

QMM_assignment5_sseetham

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```
.libPaths("C:\\Users\\Ananth\\OneDrive\\Desktop\\MSBA Kent\\Fall 2021\\Fundamentals of Machine Learning\\Assignment\\Ass 2") install.packages("Benchmarking")
```

```
library(Benchmarking)

X <- matrix(c(150,400,320,520,350,320,0.2,0.7,1.2,2.0,1.2,0.7),ncol = 2)
# <- matrix(c(0.2,0.7,1.2,2.0,1.2,0.7))
Y <- matrix(c(14000,14000,42000,28000,19000,14000,3500,21000,10500,42000,25000,15000),ncol=2)

colnames(Y) <- c("TP","RP")
colnames(X) <- c("staffing labor per hour", "cost of supplies per day")
X
```

```
##      staffing labor per hour cost of supplies per day
## [1,]                150                0.2
## [2,]                400                0.7
## [3,]                320                1.2
## [4,]                520                2.0
## [5,]                350                1.2
## [6,]                320                0.7
```

Y

```
##      TP      RP
## [1,] 14000  3500
## [2,] 14000 21000
## [3,] 42000 10500
## [4,] 28000 42000
## [5,] 19000 25000
## [6,] 14000 15000
```

FDH

```
FDH <- dea(X,Y,RTS = "fdh")
FDH
```

```
## [1] 1 1 1 1 1 1
```

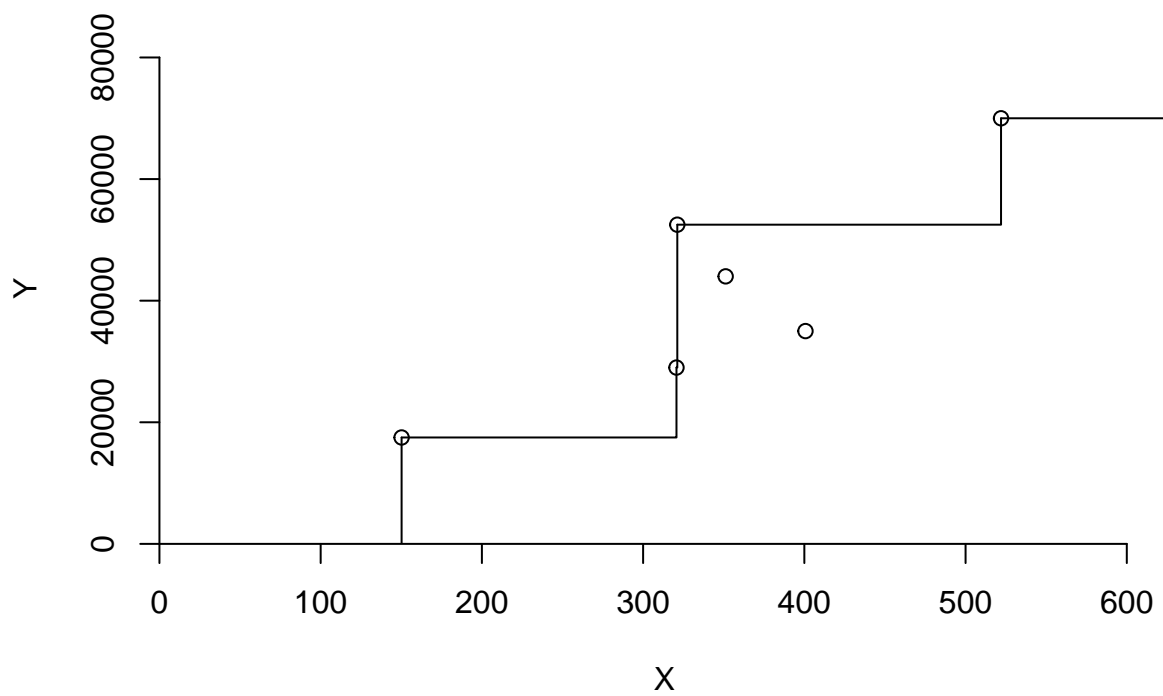
```
peers(FDH)
```

```
##      peer1
## [1,]      1
## [2,]      2
## [3,]      3
## [4,]      4
## [5,]      5
## [6,]      6
```

```
lambda(FDH)
```

```
##      L1 L2 L3 L4 L5 L6
## [1,]  1  0  0  0  0  0
## [2,]  0  1  0  0  0  0
## [3,]  0  0  1  0  0  0
## [4,]  0  0  0  1  0  0
## [5,]  0  0  0  0  1  0
## [6,]  0  0  0  0  0  1
```

```
dea.plot.frontier(X,Y,RTS="fdh")
```



```
summary(FDH, digits=4)
```

```
## Summary of efficiencies
## FDH technology and input orientated efficiency
## Number of firms with efficiency==1 are 6 out of 6
## Mean efficiency: 1
## ---
##   Eff range      #   %
##       E ==1    6 100
##   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##       1       1       1       1       1       1
```

CRS

```
CRS <- dea(X,Y,RTS = "crs")
CRS
```

```
## [1] 1.0000 1.0000 1.0000 1.0000 0.9775 0.8675
```

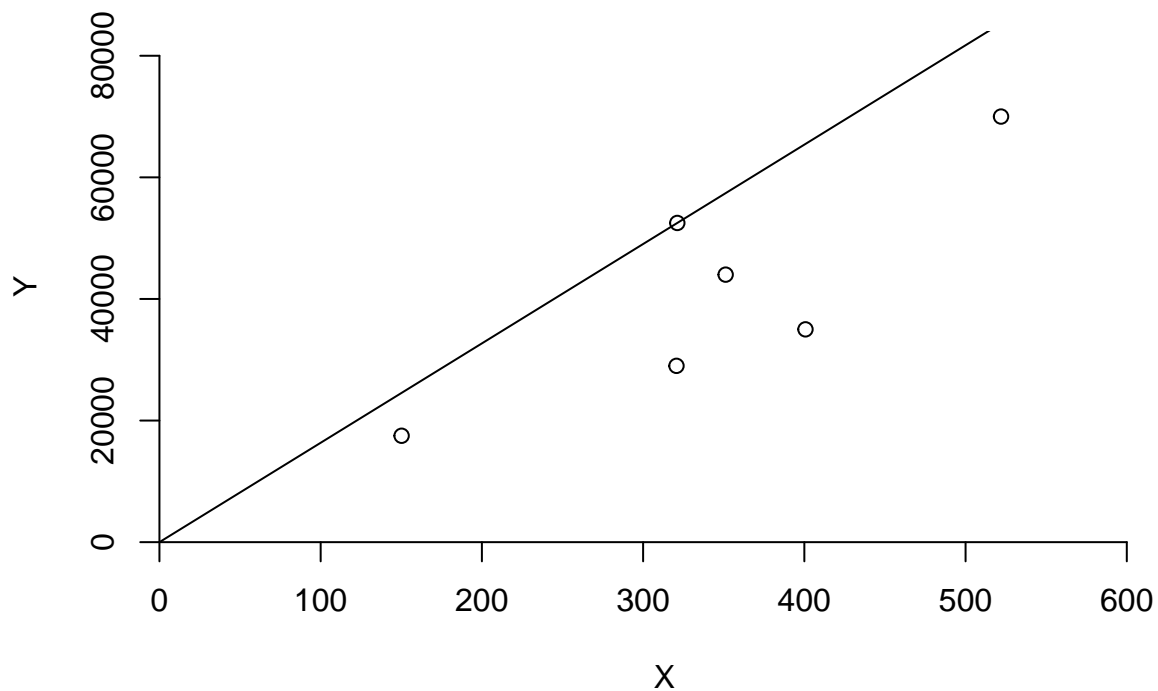
```
peers(CRS)
```

```
##      peer1 peer2 peer3
## [1,]     1    NA    NA
## [2,]     2    NA    NA
## [3,]     3    NA    NA
## [4,]     4    NA    NA
## [5,]     1     2     4
## [6,]     1     2     4
```

```
lambda(CRS)
```

```
##           L1           L2 L3           L4
## [1,] 1.0000000 0.0000000 0 0.0000000
## [2,] 0.0000000 1.0000000 0 0.0000000
## [3,] 0.0000000 0.0000000 1 0.0000000
## [4,] 0.0000000 0.0000000 0 1.0000000
## [5,] 0.2000000 0.08048142 0 0.5383307
## [6,] 0.3428571 0.39499264 0 0.1310751
```

```
dea.plot.frontier(X,Y,RTS="crs")
```



```
summary(CRS, digits=4)
```

```
## Summary of efficiencies
## CRS technology and input orientated efficiency
## Number of firms with efficiency==1 are 4 out of 6
## Mean efficiency: 0.974
## ---
##   Eff range      # %
##   0.8<= E <0.9   1 17
##   0.9<= E <1     1 17
##           E ==1   4 67
##   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.8675  0.9831  1.0000  0.9742  1.0000  1.0000
```

VRS

```
VRS <- dea(X,Y,RTS = "vrs")
VRS
```

```
## [1] 1.0000 1.0000 1.0000 1.0000 1.0000 0.8963
```

```
peers(VRS)
```

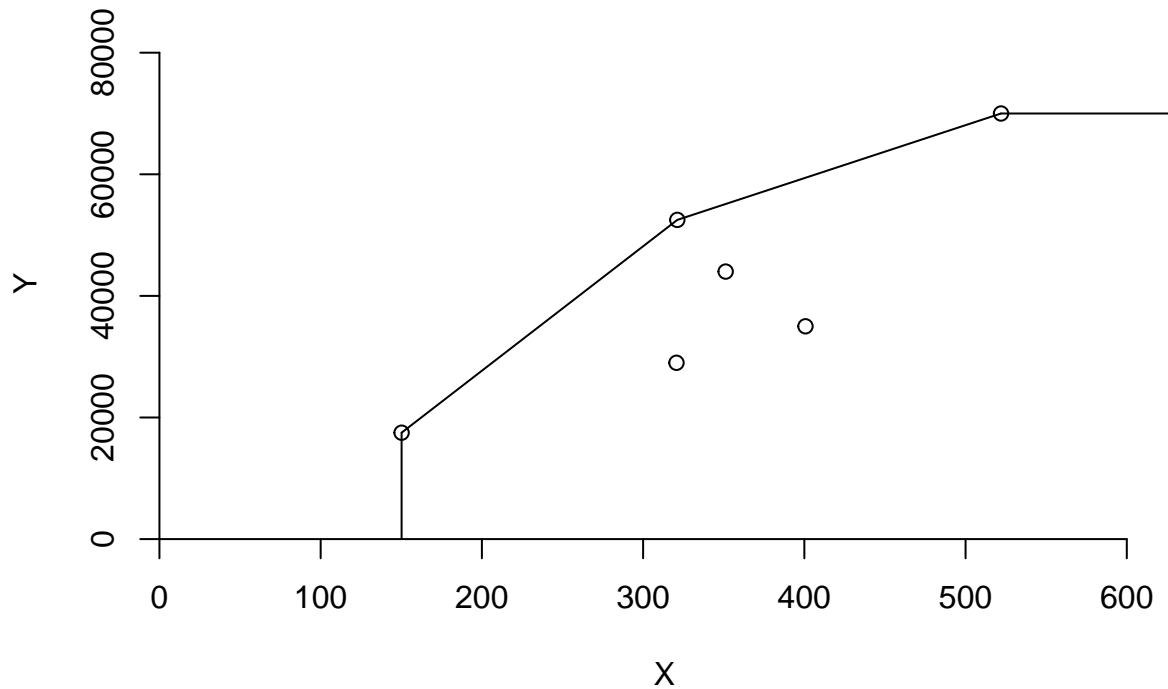
```
##      peer1 peer2 peer3
```

```
## [1,] 1 NA NA
## [2,] 2 NA NA
## [3,] 3 NA NA
## [4,] 4 NA NA
## [5,] 5 NA NA
## [6,] 1 2 5
```

```
lambda(VRS)
```

```
##          L1          L2 L3 L4          L5
## [1,] 1.0000000 0.0000000 0 0 0.0000000
## [2,] 0.0000000 1.0000000 0 0 0.0000000
## [3,] 0.0000000 0.0000000 1 0 0.0000000
## [4,] 0.0000000 0.0000000 0 1 0.0000000
## [5,] 0.0000000 0.0000000 0 0 1.0000000
## [6,] 0.4014399 0.3422606 0 0 0.2562995
```

```
dea.plot.frontier(X,Y,RTS="vrs")
```



```
summary(VRS, digits=4)
```

```
## Summary of efficiencies
## VRS technology and input orientated efficiency
## Number of firms with efficiency==1 are 5 out of 6
```

```
## Mean efficiency: 0.983
## ---
##   Eff range      #   %
##   0.8<= E <0.9   1 17
##   0.9<= E <1     0  0
##           E ==1   5 83
##   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.8963  1.0000  1.0000  0.9827  1.0000  1.0000
```

IRS

```
IRS <- dea(X,Y,RTS = "irs")
IRS
```

```
## [1] 1.0000 1.0000 1.0000 1.0000 1.0000 0.8963
```

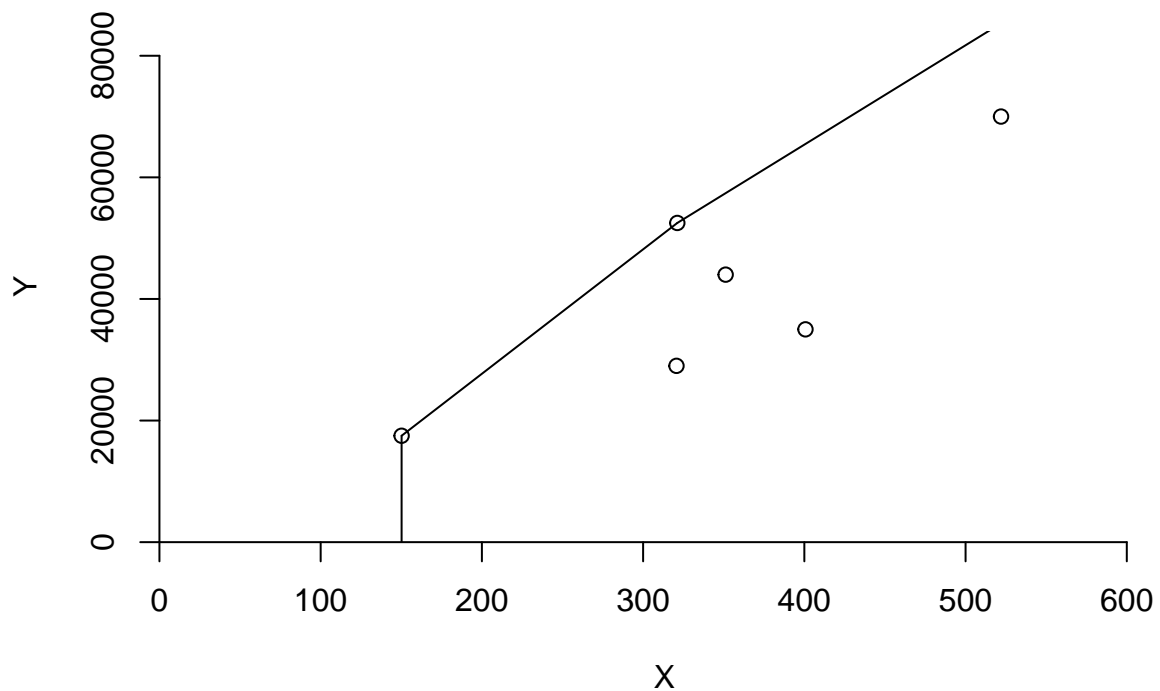
```
peers(IRS)
```

```
##      peer1 peer2 peer3
## [1,]     1    NA    NA
## [2,]     2    NA    NA
## [3,]     3    NA    NA
## [4,]     4    NA    NA
## [5,]     5    NA    NA
## [6,]     1     2     5
```

```
lambda(IRS)
```

```
##           L1           L2 L3 L4           L5
## [1,] 1.0000000 0.0000000  0  0 0.0000000
## [2,] 0.0000000 1.0000000  0  0 0.0000000
## [3,] 0.0000000 0.0000000  1  0 0.0000000
## [4,] 0.0000000 0.0000000  0  1 0.0000000
## [5,] 0.0000000 0.0000000  0  0 1.0000000
## [6,] 0.4014399 0.3422606  0  0 0.2562995
```

```
dea.plot.frontier(X,Y,RTS="irs")
```



```
summary(IRS, digits=4)
```

```
## Summary of efficiencies
## IRS technology and input orientated efficiency
## Number of firms with efficiency==1 are 5 out of 6
## Mean efficiency: 0.983
## ---
##   Eff range      #   %
##   0.8<= E <0.9   1  17
##   0.9<= E <1     0   0
##       E ==1      5  83
##   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.8963  1.0000  1.0000  0.9827  1.0000  1.0000
```

DRS

```
DRS <- dea(X,Y,RTS = "drs")
DRS
```

```
## [1] 1.0000 1.0000 1.0000 1.0000 0.9775 0.8675
```

```
peers(DRS)
```

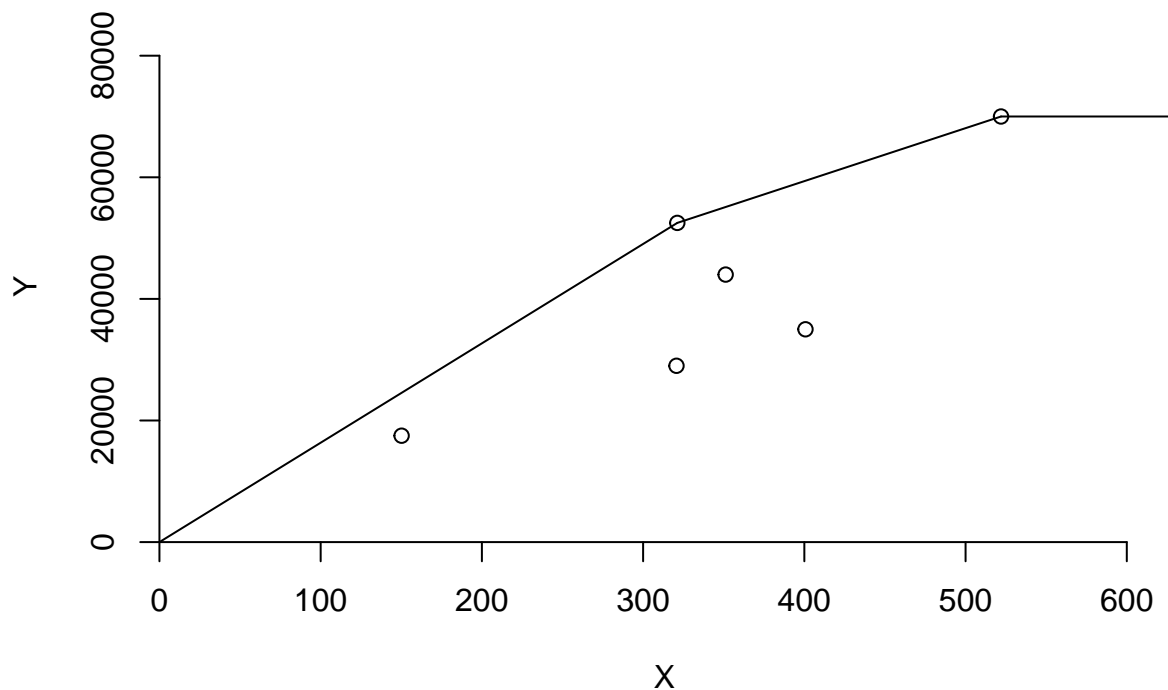
```
##      peer1 peer2 peer3
```

```
## [1,] 1 NA NA
## [2,] 2 NA NA
## [3,] 3 NA NA
## [4,] 4 NA NA
## [5,] 1 2 4
## [6,] 1 2 4
```

```
lambda(DRS)
```

```
##          L1          L2 L3          L4
## [1,] 1.0000000 0.0000000 0 0.0000000
## [2,] 0.0000000 1.0000000 0 0.0000000
## [3,] 0.0000000 0.0000000 1 0.0000000
## [4,] 0.0000000 0.0000000 0 1.0000000
## [5,] 0.2000000 0.08048142 0 0.5383307
## [6,] 0.3428571 0.39499264 0 0.1310751
```

```
dea.plot.frontier(X,Y,RTS="drs")
```



```
summary(DRS, digits=4)
```

```
## Summary of efficiencies
## DRS technology and input orientated efficiency
## Number of firms with efficiency==1 are 4 out of 6
```



```
## Mean efficiency: 0.974
## ---
##   Eff range      #   %
##   0.8<= E <0.9   1 17
##   0.9<= E <1     1 17
##           E ==1   4 67
##   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.8675 0.9831  1.0000  0.9742  1.0000  1.0000
```

FRH

```
FRH <- dea(X,Y,RTS = "add")
FRH
```

```
## [1] 1 1 1 1 1 1
```

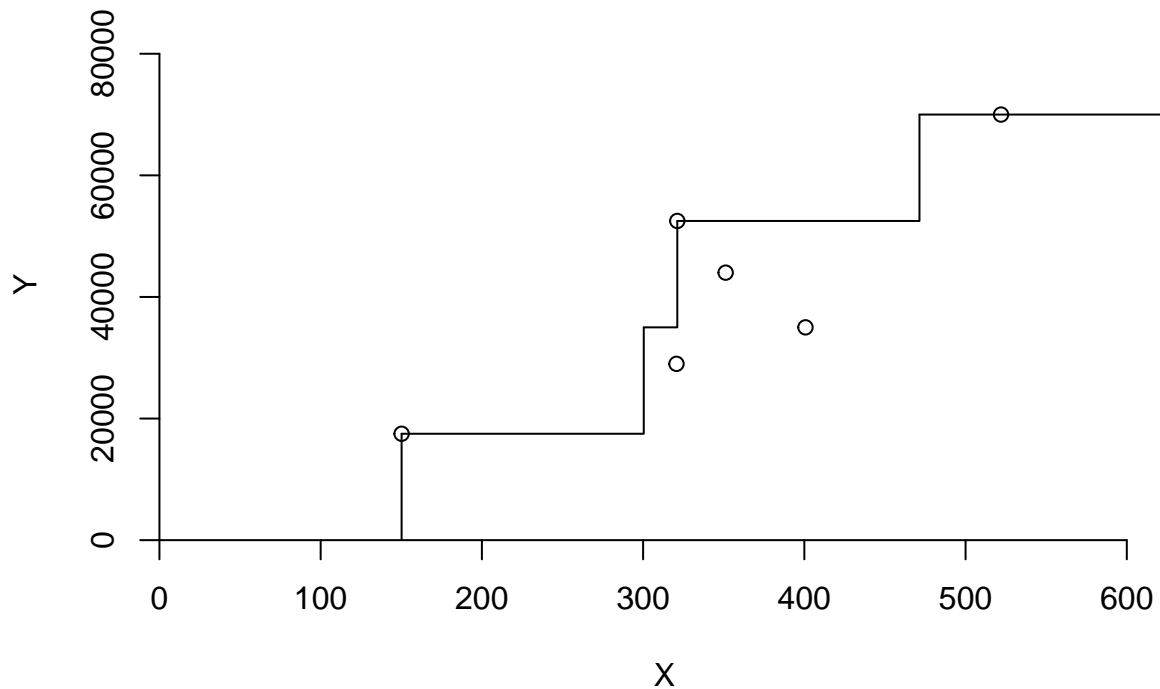
```
peers(FRH)
```

```
##      peer1
## [1,]      1
## [2,]      2
## [3,]      3
## [4,]      4
## [5,]      5
## [6,]      6
```

```
lambda(FRH)
```

```
##      L1 L2 L3 L4 L5 L6
## [1,]  1  0  0  0  0  0
## [2,]  0  1  0  0  0  0
## [3,]  0  0  1  0  0  0
## [4,]  0  0  0  1  0  0
## [5,]  0  0  0  0  1  0
## [6,]  0  0  0  0  0  1
```

```
dea.plot.frontier(X,Y,RTS="add")
```



```
summary(FRH, digits=4)
```

```
## Summary of efficiencies
## ADD technology and input orientated efficiency
## Number of firms with efficiency==1 are 6 out of 6
## Mean efficiency: 1
## ---
##   Eff range      #   %
##       E ==1      6 100
##   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##       1       1       1       1       1       1
```

4) On contrast we can see that the different DEA technologies yielded

DEA techs Efficiency FDH Number of firms with efficiency==1 are 6 out of 6, Mean efficiency: 1 CRS Number of firms with efficiency==1 are 4 out of 6, Mean efficiency: 0.974 VRS Number of firms with efficiency==1 are 5 out of 6, Mean efficiency: 0.983 IRS Number of firms with efficiency==1 are 5 out of 6, Mean efficiency: 0.983 DRS Number of firms with efficiency==1 are 4 out of 6, Mean efficiency: 0.974 FRH Number of firms with efficiency==1 are 6 out of 6, Mean efficiency: 1

QUESTION2)

Solution :

$Y1 = 6x_1 + 4x_2 + 5x_3$ -> employee $Y2 = 8x_1 + 7x_2 + 5x_3$ __> next year earning.

$Y_i = y_{i+} - y_i, i = 1, 2, 3$

$y_{i+} = \begin{cases} y_i, & Y_i \geq 0 \\ 0, & \text{otherwise} \end{cases}$

$y_i = \begin{cases} |y_i|, & y_i \leq 0 \end{cases}$

Objective function :

Max: $20x_1 + 15x_2 + 25x_3 - 6y_{1m} - 6y_{1n} - 3y_{2n}$;

s.t

$6x_1 + 4x_2 + 5x_3 - y_{1m} + y_{1n} = 50$; $8x_1 + 7x_2 + 5x_3 - y_{2m} + y_{2n} = 75$;

Initial LP

/* Objective function */

Max: $20x_1 + 15x_2 + 25x_3 - 6y_{1m} - 6y_{1n} - 3y_{2n}$;

/* Constraints */

$6x_1 + 4x_2 + 5x_3 - y_{1m} + y_{1n} = 50$; $8x_1 + 7x_2 + 5x_3 - y_{2m} + y_{2n} = 75$;

```
library(lpSolve)
```

```
Q2 <- read.lp("Q2.lp")
```

```
solve(Q2)
```

```
## [1] 0
```

```
get.objective(Q2)
```

```
## [1] 225
```

```
get.variables(Q2)
```

```
## [1] 0 0 15 25 0 0 0
```

```
get.constraints(Q2)
```

```
## [1] 50 75
```

From the results we see that the Maximize profit is 225 million dollars, and we can observe that $x_1 = 0$, $x_2 = 0$, $x_3 = 15$ and Y_+ employee is 25,000 which is not a feasible solution.

Hence, increasing the increase Y_+ to 1000(random number), to get optimal solution

Below is the new LP formulation:

```
/* Objective function */
```

```
Max: 20x1 + 15x2 + 25x3 - 1000y1m - 6y1n - 3y2n;
```

```
/* Constraints */
```

```
6x1 + 4x2 + 5x3 - y1m + y1n = 50; 8x1 + 7x2 + 5x3 - y2m + y2n = 75;
```

We can observe that increase in employment y_{1m} is set to 1000 instead 6, this will negate the increase in workforce observed in the previous lp.

```
Second = read.lp("Second.lp")  
solve(Second)
```

```
## [1] 0
```

```
get.objective(Second)
```

```
## [1] 208.3333
```

```
get.variables(Second)
```

```
## [1] 0.000000 8.333333 3.333333 0.000000 0.000000 0.000000 0.000000
```

```
get.constraints(Second)
```

```
## [1] 50 75
```

Now we can see that the Maximum profit is 208.3 Million. Emax corporation should produce 0 from product 1 and 8 of product 2 and 3 of product 3, i.e

$x_1 = 0$; $x_2 = 8$; $x_3 = 3$ and we can see that $y_{1+} = 0$, $y_{1-} = 0$, $y_{2+} = 0$, $y_{2-} = 0$. which solves the question and the constraints are 50 and 75 which optimul