CHAPTER 3

Financial Statements, Taxes, and Cash Flow

Learning Objectives

After studying this chapter you should be able to:

- ✓ Explain the contents of the balance sheet.
- ✓ Describe the contents of the statement of profit and loss.
- ✓ Derive the statement of cash flows from the balance sheet and the statement of profit and loss.
- ✓ Distinguish between profit and cash flow.
- ✓ Discuss the devices commonly used for managing the bottom line.
- ✓ Discuss the key elements of corporate and individual income tax.
- ✓ Explain the concept of free cash flow.

Managers, shareholders, creditors, and other interested groups seek answers to the following important questions about a firm:

- *What is the financial position of the firm at a given point of time?*
- *How has the firm performed financially over a given period of time?*
- *What have been the sources and uses of cash over a given period of time?*

To answer the above questions, the accountant prepares two principal statements, the balance sheet and the statement of profit and loss and an ancillary statement, the cash flow statement. The balance sheet shows the financial position (or condition) of the firm at a given point of time. It provides a snapshot and may be regarded as a static picture. The statement of profit and loss reflects the performance of the firm over a period of time. Finally, the cash flow statement displays the sources and uses of cash during the period.

In India, the Companies Act 2013 lays down the form and content of financial statements of companies. These statements have to conform to Indian Accounting Standards (Ind AS) notified by the Ministry of Corporate Affairs under the Companies Act 2013. These standards are based on the

recommendations of the *National Financial Reporting Authority*, a body constituted by the central government. In addition, SEBI requires that listed companies should prepare cash flow statements, quarterly financial results, and consolidated financial statements.

Financial statements serve important functions: (a) They provide information on how the firm has performed in the past and what is its current financial position. (b) They are a convenient device for the stakeholders (shareholders, creditors, regulators, and others) to set performance norms and impose restrictions on the management of the firm. (c) They provide templates for financial forecasting and planning.

Financial statements are often an important source of information for financial decisions. So we examine financial statements in this chapter. Our emphasis is not on preparing financial statements—which is the job of accountants—but on understanding the kind of information found in these statements.

This chapter provides a primer on financial statements, distinguishes between profit and cash flow, briefly touches on taxation, and explains how you can cull cash flow information from financial statements.

3.1 BALANCE SHEET

The balance sheet shows the financial condition of a business at a given point of time. Exhibit 3.1 shows the balance sheet of a hypothetical firm Horizon Limited as at March 31, 20X1, prepared as per the format prescribed under the Companies Act.

The format for the balance sheet under Companies (Indian Accounting Standards) Rules, 2015 is very detailed and elaborate. It is given in the Supplementary Notes. Exhibit 3.1 shows the balance sheet of a hypothetical firm Horizon Limited as at March 31, 20X1, which reflects the most important items found commonly in balance sheets.

Exhibit 3.1 Balance Sheet of Horizon Limited as at March 31, 20X1

₹ in million

	20X1	*20X0*
ASSETS		
Non-current Assets	600	550
■ Property, plant, and equipment	500	450
■ Investments	50	40
■ Long-term loans and advances	50	60
Current Assets	400	350
■ Inventories	20	20
■ Investments	160	140
■ Trade receivables	140	120
■ Cash and cash equivalents	60	50
■ Loans	20	20
	1000	900
EQUITY AND LIABILITIES		
Equity	500	450
■ Equity share capital (Par value ₹ 10)	100	100
■ Other equity	400	350
Non-current Liabilities	300	270
■ Borrowings	200	180
■ Provisions	50	45
■ Deferred tax liabilities (net)	50	45
Current Liabilities	200	180
■ Borrowings	40	30
■ Trade payables	120	110
■ Other current liabilities	30	30
■ Short-term provisions	10	10
	1,000	900

Assets

Assets are resources 'owned' by the firm which are expected to provide the firm with future economic benefits, by way of higher cash inflows or lower cash outflows. Resources are recognised as assets in accounting when (a) the firm acquires rights over them as a result of a past transaction, and (b) the firm can quantify future economic benefits with a fair degree of accuracy.

Assets are classified as follows under the format prescribed by the Companies Act:

- Non-current assets
- Current assets

Non-current Assets Non-current assets are relatively long-lives assets. They are assets other than current assets. The important types of non-current assets are: property, plant, and equipment, capital work-in-progress, goodwill, other intangible assets, and financial assets. **Property, plant, and equipment**, often the most important non-current asset, comprises of land, buildings, plant and equipment, furnitures and fixtures, vehicles, office equipment, and so on. They are reported in the balance sheet at their net book value, which is simply the gross value (the cost of acquiring the asset) less accumulated depreciation.

Capital work-in progress reflects the value of plant and equipment which is under construction.

Goodwill arises when a company acquires another company and the purchase consideration is greater than the net book value of assets over liabilities. Goodwill has to be amortised over a period of time.

Other intangible assets include items like brands or trademarks, computer software, mining rights, copyrights, patents, and other intellectual property rights, licenses and franchises, and so on.

Financial assets comprise mainly of investments and loans. *Investments* consist mainly of equity and preference securities of associate companies, joint venture companies, and subsidiary companies. These investments are meant to be held for a long period and are made for the purpose of income and control. They are stated at cost less any diminution of value which is regarded as permanent in the opinion of management. *Loans* are usually loans and advances to associate companies, subsidiary companies, employees, and others for a period of more than one year.

Current Assets An asset is classified as a current asset when it satisfies any of the following criteria: (a) it is expected to be realised in, or is intended for sale or consumption in the company's normal operating cycle, (b) it is held primarily for the purpose of being traded, (c) it is expected to be realised within twelve months after the reporting date, or (d) it is cash or cash equivalent unless it is restricted from being exchanged or used to settle a liability for at least twelve months after the reporting date. All other assets are classified as non-current.

Current assets include inventories, investments, trade receivables, cash and cash equivalents, loans, and other current assets. **Inventories** (also called stocks) comprise of raw materials, work-in-progress, finished goods, packing materials, and stores and spares. Inventories are generally valued at cost or net realisable value, whichever is lower. The cost of inventories includes purchase cost, conversion cost, and other costs incurred to bring them to their respective present location and condition. The cost of raw materials, stores and spares, packing materials, trading and other products is generally determined on a weighted average basis. The cost of work-in-progress and finished goods is generally determined on absorption costing basis—this means that the cost figure includes allocation of manufacturing overheads.

Investments consists of investments in equity instruments, investment in preference shares, investment in government securities, investment in debentures or bonds, investments in mutual funds, investments in partnership firms, and other investments. These investments are made primarily to generate income from short-term surpluses of the firm. Current investments are carried at cost or market (fair) value, whichever is lower.

Trade receivables (also called accounts receivable or sundry debtors) represent the amounts owed to the firm by its customers (who have bought goods and services on credit) and others. Trade receivables are classified into two categories viz., debts outstanding for a period exceeding six months and other debts. Further, trade receivables are classified as debts considered good and debts considered doubtful. Generally, firms make a provision for doubtful debts which is equal to debts considered doubtful. The net figure of trade receivables is arrived at after deducting the provision for doubtful debts.

Cash and cash equivalents comprise of cash on hand and credit balances with scheduled banks and non-scheduled banks.

Short-term loans and advances comprise of loans and advances given to suppliers, employees, and other companies that are recoverable within a

year. The net figure of short-term loans and advances is arrived at after deducting a provision for doubtful advances, if any.

Other current assets comprise of items such as interest accrued on investments, dividends receivable, and fixed assets held for sale (the last item is valued at net book value or estimated net realisable value, whichever is lower).

Equity and Liabilities

Equity and liabilities represent what the firm 'owes' others. The format prescribed in the Companies Act classifies equity and liabilities as follows:

- Equity
- Non-current liabilities
- Current liabilities

Equity Equity represents the contributions made by shareholders in some form or the other. It includes equity share capital and other equity. **Equity share capital** includes equity capital as well as preference capital.

Other equity, often the most significant item on the balance sheet, represents *reserves and surplus* (such as, securities premium reserve, retained earnings, capital reserve, and general reserve) and *other comprehensive income* (such as exchange differences on translating the financial statements of foreign operation and cash flow hedge reserve).

Non-current Liabilities Non-current liabilities are liabilities which are expected to be settled after one year of the reporting date. They include financial liabilities, provisions, deferred tax liabilities (net), and other non-current liabilities. **Financial liabilities** are obligations on account of borrowings, trade payable, and other non-current liabilities. *Borrowings* generally comprise of term loans from financial institutions and banks in India and abroad, rupee bonds (debentures) and foreign currency bonds, and public deposits. *Trade payables* are amounts owed to suppliers who have sold goods and services on credit.

Deferred tax liabilities (or assets) arise because of the temporary differences between taxable income and accounting profit. A deferred tax liability (asset) is recognised when the charge in the financial statements is less (more) than the amount allowed for tax purposes.

Provisions include provisions for employee benefits such as provident fund, gratuity, superannuation, and leave encashment and other provisions.

Current Liabilities Current liabilities are liabilities which are due to be settled within 12 months after the reporting date. They include financial liabilities, other current liabilities, provisions, and current tax liabilities (net). **Financial liabilities** are obligations on account of borrowings, trade payables, and other financial liabilities. *Borrowings* are mainly in the form of working capital loans (rupee loans as well as well as foreign currency loans) and other loans and advances. *Trade payables* are amounts owed to

suppliers who have sold goods and services on credit. *Other financial liabilities* include current maturities of long-term debt, current maturities of finance lease obligation, interest accrued but not yet due, and so on.

Other current liabilities include items like statutory dues and advances from customers.

Provisions include items like provisions for employee benefits.

Current tax liabilities (net) represent items like income tax liabilities (net).

Accounting Values versus Economic Values

Accounting values and economic values ought to be similar, at least in theory. In reality, however, the two diverge very often. There are three main reasons for such a discrepancy.

Use of the Historical Cost Principle For purposes of valuation, accountants often use the historical cost as the basis. The value of an asset is shown at its historical cost less accumulated depreciation. Likewise, the value of a liability reflects a historical number. Hence accounting values may differ significantly from current economic values.

Exclusion of Intangible Assets Intangible assets like technical know-how, brand equity, managerial capability, and goodwill with suppliers often have substantial economic value. Yet they are ignored in financial accounting because it is difficult to objectively value them.

Understatement or Omission of Certain Liabilities Firms usually understate or even wholly omit certain liabilities that are of a contingent nature. They may be mentioned by way of a footnote to the balance sheet but they are not recorded on the main balance sheet. Sometimes such liabilities can be substantial.

3.2 STATEMENT OF PROFIT AND LOSS

The statement of profit and loss presents a summary of the operating and financial transactions which have contributed to the change in the owners' equity during the accounting period. Revenues are transactions that augment owners' equity and expenses are transactions that diminish owners' equity. Hence, the net change in owners' equity during an accounting period, called as profit after tax, is:

$$\text{Profit after tax} = \text{Revenues} - \text{Expenses}$$

This relationship is the basis for constructing the statement of profit and loss which first reports revenues, then expenses, and finally the profit after tax.

Among the various principles underlying the financial accounting model, two are of particular significance for understanding the statement of profit and loss, viz., the realisation principle and the matching principle.

According to the realisation principle, a revenue is recognised when the transaction generating the revenue takes place and not when the cash for the transaction is received. To illustrate this principle, let us consider an example. Suppose a firm sells goods worth ₹ 10,000 on credit to a customer. The revenue is recognised when the sale takes place even though cash may be received later. When the firm receives cash, it will adjust its balance sheet by decreasing the trade receivables and increasing the cash.

The matching principle says that the expenses associated with a product or service are recognised when the product or service is sold and not when the cash payment is made. For example, consider a retail firm that purchases an item from a wholesaler, stocks it, and finally sells it. The expense will be recognised when the item is sold, not when it is purchased or when it is paid for.

Together, the realisation and matching principles form the basis for what is called **accrual accounting**. Thanks to accrual accounting, the profit after tax of a firm is generally different from its net cash flow.

Exhibit 3.2 shows the statement of profit and loss for Horizon Limited for the year ending March 31, 20X1, prepared as per the format prescribed by the Companies Act.

Revenues from operations represent revenues from (a) sales of products and services less excise duties, and (b) other operating income.

For a finance company, revenues from operations consist of revenues from interest and financial services.

Other income consists of the following: (a) interest income (in case of a company other than a finance company), (b) dividend income, (c) net gain/loss on sale of investments, and (d) other non-operating income (net of expenses directly attributable to such income).

Expenses comprise of material expenses, employee benefit expenses, finance costs, depreciation and amortisation expenses, and other expenses. **Material expenses** equal the cost of materials consumed plus purchase of stock-in-trade minus (plus) increase (decrease) in inventories of finished goods, work-in-progress, and stock-in-trade.

Exhibit 3.2 **Statement of Profit and Loss for Horizon Limited for Year Ending March 31, 20X1**

₹ in million

	20X1	20X0
Revenues from Operations	1290	1172
Other Income	10	8
Total Income	1300	1180
Expenses		
Material expenses	600	560
Employee benefit expenses	200	180
Finance costs	30	25
Depreciation and amortisation expenses	50	45
Other expenses	240	210
Total Expenses	1120	1020
Profit before Exceptional Items and Tax	180	160
Exceptional Items	-	-
Profit Before Tax	180	160
Tax Expense	50	40
Profit (Loss) for the Period from Continuing Operation	130	120
Profit (Loss) from Discontinued Operations (after Tax)	-	-
Other Comprehensive Income	-	-
Total Comprehensive Income for the Year	130	120
Earnings Per Equity Share		
Basic	₹ 13	
Diluted	₹ 13	

Employee benefit expenses are classified as follows: (a) salaries and wages, (b) contribution to provident and other funds, (c) expenses on

employee stock option plan (ESOP) and employee stock purchase plan (ESPP), and (d) staff welfare expenses.

Finance costs are classified as follows: (a) interest, (b) dividend on redeemable preferences shares, (c) exchange differences regarded as an adjustment to borrowing costs, and (d) other borrowings costs.

Depreciation represents the allocation of the cost of tangible fixed assets to various accounting periods that benefit from their use; likewise, amortisation represents the allocation of the cost of intangible fixed assets to various accounting periods that benefit from their use.

Exceptional items are material items which are infrequent, but not unusual, and they have to be disclosed separately by virtue of their size and incidence, for financial statements to present a true and fair view. Schedule III requires the presentations of 'exceptional items' on the face of the statement of profit and loss, without defining that term. Generally, they include items of income and expenses such as write-down of inventories, litigation settlements, and restructuring.

Tax expense consists of current tax and deferred tax. **Current tax** is computed by multiplying the taxable income, as reported to the tax authorities, by the appropriate tax rate.

Deferred tax, also called future income tax, is an accounting concept that arises on account of temporary difference (also called timing difference) caused by items which are included for calculating taxable income and accounting profit but in a different manner over time. For example, depreciation is charged as per the written down value for the taxable income but usually as per the straight line method for calculating the accounting profit. As a result, there are differences in the year-to-year depreciation charges under the two methods, but the total depreciation charges over the life of the asset would be the same under both the methods.

Other comprehensive income includes items like changes in revaluation surplus, fair value changes relating to own credit risk of financial liabilities designated at fair value through profit or loss, exchange differences in translating the financial statements of a foreign operation, the effective portion of gains and loss on hedging instruments in a cash loss on hedging instruments in a cash loss on hedging instruments in a cash flow hedge, and share of comprehensive income in associates and joint ventures.

Profit (Loss) from discontinued Operations (after tax) If during the accounting period, the company has discontinued operations, the profit

(loss) from discontinued operations (after tax) has to be shown separately.

Basic earnings per share is the net profit or loss for the period attributable to equity shareholders divided by the weighted average number of equity shares during the period.

Diluted earnings per share is the net profit or loss for the period attributable to equity shareholders divided by the weighted average number of shares outstanding during the period, adjusted for the potential dilution arising from conversion of debt into equity, exercise of warrants and stock options, and so on.

When a company has some operations that have been discontinued during the period, the **profit (loss) from discontinued operations** have to be considered before arriving at the **profit (loss) for the period.**

Accounting Income versus Economic Income

The economic income of a period is defined as the change in wealth during the period. Suppose you buy a share for ₹ 50 at the beginning of a year. If you receive a dividend of ₹ 2 and the price of the share moves up to ₹ 60 at the end of the year then the economic income from the share is ₹ 12, the increase in your wealth.

While it is easy to measure the change in the wealth of an investor, it is somewhat difficult to measure the change in the value of a firm. The statement of profit and loss represents the accountant's attempt to measure the change in the wealth of shareholders. Accounting income, however, diverges from economic income due to the following reasons:

Use of the Accrual Principle The accountant uses the accrual principle and not the cash principle. Hence the computation of accounting income is not based on cash flows, even though it is cash that really matters in the determination of economic income.

Omission of Changes in Value The accountant records only those changes in value which arise from definite transactions. He does not bother about things like development of new products, emergence of competition, and changes in regulation that significantly alter the future revenues and costs of the firm and, hence, its value.

Depreciation Economic depreciation represents the decline in the value of asset during the year. Since it is difficult to measure economic depreciation, the accountant often follows a fairly straight forward method for allocating the historical cost of the assets over its useful life. For example, under the straight line method-a commonly adopted method-the historical cost of the asset is allocated evenly over its life. Understandably, there is often a discrepancy between economic depreciation (loss of economic value) and accounting depreciation (allocation of historical cost using some arbitrary rule).

Treatment of R&D and Advertising Expenditures R&D expenditures increase a firm's technical know-how which enhances revenues and lowers costs in the future; likewise, advertising expenditures that build brand equity benefit the firm over a period of time. Hence these expenditures are akin to capital expenditures. Yet, for purposes of accounting, these expenditures are typically written off in the year in which

they are incurred. This naturally causes a discrepancy between accounting income and economic income.

Inflation Inflation raises the market value of the firm's assets. However, under historical cost accounting this is not acknowledged. Hence, the depreciation charge is based on the historical cost, and not the replacement cost, of assets. This leads to a divergence between accounting income and economic income.

Creative Accounting Firms may manage their accounting income by resorting to various creative accounting techniques like change in the method of stock valuation, change in the method and rate of depreciation, and sale and leaseback arrangement. Generally, the motive for creative accounting is to artificially boost the reported income. Obviously, such tactics cause a discrepancy between accounting income and economic income.

Unaudited Quarterly Financial Results

Under the SEBI (Listing Obligations and Disclosure Requirements) regulations, 2015 listed companies are required to publish quarterly results and half-yearly statement of assets and liabilities/balance sheet. The formats for unaudited/audited quarterly results and the unaudited/audited half-yearly statement of assets and liabilities/balance sheet will have to conform to the formats for the balance sheet and statement of profit and loss (excluding notes and detailed sub-classification) prescribed in Schedule III of the Companies Act, 2013. (Banking and insurance companies are required to follow the formats prescribed under the acts/regulations specified by their respective regulators).

The extracts of Unaudited Financial results for the Quarter are presented in the following format.

1. Total Income from Operations (net)
2. Net Profit/(Loss) for the period (before tax, Exceptional and/or Extraordinary items)
3. Net Profit/(Loss) for the period before tax (after Exceptional and/or Extra-ordinary items)
4. Net Profit/(Loss) for the period after tax (after Exceptional and/or Extraordinary items)
5. Total Comprehensive Income for the period [Comprising Profit/(Loss) for the period (after tax) and Other Comprehensive Income (after tax)]
6. Equity Share Capital
7. Reserves (excluding Revaluation Reserve) as shown in the Audited Balance Sheet
8. Earnings per Share (for continuing and discontinued operations)
    (a) Basic
    (b) Diluted

The pro forma requires a company to give financial results for the quarter ended, for the corresponding quarter of the previous year, and for the previous accounting year.

3.3 PROFITS VERSUS CASH FLOW

It is important to distinguish between profits and cash flow. There are several reasons why profits and cash flow are not the same.

1. When preparing the statement of profit and loss, the accountant does not simply count the cash receipts and cash payments. Instead, he starts with cash payments and then divides them into two parts, namely current expenditures (such as wages) and capital expenditures (such as purchase of machinery). While current expenditures are deducted from current revenues, capital expenditures are written off (depreciated) over the economic life of the assets (which is normally 3 to 15 years). This means that when calculating the profits of a given year, the accountant deducts depreciation applicable on capital assets purchased including those in previous years, even though no cash is paid out during the year. Therefore, to calculate the cash flow of the year, you have to add back the depreciation charge (which is not a cash payment) and deduct the expenditures on new capital equipments (which entail a cash payment).
2. Accounting is based on the accrual principle and the matching principle. According to the accrual principle (a) revenue is recognised when it is earned irrespective of when cash is received and (b) expenditure is recognised as an asset or as an expense, when it is incurred, irrespective of when cash is paid. According to the matching principle, expenses are matched to revenues.

To understand the difference between profit and cash flow, consider the following situation. In period 1, firm A produces goods that cost 150,000; in period 2 it sells the goods for 200,000 on credit; in period 3 it collects receivables. There are no other transactions. The profit and cash flow for the three year period are shown below:

		1	2	3
1	Sales	0	200,000	0
2	Change in accounts receivable	0	200,000	(200,000)
3	Cost of goods sold	0	150,000	0
4	Change in inventories	150,000	(150,000)	0
5	Profit: (1) – (3)	0	50,000	0
6	Cash inflow: (1) – (2)	0	0	200,000
7	Cash outflow: (3) + (4)	150,000	0	0
8	Net cash flow: (6) – (7)	(150,000)	0	200,000

Net Cash Flow

When we looked at the statement of profit and loss, the emphasis was on profit after tax (also called the bottom line). In finance, however, the focus is on cash flow.

1. A firm's cash flow generally differs from its profit after tax because some of the revenues/expenses shown on its statement of profit and loss may not have been received/paid in cash during the year. The relationship between net cash flow and profit after tax is as follows:

$$\text{Net cash flow} = \text{Profit after tax} - \text{Non-cash revenues} + \text{Non-cash expenses}$$

An example of non-cash revenue is accrued interest income that has not yet been received. It increases the bottom line but is not matched by a cash inflow during the accounting period – the cash inflow would occur in a subsequent period. An example of a noncash expense is depreciation.

In practice, analysts generally define the net cash flow as:

$$\text{Net cash flow} = \text{Profit after tax} + \text{Depreciation} + \text{Amortisation}$$

However, note that the above expression will not reflect net cash flow accurately if there are significant noncash items beyond depreciation and amortisation.

3.6 OTHER ITEMS IN THE ANNUAL REPORT

The annual report of the company is perhaps the most important source of information about the affairs of the company.

In addition to the three financial statements, which form its core, the annual report contains the following:

- Auditor's Report
- Directors' Report
- Management Discussion and Analysis
- Report on Corporate Governance
- Notes Forming Part of the Financial Statements

Auditor's Report The financial statements of a joint stock company must be audited by an independent chartered account. After examining the accounting records of the company, the auditor renders an opinion as to whether the company's financial statements represent a "true and fair" view of its financial affairs and conform to Generally Accepted Accounting Principles (GAAP).

It must be emphasised that the auditor does not examine each and every transaction but examines evidence on a test basis using statistical sampling. Hence, audit is meant to provide "reasonable assurance" not "absolute assurance" about the fairness of financial statements.

Directors' Report The Directors' Report gives a summary of financial performance, recommends a dividend, provides information on appointment of directors, auditors, and cost auditors, and carries a Directors' Responsibility Statement. In addition, it contains information on credit rating, fixed deposits, employee stock option scheme, strategic acquisitions and alliances, human resources development, subsidiary companies, corporate social responsibilities, conservation of energy, technology absorption, and foreign exchange earnings and outgo.

Management Discussion and Analysis The section on Management Discussion and Analysis provides an overview of the industry, spells out the strategy and thrust areas of the company, dwells on the risks faced by the company and its risk mitigation initiatives, presents highlights of the company's financial performance, and gives an idea of its internal control system.

Report on Corporate Governance Required under Clause 49 of the Listing Agreement with stock exchanges, the corporate governance report gives information on the company's philosophy on corporate governance, board of directors, audit committee, shareholders/investors grievance committee, other board committees, remuneration of directors, and general shareholder information.

Notes Forming Part of the Financial Statements The financial statements, stand-alone as well as consolidated, are supported by notes which technically form part of them. These notes provide information on significant accounting policies and details relating to various items in the balance sheet and statement of profit and loss.

3.7 MANIPULATION OF THE BOTTOM LINE

Within the provisions of GAAP, corporate managements have some discretion in influencing the occurrence, measurement, and reporting of various items. They may use this latitude to paint a desired (negative or positive, as the case may be) picture of a firm's finance. Put differently, they may resort to creative accounting which, unlike cooking the books, is generally legal. It is euphemistically also called financial engineering or earnings management. The devices commonly used for earnings management are as follows.

- Inflate the sales for the current year by advancing the sales from the following year.
- Sell an asset (whose market value is high but book value is low) to create non-operating profit that boosts the bottom line.
- Fiddle with the method and rate of depreciation. (A switch may be effected from the written down value method to the straight line method or vice versa).
- Change the method of stock valuation.
- Capitalise certain expenses like research and development costs and product promotion costs that are ordinarily written off in the statement of profit and loss.
- Defer discretionary expenditures (like repairs, advertising, research and development) to the following year.
- Make inadequate provision for certain known liabilities (gratuity, etc.) and treat certain liabilities as 'contingent liabilities.'
- Recognise the diminution in the carrying value of some investment in a period when the operating profit is high.
- Make extra provision during prosperous years and write them back in lean years.
- Revalue assets to create the impression of substantial reserves.
- Write off expenses directly from reserves. The Companies Act permits companies to write off expenses directly from share premium reserve, after seeking approval of the court.
- Lengthen the accounting year in an attempt to cover poor performance.

Accounting Manipulation: Some Examples

Here are some conspicuous examples of accounting manipulation.

Enron Enron, which was involved in perhaps the most notorious accounting scandal of the early 2000s, filed for bankruptcy in December 2001-the largest bankruptcy filing in the U.S. history.

Starting as an operator of natural gas pipelines, Enron evolved into a global trader of gas, oil, electricity, and broad band internet capacity. Interestingly, during 1990s Enron was hailed as one of the most profitable and successful companies in the U.S. It was rated by *Fortune* magazine as 'The Most Innovative Company in America' for six consecutive years, from 1995 to 2000. While many facets of Enron's business were truly innovative and successful, Enron executives manipulated its financial statements to inflate the reported earnings-for example, in 2000, 96 percent of reported earnings were the result of accounting jugglery.

Enron employed sophisticated methods of accounting manipulation, but the essence of most of its deception was quite simple. Enron sold assets at inflated prices to other entities (many of them being fictitious entities created by Enron's CFO Andrew Fastow), along with a promise to repurchase those assets at a higher price in future. While Enron recorded the profits from the sale of those assets, it cleverly hid the promises to buy them back in various ways. Much of Enron's growth in revenues and profits in the late 1990s stemmed from such manipulation.

WorldCom On July 21, 2002, WorldCom filed for bankruptcy. It shocked investors because not long back WorldCom commanded a market capitalisation of $120 billion. Beginning in 1998, WorldCom management resorted to a series of accounting manipulation which hid the firm's financial problems.

WorldCom's accounting fraud was in the form of reclassification of $3.85 billion of operating expenses as long-term investment. It boosted WorldCom's reported earnings because while long-term investments are depreciated slowly over time, operating expenses are deducted from revenues immediately.

Some investors expressed concern over WorldCom's excessive investment relative to others in the industry. As Robert Olstein commented, "Red flags(were) things like big deviations between reported earnings and excess cash flow… (and) excessive capital expenditures for a long time."

Satyam On January 7, 2009, Ramalinga Raju, Chairman of Satyam Computer Services, the fourth largest Indian information technology company with sales of over ₹ 80 billion, sent a letter to the members of Satyam's board stating that over $1 billion of cash on Satyam's balance sheet was fictitious and that the company had been, for several years overstating its earnings. Some speculate that perhaps the earnings may not have been overstated but pilfered from the company. On the same day, Satyam's market capitalisation fell from ₹ 12,068 crores to ₹ 2,689 crores, decimating the value of the firm by about 78 percent.

To save the company, the Government of India swiftly stepped in and appointed a new board to oversee the company. Backed by the government, the new board restored the confidence of customers and employees, invited bids from companies interested in acquiring a controlling stake in Satyam. On April 13, 2009 in less than 100 days after Ramalinga Raju's admission of fraud, Tech Mahindra emerged as the highest bidder. It paid ₹ 1,756 crore to acquire a 31 percent stake in the company (this was in the form to additional capital issued by the company). As per SEBI norms, Tech Mahindra made an open offer for another 20 percent at the bid price of ₹ 58. Finally, Tech Mahindra acquired 51 percent stake at a total cost of ₹ 2889 crore.

Why Do Companies Manipulate or Smoothen Earnings

A variety of motives prompt firms to manipulate or smoothen earnings. The more common ones are:

- To project an image that the company is a low risk company (It is assumed that financial analysts regard earnings variability as a key factor in risk evaluation).
- To enhance managerial compensation, if the same is influenced in some way by reported earnings.
- To promote a perception that the management of the firm is competent.
- To communicate more meaningfully about the long-term prospects of the firm.

What Can You Do

What can you do to read between the lines when corporate managements tend to manage the 'bottom line' by employing a variety of ingenious devices? Our suggestions are as follows:

- Acquire greater knowledge of how accountants prepare financial statements and what are the current financial reporting practices.
- Carefully peruse the notes to accounts in order to: (a) discover changes in accounting policies; and (b) learn about the nature and magnitude of contingent liabilities.
- Read the auditor's report and understand the implications of the qualifications in that report.
- Look at the performance of the company over a period of time and do not attach much importance to the figures for one year. Remember that while manipulation may pay for a year or two, it tends to be a self-defeating exercise in the long run. This indeed is your best safeguard against corporate accounting gimmickry.

Potential Red Flags

As an analyst, you should learn to identify potential red flags. Here is a list of common red flags.

- A qualified audit opinion.
- A change in accounting policy that is not satisfactorily explained.
- An unusual increase in accruals.
- A widening gap between reported income and cash flow from operations.
- Large adjustments in the fourth quarter.
- An abrupt change in the external or internal auditor.
- An increase in transactions with related parties.
- An unusual increase in short-term financing or lending.

3.8 TAXES

Taxes are often a major cash outflow for a firm. The magnitude of the tax burden is determined by the tax code, which is often amended. If the rules of taxation seem somewhat odd to you, remember that the tax code is significantly influenced by political forces. Hence it may not always make economic sense.

Taxes may be divided into two broad categories: direct taxes and indirect taxes. A tax is referred to as a direct tax if the impact and incidence of the tax is on the same person. Income tax, wealth tax, and gift tax are examples of direct taxes. A tax is regarded as an indirect tax if the impact and incidence of the tax is on different persons (the impact is on one person but through the process of shifting the incidence is on another). Excise duty, sales tax, and customs duty are the three important indirect taxes.

Corporate Income Tax

A company's taxable income is determined after taking into account its revenues, expenses, and deductions on account of various incentives and reliefs. The taxable income is subject to a tax rate of 30 percent for domestic companies and 40 percent for foreign companies. In addition, there is a surcharge of 5 percent on such income tax and an educational cess on both the tax and surcharge (at present 3 percent).

Depreciation Depreciation is charged on blocks of assets which represent a group of assets, within the broad class of assets such as buildings, plant, machinery, and furniture, for which a common rate of depreciation is applicable. Depreciation is calculated by applying the prescribed rate (which varies between 5 percent and 100 percent) on the written down value (WDV) of the entire block. When an asset is sold the amount realised from the sale of that asset (after deducting expense on sales) will simply be deducted from the WDV of that block. If the amount realised is greater than the WDV of the block, the difference will be treated as a short term capital gain. In a case where all the assets in the block are disposed off and there is still a balance in the account of the block, such amount will be treated as short term capital loss.

It may be noted that when an asset is acquired and put to use during the previous year for a period less than 180 days then depreciation will be allowed only to the extent of 50 percent of the prescribed rate for that asset in the year of acquisition.

Interest Expense versus Dividend Payment While interest on borrowings is a tax-deductible expense, meaning that it can be deducted before determining the taxable income, dividend on share capital (equity as well as preference) is not a tax-deductible payment.

Dividend Income When a domestic company receives dividend from another domestic company, it is allowed a deduction of an amount equal to the amount of dividend received from another company provided it distributes that to its shareholders.

Unabsorbed Business Loss and Depreciation Unabsorbed business loss (other than speculation business loss) of any year can be carried forward and set off against income under the head of business income of subsequent years. Such carry forward can be done for eight subsequent years from the year in which the loss was computed.

Unabsorbed depreciation can be carried forward and set off against the income from any other head of subsequent years without any limitation as to the number of years.

Exemptions and Deductions A variety of exemptions and deductions are granted under the Income Tax Act. The important ones are: exemption of profits and gains from the export of articles or things or software from a unit established in a Free Trade Zone; exemption of profits and gains from the export of articles or things or computer software from a 100 percent exported oriented unit; deduction in respect of profits and gains from a new industrial undertaking; deduction in respect of profits from an industrial undertaking established in an industrially backward state.

Minimum Alternate Tax If the income tax payable on the total income of a company, as computed under the Income Tax Act, is less than 18.5 percent of its book profit, the tax payable shall be deemed to be 18.5 percent of such book profit. That is every company has to pay at least 18.5 percent of the book profit as tax. Book profit means the net profit shown in the profit and loss account prepared for company law purposes, subject to certain adjustments.

Advance Tax Advance tax is payable on the current income of the company in four instalments during the financial year. Specifically, companies are required to pay 15 percent of their estimated tax liability by June 15, 45 percent by September 15, 75 percent by December 15, and 100 percent by March 15.

Individual Income Tax

Individuals pay taxes on salaries, investment income, and other incomes. The salient features of individual taxation are described below briefly.

Progressive Tax structure Tax rates on individual income are progressive, implying that the higher the income, the larger the percentage paid in taxes. For the assessment year 2020-21, the individual tax rates are as follows:

Income Range	*Tax Rate*
₹ 0 – ₹ 250,000	0%
₹ 250,001 – ₹ 500,000	5%
₹ 500,001 – ₹ 10,00,000	20%
₹ 1,000,000 and above	30%

Interest and Dividend Income Subject to certain exemptions and deductions, interest received by an individual from bank deposits, company debentures, government securities, and so on is added to other income for tax purposes. Likewise, dividend received by an individual from companies and mutual funds, subject to certain exemptions, is added to other income for tax purposes.

Capital Gains Assets such as shares, debentures, and real estate are called capital assets. If you buy a capital asset and later sell it at a price greater than your cost, the gain is called a capital gain; if you sell it at a loss, it is called a capital loss.

Capital gains are classified as long-term and short-term, depending on the period of holding of the capital asset. If the asset is held for more than 12 months in the case of listed shares and securities (or more than 36 months in the case of other assets), the gain is treated as a long-term capital gain; otherwise the gain is treated as a short-term capital gain.

Long-term capital gains, after the benefit of indexation, are taxed at a flat rate of 20 percent. However, long-term capital gains arising from the sale of equity shares or units of an equity-oriented mutual fund are exempt from tax, provided the transaction is chargeable to securities transaction tax.

Short-term capital gains from the sale of equity shares and units of an equity-oriented mutual fund are taxed at 15 percent provided the transaction

is chargeable to security transaction tax. Other short-term capital gains are taxed at the rate applicable to the assessee.

Indirect Taxes

Historically, indirect taxes in India consisted of central excise duty, state level value added tax, octroi, and service tax. With the introduction of Goods and Services Tax (GST) from July 1, 2017, all these indirect taxes have been subsumed in GST. The salient features of the GST are:

- Barring some exceptions, the same commodity has the same rate, pan-India.
- Refund is given for taxes paid on input. This is done at each stage, so that there is no cascading tax on tax.
- For commodities, only two buckets are there to claim input tax credit —one for State GST and another for Central GST. For services there is only one bucket.
- There are five slabs for GST: 0%, 5%, 12%, 18%, and 28%. These rates are all-inclusive, except for the cess on demerit, luxury and polluting items such as tobacco, luxury cars, aerated drinks, and coal.
- Real estate (except for land leasing), alcohol, petroleum products, and electricity are not included in GST.
- The GST council is the body to recommend any changes in GST, including rates. It is chaired by the Union finance minister and has state finance ministers are members.

on the same person. A tax is regarded as an indirect tax if the impact and incidence of the tax is on different persons.

- For corporate decision making and fundamental equity valuation, we have to work with modified accounting data and use measures like operating assets, NOPAT, and free cash flow.
- **Free cash flow** represents the cash available for distribution to investors after meeting the investment needs of the company.

QUESTIONS

1. List the important functions performed by the financial statements.
2. Present the format of the balance sheet.
3. Describe the various asset accounts and liability accounts found on a company's balance sheet.
4. "Accounting and economic values tend to differ." Why?
5. Discuss the important items found on the statement of profit and loss.
6. Explain the sources of divergence between accounting income and economic income.
7. List the details provided in the unaudited quarterly results.
8. Why do profits and cash flow differ.
9. What are the sources of cash and what are the uses of cash?
10. Give the format for the cash flow statement.
11. What devices are commonly employed to manage the bottom line?
12. Why do companies manipulate earnings?
13. What are the salient features of corporate income tax?
14. What are the salient features of individual income tax?
15. Explain the following terms: operating capital, NOPAT, ROIC, and free cash flow.

SOLVED PROBLEMS

3.1 The financial statements of Zenith Limited are shown below:

Balance Sheet of Zenith Ltd. as at March 31, 20X2

		₹ in Million
	20X2	20X1
ASSETS		
Non-current assets	643	610

6.2 FUTURE VALUE OF A SINGLE AMOUNT

Suppose you invest ₹ 1,000 for three years in a savings account that pays 10 percent interest per year. If you let your interest income be reinvested, your investment will grow as follows:

First year :	Principal at the beginning	1,000
	Interest for the year (₹ 1,000 × 0.10)	100
	Principal at the end	1,100
Second year :	Principal at the beginning	1,100
	Interest for the year (₹ 1,100 × 0.10)	110
	Principal at the end	1,210
Third year :	Principal at the beginning	1,210
	Interest for the year (₹ 1,210 × 0.10)	121
	Principal at the end	1,331

Formula

The process of investing money as well as reinvesting the interest earned thereon is called compounding. The future value or compounded value of an investment after *n* years when the interest rate is *r* percent is:

$$FV_n = PV(1 + r)^n \tag{6.1}$$

In this equation $(1 + r)^n$ is called the future value interest factor or simply the future value factor.

To solve future value problems you have to find the future value factors. You can do it in different ways. In the example given above, you can multiply 1.10 by itself three times or more generally (1 + *r*) by itself *n* times. This becomes tedious when the period of investment is long.

Fortunately, you have an easy way to get the future value factor. Most calculators have a key labelled 'y^x'. So all that you have to do is to enter 1.10, press the key labelled y^x, enter 3, and press the '=' key to obtain the answer.

Alternatively, you can consult a future value interest factor (FVIF) table. Exhibit 6.2 presents one such table showing the future value factors for certain combinations of periods and interest rates. A more comprehensive table is given in Appendix A at the end of the book.

Suppose you deposit ₹ 1,000 today in a bank which pays 10 percent interest compounded annually. How much will the deposit grow to after 8 years and 12 years?

The future value 8 years hence will be:

$$₹\ 1{,}000\ (1.10)^8 = ₹\ 1{,}000\ (2.144)$$
$$= ₹\ 2{,}144$$

The future value 12 years hence will be:

$$₹\ 1{,}000\ (1.10)^{12} = ₹\ 1{,}000\ (3.138)$$
$$= ₹\ 3{,}138$$

Exhibit 6.2 **Value of $FVIF_{r,n}$ for Various Combinations of *r* and *n***

n/r	*6%*	*8%*	*10%*	*12%*	*14%*
2	1.124	1.166	1.210	1.254	1.300
4	1.262	1.360	1.464	1.574	1.689
6	1.419	1.587	1.772	1.974	2.195
8	1.594	1.851	2.144	2.476	2.853
10	1.791	2.159	2.594	3.106	3.707
12	2.012	2.518	3.138	3.896	4.817

While tables are easy to use they have a limitation as they contain values only for a small number of interest rates. So often you may have to use a calculator or a spreadsheet – the use of spreadsheet is illustrated later.

Compound and Simple Interest

So far we assumed that money is invested at compound interest which means that each interest payment is reinvested to earn further interest in future periods. By contrast, if no interest is earned on interest the investment earns only simple interest. In such a case the investment grows as follows:

Future value = Present value [1 + Number of years x Interest rate]

For example, an investment of ₹ 1,000, if invested at 12 percent simple interest rate, will in 5 years time become:

1,000 [1 + 5 × 0.12] = ₹ 1,600

Exhibit 6.3 shows how an investment of ₹ 1,000 grows over time under simple interest as well as compound interest when the interest rate is 12 percent. From this exhibit you can feel the power of compound interest. As Albert Einstein once remarked: 'I don't know what the seven wonders of the world are, but I know the eighth -compound interest'. You may be wondering why your ancestors did not display foresight. Hopefully, you will show concern for the posterity.

Exhibit 6.3 **Value of ₹ 1000 Invested at 10 percent Simple and Compound Interest**

	Simple Interest					*Compound Interest*				
Year	*Starting Balance*	+	*Interest*	=	*Ending Balance*	*Starting Balance*	+	*Interest*	=	*Ending Balance*
1	1000	+	100	=	1100	1000	+	100	=	1100
5	1400	+	100	=	1500	1464	+	146	=	1610
10	1900	+	100	=	2000	2358	+	236	=	2594
20	2900	+	100	=	3000	6116	+	612	=	6728
50	5900	+	100	=	6000	106,718	+	10672	=	117,390
100	10,900	+	100	=	11,000	12,527,829	+	1,252,783	=	13,780,612

Exhibit 6.4 shows graphically how money grows under simple interest and compound interest. Note that under simple interest the growth is linear and under compound interest the growth is exponential.

Power of Compounding

The power of compounding is often illustrated with the sale of Manhattan Island in 1626. It was sold by Red Indians to Peter Minuit for $24. Looking at the New York real estate prices today, it appears that Peter Minuit got a real bargain. But consider

the future value of $24 in 2010 if Red Indians had invested for 384 years (2010 minus 1626) at an interest rate of 8 percent per year:

$$\$ 24 \times (1.08)^{384} = \$ 164{,}033{,}800{,}000{,}000$$
$$= \$ 164 \text{ trillion}$$

The total value of land on Manhattan in 2010 may perhaps have not been more than $ 500 billion. So the deal was not really a bargain.

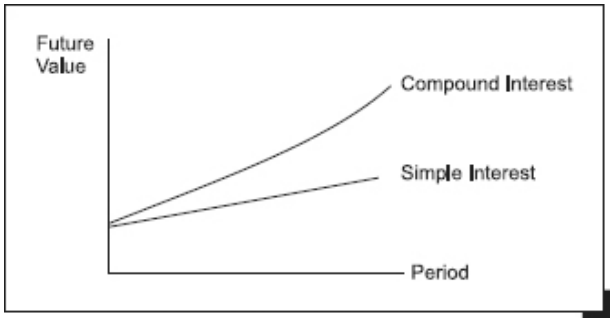
Although interesting, this comparison is misleading. First, judged by historical standards, the 8 percent rate is much higher. If we use a 3.5 percent rate, which is more consistent with historical experience, the future value of $ 24 would be just $\$24 \times (1.035)^{384} = \$$ 13,10 1,034. Second, we ignored the rental income to Peter Minuit and his successors' over the last 384 years.

Considering everything, Peter Minuit got a real good deal.

Doubling Period

Investors commonly ask the question: How long would it take to double the amount at a given rate of interest? To answer this question we may look at the future value interest factor table. Looking at Exhibit 6.2 we find that when the interest rate is 12 percent it takes about 6 years to double the amount, when the interest is 6 percent it takes about 12 years to double the amount, so on and so forth. Is there a rule of thumb which dispenses with the use of the future value interest factor table? Yes, there is one and it is called the rule of 72. According to this rule of thumb, the doubling period is obtained by dividing 72 by the interest rate. For example, if the interest rate is 8 percent, the doubling period is about 9 years (72/8). Likewise, if the interest rate is 4 percent the doubling period is about 18 years (72/4). Though somewhat crude, it is a handy and useful rule of thumb.

Exhibit 6.4 **Graphic View of Simple and Compound Interest**



If you are inclined to do a slightly more involved calculation, a more accurate rule of thumb is the rule of 69. According to this rule of thumb, the doubling period is equal to:

$$0.35 + \frac{69}{\text{Interest Rate}}$$

As an illustration of this rule of thumb, the doubling period is calculated for two interest rates, 10 percent and 15 percent.

Interest Rate	*Doubling Period*
10 percent	$0.35 + \frac{69}{10} = 7.25$ years
15 percent	$0.35 + \frac{69}{15} = 4.95$ years

Finding the Growth Rate

The formula we used to calculate future value is quite general and it can be applied to answer other types of questions related to growth. Suppose your company currently has 5,000 employees and this number is expected to grow by 5 percent per year. How many employees will your company have in 10 years? The number of employees 10 years hence will be:

$$5{,}000 \times (1.05)^{10} = 5000 \times 1.629 = 8{,}145$$

Consider another example. Phoenix Limited had revenues of ₹ 100 million in 2000 which increased to ₹ 1000 million in 2010. What was the compound growth rate in revenues? The compound growth rate may be calculated as follows:

$$\begin{aligned} 100\,(1+g)^{10} &= 1{,}000 \\ (1+g)^{10} &= \frac{1000}{100} = 10 \\ (1+g) &= 10^{1/10} \\ g &= 10^{1/10} - 1 \\ &= 1.26 - 1 = 0.26 \text{ or } 26 \text{ percent} \end{aligned}$$

6.3 PRESENT VALUE OF A SINGLE AMOUNT

Suppose someone promises to give you ₹ 1,000 three years hence. What is the present value of this amount if the interest rate is 10 percent? The present value can be calculated by discounting ₹ 1,000, to the present point of time, as follows :

$$\text{Value three years hence} = ₹\ 1{,}000$$

$$\text{Value two years hence} = ₹\ 1{,}000\left[\frac{1}{1.10}\right]$$

$$\text{Value one year hence} = ₹\ 1{,}000\left[\frac{1}{1.10}\right]\left[\frac{1}{1.10}\right]$$

$$\text{Value now} = ₹\ 1{,}000\left[\frac{1}{1.10}\right]\left[\frac{1}{1.10}\right]\left[\frac{1}{1.10}\right]$$

Formula

The process of discounting, used for calculating the present value, is simply the inverse of compounding. The present value formula can be readily obtained by manipulating the compounding formula:

$$FV_n = PV\ (1 + r)^n \tag{6.2}$$

Dividing both the sides of Eq. (6.2) by $(1 + r)^n$, we get:

$$PV = FV_n\ [1\ /\ (1 + r)^n] \tag{6.3}$$

The factor $1/(1 + r)^n$ in Eq. (6.3) is called the discounting factor or the present value interest factor ($PVIF_{r,n}$). Exhibit 6.5 gives the value of $PVIF_{r,n}$ for several combinations of r and n. A more detailed table of $PVIF_{r,n}$ is given in Appendix A at the end of the book.

What is the present value of ₹ 1,000 receivable 6 years hence if the rate of discount is 10 percent?

The present value is:

$$₹\ 1{,}000 \times PVIF_{10\%,6} = ₹\ 1{,}000(0.565) = ₹\ 565$$

What is the present value of ₹ 1,000 receivable 20 years hence if the discount rate is 8 percent? Since Exhibit 6.5 does not have the value of $PVIF_{8\%,20}$ we obtain the answer as follows:

$$₹\ 1{,}000\left[\frac{1}{1.08}\right]^{20} = ₹\ 1{,}000\left[\frac{1}{1.08}\right]^{10}\left[\frac{1}{1.08}\right]^{10}$$
$$= ₹\ 1{,}000\ (PVIF_{8\%,10})(PVIF_{8\%,10})$$
$$= ₹\ 1{,}000\ (0.463)(0.463) = ₹\ 214$$

Exhibit 6.5 **Value of $PVIF_{r,n}$ for Various Combinations of r and n**

n/r	6%	8%	10%	12%	14%
2	0.890	0.857	0.826	0.797	0.770
4	0.792	0.735	0.683	0.636	0.592
6	0.705	0.630	0.565	0.507	0.456
8	0.626	0.540	0.467	0.404	0.351
10	0.558	0.463	0.386	0.322	0.270
12	0.497	0.397	0.319	0.257	0.208

Present Value of an Uneven Series

In financial analysis we often come across uneven cash flow streams. For example, the cash flow stream associated with a capital investment project is typically uneven. Likewise, the dividend stream associated with an equity share is usually uneven and perhaps growing.

The present value of a cash flow stream - uneven or even - may be calculated with the help of the following formula:

$$PV_n = \frac{A_1}{(1+r)} + \frac{A_2}{(1+r)^2} + \cdots + \frac{A_n}{(1+r)^n} = \sum_{t=1}^{n} \frac{A_t}{(1+r)^t} \qquad (6.4)$$

where, PV_n is the present value of a cash flow stream, A_t is the cash flow occurring at the end of year t, r is the discount rate, and n is duration of the cash flow stream.

Exhibit 6.6 shows the calculation of the present value of an uneven cash flow stream, using a discount rate of 12 percent.

Exhibit 6.6 **Present Value of an Uneven Cash Flow Stream**

Year	*Cash Flow* ₹	*$PVIF_{12\%,n}$*	*Present Value of Individual Cash Flow*
1	1,000	0.893	893
2	2,000	0.797	1,594
3	2,000	0.712	1,424
4	3,000	0.636	1,908
5	3,000	0.567	1,701
6	4,000	0.507	2,028
7	4,000	0.452	1,808
8	5,000	0.404	2,020
Present Value of the Cash Flow Stream			13,376

Spreadsheet Analysis To calculate the present value of the cash flow stream given in Exhibit 6.6, you can use the Excel spreadsheet as shown below:

	A	B	C	D	E	F	G	H	I
1	Year	1	2	3	4	5	6	7	8
2	Cash flow	1,000	2,000	2,000	3,000	3,000	4,000	4,000	5,000
3	Discount rate	12%	NPV	=NPV(B3, B2:12)				→	13,375

Type the cash flows for years 1 through 8 in the cells B2 to I2 and the discount rate in the cell B3. If you want the present value in say cell I3, select I3 and type = NPV(B3,B2:I2) and press enter and the value will

appear therein. Here it should be noted that in Excel the term NPV is used to denote the net result of adding the present values of a stream of future cash flows unlike our usual practice of using the term NPV, net present value, to denote the excess of the total present value of the future receipts (payments) over the initial investment (cash inflow).

6.4 FUTURE VALUE OF AN ANNUITY

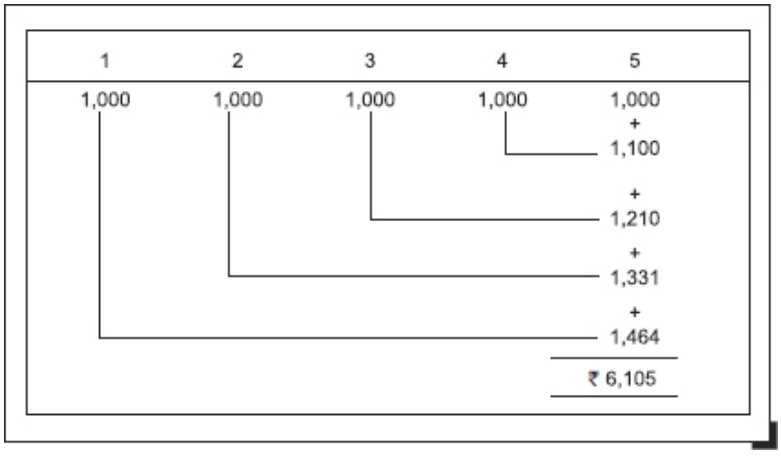
An annuity is a stream of constant cash flows (payments or receipts) occurring at regular intervals of time. The premium payments of a life insurance policy, for example, are an annuity. When the cash flows occur at the end of each period, the annuity is called an **ordinary annuity** or a **deferred annuity**. When the cash flows occur at the beginning of each period, the annuity is called an **annuity due**. Our discussion here will focus on a deferred annuity. The formula for an annuity due is simply (1 + *r*) times the formula for the corresponding ordinary annuity.

Suppose you deposit ₹ 1,000 annually in a bank for 5 years and your deposits earn a compound interest rate of 10 percent. What will be the value of this series of deposits (an annuity) at the end of 5 years? Assuming that each deposit occurs at the end of the year, the future value of this annuity will be:

$$\begin{aligned}
&₹\,1{,}000(1.10)^4 + ₹\,1{,}000(1.10)^3 + ₹\,1{,}000(1.10)^2 + ₹\,1{,}000(1.10) + ₹\,1{,}000 \\
&= ₹\,1{,}000(1.464) + ₹\,1{,}000(1.331) + ₹\,1{,}000(1.21) + ₹\,1{,}000(1.10) + ₹\,1{,}000 \\
&= ₹\,6{,}105
\end{aligned}$$

The time line for this annuity is shown in Exhibit 6.7.

Exhibit 6.7 **Time Line for an Annuity**



Formula

In general terms the future value of an annuity is given by the following formula:

$$\begin{aligned} FVA_n &= A\,(1+r)^{n-1} + A\,(1+r)^{n-2} + \cdots + A \\ &= A\,[(1+r)^n - 1] \,/\, r \end{aligned} \qquad (6.5)$$ [1]

where FVA_n is the future value of an annuity which has a duration of n periods, A is the constant periodic flow, r is the interest rate per period, and n is the duration of the annuity.

The term $[(1 + r)^n - 1] / r$ is referred to as the future value interest factor for an annuity ($FVIFA_{r,n}$). The value of this factor for several combinations of r and n is given in Exhibit 6.8. A more detailed table is given in Appendix A at the end of this book.

Exhibit 6.8 **Value of $FVIFA_{r,n}$ for Various Combinations of r and n**

n/r	*6%*	*8%*	*10%*	*12%*	*14%*
2	2.060	2.080	2.100	2.120	2.140
4	4.375	4.507	4.641	4.779	4.921
6	6.975	7. 336	7.716	8.115	8.536
8	9.897	10.636	11.436	12.299	13.232
10	13.181	14.487	15.937	17.548	19.337
12	16.869	18.977	21.384	24.133	27.270

Applications

The future value annuity formula can be applied in a variety of contexts. Its important applications are illustrated below.

Knowing What Lies in Store for You Suppose you have decided to deposit ₹ 30,000 per year in your Public Provident Fund Account for 30 years. What will be the accumulated amount in your Public Provident Fund Account at the end of 30 years if the interest rate is 8 percent?

The accumulated sum will be:

$$\begin{aligned} &₹\ 30{,}000\ (\text{FVIFA}_{8\%,30\text{yrs}}) \\ &= ₹\ 30{,}000\left[\frac{(1.08)^{30}-1}{.08}\right] \\ &= ₹\ 30{,}000\ [\ 113.283] \\ &= ₹\ 3{,}398{,}490 \end{aligned}$$

Use of Excel Spreadsheet Time value calculations can be easily done using a spreadsheet. In Excel, there are customised notations and functions for the various time value parameters as shown below:

Parameter	*Notation/Symbol*	*Built in Formula in Excel*
Present value	PV	=PV(rate,nper,pmt,[fv],[type])
Future value	FV	=FV(rate,nper,pmt,[pv],[type])
No. of continuous successive periods	NPER	=NPER(rate,pmt,pv,[fv],[type])
Payment per period	PMT	=PMT(rate,nper,pv,[fv],[type])
Interest rate	RATE	=RATE(nper,pmt,pv,[fv],[type])

The following printout of an Excel worksheet, may be used to understand how Excel is used, to calculate the accumulated sum in the above illustration.

	A		*B*
1	Amount of deposit per period(PMT)	₹	30,000
2	No. of periods(NPER)	Years	30
3	Interest rate (RATE)	p.a	8%
4	Accumulated amount(FV)	₹	3,398,496
5		Formula used	=FV(B3, B2, –B1)

Open the worksheet and input the respective given values for the various parameters inside the cells A1 to A3, respectively inside cells B1 to

B3 as shown above. To get the future value in cell B4, select B4 and type = FV (and even as you type this, the formula template, viz. FV(rate,nper,pmt, [pv].[type]) will become visible nearby (by way of a tip) to guide you further. What you have to do thereafter is just give the cell reference numbers of the respective parameter values inside the parenthesis in the order cited, duly separated by commas. Thus in the place for rate type B3(just left click the mouse in the cursor on cell B3 and that cell no. will get typed automatically), in the place for nper type B2, and in the place for pmt type −B1. Note that if there is a payment/outflow, a − sign should precede the cell reference.

Where a notation is inside a square bracket, it indicates that if you skip that place, Excel will take that the value is 0. In our case as there is no PV figure in the given data just skip that place. The notation 'type' can take only one of the two values viz, 0 if the outflow/inflow takes place at the end of each period or 1 if that takes place in the beginning of each period. Again, if nothing is typed in the space marked [type], the value would be 0.

To sum up, type =FV(B3,B2,−B1) inside B4 and press enter and the future value will automatically appear in that cell.

How Much Should You Save Annually You want to buy a house after 5 years when it is expected to cost ₹ 2 million. How much should you save annually if your savings earn a compound return of 12 percent?

The future value interest factor for a 5 year annuity, given an interest rate of 12 percent, is:

$$\text{FVIFA}_{n=5,\, r=12\%} = \frac{(1+0.12)^5 - 1}{0.12} = 6.353$$

The annual savings should be:

$$\frac{₹\ 2000{,}000}{6.353} = ₹\ 314{,}812$$

Annual Deposit in a Sinking Fund Futura Limited has an obligation to redeem ₹ 500 million bonds 6 years hence. How much should the company deposit annually in a sinking fund account wherein it earns 14 percent interest, to cumulate ₹ 500 million in 6 years time?

The future value interest factor for a 6 year annuity, given an interest rate of 14 percent is:

$$\text{FVIFA}_{n=6,\, r=14\%} = \frac{(1+0.14)^6 - 1}{0.14} = 8.536$$

The annual sinking fund deposit should be:

$$\frac{₹\ 500\ \text{million}}{8.536} = ₹\ 58.575\ \text{million}$$

Finding the Interest Rate A finance company advertises that it will pay a lump sum of ₹ 8,000 at the end of 6 years to investors who deposit annually ₹ 1,000 for 6 years. What interest rate is implicit in this offer?

The interest rate (also called the internal rate of return or IRR) may be calculated in two steps:

1. Find the $FVIFA_{r,6}$ for this contract as follows:

$$₹\ 8{,}000 = ₹\ 1{,}000 \times FVIFA_{r,6}$$

$$FVIFA_{r,6} = \frac{₹\ 8{,}000}{₹\ 1{,}000} = 8.000$$

2. Look at the $FVIFA_{r,n}$ table and read the row corresponding to 6 years until you find a value close to 8.000. Doing so, we find that

$$FVIFA_{12\%,6} \text{ is } 8.115$$

So, we conclude that the interest rate is slightly below 12 percent.

An Excel worksheet of the above is as under:

	A	B	C	D	E
1	Future value (Fv)	8,000			
2	Periods in years (Nper)	6	Rate	→	11.43%
3	Periodic payment (Pmt)	1000	= (B2, –B3, B1)		

In the rate formula, viz. =RATE (nper,pmt,pv,[fv],[type]), the value for pv here is nil. In typing out the rate formula you may either type a 0 in the place for pv or just put two commas between B3 and B1.

How Long Should You Wait You want to take up a trip to the moon which costs ₹ 1,000,000 – the cost is expected to remain unchanged in nominal terms. You can save annually ₹ 50,000 to fulfill your desire. How long will you have to wait if your savings earn an interest of 12 percent?

The future value of an annuity of ₹ 50,000 that earns 12 percent is equated to ₹ 1,000,000.

$$50{,}000 \times FVIFA_{n=?,12\%} = 1{,}000{,}000$$

$$50{,}000 \times \left[\frac{1.12^n - 1}{0.12}\right] = 1{,}000{,}000$$

$$1.12^n - 1 = \frac{1{,}000{,}000}{50{,}000} \times 0.12 = 2.4$$

$$1.12^n = 2.4 + 1 = 3.4$$

$$n \log 1.12 = \log 3.4$$

$$n \times 0.0492 = 0.5315$$

$$n = \frac{0.5315}{0.0492} = 10.8 \text{ years}$$

You will have to wait for about 11 years.

6.5 PRESENT VALUE OF AN ANNUITY

Suppose you expect to receive ₹ 1,000 annually for 3 years, each receipt occurring at the end of the year. What is the present value of this stream of benefits if the discount rate is 10 percent? The present value of this annuity is simply the sum of the present values of all the inflows of this annuity:

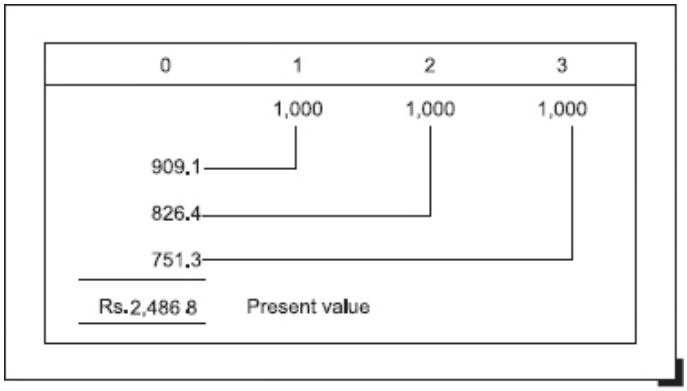
$$₹\,1{,}000\left[\frac{1}{1.10}\right] + ₹\,1{,}000\left[\frac{1}{1.10}\right]^2 + ₹\,1{,}000\left[\frac{1}{1.10}\right]^3$$

$$= ₹\,1{,}000 \times 0.9091 + ₹\,1{,}000 \times 0.8264 + ₹\,1{,}000 \times 0.7513$$

$$= ₹\,2{,}486.8$$

The time line for this problem is shown in Exhibit 6.9.

Exhibit 6.9 **Time Line**



Formula

In general terms the present value of an annuity may be expressed as follows:

$$\begin{aligned} \text{PVA}_n &= \frac{A}{(1+r)} + \frac{A}{(1+r)^2} + \cdots + \frac{A}{(1+r)^{n-1}} + \frac{A}{(1+r)^n} \\ &= A\left[\frac{1}{(1+r)} + \frac{1}{(1+r)^2} + \ldots + \frac{1}{(1+r)^{n-1}} + \frac{1}{(1+r)^n}\right] \\ &= A\,[\{1-(1/1+r)^n\}/r] \end{aligned} \qquad (6.6)^2$$

where PVA_n is the present value of an annuity which has a duration of n periods, A is the constant periodic flow, and r is the discount rate.

$[\{1-(1/1+r)^n\}/r]$ is referred to as the present value interest factor for an annuity ($\text{PVIFA}_{r,n}$). It is, as can be seen clearly, simply equal to the product of the future value interest factor for an annuity ($\text{FVIFA}_{r,n}$) and the present value interest factor ($\text{PVIF}_{r,n}$). Exhibit 6.10 shows the value of $\text{PVIFA}_{r,n}$ for several combinations of r and n. A more detailed table of $\text{PVIFA}_{r,n}$ values is found in Appendix A at the end of this book.

Exhibit 6.10 **Value of $\text{PVIFA}_{r,n}$ for Different Combinations of r and n**

n/r	*6%*	*8%*	*10%*	*12%*	*14%*
2	1.833	1.783	1.737	1.690	1.647
4	3.465	3.312	3.170	3.037	2.914
6	4.917	4.623	4.355	4.111	3.889
8	6.210	5.747	5.335	4.968	4.639
10	7.360	6.710	6.145	5.650	5.216
12	8.384	7.536	6.814	6.194	5.660

Applications

The present value annuity formula can be applied in a variety of contexts. Its important applications are discussed below.

How Much Can You Borrow for a Car After reviewing your budget, you have determined that you can afford to pay ₹ 12,000 per month for 3 years toward a new car. You call a finance company and learn that the going rate of interest on car finance is 1.5 percent per month for 36 months. How much can you borrow?

To determine how much you can borrow, we have to calculate the present value of ₹ 12,000 per month for 36 months at 1.5 percent per month.

Since the loan payments are an ordinary annuity, the present value interest factor of annuity is:

$$\text{PVIFA}_{r,n} = \frac{1-\frac{1}{(1+r)^n}}{r} = \frac{1-\frac{1}{(1.015)^{36}}}{0.015} = 27.66$$

Hence the present value of 36 payments of ₹ 12,000 each is:

Present value = ₹ 12,000 × 27.66 = ₹ 331,920

You can, therefore, borrow ₹ 331,920 to buy the car.

The above can be worked out in a spreadsheet as shown below:

	A	B	C	D	E
1	Monthly payment (Pmt) ₹	12,000			
2	Period in months (Nper)	36	Present value	→	331,928
3	Rate of interest per month (Rate)	1.50%	= PV (B3, B2, –B1)		

Period of Loan Amortisation You want to borrow ₹ 1,080,000 to buy a flat. You approach a housing finance company which charges 12.5 percent interest. You can pay ₹ 180,000 per year toward loan amortisation. What should be the maturity period of the loan?

The present value of annuity of ₹ 180,000 is set equal to ₹ 1,080,000.

$$180{,}000 \times \text{PVIFA}_{n,r} = 1{,}080{,}000$$

$$180{,}000 \times \text{PVIFA}_{n=?,\, r=12.5\%} = 1{,}080{,}000$$

$$180{,}000 \left[\frac{1-\frac{1}{(1.125)^n}}{0.125}\right] = 1{,}080{,}000$$

Given this equality the value of *n* is calculated as follows:

$$\frac{1-\frac{1}{(1.125)^n}}{0.125} = \frac{1,080,000}{180,000} = 6$$

$$\frac{1}{(1.125)^n} = 0.25$$

$$1.125^n = 4$$

$$n \log 1.125 = \log 4$$

$$n \times 0.0512 = 0.6021$$

$$n = \frac{0.6021}{0.0512} = 11.76 \text{ years}$$

You can perhaps request for a maturity of 12 years.

Determining the Loan Amortisation Schedule Most loans are repaid in equal periodic instalments (monthly, quarterly, or annually), which cover interest as well as principal repayment. Such loans are referred to as **amortised loans**.

For an amortised loan we would like to know (a) the periodic instalment payment and (b) the loan amortisation schedule showing the break up of the periodic instalment payments between the interest component and the principal repayment component. To illustrate how these are calculated, let us look an example.

Suppose a firm borrows ₹ 1,000,000 at an interest rate of 15 percent and the loan is to be repaid in 5 equal instalments payable at the end of each of the next 5 years. The annual instalment payment A is obtained by solving the following equation.

$$\text{Loan amount} = A \times \text{PVIFA}_{n=5,\, r=15\%}$$

$$1,000,000 = A \times 3.3522$$

$$\text{Hence } A = 298,312$$

The amortisation schedule is shown in Exhibit 6.11. The interest component is the largest for year 1 and progressively declines as the outstanding loan amount decreases.

Exhibit 6.11 Loan Amortisation Schedule

Year	Beginning Amount (1)	Annual Instalment (2)	Interest (3)	Principal Repayment (2)-(3) = (4)	Remaining Balance (1)-(4) = (5)
1	1,000,000	298,312	150,000[a]	148,312[b]	851,688
2	851,688	298,312	127,753	170,559	681,129
3	681,129	298,312	102,169	196,143	484,986
4	484,986	298,312	72,748	225,564	259,422
5	259,422	298,312	38,913	259,399	23*

a. Interest is calculated by multiplying the beginning loan balance by the interest rate.
b. Principal repayment is equal to annual instalment minus interest.
* Due to rounding off error a small balance is shown.

The above schedule can be set up using a spreadsheet as below:

	A	B	C	D	E	F
1		Present value	Interest rate	No. of instalments (in years)	Annual instalment amount	
2		1,000,000	15%	5	298,316	
3	Year	Beginning amount	Annual instalment	Interest	Principal repayment	Remaining balance
4	1	1,000,000	298,316	150000	148,316	851,684
5	2	851,684	298,316	127,753	170,563	681,121
6	3	681,121	298,316	102,168	196,148	484,973
7	4	484,973	298,316	72,746	225,570	259,403
8	5	259,403	298,316	38,910	259,406	–3

To create the above spreadsheet, proceed as follows: In B4, type =B2 to get the beginning amount. To get the instalment amount in C4, type =E2 and press F4. A $ sign will appear before E and 2 (E2). This will make the value in this cell absolute, that is, constant throughout. Use the formula =B4*C2 to get interest amount in D4 (note that C2 here is made absolute by pressing F4). Fill in the principal repayment amount in E4 using the formula =C4-D4 and the remaining balance in F4 using the formula =B4-E4. Copy this value to B5 by typing =F4. Next, click on C4. Observe that there is a tiny black box at the lower right corner of the cell. This is called a fill handle. Point the cursor to the fill handle (it will turn into a black cross) and drag it down upto C8. This will autofil the value in C4 (whether an absolute value or a formula) upto C8. Use the fill handle to autofil all the remaining cells by dragging down the values in the respective cells above them.

Determining the Periodic Withdrawal Your father deposits ₹ 300,000 on retirement in a bank which pays 10 percent annual interest. How much can be withdrawn annually for a period of 10 years?

$$A = ₹\ 300{,}000 \times \frac{1}{PVIFA_{10\%10}}$$

$$= ₹\ 300{,}000 \times \frac{1}{6.145}$$

$$= ₹\ 48{,}820$$

A spreadsheet calculation of the above is as under.

	A	B	C	D	E
1	Initial deposit ₹	300,000			
2	Interest rate	10%	Annual withdrawal	→	48,824
3	Period years	10	= PMT (B2, B3, –B1)		

Finding the Interest Rate Suppose someone offers you the following financial contract: If you deposit ₹ 10,000 with him he promises to pay ₹ 2,500 annually for 6 years. What interest rate do you earn on this deposit? The interest rate may be calculated in two steps:

Step 1 Find the $PVIFA_{r,6}$ for this contract by dividing ₹ 10,000 by ₹ 2,500

$$PVIFA_{r,6} = \frac{₹\ 10{,}000}{₹\ 2{,}500} = 4.000$$

Step 2 Look at the PVIFA table and read the row corresponding to 6 years until you find a value close to 4.000. Doing so, you find that

$$PVIFA_{12\%,6} \text{ is } 4.111 \text{ and } PVIFA_{14\%,6} \text{ is } 3.889$$

Since 4,000 lies in the middle of these values the interest rate lies (approximately) in the middle. So, the interest rate is 13 percent.

Valuing an Infrequent Annuity Raghavan will receive an annuity of ₹ 50,000, payable once every two years. The payments will stretch out over 30 years. The first payment will be received at the end of two years. If the annual interest rate is 8 percent, what is the present value of the annuity?

The interest rate over a two-year period is:

$$(1.08) \times (1.08) - 1 = 16.64 \text{ percent}$$

This means that ₹ 100 invested over two years will yield ₹ 116.64.

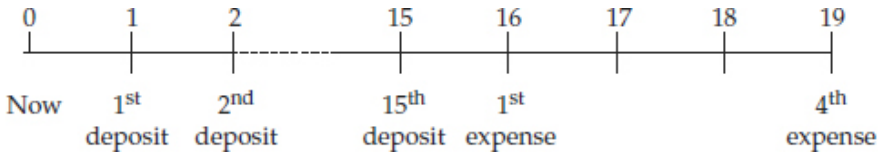
We have to calculate the present value of a ₹ 50,000 annuity over 15 periods, with an interest rate of 16.64 percent per period. This works out to:

$$₹\ 50{,}000\ [\{\ 1 - (1/1.1664)^{15}\ \}]\ /\ 0.1664 = ₹\ 270{,}620$$

Equating Present Value of Two Annuities Ravi wants to save for the college education of his son, Deepak. Ravi estimates that the college education expenses will be rupees one million per year for four years when his son reaches college in 16 years – the expenses will be payable at the beginning of the years. He expects the annual interest rate of 8 percent

over the next two decades. How much money should he deposit in the bank each year for the next 15 years (assume that the deposit is made at the end of the year) to take care of his son's college education expenses?

The time line for this problem is as follows:



The present value of college education expenses when his son becomes 15 years old is:

$$₹\ 1{,}000{,}000 \times \text{PVIFA (4 years, 8\%)}$$
$$= ₹\ 1{,}000{,}000 \times 3.312 = ₹\ 3{,}312{,}000$$

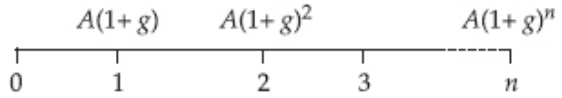
The annual deposit to be made so that the future value of the deposits at the end of 15 years is ₹ 3,312,000 is:

$$A = \frac{₹\ 3{,}312{,}000}{\text{FVIFA(15 years, 8\%)}} = \frac{₹\ 3{,}312{,}000}{27.152}$$

$$= ₹\ 121{,}980$$

Present Value of a Growing Annuity

A cash flow that grows at a constant rate for a specified period of time is a growing annuity. If *A* is the current cash flow, the time line of a growing annuity is shown below:



The present value of a growing annuity can be determined using the following formula:

$$\text{PV of a Growing Annuity} = A(1+g)\left[\frac{1-\frac{(1+g)^n}{(1+r)^n}}{r-g}\right] \quad (6.7)$$

3

The above formula can be used when the growth rate is less than the discount rate ($g < r$) as well as when the growth rate is more than the discount rate ($g > r$). However, it does not work when the growth rate is equal to the discount rate ($g = r$) - in this case, the present value is simply equal to $n\,A$.

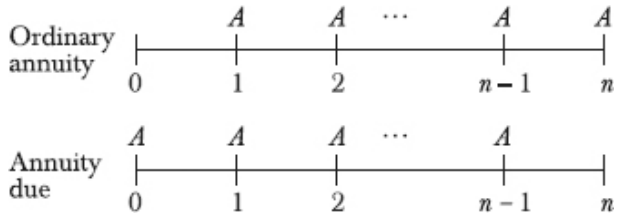
For example, suppose you have the right to harvest a teak plantation for the next 20 years over which you expect to get 100,000 cubic feet of teak per year. The current price per cubic foot of teak is ₹ 500, but it is expected to increase at a rate of 8 percent per year. The discount rate is 15 percent. The present value of the teak that you can harvest from the teak plantation can be determined as follows:

$$\text{PV of teak} = ₹\ 500 \times 100{,}000\ (1.08)\left[\frac{1-\frac{1.08^{20}}{1.15^{20}}}{0.15-0.08}\right]$$

$$= ₹\ 551{,}736{,}683$$

A Note on Annuities Due

So far we discussed ordinary annuities in which cash flows occur at the end of each period. There is a variation, which is fairly common, in which cash flows occur at the beginning of each period. Such an annuity is called an **annuity due**. For example, when you enter into a lease for an apartment, the lease payments are due at the beginning of the month. The first lease payment is made at the beginning; the second lease payment is due at the beginning of the second month, so on and so forth. The time lines for ordinary annuity and annuity due are shown below:



Since the cash flows of an annuity due occur one period earlier in comparison to the cash flows on an ordinary annuity, the following relationship holds:

$$\text{Annuity due value} = \text{Ordinary annuity value} \times (1 + r)$$

This applies for both present and future values. So, two steps are involved in calculating the value of an annuity due. First, calculate the present or future value as though it were an ordinary annuity. Second, multiply your answer by (1 + *r*).

QUESTIONS

1. Why does money have time value?
2. State the general formula for the future value of a single amount.
3. What is the difference between compound and simple interest?
4. Explain the rule of 72.
5. Explain the rule of 69. How does it compare with the rule of 72?
6. State the general formula for calculating the present value of a single amount.
7. What is an annuity? What is the difference between an ordinary annuity and an annuity due?
8. State the formula for the future value of an annuity.
9. State the formula for the present value of an annuity.
10. What is a growing annuity? What is the formula for finding the present value of a growing annuity?
11. What is the formula for the present value of a perpetuity?
12. State the formula for the future value of a single cash flow after *n* years when compounding is done *m* times a year.
13. What is the relationship between the effective interest rate and the stated interest rate?
14. State the formula for calculating the present value of a single cash flow when discounting is done *m* times a year.
15. A firm's earnings grew from ₹ 1 per share to ₹ 3 per share over a period of 10 years. The total growth was 200 percent, but the annual compound growth rate was less than 20 percent. Why?

SOLVED PROBLEMS

6.1 If you invest ₹ 5,000 today at a compound interest of 9 percent, what will be its future value after 75 years?

Solution The future value of ₹ 5,000 after 75 years, when it earns a compound interest of 9 percent, is

$$₹\ 5{,}000\ (1.09)^{75}$$

Since the FVIF table given in Appendix A has a maximum period of 30, the future value expression may be stated as

$$₹\ 5{,}000\ (1.09)^{30}\ (1.09)^{30}\ (1.09)^{15}$$

The above product is equal to

$$₹\ 5{,}000\ (13.268)\ (13.268)\ (3.642) = ₹\ 32{,}05{,}685.1$$

6.2 If the interest rate is 12 percent, what are the doubling periods as per the rule of 72 and the rule of 69 respectively?

Solution As per the rule of 72 the doubling period will be

$$72 / 12 = 6 \text{ years}$$

As per the rule of 69, the doubling period will be

$$0.35 + \frac{69}{12} = 6.1 \text{ years}$$

6.3 A borrower offers 16 percent nominal rate of interest with quarterly compounding. What is the effective rate of interest?

Solution The effective rate of interest is

$$\left[1+\frac{0.16}{4}\right]^4 - 1 = (1.04)^4 - 1$$
$$= 1.17 - 1$$
$$= 0.17 = 17 \text{ percent}$$

6.4 Fifteen annual payments of ₹ 5,000 are made into a deposit account that pays 14 percent interest per year. What is the future value of this annuity at the end of 15 years?

Solution The future value of this annuity will be:

$$₹\ 5{,}000\ (\text{FVIFA}_{14\%,15}) = ₹\ 5{,}000\ (43.842)$$
$$= ₹\ 2{,}19{,}210$$

6.5 A finance company advertises that it will pay a lumpsum of ₹ 44,650 at the end of five years to investors who deposit annually ₹ 6,000 for 5 years. What is the interest rate implicit in this offer?

Solution The interest rate may be calculated in two steps

(a) Find the FVIFA for this contract as follows:

$$₹\ 6{,}000\ (\text{FVIFA}) = ₹\ 44{,}650$$
$$\text{So}$$
$$\text{FVIFA} = \frac{₹\ 44{,}650}{₹\ 6{,}000} = 7.442$$

(b) Look at the FVIFA table and read the row corresponding to 5 years until 7.442 or a value close to it is reached. Doing so we find that

$\text{FVIFA}_{20\%,5\text{yrs}}$ is 7.442

So, we conclude that the interest rate is 20 percent.

6.6 What is the present value of ₹ 1,000,000 receivable 60 years from now, if the discount rate is 10 percent?

Solution The present value is

$$₹\ 1{,}000{,}000 \left[\frac{1}{1.10}\right]^{60}$$

This may be expressed as

$$₹\ 1{,}000{,}000\left[\frac{1}{1.10}\right]^{30}\left[\frac{1}{1.10}\right]^{30}$$

$$= ₹\ 1{,}000{,}000\ (0.057)\ (0.057) = ₹\ 3249$$

6.7 A 12 – payment annuity of ₹ 10,000 will begin 8 years hence. (The first payment occurs at the end of 8 years). What is the present value of this annuity if the discount rate is 14 percent?

Solution This problem may be solved in two steps.

Step 1 Determine the value of this annuity a year before the first payment begins, i.e., 7 years from now. This is equal to:

$$₹\ 10{,}000\ (\text{PVIFA}_{14\%,12\ \text{years}}) = ₹\ 10{,}000\ (5.660)$$
$$= ₹\ 56{,}600$$

Step 2 Compute the present value of the amount obtained in Step1:

$$₹\ 56{,}600\ (\text{PVIF}_{14\%,\ 7\ \text{years}}) = ₹\ 56{,}600\ (0.400)$$
$$= ₹\ 22{,}640$$

6.8 What is the present value of the following cash stream if the discount rate is 14 percent?

Year	*0*	*1*	*2*	*3*	*4*
Cash flow	5,000	6,000	8,000	9,000	8,000

Solution The present value of the above cash flow stream is:

$$5{,}000\,(1.000) + 6{,}000\,(0.877) + 8{,}000\,(0.769) + 9{,}000\,(0.675) + 8{,}000\,(0.592) = 27{,}225$$

6.9 Mahesh deposits ₹ 200,000 in a bank account which pays 10 percent interest. How much can he withdraw annually for a period of 15 years?

Solution The annual withdrawal is equal to:

$$\frac{₹\ 200{,}000}{\text{PVIFA}_{10\%,\ 15\ \text{yrs}}} = \frac{₹\ 200{,}000}{7{,}606} = ₹\ 26{,}295$$

6.10 You want to take a world tour which costs ₹ 1,000,000 – the cost is expected to remain unchanged in nominal terms. You are willing to save annually ₹ 80,000 to fulfill your desire. How long will you have to wait if your savings earn a return of 14 percent per annum?

Solution The future value of an annuity of ₹ 80,000 that earns 14 percent is equated to ₹ 1,000,000.

$$80{,}000 \times \text{FVIFA}_{n\ =\ ?,14\%} = 1{,}000{,}000$$

$$80{,}000\left[\frac{1.14^n - 1}{0.14}\right] = 1{,}000{,}000$$

$$1.14^n - 1 = \frac{1{,}000{,}000}{80{,}000} \times 0.14 = 1.75$$

$$1.14^n = 1.75 + 1 = 2.75$$

$$n \log 1.14 = \log 2.75$$

$$n \times .0569 = 0.4393$$

$$n = 0.4393\ /\ 0.0569 = 7.72\ \text{years}$$

You will have to wait for 7.72 years.

6.11 Shyam borrows ₹ 80,000 for a musical system at a monthly interest of 1.25 percent. The loan is to be repaid in 12 equal monthly instalments, payable at the end of each month. Prepare the loan amortisation schedule.

Solution

The monthly instalment A is obtained by solving the equation:

$$80{,}000 = A \times \text{PVIFA}_{n=12,\, r=1.25\%}$$

$$80{,}000 = A \times \frac{1 - \frac{1}{(1+r)^n}}{r}$$

$$80{,}000 = A \times \frac{1 - \frac{1}{(1.0125)^{12}}}{.0125}$$

$$= A \times 11.0786$$

Hence A = 80,000 / 11.0786 = ₹ 7221

The loan amortisation schedule is shown below:

Loan Amortisation Schedule

Month	*Beginning Amount* (1)	*Monthly Instalment* (2)	*Interest* (3)	*Principal Repayment* (2)–(3) = (4)	*Remaining Balance* (1)–(4) = (5)
1	80,000	7221	1000	6221	73779
2	73,779	7221	922.2	6298.8	67480.2
3	67,480.2	7221	843.5	6377.5	61102.7
4	61102.7	7221	763.8	6457.2	54645.5
5	54645.5	7221	683.1	6537.9	48107.6
6	48107.6	7221	601.3	6619.7	41487.9
7	41487.9	7221	518.6	6702.4	34785.5
8	34785.5	7221	434.8	6786.2	27999.3
9	27999.3	7221	350.0	6871.0	21128.3
10	21128.3	7221	264.1	6956.9	14171.4
11	14171.4	7221	177.1	7043.9	7127.1
12	7127.1	7221	89.1	7131.9	– 4.8@

@ Rounding off error

PROBLEMS

6.1 Future Value Calculate the value 5 years hence of a deposit of ₹ 1,000 made today if the interest rate is (a) 8 percent, (b) 10 percent, (c) 12 percent, and (d) 15 percent.

6.2 Rule of 72 If you deposit ₹ 5,000 today at 12 percent rate of interest in how many years (roughly) will this amount grow to ₹ 1,60,000? Work out this

problem using the *rule of* 72–do not use tables.

6.3 **Rule of 69** A finance company offers to give ₹ 8,000 after 12 years in return for ₹ 1,000 deposited today. Using the *rule of 69*, figure out the approximate interest offered.

6.4 **Future Value** You can save ₹ 2,000 a year for 5 years, and ₹ 3,000 a year for 10 years thereafter. What will these savings cumulate to at the end of 15 years, if the rate of interest is 10 percent?

6.5 **Annual Savings** Mr. Vinay plans to send his son for higher studies abroad after 10 years. He expects the cost of these studies to be ₹ 1,000,000. How much should he save annually to have a sum of ₹ 1000,000 at the end of 10 years, if the interest rate is 12 percent?

6.6 **Interest Rate** A finance company advertises that it will pay a lump sum of ₹ 10,000 at the end of 6 years to investors who deposit annually ₹ 1,000. What interest rate is implicit in this offer?

6.7 **Interest Rate** Someone promises to give you ₹ 5,000 after 10 years in exchange for ₹ 1,000 today. What interest rate is implicit in this offer?

6.8 **Present Value** Find the present value of ₹ 10,000 receivable after 8 years if the rate of discount is (i) 10 percent, (ii) 12 percent, and (iii) 15 percent.

6.9 **Present Value** What is the present value of a 5-year annuity of ₹ 2,000 at 10 percent?

6.10 **Retirement Plan** On retirement, Mr. Jingo is given a choice between two alternatives: (a) an annual pension of ₹ 10,000 as long as he lives, and (b) a lump sum amount of ₹ 50,000. If Mr. Jingo expects to live for 15 years and the interest rate is 15 percent, which option is more attractive?

6.11 **Annual Withdrawal** Mr. X deposits ₹ 1,00,000 in a bank which pays 10 percent interest. How much can he withdraw annually for a period of 30 years. Assume that at the end of 30 years the amount deposited will whittle down to zero.

6.12 **Present Value** What is the present value of an income stream which provides ₹ 1,000 at the end of year one, ₹ 2,500 at the end of year two, and ₹ 5,000 during each of the years 3 through 10, if the discount rate is 12 percent?

6.13 **Present Value** What is the present value of an income stream which provides ₹ 2,000 a year for the first five years and ₹ 3,000 a year forever thereafter, if the discount rate is 10 percent?

Hint: The present value for a perpetual annuity is derived by dividing the constant annual flow by the discount factor.

6.14 **Deposit** What amount must be deposited today in order to earn an annual income of ₹ 5,000 beginning from the end of 15 years from now? The deposit earns 10 percent per year.

6.15 **Interest Rate** Suppose someone offers you the following financial contract. If you deposit ₹ 20,000 with him he promises to pay ₹ 4,000 annually for 10 years. What is the interest rate?

6.16 **Present Value** What is the present value of the following cash flow streams?

Year	1	2	3	4	5	6	7	8	9	10
A	100	200	300	400	500	600	700	800	900	1000
B	1000	900	800	700	600	500	400	300	200	100
C	500	500	500	500	500	500	500	500	500	500

The discount rate is 12 percent.

6.17 **Future Value** Suppose you deposit ₹ 10,000 with an investment company which pays 16 percent interest with quarterly compounding. How much will this deposit grow to in 5 years?

6.18 **Future Value** How much would a deposit of ₹ 5,000 at the end of 5 years be, if the interest rate is 12 percent and if the compounding is done quarterly?

6.19 **EAR and APR** What is the difference between the effective rate of interest and annual percentage rate in the following cases:

Case A: APR 12 percent and the frequency of compounding is six times a year.

Case B: APR is 24 percent and the frequency of compounding is four times a year.

Case C: APR is 24 percent and the frequency of compounding is twelve times a year.

6.20 **Investment** If the interest rate is 12 percent how much investment is required now to yield an in come of ₹ 12,000 per year from the beginning of the 10^{th} year and continuing thereafter forever?

6.21 **Preference** You have a choice between ₹ 5,000 now and ₹ 20,000 after 10 years. Which would you choose? What does your preference indicate?

6.22 **Value** Mr. Raghu deposits ₹ 10,000 in a bank now. The interest rate is 10 percent and compounding is done semi-annually. What will the deposit grow to after 10 years? If the inflation rate is 8 percent per year, what will be the value of the deposit after 10 years in terms of the current rupee?

6.23 **Deposit** How much should be deposited at the beginning of each year for 10 years in order to provide a sum of ₹ 50,000 at the end of 10 years?

6.24 **Deposit** A person requires ₹ 20,000 at the beginning of each year from 2035 to 2039. How much should he deposit at the end of each year from 2025 to 2030? The interest rate is 12 percent.

6.25 **Present Value** What is the present value of ₹ 2,000 receivable annually for 30 years? The first receipt occurs after 10 years and the discount rate is 10 percent.

6.26 **Borrowing** After five years Mr. Ramesh will receive a pension of ₹ 6000 per month for 15 years. How much can Mr. Ramesh borrow now at 12 percent interest so that the borrowed amount can be paid with 30 percent of the pension amount? The interest will be accumulated till the first pension amount becomes receivable.

6.27 **Interest Rate** Mr. Prakash buys a motorcycle with a bank loan of ₹ 60,000. An instalment of ₹ 3000 is payable to the bank for each of 24 months towards the repayment of loan with interest. What interest rate does the bank charge?

6.28 **Sinking Fund Deposit** A Ltd. has to retire ₹ 1000 million of debentures each at the end of 8, 9, and 10 years from now. How much should the firm deposit in a sinking fund account annually for 5 years, in order to retire the debenture? The net interest rate earned is 8 percent.

6.29 **Period of Withdrawal** B receives a provident fund amount of ₹ 1,000,000. He deposits it in a bank which pays 10 percent interest. If he withdraws annually ₹ 200,000, how long can he do so?

6.30 **Loan Amortisation** Phoenix Company borrows ₹ 500,000 at an interest rate of 14 percent. The loan is to be repaid in 4 equal annual instalments payable at the end of each of the next 4 years. Prepare the loan amortisation schedule.

6.31 **Maturity Period** You want to borrow ₹ 1,500,000 to buy a flat. You approach a housing company which charges 13 percent interest. You can pay ₹ 200,000 per year toward loan amortisation. What should be the maturity period of the loan?

6.32 **Present Value of Growing Annuity** You are negotiating with the government the right to mine 100,000 tons of iron ore per year for 15 years. The price per ton of iron ore is expected to be ₹ 3,000 at the end of year 1 and increase thereafter at the rate of 6 percent per year. What is the present value of the iron ore that you can mine if the discount rate is 16 percent?

6.33 **Present Value** As a winner of a competition, you can choose one of the following prizes:

a. ₹ 500,000 now

b. ₹ 1,000,000 at the end of 6 years

c. ₹ 60,000 a year forever

d. ₹ 100,000 per year for 10 years

e. ₹ 35,000 next year and rising thereafter by 5 percent per year forever.

If the interest rate is 10 percent, which prize has the highest present value.

6.34 **Present Value of a Decreasing Annuity** Pipe India owns an oil pipeline which will generate ₹ 12 crore of cash income in the coming year. It has a very long life with virtually negligible operating costs. The volume of oil shipped, however, will decline over time and, hence, cash flows will decrease by 3 percent per year. The discount rate is 12 percent.

a. If the pipeline is used forever, what is the present value of its cash flows?

b. If the pipeline is scrapped after 25 years, what is the present value of its cash flows?

6.35 **Present Value of a Decreasing Annuity** An oil well presently produces 50,000 barrels per year. It will last for 15 years more, but the production will fall by 5 percent per year. Oil prices are expected to increase by 3 percent per year. Presently the price of oil is $50 per barrel. What is the present value of the well's production if the discount rate is 10 percent?

6.36 **Present Value of a Decreasing Annuity** An oil well presently produces 80,000 barrels per year. It will last for 20 years more, but the production will fall by 6

percent per year. Oil prices are expected to increase by 4 percent per year. Currently the price of oil is $60 per barrel. What is the present value of the well's production if the discount rate is 12 percent?

6.37 **Future Value** You are considering whether your savings will be enough to meet your retirement needs. You saved ₹ 100,000 last year and you expect your annual savings to increase by 8 percent per year for the next 20 years. If your savings can be invested at 9 percent, how much would you have at the end of the twentieth year? Hint: Future Value Growing Annuity = PVGA $(1 + r)^n$

6.38 **EAR** A bank offers an interest rate of 8 percent on deposits made with it. If the compounding is done on a weekly basis, what is the effective interest rate?

6.39 **Interest Rate** Apna Bank's Kuber deposit plan offers to double your deposit in 7 years under its special daily compounding of interest scheme. What is the interest rate involved?

6.40 **Amortisation Schedule** Monisha has bought an iPhone costing ₹ 100,000 at 9.5 percent p.a, repayable in 5 equated annual instalments. Draw the amortisation schedule for the loan.

6.41 **Waiting Period** James has now joined as a finance manager in an MNC. He can save every year 60 percent of his annual salary of ₹ 10 lakhs (that will be received at the end of the year) and which is expected to increase at the rate of 10 percent every year. He has decided to marry only once his savings crosses ₹ 1 crore mark. If he keeps his savings in a bank that offers 8 percent interest, how many years should he wait to get married?

6.42 **Growing Annuities** Prakash plans to retire after 20 years with a corpus of ₹ 50 million. He receives salary annually and he expects to receive ₹ 3 million at the end of the current year. His salary will increase at the rate of 10 percent per year and he can earn 9 percent on his investment. He plans to save a constant percentage of his salary; what should that percentage be?

6.43 **Discount Interest Loans** You borrow ₹ 100,000 for one year at an interest rate of 15 percent from a lender who deducts the interest in advance from the loan upfront and gives you ₹ 85000. What is the EAR?

6.44 **Break-even Investment Returns** Your investment company offers two different investment plans. Plan A offers a ten-year annuity of ₹ 500,000 whereas plan B offers an annual perpetuity of ₹ 30,0000. Both plans make their first payment a year from today. What discount rate will make you indifferent between the two options?

6.45 **Present Value of Infrequent Annuity** An investment pays ₹ 1,000,000 every four years for 40 years. The first payment will occur after four years. What is the value of this investment if the discount rate is 12 percent and the discounting is continuous?

MINICASE - I

As an investment advisor, you have been approached by a client called Ramesh, who wants some help in investment related matters.

Ramesh is currently 45 years old and has ₹ 600,000 in the bank. He plans to work for 15 more years and retire at the age of 60. Ramesh's present salary is ₹ 400,000 per year. He expects his salary to increase at the rate of 12 percent per year until his retirement.

Ramesh has decided to invest his bank balance and future savings in a portfolio in which stocks and bonds would be equally weighted. For the sake of simplicity, assume that these proportions will be maintained by him throughout. He also believes that bonds would provide a return of 7 percent and stocks a return of 13 percent. You concur with his assessment.

Once Ramesh retires at the age of 60 he would like to withdraw ₹ 500,000 per year from his investments for the following 15 years as he expects to live upto the age of 75 years. He also wants to bequeath ₹ 1,000,000 to his children at the end of his life. How much money would he need 15 years from now?

How much should Ramesh save each year for the next 15 years to be able to meet his investment objectives spelt out above? Assume that the savings will occur at the end of each year.

Suppose Ramesh wants to donate ₹ 200,000 each year in the last three years of his life to a charitable cause. Each donation would he made at the beginning of the year. How much money would he need when he reaches the age of 60 to meet this specific need?

Ramesh recently attended a seminar on human capital where the speaker talked about a person's human capital as the present value of his life time earnings. Ramesh is curious to find out the present value of his lifetime salary. For the sake of simplicity assume that his present salary of ₹ 400,000 will be paid exactly one year from now, and his salary will be paid in annual installments. What is the present value of his life time salary, if the discount rate is 8 percent? Remember that Ramesh expects his salary to increase at the rate of 12 percent per year until his retirement.

In answering the above questions, ignore the tax factor.

MINICASE - II

Sardar Kartar Singh is a resident of Thailand for the past two decades and is the owner of a flourishing business there. He has a son, Satnam, 10 years old and a baby girl Jasleen who will be one year old this day. The family has come to India to celebrate her birthday in Punjab. Also, Kartar's wife has made some grand plans for the future financial security of the family and they intend to use their present visit for placing suitable deposits with their bank in New Delhi as per those plans.

According to the plan, Satnam would be doing his MBA after 10 years. It would be a two year course in a premier private business school in India. For that the all inclusive expenditure at present rates would be ₹ 20 lakhs and ₹ 25 lakhs in the beginning of the first and second year respectively. Jasleen would marry at the end of

her 21st year and for that an amount of ₹ 3 crores would then be needed. Kartar's wife is insistent that her presence would be essential in India in the best interests of both the children-to keep a watchful eye on Satnam during his stint at the business school and most importantly, to have ample time to renew the old network with family and friends for ensuring a very good match for the girl. Funds would have to be tied up for her and children's relocation to India at the end of ten years from now.

Kartar Singh always had great respect for his wife's commonsense and logic (though he was always shy of acknowledging it!). To arrange the funds, he has very recently sold one of his investments, a flat in a prime locality in Bangkok, for a hefty sum. For Satnam's MBA he has decided to open two recurring deposit accounts, maturing on the 10^{th} and 11^{th} years respectively. For Jasleen's marriage he wants to open a cumulative term deposit for 20 years. For family maintenance in India after 10 years, he wants to open another cumulative term deposit for 10 years with the maturity value of which he could immediately purchase an annuity due for the following 10 years. It is expected that after 10 years the family in India would need ₹ 12 lakhs per year without taking inflation into consideration.

To make the calculations on the specific amounts needed he has called you, an upcoming financial consultant. He asks you to make the calculations in such a way that he could easily understand the logic thereof. You understand from him that as all the deposits would be made out of his NRE account with the bank, it would not deduct any tax amount from the interest to be earned.

Specifically you are required to calculate the amounts that need to be deposited now in:

(i) the two recurring deposit accounts, in the beginning of each month.

(ii) a cumulative fixed deposit for meeting the cost of Jasleen's marriage.

(iii) a cumulative fixed deposit with the bank for purchasing the annuity due needed by the family in India after 10 years from an insurance company which is expected to give a return of 10 percent per year.

You set to work with the following data:

For both cumulative fixed deposit and Recurring deposit, nominal interest rate for periods of more than 5 years is 8 percent and compounding is done once in a quarter. Inflation in India after 10 years is expected to be 5 percent for the next ten years. The MBA course expenses are likely to grow at 5 percent per annum.

Show your detailed working.

7.1 VALUATION CONCEPTS

The term value is used in different senses. Hence, let us briefly review the differences that exist among the major concepts of value.

Liquidation Value versus Going Concern Value The **liquidation value** is the amount that can be realised when an asset, or a group of assets representing a part or even the whole of a firm, is sold separately from the operating organisation to which it belongs. In contrast, the **going concern value** represents the amount that can be realised if the firm is sold as a continuing operating entity.

In general, security valuation models assume a going concern, an operating business entity that generates cash flows to its security holders. When the going concern assumption is not appropriate as in the case of an impending bankruptcy, liquidation value of assets is more relevant in determining the worth of the firm's financial securities.

Book Value versus Market Value The **book value** of an asset is the accounting value of the asset, which is simply the historical cost of the asset less accumulated depreciation or amortisation as the case may be. The book value of a firm's equity is equal to the book value of its assets minus the book value of its liabilities. Because book value reflects a historical accounting value it may diverge significantly from market value. However, under IFRS accounting, which is expected to be adopted in India, more assets are likely to be reported at "fair values." So, the traditional divergence between book value and market value may diminish.

The **market value** of an asset is simply the market price at which the asset trades in the market place. Often the market value is greater than the book value.

Market Value versus Intrinsic Value As the nomenclature suggests, the market value of a security is the price at which the security trades in the financial market.

The **intrinsic value** of a security is the present value of the cash flow stream expected from the security, discounted at a rate of return appropriate for the risk associated with the security. Put differently, intrinsic value is economic value. If the market is reasonably efficient, the market price of the security should hover around its intrinsic value. The focus of this chapter is on establishing a security's intrinsic value.

7.2 BOND VALUATION

A bond represents a contract under which a borrower promises to pay interest and principal on specific dates to the holders of the bond.

Bonds are issued by a variety of organisations. The principal issuers of bonds in India are the central government, state governments, public sector undertakings, private sector companies, and municipal bodies.

Bonds issued by the central government are called *Treasury bonds.* These are bonds which have maturities ranging upto 30 years. These bonds generally pay interest semi-annually. Presently, Treasury bonds dominate the Indian bond market in terms of market capitalisation, liquidity, and turnover.

State government bonds are issued by the state governments. These bonds have maturities that generally range from 3 to 20 years and pay interest semi-annually.

Bonds issued by companies are classified into two types: PSU (public sector undertakings) bonds and private sector bonds. *PSU bonds* are bonds issued by companies in which the central or state governments have an equity stake in excess of 50 percent. Some of these bonds enjoy a tax-free status whereas others are taxable.

Private sector bonds are bonds issued by private sector companies. Bonds issued by companies, PSU bonds as well as private sector bonds, generally have maturity ranging from 1 year to 15 years and pay interest semi-annually.

Terminology In order to understand the valuation of bonds, we need familiarity with certain bond related terms.

Par Value This is the value stated on the face of the bond. It represents the amount the firm borrows and promises to repay at the time of maturity. Usually the par or face value of bonds issued by business firms is ₹ 100. Sometimes it is ₹ 1,000.

Coupon Rate and Interest A bond carries a specific interest rate which is called 'the coupon rate'. The interest payable to the bond holder is simply: par value of the bond × coupon rate. Most bonds pay interest semi-annually. For example, a government security which has a par value of ₹ 1,000 and a coupon rate of 8 percent pays an interest of ₹ 40 every six months.

Maturity Period Typically bonds have a maturity period of 1-15 years; sometimes they have longer maturity. At the time of maturity the par (face) value plus perhaps a nominal premium is payable to the bondholder.

Valuation Model The value of a bond - or any asset, real or financial - is equal to the present value of the cash flows expected from it. Hence determining the value of a bond requires:

- An estimate of expected cash flows
- An estimate of the required return

To simplify our analysis of bond valuation we will make the following assumptions:

- The coupon interest rate is fixed for the term of the bond.
- The coupon payments are made annually and the next coupon payment is receivable exactly a year from now.
- The bond will be redeemed at par on maturity.

Given these assumptions, the cash flow for a non-callable bond (a bond that cannot be prematurely retired) comprises of an annuity of a fixed coupon interest and the principal amount payable at maturity. Hence the value of the bond is:

$$P = \sum_{t=1}^{n} \frac{C}{(1+r)^t} + \frac{M}{(1+r)^n} \tag{7.1}$$

where P is the value (in rupees), n is the number of years to maturity, C is the annual coupon payment (in rupees), r is the periodic required return, M is the maturity value, and t is the time when the payment is received.

Since the stream of coupon payments is an ordinary annuity, we can apply the formula for the present value of an ordinary annuity. Hence the bond value is given by the formula:

$$P = C \times \text{PVIFA}_{r,n} + M \times \text{PVIF}_{r,n} \tag{7.1a}$$

To illustrate how to compute the price of a bond, consider a 10-year, 12% coupon bond with a par value of ₹ 1,000. Let us assume that the required yield on this bond is 13%. The cash flows for this bond are as follows:

- 10 annual coupon payments of ₹ 120
- ₹ 1000 principal repayment 10 years from now

The value of the bond is:

$$\begin{aligned} P &= 120 \times \text{PVIFA}_{13\%,10\text{yrs}} + 1{,}000 \times \text{PVIF}_{13\%,10\text{yrs}} \\ &= 120 \times 5.426 + 1{,}000 \times 0.295 \\ &= 651.1 + 295 = ₹\,946.1 \end{aligned}$$

Bond Values with Semi-annual Interest Most of the bonds pay interest semi-annually. To value such bonds, we have to work with a unit period of six months, and not one year. This means that the bond valuation equation has to be modified along the following lines:

- The annual interest payment, *C*, must be divided by two to obtain the semi-annual interest payment.
- The number of years to maturity must be multiplied by two to get the number of half-yearly periods.
- The discount rate has to be divided by two to get the discount rate applicable to half-yearly periods.

With the above modifications, the basic bond valuation becomes:

$$P = \sum_{t=1}^{2n} \frac{C/2}{(1+r/2)^t} + \frac{M}{(1+r/2)^{2n}}$$

$$= C/2\,(\text{PVIFA}_{r/2,2n}) + M(\text{PVIF}_{r/2,2n}) \qquad (7.2)$$

where *P* is the value of the bond, *C/2* is the semi-annual interest payment, *r/2* is the discount rate applicable to a half-year period, *M* is the maturity value, and *2n* is the maturity period expressed in terms of half-yearly periods.

As an illustration, consider an 8 year, 12 percent coupon bond with a par value of ₹ 100 on which interest is payable semi-annually. The required return on this bond is 14 percent.

Applying Eq.(7.2), the value of the bond is:

$$P = \sum_{t=1}^{16} \frac{6}{(1.07)^t} + \frac{100}{(1.07)^{16}}$$

$$= 6(\text{PVIFA}_{7\%,16}) + 100(\text{PVIF}_{7\%,16})$$

$$= ₹\,6(9.447) + ₹\,100(0.339) = ₹\,90.6$$

Let us recalculate the above using the Excel financial function PRICE (settlement, maturity, rate, yield, redemption, frequency, basis), as follows:

	A	B	C	D	E	F	G	H	I	J
1	Settlement	1/1/2015	This is the date of purchase. If not specified fill in any date							
2	Maturity	30/12/2022	The formula in this case is = B1+ 365*8, as the maturity period is 8 years							
3	Rate	12%	The annual coupon rate							
4	Yield	14%	The required return per annum							
5	Redemption	100	Fill in the redemption value as a percentage of the par value							
6	Frequency	2	This represents the number of times interest is paid in a year							
7	Basis	3	3 represents the day count convention, actual no. of days/365, in interest calculation							
8	Price	90.55	To get the result in B8, use the function = PRICE(B1, B2, B3, B4, B5, B6, B7)							
9	Bond price is obtained per ₹ 100 of the face value of the bond. Here, the redemption value being ₹ 100, the price would be ₹ 90.55 × 100/100 = ₹ 90.55									

Relationship between Coupon Rate, Required Yield, and Price A basic property of a bond is that its price varies inversely with yield. The reason is simple. As the required yield decreases, the present value of the cash flow increases; hence the price increases. Conversely, when the required yield increases, the present value of the cash flow decreases.

The price-yield relationship may be illustrated with an example. Consider a bond carrying a coupon rate of 14 percent issued 3 years ago for ₹ 1000 (its par value) by Signal Corporation. The original maturity of the bond was 10 years, so its residual maturity now is 7 years. The interest rate has fallen in the last 3 years and investors now expect a return of 10 percent from this bond. The price of this bond now would be

$$P_0 = \sum_{t=1}^{7} \frac{140}{(1.10)^t} + \frac{1000}{(1.10)^7} = ₹\,1194.7$$

The arithmetic of the bond price increase is clear. What is the logic behind it? The fact that the required return on such a bond has fallen to 10 percent means that if you had ₹ 1,000 to invest, you can buy new bonds like Signal's except that these new bonds would pay ₹ 100, rather than ₹ 140, by way of interest. Naturally, as an investor you would prefer ₹ 140 to ₹ 100, so you would be willing to pay more than ₹ 1,000 for a Signal bond to enjoy its higher coupons. All investors would behave similarly and consequently the bond of Signal would be bid up in price to ₹ 1194.7. At that price it would provide a return of 10 percent, the rate the new bonds offer.

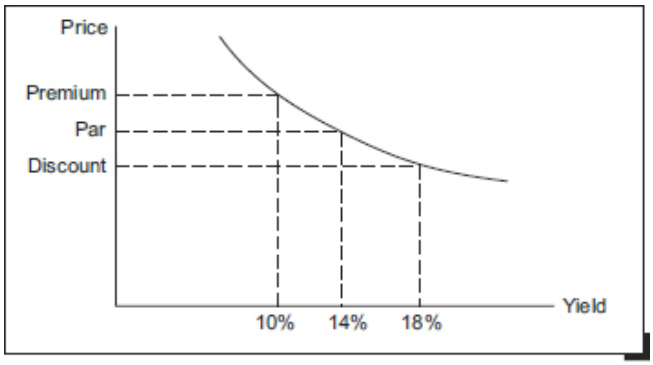
Now let us look at what happens when the interest rate rises after the bond has been issued. Assume that because of a rise in interest rates,

investors now expect a return of 18 percent from the Signal bond. The price of the bond would be:

$$P_0 = \sum_{t=1}^{7} \frac{140}{(1.18)^t} + \frac{1000}{(1.18)^7} = ₹\ 847.5$$

The graph of the price-yield relationship for the bond has a convex shape as shown in Exhibit 7.1.

Exhibit 7.1 **Price–Yield Relationship**



To sum up, the relationship between the coupon rate, the required yield, and the price is as follows:

Coupon rate > Required yield ⟷ Price > Par (Premium bond)
Coupon rate = Required yield ⟷ Price = Par
Coupon rate < Required yield ⟷ Price > Par (Discount bond)

Relationship between Bond Price and Time Since the price of a bond must typically be equal to its par value at maturity (assuming that there is no risk of default), the bond price changes with time. For example, a bond that is redeemable for ₹ 1,000 (which is its par value) after 5 years when it matures, will have a price of ₹ 1,000 at maturity, no matter what the current price is. If its current price is, say, ₹ 1,100, it is said to be a premium bond. If the required yield does not change between now and the maturity date, the premium will decline over time as shown by curve A in Exhibit 7.2. On the other hand, if the bond has a current price of say ₹ 900, it is said to be a discount bond. The discount too will disappear over time as shown by curve B in Exhibit 7.2. Only when the current price is equal to par value - in such a case the bond is said to be a par bond - there is no change in price as time passes, assuming that the required yield does not change between now and the maturity date. This is shown by the dashed line in Exhibit 7.2.

7.6 EQUITY VALUATION: DIVIDEND DISCOUNT MODEL

According to the dividend discount model, the value of an equity share is equal to the present value of dividends expected from its ownership plus the present value of the sale price expected when the equity share is sold. For applying the dividend discount model, we will make the following assumptions: (i) dividends are paid annually; and (ii) the first dividend is received one year after the equity share is bought.

Single-period Valuation Model Let us begin with the case where the investor expects to hold the equity share for one year. The price of the equity share will be:

$$P_0 = \frac{D_1}{(1+r)} + \frac{P_1}{(1+r)} \tag{7.7}$$

where P_0 is the current price of the equity share, D_1 is the dividend expected a year hence, P_1 is the price of the share expected a year hence, and r is the rate of return required on the equity share.

Example Prestige's equity share is expected to provide a dividend of ₹ 2.00 and fetch a price of ₹ 18.00 a year hence. What price would it sell for now if investors' required rate of return is 12 percent? The current price will be:

$$P_0 = \frac{2.0}{(1.12)} + \frac{18.00}{(1.12)} = ₹\ 17.86$$

What happens if the price of the equity share is expected to grow at a rate of g percent annually? If the current price, P_0, becomes $P_0(1+g)$ a year hence, we get:

$$P_0 = \frac{D_1}{(1+r)} + \frac{P_0(1+g)}{(1+r)} \tag{7.8}$$

Simplifying Eq.(7.8) we get:

$$P_0 = \frac{D_1}{r-g} \tag{7.9}_1$$

Example The expected dividend per share on the equity share of Roadking Limited is ₹ 2.00. The dividend per share of Roadking Limited has grown over the past five years at the rate of 5 percent per year. This growth

rate will continue in future. Further, the market price of the equity share of Roadking Limited, too, is expected to grow at the same rate. What is a fair estimate of the intrinsic value of the equity share of Roadking Limited if the required rate is 15 percent?

Applying Eq.(7.9) we get the following estimate:

$$P_0 = \frac{2.00}{0.15 - .05} = ₹\ 20.00$$

Expected Rate of Return In the preceding discussion we calculated the intrinsic value of an equity share, given information about (i) the forecast values of dividend and share price, and (ii) the required rate of return. Now we look at a different question: What rate of return can the investor expect, given the current market price and forecast values of dividend and share price? The expected rate of return is equal to:

$$r = D_1 / P_0 + g \tag{7.10}$$

Example The expected dividend per share of Vaibhav Limited is ₹ 5.00. The dividend is expected to grow at the rate of 6 percent per year. If the price per share now is ₹ 50.00, what is the expected rate of return?

Applying Eq. (7.10), the expected rate of return is:

$$r = 5/50 + 0.06 = 16 \text{ percent}$$

Multi-period Valuation Model Having learnt the basics of equity share valuation in a single-period framework, we now discuss the more realistic, and also the more complex, case of multi-period valuation.

Since equity shares have no maturity period, they may be expected to bring a dividend stream of infinite duration. Hence the value of an equity share may be put as:

$$P_0 = \frac{D_1}{(1+r)} + \frac{D_2}{(1+r)^2} + \ldots + \frac{D_\infty}{(1+r)^\infty} = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t} \tag{7.11}$$

where P_0 is the price of the equity share today, D_1 is the dividend expected a year hence, D_2 is the dividend expected two years hence,…. D_∞ is the dividend expected at the end of infinity, and r is the expected return.

Equation (7.11) represents the valuation model for an infinite horizon. Is it applicable to a finite horizon? Yes. To demonstrate this, consider how an equity share would be valued by an investor who plans to hold it for n years and sell it thereafter for a price of P_n. The value of the equity share to him is:

$$P_0 = \frac{D_1}{(1+r)^1} + \frac{D_2}{(1+r)^2} + \ldots + \frac{D_n}{(1+r)^n} + \frac{P_n}{(1+r)^n}$$

$$= \sum_{t=1}^{n} \frac{D_t}{(1+r)^t} + \frac{P_n}{(1+r)^n} \tag{7.12}$$

Now, what is the value of P_n in Eq.(7.12)? Applying the dividend capitalisation principle, the value of P_n would be the present value of the dividend stream beyond the nth year, evaluated as at the end of the nth year. This means:

$$P_n = \frac{D_{n+1}}{(1+r)^1} + \frac{D_{n+2}}{(1+r)^2} + \ldots + \frac{D_\infty}{(1+r)^\infty} \tag{7.13}$$

Substituting this value of P_n in Eq. (7.12) we get:

$$P_0 = \frac{D_1}{(1+r)^1} + \frac{D_2}{(1+r)^2} + \ldots + \frac{D_n}{(1+r)^n} + \frac{1}{(1+r)^n}\left[\frac{D_{n+1}}{(1+r)} + \frac{D_{n+2}}{(1+r)^2} + \ldots + \frac{D_\infty}{(1+r)^\infty}\right]$$

$$= \frac{D_1}{(1+r)} + \frac{D_2}{(1+r)^2} + \ldots + \frac{D_n}{(1+r)^n} + \frac{D_n}{(1+r)^{n+1}} + \ldots + \frac{D_\infty}{(1+r)^\infty}$$

$$= \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t} \tag{7.14}$$

This is the same as Eq.(7.11) which may be regarded as a generalised multi-period valuation formula. Eq.(7.11) is general enough to permit any dividend pattern — constant, rising, declining, or randomly fluctuating. For practical applications it is helpful to make simplifying assumptions about the pattern of dividend growth. The more commonly used assumptions are as follows:

- The dividend per share remains constant forever, implying that the growth rate is nil (the zero growth model).
- The dividend per share grows at a constant rate per year forever (the constant growth model).
- The dividend per share grows at a constant rate for a finite period, followed by a constant normal rate of growth forever thereafter (the two-stage model).
- The dividend per share, currently growing at an above-normal rate, experiences a gradually declining rate of growth for a while. Thereafter, it grows at a constant normal rate (the *H* model).

Zero Growth Model If we assume that the dividend per share remains constant year after year at a value of D, Eq.(7.11) becomes:

$$P_0 = \frac{D}{(1+r)} + \frac{D}{(1+r)^2} + \ldots + \frac{D}{(1+r)^n} + \ldots \infty \quad (7.15)$$

Equation (7.15), on simplification, becomes:

$$P_0 = \frac{D}{r} \quad (7.16)$$

This is an application of the present value of perpetuity formula.

Constant Growth Model One of the most popular dividend discount models assumes that the dividend per share grows at a constant rate (g). The value of a share, under this assumption, is:

$$P_0 = \frac{D_1}{(1+r)} + \frac{D_1(1+g)}{(1+r)^2} + \ldots \frac{D_1(1+g)^n}{(1+r)^{n+1}} + \ldots \quad (7.17)$$

Applying the formula for the sum of a geometric progression, the above expression simplifies to:

$$P_0 = \frac{D_1}{r-g} \quad (7.18)_2$$

Example Ramesh Engineering Limited is expected to grow at the rate of 6 percent per annum. The dividend expected on Ramesh's equity share a year hence is ₹ 2.00. What price will you put on it if your required rate of return for this share is 14 percent?

The price of Ramesh's equity share would be:

$$P_0 = \frac{2.00}{0.14 - 0.06} = ₹\ 25.00$$

What Drives Growth Most stock valuation models are based on the assumption that dividends grow over time. What drives this growth? The two major drivers of growth are: (a) ploughback ratio and (b) return on equity (ROE). To see why this is so let us consider an example. Omega Limited has an equity (net worth) base of 100 at the beginning of year 1. It earns a return on equity of 20 percent. It pays out 40 percent of its equity earnings and ploughs back 60 percent of its equity earnings. Its financials for a 3 year period are shown in Exhibit 7.3, from which we find that dividends grow at a rate of 12 percent. The growth figure is a product of: Ploughback ratio × Return on equity = 0.6 × 20% = 12%

Exhibit 7.3 **Financials of Omega Limited**

	Year 1	Year 2	Year 3
Beginning equity	100	112	125.44
Return on equity	20%	20%	20%
Equity earnings	20	22.4	25.1
Dividend payout ratio	0.4	0.4	0.4
Dividends	8	8.96	10.04
Ploughback ratio	0.6	0.6	0.6
Retained earnings	12	13.44	15.06

Two Stage Growth Model The simplest extension of the constant growth model assumes that the extraordinary growth (good or bad) will continue for a finite number of years and thereafter the normal growth rate will prevail indefinitely.

Assuming that the dividends move in line with the growth rate, the price of the equity share will be:

$$P_0 = \left[\frac{D_1}{(1+r)} + \frac{D_1(1+g_1)}{(1+r)^2} + \frac{D_1(1+g_1)^2}{(1+r)^3} \ldots + \frac{D_1(1+g_1)^{n-1}}{(1+r)^n}\right] + \frac{P_n}{(1+r)^n} \tag{7.19}$$

where P_0 is the current price of the equity share, D_1 is the dividend expected a year hence, g_1 is the extraordinary growth rate applicable for n years, and P_n is the price of the equity share at the end of year n.

The first term on the right hand side of Eq.(7.19) is the present value of a growing annuity. Its value is equal to:

$$D_1\left[\frac{1-\left[\frac{1+g_1}{1+r}\right]^n}{r-g_1}\right] \tag{7.20}$$

Remember that this is a straightforward application of Eq.(6.7) developed in the previous chapter.

Hence

$$P_0 = D_1\left[\frac{1-\left[\frac{1+g_1}{1+r}\right]^n}{r-g_1}\right] + \frac{P_n}{(1+r)^n} \tag{7.21}$$

Since the two-stage growth model assumes that the growth rate after n years remains constant, P_n will be equal to:

$$\frac{D_{n+1}}{r-g_2} \tag{7.22}$$

where D_{n+1} is the dividend for year n+1 and g_2 is the growth rate in the second period.

D_{n+1}, the dividend for year n+1, may be expressed in terms of the dividend at the end of the first stage and growth rate in the second stage:

$$D_{n+1} = D_1 (1+g_1)^{n-1} (1+g_2) \tag{7.23}$$

Substituting the above expression, we have:

$$P_0 = D_1 \left[\frac{1-\left[\frac{1+g_1}{1+r}\right]^n}{r-g_1} \right] + \left[\frac{D_1 (1+g_1)^{n-1} (1+g_2)}{r-g_2} \right] \left[\frac{1}{(1+r)^n} \right] \tag{7.24}$$

Example The current dividend on an equity share of Vertigo Limited is ₹ 2.00. Vertigo is expected to enjoy an above-normal growth rate of 20 percent for a period of 6 years. Thereafter the growth rate will fall and stabilise at 10 percent. Equity investors require a return of 15 percent. What is the intrinsic value of the equity share of Vertigo?

The inputs required for applying the two-stage model are:

$g_1 =$ 20 percent
$g_2 =$ 10 percent
$n =$ 6 years
$r =$ 15 years
$D_1 = D_0 (1+g_1) =$ ₹ 2(1.20) = 2.40

Plugging these inputs in the two-stage model, we get the intrinsic value estimate as follows:

$$P_0 = 2.40 \left[\frac{1-\left[\frac{1.20}{1.15}\right]^6}{.15-.20} \right] + \left[\frac{2.40\,(1.20)^5\,(1.10)}{.15-.10} \right] \left[\frac{1}{(1.15)^6} \right]$$

$$= 2.40 \left[\frac{1-1.291}{-0.05} \right] + \left[\frac{2.40\,(2.488)(1.10)}{.05} \right] [0.432]$$

$$= 13.96 + 56.80$$

$$= ₹\ 70.76$$

The Excel spreadsheet for the two stage growth model is as under:

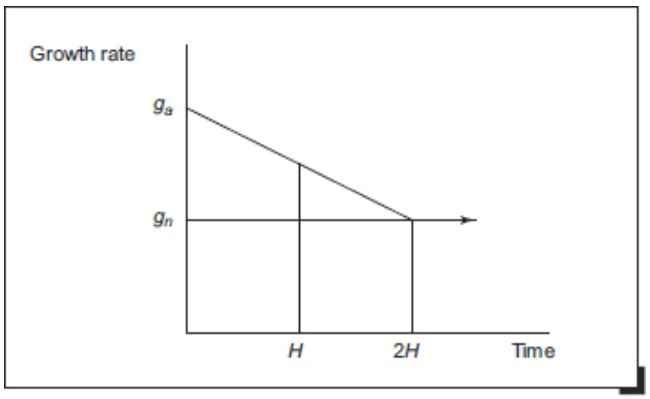
	A	B	C	D	E
1	g_1	g_2	n(years)	r	D_0(₹)
2	20%	10%	6	15%	2
3	P_0(₹)	Formula used = E2*(1 + A2)*(1–((1 + A2)/(1 + D2))^C2)/(D2 – A2) +E2* (1 +A2)* (1 + A2)^(C2 – 1)* (1 + B2)/(D2 – B2)/(1 + D2)^C2			70.76

H Model The *H* model of equity valuation is based on the following assumptions:

- While the current dividend growth rate, g_a, is greater than g_n, the normal long-run growth rate, the growth rate declines linearly for $2H$ years.
- After $2H$ years the growth rate becomes g_n.

The graphical representation of the dividend growth rate pattern for the *H*-model is shown in Exhibit 7.4.

Exhibit 7.4 **Dividend Growth Rate Pattern for the *H* model**



While the derivation of the *H* model is rather complex, the valuation equation for the *H* model is quite simple:

$$P_0 = \frac{D_0[(1+g_n)+H(g_a-g_n)]}{r-g_n} \tag{7.25}$$

where P_0 is the intrinsic value of the share, D_0 is the current dividend per share, r is the rate of return expected by investors, g_n is the normal long-run growth rate, g_a is the current growth rate, and H is one-half of the period during which g_a will level off to g_n.

Equation (7.25) may be re-written as:

$$P_0 = \frac{D_0(1+g_n)}{r-g_n} + \frac{D_0H(g_a-g_n)}{r-g_n} \tag{7.26}$$

Expressed this way, the *H* model may be interpreted in a simple, intuitive manner. The first term on the right hand side of Eq. (7.26)

$$\frac{D_0(1+g_n)}{r-g_n}$$

represents the value based on the normal growth rate, whereas the second term

$$\frac{D_0 H(g_a - g_n)}{r - g_n}$$

reflects the premium due to abnormal growth rate.

Example The current dividend on an equity share of International Computers Limited is ₹ 3.00. The present growth rate is 50 percent. However, this will decline linearly over a period of 10 years and then stabilise at 12 percent. What is the intrinsic value per share of International Computers Limited, if investors require a return of 16 percent?

The inputs required for applying the *H*-model are:

$D_0 =$ ₹ 3.00
$g_a =$ 50 percent
$H =$ 5 years
$g_n =$ 12 percent
$r =$ 16 percent

Plugging these inputs in the *H*-model we get the intrinsic value estimate as follows:

$$P_0 = \frac{300[(1.12) + 5(0.50 - 0.12)]}{.16 - .12} = ₹\ 226.5$$

The Excel illustration of the *H*-model is under:

	A	B	C	D	E
1	g_a	g_n	H(years)	r	D_0(₹)
2	50%	12%	5	16%	3
3	P_0(₹)	Formula used = E2* ((1+B2) + C2*(A2–B2)) / (D2-B2)			226.50

Impact of Growth on Price, Returns, and P/E Ratio The expected growth rates of companies differ widely. Some companies are expected to remain virtually stagnant or grow slowly; other companies are expected to show normal growth; still others are expected to achieve supernormal growth rate.

Assuming a constant total required return, differing expected growth rates mean differing stock prices, dividend yields, capital gains yields, and price-earnings ratios. To illustrate this, consider three cases:

	Growth rate (%)
Low growth firm	5
Normal growth firm	10
Supernormal growth firm	15

The expected earnings per share and dividend per share of each of the three firms for the following year are ₹ 3.00 and ₹ 2.00 respectively. Investors' required total return from equity investment is 20 percent.

Given the above information, we may calculate the stock price, dividend yield, capital gains yield, and price-earnings ratio for the three cases as shown in Exhibit 7.5.

The results in Exhibit 7.5 suggest the following points:

1. As the expected growth in dividend increases, other things being equal, the expected return[3] depends more on the capital gains yield and less on the dividend yield.
2. As the expected growth rate in dividend increases, other things being equal, the price-earnings ratio increases.
3. High dividend yield and low price-earnings ratio imply limited growth prospects.
4. Low dividend yield and high price-earnings ratio imply considerable growth prospects.

Exhibit 7.5 **Price, Dividend Yield, Capital Gains Yield, and Price-Earnings Ratio under Differing Growth Assumptions**

	Price $P_0 = \frac{D_1}{r-g}$	*Dividend yield* (D_1/P_0)	*Capital gains yield* $(P_1-P_0)/P_0$	*Price earnings ratio* (P/E)
Low growth firm	$P_0 = \frac{₹\ 2.00}{0.20-0.05} = ₹\ 13.33$	15.0%	5.0%	4.44
Normal growth firm	$P_0 = \frac{₹\ 2.00}{0.20-0.10} = ₹\ 20.00$	10.0%	10.0%	6.67
Supernormal growth firm	$P_0 = \frac{₹\ 2.00}{0.20-0.15} = ₹\ 40.00$	5.0%	15.0%	13.33

Is the Stock Market Shortsighted? Many managers believe that the stock market is myopic and obsessed with short-term performance. Let's test this assertion with the help of the constant growth model. Sun Pharma was quoting at ₹ 591.80 on May 26, 2014. Sun Pharma's most recent dividend was ₹ 10 per share and the dividend growth rate in the previous five years was 19 percent per year. If we assume that the dividend per share continues to grow at the same rate for the next five years and apply a

discount rate of 13 percent, the present value of the projected dividends for the following five years would be:

$$PV = \frac{₹\,10(1.19)}{(1.13)} + \frac{₹\,10(1.19)^2}{(1.13)^2} + \frac{₹\,10(1.19)^3}{(1.13)^3} + \frac{₹\,10(1.19)^4}{(1.13)^4} + \frac{₹\,10(1.19)^5}{(1.13)^5}$$

$$= 10.53 + 11.09 + 11.68 + 12.30 + 12.95 = ₹\,58.55$$

Recall that Sun Pharma's stock price was ₹ 591.80. Therefore, less than 10 percent of the current stock price is attributable to the projected cash flows (by way of dividends) for the following five years. This means that Sun Pharma managers should focus on increasing long-term cash flows. This is true for most companies. Indeed, many researchers and consultants have found that for a typical company more than 80 percent of current stock price is accounted for by cash flows beyond five years.

If long-term cash flows account for the bulk of a stock's value, why are managers and analysts obsessed with quarterly earnings? The primary reason seems to be the informational content of short-term earnings. While the quarterly earnings by themselves may not be important, the information they convey about long-term prospects may be very significant. If the quarterly earnings are lower than expected because the new products launched by the company have failed, the long–term cash flows of the company would be negatively impacted. On the other hand, if the reason is that the company has significantly increased its R&D outlay on promising projects, the market may greet it positively rather than negatively. Another reason for managerial focus on short-term earnings may be that the bonus of managers is linked to reported earnings.

7.7 EQUITY VALUATION: THE P/E RATIO APPROACH

An approach to valuation, practised widely by investment analysts, is the P/E ratio or earnings multiplier approach. The value of a stock, under this approach, is estimated as follows:

$$P_0 = E_1 \times P_0 / E_1 \tag{7.27}$$

where P_0 is the estimated price, E_1 is the estimated earnings per share, P_0/E_1 is the justified price-earnings ratio.

Determinants of the P/E Ratio The determinants of the P/E ratio can be derived from the dividend discount model, which is the foundation for valuing equity stocks.

Let us start with the constant growth dividend discount model:

$$P_0 = \frac{D_1}{r-g} \tag{7.28}$$

In this model $D_1 = E_1 (1-b)$. b stands for the ploughback ratio and g = ROE × b. Note that ROE is return on equity. Making these substitutions we find that:

$$P_0 = \frac{E_1(1-b)}{r-\text{ROE}\times b} \tag{7.29}$$

Dividing both the sides by E_1, we get:

$$P_0/E_1 = \frac{(1-b)}{r-\text{ROE}\times b} \tag{7.30}$$

Equation (7.30) indicates that the factors that determine the P/E ratio are:

- The dividend payout ratio, $(1-b)$
- The required rate of return, r
- The expected growth rate, ROE × b

P/E Ratio and Ploughback Ratio Note that b, the ploughback ratio, appears in the numerator as well as the denominator of the ratio on the right hand side of Eq. (7.30). What is the effect of a change in b on the P/E ratio? It depends on how ROE compares with r. If ROE is greater than r, an increase in b leads to an increase in P/E; if ROE is equal to r an increase in

b has no effect on P/E; if ROE is less than *r* an increase in *b* leads to decrease in P/E.

P/E Ratio and Interest Rate The required rate of return on equity stocks reflects interest rate and risk. When interest rates increase, required rates of return on all securities, including equity stocks, increase, pushing security prices downward. When interest rates fall security prices rise. Hence there is an inverse relationship between P/E ratios and interest rates.

P/E Ratio and Risk Other things being equal, riskier stocks have lower P/E multiples. This can be seen easily by examining the formula for the P/E ratio of the constant growth model:

$$P/E = \frac{1-b}{r-g} \tag{7.31}$$

Riskier stocks have higher required rates of return (*r*) and hence lower P/E multiples. This is true in all cases, not just the constant growth model. For any expected earnings and dividend stream, the present value will be lower when the stream is considered to be riskier. Hence the P/E multiple will be lower.

P/E Ratio and Liquidity Other things being equal, stocks which are highly liquid command higher P/E multiples and stocks which are highly illiquid command lower P/E multiples. The reason for this is not far to seek. Investors value liquidity just the way they value safety and hence are willing to give higher P/E multiples to liquid stocks.

- If the dividend per share grows at a constant rate, the value of the share is:

$$P_0 = D_1/(r-g)$$

- The two key drivers of dividend growth are (a) ploughback ratio and (b) return on equity.
- The value per share, according to the *H* model is:

$$P_0 = \frac{D_0(1+g_n)}{r-g_n} + \frac{D_0 H(g_a - g_n)}{r-g_n}$$

- An approach to valuation, practised widely by investment analysts, is the P/E ratio approach. The value of an equity share, under this approach, is estimated as follows:

$$P_0 = E_1 \times P_0/E_1$$

- The stock price may be considered as the capitalised value of the earnings under the assumption of no growth plus the present value of growth opportunities.
- The stock market consists of a primary segment and a secondary segment. The principal bourses are the National Stock Exchange and the Bombay Stock Exchange, accounting for the bulk of the trading on the Indian stock market.

QUESTIONS

1. Describe briefly the various concepts of value.
2. Discuss the basic bond valuation formula.
3. State the formula for a bond which pays interest semi-annually.
4. What is the relationship between coupon rate, required yield, and price?
5. Explain and illustrate the following yield measures: current yield, yield to maturity, and yield to call.
6. State and illustrate the formula to find the approximate YTM on a bond.
7. Discuss the constant growth dividend discount model.
8. Explain the two stage dividend discount model.
9. Discuss the *H* model.
10. What is the impact of growth on price, dividend yield, capital gains yield, and price-earnings ratio?
11. Discuss the P/E ratio approach to stock valuation.
12. How is the E/P linked to the required return and the present value of growth opportunities?
13. Explain how the price-earnings ratio is related to growth, dividend payout ratio, and the required return.
14. Discuss the transformation of the Indian stock market from mid 1990s.
15. Discuss the salient features of the National Stock Exchange.
16. Discuss the salient features of the Bombay Stock Exchange.

17. How is stock price reported?

SOLVED PROBLEMS

7.1 A ₹ 100 par value bond bearing a coupon rate of 12 percent will mature after 5 years. What is the value of the bond, if the discount rate is 15 percent?

Solution Since the annual interest payment will be ₹ 12 for 5 years and the principal repayment will be ₹ 100 after 5 years, the value of the bond, at a discount rate of 15 percent, will be

$$\begin{aligned} V &= ₹\ 12\ (\text{PVIFA}_{15\%,\,5\ \text{yrs}}) + ₹\ 100\ (\text{PVIF}_{15\%,\,5\ \text{yrs}}) \\ &= ₹\ 12\ (3.352) + ₹\ 100\ (0.497) \\ &= 40.22 + 49.70 = ₹\ 89.92 \end{aligned}$$

7.2 The market price of a ₹ 1,000 par value bond carrying a coupon rate of 14 percent and maturing after 5 years is ₹ 1050. What is the yield to maturity (YTM) on this bond? What is the approximate YTM?

Solution The YTM is the value of r in the following equation:

$$\begin{aligned} 1{,}050 &= \sum_{t=1}^{5} \frac{140}{(1+r)^t} + \frac{1{,}000}{(1+r)^5} \\ &= 140\ (\text{PVIFA}_{r,\,5\ \text{yrs}}) + 1{,}000\ (\text{PVIF}_{r,\,5\ \text{yrs}}) \end{aligned}$$

Let us try a value of 13 percent for r. The right hand side of the above expression becomes:

$$\begin{aligned} &140\ (\text{PVIFA}_{13\%,\,5\ \text{yrs}}) + 1{,}000\ (\text{PVIF}_{13\%,\,5\ \text{yrs}}) \\ &= 140\ (3.517) + 1{,}000\ (0.543) \\ &= 492.4 + 543.0 = ₹\ 1035.4 \end{aligned}$$

Since this is less than ₹ 1,050, we try a lower value for r. Let us try r = 12 percent. This makes the right-hand side equal to:

$$\begin{aligned} &140\ (\text{PVIFA}_{12\%,\,5\ \text{yrs}}) + 1{,}000\ (\text{PVIF}_{12\%,\,5\ \text{yrs}}) \\ &= 140\ (3.605) + 1{,}000\ (0.567) \\ &= 504.7 + 567.0 = ₹\ 1071.7 \end{aligned}$$

Thus, r lies between 12 percent and 13 percent. Using a linear interpolation in this range, we find that r is equal to:

$$12\% + (13\% - 12\%)\ \frac{1071.7 - 1050.0}{1071.7 - 1035.4} = 12.60\ \text{percent}$$

(b) The approximate YTM works out to:

$$\text{YTM} = \frac{140 + (1{,}000 - 1{,}050)/5}{0.40 \times 1000 + 0.6 \times 1050} = 12.62\ \text{percent}$$

7.3 A ₹ 100 par value bond bears a coupon rate of 14 percent and matures after 5 years. Interest is payable semi-annually. Compute the value of the bond if the required rate of return is 16 percent.

Solution

In this case the number of half-yearly periods is 10, the half-yearly interest payment is ₹ 7, and the discount rate applicable to a half-yearly period is 8 percent. Hence, the value of the bond is:

$$V = \sum_{t=1}^{10} \frac{7}{(1.08)^t} + \frac{100}{(1.08)^{10}}$$
$$= 7\,(\text{PVIFA}_{8\%,\,10\,\text{yrs}}) + 100\,(\text{PVIF}_{8\%,\,10\,\text{yrs}})$$
$$= 7\,(6.710) + 100\,(0.463)$$
$$= 46.97 + 46.30$$
$$= ₹\,93.27$$

7.4 The equity stock of Rax Limited is currently selling for ₹ 30 per share. The dividend expected next year is ₹ 2.00. The investors' required rate of return on this stock is 15 percent. If the constant growth model applies to Rax Limited, what is the expected growth rate?

Solution

According to the constant growth model

$$P_0 = \frac{D_1}{r - g}$$

This means

$$g = r - D_1 / P_0$$

Hence, the expected growth rate (g) for Rax Limited is:

$$g = 0.15 - \frac{2.00}{30.00} = .083 \text{ or } 8.3 \text{ percent}$$

7.5 Vardhman Limited's earnings and dividends have been growing at a rate of 18 percent per annum. This growth rate is expected to continue for 4 years. After that the growth rate will fall to 12 percent for the next 4 years. Thereafter, the growth rate is expected to be 6 percent forever. If the last dividend per share was ₹ 2.00 and the investors' required rate of return on Vardhman's equity is 15 percent, what is the intrinsic value per share?

Solution The intrinsic value per share of Vardhman may be computed using a 3-step procedure.

Step 1: The dividend stream during the first eight years when Vardhman would enjoy a relatively high rate of growth will be:

$$D_1 = 2.00\,(1.18) = 2.36$$
$$D_2 = 2.00\,(1.18)^2 = 2.78$$
$$D_3 = 2.00\,(1.18)^3 = 3.29$$
$$D_4 = 2.00\,(1.18)^4 = 3.88$$
$$D_5 = 2.00\,(1.18)^4\,(1.12) = 4.34$$
$$D_6 = 2.00\,(1.18)^4\,(1.12)^2 = 4.86$$
$$D_7 = 2.00\,(1.18)^4\,(1.12)^3 = 5.45$$
$$D_8 = 2.00\,(1.18)^4\,(1.12)^4 = 6.10$$

The present value of this dividend stream is:

$$2.36\,(0.870) + 2.78\,(0.756) + 3.29\,(0.658) + 3.88\,(0.572)$$
$$+ 4.34\,(0.497) + 5.45\,(0.432) + 6.10\,(0.376) = ₹\ 16.83$$

Step 2: The price of the share at the end of 8 years, applying the constant growth model at that point of time, will be:

$$P_8 = \frac{D_9}{r - g_n} = \frac{D_8(1+g_n)}{r - g_n}$$
$$= \frac{2.00(1.18)^4(1.12)^4(1.06)}{0.15 - 0.06} = ₹\ 71.84$$

The present value of this price is:

$$\frac{71.84}{(1.15)^8} = 23.49$$

Step 3: The sum of the above components is:

$$P_0 = ₹\ 16.83 + ₹\ 23.49 = ₹\ 40.32$$

7.6 The current dividend on an equity share of Pioneer Technology is ₹ 3.00. Pioneer is expected to enjoy an above-normal growth rate of 40 percent for 5 years. Thereafter, the growth rate will fall and stabilise at 12 percent. Equity investors require a return of 15 percent from Pioneer's stock. What is the intrinsic value of the equity share of Pioneer?

Solution The inputs required for applying the two-stage growth model are:

g_1 = 40%, g_2 = 12%, n = 5 years, r = 15%

$D_1 = D_0(1+g_1)$ = ₹ 3 (1.40) = ₹ 4.20

Plugging these inputs in the two-stage growth model, we get the intrinsic value estimate as follows:

$$P_0 = 4.20\left[\frac{1-\left[\frac{1.40}{1.15}\right]^5}{0.15-0.40}\right] + \left[\frac{4.20(1.40)^4(1.12)}{0.15-0.12}\right]\left[\frac{1}{(1.15)^5}\right]$$
$$= 28.12 + 299.48 = ₹\ 327.60$$

7.7 The current dividend on an equity share of National Computers Limited is ₹ 5.00. The present growth rate is 50 percent. However, this will decline linearly over a period of 8 years and then stabilise at 10 percent. What is the intrinsic value per share of National Computers, if investors require a return of 18 percent from its stock?

Solution The inputs required for applying the *H*-model are:

D_0 = ₹ 5.00, g_a = 50%, H = 4 years, g_n = 10 years, r = 18%

Plugging these inputs in the *H*-model we get the intrinsic value estimate as follows:

$$P_0 = \frac{5.00[(1.10) + 4(0.50 - 0.10)]}{0.18 - 0.10} = ₹\ 168.75$$

PROBLEMS

7.1 **Bond Value** A ₹ 100 par value bond, bearing a coupon rate of 11 percent will mature after 5 years. What is the value of the bond, if the discount rate is 15 percent?

7.2 **Bond Value** A ₹ 100 par value bond, bearing a coupon rate of 12 percent, will mature after 7 years. What is the value of the bond if the discount rate is 14 percent? 12 percent?

7.3 **YTM** The market value of a ₹ 1,000 par value bond, carrying a coupon rate of 12 percent and maturing after 7 years, is ₹ 750. What is the yield to maturity on this bond?

7.4 **YTM** The market value of a ₹ 100 par value bond, carrying a coupon rate of 14 percent and maturing after 10 years, is ₹ 80. What is the yield to maturity on this bond?

7.5 **Bond Value** A ₹ 100 par value bond bears a coupon rate of 12 percent and matures after 6 years. Interest is payable semi-annually. Compute the value of the bond if the required rate of return is 16 percent, compounded semi-annually.

7.6 **YTM** You are considering investing in one of the following bonds:

	Coupon rate	*Maturity*	*Price/₹ 100 par value*
Bond A	12%	10yrs	₹ 70
Bond B	10%	6yrs	₹ 60

Your income tax rate is 30 percent and your capital gains tax is effectively 10 percent. Capital gains taxes are paid at the time of maturity on the difference between the purchase price and par value. What is your post-tax yield to maturity from these bonds?

7.7 **Bond Value** A company's bonds have a par value of ₹ 100, mature in 7 years, and carry a coupon rate of 12 percent payable semi-annually. If the appropriate discount rate is 16 percent, what price should the bond command in the market place?

7.8 **Stock Price** The share of a certain stock paid a dividend of ₹ 2.00 last year (D_0 = ₹ 2.00). The dividend is expected to grow at a constant rate of 6 percent in the future. The required rate of return on this stock is considered to be 12 percent. How much should this stock sell for now? Assuming that the expected growth rate and required rate of return remain the same, at what price should the stock sell 2 years hence?

7.9 **Stock Price** Sherief Corporation's previous dividend was ₹ 12.00. Earnings and dividends are expected to grow at a rate of 10 percent. The required rate of return on Sherief's stock is 15 percent. What should be the market price of Sherief's stock now?

7.10 **Growth Rate** The equity stock of Max Limited is currently selling for ₹ 32 per share. The dividend expected next is ₹ 2.00. The investors' required rate of return on this stock is 12 percent. Assume that the constant growth model applies to Max Limited. What is the expected growth rate of Max Limited?

7.11 Rate of Return Fizzle Limited is facing gloomy prospects. The earnings and dividends are expected to decline at the rate of 4 percent. The previous dividend was ₹ 1.50. If the current market price is ₹ 8.00, what rate of return do investors expect from the stock of Fizzle Limited?

7.12 Variable Growth Rate The Commonwealth Corporation's earnings and dividends have been growing at the rate of 12 percent per annum. This growth rate is expected to continue for 4 years. After that the growth rate would fall to 8 percent for the next four years. Beyond that the growth rate is expected to be 5 percent forever. If the last dividend was ₹ 1.50 and the investors' required rate of return on the stock of Commonwealth is 14 percent, how much should be the market value per share of Commonwealth Corporation's equity stock?

7.13 Two Stage Growth Model Determine the intrinsic value of an equity share, given the following data:

Last dividend (D_0)	: ₹ 2.00
Growth rate for the next five years	: 15 percent
Growth rate beyond 5 years	: 10 percent

Assume a required rate of return.

7.14 YTM You can buy a ₹ 1000 par value bond carrying an interest rate of 14 percent (payable annually) and maturing after 4 years for ₹ 900. If the re-investment rate applicable to the interest receipts from this bond is 16 percent, what will be your yield to maturity?

7.15 Two Stage Growth Model The current dividend on an equity share of Dizzy Limited is ₹ 2.00. Dizzy is expected to enjoy an above-normal growth rate of 18 percent for 6 years. Thereafter the growth rate will fall and stabilise at 12 percent. Equity investors require a return of 16 percent from Dizzy's stock. What is the intrinsic value of the equity share of Dizzy?

7.16 H Model The current dividend on an equity share of International Chemicals Limited is ₹ 4.00. The present growth rate is 20 percent. However, this will decline linearly over a period of 8 years and stabilise at 10 percent. What is the intrinsic value per share of International Chemicals Limited if investors require a return of 18 percent?

7.17 PVGO Mahaveer Electronics is expected to give a dividend of ₹ 8 next year and the same would grow by 12 percent per year forever. Mahaveer pays out 40 percent of its earnings. The required rate of return on Mahaveer's stock is 15 percent. What is the PVGO?

7.18 YTM Rajesh has invested ₹ 2 crores in ₹ 100 par 5 year bonds of a company which pays semi-annual interest of ₹ 5.5 which he regularly invests in bank deposits carrying interest at 9 percent. What is the yield to maturity of this arrangement?

7.19 YTM Ashok has paid ₹ 880 per bond to buy ₹ 1000 par bonds maturing after 8 years that pay an annual coupon of 10 percent. He falls in the income tax bracket of 30 percent. The capital gains tax is effectively 9 and has to be paid at

the time of maturity on the difference between the purchase price and par value. What is his post-tax yield to maturity from these bonds?

7.20 **PVGO** Bio Synthetics has a policy of maintaining a payout ratio of 40 percent. Their net profit margin is steady at 10 percent and sales growth rate is 9 percent. If the number of outstanding equity shares is 10 million and the forecasted sales for the year is ₹ 800 million, what is their present value of growth opportunities? The rate of return required by the investors in Bio Synthetics is 14 percent.

7.21 **Stock Valuation** Investors require a rate of return of 16 percent from the stock of Evergreen Industries, which has a strict policy of paying just 10 percent of their net profits as dividend on their outstanding equity shares of 0.8 crore. Their net profit margin is steady at 12 percent. The revenues of the company is likely to grow at a high growth rate of 30 percent for the next 3 years and thereafter at a modest rate of just 10 percent. If the sales now has just reached ₹ 200 crore, what is the current intrinsic value of their equity stock?

7.22 **Market Price** A dividend of ₹ 6 per share has been paid on the equity shares of Cosmos International and according to an analyst forecast the dividend and stock price are expected to grow at 5 percent in the future. If that forecast is reliable, and investors require a return of 20 percent from Cosmos what would be the likely market price per share after 2 years?

7.23 **Bond Yield** A ₹ 1000 par bond has a coupon rate of 10 percent paid annually. It matures in 12 years. It is currently selling for ₹ 1050. What is its yield to maturity? Use the approximate formula.

7.24 **Components of Bond Returns** Bond A caries an annual coupon of 10 percent and has a residual maturity of 5 years. Bond B carries an annual coupon of 8 percent and it too has a residual maturity of 5 years. Both the bond have a par value of ₹ 1000 and a YTM of 9 percent. Bond A sells at a discount. What is the current yield for Bond A and Bond B? What is the expected capital gains yield over the next one year for Bond A and Bond B?

7.25 **Deep Discount Bonds** On January 1, 2010 ABC Limited issues a 20-year deep discount bond maturing on December 31, 2030. The par value of the bond is ₹ 100000 and it was issued at ₹ 10000. On January 1, 2019 the bond was trading at ₹ 30,040. What was the implicit yield at the time of issues? What is the return to an investor who bought at the time of issue and sold on January 1, 2019? What return can an investor who buys on January 1, 2019 and holds it till maturity expect? Assume that there will be no default.

7.26 **Dividend Per Share** Shakti Limited stock currently sells for ₹ 90 per share. Investors, require a return of 14 percent on the firms stock. If the constant dividend growth rate applicable to the company is 8 percent, what was the dividend paid per share on the stock recently?

MINICASE - I

You have recently graduated from a business school and joined SMART INVEST as a financial analyst. Your job is to help clients in choosing a portfolio of bonds and stocks. Dinshaw Mistry, a prospective client, seeks your help in understanding how bonds and stocks are valued and what rates of return they offer. In particular, you have to answer the following questions.

a. How is the value of a bond calculated?
b. What is the value of a 5-year, ₹ 1,000 par value bond with a 10 percent annual coupon, if the required rate of return is 8 percent?
c. What is the approximate yield to maturity of an 8-year, ₹ 1,000 par value bond with a 10 percent annual coupon, if it sells for ₹ 1,060.
d. What is the yield to call of the bond described in part (c), if the bond can be called after 2-years at a premium of ₹ 1,050.
e. What is the general formula for valuing any stock, irrespective of its dividend pattern?
f. How is a constant growth stock valued?
g. Magnum chemicals is a constant growth company which paid a dividend of ₹ 6.00 per share yesterday (D_o = ₹ 6.00) and the dividend is expected to grow at a rate of 12 percent per year forever. If investors require a rate of return of 15 percent (i) what is the expected value of the stock a year from now? (ii) what is the expected dividend yield and capital gains yield in the first year?
h. Zenith Electronics paid a dividend of ₹ 10.00 per share yesterday (D_o = ₹ 10.00). Zenith Electronics is expected to grow at a supernormal growth rate of 25 percent for the next 4 years, before returning to a constant growth rate of 10 percent thereafter. What will be the present value of the stock, if investors require a return of 16 percent?
i. The earnings and dividends of Ravi Pharma are expected to grow at a rate of 20 percent for the next 3 years. Thereafter, the growth rate is expected to decline linearly for the following 5 years before settling down at 10 percent per year forever. Ravi Pharma paid a dividend of ₹ 8.00 per share yesterday (D_o = ₹ 8.00). If investors require a return of 14 percent from the equity of Ravi Pharma, what is the intrinsic value per share?

MINICASE - II

Jagan Reddy, the MD of Reddy Lifestyle was much dejected when his bankers simply refused any additional funding for his company. Somehow they didn't seem to share his enthusiasm over the company's prospects. Coming out of the bank, he called his CFO and close confidante Ram Rao. After showering a couple of choice adjectives on the bank manager he sobered down: 'What is the point in blaming the bank? Anyone can see that our stock is one of the worst performers in the market. Any idea why it is jinxed? Frankly, I have had enough of this useless furniture business. It can take us only thus far. Now, here is a secret-just keep it strictly to yourself: I think the time has come to unlock value in our old land investments. We can easily diversify into realty

business by the end of this year. We will then raise the needed funds by placing equity privately at a good premium. We can flaunt a growth rate as high as forty percent for the first four years and a fair twelve percent thereafter. All that is needed is a bit of guts! I will give you a whole six months' time to work on those hardnosed directors to make them see the writing on the wall, so that when I eventually come up with the real estate idea, they would jump for it. Enough for the day. Tomorrow we will discuss these in detail. Specifically I want you to come up with some answers, even if approximate for the following:

1. For our immediate need of ₹ 10 crores, I think the only way left is to go for a new series of unsecured debentures. Could you figure out the coupon rate we will have to offer for a five year issue now at par?
2. What should be our P/E ratio if we go for the new debenture issue? Also, based on our current earnings prospects, come up with some convincing calculations to show why our stock would continue to be a laggard in the market if we just stick to the present furniture business.
3. At what possible price would we be able to place the shares privately after a year, assuming that the board approves the diversification? Also let me know what would be the present value of growth opportunities then?'

   If you were the CFO, how would you have worked out the solutions for the above queries with the following data?

   Currently the company's 8 percent coupon debentures of face value of ₹ 100, with a remaining maturity of five years are trading at ₹ 90 per debenture. The current market price per equity share of face value ₹ 10 of the company is ₹ 24.70 and the average P/E multiple for the industry is 14. For simplicity you assume that the profitability, payout and turnover ratios remain unchanged. You decide to use a discount rate of 15 percent for the diversified company. The summarised financial statements of the company for the year ended just now are as under:

			(₹ in millions)
Net sales	625	Equity capital	250
		Reserve & surplus	80
Cost of goods	495	Loan funds	200
Gross profit	130	Total	530
PBIT	92		
Interest	20	Fixed assets	410
PBT	72	Investments	20
Tax	22	Net current assets	100
PAT	50	Total	530
Dividend	30		

PRACTICAL ASSIGNMENT

Value the equity share of the company of your choice using the two-stage growth model.

Make suitable assumptions along with justification with respect to g_1, n, g_2, and r. Compare your value with the prevailing market price and explain the discrepancy, if any.

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[1] The steps in simplification are:

$$P_0 = \frac{D_1}{(1+r)} + \frac{P_0(1+g)}{(1+r)} \quad (1)$$

$$P_0 = \frac{D_1 + P_0(1+g)}{(1+r)} \quad (2)$$

$$P_0(1+r) = D_1 + P_0(1+g) \quad (3)$$

$$P_0(1+r) - P_0(1+g) = D_1 \quad (4)$$

$$P_0(r-g) = D_1 \quad (5)$$

$$P_0 = \frac{D_1}{r-g} \quad (6)$$

[2] Start with

$$P_0 = \frac{D_1}{(1+r)} + \frac{D_2}{(1+r)^2} + \ldots + \frac{D_\infty}{(1+r)^\infty} = \frac{D_1}{(1+r)} + \frac{D_1(1+g)}{(1+r)^2} + \ldots \quad (1)$$

Multiplying both the sides of (1) by [(1 + g)/(1 + r)] gives:

$$P_0\left[\frac{1+g}{1+r}\right] = \frac{D_1(1+g)}{(1+r)^2} + \frac{D_1(1+g)^2}{(1+r)^3} + \ldots + \frac{D_1(1+g)^{n+1}}{(1+r)^{n+2}} \quad (n \to \infty) \quad (2)$$

Subtracting (2) from (1) yields:

$$\frac{P_0(r-g)}{(1+r)} = D_1\left[\frac{1}{(1+r)} - \frac{(1+g)^{n+1}}{(1+r)^{n+2}}\right] \quad (n \to \infty) \quad (3)$$

$$\text{As } (n \to \infty),\ \frac{(1+g)^{n+1}}{(1+r)^{n+2}} \to 0 \text{ because } g < r$$

Hence (2) becomes:

$$\frac{P_0(r-g)}{(1+r)} = \frac{D_1}{(1+r)} \quad (4)$$

This means:

$$P_0 = \frac{D_1}{r-g} \quad (5)$$

[3] Note that total return is the sum of the dividend yield and capital gain yield:

$$\frac{D_t + P_t - P_{t-1}}{P_{t-1}} = \frac{D_t}{P_{t-1}} + \frac{P_t - P_{t-1}}{P_{t-1}}$$

Total return = Dividend yield + Capital gains yield

Online Resources

http://highered.mheducation.com/sites/9353166527/student_view0/chapter7/index.html

- Additional Self-Test Problems
- Additional Solved Problems
- Chapters Excel
- Excel on Solved Problems
- Answer Key

CHAPTER 5			
Problem no.			**Answer**
5.1	Retained earnings		9
5.2	Retained earnings		38
5.3	*EFR*		5
5.4	1		₹ 144
5.5	(a)		52.5
	(c)	For 20X0	For 20X1
	Current ratio	1.5	1.8
	Debt to total assets ratio	0.53	0.54
	Return on equity	14.30%	14.50%
	(d) EFR For	20X1	9.38
		20X2	8.75
		20X3	8.11
		20X4	7.49
5.6			7.14%
5.7	(a)		₹ 50
	(b) (i)	STL <=	₹ 102.50
	(ii) Short-term borrowings		₹ 42.5
	Long-term loans		7.5
5.8			4140
5.9	a. g	=	3.70%
	b. d must be reduced to		46.60%
	c. A/E ratio to be increased	to	3.33
	d. m should be increased	to	7.92%
Minicase	Additional term loan		389.6
	Additional equity		100
CHAPTER 6			
Problem no.			**Answer**
6.1	a		1469
	b		1611
	c		1762
	d		2011
6.2			30 years
6.3			18.00%
6.4			₹ 79,481
6.5			₹ 56,983
6.6			20.30%
6.7			17.40%
6.8	(i)		₹ 4,670
	(ii)		₹ 4,040
	(iii)		₹ 3,270
6.9			₹ 7,582
6.10			a
6.11			₹ 10,608
6.12			₹ 22,683
6.13			₹ 26,212
6.14			₹ 13,165
6.15			15.10%
6.16	Stream A		₹ 2,590.90
	Stream B		₹ 3,625.20
	Stream C		₹ 2,825.10
6.17			₹ 21,910
6.18			₹ 9,030
6.19	a		0.60%
	b		2.20%
	c		2.80%
6.20.			₹ 40,388
6.21	For 10 years		₹ 20,000
6.22			₹ 26530
	With inflation		₹ 12,283
6.23			₹ 2,544
6.24			₹ 6,326
6.25			₹ 7,994
6.26			₹ 82,540
6.27			20% p.a.
6.28		₹ Million	376.68
6.29			7.3 years
6.31			30 years
6.32			₹ 2,224 mn.
6.33	a		₹ 700,000
6.34	a. Option e		₹ 80 crore.
	b.		₹ 77.8 crore.

6.35			$ 16,654,633
6.36			$ 30,781,329
6.37			₹ 9,434,536
6.38			8.32%
6.39			9.90%
6.41			7 years
6.42			14.84%
6.43			17.65%
6.44			9.60%
6.45			₹ 1,609,757
Minicase-1	1. Money needed 15 years hence		₹ 4,042,000
	2. Investment savings		₹ 48,338
	3. Donation need		₹ 157,676
	4. PV.life time salary		₹ 7,254,962
Minicase-2	1. Monthly deposit in RD 1		₹ 17,742
	in RD 2		₹ 20,236
	2.Deposit		₹ 61,53,292
	3. Deposit		₹ 45,95,432

CHAPTER 7

Problem no.			Answer
7.1			₹ 86.7
7.2	disc.14%		₹ 91.46
	disc.12%		₹ 100
7.3			18.70%
7.4			18.56%
7.5			₹ 84.92
7.6	Bond A		13.73%
	Bond B		17.47%
7.7			₹ 83.56
7.8	P_0		₹ 35.33
	P_2		₹ 39.70
7.9			₹ 264
7.10			5.75%
7.11			14%
7.12			₹ 23.77
7.13			₹ 136.37
7.14			17.39%
7.15			₹ 74.80
7.16			₹ 75
7.17			₹ 133.4
7.18			10.88%
7.19			9.06%
7.20			₹ 6.86
7.21			₹ 88.76
7.22			₹ 46.3
7.23			9.30%
7.24		A	B
	Current yield	9.62%	8.32%
	Capital gains yield next year	0.67%	0.65%
7.25	Return for seller		13%
	Return for buyer		11.55%
7.26			₹ 5
Minicase-1	b		₹ 1,080.30
	c		8.93%
	d		9%
	g (i)		₹ 250.88
	(ii) Dividend yield		3%
	Cap.gain yield		12%
	h		₹ 295.67
	i		₹ 250
Minicase-2	1		10.64%
	2. P/E ratio		10
	3. Possible price		₹ 131.47
	PVGO		₹ 108.42