Dear all, Please do not panic looking at the questions. Take a deep breath and read the questions carefully. You can solve all 20 of them easily. All the questions require you to don your thinking hat. I am sure you will all do well. All the best and Happy times solving the problems!!!

1. Which of the following represents a system with unique solution?
   A. $\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$ B. $\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$ C. $\begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$ D. $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
2. Which of the following is not an invertible system?
   A. $\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$ B. $\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$ C. $\begin{bmatrix} 3 & 3 \\ 3 & 0.1 \end{bmatrix}$ D. $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
3. The system
   $$\begin{aligned} 3x_1 + 2x_2 &= 5 \\ 3x_1 + 2x_2 &= 6 \end{aligned}$$
   has
   A. has unique solution B. infinitely many solution C. no solution
   D. has solution but I am not confident if it is unique or infinitely many.
4. The rank of the matrix
   $$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$
   is
   A. 0 B. 1 C. 2 D. 3
5. The number of linearly independent columns in the matrix
   $$A = \begin{pmatrix} 4 & 5 \end{pmatrix}$$
   is
   A. 0 B. 1 C. 2 D. indeterminate as we need more information.
6. Identify the linearly independent set of vectors among the following:
   A. $\left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right\}$ B. $\left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right\}$ C. $\left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$
   D. $\left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$

7. The dimension of the vector space spanned by the vector $\left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right\}$ is

   A. 0 B. 1 C. 2 D. indeterminate as we need more information.

8. Let $\mathbf{u} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, then the vector $\mathbf{w} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ can be expressed as a linear combination of $\mathbf{u}$ and $\mathbf{v}$ as $\mathbf{w} = x_1\mathbf{u} + x_2\mathbf{v}$, where x_1, x_2 respectively are

   A. 1, 2 B. 1.5, 2.5 C. 2.5, 1.5 D. 2, 3

9. The vector space spanned by the vectors $\left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right\}$ is

   A. a line passing through the origin in $\mathbb{R}^2$ B. entire $\mathbb{R}^2$ C. a plane passing through the origin in $\mathbb{R}^3$ D. a line passing through the origin in $\mathbb{R}^3$

10. The null space associated with the matrix $\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$ is

    A. a line passing through the origin in $\mathbb{R}^2$ B. entire $\mathbb{R}^2$ C. the zero dimensional subspace of $\mathbb{R}^2$ D. non existent as the two column vectors are linearly independent and hence span the entire $\mathbb{R}^2$

11. The null space associated with the matrix $\begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 3 & 1 \end{bmatrix}$ is

    A. a line passing through the origin in $\mathbb{R}^2$ B. entire $\mathbb{R}^2$ C. the zero dimensional subspace of $\mathbb{R}^2$ D. the zero dimensional subspace of $\mathbb{R}^3$

12. The subspace spanned by the vector

$$A = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

    is

    A. line through the origin in $\mathbb{R}^2$ B. entire $\mathbb{R}^2$ C. indeterminate with the given information D. non existent as a single vector cannot span a vector space.

13. Let $\mathcal{V} = \left\{ \begin{pmatrix} a \\ b \\ 0 \\ c \\ a - 2b + c \end{pmatrix} \right\}$ in $\mathbb{R}^5$. Is $\mathcal{V}$ a vector subspace in $\mathbb{R}^5$, under the

usual operations of $+$ and $\bullet$?

14. Consider the set $\mathbb{R}^2$ with operations $\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \oplus \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} u_1 + v_1 - 2 \\ u_2 + v_2 + 3 \end{pmatrix}$ and a real scalar α, such that $\alpha \odot \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} \alpha u_1 - 2\alpha + 2 \\ \alpha u_2 + 3\alpha - 3 \end{pmatrix}$. Is the set, with the operations defined as above, a vector space? If yes, what is the zero vector $\mathbf{0}$. If no, why is it not a vector space?

15. Let $\mathcal{W}$ be a subspace of a vector space $\mathcal{V}$. Consider the set $\mathcal{W}^c$, the complement of the set $\mathcal{W}$, which contains all the vectors in $\mathcal{V}$ but not the ones in $\mathcal{W}$. Is the set $\mathcal{W}^c$ a vector subspace? Justify your choice.

16. Given a vector space of polynomials of degree ≤ 3 with real coefficients, find a basis for the vector space.

17. Identify if each of the statements below is TRUE or FALSE. Do justify your choice.

    (a) Every vector space has at least one subspace.

    (b) Any plane in $\mathbb{R}^3$ is a subspace of $\mathbb{R}^3$.

    (c) A subset $\mathcal{S} = \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$ of a vector space $\mathcal{V}$ is a linearly dependent set.

    (d) A 3-dimensional subspace of $\mathbb{R}^5$ has exactly 3 linearly independent vectors in its basis.

    (e) Any set of 3 vectors in $\mathbb{R}^2$ is always a linearly dependent set.

    (f) If $\mathcal{W}$ is a subspace of a vector space $\mathcal{V}$, then $dim(\mathcal{W}) < dim(\mathcal{V})$.

    (g) There exists a set of 7 vectors whose linear combination gives all vectors in $\mathbb{R}^9$.

    (h) The zero vector space has dimension 0.

    (i) Every linearly dependent set contains the zero vector.

18. Give an example where union of two subspaces is not a subspace.

19. Given an example where union of two subspaces is a subspace.

20. Consider the vector space of all 2×2 real matrices, $\mathcal{M}^{2\times 2}$. Find a basis for this vector space.