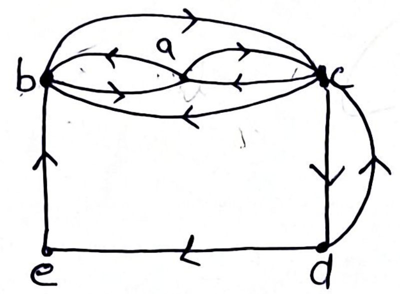
Communication network

a R b means a can communicate with b. (If an edge is directed from a to b It means that a can communicate with b)
Consider the problem of determining the importance of each individual in a given network.



We assume that if an individual can send message directly to n individuals, Then he will message to any one with probability $1/n$.

The following matrix shows all such probabilities.

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array}\begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 0 & 1/2 & 1/2 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 \\ 1/3 & 1/3 & 0 & 1/3 & 0 \\ 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

No individual is to send a message to himself

~~To find out what~~

Since all elements of the matrix are non-negative and the sum of elements of every row is unity, the matrix is a transition matrix. ~~and~~

To find out what fraction of messages pass through them in the long run, we find the fixed point v (or fixed probability vector) (eigenvalue 1)

Such that $vP = v$

Let $v = [a \quad b \quad c \quad d \quad e]$

Then $$[a \; b \; c \; d \; e]\begin{bmatrix} 0 & 1/2 & 1/2 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 \\ 1/3 & 1/3 & 0 & 1/3 & 0 \\ 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} = [a \; b \; c \; d \; e]$$

$\Rightarrow \frac{1}{2}b + \frac{c}{3} = a$

$\frac{1}{2}a + \frac{1}{3}c + e = b$

$\frac{1}{2}a + \frac{1}{2}b + \frac{1}{2}d = c \Rightarrow a + b + d = 2c$

$\frac{1}{3}c = d$

$\frac{1}{2}d = e$

$\Rightarrow d = 2e$

$c = 3d$

$a + b = 2(3d) - d = 5d$

$= 10e$

$d = \frac{2}{19}$ $\quad c = \frac{6}{19}$

Also W.K.T, $a + b + c + d + e = 1$

$10e + 6e + 2e + e = 1$

$19e = 1$

$e = 1/19$

$\frac{a}{2} + \frac{2}{19} + \frac{1}{19} = b$

$\frac{a}{2} - b = -\frac{3}{19}$

$a - 2b = -\frac{6}{19}$

$a - \frac{1}{2}b = \frac{2}{19}$

$-\frac{1}{2}b + 2b = \frac{8}{19}$

$\frac{3b}{2} = \frac{8}{19}$

$b = \frac{16}{57}$

$a = \frac{14}{57}$

$$v = \left[\frac{14}{57}, \frac{16}{57}, \frac{6}{19}, \frac{2}{19}, \frac{1}{19}\right]$$

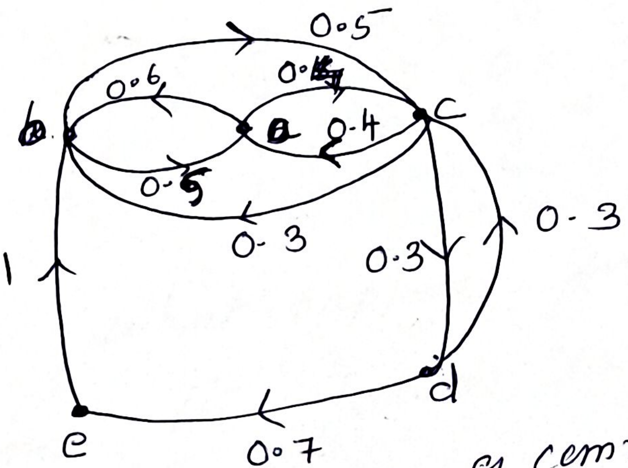
$$\text{or } v = \left[\underset{\text{III}}{\frac{14}{57}}, \underset{\text{II}}{\frac{16}{57}}, \underset{\text{I}}{\frac{18}{57}}, \underset{\text{IV}}{\frac{6}{57}}, \underset{\text{V}}{\frac{3}{57}}\right]$$

This gives the fraction of messages that pass through each individual in the long run.

$\therefore$ c is the most important individual in this network.

Mathematical Modelling in terms of weighted digraphs.

Communication network with known probabilities of communication.



Given that a's chances of communicating with b and c are in ratio 3 : 2

b's chances ,, ,, a and c are in ratio 1 : 1

c's chances of communicating with b a & d are in the ratio 3 : 4 : 3

d's chances of comunicating with c and e are in the ratio 3 : 7

e comunicates with b only.

In the long run, find the importance of each individual.

Solu. graph.

From data, the probabilities (weights) for the directed edges are as shown in the figure. Thus, we have the transition probability matrix:

$$
A = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array}
\begin{array}{c}
\begin{array}{ccccc} a & b & c & d & e \end{array} \\
\begin{bmatrix}
0 & 0.6 & 0.4 & 0 & 0 \\
0.5 & 0 & 0.5 & 0 & 0 \\
0.4 & 0.3 & 0 & 0.3 & 0 \\
0 & 0 & 0.3 & 0 & 0.7 \\
0 & 1 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

To be what happens in the long run

we find the fixed vector

$$VA = V$$

Let $V = [a \ b \ c \ d \ e]$

$$
[a \ b \ c \ d \ e] \begin{bmatrix}
0 & 0.6 & 0.4 & 0 & 0 \\
0.5 & 0 & 0.5 & 0 & 0 \\
0.4 & 0.3 & 0 & 0.3 & 0 \\
0 & 0 & 0.3 & 0 & 0.7 \\
0 & 1 & 0 & 0 & 0
\end{bmatrix} = [a \ b \ c \ d \ e]
$$

$$0.5b + 0.4c = a$$

$$0.6a + 0.3c + e = b$$

$$0.4a + 0.5b + 0.3d = c$$

$$0.3c = d \Rightarrow c = \frac{d}{0.3} = \frac{e}{0.21}$$

$$0.7d = e \qquad d = \frac{e}{0.7}$$

$$a - 0.5b = 0.4c$$

$$= 0.4 \cdot \frac{e}{0.21}$$

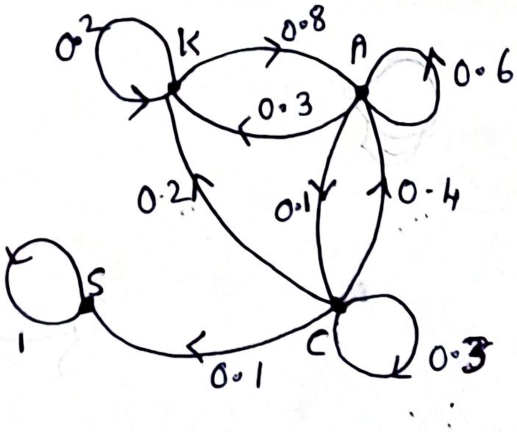
$$0.6a - b = -e - 0.3\frac{e}{0.21}$$

$$= -\left[1 + \frac{0.3}{0.21}\right]e$$

$$= -2.4285e$$

$$\left[\frac{655}{2462} \quad \frac{375}{1231} \quad \frac{350}{1231} \quad \frac{105}{1231} \quad \frac{147}{2462}\right]$$

write tPM for the following markov chain.



$$\begin{array}{c} \\ K \\ A \\ C \\ S \end{array} \begin{array}{c} \begin{array}{cccc} K & A & C & S \end{array} \\ \begin{bmatrix} 0.2 & 0.8 & 0 & 0 \\ 0.3 & 0.6 & 0.1 & 0 \\ 0.2 & 0.4 & 0.3 & 0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$