RV College of Engineering®
Mysore Road, RV Vidyaniketan Post,
Bengaluru - 560059, Karnataka, India

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R V College of Engineering

R V Vidyanikethan Post
Mysuru Road Bengaluru - 560 059

IV Semester BE Regular / Supplementary Examinations June/July -2025
(Common to CS, IS, CD, CY and AIML)
Course : Discrete Mathematical Structures and Combinatorics-CS241AT

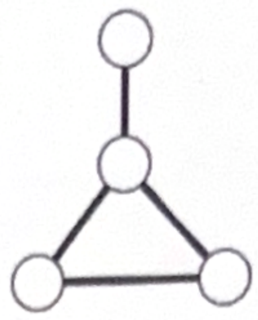
Time : 3 Hours **Maximum Marks : 100**

Instructions to the students

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2, is compulsory. Answer any one full question from 3 & 4, 5 & 6, 7 & 8, and 9 & 10.

Part A

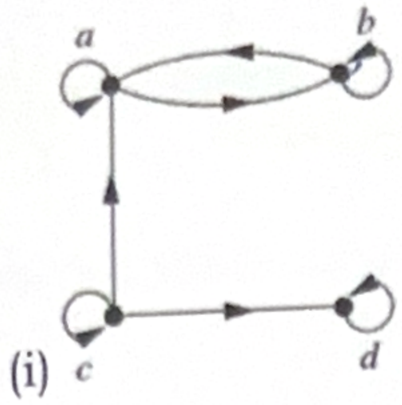
Question No	Question	M	CO	BT
1.1	If $G(x)$ is the generating function of the sequence $(a_n) = (a_0, a_1, a_2, a_3, \ldots)$ then the generating function for the sequence $(a_0, 0, a_1, 0, a_2, 0, \ldots)$ is _____	02	1	2
1.2	Number of derangements of $\{A, B, C, D\}$ when A is in the second position is _____.	02	1	1
1.3	Translate the statement "Some drivers do not obey the speed limit" into logical expression using quantifiers and express the negation of the statement in terms of quantifiers.	02	1	2
1.4	Without using truth table, prove that $\neg(p \vee (q \wedge r)) \iff \neg p \wedge (\neg q \vee \neg r)$.	02	2	2
1.5	If $R = \{(1,1),(1,2),(2,1),(2,2),(3,4),(4,3),(3,3),(4,4)\}$ is defined on the set $A = \{1,2,3,4\}$ determine the partition induced by R.	02	1	1
1.6	If there are 60 one-one functions from A to B and $\|A\| = 3$, then $\|B\| =$ _____.	02	1	1
1.7	For $n = 3$ and $x = 101 \in \mathbb{Z}_2^3$, the sphere $S(x, 2)$ is _____.	02	2	2
1.8	Verify if $f : \langle \mathbb{R}, + \rangle \longrightarrow \langle \mathbb{R}, + \rangle$ defined by $f(x) \doteq 3x + 1, \forall x \in \mathbb{R}$ is a homomorphism.	02	2	2

1.9	The vertex and edge chromatic number of the following graph are _____ and _____ respectively. 	02	2	1
1.10	Number of zeros in the incidence matrix of K_{17} is _____.	02	2	2

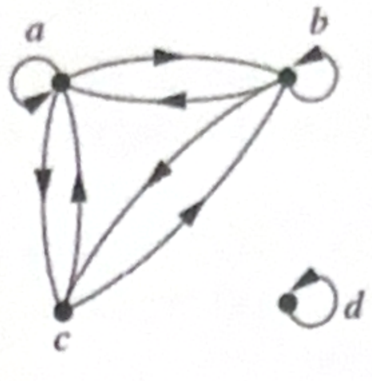
Part B

Question No	Question	M	CO	BT
2a	Determine the number of derangements of $\{1, 2, 3, 4, 5, 6\}$ which (i) begin with integers 1, 2, and 3 in some order. (ii) end with integers 1, 2, and 3 in some order.	05	3	3
2b	A bank pays a certain annual interest rate on deposits, with the interest compounded quarterly (i.e., once every 3 months). A deposit doubles in 6 years and 6 months. Obtain the recurrence relation for the value of the deposit at the end of n^{th} quarter and hence determine the annual percentage of interest rate paid by the bank.	06	3	4
2c	Determine the number of six- digit positive integers (without repetition of digits) which are (i) even (ii) divisible by 5.	05	2	2
3a	Verify whether $(p \wedge q) \vee r$ is logically equivalent to $(p \vee r) \wedge (q \vee r)$ using a truth table.	05	1	1
3b	Let $M(x, y)$ be "x has sent y an e-mail message" and $T(x, y)$ be "x has telephoned y" where the domain consists of all students in your class. i) Use quantifiers and translate the statement "There is a student in your class who has not received an e-mail message from anyone else in the class and who has not been called by any other student in the class" to logical expression. ii) Write the negation of the above statement in logical expression and in words.	05	3	3

3c	Write the following argument in symbolic form. Verify the validity of the argument using rules of inference. "All movies produced by John Sayles are wonderful. John Sayles produced a movie about coal miners. Therefore, there is a wonderful movie about coal miners."	06	3	3	
	OR				
4a	Write the converse, inverse and contrapositive of the statement: If $x > 0$ then $x^2 > 0$. Also, write the truth value of all the three statements.	06	2	2	
4b	Use a direct proof and proof by contraposition to show that if m is an even integer then $m + 7$ is odd.	05	4	3	
4c	Prove that $\neg[\neg((p \vee q) \wedge r) \vee \neg q] \iff q \wedge r$ using laws of logic.	05	3	3	
5a	Prove that the $f : \mathbb{R} \to \mathbb{R}^+$ defined as $f(x) = e^{2x+5}$ is one-one and onto function. Also compute $f^{-1}(x)$.	06	3	4	
5b	Draw the Hasse diagram for the partial order R on the set $A = \{1, 2, 3, 4, 5\}$ whose matrix is given below. Also, write the relation R in each case. i) $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ ii) $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	05	2	2	
5c	Let R be a relation on the set of all bit strings of length three or more, where two bit strings x and y are related if and only if their first three bits are equal. Show that R is an equivalence relation. Describe the equivalence classes of this relation. In particular, determine the equivalence class of the bit strings: (i) 11111, (ii) 01010101.	05	4	4	
	OR				
6a	Consider the functions $f, g, h : \mathbb{Z} \to \mathbb{Z}$ defined as $f(x) = x - 1$, $g(x) = 3x$, $h(x) = \begin{cases} 0 \text{ if } x \text{ is even} \\ x \text{ if } x \text{ is odd.} \end{cases}$ Then compute $f \circ g$, $g \circ f$, $g \circ h$, $h \circ g$, $f \circ (g \circ h)$, $(f \circ g) \circ h$.	06	3	3	
6b	Determine whether the relations represented by the directed graphs shown in (i) and (ii) are reflexive, irreflexive, symmetric, antisymmetric, and transitive. Justify your answer.	05	2	3	



(i)



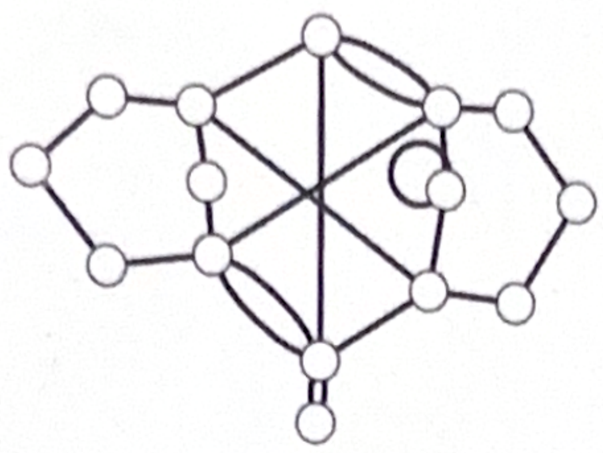
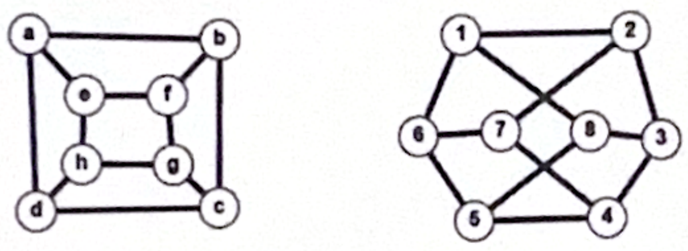
(ii)

6c	Let R and S be two relations on a set of positive integers, where $R = \{(x, 2x)\|x \in Z^+\}$, $S = \{(x, 7x)\|x \in Z^+\}$. Find $R \circ S$, $S \circ R$, R^2.	05	2	3
7a	Let G be a cyclic groups. Prove that If $\|G\|$ is finite , then G is isomorphic to $(\mathbb{Z}_n, +_n)$.	05	3	3
7b	Show that $\langle \mathbb{Z}, * \rangle$, where $\mathbb{Z}$ is the set of integers and $a * b = a + b - 1 \forall a, b \in \mathbb{Z}$ is a group.	05	3	3
7c	The $(5m, m)$ five times repetition code has encoding function $E : \mathbb{Z}_2^m \to \mathbb{Z}_2^{5m}$, where $E(w) = wwwww$. The decoding function $D : \mathbb{Z}_2^{5m} \to \mathbb{Z}_2^m$ is accomplished by the majority rule. Assume the probability of incorrect transmission as $p = 0.03$. With $m = 3$ (i) what is the probability for the transmission and correct decoding of the signal 011, (ii) decode the received word 001011111101110, (iii) find two received words r, where $D(r) = 101$.	06	4	4
	OR			
8a	The $(5m, m)$ five times repetition code has encoding function $E : \mathbb{Z}_2^m \to \mathbb{Z}_2^{5m}$, where $E(w) = wwwww$. The decoding function $D : \mathbb{Z}_2^{5m} \to \mathbb{Z}_2^m$ is accomplished by the majority rule. Assume the probability of incorrect transmission as $p = 0.01$. With $m = 2$ (i) what is the probability for the transmission and correct decoding of the signal 11, (ii) decode the received word 1010100001, (iii) find two received words r, where $D(r) = 11$.	06	4	4
8b	Show that $\langle \mathbb{Z}_5, +_5 \rangle$, where $\mathbb{Z}_5 = \{0, 1, 2, 3, 4\}$ is a group.	05	3	3
8c	Find the index of H in $\langle G, \times_{13} \rangle$ and all the distinct right cosets of $H = \{1, 3, 9\}$ in the group G where $G = \mathbb{Z}_{13}^*$.	05	3	3
9a	Eight cities A, B, C, D, E, F, G, and H are required to be connected by a new road network. The possible roads and the cost involved to lay them (in crores of rupees) are summarized in the following table:	06	3	3

Between	Cost	Between	Cost
A and B	140	D and F	120
A and D	135	E and F	140
A and G	120	F and G	150
B and C	135	F and H	140
C and D	140	G and H	150
C and E	85	D and F	80

(i) Draw a weighted graph which represents the new road network.

(ii) Further determine a road network of minimal cost that connects all the cities using Kruskal's algorithm. Also mention the minimum cost.

9b	Briefly outline the procedure for detection of planarity of a graph using elementary reduction. Hence verify whether the following graph is planar or not? (Justify your answer) 	05	3	4	
9c	If T is a tree with n vertices and m edges then prove that $m = n - 1$.	05	2	2	
	OR				
10a	Give the conditions on which two graphs can be said to be isomorphic to each other. Establish a one-one correspondence between the vertices and edges to show that the following graphs are isomorphic. 	06	2	4	
10b	Seven volleyball teams from Andhra Pradesh (AP), Goa (GO), Tamilnadu (TA), Delhi (DE), Kerala (KE), Maharashtra (MA) and Orissa (OR) have been invited to participate in tournaments, where each team is scheduled to play a certain number of other teams (given below). No team is to play more than one game each day. Using graph theory set up a schedule of games over the smallest number of days. **AP:** GO, TA, MA, OR ; **GO:** AP, TA, OR ; **TA:** AP, GO, DE, KE ; **DE:** TA, KE, MA, OR; **KE:** TA, DE, MA ; **MA:** AP, DE, KE, OR ; **OR:** AP, GO, DE, MA .	05	4	3	
10c	If G is a graph with n vertices, m edges and f regions, then prove that $n - m + f = 2$.	05	2	2	