}{H} = 376.72 \text{ ohm}$$

$$\boxed{\frac{E}{H} \approx 377\,\Omega}$$

v. imp

Q.1 *If the magnitude of H in a plane wave is 1 amp / meter. Find the magnitude of E for plane wave in free space*

Given

$H = 1\ A/m$

We know that

$$\frac{E}{H} = 377\ \Omega$$

$$\frac{E}{1} = 377$$

$$\boxed{E = 377\ V/m}$$

Q.2 *Earth receives solar energy from the Sun which is 2 cal/cm^2 Min. What are the amplitude of electric and magnetic fields of radiation?*

Given

$S = 2 \text{ cal/cm}^2 \text{ Min}$

$S = \dfrac{2 \times 4.2}{10^{-4} \times 60}$

$S = 1400 \text{ J/m}^2\text{s}$

$EH = 1400$ —— (1)

Also

$\dfrac{E}{H} = 377$ —— (2)

Multiply (1) and (2)

$E\cancel{H} \times \dfrac{E}{\cancel{H}} = 1400 \times 377$

$E^2 = 1400 \times 377$

$\boxed{E = 726.49}$ V/m

$\boxed{H = 1.92 \text{ A/m}}$

Peak values

$E_0 = E\sqrt{2}$

$\boxed{E_0 = 1027.4 \text{ V/m}}$

$H_0 = H\sqrt{2}$

$\boxed{H_0 = 2.71 \text{ A/m}}$

Q.3 *If the upper atmosphere of earth receives the energy 1.38 kW/m² from the sun. What will be the peak values of electric and magnetic fields at the layer.*

Given

$S = 1.38\ kW/m^2$

$S = 1380\ W/m^2$

$E \times H = 1380\ W/m^2$ —— (1)

We know that

$\frac{E}{H} = 377\ \Omega$ —— (2)

Multiply (1) and (2)

$E \times \cancel{H} \times \frac{E}{\cancel{H}} = 1380 \times 377$

$E^2 = 520260$

$\boxed{E = 721.29\ V/m}$

$\boxed{H = 1.91\ A/m}$

Peak values

$E_0 = E\sqrt{2}$

$E_0 = 1020.06\ V/m$

$\boxed{E_0 = 1.02\ kV/m}$

$H_0 = H\sqrt{2}$

$\boxed{H_0 = 2.70\ A/m}$

Q.4 *A 100 Watt sodium lamp radiating its power. Calculate the electric field and magnetic fields strength at a distance of 5m from the lamp.*

Given

$P = 100W$

$r = 5m$

By pointing vector

$$S = \frac{P}{4\pi r^2}$$

$$S = \frac{\cancel{100}\,1}{\cancel{4}\pi \cancel{25}}$$

$$S = \frac{1}{\pi}\ J/m^2 s$$

$$E \times H = \frac{1}{\pi} \quad \text{——— (1)}$$

We know that

$$\frac{E}{H} = 377 \quad \text{——— (2)}$$

Multiply (1) and (2)

$$E \times \cancel{H} \times \frac{E}{\cancel{H}} = \frac{1}{\pi} \times 377$$

$$E^2 = \frac{377}{\pi}$$

$$E^2 = 120.0028$$

$$\boxed{E = 10.95\ V/m}$$

$$\boxed{H = 0.029\ amp/m}$$

DPP-5

UNIT : Electromagnetic Field Theory

Q.1 *Write down the Maxwell's equation in free space and using these equations derive wave equations for both electric and magnetic fields.*

OR

Derive the equation for propagation of plane electromagnetic wave in free space. Show that the velocity of plane electromagnetic waves in free space in given by $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

OR

Derive the electromagnetic waves equation in free space. Prove that the electromagnetic wave propagate with the speed of light in free space

Q.2 *Show electromagnetic waves are transverse in nature.*

OR

Show that electric field and magnetic field are normal to the direction of propagation of electromagnetic wave

OR

Show that $\vec{E}, \vec{H}$ *and direction of propagation form a set of orthgonal vectors*

Q.3 *Find the relation between E and H ?*

Q.4 *Define characteristic impedance or wave impedance of free space*

B.TECH : Engineering Physics

UNIT : Electromagnetic Field Theory (EMFT)

Lecture-6

Today's target

- Plane Electromagnetic Waves in Conducting Medium
- Skin Depth or Depth of Penetration
- DPP
- PYQ

Maxwell's Equation in conducting Medium

$$\nabla \cdot \vec{E} = 0 \quad \text{—— (1)}$$

$$\nabla \cdot \vec{H} = 0 \quad \text{—— (2)}$$

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad \text{—— (3)}$$

$$\nabla \times \vec{H} = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t} \quad \text{—— (4)}$$

Taking curl of equation (3)

$$\nabla \times (\nabla \times \vec{E}) = -\mu \frac{\partial}{\partial t}(\nabla \times \vec{H}) \quad \text{—— (5)}$$

Using eq^n (4) in (5)

$$\nabla \times (\nabla \times \vec{E}) = -\mu \frac{\partial}{\partial t}\left(\sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t}\right)$$

$$\nabla \times (\nabla \times \vec{E}) = -\left(\mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}\right)$$

Using vector identity

$$\boxed{\nabla \times (\nabla \times \vec{E}) = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E}}$$

$$\nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\left(\mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}\right)$$

$$0+\nabla^2\vec{E} = +\left(\mu\sigma\frac{\partial \vec{E}}{\partial t}+\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}\right)$$

$$\boxed{\nabla^2\vec{E} = \mu\sigma\frac{\partial \vec{E}}{\partial t}+\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}} \quad \text{— (6)}$$

Similarly,

Taking curl of equation (4) we get

$$\boxed{\nabla^2\vec{H} = \mu\sigma\frac{\partial \vec{H}}{\partial t}+\mu\varepsilon\frac{\partial^2\vec{H}}{\partial t^2}} \quad \text{— (7)}$$

Eq^n (6) and (7) represent EM waves in conducting Medium

solution of EM waves in conducting Medium

$$\boxed{\vec{E} = E_0 e^{-\gamma z}} \quad \text{— (8)}$$

$$\boxed{\gamma = \alpha + i\beta}$$

α = Attenuation constant

$$\boxed{\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}-1\right)}}$$

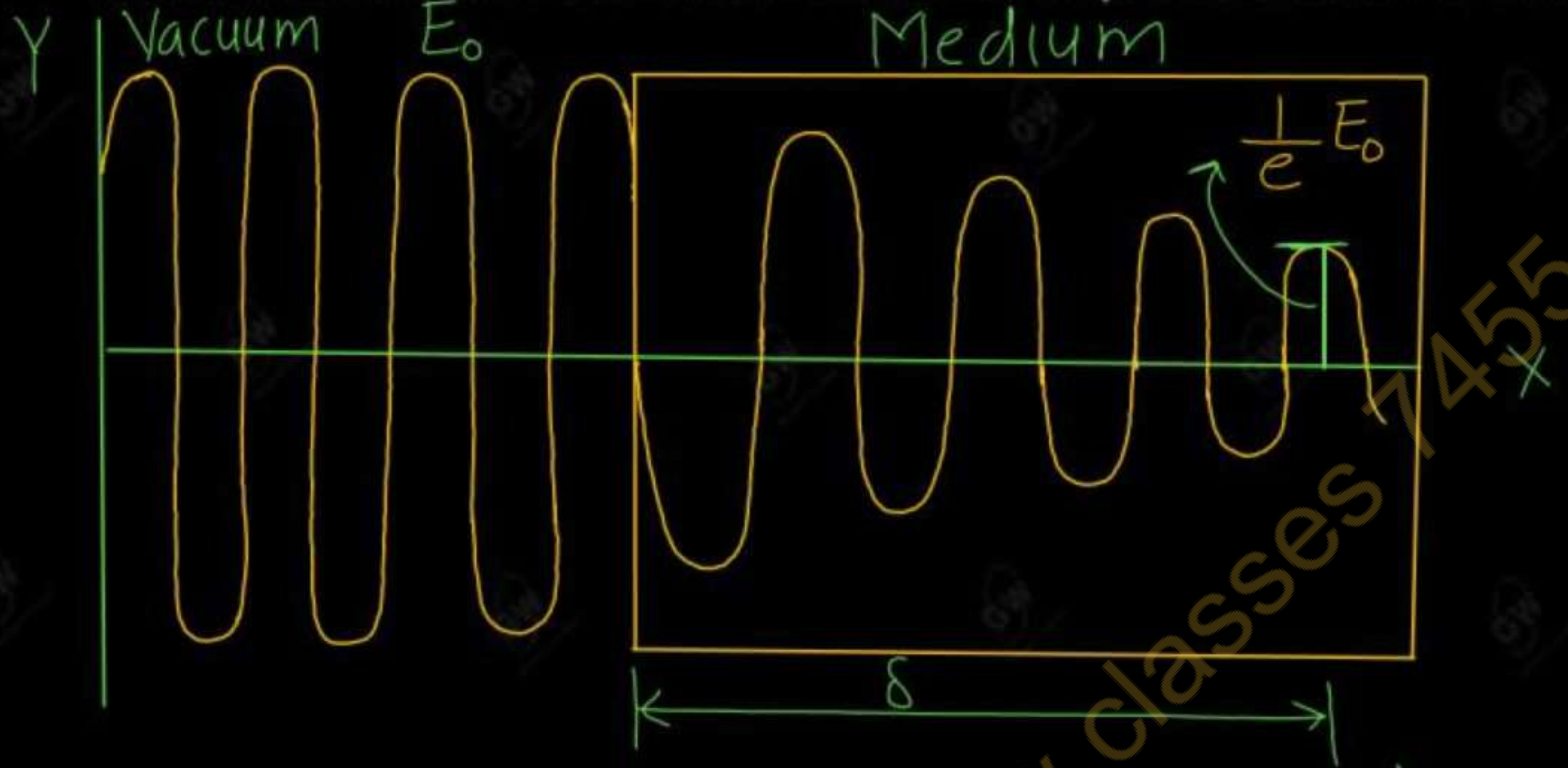
V.imp

β = phase constant

$$\boxed{\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}+1\right)}}$$

Attenuation

When an electromagnetic wave propagate in a medium, its amplitude decrease with the distance Inside the medium from the surface. This phenomenon is known as Attenuation.



When EM waves enter into a medium

$$\boxed{\vec{E} = E_0 e^{-\alpha x}}$$

Where

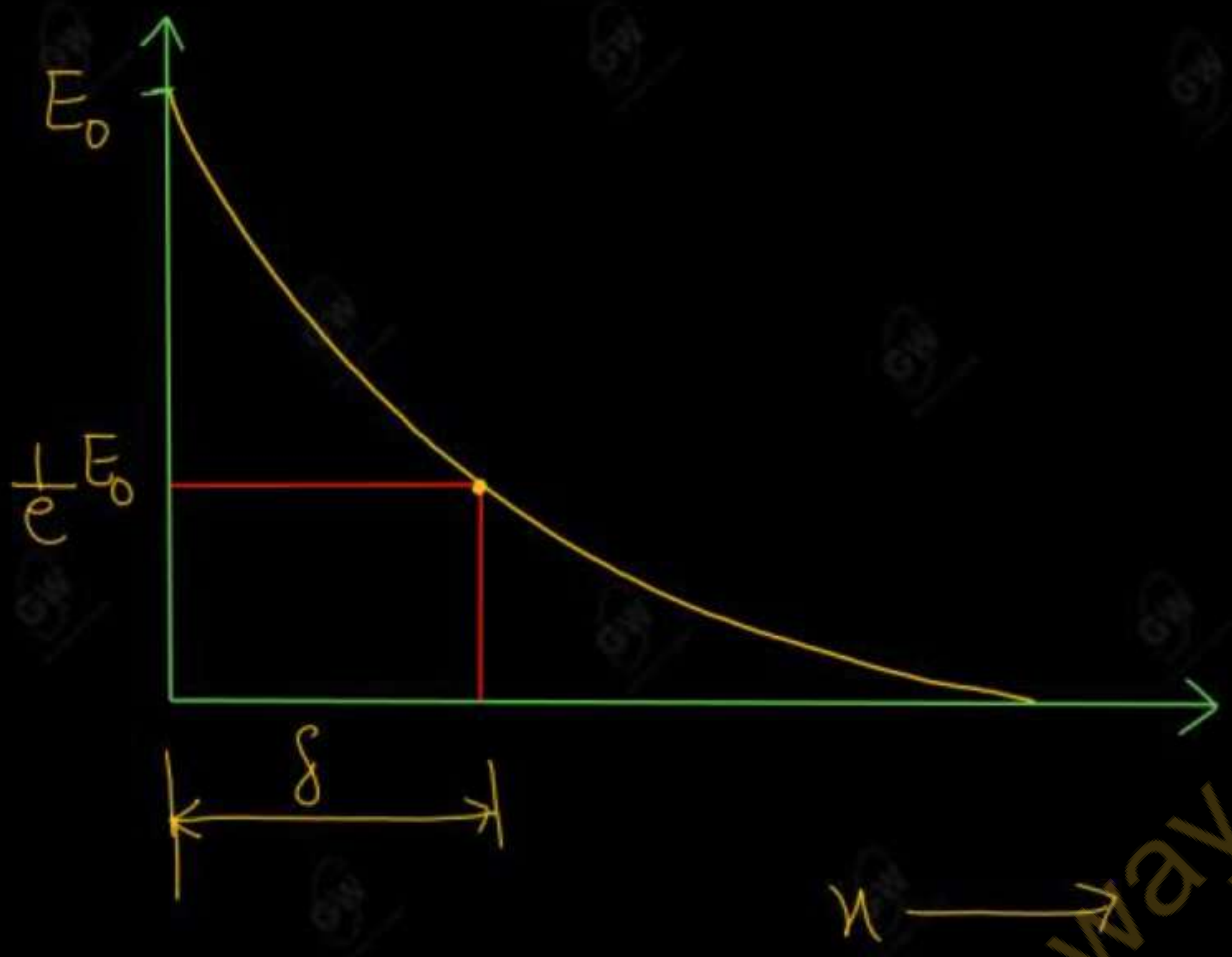
E_0 = Amplitude at the surface of Medium

$\alpha \rightarrow$ Attenuation constant

$$\boxed{\alpha = \omega\left[\frac{\mu\varepsilon}{2}\left\{\left(1+\frac{\sigma^2}{\omega^2\varepsilon^2}\right)^{1/2}-1\right\}\right]^{1/2}}$$

Skin Depth or Depth of Penetration (δ)

Skin Depth is the distance travelled by electromagnetic wave when it enters in to a medium and attenuates by $\frac{1}{e}$ times of its initial value.



When EM waves enters into a Medium

$$\vec{E} = \vec{E_0}\, e^{-\alpha x} \quad \text{--- (1)}$$

At Skin Depth

$$x = \delta$$

$$E = \frac{1}{e} E_0$$

From (1)

$$\frac{1}{e} \cancel{E_0} = \cancel{E_0}\, e^{-\alpha \delta}$$

$$e^{-1} = e^{-\alpha\delta}$$

$$+1 = +\alpha\delta$$

$$\boxed{\delta = \frac{1}{\alpha}}$$

It is the relation between skin depth and attenuation

Where

$$\alpha = \omega\left[\frac{\mu\varepsilon}{2}\left\{\left(1+\frac{\sigma^2}{\omega^2\varepsilon^2}\right)^{\frac{1}{2}}-1\right\}\right]^{\frac{1}{2}}$$

For Good conductors

$$\frac{\sigma}{\omega\varepsilon} >>> 1 \quad (\text{Neglect } 1)$$

$$\alpha = \omega\left[\frac{\mu\varepsilon}{2}\left\{\left(\frac{\sigma^2}{\omega^2\varepsilon^2}\right)^{\frac{1}{2}}-1\right\}\right]^{\frac{1}{2}}$$

$$\alpha = \omega\left[\frac{\mu\varepsilon}{2}\left(\frac{\sigma}{\omega\varepsilon}-1\right)\right]^{\frac{1}{2}}$$

$$\alpha = \omega\left(\frac{\mu\cancel{\varepsilon}}{2}\times\frac{\sigma}{\omega\cancel{\varepsilon}}\right)^{\frac{1}{2}}$$

$$\alpha = \omega\sqrt{\frac{\mu\sigma}{2\omega}}$$

$$\alpha = \sqrt{\frac{\omega^2 \mu\sigma}{2\cancel{\omega}}}$$

$$\alpha = \sqrt{\frac{\mu\sigma\omega}{2}}$$

Skin depth

$$\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\frac{\mu\sigma\omega}{2}}}$$

$$\boxed{\delta = \sqrt{\frac{2}{\mu\sigma\omega}}}$$

Where

$$\omega = 2\pi f$$

$$\delta = \sqrt{\frac{\cancel{2}}{\mu\sigma \times \cancel{2}\pi f}}$$

$$\boxed{\delta = \sqrt{\frac{1}{\mu\sigma\pi f}}}$$

V.V.imp

Skin depth for good conductors depend on the frequency of EM waves

For insulators

$$\frac{\sigma}{\omega\varepsilon} <<< 1$$

$$\therefore \left(1 + \frac{\sigma^2}{\omega^2\varepsilon^2}\right)^{\frac{1}{2}} \approx 1 + \frac{1}{2}\frac{\sigma^2}{\omega^2\varepsilon^2}$$

$$\alpha = \omega\left[\frac{\mu\varepsilon}{2}\left(\cancel{1} + \frac{\sigma^2}{2\omega^2\varepsilon^2} - \cancel{1}\right)\right]^{1/2}$$

$$\alpha = \omega\left[\frac{\mu\cancel{\varepsilon}}{2} \times \frac{\sigma^2}{2\omega^2\varepsilon^{\cancel{2}}}\right]^{1/2}$$

$$\alpha = \omega\sqrt{\frac{\mu\sigma^2}{4\omega^2\varepsilon}}$$

$$\alpha = \sqrt{\frac{\mu\sigma^2\cancel{\omega^2}}{4\cancel{\omega^2}\varepsilon}}$$

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

Skin Depth

$$\delta = \frac{1}{\alpha}$$

$$\delta = \frac{1}{\frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}}$$

$$\boxed{\delta = \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}}$$

Skin depth for insulators does not depend on the frequency of EM waves

Q.1 *For silver, $\mu = \mu_0$ and $\sigma = 3 \times 10^7$ mho/m. Calculate the skin depth at 10^8 Hz frequency. Given, $\mu_0 = 4\pi \times 10^{-7}\ N/A^2$.*

Given

$$\mu = 4\pi \times 10^{-7}\ N/A^2$$

$$\sigma = 3 \times 10^{7}\ mho/m$$

$$f = 10^{8}\ Hz$$

Skin Depth (δ)

$$\delta = \sqrt{\frac{1}{\mu \sigma \pi f}}$$

$$\delta = \sqrt{\frac{1}{4\pi \times \cancel{10^{-7}} \times 3 \times \cancel{10^{7}} \times \pi \times 10^{8}}}$$

$$\boxed{\delta = 9.18 \times 10^{-6}\ m}$$

Q.2 *Find the skin depth at frequency 71.6 MHz in aluminium. The related parameters for aluminium are* $\mu = \mu_0 = 4\pi \times 10^{-7}\ N/A^2$ *and* $\sigma = 3.58 \times 10^{7}$ *siemen/m.*

Given

$$f = 71.6\,MHz = 71.6 \times 10^{6}\,Hz$$

$$\mu = 4\pi \times 10^{-7}\ N/A^2$$

$$\sigma = 3.58 \times 10^{7}\ S/m$$

Skin Depth

$$\delta = \sqrt{\frac{1}{\mu \sigma \pi f}}$$

$$\delta = \sqrt{\frac{1}{4\pi \times 10^{-7} \times 3.58 \times 10^{7} \times \pi \times 71.6}}$$

$$\boxed{\delta = 9.94 \times 10^{-6}\ m}$$

Engineering Physics by Gulshan Sir

DPP-6

UNIT : Electromagnetic Field Theory

Q.1 *What do you mean by Depth of penetration.*

Q.2 *Show that skin depth for good conductors depend upon frequency of EM waves.*

Q.3 *Show that the skin depth for insulator (poor conductors) does not depend upon frequency of EM waves.*

Engineering Physics (Unit-II)

I Semester Theory Examination – 2023 – 24

Section – A

b. *Write down the physical significance of Poynting vector ?*

Section – B

b. *Assuming that all the energy from a 1000 – Watt lamp is radiated uniformly; calculate the average values of the intensities of electric and magnetic fields of readiation at a distance of 2 m from the lamp.*

Section – C

4. Attempt any one part of the following:

(a) *What is Maxwell fourth equation modifying on the basic of displacement current? When an ideal capacitor is charged by a dc battery, no current flows. However, when an ac source is used, the current flows continouusly. How does one explain this, based on the concept of displacement current ?*

(b) *Derive the Poynting or work – energy theorem for the flow of energy in an electromagnetic field. Also give the physical interpretation.*

Engineering Physics (Unit-II)

I Semester Theory Examination – 2022 – 23

Section – A

c. *Write down the expression for Continuity Equation in differential form.*

Section – B

b. *What do you understand by Displacement current and skin depth ?*

Section – C

4. Attempt any one part of the following:

(a) *Derive equation for simple plain electromagnetic wave starting from Maxwell's equations in free space. Show that the electromagnetic wave in free space is transverse in nature.*

(b) *Prove the Poynting theorem in electrodynamics and explain the physical significance of each of the term appearing in the final expression of the theorem.*

II Semester Theory Examination – 2022 – 23

Section – A

c. Differentiate between conduction current and displacement current.

Section – B

b. State and explain Stoke's theorem and Divergence theorem.

Section – C

4. Attempt any one part of the following:

(a) *Derive the electromagnetic wave equations in free space. Calculate the amplitude of electric and magnetic fields E_0 and H_0, at a distance of 5m from an oscillator which radiates energy isotropically at 1000W.*

(b) *Define skin depth. Write the necessary formula for the skin depth for conducting and non – conducting media. Calculate the skin depth for silver at 10^8 Hz frequency.*

Given – for silver $\mu = \mu_o$, $\mu_o = 4\pi \times 10^{-7}\ N/A^2$, $\sigma = 3 \times 10^7 mhos/m$.

I Semester Theory Examination – 2021 – 22

Section – A

c. *Write the similarities and dissimilarities between conduction and displacement current.*

d. *Define the Poynting vector and write its unit.*

Section – B

b. *Find the skin depth* δ *at a frequency of* 3.0×10^6 *Hz in aluminium where* $\sigma = 38.0 \times 10^6 S/m$ *and* $\mu_r = 1$

Section – C

4. Attempt any one part of the following:

(a) *Establish the e-m waves' equations in free space and solve them to show that they travel with the speed of light in free space and are transverse in nature.*

(b) *State and prove the Poynting theorem. Show that* $E/H = 377\ Ohm$.

II Semester Theory Examination – 2021 – 22

Section – A

c. *In an electromagnetic wave, the electric and magnetic fields are* $100V/m$ *and* $0.265A/m$.
What is the maximum energy flow

Section – B

b. *Assuming that all the energy from a 1000 watt lamp is radiated uniformly; calculate the average values of the intensities of electric and magnetic fields of radiation at a distance of 2m from lamp.*

Section – C

4. Attempt any one part of the following:

(a) *Deduce the Maxwell's equations for free space and prove that electromagnetic waves are transverse in nature*

Engineering Physics (Unit-II)

I Semester Theory Examination – 2020 – 21

Section – A

c. *What is displacement current.*

d. *Show that magnetic monopoles do not exist.*

Section – B

b. *Find the conduction current density and displacement current density for a solid with conductivity,* $\sigma = 10^{-3}\ S/m$ *and* $\varepsilon_r = 2.5$. *Electric field intensity,* $E = 4.5 \times 10^{-6} \sin(10^9 t)$.

Section – C

4. Attempt any one part of the following:

(a) *Write Maxwell's equations in free space. Also show that the electric and magnetic vectors are normal to the direction of propagation of the electromagnetic wave.*

(b) *State and deduce Poynting theorem for the flow of energy in an electromagnetic field. Discuss the physical significance of Poynting theorem.*

I Semester Theory Examination – 2019 – 20

Section – A

c. *What do you mean by impedance of a wave ?*

Section – B

b. *The sunlight strikes the upper atmosphere of earth with energy flux* $1.38\ kWm^2$. *What will be the peak values of electric and magnetic field at the points ?*

Section – C

4. Attempt any one part of the following:

(a) *Deduce four Maxwell equations in free space. Explain the concept of displacement current and show how it led to modification of Ampere law.*

(b) *State and deduce poynting theorem for the flow of energy in an electromagnetic field.*