USN | | | | | | | | | |

RV COLLEGE OF ENGINEERING®

(An Autonomous Institution affiliated to VTU)

V Semester B. E. Regular Examinations Feb/Mar-2025

Artificial Intelligence and Machine Learning

MATHEMATICAL ALGORITHMS FOR ARTIFICIAL INTELLIGENCE

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.

PART-A

			M	BT	CO
1	1.1	If the null space of an 8×4 matrix A is 3 dimensional, give the dimensions of the other three fundamental subspaces.	02	2	1
	1.2	Assert or Reject the following statement with justification: Statement: The set of all 2×2 symmetric matrices is a vector space.	02	3	2
	1.3	Given the Euclidean norms $\|u\| = 4, \|v\| = 3$. If u and v are perpendicular, determine $\|u+v\|$?	02	2	3
	1.4	Determine a real symmetric matrix whose characteristic equation is $\lambda(\lambda^2-4)(\lambda-1)=0$	02	2	1
	1.5	Define heteroscedasticity.	02	1	1
	1.6	How is the ordinary least squares problem different from a weighted least squares case?	02	1	1
	1.7	The position vector of point P at time t is $r = (3\cos t, 5\sin t, 4\, cos\, t)$. What is the acceleration?	02	3	1
	1.8	Obtain the gradient of the function $f(x,y)=\sin(x+y)$.	02	3	2
	1.9	Consider a 2×3 real matrix A, with rank of $A = 2$. What is the pseudoinverse of A?	02	2	2
	1.10	Consider minimizing a function using gradient descent. If you notice that the function values are not decreasing in consecutive steps, what could be an issue?	02	2	2

PART-B

2	a	Construct a 3×2 matrix with a column space that equals XY plane. Find the other three subspaces associated with it. Mention one basis each for each of the subspaces.	10	3	1
	b	Assert or Reject the following statements with justification. Statement 1: For a given $n\times n$ real matrix A, two distinct eigenvectors (one is not the scalar multiplication of the other) can have the same eigenvectors. Statement 2: $\mathbb{R}^2$ is isomorphic to XY plane of $\mathbb{R}^3$	06	3	1

3	a	Given the following vectors $u_1 = [1 \quad 1 \quad 0]^T, u_2 = [1 \quad -1 \quad 1]^T, u_3 = [1 \quad -1 \quad -2]^T$ i) Are the vectors u_1, u_2, u_3 orthogonal? ii) Are the vectors u_1, u_2, u_3 orthonormal? iii) Express the vector $x = [1 \quad 2 \quad 3]^T$ as a linear combination of the vectors u_1, u_2, u_3.	08	3	2
	b	i) If P is a projection matrix, prove that $P^2 = P$. ii) Obtain the matrix that projects every vector in $\mathbb{R}^3$ along the line passing through origin along the vector $[1 \quad -1 \quad 1]^T$	08	3	1
		OR			
4	a	Obtain the orthogonal complement subspace of the line passing through origin along the vector $[1 \quad -1 \quad 1]^T$.	06	3	3
	b	Consider the four sample points of a two dimensional Data: $\{[1,1], [1,1], [2,0], [0,2]\}$ We want to represent the data in only one dimension. Compute the unit-length principal component directions of X, and state which one the PCA algorithm would choose if you request just one principal component. Obtain the projections of all four sample points onto the principal direction.	10	3	1
5	a	Suppose you measure your pulse rate x three times and get values $x = b_1, x = b_2, x = b_3$. If each measurement is independent with zero mean and unit variances, find the best estimate $\hat{x}$ based on b_1, b_2, b_3. What is the corresponding variance?	10	3	3
	b	List out 3 limitations of Gaussian Mixture Models.	06	2	3
		OR			
6	a	List out the four limitations of Expectation Maximization algorithm.	12	2	1
	b	List four applications of Kalman filters.	04	2	1
7		A general second degree vector polynomial is of the form $f(x) = x^T A x + b^T x$ where x is a vector $[x_1, x_2, \ldots\ldots x_d]^T$, b is a vector $[b_1, b_2, \ldots\ldots b_d]^T$ and A is a matrix with (i,j)-th element A_{ij}. A and b are constants and not functions of x. i) Evaluate $f(x)$ for two dimensional $x = [x, y]^T$. ii) Which definition of the derivative do we need in order to evaluate $d(f(x))/dx$? iii) Assume $x, b \in \mathbb{R}^2$ and $A \in \mathbb{R}^{2\times 2}$. Evaluate $d(f(x))/dx$. iv) Generalize the previous result to when $x, b \in \mathbb{R}^n$ and $A \in \mathbb{R}^{n\times n}$ and evaluate $d(f(x))/dx$. Can you express the result in matrix form? v) What happens when A is a symmetric matrix, i.e., $A^T = A$?	16	3	2
		OR			

8		Suppose the 2×1 function f of the 3×1 vector x is given by $f(x) = [x^T A x \; b^T x]^T$ (A is a matrix and b is a vector) and the 2×1 function g of the 2×1 vector z is given by $g(z) = z^T z$ Compute $d(f(x))/dx$ Where $y(x)$ is the composite function defined by $y(x) = g\big(f(x)\big)$.	16	3	2
9	a	The function $f(x, y, z) = 2x^2 + 2y^2 + z^2 - x - y - 2xy$ defined on $\mathbb{R}^3$ has only one stationary point. Determine it and show that it is a local minimum.	06	3	2
	b	Find the least square solution to the following system of equations: $x_1 + x_2 = 2; x_1 - x_2 = 0, 2x_1 + x_2 = 4.$	10	3	2
		OR			
10		Use the method of Lagrange multipliers to find the minimum value of $x^2 + y^2 - 4x - 4y$ subject to the constraint $x + y = 2$.	16	3	2