AS5401: Data-driven modeling of complex aerospace systems and fluid flows

Project Proposal

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1 Aim and Motivation

Reduced Order Models (ROM) has been an great tools for digital twins, solving inverse problems, reducing the complexity of system for better understanding. Electrohydrodynamics (EHD) is multidisciplinary field which studies on fluid flow based on electric field and how well the flow can be manipulated for enhancement of heat transfer, retarding flow separation, mixing, pumping in micro channels where conventional pumps fails/ produces noise. To fully understand the EHD, one has to understand the underlying PDE's which are non-linear, coupled. Decoupling or linearizing these equations are extremely tedious. Thus, we take an advantage of ROM to better understand the EHD equations. The overall aim of this project is to formulate the ROM such that inverse problems can be solved with reduced computational complexity.

2 Introduction

EHD deals with calculation of charge density (q) (estimation of count of charges- be it positive and/ or negative per unit volume). Unipolar and bipolar charge describes the kind of charges being considered, uni (single) denotes distribution either positive or negative (usually when positive and negative ions have similar sizes) and bi (double) denotes distribution of both positive and negative charges. EHD equations are coupled and non-linear and are often coupled with Navier Stokes (NS) equations. Navier Stokes equation describes how the fluid behaves under given conditions. For this project, we shall decouple the NS, the reason being that charges moving in the medium barely accelerates the fluid and velocity typically remains in $O(10^{-4})$. Further EHD domain is subjected to certain potential difference (ϕ). Unipolar solves for Gauss law (that solves potential difference) and (one) species transport of charge density but bipolar accounts for solving two species transport equations (which is expensive but accurate). Throughout this project, we shall consider unipolar injection.

One can solve these equations numerically (Finite Difference Methods, Spectral Methods) but accounts to higher computational complexity. The reason being that calculating the gradients of certain parameters are important for derived variables. For example, calculating the velocity or temperature or charge density gradients results in accurate estimation of pressure drop or heat transfer coefficients or current respectively. Calculation of current is crucial as this determines the power input of the system and one tries to minimize the power for sustainability would result in wrong configuration otherwise. Throughout this project, we shall consider non-dimensioned variables unless specified.

The idea of the whole proposal is to estimate the charge density ($q(y)$) along the domain which is one-dimensional based on conduction number (C). Conduction number denotes how strong or the weak is the charge injection at the boundary $y = 0$ as in Fig. 1. According to knowledge, there are no such such studies being conducted for EHD based equations to study the charge density distribution using parameterized ROM's.

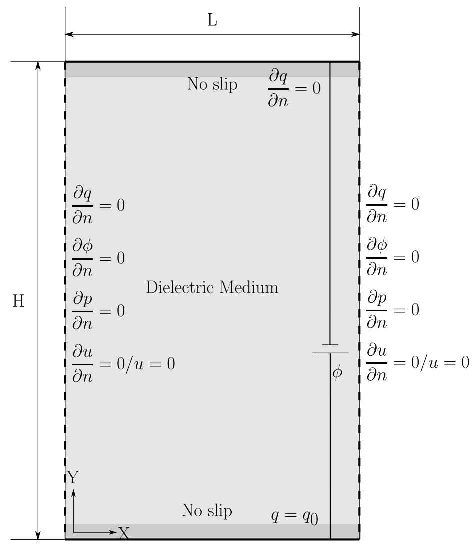


Figure 1: Schematic for validation of unipolar injection. Adapted from **Vázquez and Castellanos** [1]

3 Unipolar Injection

Gauss law defines the variation of potential difference (ϕ) with charge density (q) as in Eq. (1)

$$\nabla^2 \phi = -q \tag{1}$$

The charge density species solver is written as in Eq. (2)

$$\frac{\partial q}{\partial t} + \nabla \cdot (q\mathbf{E}) = 0 \tag{2}$$

where $\mathbf{E}$ denotes the electric field which is the derivative of potential difference as in Eq. (3)

$$\mathbf{E} = -\nabla \phi \tag{3}$$

As the equations are coupled and non-linear, we shall account for certain assumptions which doesn't change the physics of the problem. The interest is to calculate charge density variation at steady state (so, $\frac{\partial q}{\partial t} = 0$ from Eq. (2)). We shall try to formulate or combine Eqs. (1) to (3) either in terms of q or ϕ such that full order models (FOM) can be calculated for snapshot matrices which is formulated along with boundary condition as,

$$\frac{\partial^3 \phi}{\partial x^3}\frac{\partial \phi}{\partial x} + \left(\frac{\partial^2 \phi}{\partial x^2}\right)^2 = 0, \quad \phi(y=0) = \phi_0, \quad \phi(y=H) = 0, \quad \frac{\partial^3 \phi}{\partial y^3} = 0 \tag{4}$$

where $\phi \in \mathbb{R}^N$ and is non-linear and higher order PDE. Further conduction number is defined as,

$$C = \frac{q_0 H^2}{\epsilon \phi_0} \tag{5}$$

where H denotes the height of channel, q_0 is the injected charge density ($= q(y = 0) = q_0$), ϕ_0 is the potential difference between $y = 0$ and $y = H$ and ϵ is the permittivity of the medium. From Eq. (5) and boundary conditions of Eq. (4), its clear that $C \in \mathbb{R}^1$ serves are the parameter in terms of ϕ_0.

4 ROM Methodology

We shall use Finite Difference Method for discretization of domain to solve Eq. (4), Further for generating the snapshot matrices, $\Phi = [\phi(C_{\mu_1}), \phi(C_{\mu_2}), \cdots, \phi(C_{\mu_n})]_{N\times n}$ where $C_{\mu_n} \in \mathbb{R}^n$ is possible with full order model. This is further reduced through SVD to extract the modes of the system,

$$\Phi \approx U_r \Sigma_r V_r^\top \tag{6}$$

where U_r denotes the modes for reduced system. Through Galerkin projection, one can project the actual subspace (ϕ) to reduced subspace ($\hat{\phi}$),

$$\hat{\phi} = U_r^\top \phi \tag{7}$$

Since we have the governing equation (Eq. (4)) already and can be transformed to projected subspace,

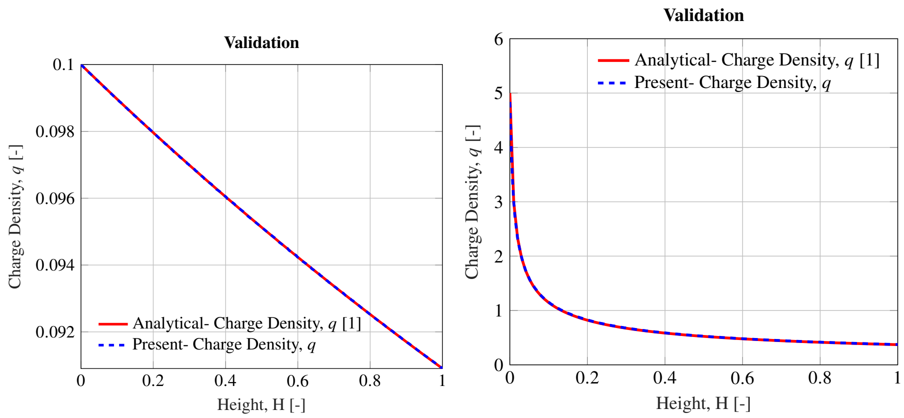
$$U_r^\top \left[U_r \frac{\partial^3 \hat{\phi}}{\partial x^3} \cdot U_r \frac{\partial \hat{\phi}}{\partial x} + \left(U_r \frac{\partial^2 \hat{\phi}}{\partial x^2} \right)^2 \right] = 0 \tag{8}$$

and can be solved through finite difference method as it's possible to evaluate $\hat{\phi}(y = 0)$ for solving.

The overall idea is for Intrusive- ROM through parametric study is to do inverse problem (say, for given charge density profile estimating the C would be much easier with ROM than with FOM). This accounts for decrease in computational complexity without loss of accuracy.

5 Conclusions

The idea has been formulated to reformulate EHD equations and further to perform inverse problems (similar to optimization) to further reduce computational complexity



(a) Weak injection (C = 0.1)

(b) Strong injection (C = 10)

Figure 2: Validation of charge density (unipolar injection)

(both in time and space) and final metrics of wall clock time, storage shall dictate the success of this project. Further as next steps, multiple parameters shall be added to study on parameter interactions, computing global sensitivity of systems.

A Validation

The EHD equations (Eqs. (1) to (3)) has been validated (Figs. 2a and 2b) in OpenFOAM (Finite Volume based solver) for $C = 0.1, 10$ and this shall provide the confidence on the prepared solver. This poses an alternative to finite difference solution methods with higher confidence.

References

[1] P.A. Vázquez and A. Castellanos. Numerical simulation of ehd flows using discontinuous galerkin finite element methods. *Computers &; Fluids*, 84:270–278, September 2013. ISSN 0045-7930. doi:10.1016/j.compfluid.2013.06.013.