

CHAPTER 15: Database Design Theory: Normalization Algorithms**Answers to Selected Exercises**

15.17 - Show that the relation schemas produced by Algorithm 15.4 are in 3NF.

Answer:

We give a proof by contradiction. Suppose that one of the relations R_i resulting from Algorithm 15.1 is not in 3NF. Then a FD $Y \rightarrow A$ holds in R_i where: (a) Y is not a superkey of R_i , and (b) A is not a prime attribute. But according to step 2 of the algorithm, R_i will contain a set of attributes $X \cup A_1 \cup A_2 \cup \dots \cup A_n$, where $X \rightarrow A_i$ for $i=1, 2, \dots, n$, implying that X is a key of R_i and the A_i are the only non-prime attributes of R_i . Hence, if an FD $Y \rightarrow A$ holds in R_i where A is non-prime and Y is not a superkey of R_i , Y must be a proper subset of X (otherwise Y would contain X and hence be a superkey). If both $Y \rightarrow A$ and $X \rightarrow A$ hold and Y is a proper subset of X , this contradicts that $X \rightarrow A$ is a FD in a minimal set of FDs that is input to the algorithm, since removing an attribute from X leaves a valid FD, thus violating one of the minimality conditions. This produces a contradiction of our assumptions. Hence, R_i must be in 3NF.

15.18 - Show that, if the matrix S resulting from Algorithm 15.3 does not have a row that is all "a" symbols, then projecting S on the decomposition and joining it back will always produce at least one spurious tuple.

Answer:

The matrix S initially has one row for each relation R_i in the decomposition, with "a" symbols under the columns for the attributes in R_i . Since we never change an "a" symbol into a "b" symbol during the application of the algorithm, then projecting S on each R_i at the end of applying the algorithm will produce one row consisting of all "a" symbols in each $S(R_i)$. Joining these back together again will produce at least one row of all "a" symbols (resulting from joining the all "a" rows in each projection $S(R_i)$). Hence, if after applying the algorithm, S does not have a row that is all "a", projecting S over the R_i 's and joining will result in at least one all "a" row, which will be a spurious tuple (since it did not exist in S but will exist after projecting and joining over the R_i 's).

15.19 - Show that the relation schemas produced by Algorithm 15.5 are in BCNF.

Answer:

This is trivial, since the algorithm loop will continue to be applied until all relation schemas are in BCNF.

15.20 – No Solution Provided

15.21 - Specify a template dependency for join dependencies.

Answer:

The following template specifies a join dependency $JD(X,Y,Z)$.

	$R = \{ A, B, C \}$			$X = \{ A, B \}$
	a	b	c_1	$Y = \{ B, C \}$
hypothesis	a	b_1	c	$Z = \{ A, C \}$
	a_1	b	c	
conclusion	a	b	c	

15.22 - Specify all the inclusion dependencies for the relational schema of Figure 3.5.

Answer:

The inclusion dependencies will correspond to the foreign keys shown in Figure 3.7.

15.23 - Prove that a functional dependency satisfies the formal definition of multi-valued dependency.

Answer:

Suppose that a functional dependency $X \rightarrow Y$ exists in a relation $R = \{X, Y, Z\}$, and suppose there are two tuples with the same value of X . Because of the functional dependency, they must also have the same value of Y . Suppose the tuples are $t_1 = \langle x, y, z_1 \rangle$ and $t_2 = \langle x, y, z_2 \rangle$. Then, according to the definition of multivalued dependency, we must have two tuples t_3 and t_4 (not necessarily distinct from t_1 and t_2) satisfying: $t_3[X] = t_4[X] = t_1[X] = t_2[X]$, $t_3[Y] = t_2[Y]$, $t_4[Y] = t_1[Y]$, $t_3[Z] = t_1[Z]$, and $t_4[Z] = t_2[Z]$. Two tuples satisfying this are t_2 (satisfies conditions for t_4) and t_1 (satisfies conditions for t_3). Hence, whenever the condition for functional dependency holds, so does the condition for multivalued dependency.

15.24 - 15.31: No solutions provided.

15.32 - Consider the relation REFRIG(MODEL#, YEAR, PRICE, MANUF_PLANT, COLOR), which is abbreviated as REFRIG(M, Y, P, MP, C), and the following set of F of functional dependencies: $F = \{M \rightarrow MP, \{M, Y\} \rightarrow P, MP \rightarrow C\}$

- Evaluate each of the following as a candidate key for REFRIG, giving reasons why it can or cannot be a key: $\{M\}$, $\{M, Y\}$, $\{M, C\}$
- Based on the above key determination, state whether the relation REFRIG is in 3NF and in BCNF, giving proper reasons.

- (c) Consider the decomposition of REFRIG into $D=\{R1(M,Y,P), R2(M,MP,C)\}$. Is this decomposition lossless? Show why. (You may consult the test under Property LJ1 in Section 15.2.4)

Answers:

(a)

- $\{M\}$ IS NOT a candidate key since it does not functionally determine attributes Y or P.
- $\{M, Y\}$ IS a candidate key since it functionally determines the remaining attributes P, MP, and C.

i.e.

$\{M, Y\} \rightarrow P$, But $M \not\rightarrow MP$

By augmentation $\{M, Y\} \rightarrow MP$

Since $MP \rightarrow C$, by transitivity $M \rightarrow MP$, $MP \rightarrow C$, gives $M \rightarrow C$

By augmentation $\{M, Y\} \rightarrow C$

Thus $\{M, Y\} \rightarrow P, MP, C$ and $\{M, Y\}$ can be a candidate key

- $\{M, C\}$ IS NOT a candidate key since it does not functionally determine attributes Y or P.

(b)

REFRIG is not in 2NF, due to the partial dependency $\{M, Y\} \rightarrow MP$ (since $\{M\} \rightarrow MP$ holds). Therefore REFRIG is neither in 3NF nor in BCNF.

Alternatively: BCNF can be directly tested by using all of the given dependencies and finding out if the left hand side of each is a superkey (or if the right hand side is a prime attribute). In the two fields in REFRIG: $M \rightarrow MP$ and $MP \rightarrow C$. Since neither M nor MP is a superkey, we can conclude that REFRIG is is neither in 3NF nor in BCNF.

(c)

$$R = \{M, Y, P, MP, C\}$$

$$R1 = \{M, Y, P\}$$

$$R2 = \{M, MP, C\}$$

$$F = \{M \rightarrow MP, \{M, Y\} \rightarrow P, MP \rightarrow C\}$$

$$F^+ = \{ \{M\}^+ \rightarrow \{M, MP, C\}, \\ \{M, Y\}^+ \rightarrow \{M, Y, P, MP, C\}, \\ \{MP\}^+ \rightarrow \{MP, C\} \}$$

$$R1 \cap R2 = M$$

$$R2 - R1 = \{MP, C\}$$

$D(R1, R2)$ has the lossless join property since

Property: *LJ1*: $FD((R1 \cap R2) \rightarrow (R2 - R1))$ is in F^+
is satisfied (due to $M \rightarrow \{MP, C\}$).