8.2 EXPECTED RETURN AND RISK OF A SINGLE ASSET

So far we looked at past returns. Now we discuss prospective returns. When you invest in a stock you know that the return from it can take various possible values. For example, it may be –5 percent, or 15 percent, or 35 percent. Further, the likelihood of these possible returns can vary. Hence, you should think in terms of a probability distribution.

The probability of an event represents the likelihood of its occurrence. Suppose you say that there is a 4 to 1 chance that the market price of a stock A will rise during the next fortnight. This implies that there is an 80 percent chance that the price of stock A will increase and a 20 percent chance that it will not increase during the next fortnight. Your judgment can be represented in the form of a probability distribution as follows:

Outcome	*Probability*
Stock price will rise	0.80
Stock price will not rise	0.20

Another example may be given to illustrate the notion of probability distribution. Consider two equity stocks, Bharat Foods stock and Oriental Shipping stock. Bharat Foods stock may provide a return of 16 percent, 11 percent, or 6 percent with certain probabilities associated with them, based on the state of the economy. The second stock, Oriental Shipping stock, being more volatile, may earn a return of 40 percent, 10 percent, or –20 percent with the same probabilities, based on the state of the economy. The probability distributions of the returns on these two stocks are shown in Exhibit 8.2.

When you define the probability distribution of the rate of return (or for that matter any other variable) remember that:

- The possible outcomes must be mutually exclusive and collectively exhaustive.
- The probability assigned to an outcome may vary between 0 and 1 (An impossible event is assigned a probability of 0, a certain event is assigned a probability of 1, and an uncertain event is assigned a probability somewhere between 0 and 1).
- The sum of the probabilities assigned to various possible outcomes is 1.

Exhibit 8.2 **Probability Distributions of the Rate of Return on Bharat Foods Stock and Oriental Shipping Stock**

		Rate of Return (%)	
State of the Economy	*Probability of Occurrence*	*Bharat Foods*	*Oriental Shipping*
Boom	0.30	16	40
Normal	0.50	11	10
Recession	0.20	6	–20

Based on the probability distribution of the rate of return, you can compute two key parameters, the expected rate of return and the standard deviation of rate of return.

Expected Rate of Return The expected rate of return is the weighted average of all possible returns multiplied by their respective probabilities. In symbols,

$$E(R) = \sum_{i=1}^{n} p_i R_i \tag{8.5}$$

where $E(R)$ is the expected return, R_i is the return for the ith possible outcome, p_i is the probability associated with R_i, and n is the number of possible outcomes.

From Eq. (8.5), it is clear that $E(R)$ is the weighted average of possible outcomes – each outcome is weighted by the probability associated with it. The expected rate of return on Bharat Foods stock is:

$$E(R_b) = (0.30)(16\%) + (0.50)(11\%) + (0.20)(6\%) = 11.5\%$$

Similarly, the expected rate of return on Oriental Shipping stock is:

$$E(R_o) = (0.30)(40\%) + (0.50)(10\%) + (0.20)(-20\%) = 13.0\%$$

Standard Deviation of Return Risk refers to the dispersion of a variable. It is commonly measured by the variance or the standard deviation. The variance of a probability distribution is the sum of the squares of the deviations of actual returns from the expected return, weighted by the associated probabilities. In symbols,

$$\sigma^2 = \Sigma p_i (R_i - E(R))^2 \tag{8.6}$$

where σ^2 is the variance, R_i is the return for the ith possible outcome, p_i is the probability associated with the ith possible outcome, and $E(R)$ is the expected return.

Since variance is expressed as squared returns, it is somewhat difficult to grasp. So its square root, the standard deviation, is employed as an equivalent measure.

$$\sigma = (\sigma^2)^{1/2} \tag{8.7}$$

where σ is the standard deviation.

As an illustration, the standard deviation of returns on Bharat Foods stock and Oriental Shipping stock are calculated in Exhibit 8.3.

Exhibit 8.3 **Illustration of the Calculation of Standard Deviation**

			Bharat Foods Stock			
State of the Economy	p_i	R_i	p_iR_i	$R_i-E(R)$	$(R_i-E(R))^2$	$p_i(R_i-E(R))^2$
1. Boom	0.30	16	4.8	4.5	20.25	6.075
2. Normal	0.50	11	5.5	–0.5	0.25	0.125
3. Recession	0.20	6	1.2	–5.5	30.25	6.050
			$\Sigma p_iR_i = 11.5$		$\Sigma p_i(R_i - E(R))^2 = 12.25$	
			$\sigma = [\Sigma p_i (R_i - E(R))^2]^{1/2} = (12.25)^{1/2} = 3.5\%$			
			Oriental Shipping Stock			
1. Boom	0.30	40	12.0	27.0	729.0	218.7
2. Normal	0.50	10	5.0	–3.0	9.0	4.5
3. Recession	0.20	–20	–4.0	–33.0	1089.00	217.8
			$\Sigma p_iR_i = 13.0$		$\Sigma p_i (R_i - E(R))^2 = 441.0$	
			$\sigma = [\Sigma p_i(R_i - E(R))^2]^{1/2} = (441.0)^{1/2} = 21.0\%$			

Features of Standard Deviation Looking at the calculation of standard deviation, we find that it has the following features:

- The differences between the various possible values and the expected value are squared. This means that values which are far away from the expected value have a much more effect on standard deviation than values which are close to the expected value.
- The squared differences are multiplied by the probabilities associated with the respective values. This means that the smaller the probability that a particular value will occur, the lesser its effect on standard deviation.
- The standard deviation is obtained as the square root of the sum of squared differences (multiplied by their probabilities). This means that the standard deviation and expected value are measured in the same units and hence the two can be directly compared.

Rationale for Standard Deviation Why is standard deviation employed commonly in finance as a measure of risk? The principal reasons for using standard deviation seem to be:

- If a variable is normally distributed, its mean and standard deviation contain all the information about its probability distribution.
- If the utility of money is represented by a quadratic function (a function commonly suggested to represent diminishing utility of wealth), then the expected utility is a function of mean and standard deviation.
- Standard deviation is analytically more easily tractable.

Continuous Probability Distributions The probability distribution of returns on Bharat Foods stock (or Oriental Shipping stock) is a discrete distribution because probabilities have been assigned to a finite number of specific values. In finance, however, probability distributions are commonly regarded as continuous, even though they may actually be discrete. In a continuous probability distribution, probabilities are not assigned to individual points as in the case of a discrete distribution. Instead, probabilities are assigned to intervals between two points on a continuous curve. Hence, when a continuous probability distribution is used, the following kinds of questions are answered: What is the probability that the rate of return will fall between, say, 10 percent and 20 percent? What is the probability that the rate of return will be less than 0 percent or more than 25 percent?

The Normal Distribution The normal distribution, a continuous probability distribution, is the most commonly used probability distribution in finance. As shown in Exhibit 8.4, the normal distribution, resembles a bell shaped curve. It appears that stock returns, at least over short time intervals, are approximately normally distributed. The following features of the normal distribution may be noted:

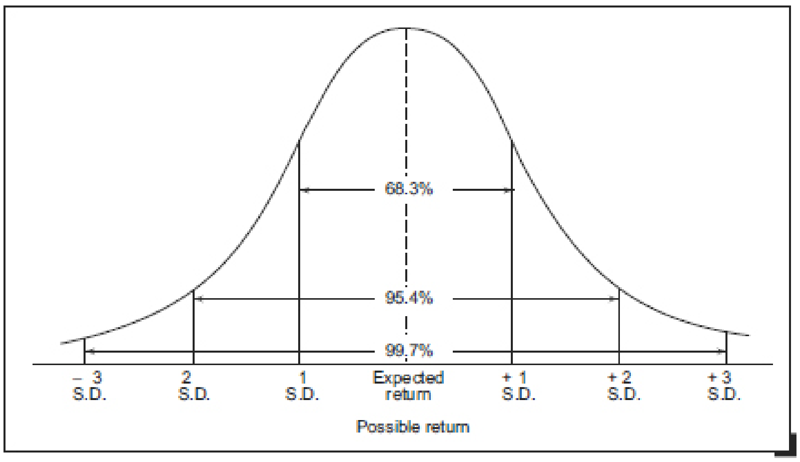
- It is completely characterised by just two parameters, viz., expected return and standard deviation of return.
- A bell-shaped distribution, it is perfectly symmetric around the expected return.
- The probabilities for values lying within certain bands are as follows:

Band	*Probability*
± One standard deviations from the expected return	68.3%
± Two standard deviations from the expected return	95.4%

± Three standard deviations from the expected return	99.7%

Exhibit 8.4 displays this graphically. Detailed probability ranges are given in an appendix at the end of the book.

Exhibit 8.4 **Normal Distribution**



Risk Aversion and Required Returns You are lucky to be invited by the host of a television game show. After the usual introduction, the host shows two boxes to you. He tells you that one box contains ₹ 10,000 and the other box is empty. He does not tell you which one is which.

The host asks you to open any one of the two boxes and keep whatever you find in it. You are not sure which box you should open. Sensing your vacillation, he says he will offer you a certain ₹ 3,000 if you forfeit the option to open a box. You don't accept his offer. He raises his offer to ₹ 3,500. Now you feel indifferent between a certain return of ₹ 3,500 and a risky (uncertain) expected return of ₹ 5,000. This means that a certain amount of ₹ 3,500 provides you with the same satisfaction as a risky expected value of ₹ 5,000. Thus your certainty equivalent (₹ 3,500) is less than the risky expected value (₹ 5,000).

Empirical evidence suggests that most individuals, if placed in a similar situation, would have a certainty equivalent which is less than the risky expected value.

The relationship of a person's certainty equivalent to the expected monetary value of a risky investment defines his attitude toward risk. If the certainty equivalent is less than the expected value, the person is *risk-averse*; if the certainty equivalent is equal to the expected value, the person is *risk-neutral*; finally, if the certainty equivalent is more than the expected value, the person is *risk-loving*.

In general, investors are risk-averse. This means that risky investments must offer higher expected returns than less risky investments to induce people to invest in them. Remember, however, that we are talking about *expected* returns; the actual return on a risky investment may well turn out to be less than the actual return on a less risky investment.

Put differently, risk and return go hand in hand. This indeed is a well-established empirical fact, particularly over long periods of time. For example, the average annual rates and annual standard deviations for Treasury bills, bonds, and common stocks in the U.S. over a 75 year period (1926-2000) as calculated by Ibbotson Associates have been as shown in Exhibit 8.5[2].

Exhibit 8.5 **Return and Risk Performance of Different Categories of Financial Assets in the U.S. Over a 75 Year Period (1926 – 2000)**

	Average Annual Rate of Return (%)	*Standard Deviation (%)*
Portfolio		
Treasury bills	3.9	3.2
Government bonds	5.7	9.4
Corporate bonds	6.0	8.7
Common stocks (S&P 500)	13.0	20.2
Small-firm common stocks	17.3	33.4

From the above it is clear that: (a) Treasury bills, the least risky of financial assets, earned the lowest average annual rate of return. (b) Common stocks, the most risky of financial assets, earned the highest average annual rate of return. (c) Bonds which occupy a middling position on the risk dimension earned a middling average annual return.

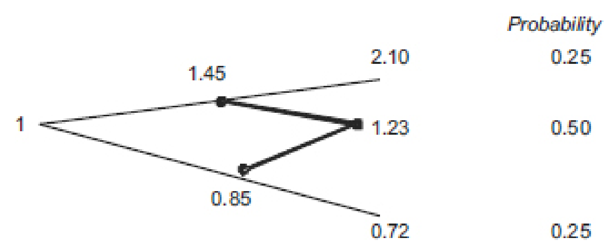
Arithmetic Mean versus Geometric Mean

It may be noted that the returns shown in Exhibit 8.5 are arithmetic means. Put differently, Ibbotson Associates just added the 75 annual returns and divided the sum by 75.

What is the rationale for the arithmetic mean? Why should the arithmetic mean be preferred to the geometric mean?

To answer these questions, let us consider an example. Suppose the equity share of Modern Pharma has an expected return of 15 percent in each year with a standard deviation of 30 percent. Assume that there are two equally possible outcomes each year, +45 percent and -15 percent (that is, the mean plus or minus one standard deviation). The arithmetic mean of these returns is 15 percent, (45-15)/2, whereas the geometric mean of these returns is 11.0 percent, $[(1.45)\,(0.85)]^{1/2} - 1$.

An investment of one rupee in the equity share of Modern Pharma would grow over a two year period as follows:



Notice that the median (middle outcome) and mode (most common outcome) are given by the geometric mean (11.0 percent), which over a two-year period compounds to 23 percent ($1.11^2 = 1.23$). The expected value of all possible outcomes, however, is equal to:

$$(0.25 \times 2.10) + (0.50 \times 1.23) + (0.25 \times 0.72) = 1.32$$

Now 1.32 is equal to $(1.15)^2$. This means that the expected value of the terminal wealth is obtained by compounding up the arithmetic mean, not the geometric mean. Hence the arithmetic mean is the appropriate discount rate.

Put differently, the arithmetic mean is the appropriate mean because an investment that has uncertain returns will have a higher expected terminal value than an investment that earns its compound or geometric mean with certainty every year. In the above example, compounding at the rate of 11 percent for two years produces a terminal value of ₹ 1.23, for an investment of ₹ 1.00. But holding the uncertain investment which yields high returns (45 percent per year for two years in a row) or low returns (-15 percent per year for two years in a row), yields a higher expected terminal value, ₹ 1.32. This happens because the gains from higher-than-expected returns are greater than the losses from lower-than-expected returns. As Roger G. Ibbotson and Rex A. Sinquefield put it: “Therefore, in the investment markets, where returns are described by a probability distribution, the arithmetic mean is the measure that accounts for uncertainty, and is the appropriate one for estimating discount rates and the cost of capital.”

8.3 RISK AND RETURN OF A PORTFOLIO

Most investors invest in a portfolio of assets, as they do not want to put all their eggs in one basket. Hence, what really matters to them is not the risk and return of stocks in isolation, but the risk and return of the portfolio as a whole.

Expected Return on a Portfolio The expected return on a portfolio is simply the weighted average of the expected returns on the assets comprising the portfolio. For example, when a portfolio consists of two securities, its expected return is:

$$E(R_p) = w_1 E(R_1) + (1 - w_1) E(R_2) \tag{8.8}$$

where $E(R_p)$ is the expected return on a portfolio, w_1 is the proportion of a portfolio invested in security 1, $E(R_1)$ is the expected return on security 1, $(1-w_1)$ is the proportion of portfolio invested in security 2[3], $E(R_2)$ is the expected return on security 2.

To illustrate, consider a portfolio consisting of two securities, A and B. The expected return on these two securities are 10 percent and 18 percent respectively. The expected return on the portfolio, when the proportions invested in A and B are 0.4 and 0.6, is simply: 0.4 x 10 + 0.6 x 18 = 14.8%.

In general, when a portfolio consists of *n* securities, the expected return on the portfolio is:

$$E(R_p) = \Sigma w_i E(R_i) \tag{8.9}$$

where $E(R_p)$ is the expected return on portfolio, w_i is the proportion of portfolio invested in security *i*, and $E(R_i)$ is the expected return on security *i*.

To illustrate, consider a portfolio consisting of five securities with the following expected returns: $E(R_1)$ = 10 percent, $E(R_2)$ = 12 percent, $E(R_3)$ = 15 percent, $E(R_4)$ = 18 percent, $E(R_5)$ = 20 percent. The portfolio proportions invested in these securities are: w_1 = 0.1, w_2 = 0.2, w_3 = 0.3, w_4 = 0.2, w_5 = 0.2. The expected portfolio return is:

$$\begin{aligned} E(R_p) &= w_1 E(R_1) + w_2 E(R_2) + w_3 E(R_3) + w_4 E(R_4) + w_5 E(R_5) \\ &= 0.1 \times 10 + 0.2 \times 12 + 0.3 \times 15 + 0.2 \times 18 + 0.2 \times 20 \\ &= 15.5 \text{ percent} \end{aligned}$$

Diversification and Portfolio Risk Suppose you have ₹ 100,000 to invest and you want to invest it equally in two stocks, A and B. The returns on these stocks depend on the state of the economy. Your assessment suggests that the probability distributions of the returns on stocks A and B are as shown in Exhibit 8.6. For the sake of simplicity, all the five states of the economy are assumed to be equiprobable. The last column of Exhibit 8.6 shows the return on a portfolio consisting of stocks A and B in equal proportions. The expected return and standard deviation of return on stocks A and B and the portfolio consisting of A and B in equal proportions are calculated in Exhibit 8.7.

Exhibit 8.6 **Probability Distribution of Returns**

State of the Economy	*Probability*	*Return on Stock A(%)*	*Return on Stock B(%)*	*Return on Portfolio(%)*
1	0.20	15	–5	5
2	0.20	–5	15	5
3	0.20	5	25	15
4	0.20	35	5	20
5	0.20	25	35	30

Exhibit 8.7 shows that if you invest only in stock A, the expected return is 15 percent and the standard deviation is 14.14 percent. Likewise, if you invest only in stock B, the expected return is 15 percent and the standard deviation is 14.14 percent. What happens if you invest in a portfolio consisting of stocks A and B in equal proportions? While the expected return remains at 15 percent, the same as that of either stock individually, the standard deviation of the portfolio return, 9.49 percent, is lower than that of either stock individually. Thus, in this case diversification reduces risk.

Exhibit 8.7 **Expected Return and Standard Deviation**

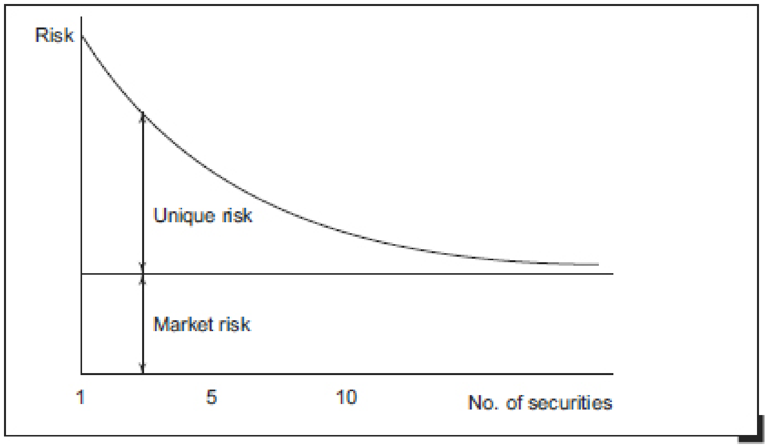
Expected Return	
Stock A	: 0.2(15%) + 0.2(–5%) + 0.2(5%) +0.2(35%) + 0.2(25%) = 15%
Stock B	: 0.2(–5%) + 0.2(15%) + 0.2(25%) + 0.2(5%) + 0.2(35%) = 15%
Portfolio of A and B	: 0.2(5%) + 0.2(5%) + 0.2(15%) + 0.2(20%) + 0.2(30%) = 15%

Standard Deviation				
Stock A	:	σ_A^2	=	$0.2(15-15)^2 + 0.2(-5-15)^2 + 0.2(5-15)^2 + 0.2(35-15)^2 + 0.20\,(25-15)^2$
			=	200
		σ_A	=	$(200)^{1/2} = 14.14\%$
Stock B	:	σ_B^2	=	$0.2(-5-15)^2 + 0.2(15-15)^2 + 0.2(25-15)^2 + 0.2(5-15)^2 + 0.2\,(35-15)^2$
			=	200
		σ_B	=	$(200)^{1/2} = 14.14\%$
Portfolio	:	$\sigma^2_{(A+B)}$	=	$0.2(5-15)^2 + 0.2(5-15)^2 + 0.2(15-15)^2 + 0.2(20-15)^2 + 0.2(30-15)^2$
			=	90
		σ^2_{A+B}	=	$(90)^{1/2} = 9.49\%$

In general, if returns on securities do not move in perfect lockstep, diversification reduces risk. In technical terms, diversification reduces risk if returns are not perfectly positively correlated.

The relationship between diversification and risk is shown graphically in Exhibit 8.8. When the portfolio has just one security, say stock 1, the risk of the portfolio σ_p, is equal to the risk of the single stock included in it, σ_1. As a second security – say stock 2 is added, the portfolio risk decreases. As more and more securities are added, the portfolio risk decreases, but at a decreasing rate, and reaches a limit. Empirical studies suggest that the bulk of the benefit of diversification, in the form of risk reduction, is achieved by forming a portfolio of about 15 to 20 securities. Thereafter, the gain from diversification tends to be negligible.

Exhibit 8.8 **Relationship between Diversification and Risk**



Market Risk versus Unique Risk Notice that the portfolio risk does not fall below a certain level, irrespective of how wide the diversification is. Why? The answer lies in the following relationship which represents a basic insight of modern portfolio theory.

Total risk = Unique risk + Market risk

The *unique* risk of a security represents that portion of its total risk which stems from firm-specific factors like the development of a new product, a labour strike, or the emergence of a new competitor. Events of this nature primarily affect the specific firm and not all firms in general. Hence, the unique risk of a stock can be washed away by combining it with other stocks. In a diversified portfolio, unique risks of different stocks tend to cancel each other – a favourable development in one firm may offset an adverse happening in another and vice versa. Hence, unique risk is also referred to as diversifiable risk or unsystematic risk.

The *market* risk of a stock represents that portion of its risk which is attributable to economy-wide factors like the growth rate of GNP, the level of government spending, money supply, interest rate structure, and inflation rate. Since these factors affect all firms to a greater or lesser degree, investors cannot avoid the risk arising from them, however diversified their portfolios may be. Hence, it is also referred to as systematic risk (as it affects all securities) or non-diversifiable risk.

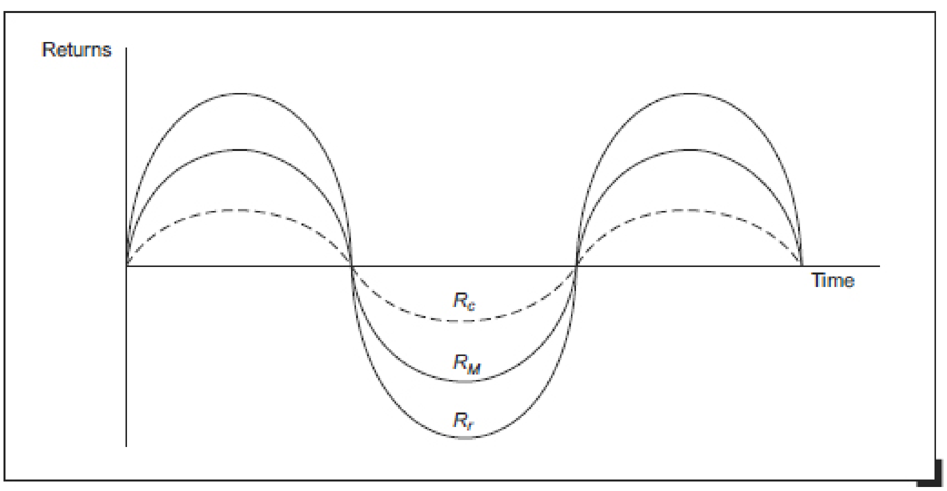
8.4 MEASUREMENT OF MARKET RISK

If you want to figure out the contribution of an individual stock to the risk of a well-diversified portfolio, you have to look at its market risk, and not its risk if it is held in isolation.

The market risk of a security reflects its sensitivity to market movements. Different securities seem to display differing sensitivities to market movements. This is illustrated graphically in Exhibit 8.9 which shows the returns on the market portfolio (R_M) over time, along with returns on two other securities - a risky security, whose return is denoted by (R_r) and a conservative security, whose return is denoted by (R_c). It is evident that R_r is more volatile than R_M whereas R_c is less volatile than R_M.

The sensitivity of a security to market movements is called beta (β). By definition, the beta for the market portfolio is 1. A security which has a beta of, say, 1.5 experiences greater fluctuation than the market portfolio. More precisely, if the return on market portfolio is expected to increase by 10 percent, the return on the security with a beta of 1.5 is expected to increase by 15 percent (1.5 x 10 percent). On the other hand, a security which has a beta of, say, 0.8 fluctuates lesser than the market portfolio. Individual security betas generally fall in the range 0.30 to 2.00 and rarely, if ever, assume a negative value.

Exhibit 8.9 **Behaviour of Returns Over Time**



Calculation of Beta

For calculating the beta of a security, the following market model is employed:

$$R_{jt} = \alpha_j + \beta_j R_{Mt} + e_j \tag{8.10}$$

where R_{jt} is the return on security j in period t, α_j is the intercept term, alpha, β_j is the regression coefficient, beta, R_{Mt} is the return on market portfolio in period t, and e_j is the random error term. Remember that you must regress the returns on the security on the returns on the market index as shown by the characteristic line in Exhibit 8.11. As the dividend yield is often a very small component of returns, you will get a reasonable estimate if you regress the percentage changes in the security price on the percentage change in the market index. However, you will get absurd results, if you regress the price level of the security on the level of the index.

Beta reflects the slope of the above regression relationship. It is equal to:

$$\beta_j = \frac{Cov(R_j, R_M)}{\sigma^2_M} = \frac{\rho_{jM}\sigma_j\sigma_M}{\sigma^2_M} = \frac{\rho_{jM}\sigma_j}{\sigma_M} \tag{8.11}$$

where Cov is the covariance between the return on security j and the return on market portfolio M. It is equal to:

$$\sum_{i=1}^{n}(R_{jt} - \overline{R}_j)(R_{Mt} - \overline{R}_M)/(n-1)$$

where σ^2_M is the variance of return on the market portfolio, ρ_{jM} is the correlation coefficient between the return on jth security and the return on the market portfolio, σ_j is the standard deviation of return on the jth security, and σ_M is the standard deviation of return on the market portfolio.

An example will help in understanding what β_j is and how it is calculated. The returns on security j and the market portfolio for a 10-year period are given below:

Year	*Return on Security j (%)*	*Return on Market Portfolio (%)*
1	10	12
2	6	5

3	13	18
4	−4	−8
5	13	10
6	14	16
7	4	7
8	18	15
9	24	30
10	22	25

The beta for security *j*, β_j is calculated in Exhibit 8.10. For the sake of completeness, the intercept term, α_j, has also been computed in Exhibit 8.10.

Given the values of β_j (0.76) and α_j (2.12 percent), the regression relationship between the return on security $j(R_j)$ and the return on market portfolio (R_M) is shown graphically in Exhibit 8.11. The graphic representation is commonly referred to as the characteristic line. Since security *j* has a beta of 0.76, we infer that its return is less volatile than the return on the market portfolio. If the return on market portfolio rises/falls by 10 percent, the return on security *j* would be expected to increase/decrease by 7.6 percent (0.76x10%). The intercept term for security *j* (α_j) is equal to 2.12 percent. It represents the expected return on security *j* when the return on the market portfolio is zero.

Exhibit 8.10 **Calculation of Beta**

Year	R_{jt}	R_{Mt}	$R_{jt} - \bar{R}_j$	$R_{Mt} - \bar{R}_M$	$(R_{jt} - \bar{R}_j)(R_{Mt} - \bar{R}_M)$	$(R_{Mt} - \bar{R}_M)^2$
1	10	12	–2	–1	2	1
2	6	5	–6	–8	48	64
3	13	18	1	5	5	25
4	–4	–8	–16	–21	336	441
5	13	10	1	–3	–3	9
6	14	16	2	3	6	9
7	4	7	–8	–6	48	36
8	18	15	6	2	12	4
9	24	30	12	17	204	289
10	22	25	10	12	120	144

$\Sigma R_{jt} = 120$ $\quad \Sigma R_{Mt} = 130$ $\quad \sum (R_{jt} - \bar{R}_j)(R_{Mt} - \bar{R}_M) = 778$ $\quad \sum (R_M - \bar{R}_M)^2 = 1022$

$\bar{R}_j = 12$ $\quad \bar{R}_M = 13$ $\quad \text{Cov}(R_{jt}, R_{Mt}) = 778/9 = 86.4$ $\quad \sigma_M^2 = 1022/9 = 113.6$

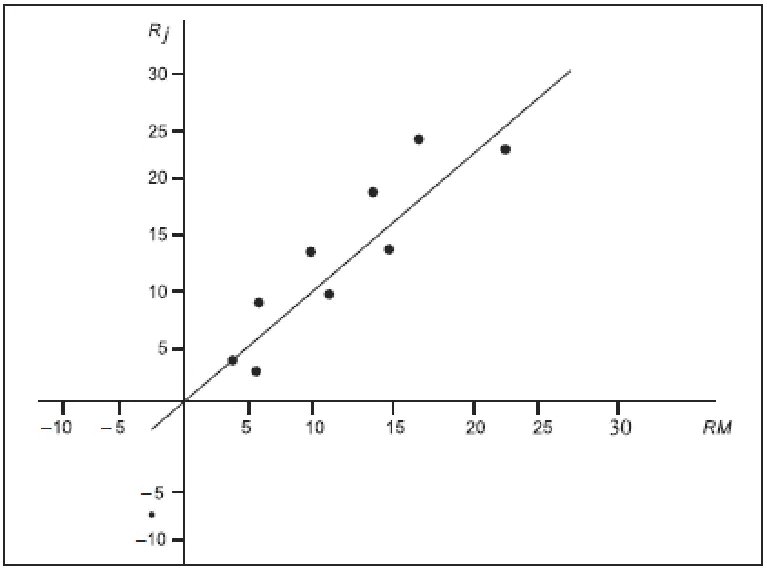
$$\text{Beta}: \beta_j = \frac{\text{Cov}(R_j, R_M)}{\sigma_M^2} = \frac{86.4}{113.6} = 0.76$$

$$\text{Alpha}: \alpha_j = \bar{R}_j - \beta_j \bar{R}_M = 12 - (0.76)(13) = 2.12\%$$

Spreadsheet Calculation You can calculate the beta and alpha using the Excel. Enter the returns on security *j* for the years 1 to 10 in the cells B2 to K2 and enter the returns on the market portfolio in the cells B3 to K3. Select F4 and type =slope ( ). Excel will prompt you to fill inside the bracket known *y*'s and known *x*'s which mean the y and x coordinate values, which in this case are between columns B and K. So, what you type inside F4 will be =slope(B2:K2,B3:K3). Press Enter and you will get the result. Similarly to get the alpha in K4, inside that cell type =intercept(B2:K2,B3:K3) and press Enter. Alternatively you may get the same results by going to the menu item Insert and then selecting the statistical functions slope and intercept. The spreadsheet is shown below.

	A	B	C	D	E	F	G	H	I	J	K
1	Year	1	2	3	4	5	6	7	8	9	10
2	Return on security *j* (%)	10	6	13	-4	13	14	4	18	24	22
3	Return on market portfolio (%)	12	5	18	-8	10	16	7	15	30	25
4	SLOPE	= SLOPE (B2:K2,B3:K3)			→	0.76	INTERCEPT	=INTERCEPT (B2:K2,B3:K3)		→	2.10

Exhibit 8.11 **Characteristic Line for Security *j***



Unsystematic Risk Revisited The dispersion of the data points around the characteristic line reflects the unsystematic risk of the stock. The wider the dispersion of the data points around the characteristic line, the greater the unsystematic risk. This also implies that the correlation between the stock's return and the market return is lower. On the other hand, the narrower the dispersion of the data points around the characteristic line, the smaller the unsystematic risk and the higher the correlation between the stock's return and the market return.

As we have learnt earlier, unsystematic risk can be reduced or even eliminated through diversification. If a portfolio has about 20 stocks, the portfolio data points would hover closely around the portfolio characteristic line.

Beta of Sensex Stocks Betas of Sensex stocks as of December 2018 are given below.

Company	*Beta*	*Company*	*Beta*	*Company*	*Beta*
Asian Paints	0.94	Housing Development Finance Corporation	1.25	Oil and Natural Gas Corporation	1.01
Axis Bank	1.26	ICICI Bank	1.33	Power Grid Corporation of India	0.54
Bajaj Auto	1.03	IndusInd Bank	0.96	Reliance Industries	1.37
Bajaj Finance	1046	Infosys	0.58	State Bank of India	1.41
Bharti Airtel	1.11	ITC	0.85	Sun Pharmaceutical	0.87
Coal India	0.5	Kotak Mahindra Bank	1.04	Tata Consultancy Service	0.44
HDFC Bank	0.69	Larsen & Toubro	1.09	Tata Motors	1.27
HCL Technologies	0.53	Mahindra & Mahindra	1.17	Tata Steel	1.37
Hero MotoCorp	0.89	Maruti Suzuki	1.12	Vedanta	1.36
Hindustan Unilever	0.75	NTPC	0.61	Yes Bank	1.33

8.6 RELATIONSHIP BETWEEN RISK AND RETURN

Before proceeding further, let us pause for a while and recapitulate the key elements of our story so far:

- Securities are risky because their returns are variable.
- The most commonly used measure of risk or variability in finance is standard deviation.
- The risk of a security can be split into two parts: unique risk and market risk.
- Unique risk stems from firm-specific factors, whereas market risk emanates from economy-wide factors.
- Portfolio diversification washes away unique risk, but not market risk. Hence, the risk of a fully diversified portfolio is its market risk.
- The contribution of a security to the risk of a fully diversified portfolio is measured by its beta, which reflects its sensitivity to the general market movements.

Since beta is the relevant measure of a security's risk, the next logical question is: What is the relationship between the risk of a security, as measured by its beta, and its expected return? The capital asset pricing model (CAPM), a seminal theory in modern finance developed more or less simultaneously by William Sharpe, John Lintner, and Jack Treynor, answers this question.

Security Market Line

According to the CAPM, risk and return are related in a linear fashion:

$$E(R_j) = R_f + \beta_j [E(R_M) - R_f] \tag{8.16}$$

where $E(R_j)$ is the expected return on security j, R_f is the risk-free return, β_j is the beta of security j, and $E(R_M)$ is the expected return on the market portfolio.

As per the above relationship, referred to as the security market line, the required return on a security consists of two components:

Risk-free return	: R_f
Risk-premium	: $\beta_j [E(R_M)-R_f]$

Note that the risk premium is a product of the level of risk, β_j, and the compensation per unit of risk, $[E(R_M)-R_f]$.

To illustrate, let us consider an example. Stock j has a beta of 1.4. If the risk-free rate is 10 percent and the expected return on the market portfolio is 15 percent, the expected return on stock j is:

$$10 + 1.4\,(15 - 10) = 17 \text{ percent}$$

It is obvious that, ceteris paribus, the higher the beta, the higher the expected return, and vice versa.

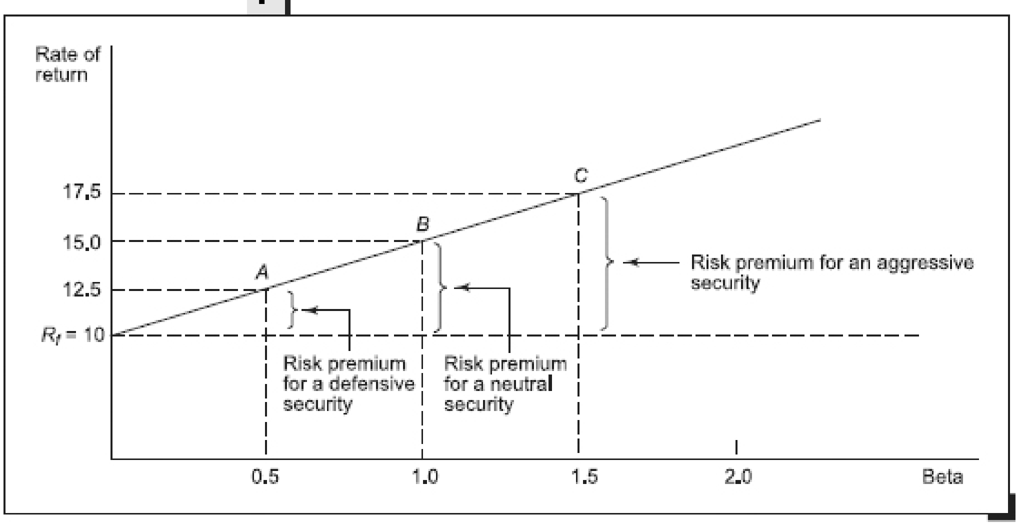
Exhibit 8.14 shows the security market line for the basic data given above. In this figure, the expected return on three securities A, B and C is shown. Security A is a defensive security with a beta of 0.5. Its expected rate of return is 12.5 percent. Security B is a neutral security with a beta of 1. Its expected rate of return is equal to the expected rate of return on the market portfolio namely 15.0 percent. Security C is an aggressive security with a beta of 1.5. Its expected rate of return is 17.5 percent. (In general, if the beta of a security is less than 1 it is characterised as defensive; if it is equal to 1 it is characterised as neutral; and if it is more than 1 it is characterised as aggressive.)

Changes in Security Market Line

The two parameters defining the security market line are the intercept (R_f) and the slope [$E(R_M)$]−R_f]. The intercept represents the nominal rate of return on the risk-free security. It is expected to be equal to the risk-free real rate of return plus the inflation rate. For example, if the risk-free real rate of return is 4 percent and the inflation rate is 8 percent, the nominal rate of return on the risk-free security is expected to be 12 percent. The slope represents the price per unit of risk and is a function of the risk-aversion of investors.

If the real risk-free rate of return and/or the inflation rate changes, the intercept of the security market line changes. If the risk aversion of investors changes, the slope of the security market line changes. Exhibit 8.15 shows the change in the security market line when the inflation rate increases and Exhibit 8.16 shows the change in the security market line when the risk-aversion of investors decreases.

Exhibit 8.14 **Security Market Line**



Security Market Equilibrium

Suppose the required return on stock A is 15 percent, calculated as follows:

$$R_A = R_f + \beta_A(R_M - R_f)$$
$$= 10\% + 1.25(14\% - 10\%) = 15\%$$

After assessing the prospects of stock A, investors conclude that its earnings, dividends, and price will continue to grow at the rate of 6 percent annum. The previous dividend per share, D_o was ₹ 1.70. The dividend per share expected a year hence is:

$$D_1 = ₹\ 1.70(1.06) = 1.80$$

The market price per share happens to be ₹ 22. What would investors, in general, do? Investors would calculate the expected return from stock A as follows:

$$\text{Expected return} = \text{Dividend yield} + \text{Growth rate}$$
$$= 1.80/22 + 6\%$$
$$= 8.2\% + 6\% = 14.2\%$$

Finding that the expected return is less than the required rate, investors, in general, would like to sell the stock. However, as there would be no demand for the stock at ₹ 22 per share, existing owners will have to lower the price to such a level that it fetches a return of 15 percent, its required return. That price, its equilibrium price, is the value of P_A in the following equation:

$$15\% = \frac{1.80}{P_A} + 6\%$$

Solving the above equation for P_A, we find that the equilibrium price is ₹ 20.00. If the market price initially had been lower than ₹ 20.00, investors, finding its return to be greater than required return, would seek to buy it. In this process the price will be pushed up to ₹ 20.00, its equilibrium price.

Exhibit 8.15 Change in the Security Market Line Caused by an Increase in Inflation

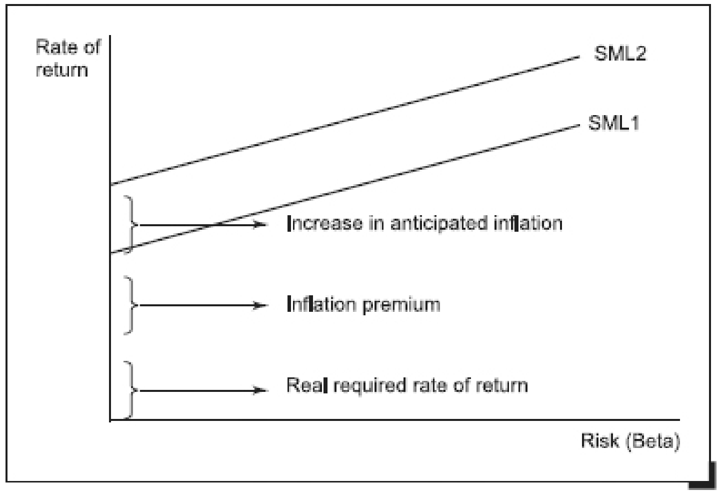
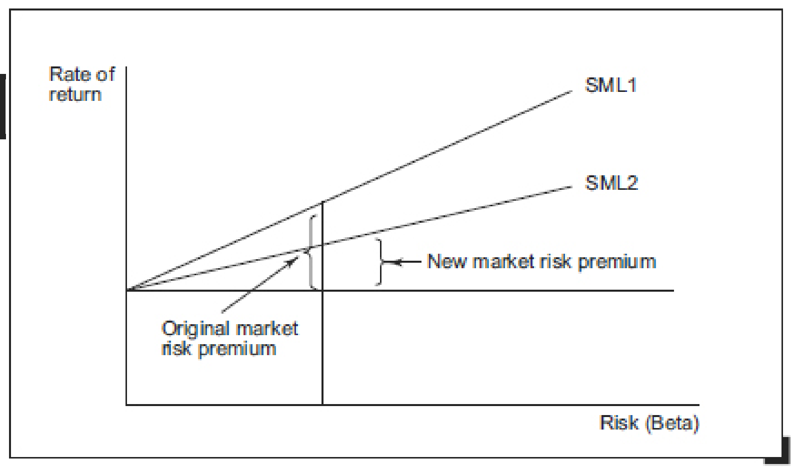


Exhibit 8.16 Changes in the Security Market Line Caused by a Decrease in Risk Aversion



Changes in Equilibrium Stock Prices

Stock market prices tend to change in response to changes in the underlying factors. To illustrate, let us assume that stock A, described above, is in equilibrium and sells at a price of ₹ 20.00 per share. If the expectation with respect to this stock is fulfilled, its equilibrium price a year hence will be ₹ 21.20, six percent higher than the current price. However, several factors could change in the course of a year and alter its equilibrium price. Suppose the values of underlying factors change as follows:

	Value of the Underlying Factor	
	Original	*Revised*
R_f (Riskless rate)	10%	9%
$R_M - R_f$ (Market risk premium)	4%	3%
β_A (Stock A's beta coefficient)	1.25	1.33
g (Stock A's expected growth rate)	6%	8%

The changes in the first three factors cause R_A to change from 15 percent to 13 percent.

Original : $R_A = 10\% + 1.25\ (4\%) = 15\%$

Revised : $R_A = 9\% + 1.33\ (3\%) = 13\%$

The change in the expected growth rate along with the change in the expected return alters the equilibrium price.

$$\text{Original} : P_o = \frac{1.70(1.06)}{0.15 - 0.06} = \frac{1.80}{0.09} = ₹\ 20.00$$

$$\text{Revised} : P_o = \frac{1.70(1.08)}{0.13 - 0.08} = \frac{1.84}{0.05} = ₹\ 36.80$$

Investment Implications

We have learnt about the components of risk, consequences of diversification, and the relationship between risk and return. The investment implications of our discussion are as follows:

- Diversification is important. Owning a portfolio dominated by a small number of stocks is a risky proposition.
- While diversification is desirable, an excess of it is not. There is hardly any gain in extending diversification beyond 15 to 20 stocks.
- The performance of a well-diversified portfolio more or less mirrors the performance of the market as a whole.
- In a well-ordered market, investors are compensated primarily for bearing market risk, but not unique risk. To earn a higher expected rate of return, one has to bear a higher degree of market risk.

SUMMARY

- Risk is present in virtually every decision. Assessing risk and incorporating the same in the final decision is an integral part of financial analysis.
- The **rate of return** on an asset for a given period (usually a period of one year) is defined as follows:

$$\text{Rate of return} = \frac{\text{Annual income} + \text{Ending price} - \text{Beginning price}}{\text{Beginning price}}$$

- Based on the probability distribution of the rate of return, two key parameters may be computed: expected rate of return and standard deviation.
- The **expected rate of return** is the weighted average of all possible returns multiplied by their respective probabilities. In symbols,

$$E(R_p) = \sum w_i E(R_i)$$

- Risk refers to the dispersion of a variable. It is commonly measured by the variance or the standard deviation.
- The **variance** of a probability distribution is the sum of the squares of the deviations of actual returns from the expected return, weighted by the associated probabilities. In symbols,

$$\sigma^2 = \sum_{i=1}^{n} p_i (R_i - \bar{R})^2$$

- **Standard deviation** is the square root of variance.
- The **normal distribution** is the most commonly used probability distribution in finance. It resembles a bell-shaped curve.

- The expected return on a portfolio is simply the weighted average of the expected returns on the assets comprising the portfolio. In general, when the portfolio consists of *n* securities, its expected return is:

$$\bar{R}_p = \sum_{i=1}^{n} w_i \bar{R}_i$$

- If returns on securities do not move in perfect lockstep, diversification reduces risk.
- As more and more securities are added to a portfolio, its risk decreases, but at a decreasing rate. The bulk of the benefit of diversification is achieved by forming a portfolio of about 15–20 securities.
- The following relationship represents a basic insight of modern portfolio theory:

$$\text{Total risk} = \text{Unique risk} + \text{Market risk}$$

- The **unique risk** of a security represents that portion of its total risk which stems from firm-specific factors. It can be washed away by combining it with other securities. Hence, unique risk is also referred to as diversifiable risk or **unsystematic risk**.
- The market risk of a security represents that portion of its risk which is attributable to economy-wide factors. It is also referred to as **systematic risk** (as it affects all securities) or non-diversifiable risk (as it cannot be diversified away).
- The market risk of a security reflects its sensitivity to market movements. It is called **beta**.

QUESTIONS

1. How is the rate of return on an asset defined?
2. What is standard deviation? What are its important features?
3. Why is standard deviation employed commonly in finance as a measure of risk?
4. Describe the key features of normal distribution.
5. Explain how diversification influences risk.
6. Distinguish between unique risk and market risk.
7. Define the standard deviation of the returns on a two-security portfolio.
8. How is beta calculated?
9. What is the relationship between risk and return as per CAPM?
10. What is a defensive security, neutral security, and aggressive security?
11. What is the effect of change in risk aversion on the security market line?
12. ‘The increase in the risk-premium of all stocks, irrespective of their beta, is the same, when risk aversion increases’. Comment.
13. What are the investment implications of CAPM?

SOLVED PROBLEMS

8.1 The stock of Box Limited performs well relative to other stocks during recessionary periods. The stock of Cox Limited, on the other hand, does well during growth periods. Both the stocks are currently selling for ₹ 100 per share. You assess the rupee return (dividend plus price) of these stocks for the next year as follows:

	Economic Condition			
	High growth	*Low growth*	*Stagnation*	*Recession*
Probability	0.3	0.4	0.2	0.1
Return on Box's stock	100	110	120	140
Return on Cox's stock	150	130	90	60

Calculate the expected return and standard deviation of investing:

(a) ₹ 1,000 in the equity stock of Box Limited

(b) ₹ 1,000 in the equity stock of Cox Limited

(c) ₹ 500 each in the equity stock of Box Limited and Cox Limited.

Solution

(a) 10 equity shares of Box Limited can be bought for ₹ 1,000. The probability distribution of overall return, when 10 equity shares of Box Limited are purchased will be as follows:

Economic condition	*Overall return*	*Probability*
High growth	10(100) = ₹ 1,000	0.3
Low growth	10(110) = ₹ 1,100	0.4
Stagnation	10(120) = ₹ 1,200	0.2
Recession	10(140) = ₹ 1,400	0.1

The expected return is:

0.3 (1,000) + 0.4 (1,100) + 0.2 (1,200) + 0.1 (1,400) = 300 + 440 + 240 + 140 = ₹ 1,120

The standard deviation of return is:

$[0.3\ (1{,}000 - 1{,}120)2 + 0.4\ (1{,}100 - 1{,}120)2 + 0.2\ (1{,}200 - 1{,}120)^2 +$
$0.1(1{,}400 - 1{,}120)\ ^2]^{1/2}$

$= [0.3\ (14{,}400) + 0.4\ (400) + 0.2\ (6{,}400) + 0.1\ (78{,}400)]^{1/2}$

$= [4{,}320 + 160 + 1{,}280 + 7{,}840]^{1/2}$

$= [13{,}600]^{1/2} =$ ₹ 116.6

(b) 10 equity shares of Cox Limited can be bought for ₹ 1,000. The probability distribution of overall return, when 10 equity shares of Cox Limited are purchased will be as follows:

Economic condition	*Overall return*	*Probability*
High growth	10(150) = ₹ 1,500	0.3
Low growth	10(130) = ₹ 1,300	0.4
Stagnation	10(90) = ₹ 900	0.2
Recession	10(60) = ₹ 600	0.1

The expected return is:

0.3 (1,500) + 0.4 (1,300) + 0.2 (900) + 0.1 (600) = 450 + 520 + 180 + 60 = ₹ 1,210

The standard deviation of return is:

$[0.3\ (1{,}500 - 1{,}210)^2 + 0.4\ (1{,}300 - 1{,}210)^2 + 0.2\ (900 - 1{,}210)^2 + 0.1(600 - 1{,}210)^2]^{1/2}$

$= [0.3\ (84{,}100) + 0.4\ (8{,}100) + 0.2\ (96{,}100) + 0.1\ (372{,}100)]^{1/2}$

$= [25{,}230 + 3{,}240 + 19{,}220 + 37{,}210]^{1/2}$

$= [84{,}900]^{1/2}$ = ₹ 291.4

(c) If ₹ 500 each are invested in the equity stocks of Box Limited and Cox Limited, 5 shares will be bought of each company. The probability distribution of overall return, on this portfolio will be as follows:

Economic condition	*Overall return*	*Probability*
High growth	5(100) + 5(150) = ₹ 1,250	0.3
Low growth	5(110) + 5(130) = ₹ 1,200	0.4
Stagnation	5(120) + 5(90) = ₹ 1,050	0.2
Recession	5(140) + 5(60) = ₹ 1,000	0.1

The expected return is:

0.3 (1,250) + 0.4 (1,200) + 0.2 (1,050) + 0.1 (1,000)

= 375 + 480 + 210 + 100

= ₹ 1,165

The standard deviation of return is:

$[0.3\ (1{,}250 - 1{,}165)^2 + 0.4\ (1{,}200 - 1{,}165)^2 + 0.2\ (1{,}050 - 1{,}165)^2 + 0.1(1{,}000 - 1{,}165)^2]^{1/2}$

$= [0.3\ (7{,}225) + 0.4\ (1{,}225) + 0.2\ (13{,}225) + 0.1\ (27{,}225)]^{1/2}$

$= [2167.5 + 490 + 2{,}645 + 2{,}722.5]^{1/2}$

$= [8{,}025]^{1/2}$ = ₹ 89.6

8.2 The risk-free rate of return R_f is 9 percent. The expected rate of return on the market portfolio R_M is 13 percent. The expected rate of growth for the dividend of firm *A* is 7 percent. The last dividend paid on the equity stock of firm *A* was ₹ 2.00. The beta of firm *A*'s equity stock is 1.2 (a) What is the equilibrium price of the equity stock of firm *A*? (b) How would the equilibrium price change when (i) the inflation premium increases by 2 percent, (ii) the expected growth rate increases by 3 percent, and (iii) the beta of *A*'s equity rises to 1.3.

Solution

(a) To calculate the equilibrium price of A's equity, we have to first obtain its required rate of return, R_A

$$R_A = R_f + \beta_A(R_M - \overline{R}_f) = 9 + 1.2(13 - 9) = 13.8 \text{ percent}$$

Combining this value of R_A with the given values of D_0(₹ 200) and g (7 percent), we get the equilibrium price of *A*'s equity stock, P_A, as follows:

$$P_A = \frac{D_0(1+g)}{R_A - g} = \frac{₹\ 200(1.07)}{0.138 - .07}$$

$$= ₹\ 31.47$$

(b) The change in the equilibrium price in response to variations in certain underlying factors is as follows:

(i) Inflation premium increases by 2 percent. This raises R_A to 15.8 percent. Hence the new equilibrium price is

$$P_A = \frac{₹\ 2.00(1.07)}{0.158 - 0.07} = ₹\ 24.32$$

(ii) The expected growth rate increases by 3 percent. This means that the growth rate, g, becomes 10 percent. As a result, the new equilibrium price is:

$$P_A = \frac{₹\ 2.00(1.10)}{0.138 - 0.10} = ₹\ 57.89$$

(iii) The beta of A's equity rises to 1.3 This makes the required return on A's equity equal to: $R_A = 9 + 1.3(13 - 9) = 14.2$ percent. As a result, the new equilibrium price is:

$$P_A = \frac{₹\ 2.00(1.07)}{0.142 - 0.07} = ₹\ 29.72$$

8.3 The rates of return on stock A & market portfolio for 15 periods are given below

Period	Return on stock A(%)	Return on market portfolio (%)	Period	Return on stock A(%)	Return on market portfolio (%)
1	10	12	9	–9	1
2	15	14	10	14	12
3	18	13	11	15	–11
4	14	10	12	14	16
5	16	9	13	6	8
6	16	13	14	7	7
7	18	14	15	–8	10
8	4	7			

What is the beta for stock A?

What is the characteristic line for stock A?

Solution

Period	$R_A(\%)$	$R_M(\%)$	$R_A - \bar{R}_A$	$R_M - \bar{R}_M$	$(R_A - \bar{R}_A)(R_M - \bar{R}_M)$	$(R_M - \bar{R}_M)^2$
1	10	12	0	3	0	9
2	15	14	5	5	25	25
3	18	13	8	4	32	16
4	14	10	4	1	4	1
5	16	9	6	0	0	0
6	16	13	6	4	24	16
7	18	14	8	5	40	25
8	4	7	–6	–2	12	4
9	–9	1	–19	–8	152	64
10	14	12	4	3	12	9
11	15	–11	5	–20	–100	400
12	14	16	4	7	28	49
13	6	8	–4	–1	4	1
14	7	7	–3	–2	6	4
15	–8	10	–18	1	–18	1
$\Sigma R_A=150$ $\bar{R}_A=10$		$\Sigma R_M=135$ $\bar{R}_M=9$	$\Sigma(R_A - \bar{R}_A)(R_M - \bar{R}_M)=221$		$\Sigma(R_M - \bar{R}_M)^2=575$	

$$\sigma_M^2 = \frac{\Sigma(R_M - \bar{R}_M)^2}{n-1} = \frac{575}{14} = 41.07 \qquad \text{Cov}_{A,M} = \frac{\Sigma(R_A - \bar{R}_A)(R_M - \bar{R}_M)}{n-1}$$

$$= \frac{221}{14} = 15.79$$

$$\text{Beta} : \beta_A = \frac{\text{Cov}_{A,M}}{\sigma_M^2} = \frac{15.79}{41.07} = 0.384$$

$$\text{Alpha} = \alpha_A = \bar{R}_A - \beta_A \bar{R}_M = 10 - 0.384 \times 9 = 6.54\%$$

The characteristic line for stock *A* is : $R_A = 6.54 + 0.384\, R_M$

PROBLEMS

8.1 Rate of Return You are considering purchasing the equity stock of MVM Company. The current price per share is ₹ 10. You expect the dividend a year hence to be ₹ 1.00. You expect the price per share of MVM stock a year hence to have the following probability distribution.

Price a year hence	₹ 10	11	12
Probability	0.4	0.4	0.2

(a) What is the expected price per share a year hence?

(b) What is the probability distribution of the rate of return on MVM's equity stock?

8.2 Expected Return and Standard Deviation The stock of Alpha Company performs well relative to other stocks during recessionary periods. The stock of Beta Company, on the other hand, does well during growth periods. Both the stocks are currently selling for ₹ 50 per share. The rupee return (dividend plus price change) of these stocks for the next year would be as follows:

	Economic condition			
	High growth	*Low growth*	*Stagnation*	*Recession*
Probability	0.3	0.3	0.2	0.2
Return on Alpha stock	55	50	60	70
Return on Beta stock	75	65	50	40

Calculate the expected return and standard deviation of:

(a) ₹ 1,000 in the equity stock of Alpha;

(b) ₹ 1,000 in the equity stock of Beta;

(c) ₹ 500 in the equity stock of Alpha and ₹ 500 in the equity stock of Beta;

(d) ₹ 700 in the equity stock of Alpha and ₹ 300 in the equity of Beta.

Which of the above four options would you choose? Why?

8.3 Beta and Characteristic Line The returns on the equity stock of Auto Electricals Limited and the market portfolio over a 11 year period are given below:

Year	*Return on Auto Electricals Ltd. (%)*	*Return on Market Portfolio (%)*
1	15	12
2	−6	1
3	18	14
4	30	24
5	12	16
6	25	30
7	2	−3
8	20	24

9	18	15
10	24	22
11	8	12

(a) Calculate the beta for the stock of Auto Electricals Limited.

(b) Establish the characteristic line for the stock of Auto Electricals Limited.

8.4 Intrinsic Value The risk-free return is 10 percent and the return on market portfolio is 15 percent. Stock A's beta is 1.5; its dividends and earnings are expected to grow at the constant rate of 8 percent. If the previous dividend per share of stock A was ₹ 2.00, what should be the intrinsic value per share of stock A?

8.5 Beta The risk-free return is 8 percent and the expected return on a market portfolio is 12 percent. If the required return on a stock is 15 percent, what is its beta?

8.6 Marked Return The risk-free return is 9 percent. The required return on a stock whose beta is 1.5 is 15 percent. What is the expected return on the market portfolio?

8.7 Stock Price The required return on the market portfolio is 12 percent. The beta of stock X is 2.0. The required return on the stock is 18 percent. The expected dividend growth on stock X is 5 percent. The price per share of stock X is ₹ 30. What is the expected dividend per share of stock X next year?

What will be the combined effect of the following on the price per share of stock X?

(a) The inflation premium increases by 2 percent.

(b) The decrease in the degree of risk-aversion reduces the differential between the return on market portfolio and the risk-free return by one-third.

(c) The expected growth rate of dividend on stock X decrease to 4 percent.

(d) The beta of stock X falls to 1.8.

8.8 Asset Beta A firm's equity beta is 1.1. Its tax rate is 30 percent and debt-equity ratio is 4:5. What is its asset beta?

8.9 Average Return and Variability Given the following returns, calculate the average returns, variances, and standard deviation of **A** and **B**.

Year	1	2	3	4	5	6
A	10%	18	–12	16	3	24
B	15%	22	2	–18	12	17

8.10 Arithmetic and Geometric Returns A stock generated the following returns over the last five years: 6%, 42%, −10%, 25%, −5%. Calculate the arithmetic mean return and geometric mean return.

8.11 CAPM The risk-free rate is 7 percent and the expected return on the market is 13 percent. What is the expected return on a stock that has a beta of 1.2?

8.12 **CAPM** The expected return on a stock that has a beta of 0.90 is 13.3 percent. The expected return on the market is 14 percent. What is the risk-free return?

MINICASE - I

The monthly closing share prices for 31 months for Tata Motors, Hero MotoCorp, Hindustan Unilever Ltd and Nifty are given below:

Month	*Tata Motors*	*Hero MotoCorp*	*Hindustan Unilever*	*Nifty*
2016 January	337	2565.65	817	7564
February	300	2499.9	830	6987
March	387	2945.7	870	7738
April	408	2897.4	868	7850
May	460	3098.9	848	8160
June	459	3178.25	899	8288
July	503	3203.15	923	8639
August	538	3541.35	917	8786
September	535	3413.7	868	8611
October	532	3352.1	839	8626
November	459	3166.05	844	8225
December	472	3043.65	826	8186
2017 January	524	3172.35	855	8561
February	457	3138.1	866	8880
March	466	3221.95	912	9174
April	459	3318.6	935	9304
May	476	3745.45	1067	9621
June	433	3701.35	1080	9521
July	445	3655.75	1156	10077
August	377	3995.1	1220	9918
September	402	3774.55	1174	9789
October	428	3849.9	1237	10335
November	404	3633.65	1272	10227
December	432	3785.15	1368	10531
2018 January	399.5	3691.45	1369	11028
February	369.9	3596.7	1318	10493
March	326.85	3542.8	1333	10114
April	340.4	3732.25	1509	10739
May	282.5	3544.9	1611	10736
June	269.3	3473.5	1641	10714
July	264.1	3294.45	1732	11357

(a) What are the monthly returns on Tata Motors, Hero MotoCorp, Hindustan Unilever and Nifty? You may ignore the dividend yield.

(b) What are the average returns (arithmetic and geometric) on Tata Motors, Hero MotoCorp, Hindustan Unilever Ltd and Nifty?

(c) What are the standard deviations of the returns on Tata Motors, Hero MotoCorp, Hindustan Unilever and Nifty?

(d) Calculate the betas of Tata Motors, Hero MotoCorp and Hindustan Unilever.

MINICASE - II

The following is the information on price per share and dividend per share for NTPC Ltd, Escorts Ltd and MRF Ltd at the end of the financial years from 2008 to 2018:

Financial year ended	NTPC		ESCORTS		MRF	
	Price per share	Dividend per share	Price per share	Dividend per share	Price per share	Dividend per share
2008	196.60	3.50	85.70	0.00	3989.05	20.00
2009	179.85	3.60	35.25	0.00	1738.75	20.00
2010	207.25	3.80	149.65	1.00	6784.35	25.00
2011	193.10	3.80	141.65	1.50	6271.10	50.00
2012	162.75	4.30	68.60	1.50	9932.65	25.00
2013	141.95	4.25	49.50	1.20	11971.80	25.00
2014	119.90	6.00	115.25	1.20	21788.75	30.00
2015	146.85	2.50	127.35	0.60	38750.65	50.00
2016	128.85	3.35	139.25	1.20	38296.75	6.00
2017	166.00	4.36	538.75	1.20	60954.45	100.00
2018	169.70	4.90	818.10	1.50	73122.45	60.00

For each company calculate the following: (i) Annual returns for each year. (ii) Annual dividend yield and capital gain/loss percentage for each year. (iii) Arithmetic and geometric mean returns. (iv) Standard deviation of the rates of return.

PRACTICAL ASSIGNMENT

Calculate the monthly returns for the past 30 months, ignoring the dividend yield for the company that you have chosen and Nifty. What are the average returns (arithmetic and geometric) for the company and Nifty? What is the standard deviation of the returns on the company and Nifty? What is the beta of the company? Can you explain the beta in terms of the company characteristics?

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1 Note that

$$\sum_{i=1}^{n}(R_i - \bar{R})^2$$

is divided by n - 1 not n. This is done technically to correct for the loss of one degree of freedom.

2 Source: Ibbotson Associates, Inc. 2001 Yearbook

3 Since the portfolio in this case consists of only two securities, the proportion invested in security 2 is simply one minus the proportion invested in security 1.

Online Resources

http://highered.mheducation.com/sites/9353166527/student_view0/chapter8/index.html

- Additional Self-Test Problems
- Additional Solved Problems
- Chapters Excel
- Excel on Solved Problems
- Answer Key

11

CHAPTER

Techniques of Capital Budgeting

Learning Objectives

After studying this chapter you should be able to:

- ✓ Discuss DCF criteria like net present value, benefit-cost ratio, internal rate of return, and modified internal rate of return.
- ✓ Explain the properties of the NPV rule.
- ✓ Discuss the problems with IRR.
- ✓ Discuss non-DCF criteria like the payback period and accounting rate of return.

A truck manufacturer is considering investment in a new plant; an airliner is planning to buy a fleet of jet aircrafts; a commercial bank is thinking of an ambitious computerisation programme; a pharmaceutical firm is evaluating a major R&D programme. All these situations involve a capital expenditure decision. Essentially, each of them represents a scheme for investing resources which can be analysed and appraised reasonably independently. The basic characteristic of a capital expenditure (also referred to as capital investment or capital project or just project) is that it typically involves a current outlay (or current and future outlays) of funds in the expectation of a stream of benefits extending far into future.

This definition of capital expenditure is not necessarily synonymous with how capital expenditure is defined in accounting. A capital expenditure, from the accounting point of view, is an expenditure that is shown as an asset on the balance sheet. This asset, except in the case of a non-depreciable asset like land, is depreciated over its life. In accounting, the classification of an expenditure as capital expenditure or revenue expenditure is governed by certain conventions, by some provisions of law, and by the management's desire to enhance or depress reported profits. Often, outlays on research and development, major advertising campaign, and reconditioning of plant and machinery may be treated as revenue expenditure for accounting purposes, even though they are expected to generate a stream of benefits in future and, therefore, qualify for being capital expenditures as per our definition.

Capital expenditures represent the growing edge of a business. Capital expenditures have three distinctive features: (i) They have long-term consequences. (ii) They often involve substantial outlays. (iii) It may be difficult or expensive to reverse them.

Thanks to these characteristics, capital budgeting is perhaps the most important issue in corporate finance. How a firm finances its investments (the capital structure decision) and how it manages its short-term operations (the working capital decision) are definitely issues of concern, but how it allocates its capital (the capital budgeting decision) really reflects its strategy and its business. That is why the process of capital budgeting is also referred to as strategic asset allocation.

Given the crucial significance of capital budgeting decisions, it is not surprising that firms spend considerable time in planning these decisions and involve top executives from production, engineering, marketing, and so on, in evaluating capital expenditure proposals—these decisions are too important to be left to financial managers alone.

Most firms have numerous investment opportunities before them. Some are valuable while others are not. The essence of financial management is to identify which are which. The primary goal of this chapter is to introduce you to techniques of capital budgeting which are helpful in identifying valuable investment opportunities.

11.1 CAPITAL BUDGETING PROCESS

Capital budgeting is a complex process which may be divided into the following phases:

- Identification of potential investment opportunities
- Assembling of proposed investments
- Decision making
- Preparation of capital budget and appropriations
- Implementation
- Performance review

Identification of Potential Investment Opportunities The capital budgeting process begins with the identification of potential investment opportunities. Typically, the planning body (it may be an individual or a committee organised formally or informally) develops estimates of future sales which serve as the basis for setting production targets. This information, in turn, is helpful in identifying required investments in plant and equipment, research and development, distribution, and so on.

For imaginative identification of investment ideas it is helpful to (i) monitor external environment regularly to scout investment opportunities, (ii) formulate a well-defined corporate strategy based on a thorough analysis of strengths, weaknesses, opportunities, and threats, (iii) share corporate strategy and perspectives with persons who are involved in the process of capital budgeting, and (iv) motivate employees to make suggestions.

Assembling of Investment Proposals Investment proposals identified by the production department and other departments are usually submitted in a standardised capital investment proposal form. Generally, most of the proposals, before they reach the capital budgeting committee or somebody which assembles them, are routed through several persons. The purpose of routing a proposal through several persons is primarily to ensure that the proposal is viewed from different angles. It also helps in creating a climate for bringing about co-ordination of interrelated activities.

Investment proposals are usually classified into various categories for facilitating decision-making, budgeting, and control. An illustrative classification is given below.

1. Replacement investments
2. Expansion investments

3. New product investments
4. Obligatory and welfare investments

Decision Making A system of rupee gateways usually characterises capital investment decision making. Under this system, executives are vested with the power to okay investment proposals up to certain limits. For example, in one company the plant superintendent can okay investment outlays up to ₹ 200,000, the works manager up to ₹ 500,000, and the managing director up to ₹ 2,000,000. Investments requiring higher outlays need the approval of the board of directors.

Preparation of Capital Budget and Appropriations Projects involving smaller outlays and which can be decided by executives at lower levels are often covered by a blanket appropriation for expeditious action. Projects involving larger outlays are included in the capital budget after necessary approvals. Before undertaking such projects an appropriation order is usually required. The purpose of this check is mainly to ensure that the funds position of the firm is satisfactory at the time of implementation. Further, it provides an opportunity to review the project at the time of implementation.

Implementation Translating an investment proposal into a concrete project is a complex, time-consuming, and risk-fraught task. Delays in implementation, which are common, can lead to substantial cost-overruns. For expeditious implementation at a reasonable cost, the following are helpful.

Adequate formulation of projects The major reason for delay is inadequate formulation of projects. Put differently, if necessary homework in terms of preliminary studies and comprehensive and detailed formulation of the project is not done, many surprises and shocks are likely to spring on the way. Hence, the need for adequate formulation of the project cannot be over-emphasised.

Use of the principle of responsibility accounting Assigning specific responsibilities to project managers for completing the project within the defined time-frame and cost limits is helpful for expeditious execution and cost control.

Use of network techniques For project planning and control several network techniques like PERT (Programme Evaluation Review Technique) and CPM (Critical Path Method) are available. With the help of these techniques, planning and, monitoring becomes easier.

Performance Review Performance review, or post-completion audit, is a feedback device. It is a means for comparing actual performance with projected performance. It may be conducted, most appropriately, when the operations of the project have stabilised. It is useful in several ways: (i) it throws light on how realistic were the assumptions underlying the project; (ii) it provides a documented log of experience that is highly valuable for future decision-making; (iii) it helps in uncovering judgmental biases; and (iv) it induces a desired caution among project sponsors.

11.2 PROJECT CLASSIFICATION

Project analysis entails time and effort. The costs incurred in this exercise must be justified by the benefits from it. Certain projects, given their complexity and magnitude, may warrant a detailed analysis; others may call for a relatively simple analysis. Hence firms normally classify projects into different categories. Each category is then analysed somewhat differently.

While the system of classification may vary from one firm to another, the following categories are found in most classifications.

Mandatory Investments These are expenditures required to comply with statutory requirements. Examples of such investments are pollution control equipment, medical dispensary, fire-fighting equipment, creche in factory premises, and so on. These are often non-revenue producing investments. In analysing such investments the focus is mainly on finding the most cost-effective way of fulfilling a given statutory need.

Replacement Projects Firms routinely invest in equipments meant to replace obsolete and inefficient equipments, even though they may be in a serviceable condition. The objective of such investments is to reduce costs (of labour, raw material, and power), increase yield, and improve quality. Replacement projects can be evaluated in a fairly straightforward manner, though at times the analysis may be quite detailed.

Expansion Projects These investments are meant to increase capacity and/or widen the distribution network. Such investments call for an explicit forecast of growth. Since this can be risky and complex, expansion projects normally warrant more careful analysis than replacement projects. Decisions relating to such projects are taken by the top management.

Diversification Projects These investments are aimed at producing new products or services or entering into entirely new geographical areas. Often diversification projects entail substantial risks, involve large outlays, and require considerable managerial effort and attention. Given their strategic importance, such projects call for a very thorough evaluation, both quantitative and qualitative. Further, they require a significant involvement of the board of directors.

Research and Development Projects Traditionally, R&D projects absorbed a very small proportion of capital budget in most Indian companies. Things, however, are changing. Companies are now allocating more funds to R&D projects, more so in knowledge-intensive industries. R&D projects are characterised by numerous uncertainties and typically involve sequential decision making. Hence the standard discounted cash flow analysis is not applicable to them. Such projects are decided on the basis of managerial judgment. Firms which rely more on quantitative methods use decision tree analysis and option analysis to evaluate R&D projects.

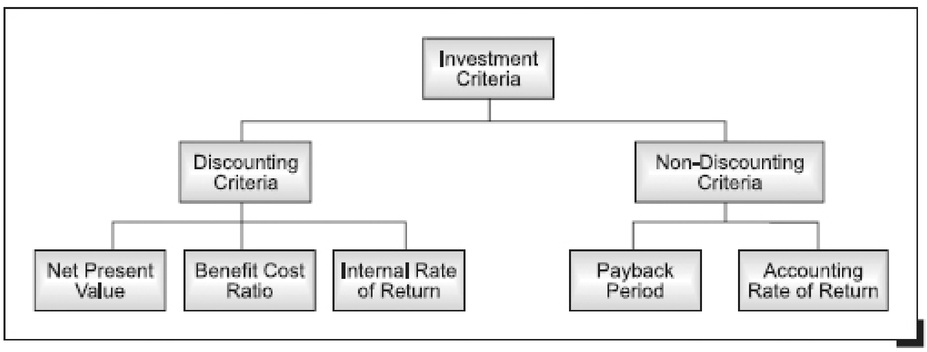
Miscellaneous Projects This is a catch-all category that includes items like interior decoration, recreational facilities, executive aircrafts, landscaped gardens, and so on.

Corporate social responsibility (CSR) projects or contributions, which are now assuming greater significance, may also be included here. There is no standard approach for evaluating these projects and decisions regarding them are based on personal preferences of top management.

11.3 INVESTMENT CRITERIA

A wide range of criteria has been suggested to judge the worthwhileness of investment projects. The important investment criteria, classified into two broad categories - discounting criteria and non-discounting criteria - are shown in Exhibit 11.1. The discounting criteria take into account the time value of money whereas the non-discounting criteria ignore the time value of money. Subsequent sections describe and evaluate these criteria in some detail.

Exhibit 11.1 **Investment Criteria**



11.4 NET PRESENT VALUE

Net present value is perhaps the most important concept of finance. It is used to evaluate investment and financing decisions that involve cash flows occurring over multiple periods. The net present value (NPV) of a project is the sum of the present values of all the cash flows - positive as well as negative - that are expected to occur over the life of the project. The general formula of NPV is:

$$\text{NPV} = \sum_{t=1}^{n} \frac{C_t}{(1+r)^t} - \text{Initial investment} \tag{11.1}$$

where C_t is the cash flow at the end of year *t, n* is the life of the project, and *r* is the discount rate.

To illustrate the calculation of net present value, consider a project which has the following cash flow stream:

Year	*Cash flow*
0	₹ (1,000,000)
1	200,000
2	200,000
3	300,000
4	300,000
5	350,000

The cost of capital[1], *r*, for the firm is 10 percent. The net present value of the proposal is:

$$\text{NPV} = \frac{200,000}{(1.10)^1} + \frac{200,000}{(1.10)^2} + \frac{300,000}{(1.10)^3} + \frac{300,000}{(1.10)^4} + \frac{350,000}{(1.10)^5} - 1,000,000 = -5,272$$

The net present value represents the net benefit over and above the compensation for time and risk. Hence the decision rule associated with the net present value criterion is: Accept the project if the net present value is positive and reject the project if the net present value is negative. (If the net present value is zero, it is a matter of indifference.)

A spreadsheet calculation of the above is as follows. Note that the formula for NPV in Excel returns only the sum of the present values of a stream of future cash flows. So, the initial outflow should be added to the

NPV formula to get the net present value in the sense that we are using that term.

	A	B	C	D	E	F	G
1	Year	0	1	2	3	4	5
2	Cash flow	−1,000,000	200,000	200,000	300,000	300,000	350,000
3	Cost of capital	10%		= NPV(B3,C2: G2) + B2		→	(5,272)

Properties of the NPV Rule

The net present value has certain properties that make it a very attractive decision criterion:

Net Present Values Are Additive The net present value of a package of projects is simply the sum of the net present values of individual projects included in the package. This property has several implications:

- The value of a firm can be expressed as the sum of the present values of projects in place as well as the net present value of prospective projects:
  Value of a firm = ∑ Present values of projects + ∑ NPV of expected future projects
  The first term on the right hand side of this equation captures the value of **assets in place** and the second term the value of **growth opportunities**.
- When a firm terminates an existing project which has a negative NPV based on its expected future cash flows, the value of the firm increases by that amount. Likewise, when a firm undertakes a new project that has a negative NPV, the value of the firm decreases by that amount.
- When a firm divests itself of an existing project, the price at which the project is divested affects the value of the firm. If the price is greater/lesser than the present value of the anticipated cash flows of the project the value of the firm will increase/decrease with the divestiture.
- When a firm makes an acquisition and pays a price in excess of the present value of the expected cash flows from the acquisition it is like taking on a negative NPV project and hence will diminish the value of the firm.
- When a firm takes on a new project with a positive NPV, its effect on the value of the firm depends on whether its NPV is in line with expectation. Hindustan Unilever Limited, for example, is expected to take on high positive NPV projects and this expectation is reflected in its value. Even if the new projects taken on by Hindustan Unilever Limited have positive NPV, the value of the firm may drop if the NPV is not in line with the high expectation of investors.

Intermediate Cash Flows Are Invested at the Cost of Capital
The NPV rule assumes that the intermediate cash flows of a project - that is,

cash flows that occur between the initiation and the termination of the project - are reinvested at a rate of return equal to the cost of capital.

NPV Calculation Permits Time Varying Discount Rates So far we assumed that the discount rate remains constant over time. This need not be always the case. The NPV can be calculated using time-varying discount rates. The general formula of NPV is as follows:

$$\text{NPV} = \sum_{t=1}^{n} \frac{C_t}{(1+r_t)^t} - \text{Initial investment} \tag{11.2}$$

where C_t is the cash flow at the end of year t, and r_t is the discount rate for the period t.

In even more general terms, NPV is expressed as follows:

$$\text{NPV} = \sum_{t=1}^{n} \frac{C_t}{\prod_{j=1}^{t}\left(1+r_j\right)} - \text{Initial investment} \tag{11.3}$$

where C_t is the cash flow at the end of year t, r_j is the one period discount rate applicable to period j, and n is the life of the project.

The discount rate may change over time for the following reasons: (a) The level of interest rates may change over time - the term structure of interest rates sheds light on expected rates in future. (b) The risk characteristics of the project may change over time, resulting in changes in the cost of capital. (c) The financing mix of the project may vary over time, causing changes in the cost of capital.

To illustrate, assume that you are evaluating a 5-year project involving software development. You believe that the technological uncertainty associated with this industry leads to higher discount rates in future.

Discount rate		14%	15%	16%	18%	20%
Investment	−12000					
Cash flow		4,000	5,000	7,000	6,000	5,000

The present value of the cash flows can be calculated as follows:

PV of C_1	= 4,000/1.14	= 3509
PV of C_2	= 5,000 / (1.14 * 1.15)	= 3814
PV of C_3	= 7,000 / (1.14 * 1.15 * 1.16)	= 4603
PV of C_4	= 6,000 / (1.14 * 1.15 * 1.16 * 1.18)	= 3344
PV of C_5	= 5,000 / (1.14 * 1.15 *	= 2322

	1.16 * 1.18 * 1.20)	
NPV of project	= 3509 + 3814 + 4603 + 3344 + 2322 − 12000	= ₹ 5592

Limitations

Despite its advantages and a direct linkage to the objective of value maximisation, the NPV rule has its opponents who point towards some limitations:

- The NPV is expressed in absolute terms rather than relative terms and hence does not factor in the scale of investment. Thus, project A may have an NPV of ₹ 5,000 while project B has an NPV of ₹ 2,500, but project A may require an investment of ₹ 50,000 whereas project B may require an investment of just ₹ 10,000. Advocates of NPV, however, argue that what matters is the surplus value, over and above the hurdle rate, irrespective of what the investment is.
- The NPV rule does not consider the life of the project. Hence, when mutually exclusive projects with different lives are being considered, the NPV rule is biased in favour of the longer term project.

11.5 BENEFIT-COST RATIO

Benefit-cost ratio, also called profitability index, may be defined in two ways:

$$\text{Benefit-cost ratio} \quad : \text{BCR} = \frac{\text{PVB}}{I} \tag{11.4}$$

$$\text{Net benefit-cost ratio: NBCR} = \frac{\text{PVB} - I}{I} = \text{BCR} - 1 \tag{11.5}$$

where PVB is the present value of benefits and *I* is the initial investment.

To illustrate the calculation of these measures, let us consider a project which is being evaluated by a firm that has a cost of capital of 12 percent.

Initial investment:		₹ 100,000
Benefits:	Year 1	25,000
	Year 2	40,000
	Year 3	40,000
	Year 4	50,000

The benefit cost ratio measures for this project are:

$$\text{BCR} = \frac{\dfrac{25{,}000}{(1.12)} + \dfrac{40{,}000}{(1.12)^2} + \dfrac{40{,}000}{(1.12)^3} + \dfrac{50{,}000}{(1.12)^4}}{100{,}000} = 1.145$$

$$\text{NBCR} = \text{BCR} - 1 = 0.145$$

The two benefit-cost ratio measures, because the difference between them is simply unity, give the same signals. The following decision rules are associated with them.

When BCR	*or NBCR*	*Rule is*
>1	>0	Accept
=1	=0	Indifferent
<1	<0	Reject

Evaluation

The proponents of benefit-cost ratio argue that since this criterion measures net present value per rupee of outlay (bang per buck), it can discriminate better between large and small investments and hence is preferable to the net present value criterion.

How valid is this argument? Henry Weingartner, who examined this criterion theoretically, finds that: (i) Under unconstrained conditions, the benefit-cost ratio criterion will accept and reject the same projects as the net present value criterion. (ii) When the capital budget is limited in the current period, the benefit-cost ratio criterion may rank projects correctly in the order of decreasingly efficient use of capital. However, its use is not recommended because it provides no means for aggregating several smaller projects into a package that can be compared with a large project. (iii) When cash outflows occur beyond the current period, the benefit-cost ratio criterion is unsuitable as a selection criterion.

11.6 INTERNAL RATE OF RETURN

The internal rate of return (IRR) of a project is the discount rate which makes its NPV equal to zero. Put differently, it is the discount rate which equates the present value of future cash flows with the initial investment. It is the value of *r* in the following equation:

$$\text{Investment} = \sum_{t=1}^{n} \frac{C_t}{(1+r)^t} \tag{11.6}$$

where C_t is the cash flow at the end of year *t, r* is the internal rate of return (IRR), and *n* is the life of the project.

In the NPV calculation we assume that the discount rate (cost of capital) is known and determine the NPV. In the IRR calculation, we set the NPV equal to zero and determine the discount rate that satisfies this condition.

To illustrate the calculation of IRR, consider the cash flows of a project being evaluated by Techtron Limited:

Year	0	1	2	3	4
Cash flow	(100,000)	30,000	30,000	40,000	45,000

The IRR is the value of *r* which satisfies the following equation:

$$100{,}000 = \frac{30{,}000}{(1+r)^1} + \frac{30{,}000}{(1+r)^2} + \frac{40{,}000}{(1+r)^3} + \frac{45{,}000}{(1+r)^4}$$

The calculation of *r* involves a process of trial and error. We try different values of *r* till we find that the right-hand side of the above equation is equal to 100,000. Let us, to begin with, try *r* = 15 percent. This makes the right-hand side equal to:

$$\frac{30{,}000}{(1.15)^1} + \frac{30{,}000}{(1.15)^2} + \frac{40{,}000}{(1.15)^3} + \frac{45{,}000}{(1.15)^4} = 100{,}801$$

This value is slightly higher than our target value, 100,000. So we increase the value of *r* from 15 percent to 16 percent. (In general, a higher *r* lowers and a smaller *r* increases the right-hand side value). The right-hand side becomes:

$$\frac{30{,}000}{(1.16)^1} + \frac{30{,}000}{(1.16)^2} + \frac{40{,}000}{(1.16)^3} + \frac{45{,}000}{(1.16)^4} = 98{,}636$$

Since this value is now less than 100,000, we conclude that the value of *r* lies between 15 percent and 16 percent. For most of the purposes this indication suffices.

If a single point estimate of *r* is needed, use the following interpolation procedure:

1. Determine the net present value of the two closest rates of return.

(NPV / 15 percent)	801
(NPV / 16 percent)	(1,364)

2. Find the sum of the absolute values of the net present values obtained in step 1:

   $$801 + 1364 = 2165$$

3. Calculate the ratio of the net present value at the smaller discount rate, identified in step 1, to the sum obtained in step 2:

   $$\frac{801}{2{,}165} = 0.37$$

4. Add the number obtained in step 3 to the smaller discount rate:

   $$15 + 0.37 = 15.37 \text{ percent}$$

The internal rate of return, calculated in this manner, is a very close approximation to the true internal rate of return.

The decision rule for IRR is as follows:

Accept : If the IRR is greater than the cost of capital
Reject : If the IRR is less than the cost of capital

The spreadsheet calculation of IRR is given below:

	A	*B*	*C*	*D*	*E*	*F*
1	Year	0	1	2	3	4
2	Cash flow	(100,000)	30,000	30,000	40,000	45,000
3		= IRR(B2:F2)		→	15.37%	

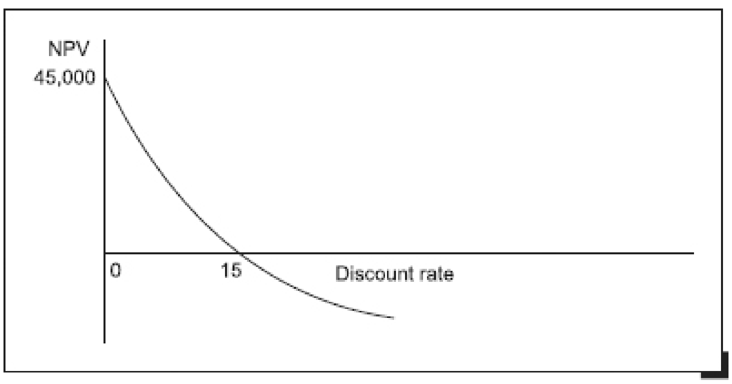
NPV and IRR

By now you may have noticed that the IRR rule is quite similar to the NPV rule. To see the link between them, let us plot the values of NPV for the project of Techtron Limited for different discount rates. The NPV profile is shown in Exhibit 11.2 where the NPV is plotted on the vertical or *y*-axis and the discount rate on the horizontal or *x*-axis. The NPV profile provides valuable insights:

- The IRR is the point at which the NPV profile crosses the *x* axis.
- The slope of the NPV profile reflects how sensitive the project is to discount rate changes.

Do the IRR and the NPV rules lead to identical decisions? Yes, provided two conditions are satisfied. First, the cash flows of the project must be **conventional**, implying that the first cash flow (initial investment) is negative and the subsequent cash flows are positive. Second, the project must be **independent**, meaning that the project can be accepted or rejected without reference to any other project.

Exhibit 11.2 **NPV Profile**



Problems with IRR

There are problems in using IRR when the cash flows of the project are not conventional or when two or more projects are being compared to determine which one is the best. In the first case it is difficult to define 'what is IRR' and in the second case IRR can be misleading. Further, IRR cannot distinguish between lending and borrowing. Finally, IRR is difficult to apply when short-term interest rates differ from long-term interest rates.

Non-conventional Cash Flows Consider a project which has the following cash flow stream associated with it:

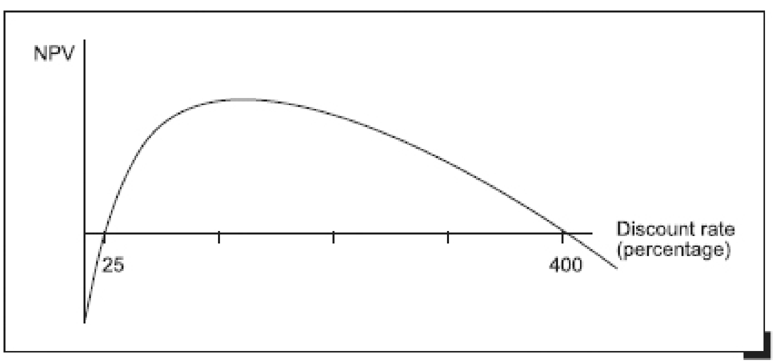
	Cash flow		
Project	C_0	C_1	C_2
M	– 160,000	+ 1,000,000	– 1,000,000

The IRR equation for this cash flow stream is:

$$-160{,}000 + \frac{1{,}000{,}000}{(1+r)} - \frac{1{,}000{,}000}{(1+r)^2} = 0$$

There are two roots of this equation viz., 1.25 and 5.00. The IRRs corresponding these roots are 25 percent and 400 percent. This is illustrated by the NPV profile shown in Exhibit 11.3.

Exhibit 11.3 **Multiple Internal Rates of Return**



In Exhibit 11.3, the NPV is zero at two discount rates, viz., 25 percent and 400 percent. Which of these is the correct IRR? We can't say. There is no unambiguously correct answer. This is the problem of multiple rate of return and in such cases the IRR rule breaks down.

As if this were not enough, there can also be cases in which no IRR exists. For example, project *P* has a positive NPV for all discount rates and hence no IRR:

	Cash flow				NPV	
Project	C_0	C_1	C_2	IRR, Percent	at 15%	at 30%
P	+ 15,000	– 45,000	+ 37,500	None	4225	2574

Several modifications of the IRR rule have been suggested for such cases. These modifications are neither adequate nor necessary, for the simple solution lies in using the NPV rule.

Mutually Exclusive Projects Often firms have to choose from two or more mutually exclusive projects. In such cases IRR can be misleading.

Consider projects *P* and *Q*

	Cash flow		IRR	NPV
Project	C_0	C_1		(assuming r = 12percent)
P	– 10,000	20,000	100%	7,857
Q	– 50,000	75,000	50%	16,964

Both the projects are good, but *Q*, with its higher NPV, contributes more to the value of the firm. Yet from an IRR point of view *P* looks better than *Q*. Hence the IRR rule seems unsuitable for ranking projects of different scale.

The IRR rule, of course, can be salvaged in such cases by considering the IRR on the incremental cash flow. Here is how we do it. Looking at *P*, the project which requires the smaller outlay, we find that it is highly attractive because its IRR is 100 percent, far above the cost of capital which is 12 percent. Now we ask: What is the rate of return on the incremental cash flow if we switch from *P* (the low-outlay project) to *Q* (the high-outlay project)? The incremental cash flow associated with such a switch is:

C_0	C_1
−40,000	55,000

The IRR of this cash flow stream is 37.5 percent, much above the cost of capital. Hence it is desirable to switch from *P* to *Q*.

Thus, unless you look at the incremental cash flow, IRR is not a reliable rule for ranking projects of different scales.

IRR is also unreliable for ranking projects which have different patterns of cash flow over time. Consider two projects, *X* and *Y*, being evaluated by a firm that has a cost of capital of 10 percent.

Project	C_0	C_1	C_2	C_3	C_4	IRR	NPV at 10%
X	–110,000	+31,000	+40,000	+50,000	+70,000	22%	36,613
Y	– 110,000	+71,000	+40,000	+40,000	+20,000	25%	31,316

Both the projects look good but *X*, with its higher NPV, contributes more to the value of the firm. Yet from an IRR point of view *Y* looks more attractive. Hence the IRR rule can be misleading when a choice has to be made between mutually exclusive projects which have different patterns of cash flow over time.

Of course, in this case too the IRR rule can be salvaged by considering the IRR on the incremental cash flow.

As the previous examples suggest, when mutually exclusive projects are evaluated it is much simpler to use the NPV rule rather than the IRR rule with such involved additional computations.

Lending versus Borrowing The IRR rule cannot distinguish between lending and borrowing and hence a high IRR need not necessarily be a desirable thing.

To illustrate this point, let us consider two projects *A* and *B*:

	Cash flow		*IRR*	*NPV at 10% discount rate*
Project	C_0	C_1		
A	– 4000	+ 6000	50%	1455
B	+ 4000	– 7000	75%	– 2364

The IRR for project *A* is 50 percent, whereas the IRR for project *B* is 75 percent. This means that *B* is a more attractive project, when *B* is actually a highly undesirable project. Why? *A* involves investing ₹ 4000 at a rate of return of 50 percent, whereas *B* involves borrowing ₹ 4000 at a rate of return of 75 percent. Yet if we go by the IRR figures, *B* appears more attractive than *A*.

Differences Between Short-term and Long-term Interest Rates Recall our general formula for calculating NPV:

$$\text{NPV} = \Sigma \frac{C_t}{\prod_{j=1}^{t}(1+r_j)} - \text{Initial investment}$$

Thus, the cash flow for year 1, C_1, is discounted at the opportunity cost of capital for year 1, r_1; the cash flow for year 2, C_2, is discounted at the opportunity cost of capital for year 2, r_2 and again at r_1; so on and so forth.

The IRR rule says that a project should be accepted if its IRR is greater than the opportunity cost of capital. But what should we do when there are several opportunity costs? Should we compare IRR with r_1 or r_2 or r_3 … or r_n? We have to, in effect, compute a complex weighted average of various rates to get a number comparable to IRR. Given the difficulty in doing so, it makes sense to ignore IRR, when short-term interest rates differ from long-term interest rates, and simply calculate NPV.

Redeeming Qualities

Despite its deficiencies, IRR is immensely popular in practice, even more than NPV. It perhaps fills a need that NPV does not. Managers as well as financial analysts are wonted to think in terms of rates of return rather than absolute rupee values. Although IRR can be misleading, the result can be readily interpreted by all parties. As Samuel Weaver says: "The resulting IRR can be mentally compared to expected inflation, the current borrowing rates, the cost of capital, an equity's portfolio return, and so on." No wonder surveys suggest that the IRR is the most popular investment evaluation technique.

Further, in certain situations, the IRR offers a practical advantage over NPV. You can't estimate the NPV unless you know the discount rate, but you can still calculate the IRR. Suppose you don't know the discount rate but you find that the project has an IRR of 35 percent. You would perhaps accept the project because it is unlikely that the discount rate would be that high. The pros and cons of IRR are summarised below:

Pros	*Cons*
■ Closely related to NPV	■ May lead to multiple rates of return
■ Easy to understand and interpret	■ May result in incorrect decisions in comparing mutually exclusive projects

11.8 PAYBACK PERIOD

The payback period is the length of time required to recover the initial cash outlay on the project. For example, if a project involves a cash outlay of ₹ 600,000 and generates cash inflows of ₹ 100,000, ₹ 150,000, ₹ 150,000, and ₹ 200,000, in the first, second, third, and fourth years, respectively, its payback period is 4 years because the sum of cash inflows during 4 years is equal to the initial outlay. When the annual cash inflow is a constant sum, the payback period is simply the initial outlay divided by the annual cash inflow. For example, a project which has an initial cash outlay of ₹ 1,000,000 and a constant annual cash inflow of ₹ 300,000 has a payback period of: ₹ 1,000,000/300,000 = $3^1/_3$ years.

According to the payback criterion, the shorter the payback period, the more desirable the project. Firms using this criterion generally specify the maximum acceptable payback period. If this is *n* years, projects with a payback period of *n* years or less are deemed worthwhile and projects with a payback period exceeding *n* years are considered unworthy.

Evaluation

A widely used investment criterion, the payback period seems to offer the following advantages:

- It is simple, both in concept and application. It does not use involved concepts and tedious calculations and has few hidden assumptions.
- It is a rough and ready method for dealing with risk. It favours projects which generate substantial cash inflows in earlier years and discriminates against projects which bring substantial cash inflows in later years but not in earlier years. Now, if risk tends to increase with futurity - in general, this may be true - the payback criterion may be helpful in weeding out risky projects.
- Since it emphasises earlier cash inflows, it may be a sensible criterion when the firm is pressed with problems of liquidity.

The limitations of the payback criterion, however, are very serious:

- It fails to consider the time value of money. Cash inflows, in the payback calculation, are simply added without suitable discounting. This violates the most basic principle of financial analysis which stipulates that cash flows occurring at different points of time can be added or subtracted only after suitable compounding/discounting.
- It ignores cash flows beyond the payback period. This leads to discrimination against projects which generate substantial cash inflows in later years. To illustrate, consider the cash flows of two projects, *A* and *B*:

Year	*Cash flow of A*	*Cash flow of B*
0	₹ (100,000)	₹ (100,000)
1	50,000	20,000
2	30,000	20,000
3	20,000	20,000
4	10,000	40,000
5	10,000	50,000
6	–	60,000

The payback criterion prefers *A*, which has a payback period of 3 years in comparison to *B* which has a payback period of 4 years, even though B has very substantial cash inflows in years 5 and 6.

- It is a measure of project's capital recovery, not profitability.

- Though it measures a project's liquidity, it does not indicate the liquidity position of the firm as a whole, which is more important. Weingartner writes: "The usually designated speculative and/or precautionary motive of firms to hold liquid or near liquid funds in order to seize upon unexpected opportunities is a different motive from that which requires each new investment separately to recover its original cost within a short period."[2]

A major shortcoming of the conventional payback period is that it does not take into account the time value of money. To overcome this limitation, the **discounted payback period** has been suggested. In this modified method, cash flows are first converted into their present values (by applying suitable discounting factors) and then added to ascertain the period of time required to recover the initial outlay on the project. Exhibit 11.4 illustrates the calculation of discounted payback period. Looking at the last column in this exhibit, we find that the discounted payback period is between 3 and 4 years.

Reasons for Popularity of Payback Period

Despite its serious shortcomings the payback period is widely used in appraising investments. Why? It appears that the payback measure serves as a proxy for certain types of information which are useful in investment decision-making.

1. The payback period may be regarded roughly as the reciprocal for the internal rate of return when the annual cash inflow is constant and the life of the project fairly long.
2. The payback period is somewhat akin to the break-even point. A rule of thumb, it serves as a useful shortcut in the process of information generation and evaluation.
3. The payback period conveys information about the rate at which the uncertainty associated with a project is resolved. The shorter the payback period, the faster the uncertainty associated with the project is resolved. The longer the payback period, the slower the uncertainty associated with the project is resolved. Decision-makers, it may be noted, prefer an early resolution of uncertainty. Why? An early resolution of uncertainty enables the decision-maker to take prompt corrective action, adjust his consumption patterns, and modify/change other investment decisions.

Exhibit 11.4 **Calculation of Discounted Payback Period**

Year	*Cash Flow*	*Discounting Factor @10%*	*Present Value*	*Cumulative Net Cash Flow after Discounting*
0	–10,000	1.000	–10,000	–10,000
1	3,000	0.909	2,727	– 7,273
2	3,000	0.826	2,478	– 4,795
3	4,000	0.751	3,004	– 1,791
4	4,000	0.683	2,732	941
5	5,000	0.621	3,105	
6	2,000	0.565	1,130	
7	3,000	0.513	1,539	

11.9 ▌ ACCOUNTING RATE OF RETURN

The accounting rate of return, also called the average rate of return, is defined as:

$$\frac{\text{Profit after tax}}{\text{Book value of the investment}}$$

The numerator of this ratio may be measured as the average annual post-tax profit over the life of the investment and the denominator as the average book value of investment in the project. To illustrate the calculation consider a project:

Year	*Book value of investment*	*Profit after tax*
1	₹ 90,000	₹ 20,000
2	80,000	22,000
3	70,000	24,000
4	60,000	26,000
5	50,000	28,000

The accounting rate of return is:

$$\frac{1/5(20,000+22,000+24,000+26,000+28,000)}{1/5(90,000+80,000+70,000+60,000+50,000)} = 34 \text{ percent}$$

Obviously, the higher the accounting rate of return, the better the project. In general, projects which have an accounting rate of return equal to or greater than a pre-specified cut-off rate of return - which is usually between 20 percent and 30 percent - are accepted; others are rejected.

Evaluation

Traditionally a popular investment appraisal criterion, the accounting rate of return has the following virtues:

- It is simple to calculate.
- It is based on accounting information which is readily available and familiar to businessmen.
- While it considers benefits over the entire life of the project, it can be used even with limited data. As one executive put it: "The discounted cash flow methods call for estimates of costs and revenues over the whole project life. This is difficult. Very often we can't estimate the life. We have been using machines longer than their life by good maintenance. Changes in costs and revenues cannot be predicted. Due to these difficulties we use the accounting rate of return. Here we take our best estimates for 2-3 years and calculate the average return. Once the project is established, the balance between cost and revenue can be maintained in normal circumstances".

Its shortcomings, however, seem to be considerable:

- It is based upon accounting profit, not cash flow.
- It does not take into account the time value of money. To illustrate this point, consider two investment proposals *A* and *B*, each requiring an outlay of ₹ 100,000. Both the pro-posals have an expected life of 4 years after which their salvage value would be nil.

	A				*B*			
Year	*Book value*	*Depreciation*	*Profit after tax*	*Cash flow*	*Book value*	*Depreciation*	*Profit after tax*	*Cash flow*
0	100,000	0	0	(100,000)	100,000	0	0	(100,000)
1	75,000	25,000	40,000	65,000	75,000	25,000	10,000	35,000
2	50,000	25,000	30,000	55,000	50,000	25,000	20,000	45,000
3	25,000	25,000	20,000	45,000	25,000	25,000	30,000	55,000
4	0	25,000	10,000	35,000	0	25,000	40,000	65,000

Both the proposals, with an accounting rate of return equal to 40 percent, look alike from the accounting rate of return point of view. However, project *A*, because it provides benefits earlier, is much more desirable. While the payback period criterion gives no weightage to more distant benefits, the accounting rate of return criterion seems to give them too much weightage.

- The accounting rate of return measure is internally inconsistent. While the numerator of this measure represents profit belonging to equity and preference stockholders, its denominator represents the total investment in the project which is supported by equity, preference, and debt.
- The accounting rate of return does not provide any guidance on what the target rate of return should be.

SUMMARY

- Investment criteria fall into two categories: discounting criteria and non-discounting criteria. Net present value **(NPV)**, benefit cost ratio **(BCR)**, and internal rate of return **(IRR)**, are the most popular discounting criteria. **Payback period** and **accounting rate of return** are the major non-discounting criteria.
- The **NPV** of a project is the sum of the present values of all the cash flows of the project. A project is worthwhile if its NPV>0; otherwise not.
- The **BCR** of a project is the present value of its benefits (cash inflows) divided by the present value of its costs (cash outflows). A project is worthwhile if its BCR>1; otherwise not.
- The **IRR** of a project is the discount rate which makes its NPV equal to zero. A project is worthwhile if its IRR exceeds the cost of capital; otherwise not.
- The **payback period** is the length of time required to recover the initial outlay on the project.
- The **accounting rate of return** is the average profit after tax divided by the average book value of the investment over the life of the project.
- NPV and IRR are the most important criteria in practice with accounting rate of return and payback period being used as supplementary criteria.

QUESTIONS

1. NPVWhy are capital expenditures deemed very important?
2. Discuss the phases of capital budgeting.
3. Describe the commonly found categories in project classification.
4. What is NPV?
5. What are the implications of the additivity property of NPV?
6. Discuss the general formula of NPV when discount rates vary over time.
7. What are the limitations of NPV?
8. What are the two ways of defining the benefit-cost ratio?
9. Evaluate the benefit-cost ratio as an investment criterion.
10. What is IRR and how is it calculated?
11. Discuss the problems associated with IRR.
12. What are the redeeming qualities of IRR?
13. How is modified IRR calculated?
14. Why is MIRR superior to the regular IRR?
15. What is payback period?
16. Evaluate payback period as an investment criterion.
17. Why is payback period so popular, despite its shortcomings.
18. What is discounted payback period?
19. How is accounting rate of return calculated?

20. What are the pros and cons of accounting rate of return?
21. Discuss how investment appraisal is done in practice.

SOLVED PROBLEMS

11.1 The expected cash flows of a project are as follows:

Year	*Cash flow*
0	− 100,000
1	20,000
2	30,000
3	40,000
4	50,000
5	30,000

The cost of capital is 12 percent. Calculate the following: (a) net present value, (b) benefit-cost ratio, (c) internal rate of return, (d) modified internal rate of return, (e) payback period, and (f) discounted payback period.

Solution

a. The net present value is:

$$-100{,}000 + 20{,}000\,/\,(1.12) + 30{,}000\,/\,(1.12)^2 + 40{,}000\,/\,(1.12)^3 + 50{,}000\,/\,(1.12)^4$$
$$+ 30{,}000\,/\,(1.12)^5$$
$$= -100{,}000 + 17{,}860 + 23{,}910 + 28{,}480 + 31{,}800 + 17{,}010 = 19{,}060$$

b. The benefit-cost ratio is:

$$119{,}060\,/\,100{,}000 = 1.19$$

c. Try a discount rate of 18 percent. The NPV at 18 percent discount rate is 1750. Try a discount rate of 19 percent. The NPV at 19 percent discount rate is:

− ₹ 780.

Hence the IRR is:

$$18\% + \frac{1750}{2530} \times 1\%$$
$$= 18.69\%$$

d. The future value of benefits when compounded at 12 percent is:

$$20{,}000\,(1.12)^4 + 30{,}000\,(1.12)^3 + 40{,}000\,(1.12)^2 + 50{,}000\,(1.12) + 30{,}000$$
$$= 209{,}790$$
$$100{,}000\,(1 + r^*)^5 = 209{,}790$$
$$r^* = 15.97\%$$

Hence the MIRR is 15.97%

e. The payback period is slightly more than 3 years.

f. The discounted payback period is slightly less than 4 years.

PROBLEMS

11.1 NPV Sulabh International is evaluating a project whose expected cash flows are as follows:

Year	*Cash flow (₹)*
0	−1000,000
1	100,000
2	200,000
3	300,000
4	600,000
5	300,000

(a) What is the NPV of the project, if the discount rate is 14 percent for the entire period?

(a) What is the NPV of the project if the discount rate is 12 percent for year 1 and rises every year by 1 percent?

11.2 IRR What is the internal rate of return of an investment which involves a current outlay of ₹ 300,000 and results in an annual cash inflow of ₹ 60,000 for 7 years?

11.3 IRR What is the internal rate of return of the following cash flow stream?

Year	*Cash flow (₹)*
0	(3,000)
1	9,000
2	(3,000)

11.4 Minimum Cash Flow If an equipment costs ₹ 500,000 and lasts 8 years, what should be the minimum annual cash inflow before it is worthwhile to purchase the equipment? Assume that the cost of capital is 10 percent.

11.5 Investment Value How much can be paid for a machine which brings in an annual cash inflow of ₹ 25,000 for 10 years? Assume that the discount rate is 12 percent.

11.6 NPV The cash flows associated with three projects, *P*, *Q*, and *R*, are given below:

	Net cash flow (₹)		
Year	*P*	*Q*	*R*
0	(2,000)	(2,000)	(2,000)
1	1,400	500	500
2	600	1,100	500
3	400	900	1,600

Calculate the net present value of each project at discount rates of 0 percent, 5 percent, 10 percent, 15 percent, 25 percent, and 30 percent. Plot the results on a graph paper.

11.7 NPV, IRR, and MIRR Phoenix Company is considering two mutually exclusive investments, Project *P* and Project *Q*. The expected cash flows of these projects are as follows:

Year	*Project P* (₹)	*Project Q* (₹)
0	(1,000)	(1,600)
1	(1,200)	200
2	(600)	400
3	(250)	600
4	2,000	800
5	4,000	100

(a) Construct the NPV profiles for Projects *P* and *Q*.
(b) What is the IRR of each project?
(c) Which project would you choose if the cost of capital is 10 percent? 20 percent?
(d) What is each project's MIRR if the cost of capital is 12 percent?

11.8 NPV and IRR Your company is considering two mutually exclusive projects, *A* and *B*. Project *A* involves an outlay of ₹ 100 million which will generate an expected cash inflow of ₹ 25 million per year for 6 years. Project *B* calls for an outlay of ₹ 50 million which will produce an expected cash inflow of ₹ 13 million per year for 6 years. The company's cost of capital is 12 percent.
(a) Calculate the NPV and IRR of each project.
(b) What is the NPV and IRR of the differential project (project *A* over *B*)?

11.9 Investment Criteria Your company is considering two projects, *M* and *N*, each of which requires an initial outlay of ₹ 50 million. The expected cash inflows from these projects are:

Year	*Project M*	*Project N*
1	11	38
2	19	22
3	32	18
4	37	10

(a) What is the payback period for *M* and *N*?

(b) What is the discounted payback period for *M* and *N* if the cost of capital is 12 percent?
(c) If the two projects are independent and the cost of capital is 12 percent, which project(s) should the firm invest in?
(d) If the two projects are mutually exclusive and the cost of capital is 10 percent, which project should the firm invest in?
(e) If the two projects are mutually exclusive and the cost of capital is 15 percent, which project should the firm invest in?
(f) If the cost of capital is 14 percent, what is the modified IRR of each project?

11.10 IRR and MIRR The estimated net cash flows of a project with an investment of ₹ 1000 million are as follows:

(Amounts in ₹ million)

Year	1	2	3	4	5
Net cash flow	120	400	480	380	300

(a) What is the IRR of the project?
(b) What is the MIRR if the cost of capital is 10 percent?

11.11 IRR ICC Projects is considering a project in which an investment of ₹ 50 lakhs will be needed at the commencement and ₹ 200 lakhs after one year. The returns will start at the rate of ₹ 80 lakhs per year commencing from the end of the second year for 5 years. What is the IRR of the project?

11.12 NPV and BCR As part of its programme of supporting start-up ventures, Angel bank has provided the full initial investment cost of ₹ 200 lakhs to Suresh at a concessional interest rate of just 6 percent per annum. The estimated net cash flows of the project are as follows:

(₹ in lakhs)

Year	1	2	3	4	5
Net cash flow	40	60	100	70	60

Calculate the net present value and the net benefit cost ratio of the project.

11.13 Multiple IRR A project costs ₹ 1,000,000 today. It will have an inflow of ₹ 2,100,000 a year from now and an outflow of ₹ 1,04,000 two years from now. What are the IRRs of this project?

11.14 NPV and IRR US International Limited is evaluating a project in Estonia. The project's cash flows are estimated as follows:

Year	0	1	2	3	4
Cash Flow	–$ 800,000	300,000	3,60,000	4,30,000	3,00,000

The Estonian government requires that cash flows generated by a foreign company are to the kept in Estonia and investment with the government for two years at a rate of 5 percent per year before repatriation. US International requires a return of 15 percent on this project. Calculate the NPV and IRR of the project.

MINICASE

Aman Limited is a leading manufacturer of automotive components. It supplies to the original equipment manufacturers as well as the replacement market. Its projects typically have a short life as it introduces new models periodically.

You have recently joined Aman Limited as a financial analyst reporting to Ravi Sharma, the CFO of the company. He has provided you the following information about three projects, *A*, *B*, and *C* that are being considered by the Executive Committee of Aman Limited:

- Project *A* is an extension of an existing line. Its cash flow will decrease over time.
- Project *B* involves a new product. Building its market will take some time and hence its cash flow will increase over time.
- Project *C* is concerned with sponsoring a pavilion at a Trade Fair. It will entail a cost initially which will be followed by a huge benefit for one year. However, in the year following that some cost will be incurred to raze the pavilion.

The expected net cash flows of the three projects are as follows.

Year	*Project A* (₹)	*Project B* (₹)	*Project C* (₹)
0	(15,000)	(15,000)	(15,000)
1	11,000	3,500	42,000
2	7,000	8,000	(4,000)
3	4,800	13,000	–

Ravi Sharma believes that all the three projects have risk characteristics similar to the average risk of the firm and hence the firm's cost of capital, viz. 12 percent, will apply to them.

You have been asked to prepare a report for the executive committee, covering the following:

(a) What is payback period and discounted payback period? Find the payback period and the discounted payback period of Projects *A* and *B*.

(b) What is net present value (NPV)? What are the properties of NPV? Calculate the NPV of projects *A*, *B*, and *C*.

(c) What is internal rate of return (IRR)? What are the problems with IRR? Calculate the IRR for Projects *A*, *B*, and *C*.

(d) What is modified internal rate of return (MIRR)? What are the pros and cons of MIRR vis-à-vis IRR and NPV? Calculate the MIRR for Projects *A*, *B*, and *C* assuming that the intermediate cash flows can be reinvested at 12 percent rate of return.

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[1] The cost of capital must reflect the risk of the project. While the details of how the cost of capital is calculated are discussed in Chapter 14, for the present we assume that the cost of capital figure is given and includes an appropriate premium for risk.

2 H.M. Weingartner, "Some Views on the Payback Period and Capital Budgeting Decisions," *Management Science*, Vol.15 (August 1969).

Online Resources

http://highered.mheducation.com/sites/9353166527/student_view0/chapter11/index.html

- Additional Self-Test Problems
- Additional Solved Problems
- Chapters Excel
- Excel on Solved Problems
- Mini Cases
- Answer Key

CHAPTER 8			
Problem no.			**Answer**
8.1	a.		₹ 10.80
8.2		E(R)	σ
	a	1,150	143.18
	b	1,200	264.58
	c	1,175	84.41
	d	1,165	57.66
8.3	a.	RA =	0.96
	b	0.52 +0.96RM	
8.4			₹ 22.74
8.5			1.75
8.6			13%
8.7			₹ 34.45
8.8			0.71
8.9		A	B
	Average return	9.83%	8.33%
	Standard deviation	12.88%	14.51%
	Variance	165.77%	210.67%
8.10	A.M		11.60%
	G.M.		10%
8.11			14.20%
8.12			7%
Minicase-1		A.M	G.M
	Tata Motors	–2.83%	3.21%
	TCS	1.73%	1.48%
	HUL	2.50%	2.38%
	Nifty	0.97%	0.91%
		Std,dev.	Beta
	Tata Motors	8.61%	1.47
	TCS	7.41%	1.15
	HUL	5.09%	0.91
	Nifty	3.66%	
Minicase-2	A.M	G.M	St.dev
	NTPC 2.27%	1.12%	16.67%
	Escorts 68.4%	26.45%	137.90%
	MRF 54.71%	34.20%	93.67%

CHAPTER 9			
Problem no.			**Answer**
9.1	a.	E(R)	σ
	Asset 1	12.90%	9.17%
	Asset 2	13%	2.24%
	b		16.7
	c		0.81
9.2	a.		7.83%
	b		8.50%
	c		8.67%
	d		8.88%
9.3			10.60%
9.4			0.21
9.5			25.62%
9.6			11.75%
Minicase	a.	Exp. Return	Std.dev.
		A 19	19.08
		B 4	15.62
		C 14	10.44
	Mkt portfolio	15	13.89
	b.	COV(A, B)	(–) 296
		COV(A, C)	199
	c.	ρ_{AB}	(–) 1
		ρ_{AC}	1
	d.	Exp. Return	Std.dev.
	Equal weights	11.50%	2%
	Other weights	12	3.61%
	e.(i)	Required Return	6%+β x9%
	(ii)	Stock A	2.20%
		Stock B	4.30%
		Stock C	(–) 0.1%
	f.	Beta	1.53

CHAPTER 10

Problem no.			Answer
10.1			₹ 18.37
10.2			102%
10.3			₹ 31.90
10.4			₹ 11.86
10.5			₹ 3.27
Minicase	b	In-the – money	Out of-the-money
	Calls with st.price.	280, 300, 320	340, 360
	Puts with st.price	340, 360	280, 300, 320
	d. Max. profit		₹ 11
	Max.loss		₹ 9
	B.E. price		₹ 349
	g		₹ 24.42

CHAPTER 11

Problem no.			Answer
11.1	a.		-44,837
	b		-27,264
11.2			9.20%
11.3		Rule breaks down	
11.4			₹ 93,721
11.5			₹ 141,256
11.7	b. P		20.13%
	Q		9.34%
	c.	in both cases choose P	
	d. P		18%
	Q		10.41%
11.8	a.	NPV	IRR
	Project	₹ 2.79 mn.	12.98%
	Project B	₹ 3.45 mn	14.40%
b	Differ.Project	₹-0.67 mn	11.53%
11.9	Project	M	N
	a	2.63 years	1.55 years
	b	3.1 years	1.92 years
	c	Both projects acceptable	
	d		Project M
	e		Project N
	f	23.01%	23.26%
11.10	a		18.32%
	b		14.96%
11.11			16.61%
11.12			0.38
11.13	IRR rule breaks down		
11.14	NPV	$	23,094
	IRR		15.75%
Minicase		Payback period	Discounted Payback period
	a. A	1.57yrs	1.93 yrs
	B	2.27 yrs	2.59 yrs
		NPV	IRR
	b & c. A	3,820	28.84%
	B	3,758	23.43%
	C	19,318	rule breaks down
			MIRR
	d. A		20.80%
	B		20.70%
	C		60.80%

CHAPTER 12

Problem no.			Answer
12.1	a. (₹ in mn.)	Year	NCF
		0	-200.00
		1	116.25
		2	113.44
		3	111.33
		4	109.75
		5	108.56
		6	107.67
		7	205.00
	b		55.17%
12.2	a. (₹ in mn.)	Year	NCF
		0	-140.00