RV College of Engineering
Mysore Road, RV Vidyaniketan Post,
Bengaluru - 560059, Karnataka, India

USN | 1 R V 2 4 C S 0 3 1

DEPARTMENT OF MATHEMATICS

Academic year 2024-2025 (Even Semester)

Date : 16 June 2025	**CIE - II**	**Max Marks : 50**
Time: 10:00 AM to 11 :30 AM	**UG**	**Duration : 90 mins**
Semester: II B. E. (AIML, CS, CY, CD, IS, BT)		
Course Title: Number Theory, Vector Calculus and Computational Methods		**Course Code: MA221TC**

Q. Nos.	Question	M	CO	BT
1	Using the Euclidean algorithm, solve the following: i) Find the greatest common divisor d of 1769 and 2378. ii) Determine integers x and y such that $1769x + 2378y = d$. iii) Show that there are infinitely many integer solutions (x, y) to the equation $1769x + 2378y = d$.	10	2	2
2a	Consider the linear congruence: $18x \equiv 42(\text{mod } 120)$ i) Verify whether the congruence has any solution. ii) If a solution exists, determine all distinct solutions modulo 120.	6	3	3
2b	Determine the number of positive integers less than 382 that have a multiplicative inverse modulo 382, and find the multiplicative inverse of 381 modulo 382, if it exists.	4	1	1
3a	Find the remainder when 7^{4986} is divided by 36.	6	2	2
3b	Find the number and sum of positive divisors of $n = 3980$.	4	1	1
4	Consider the public key $(e, n) = (7,33)$, encrypt the plain text G F L R where the alphabet: A, B…. Y, Z are assigned numbers 2, 3…27. Obtain cipher text and private key.	10	4	3
5a	A particle moves along the curve: $x = 1 - t^3,\ y = 1 + t^2,\ z = 2t - 5$. Determine its velocity and acceleration. Find the components of velocity and acceleration at $t = 1$ in the direction of vector $2\hat{i} + \hat{j} + 2\hat{k}$.	5	3	3
5b	Find the angle between tangents to the curve: $x = t^2,\ y = t^3,\ z = t^4$ at $t = 2$ and $t = 3$.	5	3	3

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars		CO1	CO2	CO3	CO4	L1	L2	L3	L4	L5	L6
	Test	Max Marks	08	16	16	10	08	16	26	-	-	-

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R V College of Engineering

R V Vidyanikethan Post
Mysuru Road Bengaluru - 560 059

II Semester B.E Regular/Supplementary Examinations July-2025.
(Common to AI, BT, CS, CD, CY, & IS)
Course : Number Theory, Vector Calculus And Computational Methods-MA221TC

Time : 3 Hours

Maximum Marks : 100

Instructions to the students

1. Answer all the questions from Part A.
Part A questions should be answered in first three pages of the answer book only.
2. Answer Five full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and four, 5 and 6, 7 and 8, 9 and 10.
3. Handbook of Mathematics is permitted.

Part A

Question No	Question	M	CO	BT
1.1	The remainder obtained when $(1! + 2! + 3! + \cdots + 100!)$ is divided by 12 is_____.	02	2	2
1.2	If $693 = 3^x\, 7^y\, 11^z$, where x, y, z are positive integers, then the product of x, y, z is_______.	02	2	2
1.3	If the vector $\vec{F} = (2x^2y^2 + z^2)\hat{i} + (3xy^3 - x^2z)\hat{j} + (\lambda xy^2z + xy)\hat{k}$ is solenoidal, then the value of the constant λ is ____.	02	1	1
1.4	If $\vec{F} = (x + y + z)\hat{i} + (x + 2y - z)\hat{j} + (x + cy + 2z)\hat{k}$ is an irrotational vector field, then the value of $c =$_________________.	02	2	1
1.5	If $curl\ \vec{F} = 0$, then for any closed curve C $\int_C \vec{F} \cdot d\vec{r}$ =_____________.	02	1	1
1.6	By Gauss divergence theorem , find $\iint_S (ax\,\hat{\imath} + by\,\hat{\jmath} + cz\,\hat{k}) \cdot \hat{n}\, dS$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$.	02	2	2
1.7	If $2 + \sqrt{3}$ and $2 - \sqrt{3}$ are the roots of an auxiliary equation then the corresponding differential equation is _______________.	02	1	1
1.8	*Solve* $\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + 25y = 0$.	02	1	1
1.9	Given $y = 2x^3 - 4x + 1$ for $x = -1, 0, 1$ then $\Delta^2\{y_0\} =$ ________________.	02	1	1
1.10	Construct the forward difference table for the polynomial $y = x^2 - 3x + 1$ where $x = -1, 0, 1$.	02	2	2

Part B

Question No	Question	M	CO	BT
2a	Given the public key $(e, n) = (11, 65)$, encrypt plain text $J\ B\ E$, where the alphabet $A, B, C, \ldots X, Y, Z$ are assigned the numbers $2, 3, \cdots, 26, 27$. Give the cipher text and also find the private key d.	08	4	4

2b	By using the Euclidean algorithm, find the greatest common divisor d of 1389 and 2567 and then find integers x and y to satisfy $1389x + 2567y = d$. Also show that x and y are not unique.	08	3	3
3a	Find a and b such that the surfaces $ax^2y + z^4 = 12$ and $5x^2 - byz = 9x$ intersect orthogonally at $(1, -1, 2)$.	06	3	3
3b	Angle between tangents to the curve $x = t^2 + 1, y = 4t - 3, z = 2t^2 - 6t$ at $t = 1$ and $t = 2$.	05	2	2
3c	Show that the divergence of the vector field $\phi(r)\vec{r}$ is $3\phi(r) + r\phi'(r)$, where $\vec{r}$ is the position vector of the point (x, y, z) and $r = \lvert\vec{r}\rvert$.	05	3	2
	OR			
4a	If $\phi = x^2yz + xyz + 4xz^2$, find div(grad ϕ) at the point (1,2,1).	05	1	1
4b	Show that the field $\vec{F} = 3x^2y\hat{i} + (x^3 - 2yz^2)\hat{j} + (3z^2 - 2y^2z)\hat{k}$ is conservative vector field. Hence determine its scalar potential $\emptyset$ such that $\vec{F} = grad\, \emptyset$.	06	4	3
4c	Show that $r^n\vec{r}$ is an irrotational vector for any value of n, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\lvert\vec{r}\rvert = r$.	05	3	2
5a	If $\vec{F} = (3x^2 + 6y)\hat{i} - 14yz\,\hat{j} + 20xz^2\hat{k}$, evaluate $\int_C \vec{F}\cdot d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the curve given by $x = t,\ y = t^2,\ z = t^3$.	08	2	2
5b	Using Stokes theorem evaluate $\int_C \vec{F}\cdot d\vec{r}$ for the vector field $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ where C is the rectangle bounded by the lines $x = -a$, $x = +a$, $y = 0$, $y = b$.	08	3	3
	OR			
6a	Apply Divergence theorem to compute $\iint_S \vec{F}\cdot\hat{n}\,dS$ where $\vec{F} = 4xz\,\hat{\imath} - y^2\,\hat{\jmath} + yz\,\hat{k}$ and S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.	08	3	3
6b	If $\vec{F} = x^2\,\hat{\imath} + xy\,\hat{\jmath}$, evaluate $\int_C \vec{F}\cdot d\vec{r}$ from $(0,0)$ to $(1,1)$ along the parabola $y = \sqrt{x}$.	08	2	2
7a	Solve the differential equation: $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = cos2x$.	08	2	2
7b	Obtain the general solution of the second-order differential equation $x^2\,y'' - xy' + y = x^2\log(x)$.	08	2	3
	OR			
8a	Solve the boundary value problem: $\frac{d^2y}{dx^2} + 4y = sin(2x), y(0) = 1, y'\left(\frac{\pi}{2}\right) = 0$.	08	4	4
8b	Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x}\log x$ by the method of variation of parameters.	08	3	3
9a	The population of a town is given by the following table:	08	3	3

Year	1951	1961	1971	1981	1991
Population in thousands	19.96	39.65	58.81	77.21	94.61

Using suitable interpolation formula, calculate the increase in the population from the year 1955 to 1985.

The following table gives the viscosity of an oil as a function of temperature. Use suitable interpolation formula to compute viscosity of oil at the temperatures of 120^0C and 140^0C.

9b

Temperature	110	130	160	190
Viscosity	10.8	8.1	5.5	4.8

08 3 2

OR

The following table shows the observed temperatures T in degree centigrade of cooling water in a vessel at different times t (in minutes).

10a

t	1	3	5	7	9
T	85.3	74.5	67.0	60.5	54.3

8 3 3

(i) Estimate the temperature at $t = 1.5$.

(ii) Estimate the approximate rate of cooling at $t = 3$.

The following data defines the sea-level concentration of dissolved oxygen for fresh water as a function of temperature:

10b

$T(^0C)$	0	8	16	24	32
$O_2(mg/L)$	14.621	11.843	9.870	8.418	7.305

08 3 2

Use appropriate Newton's interpolation formula to calculate the amount of oxygen when temperature is 10^0C and 35^0C.