UG

UG

RV College of Engineering®,
(Autonomous Institution affiliated to VTU)
Bengaluru-560 059
Course Title: Mathematical Algorithms for Artificial Intelligence
Model Question Paper

COURSE CODE:AI255TBC	**SEM**: V
COURSE TITLE : Mathematical Algorithms for AI	
Duration of Paper: 03 Hrs	

Instructions to Candidates:
1. Answer all questions from Part A
2. Any 5 Full questions from Part B choosing one from each side. (Question No.2 is compulsory)

Q.No	PART A	Marks	BTL	CO
1.1	If the column space of an 8×4 matrix A is 3 dimensional, give the dimensions of the other three fundamental subspaces.	02		
1.2	Assert or Reject the following statement with justification: **Statement: The set of all even degree polynomials (including 0) is a vector space.**	02		
1.3	Given the Euclidean norms $\|\|u\|\| = 4$, $\|\|v\|\| = 3$, $\|\|u + v\|\| = 7$, then what is the dot product **<u, v>** ?	02		
1.4	Assert or Reject the following statement with justification: **Statement: There exists a real symmetric matrix whose characteristic equation is** $\lambda(\lambda^2 + 4)(\lambda - 1) = 0$.	02		
1.5	Define heteroscedasticity.	02		
1.6	How is the ordinary least squares problem different from a weighted least squares case?	02		
1.7	The position vector of point P is r = (3 cos t, 5 sin t, 4 cos t). What is the speed?	02		
1.8	Obtain the gradient of the function $f(x, y) = (x + y)^2$	02		
1.9	Consider a 2x3 real matrix A, with rank of A = 2. What is the pseudoinverse of A?	02		
1.10	What could be the consequence of higher learning rate in gradient descent algorithm?	02		
	PART B			
2(a)	Let $$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$$ Find out all the 4 subspaces associated with A. Mention one basis each for each of the subspaces.	10		
2(b)	Assert or Reject the following statements with justification.	6		

	Statement 1: For a given nxn real matrix A, two eigenvalues can have the same eigenvectors. **Statement 2:** $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$.			
3(a)	Given the following vectors $\mathbf{u}_1 = \begin{pmatrix} 3 \\ -3 \\ 0 \end{pmatrix}, \mathbf{u}_2 = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}, \mathbf{u}_3 = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}, \mathbf{x} = \begin{pmatrix} 5 \\ -3 \\ 1 \end{pmatrix}$ (i) Are the vectors $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ orthogonal? (ii) Are the vectors $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ orthonormal? (iii) Express the vector $\mathbf{x}$ as a linear combination of the vectors $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$.	8 2+1+5		
3(b)	(i) If P is a projection matrix, prove that $P^2 = P$. (ii) Obtain the matrix that projects every vector in $\mathbb{R}^2$ along the line passing through the origin and in the direction of the vector $\mathbf{u}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$	4+4		
	OR			
4(a)	Obtain the orthogonal complement subspace of the plane $x_1 + x_2 + x_3 = 0$	6		
4(b)	Consider the following data matrix: $X = \begin{pmatrix} 4 & 1 \\ 2 & 3 \\ 5 & 4 \\ 1 & 0 \end{pmatrix}$ We want to represent the data in only one dimension. Compute the unit-length principal component directions of X, and state which one the PCA algorithm would choose if you request just one principal component. Obtain the projections of all four sample points onto the principal direction.	10		
5(a)	Suppose you measure your pulse rate x three times and get values $x = b_1$, $x = b_2$, $x = b_3$. If each right side b_i, are independent and has zero mean and variances $\sigma_1^2 = \frac{1}{16}$, $\sigma_2^2 = \frac{1}{9}$, $\sigma_3^2 = \frac{1}{36}$, find the best estimate $\hat{x}$ based on b_1, b_2, b_3.	10		
5(b)	List out 3 limitations of Gaussian Mixture Models	6		
	OR			
6(a)	List out 4 limitations of Expectation Maximization algorithm	12		
6(b)	List 4 applications of Kalman filters	4		
7(a)	The quadratic form xT Ax is a form we will encounter often.* In this question, we are interested in $d(\mathbf{x}^T A\mathbf{x})/d\mathbf{x}$. Assume that A is not a function of $\mathbf{x}$. (a) Evaluate $\mathbf{x}^T A\mathbf{x}$ when $\mathbf{x} = [x_1,x_2]^T$ and the (i,j)-th element of A is A_{ij} . Why do you think $\mathbf{x}^T A\mathbf{x}$ is called the quadratic form?	16		

	(b) Which definition of the derivative do we need in order to evaluate $d(\mathbf{x}^T A\mathbf{x})/d\mathbf{x}$? (c) Assume $\mathbf{x} \in \mathbb{R}^2$ and $A \in \mathbb{R}^{2\times 2}$. Evaluate $d(\mathbf{x}^T A\mathbf{x})/dx$. (d) Generalize the previous result to when $\mathbf{x} \in \mathbb{R}^n$ and $A \in \mathbb{R}^{n\times n}$ and evaluate $d(\mathbf{x}^T A\mathbf{x})/d\mathbf{x}$. Can you express the result in matrix form? (e) What happens when A is a symmetric matrix, i.e., $A^T = A$?			
	OR			
8	Suppose the 2x1 function **f** of the 3x1 vector **x** is given by $\mathbf{f}(\mathbf{x}) = \begin{pmatrix} x_1^2 + x_2^2 + x_3^2 \\ 2x_1 - x_2 - x_3 \end{pmatrix}$ and the 2x1 function **g** of the 2x1 vector **z** is given by $\mathbf{g}(\mathbf{z}) = \begin{pmatrix} \frac{z_2}{z_1} \\ z_1 z_2 \end{pmatrix}$ Use the chain rule to compute $\frac{\partial}{\partial \mathbf{x}^T}\mathbf{y}(\mathbf{x})$ where $\mathbf{y}(\mathbf{x})$ is the composite function defined by $\mathbf{y}(\mathbf{x}) = \mathbf{g}(\mathbf{f}(\mathbf{x}))$.	16		
9(a)	The function $f(x, y, z) = x^2 + y^2 + 3z^2 - xy + 2xz + yz$ defined on $\mathbb{R}^3$ has only one stationary point. Show that it is a local minimum.	6		
9(b)	Find the least square solution to the following system of equations: $x_1 + x_2 = 2;\ x_1 - x_2 = 0;\ 2x_1 + x_2 = -4$	10		
	OR			
10(a)	Use the method of Lagrange multipliers to find the minimum value of $x^2 + 4y^2 - 2x + 8y$ subject to the constraint $x+2y=7$	16		

Name of the Scrutinizer

Name of the BoE Chairperson

Signature of Scrutinizer

Name of the BoE Chairperson