Lecture 4: Matrix Operations

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Recap

- General System of linear equations and their solutions

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- Defined various matrices

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- General System of linear equations and their solutions
- Defined various matrices
- Proceeding further - Look at operations on matrices

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- Example:

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & -1 & 3 \end{pmatrix}, B = \begin{pmatrix} 1 & -1 & 1 & 1 \\ 2 & 3 & 4 & 5 \end{pmatrix}, C = A + B = \begin{pmatrix} 2 & 1 & 4 & 5 \\ 6 & 5 & 3 & 8 \end{pmatrix}$$

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- Subtraction of two matrices - Effected as matrix addition with (-1) multiple of B

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- Identity property of scalar multiplication: $1(A) = A$
- Note that we have only defined the scalar multiplication and not multiplication of two matrices.

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- Symmetric matrices - Wide applications in Data Science etc

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- Example: $A = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}$ is a skew-symmetric matrix.
- $A = \begin{pmatrix} 1 & 2 & 3 \\ -2 & 4 & 5 \\ -3 & -5 & 7 \end{pmatrix}$ is not a skew-symmetric matrix

Properties of Transpose Operation

For any two matrix of same size A and B

1. $(A \pm B)^T = A^T \pm B^T$
2. $(A^T)^T = A$
3. $(k_1 A)^T = k_1(A^T)$
4. For a square matrix A, $A + A^T$ is always symmetric
   Proof: $(A + A^T)^T = A^T + (A^T)^T = A^T + A = A + A^T$.
5. For a square matrix A, $A - A^T$ is always skew - symmetric.
   Proof: $(A - A^T)^T = A^T - (A^T)^T = -(A - A^T)$

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$$A + A^T = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} + \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix} = \begin{pmatrix} 2 & 6 & 10 \\ 6 & 10 & 14 \\ 10 & 14 & 18 \end{pmatrix}$$

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$$A - A^T = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} - \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix} = \begin{pmatrix} 0 & -2 & -4 \\ 2 & 0 & -2 \\ 4 & 2 & 0 \end{pmatrix}$$

Trace of a matrix

- Given a matrix A, Trace of A as the sum of the diagonal elements of A.
- $Trace(A) = \sum_{i=1}^{n} a_{ii}$
- $Tr(A)$ or $Trace(A)$.
- For example:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \Rightarrow Tr(A) = 1 + 5 + 9 = 15$$

Properties of Trace of a matrix

- For any two square matrices of the same size, A, B, $Trace(A + B) = Trace(A) + Trace(B)$
- $Trace(A^T) = Trace(A)$
- $Trace(k_1 A) = k_1(Trace(A))$, for some scalar k_1
- Trace of a matrix = sum of the eigenvalues of A

Summary

- Matrix Operations such as addition of matrices and scalar multiplication of a matrix
- Matrix Transpose, Symmetric and skew-symmetric matrix
- Trace of matrix and properties
- Matrix Multiplication, Homogeneous system of equations - Next Lecture

Namaste!!!

Lecture 5: Matrix Multiplication, Homogeneous System of Equations

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- Focus of this lecture: Matrix Multiplication, Homogeneous system of equations

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- $C = A \pm B \Rightarrow c_{i,j} = a_{i,j} \pm b_{i,j}$
- Matrix Multiplication - Not straightforward - Not element-wise multiplication
- Done slightly differently

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- For $A^{m \times n}, B^{n \times p}$, $C = AB$ is an $m \times p$ matrix

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- If A is an $m \times n$ matrix, then B must be $n \times p$ matrix
- C, then, has m rows and p columns
- For $A^{m \times n}, B^{n \times p}$, $C = AB$ is an $m \times p$ matrix
- The product AB is defined only if the number of rows of B = Number of columns of A

Matrix Multiplication - How is it done?

- Let A be a 2×3 matrix : $A = \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \end{pmatrix}$

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- Let B be a 3×2 matrix: $B = \begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \\ b_{3,1} & b_{3,2} \end{pmatrix}$

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- Matrix multiplication AB - Defined

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- Let B be a 3×2 matrix: $B = \begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \\ b_{3,1} & b_{3,2} \end{pmatrix}$
- Number of columns of $A = 3$, Number of rows of $B = 3$
- Matrix multiplication AB - Defined
- The product AB is defined, however, the product BA is not defined

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$$C = \begin{pmatrix} (a_{1,1} \times b_{1,1}) + (a_{1,2} \times b_{2,1}) + (a_{1,3} \times b_{3,1}) & (a_{1,1} \times b_{1,2} + (a_{1,2} \times b_{2,2}) + (a_{1,3} \times b_{3,2}) \\ (a_{2,1} \times b_{1,1}) + (a_{2,2} \times b_{2,1}) + (a_{2,3} \times b_{3,1}) & (a_{2,1} \times b_{1,2}) + (a_{2,2} \times b_{2,2}) + (a_{2,3} \times b_{3,2}) \end{pmatrix}$$

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- $c_{i,j} = \sum_{k=1}^{3} a_{i,k} b_{k,j}$

Matrix Multiplication - How is it done?

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- $B = \begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \\ b_{3,1} & b_{3,2} \end{pmatrix}$

$$C = \begin{pmatrix} (a_{1,1} \times b_{1,1}) + (a_{1,2} \times b_{2,1}) + (a_{1,3} \times b_{3,1}) & (a_{1,1} \times b_{1,2} + (a_{1,2} \times b_{2,2}) + (a_{1,3} \times b_{3,2}) \\ (a_{2,1} \times b_{1,1}) + (a_{2,2} \times b_{2,1}) + (a_{2,3} \times b_{3,1}) & (a_{2,1} \times b_{1,2}) + (a_{2,2} \times b_{2,2}) + (a_{2,3} \times b_{3,2}) \end{pmatrix}$$

- $c_{i,j} = \sum_{k=1}^{3} a_{i,k} b_{k,j}$
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- Example: Both A and B are square matrices of the same size

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$$A = \begin{pmatrix} 2 & 1 \\ 6 & 3 \end{pmatrix}, B = \begin{pmatrix} -1 & 0 \\ 5 & 2 \end{pmatrix}, C = \begin{pmatrix} 3 & 1 \\ -3 & 0 \end{pmatrix}$$

$$AB = \begin{pmatrix} 2 & 1 \\ 6 & 3 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 5 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 9 & 6 \end{pmatrix}$$

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- In general $(AB)^2 = (AB)(AB) \neq A^2B^2$, as $\mathrm{A}^2B^2 = AABB \neq ABAB$

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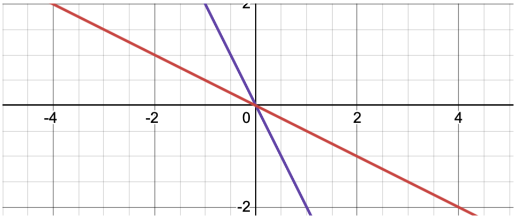

Figure: Homogeneous System with unique solution. The two lines intersect at the origin (0, 0)

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- Study of homogeneous system of linear equations and its solution - Reveals lot of information about the matrix rank, invertibility if the coefficient matrix is a square matrix etc

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Namaste!!!