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Academic Year 2024-25 (ODD Semester)

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Department of Artificial Intelligence and Machine Learning

Mathematical Algorithms for Artificial Intelligence

Course Code : AI255TBC
Semester : 5
Max Marks : 10+50=60

Date : 27/11/2024
Time : 9:30AM – 11:30AM
Duration : 120 Mins

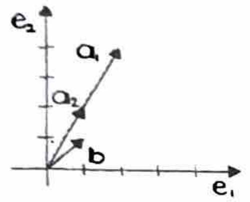
<table>
<tr><th colspan="5">CIE -1</th></tr>
<tr><th>SL.</th><th>Questions</th><th>M</th><th>B</th><th>C</th></tr>
<tr><td>1.</td><td>Let $\mathbf{a}_1, \mathbf{a}_2$ be the two columns of a 2×2 real matrix and $\mathbf{b}$ be any other vector in $\mathbb{R}^2$. Consider the figures given below. Identify which of them corresponds to a system of linear equations in two variables with
(a) unique solution
(b) no solution
In each case, justify your choice in not more than 2 sentences.


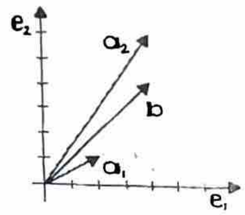
Fig Q1 (i)


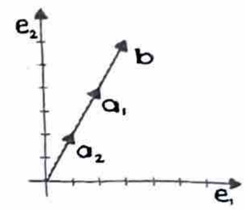
Fig Q1 (ii)


Fig Q1 (iii)</td><td>5</td><td>3</td><td>2</td></tr>
<tr><td>2.</td><td>Which among the following sets of matrices is not a vector space over $\mathbb{R}$, under the usual vector addition and scalar multiplication operations and why?
(i) $\mathcal{M}_{2\times 2}$, the set of real, invertible 2 x 2 matrices.
(ii) $\mathcal{U}_{3\times 3}$, the set of real, upper triangular matrices</td><td>5</td><td>2</td><td>1</td></tr>
<tr><td>3.</td><td>Assert or Reject each of the following 3 statements with proper justification.
Statement 1: The set containing a single vector is ALWAYS a linearly DEPENDENT set.
Statement 2: The set containing a single vector is ALWAYS a basis for some vector subspace.
Statement 3: Every vector space has at least one subspace.</td><td>10</td><td>3</td><td>2</td></tr>
<tr><td>4.</td><td>Let A be a 3×3 matrix such that $A\mathrm{x} = \begin{pmatrix}1\\0\\-1\end{pmatrix}$ has both $\begin{pmatrix}1\\2\\3\end{pmatrix}$ and $\begin{pmatrix}3\\1\\4\end{pmatrix}$ as its solutions. Find one more solution for the same.</td><td>5</td><td>3</td><td>2</td></tr>
<tr><td>5.</td><td>$A = \begin{pmatrix}1 & 1\\1 & 1\\1 & 1\end{pmatrix}$
(i) List out all the four fundamental subspaces associated with the matrix A.
(ii) List at least one basis for each of the subspaces that you found above.
(iii) Mention the dimension of each of the subspaces alongside.</td><td>10</td><td>3</td><td>2</td></tr>
<tr><td>6.</td><td>Let the set $\mathbb{R}^2$ with operations $\oplus$ and $\odot$ defined as follows. For any two vectors $\mathbf{x} = \begin{pmatrix}x_1\\x_2\end{pmatrix}, \mathbf{y} = \begin{pmatrix}y_1\\y_2\end{pmatrix}$, in the set,
vector addition is defined as $\mathbf{x} \oplus \mathbf{y} = \begin{pmatrix}x_1 + y_1 - 2\\x_2 + y_2 - 3\end{pmatrix}$
For a real scalar k, scalar multiplication is defined as $k \odot \mathbf{x} = \begin{pmatrix}kx_1 - 2k + 2\\kx_2 + 3k - 3\end{pmatrix}$
Find the zero vector Θ_2 for this vector space with the operations as defined above.</td><td>5</td><td>3</td><td></td></tr>
<tr><td>7.</td><td>Consider the following system of equation:
$x_1 + x_2 - x_3 + x_4 - x_5 = 0$
Identify the null space associated with the coefficient matrix and its dimension.</td><td>5</td><td>2</td><td></td></tr>
<tr><td>8.</td><td>Given the vector space $\mathbb{R}^3$, list out the geometry of its possible subspaces.</td><td>5</td><td>2</td><td></td></tr>
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