$$[3] = \{4k+3 \mid k \in \mathbb{Z}\}$$

$$[4] = \{4k \mid k \in \mathbb{Z}\} = [0]$$

$$[5] = \{4k+5 \mid k \in \mathbb{Z}\} = \{4k+1 \mid k \in \mathbb{Z}\} = [1]$$

$$\begin{aligned} &4k+4+1 \\ &= 4(k+1)+1 \\ &= 4k'+1 \end{aligned}$$

$$[6] = [2]$$

Observe: $\mathbb{Z} = \{-4, -3, -2, -1, 0, 1, 2, 3, 4 \ldots\}$

$$= [1] \cup [2] \cup [3] \cup [4] \rightsquigarrow \bigcup_{a \in A} [a] = A$$

union covers all of A.

Also, $[1] \cap [2] = \phi$, $[3] \cap [4] = \phi$,

$[2] \cap [3] = \phi$, $[1] \cap [4] = \phi$ $\rightsquigarrow$ pairwise disjoint

(Each equivalence class is like a 'piece' of the set)

Partition of a set :

Let A be a non-empty set. Suppose there exists non empty subsets $A_1, A_2, \ldots A_k$ of A such that the following 2 conditions hold

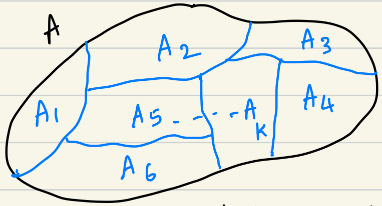
1) A is the union of $A_1, A_2, \ldots A_k$

i.e. $A = A_1 \cup A_2 \cup \ldots A_k$

$$A = \bigcup_{i=1}^{k} A_i$$

2) Any 2 of the subsets $A_1, A_2, \ldots A_k$ are disjoint i.e. $A_i \cap A_j = \phi$ for $i \neq j$

Then the set $\{A_1, A_2, \ldots A_k\}$ is called partition of A. Also $A_1, A_2, \ldots A_k$ are called the blocks or cells of partition.



Partition of set A.

Ex: Let $S = \{1, 2, 6, 7, 9, 11\}$, $A_1 = \{1, 2\}$, $A_2 = \{6\}$, $A_3 = \{9, 11, 7\}$ be subsets of S.

Clearly A_1, A_2, A_3 are disjoint subsets and

$$S = A_1 \cup A_2 \cup A_3$$

Thus $\{A_1, A_2, A_3\}$ is partition of S.

Thm: Let R be an equivalence relation on a set S. Then the equivalence classes of R form a partition of S. Conversely, given a partition $\{A_i \mid i \in I\}$ of set S, there is an equivalence relation R that has the sets $A_i, i \in I$ as its equivalence classes.

WKT $\bigcup_{a \in S} [a] = S$, Also 2 equivalence classes are equal or disjoint. Thus we say that equivalence classes partition S.

Ex: Congruence modulo 4.

$$Z = [0] \cup [1] \cup [2] \cup [3]$$

Ex: Let $S = \{1, 2, 3, 4, 5, 6\}$, $A_1 = \{1, 2, 3\}$, $A_2 = \{4, 5\}$

$A_3 = \{6\}$. List the ordered pairs in equivalence relation R.

Sol: $R = \{(1,1), (1,2), (1,3), (2,2), (2,1), (2,3), (3,1), (3,2),$

$(3,3), (4,4), (4,5), (5,4), (5,5), (6,6)\}$

Ex: Consider the set $A = \{1, 2, 3, 4, 5\}$ and the equivalence relation $R = \{(1,1), (2,2), (2,3), (3,2), (3,3), (4,4), (4,5), (5,4), (5,5)\}$ defined on A. Find the partition of A induced by R.

Sol. $[1] = \{1\}$, $[2] = \{2, 3\}$, $[3] = \{2, 3\}$,

$[4] = \{4, 5\}$, $[5] = \{4, 5\}$

From these equivalence classes only $[1], [2], [4]$ are distinct.

$\therefore$ The partition P of A induced by R is

$$P = \{[1], [2], [4]\}$$

Partial orders:

A relation R on set A is said to be a partial ordering relation or a partial order on A if
i) R is reflexive
ii) R is antisymmetric
iii) R is transitive on A.
A set A together with a partial order R is called a partially ordered set or poset and is denoted by (A, R).

not less than or equal to

Notation: Let R be a partial order on set A.
For $a, b \in A$ aRb is denoted by $a \preceq b$ and poset (A, R) is written as $(A, \preceq)$

symbol to denote relation in any poset

Defn: 2 elements a and b of a poset are called comparable iff either aRb or bRa.

Defn: If (A, R) is a poset and every 2 elements are comparable then A is called total ordered set. or linearly ordered set
A total ordered set is also called chain.
ex: The poset $(Z, \leq)$ is totally ordered. because $a \leq b$ or $b \leq a$ whenever a and b are integers.

Ex: Let R be a relation on set Z^+,
$R = \{(a, b) : a \mid b\}$. Is $\{Z^+, \mid\}$ a poset?
Ans: Yes (verify)
Is this poset totally ordered?

Ans: No $(\because 5 \nmid 7$ or $7 \nmid 5)$

Hasse diagram: (simpler graphical representation of digraph for partial order)

Procedure to construct Hasse diagram:

1) Write down all the ordered pairs which satisfy the relation R.

2) Each element is represented as a dot and all the edges are pointed upwards with no directions of edges. (smaller elements at bottom and larger ones at top)

3) Remove unnecessary edges or connections: Remove all loops and edges that appear due to transitivity.

Ex: Let $A = \{1, 2, 3, 4, 6, 12\}$. The relation R defined on A by aRb iff a divides b. P.T. R is a partial order on A. Draw the Hasse diagram for this relation.

Sol: $R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (1,12), (2,2), (2,4), (2,6), (2,12), (3,3), (3,6), (3,12), (4,4), (4,12), (6,6), (6,12), (12,12)\}$

a) Reflexive: $\forall a \in A, (a,a) \in R$

$\therefore$ R is reflexive.