Unit 5: Graph Theory

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Definition of a Graph

Definition (Graph). A graph G consists of a finite or countable vertex set $V := V(G)$ and an edge set $E := E(G) \subseteq V \times V$.

So a graph is a pair:

$$G = \{V, E\}$$

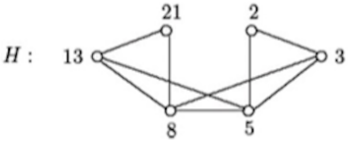
Consider the set $S = \{2, 3, 5, 8, 13, 21\}$.

Form all pairs $(a, b) \in S \times S$ such that $a + b$ or $|a - b| \in S$.

Vertices: $V(H) = \{2, 3, 5, 8, 13, 21\}$

Edges: $E(H) =$

$\{\{2, 3\}, \{2, 5\}, \{3, 5\}, \{3, 8\}, \{5, 8\}, \{5, 13\}, \{8, 13\}, \{8, 21\}, \{13, 21\}\}$



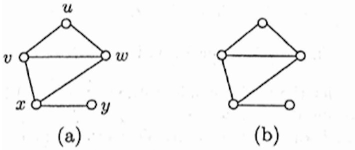
- The number of vertices in G is called the **order** n.
- The number of edges in G is called the **size** m.

Example: For the above Fibonacci graph,

Order $n = 6$, Size $m = 9$.

Graph Example:

$$V(G) = \{u, v, w, x, y\}, \quad E(G) = \{uv, uw, vw, vx, wx, xy\}$$



: A labeled graph and an unlabeled graph

Graph Terminology

- **End vertices:** Ends of an edge (u, v)
- **Parallel edges:** Edges that have the same end vertices
- **Loop:** Edge of the form (v, v)
- **Simple graph:** No loops or parallel edges
- **Empty graph:** $E = \emptyset$
- **Null graph:** $V = \emptyset$, $E = \emptyset$
- **Trivial graph:** One vertex

More Terminology

- **Adjacent edges:** Share a common end vertex
- **Adjacent vertices:** Connected by an edge
- **Degree:** Number of edges incident on a vertex
- **Pendant vertex:** Vertex with degree 1
- **Pendant edge:** Edge connected to pendant vertex
- **Isolated vertex:** Vertex with degree 0

Degree of a Vertex

- **Minimum degree** of G: $\delta(G)$
- **Maximum degree** of G: $\Delta(G)$

If G is a simple graph of order n, then:

$$0 \leq \delta(G) \leq \deg(v) \leq \Delta(G) \leq n-1$$

Theorem: Degree Sum Formula

Theorem 2 (First Theorem of Graph Theory):

If G is a graph of size m, then

$$\sum_{v \in V(G)} \deg(v) = 2m$$

This is also called the handshaking lemma.

Example: Degree Distribution

Example 3. A graph G has order 14 and size 27. Six vertices have degree 4. Find how many have degree 3 and how many have degree 5.

Let x = number of vertices with degree 3 Then,

$$3x + 4 \cdot 6 + 5(8 - x) = 2 \cdot 27 \Rightarrow x = 5$$

Answer: 5 vertices of degree 3, 3 vertices of degree 5

Theorem 4: Even Number of Odd Degree Vertices

Theorem. Every graph has an even number of vertices with odd degree.

Proof Sketch:

$$\sum_{v \in V(G)} \deg v = \sum_{v \in V_1} \deg v + \sum_{v \in V_2} \deg v = 2m$$

Where:

- V_1: vertices with odd degree
- V_2: vertices with even degree

Since both $\sum \deg v$ and $\sum_{v \in V_2} \deg v$ are even,

$$\sum_{v \in V_1} \deg v = \text{even}$$

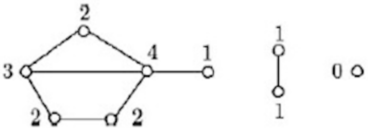
$\Rightarrow$ Number of odd-degree vertices is even.

Degree Sequences

A sequence of vertex degrees of a simple graph is called a **degree sequence**.

Example: $s = 4, 3, 2, 2, 2, 1, 1, 1, 0$; $\quad s' = 0, 1, 1, 1, 2, 2, 2, 3, 4$

$s'' = 4, 3, 2, 1, 2, 2, 1, 1, 0$ are degree sequences of the given graph.



Sequence types:

- s: non-increasing
- s': non-decreasing
- s'': neither

Example 5: Graphical Sequences

A finite non-negative sequence is said to be graphical sequence if it is a degree sequence of some simple graph.

Which of the following are graphical?

1. $s_1 = 3, 3, 2, 2, 1, 1$ **Yes**
2. $s_2 = 6, 5, 5, 4, 3, 3, 3, 2, 2$ **No**
3. $s_3 = 7, 6, 4, 4, 3, 3, 3$ **No**
4. $s_4 = 3, 3, 3, 1$ **No**

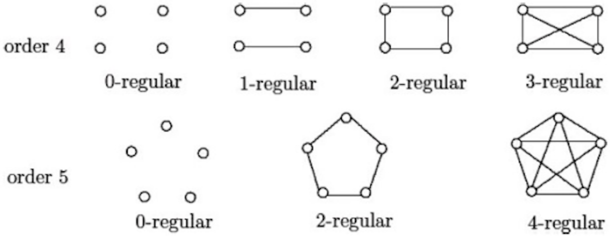
Definition: Regular Graphs

Regular Graph: A graph where all vertices have the same degree.

$$\delta(G) = \Delta(G) \Rightarrow \text{G is regular}$$

If every vertex has degree r, then G is called **r-regular**

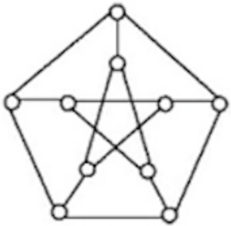
Example:



These are the only regular graphs of order 4 and 5.

Regular Graphs

- Odd-degree regular graphs of odd order are not possible
- A 3-regular graph is also called a **cubic graph**.
- Example: The Petersen graph is a well-known cubic graph.



Example

- A loop free k-regular graph with 2^k vertices is called k-dimension hypercube, denoted by Q_k.

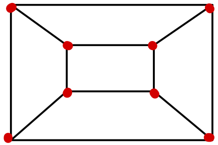


fig: Q_3

No. of vertices is 2^3

No. of edges is 12

- A k-dimensional hypercube Q_k has $k \cdot 2^{k-1}$ edges

Example

- Let G be a Cubic graph with 9 edges. Determine its order

Soln: Let n be no. of vertices and m be no. of edges.

we have

$$\sum \deg(v) = 3n = 2m$$

$$\Rightarrow \quad 3n = 2 \times 9$$

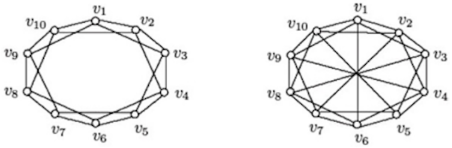
$$\Rightarrow \quad n = \frac{18}{3} = 6.$$

Existence of r-Regular Graphs

Theorem: Let r and n be integers with $0 \leq r \leq n-1$.

There exists an r-regular graph of order n if and only if at least one of r and n is even.

- Examples of 4-regular and 5-regular graphs of order 10 are shown.



(a): $H_{4,10}$ (b): $H_{5,10}$

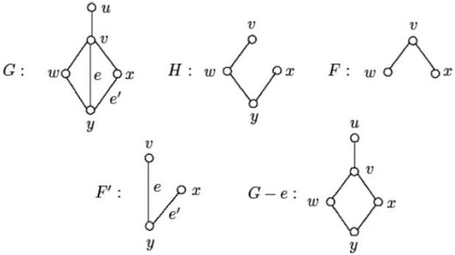
Definition

A graph H is called a **subgraph** of a graph G, written $H \subseteq G$, if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$ and each edge of H has same end vertices as in G.

- If $V(H)$ or $E(H)$ is a proper subset, then H is a **proper subgraph**.
- If H has the same vertex set as G, it is a **spanning subgraph**.
- No. of spanning subgraph with size m is 2^m. Since each edge may or may not be included in the spanning subgraph.

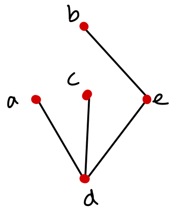
Induced Subgraphs

- A subgraph F of G is an **induced subgraph** of $\phi \neq S \subseteq V$ if for every pair of vertices u, v in S, $uv \in E(G)$ implies $uv \in E(F)$.
- If S is a non- empty subset of vertices of G, the induced subgraph by S is denoted $\langle S \rangle$.
- If X is a non- empty subset of edges of G, $\langle X \rangle$ is the **edge-induced subgraph**, consisting of all edges in X and their endpoints.

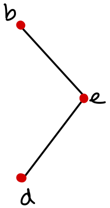


Example

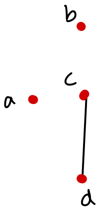
G :



G_1 :



G_2 :

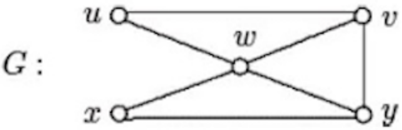


G_1 is an induced subgraph of $\{b, e, d\}$

G_2 is not an induced subgraph.

Walks in Graphs

- A $u - v$ **walk** in a graph is a sequence of vertices $W : u = v_0, v_1, ..., v_k = v$ such that $v_i v_{i+1} \in E(G)$.
- If $u = v$, the walk is **closed**; otherwise, it is **open**.
- The **length** of a walk is the number of edges in it.
- Walks may include repeated vertices or edges.
- A walk of length 0 is called trivial walk.
- In the below graph $W : x, y, w, y, v, w$ is $x - w$ walk of length 5.
- Walk $W : v$ is a trivial walk



Trails and Paths

- A **trail** is a walk in which no edge is repeated. For example, $T : u, w, y, x, w, v$ is a $u - v$ trail in G.
- A **path** is a walk in which no vertex is repeated. For example $P : u, w, y, v$ is a path in G.
- If no vertex in a walk is repeated, then no edge is repeated. Hence every path is a trial.

Theorem: If a graph G contains a u–v walk of length l, then it contains a u–v path of length at most l.

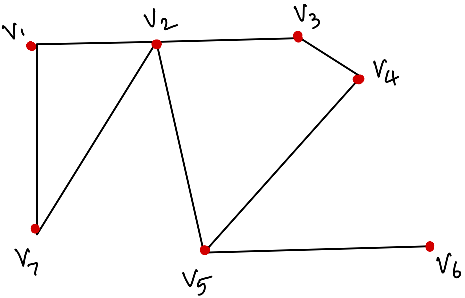
Circuits and Cycles

Definition

A **circuit** is a closed trail of length ≥ 3. It starts and ends at the same vertex, with no repeated edges.

- A **cycle** is a circuit with no repeated vertices (except the first and last).
- A **k-cycle** is a cycle of length k.
- A 3-cycle is called a **triangle**.
- Odd-length cycles are called **odd cycles**; even-length ones are **even cycles**.
- If a vertex of a cycle is deleted, then a path is obtained. This is not true for circuits.

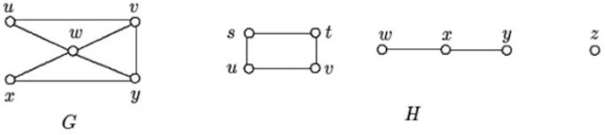
Example



- $v_1\ v_2\ v_3\ v_4\ v_5\ v_2\ v_7\ v_1$ is a 8-circuit
- $v_2\ v_3\ v_4\ v_5\ v_2$ is a 5-cycle

Connectivity in Graphs

- Two vertices u and v are **connected** if there is a u–v path in G.
- A graph is **connected** if every pair of vertices is connected.
- A graph is **disconnected** if it is not connected.
- A **component** is a maximal connected subgraph.
- A graph G is connected if and only if it has exactly one component.



Distance in a Graph

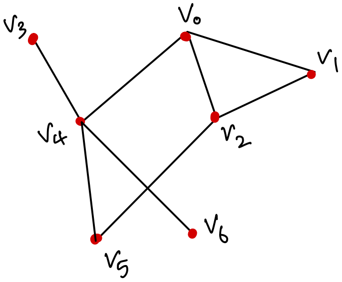
Definition:

The **distance** between two vertices u and v, denoted $d(u, v)$, is the length of the **shortest path** between them.

Properties:

- $d(u, v) = d(v, u)$ (Symmetric)
- $d(u, v) = 0$ if and only if $u = v$
- $d(u, v) = \infty$ if no path exists (disconnected graph)
- Triangle Inequality: $d(u, w) \leq d(u, v) + d(v, w)$
- Shortest u-v path is called a geodesic.

Example



- $d(V_1, V_5) = 2$

  Shortest $V_1 - V_5$ path is $V_1 V_2 V_5$

  $d(V_1, V_3) = 3$

  Shortest $V_1 - V_3$ path is $V_1 V_0 V_4 V_3$

Eccentricity

Definition:

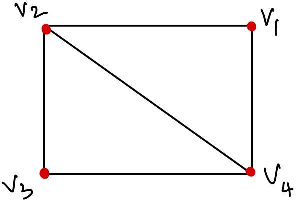
The **eccentricity** of a vertex v, denoted $e(v)$, is the **maximum distance** from v to any other vertex:

$$e(v) = \max\{d(v, u) \mid u \in V(G)\}$$

Interpretation:

Measures how far a vertex is from the farthest vertex in the graph.

Example



$d(v_1, v_2) = 1$

$d(v_1, v_4) = 1$

$d(v_1, v_3) = 2$

$\therefore \quad e(v_1) = 2$

Similarly,

$e(v_2) = 1$

$e(v_3) = 2$

$e(v_4) = 1$

Definition:

The **diameter** of a graph G, denoted diam(G), is the **maximum eccentricity** among all vertices:

$$\text{diam}(G) = \max\{e(v) \mid v \in V(G)\}$$

Interpretation:

Represents the longest shortest path between any two vertices in the graph.

If $e(v) = \text{diam}(G)$, then v is a peripheral vertex.
The set of all such vertices make the periphery of G.

Radius

Definition:

The **radius** of a graph G, denoted $\text{rad}(G)$, is the **minimum eccentricity** among all vertices:

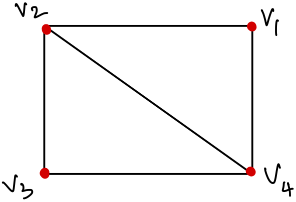
$$\text{rad}(G) = \min\{e(v) \mid v \in V(G)\}$$

Interpretation:

Represents the minimum distance needed to reach the farthest vertex from a central point.

If $e(v) = \text{rad}(G)$, then v is a central vertex.
The set of all such vertices make the center of G.

Example

G : 

$e(v_1) = 2$

$e(v_2) = 1$

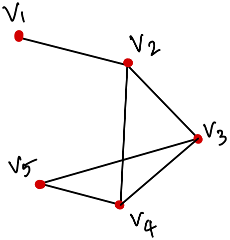
$e(v_3) = 2$

$e(v_4) = 1$

$\left.\begin{array}{l} diam(G) = 2 \\ rad(G) = 1 \end{array}\right\}$ periphery $= \{v_1, v_3\}$
Center $= \{v_2, v_4\}$

Example

G:



$e(v_1) = 3$

$e(v_2) = 2$

$e(v_3) = 2$

$e(v_4) = 2$

$e(v_5) = 3$

$\therefore$ $\text{diam}(G) = 3$ periphery is $\{v_1, v_5\}$

$\text{radius}(G) = 2$ Center is $\{v_2, v_3, v_4\}$

Key Relationships

For any connected graph:

i) $$\text{rad}(G) \leq e(v) \leq \text{diam}(G)$$

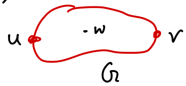
ii) $$\text{rad}(G) \leq \text{diam}(G) \leq 2 \times \text{rad}(G)$$

Pf ii): $\text{rad}(G) \leq \text{diam}(G)$ (obvious)

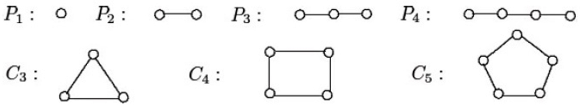
Let $u, v \in V(G)$ such that $d(u,v) = \text{diam}(G)$

Let w be central vertex. Then

$d(u,v) \leq d(u,w) + d(w,v) \leq 2\,e(w) = 2\,\text{rad}(G)$



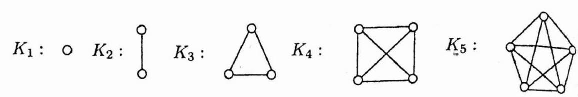
- A **path** graph of order n is a sequence of vertices $v_1, v_2, \ldots, v_n$ with edges $v_1v_2, v_2v_3, \ldots, v_{n-1}v_n$.
- A **cycle** graph of order $n \geq 3$ is a closed path: $v_1v_2, v_2v_3, \ldots, v_{n-1}v_n, v_nv_1$.
- Path graphs are denoted by P_n and cycle graphs by C_n.



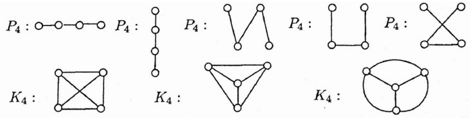
- In a path graph,
  - there exist exactly one path between any pair of vertices.
  - Degree of each vertices, except terminal vertices, is 2.

Complete Graphs

- A graph G is **complete** if every pair of distinct vertices is adjacent. (i.e There exist an edge between every pair of vertices)
- A complete graph on n vertices is denoted by K_n.
- Number of edges in K_n: $\binom{n}{2} = \frac{n(n-1)}{2}$.



The graphs can be drawn in different ways.



Radius and Diameter

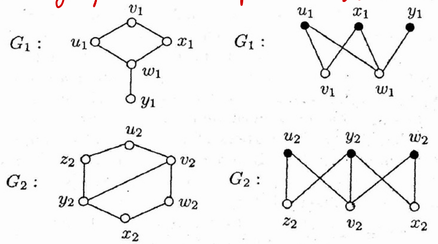
- For complete graphs: $\mathrm{diam}(K_n) = \mathrm{rad}(K_n) = 1$, for $n \geq 2$.
- For paths on n vertices:
  - $\mathrm{diam}(P_n) = n - 1$
  - $\mathrm{rad}(P_n) = \left\lceil \frac{n-1}{2} \right\rceil$
- For cycles on n vertices:
  - $\mathrm{diam}(C_n) = \left\lceil \frac{n-1}{2} \right\rceil$
  - $\mathrm{rad}(C_n) = \left\lceil \frac{n-1}{2} \right\rceil$

Bipartite Graphs

Definition

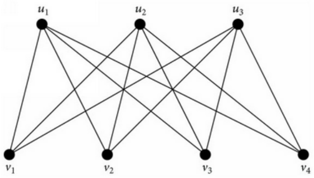
A graph G is **bipartite** if $V(G)$ can be partitioned into two sets U and W such that every edge connects a vertex in U to one in W.

- G_1 and G_2 shown are bipartite.
- A graph is bipartite if and only if each component is bipartite.
- C_5 is **not** bipartite.
- A nontrivial graph G is bipartite iff G contain no odd cycles.



Complete Bipartite Graphs

- A graph is a **complete bipartite graph** if every vertex in U is connected to every vertex in W.
- Denoted by $K_{p,q}$. (where p and q are number of vertices in partite set U and W, respectively)
- Diameter: $\text{diam}(K_{p,q}) = 2$ (if $p, q \geq 2$)
- Number of vertices of $K_{p,q}$: $n = p + q$
- Number of edges of $K_{p,q}$: $m = pq$

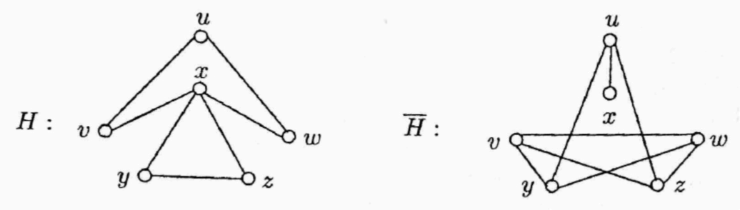


Complement of a Graph

Definition

The **complement** $\overline{G}$ of a graph G is the graph whose vertex set is $V(G)$ such that for each pair u, v of vertices of G, uv is an edge of $\overline{G}$ if and only if uv is not an edge of G.

A graph H and its complement $\overline{H}$ are shown below:



Size of Complement of G

- If G is a graph of order n and size m, then $\overline{G}$ is a graph of order n and size $\binom{n}{2} - m$.
- The graph $\overline{K_n}$ has n vertices and no edges — it is called the **empty graph** of order n.

Definition :

- The adjacency matrix of a graph G is the $n \times n$ matrix $A = (a_{ij})$, where

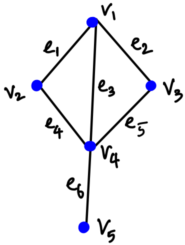
$$a_{ij} = \begin{cases} 1 & \text{if } v_i v_j \in E(G) \\ 0 & \text{otherwise} \end{cases}$$

- The incidence matrix of G is the $n \times n$ matrix $A = (b_{ij})$, where

$$b_{ij} = \begin{cases} 1 & \text{if } v_i \text{ is incident with } e_j \\ 0 & \text{otherwise} \end{cases}$$

Computer Representation of Graphs

Example: For the graph



- Adjacency matrix

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

* Arrangement of v_1, v_2, v_3, v_4, v_5 must be same in both rows and columns.

- Incidence matrix

$$B = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Isomorphism

Let G and H be simple graphs. A map $\phi : G \to H$ is an isomorphism if

i) For any $u, v \in V(G)$,

$$\underbrace{uv \in E(G) \text{ iff } \phi(u)\phi(v) \in E(H).}_{\text{it says that adjacency is preserved.}}$$

ii) ϕ is bijective.

- If there exist an isomorphism from G to H, then we say G is isomorphic H, denoted by $G \cong H$.
- If $G \cong H$ means they have same graph structure.

Isomorphism

In general, for any graph (simple or non-simple graph)

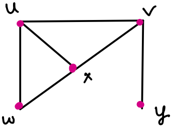
An isomorphism from G to H is a bijective ϕ that maps $V(G)$ to $V(H)$ and $E(G)$ to $E(H)$ such that each edge of G with end points u and v is mapped to an edge with end points $\phi(u)$ to $\phi(v)$.

- If $\phi : G \to H$ is isomorphism, then for any $u \in G$

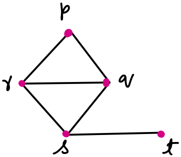
$$\deg(u) = \deg(\phi(u))$$

Example 1: Show that the following graphs G and H are isomorphic.

G :



H :



Soln: Clearly,

order of G = order of H

size of G = size of H

and

$\deg(u) = 3$	$\deg(p) = 2$
$\deg(v) = 3$	$\deg(r) = 3$
$\deg(x) = 3$	$\deg(q) = 3$
$\deg(w) = 2$	$\deg(s) = 3$
$\deg(y) = 1$	$\deg(t) = 1$

Define a map ϕ :

$$\underbrace{\phi = \begin{pmatrix} y & w & u & v & x \\ t & p & q & s & r \end{pmatrix}}_{\text{bijection.}}$$ such that

$$\left.\begin{aligned}
\phi(yv) &= ts = \phi(y)\,\phi(s) \\
\phi(wu) &= pq = \phi(w)\,\phi(u) \\
\phi(wx) &= pr = \phi(w)\,\phi(x) \\
\phi(uv) &= qs = \phi(u)\,\phi(v) \\
\phi(ux) &= qr = \phi(u)\,\phi(x) \\
\phi(vx) &= sr = \phi(v)\,\phi(x)
\end{aligned}\right\}\ \text{adjacency is preserved}$$

Thus, $G \cong H$

Isomorphism

or

Define a bijective map: $\phi = \begin{pmatrix} u & v & w & x & y \\ q & s & p & r & t \end{pmatrix}$

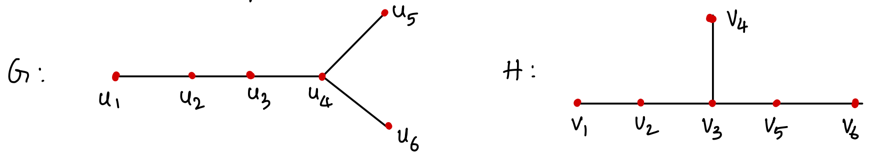
find adjacent matrices of G and H as follows:

$$M_G = \begin{array}{c} \\ u \\ v \\ w \\ x \\ y \end{array} \begin{array}{c} \begin{array}{ccccc} u & v & w & x & y \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}, \qquad M_H = \begin{array}{c} \\ q \\ s \\ p \\ r \\ t \end{array} \begin{array}{c} \begin{array}{ccccc} q & s & p & r & t \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$\because \; M_G = M_H$

$\Rightarrow \; G \cong H.$

Example 2. Show that the following graphs are not isomorphic.



Soln: In G, $\deg(u_5) = \deg(u_6) = 1$, $\deg(u_4) = 3$

and u_5 and u_6 are adjacent to u_4.

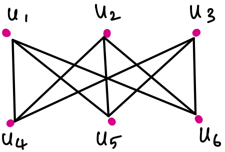
But in H, there is no two vertices of degree 1 which is adjacent to a vertex of degree 3.

$\therefore$ G is not isomorphic to H.

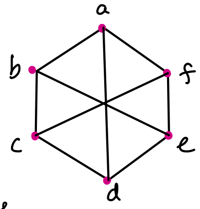
Isomorphism

Example 3 : S.T $G \cong H$.

G :



H :



Soln: Each has 6 vertices and 9 edges.

Define a map : $\phi : \begin{pmatrix} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \\ a & c & e & b & f & d \end{pmatrix}$

$$M_G = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{array} \begin{array}{c} \begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$M_H = \begin{array}{c} \\ a \\ c \\ e \\ b \\ f \\ d \end{array} \begin{array}{c} \begin{array}{cccccc} a & c & e & b & f & d \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

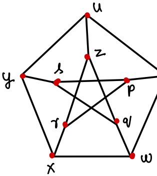
$M_G = M_H$

$\therefore G \cong H$

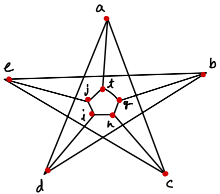
Isomorphism

Example 4 : S.T $G \cong H$

G:



H:



Soln: Each has 10 vertices and 15 edges.

Define a bijective map

$$\phi = \begin{pmatrix} u & v & w & x & y & z & p & q & r & s \\ t & g & h & i & j & a & b & c & d & e \end{pmatrix}$$ under this map

adjacency is preserved. For instance

$uv \in E(G) \Rightarrow tg \in E(H)$

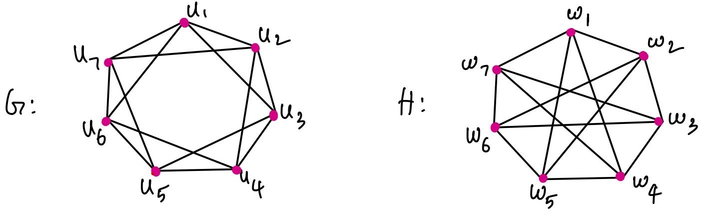
$xy \in E(G) \Rightarrow ij \in E(H)$

$zq \in E(G) \Rightarrow ac \in E(H)$ etc.

Show that adjacency matrices of G and H are equal.

Isomorphism

Example 5: Show that $G \cong H$



Soln: Both G and H have 7 vertices and 14 edges.

Define a map

$$\phi = \begin{pmatrix} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 \\ w_1 & w_4 & w_7 & w_3 & w_6 & w_2 & w_5 \end{pmatrix},$$

under this map we shall S.T adjacency is preserved by constructing adjacency matrices

Isomorphism

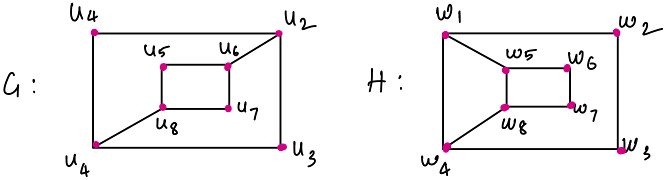
$$M_G = \begin{array}{c|ccccccc} & u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 \\ \hline u_1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ u_2 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ u_3 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ u_4 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ u_5 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ u_6 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ u_7 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \end{array}$$

$$M_H = \begin{array}{c|ccccccc} & w_1 & w_4 & w_7 & w_3 & w_6 & w_2 & w_5 \\ \hline w_1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ w_4 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ w_7 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ w_3 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ w_6 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ w_2 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ w_5 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \end{array}$$

Thus adjacency is preserved.

Thus $G \cong H$

Example 6 : Check whether following pair is isomorphic or not.



Soln : G has two cycles of length 4; whereas H has three cycles of length 4.

Thus, $G \not\cong H$.