Recall:

If $A^{n\times n}$ is a real Symmetric matrix with distinct eigenvalues, then the corresponding eigenvectors are Orthogonal to each other

$$A = P D P^T.$$

P: Matrix with Orthonormal eigenvectors

$Ax \to$

$A^{2\times 2}$, RSM, $\lambda_1$, $\lambda_2$

$\vec{u}$. $\vec{v}$

$\vec{u} \cdot \vec{u} = 1$

$\vec{u} \cdot \vec{v} = 0$

$\vec{v} \cdot \vec{v} = 1$.

$\vec{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$, $\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$$Ax = (\lambda_1 u u^T + \lambda_2 v v^T)x.$$

$\vec{u}$, $\vec{v}$ Orthonormal vectors.

$uu^T$ = 2x2 Matrix $\to$ Rank 1.

$vv^T$ = 2x2 Matrix $\to$ Rank 1.

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$$\boxed{A = \lambda_1 u u^T + \lambda_2 v v^T} \longrightarrow \text{L.C. of 2 } 2\times 2 \text{ RSM of rank 1.}$$

$$\underset{\substack{2\times 2 \\ \text{Rank 2}}}{A} = \lambda_1 \underset{\text{Rank 1}}{[\quad]^{2\times 2}} + \lambda_2 \underset{\text{Rank 1}}{[\quad]^{2\times 2}}$$

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \begin{pmatrix} u_1 & u_2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} u_1^2 & u_1 u_2 \\ u_1 u_2 & u_2^2 \end{pmatrix}$$

$$A = \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} \lambda_1 & \\ & \lambda_2 \end{bmatrix} \begin{bmatrix} u^T \\ v^T \end{bmatrix}$$

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$$A^{m\times n} = U\Sigma V^{T}.$$

$$\underset{m\times m}{\begin{bmatrix} \\ U \\ \\ \end{bmatrix}} \underset{m\times n}{\begin{bmatrix} \\ \Sigma \\ \\ \end{bmatrix}} \underset{n\times n}{\begin{bmatrix} \\ V^{T}. \\ \\ \end{bmatrix}}$$

$$\underset{m\times m}{\begin{bmatrix} u_1 & u_2 & u_3 \\ | & | & | \\ | & | & | \end{bmatrix}} \underset{m\times n}{\begin{bmatrix} \sigma_1 & & \\ & \sigma_2 \, . \, . & \\ & & \sigma_r. \end{bmatrix}} \underset{n\times n}{\begin{bmatrix} v_1^T \\ v_2^T. \\ \vdots \end{bmatrix}}$$

$$A^{m\times n} = \overset{\text{Rank 1}}{\sigma_1 u_1 v_1^T} + \sigma_2 \overset{\text{Rank 1}}{u_2 v_2^T} + \dots + \sigma_r \overset{\text{Rank 1.}}{u_r v_r^T.}$$

$A^{3\times 2}$

$$u_1 = \begin{bmatrix} u_{11} \\ u_{12} \\ u_{13} \end{bmatrix} \qquad v_1 = \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix}$$

$$u_1 v_1^T : \begin{bmatrix} u_{11} \\ u_{12} \\ u_{13} \end{bmatrix} \begin{bmatrix} v_{11} & v_{12} \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} u_{11}v_{11} & u_{11}v_{12} \\ u_{12}v_{11} & u_{12}v_{12} \\ u_{13}v_{11} & u_{13}v_{12} \end{bmatrix}^{3\times 2}}_{\text{Rank 1 Matrix.}}$$

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SVD in the Sum form $\longrightarrow$ Generating $m \times n$ matrices as a LC of rank 1 matrices

$$A^{m\times n} = \sum_{i=1}^{r} \sigma_i u_i v_i^T$$ $\longrightarrow$ Linear Combination of $r$ rank 1 $m \times n$ matrices

SVD in the product form

$$A^{m\times n} = U \Sigma V^T$$

$\underset{m\times n}{u_1 v_1^T}$ $\quad \underset{m\times n}{u_2 v_2^T}$ $\quad \ldots \quad \underset{m\times n}{u_r v_r^T}$

Linearly indep. matrices

$A^{m \times n}$ real matrix.

$$A = \sum_{i=1}^{r} \sigma_i u_i v_i^T.$$

Pseudo inverse of $A$.

$$A^{\dagger} = (A^TA)^{-1}A^T \qquad (A^{m \times n},\ \text{rank } n).$$

$$x^{\dagger} = (A^TA)^{-1}A^T b \quad \text{when } b \notin \text{col Sp}(A).$$

How do we interpret the action of $A^{m \times n}$ on an $n$-component vector $x$?

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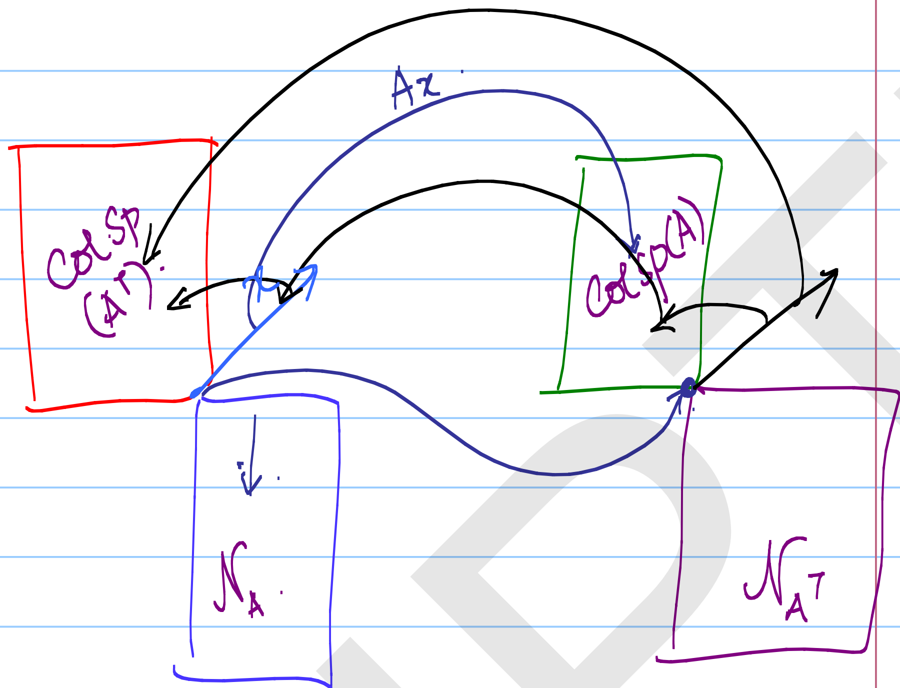
Col Sp (A^T)
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