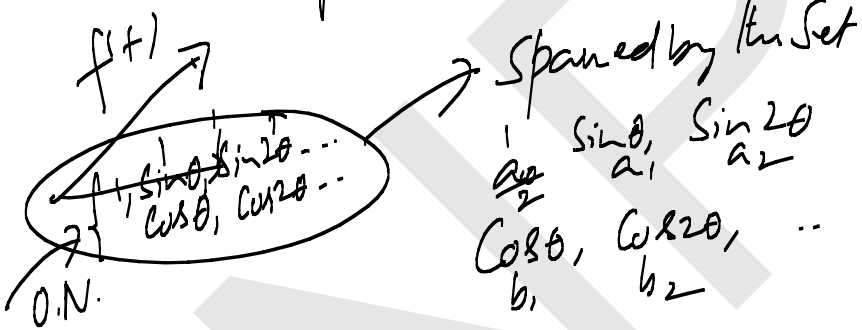
Recall: A real matrix U is
Orthogonal if $U^T U = U U^T = I$ ✓

A Complex matrix U is Hermitian
if $U^{\#} U = U U^{\#} = I$ ✓
" Orthogonality.

→ Recall imp result



$$f(t) \cong \frac{a_0}{2} + a_1 \sin\theta + a_2 \sin 2\theta \cdots + b_1 \cos\theta + b_2 \cos 2\theta \cdots$$

$\{a_0, a_1, a_2 \cdots b_1, b_2\}$ $\equiv$ Fourier Series Co-efficients for $f(t)$.

$\rightarrow$ basis vectors $\{1, \sin\theta, \sin 2\theta \cdots \cos\theta, \cos 2\theta \cdots\}$

Orthonormal !

Expanding $f(t) \equiv$ Fourier basis.

Can I look at only this basis

$\{1, \cos\theta, \cos 2\theta, \cdots\}$

Cosine terms.

Infact this set
$\{1, \cos\theta, \cos 2\theta, \cdots\}$ is also
a Orthonormal basis.

→ Cosine Series expansion.

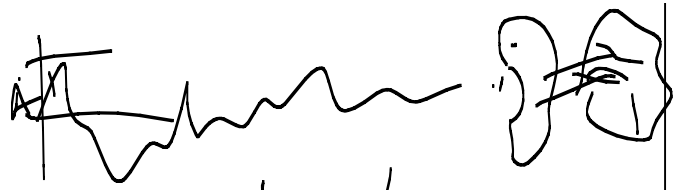
$\{1, \sin\theta, \sin 2\theta, \cdots\}$ Also.

$\{1, \cos\theta, \cos 2\theta, \cdots\}$ Cosine over Sine.

Why?

Majority of Applications choose $\{\cos n\theta\}_{n=1}^{\infty}$

Many Explanations

→ Audio 

→ What does the ear Hear!

→ Pressure waves Impinging on Ear drum.

Energy functions (not amplitude alone) $\propto A^2$

Energy is an Even Function.

Ohm's law I^2R or $\frac{V^2}{R}$; $\propto I^2$, $\propto V^2$

$y = x^2$ 

(EVEN Functions)

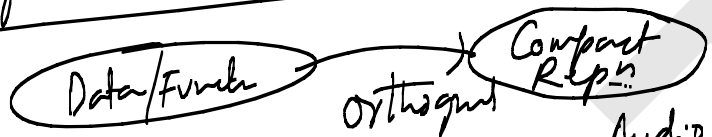
$\cos\theta$; $\cos(-\theta) = \cos\theta$

$f(x) = f(-x)$; Even fn.

$\sin\theta$: $\sin(-\theta) = -\sin\theta$, $f(-x) = -f(x)$: Odd

→ Significant Audio/Video - MM is built on using Cosine fn to code (orthonormal)

→ Fact: Orthogonal system: Representation using that is "Minimal" & "optimal" "Best"



→ Trick: Data Compression — Audio / Video } Data in "Coded" using orthonormal Basis

→ Cosine Series. (Finite) $\{\cos n\theta\}_{n=0}^{\infty}$

$\{\cos n\theta\}_{0}^{N-1}$: "N" length

N Terms.

How do we use this.?

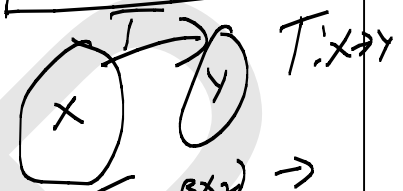
What this Idea of Frequency domain Repn.

We know $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \sin n\omega t + \sum_{n=0}^{\infty} b_n \cos n\omega t$

$f(t)$ is aperiodic = More General than periodic.

Cousin of Fourier Series : Fourier Transform.

F.T. → Linear Transform



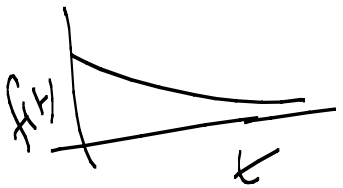
$T(\alpha x + \beta x_2) \rightarrow \alpha x_1 + \beta y_2$

α, β

$$\int_0^{\infty} f(t) e^{-j\omega t} dt \triangleq F(\omega)$$

L.T. Scaling & Superposition

Ex: $f(t) = e^{-at}$; $F(w) = \int_0^\infty f(t) e^{-jwt} dt$



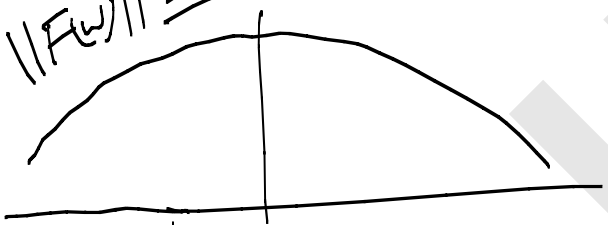
$= \int_0^\infty e^{-at} e^{-jwt} dt$

$= \frac{1}{a+jw} = F(w)$

$F(w)$ = Complex : Real Part + Imaginary Part.

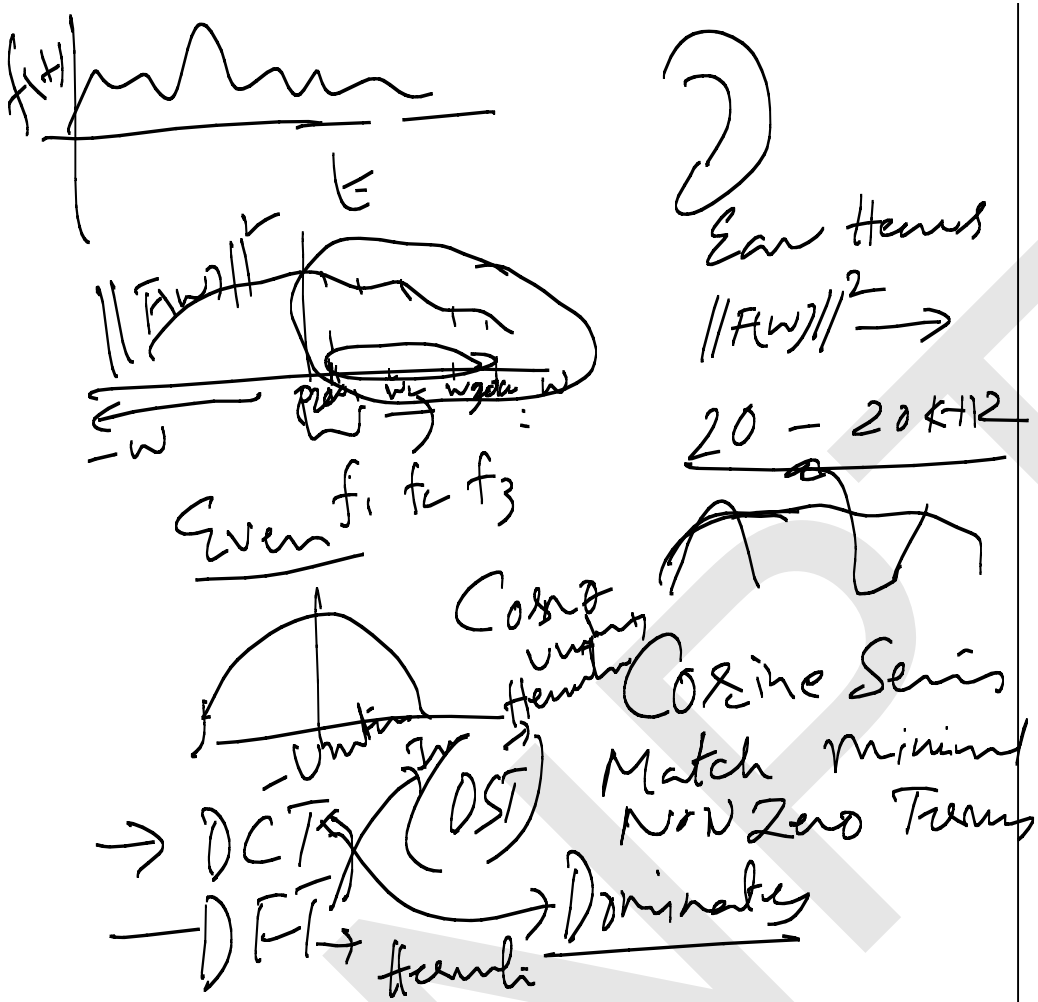
$\frac{a-jw}{a-jw}$: Energy $\frac{a-jw}{(a^2+w^2)}$ $\frac{a}{a^2+w^2}$ $-j\frac{w}{a^2+w^2}$

Re Im

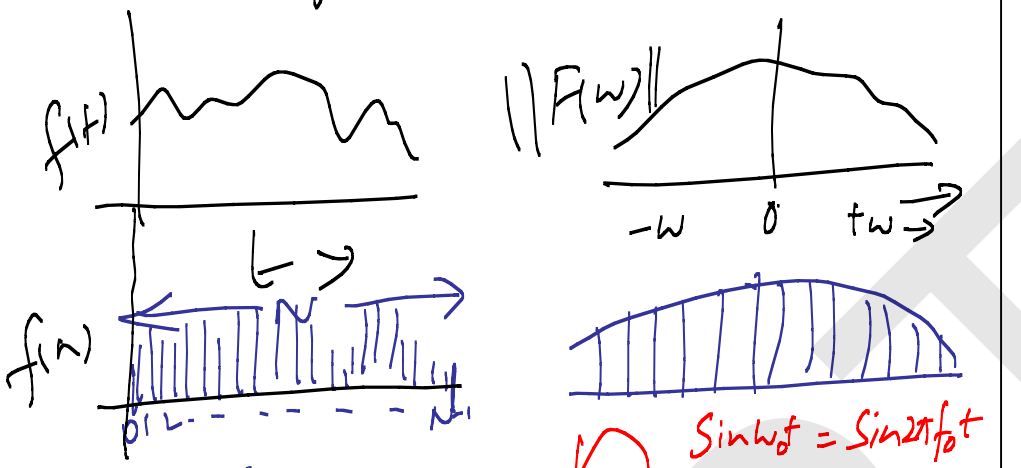


$F(w) = \frac{1}{\sqrt{a^2+w^2}}$

Even fn $\frac{1}{a^2+w^2}$

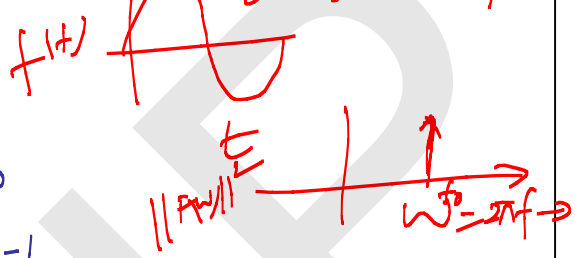

f(t)
Even
Even Terms
20 - 20K+12
Cosine
Cosine Series
Match minimal
Non Zero Terms
DCT
DST
DFT
Dominates

How do we go about Discrete version.



$\sin \omega_0 t = \sin 2\pi f_0 t$

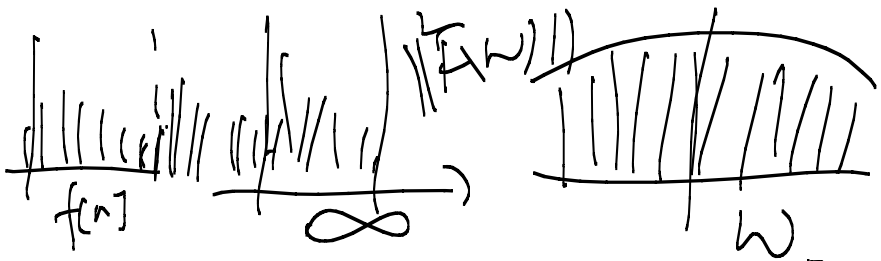
$f(t) \rightarrow$ Sequence $f[n]$



Finite $f(t) \rightarrow 0 \rightarrow \infty$

$f[n] - 0 - N-1$

Periodic in time $\equiv$ Discrete in Freq

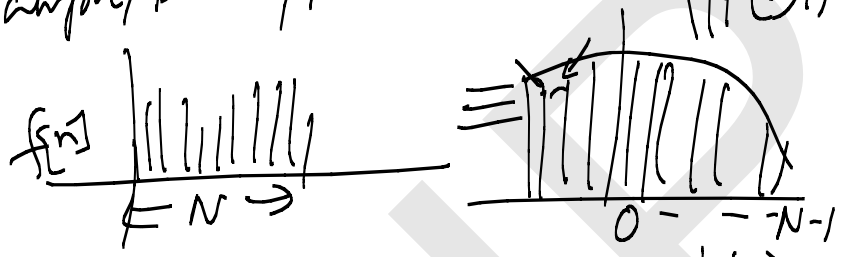


Discrete in time ; Discrete in Frequency

Finite

Make it Finite in Freq.

Sample/Truncate/Periodic Extension



$$\frac{2\pi}{N} = \Delta w$$

→ F.S → FT: → f[n] → f[n]

f[n] → F(ω) → ||F(ω)||²

↳ Even Best Rep.n by Orthonormal f.n which are Even: Cosine Seins.

→ Audio/Video App.n O.N. Basis.

KLT.