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RV COLLEGE OF ENGINEERING®

(An Autonomous Institution Affiliated to VTU)

I Semester B. E. Regular / Supplementary Examinations Feb/Mar-2025

Common to AIML / BT / CS / CY / CD / IS

FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS AND STATISTICS

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.
3. Use of Handbook of Mathematics is permitted

PART-A

			M	BT	CO
1	1.1	The Trace and determinant of matrix whose eigen values are 7,1,9, respectively _____ and _____.	02	1	1
	1.2	The coefficient of x^3 in Maclaurin series of $\sin(2x)$ is _____.	02	2	2
	1.3	The curvature of the function $f(x) = x^2 + 2x + 1$ at $x = 0$ is _____.	02	1	2
	1.4	The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 3 & 7 \end{bmatrix}$ is _____.	02	1	1
	1.5	If $f(x,y) = x \sin y$, then value of f_{xy} at(0,1) is _____.	02	2	2
	1.6	For the implicit function, $x^3 + y^3 - 3xy + y^2 = 0$, find $\frac{dy}{dx}$ using partial differentiation.	02	2	2
	1.7	Evaluate the integral $\int_0^1 \int_0^2 \int_0^3 dx\, dy\, dz$.	02	1	1
	1.8	Sketch the domain of integral $\int_0^1 \int_{x^2}^{x} f(x,y) dy\, dx$.	02	3	3
	1.9	If $r = 0.8, b_{xy} = 0.32$, then the value of b_{yx} is _____.	02	1	1
	1.10	If $\mu_2 = 8, \mu_3 = 50$ and $\mu_4 = 199$, then β_1 = _____ and β_2 = _____.	02	1	1

PART-B

2	a	Examine the consistency and solve the system of equations $x - y + z = 4$ $2x + y - 3z = 0$ $x + y + z = 0$	05	2	2
	b	Solve following system of equations by Gauss Seidel method $10x + y + z = 9$ $x - 10y + 3z = 8$ $2x + y - 10z = 3$ By taking initial approximation $X(0) = [0 \quad 0 \quad 0]^T$. Perform 4 iterations.	05	2	2
	c	Find the dominant eigen value and the corresponding eigen vector of the matrix. $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by power method by taking the initial eigen vector as $[1 \quad 1 \quad 1]^T$. Perform four iterations.	06	3	3
3	a	Show that the pair of curves intersect each other orthogonally. $r^2 \sin(2\theta) = a^2, \quad r^2 \cos(2\theta) = b^2$	08	2	2
	b	Show that the radius of curvature of the curve $r^n = a^n \cos n\theta$ varies inversely as r^{n-1}.	08	3	2
		OR			

4	a	Find the circle of curvature at the point $(3/2, 3/2)$ of the curve $x^3 + y^3 = 3xy$.	08	3	2
	b	Expand $\tan^{-1} x$ in powers of $(x-1)$ upto the term containing fourth degree.	08	2	2
5	a	If $v = e^{a\theta} cos(a \log r)$ prove that $\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2} = 0$	08	2	1
	b	The temperature 'T' at any point (x, y, z) in space is $T = 400\,xyz^2$. Find the highest temperature at the surface of the unit sphere. $x^2 + y^2 + z^2 = 1$.	08	4	4
		OR			
6	a	If $Z = f(x, y)$ where $x = r\cos\theta$, $y = r\,sin\theta$. Show that $\left(\frac{\partial Z}{\partial x}\right)^2 + \left(\frac{\partial Z}{\partial y}\right)^2 = \left(\frac{\partial Z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial Z}{\partial \theta}\right)^2$.	08	2	1
	b	If $x = r\,sin\,\theta\cos\phi$, $y = r\,sin\,\theta\sin\phi$, $z = r\,cos\theta$. Show that $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = r^2 sin\theta$.	08	4	4
7	a	Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz\,dy\,dx}{\sqrt{1-x^2-y^2-z^2}}$.	08	2	2
	b	Change the order of integration and evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dy\,dx$.	08	3	3
		OR			
8	a	Evaluate $\iint xy\,dx\,dy$ over the region bounded by x-axis, ordinate $x = 2a$ and the curve $x^2 = 4ay$.	08	2	2
	b	Find the area enclosed by the cardiod $r = a(1 + cos\theta)$ between $\theta = 0$ and $\theta = \pi$, using double integration.	08	3	3
9	a	If F is the force required to lift a load W, by means of a pulley, fit a linear expression $F = a + bW$ against the following data			

W	50	70	100	120	140	180
F	12	15	21	25	32	39

Also find the force at the load $W = 130$. 08 2 2

b Various doses of medical substances were given to groups of 25 mice and the following results were observed.

Dose mg (x)	4	6	8	10	12	14	16
No. of recoveries (y)	1	3	6	8	14	16	20

i) Find the equation of regression line of y on x which fits the data best.

ii) Estimate the number of recoveries in a group of 25 mice who received the doses of 7 mg. 08 2 3

OR

10 a Calculate the first four central moments of the following distribution.

Wages	1.5 – 2.5	2.5 – 3.5	3.5 – 4.5	4.5 – 5.5	5.5 – 6.5
f	1	3	7	3	1

08 2 2

b Fit a curve of the form $y = ab^x$ for the data and hence find the estimation for 'y' when $x = 8$.

x	1	2	3	4	5	6	7
y	87	97	113	129	202	195	193

08 2 3