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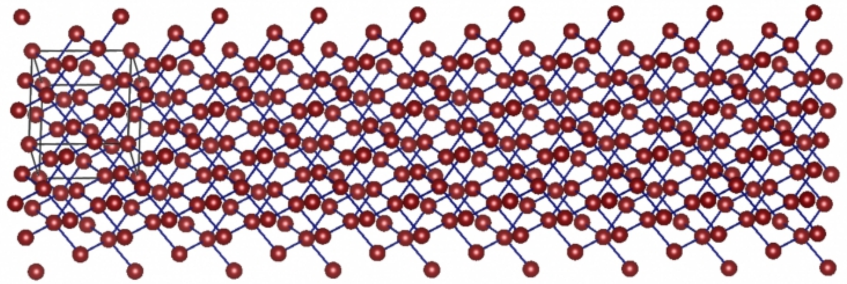

December 14 – 16 2020

An Introduction to Quantum Computing and Materials

DARTMOUTH

Quantum Information Science at Dartmouth

Introduction to Quantum Control of Open Quantum Systems: from Noisy Qubits to Qubit Sensors

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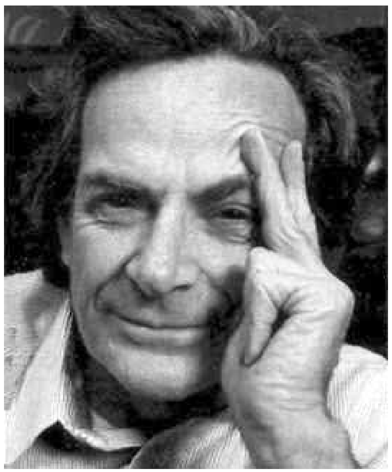
Prelude: Thinking small – 1959

An invitation to quantum control...



"I would like to describe a field... which might tell us much of great interest about the strange phenomena that occur in complex situations... and, furthermore, would have an enormous number of technical applications.

What I want to talk about is the problem of manipulating and controlling things on a small scale.*

It is something, in principle, that can be done; but in practice, it has not because we are too big.

...In the year 2000, when they look back at this age, they will wonder why it was not until the year 1960 that anybody began seriously to move in this direction."

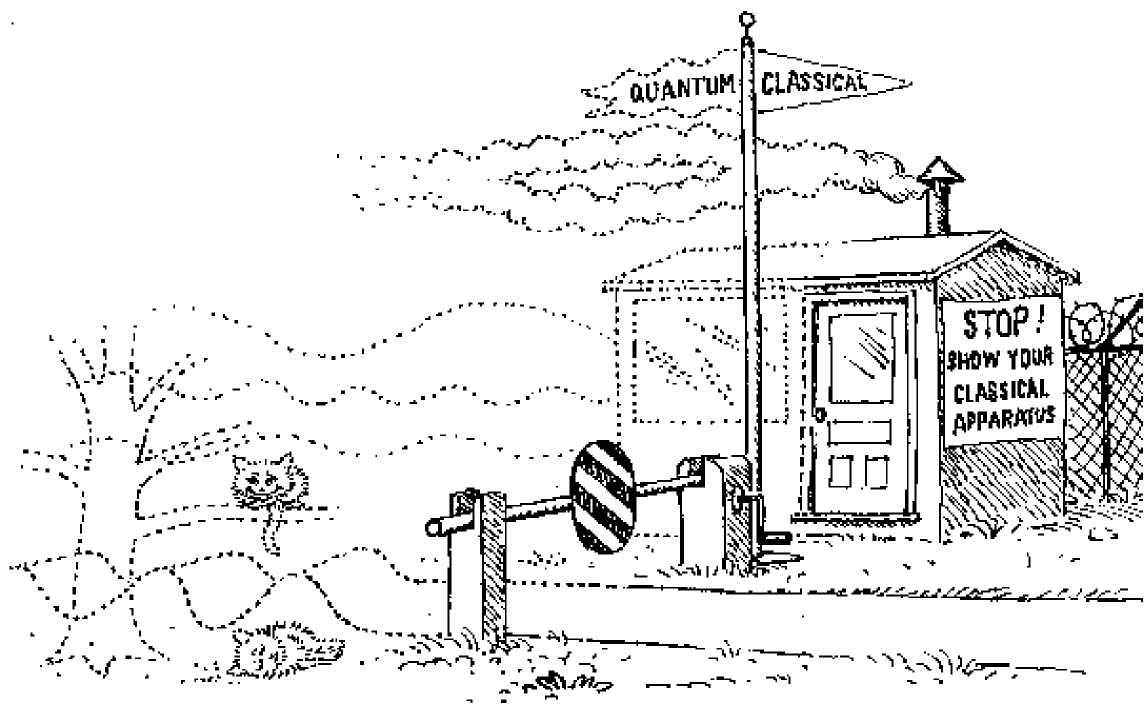
R.P. Feynman, *There's plenty of room at the bottom*
(Caltech, APS Meeting, December 1959).

*Objects on a small (nm) scale behave
like *nothing* on a large scale...

Prelude: Thinking small

Quantum control has been both curiosity-driven...

- Control at the quantum scale carries a fundamental physical significance ⇒ Central to the exploration of *non-classical states and dynamical behavior* in theory...



W.H. Zurek, *Decoherence and the transition from quantum to classical,* Phys. Today **44** (1991).

Prelude: Thinking small

Quantum control has been both curiosity-driven...

- Control at the quantum scale carries a fundamental physical significance ⇒
  Central to the exploration of *non-classical states and dynamical behavior* in the lab...

"for groundbreaking experimental methods that enable measuring and manipulation of individual quantum systems"

Prelude: Thinking small

...as well as application- and technology-driven...

- Control at the quantum scale carries an even bigger practical significance ⇒ Central to the development of *high-resolution spectroscopic and imaging techniques*...

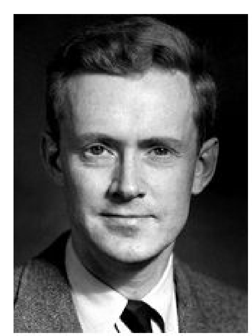

Front

The Nobel Prize in Physics 1952


Back

"for their development of new methods for nuclear magnetic precision measurements and discoveries in connection therewith "

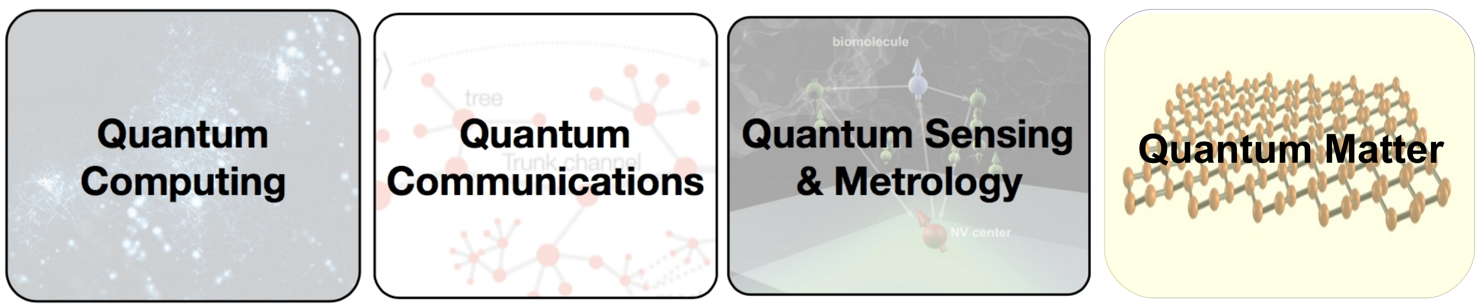

Felix Bloch


Edward Mills Purcell

Prelude: Thinking small – NISQ era

...as well as application- and technology-driven...

- Control at the quantum scale carries an even bigger practical significance ⇒ Central to the development of *quantum-based and quantum-enabled technologies*...



- Quantum control advances have been a (the?) *key factor* in making NISQ devices a reality...

Preskill, *Quantum* **2**, 79 (2018).

➞ 'Intermediate-Scale Quantum' ⇒ size of target system (> 50 qubits) is too large for complete ('pen-and-paper' or 'brute-force' numerical) descriptions...

➞ 'Noisy' ⇒ behavior of target system deviates significantly from intended one...

Realizing the full potential that NISQ era promises – and *moving beyond* – will require sustained progress in addressing emerging challenges in modeling and control...

Prelude: Thinking small – This lecture

- [My own] broad motivation: To explore challenges, limitations, and opportunities in controlling realistic, *open* quantum-dynamical systems

  ➞ To what extent (how) can control modify *seemingly irreversible* physical behavior?...
  ➞ Can we build a general *system-theoretic framework* for quantum processes?...
  ➞ Can this teach us how to *engineer novel forms* of quantum matter?...
  ➞ How can control be used to enable *robust realizations of quantum information*?...
  ⋮

*Feel free to raise your hand & ask questions!

Outline:*

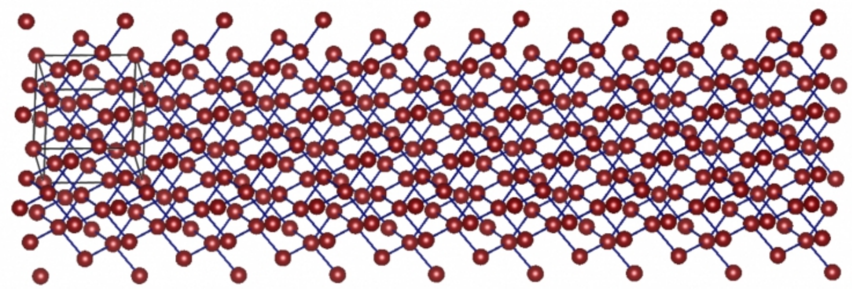
I. Controlling quantum systems – An (incomplete) overview

✔ (Some) basic concepts and terminology
✔ Quantum control for high-fidelity quantum information processing

II. Quantum control highlights – 'Open-loop Hamiltonian engineering'

✔ Noise suppression via dynamical decoupling & dynamically corrected gates
✔ Noise characterization with qubit sensors

2020 Quantum Winter School



December 14 – 16 2020

An Introduction to Quantum Computing and Materials

Part I.
Quantum Control Fundamentals

What is (and why) control?

<u>Fact</u>: Dynamical systems typically do *not* behave the way we want...

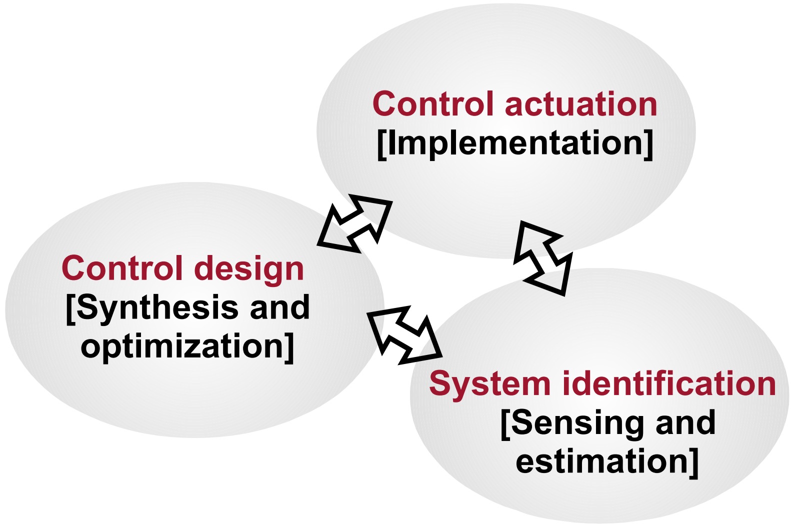
- Steps in control engineering:

  (1) Learn about behavior of 'target system' ('plant') S ⇒ *Modeling*

  (2) Design 'controller' C to be adjoined to S to modify dynamics ⇒ *Analysis and synthesis*

  (3) Actuate the system and validate performance ⇒ *Optimization*

- A naturally cross-disciplinary discipline...
  - ➡ Physics and applied math (dynamics modeling)
  - ➡ Computer science (information, software)
  - ➡ Operation research (optimization, networks)



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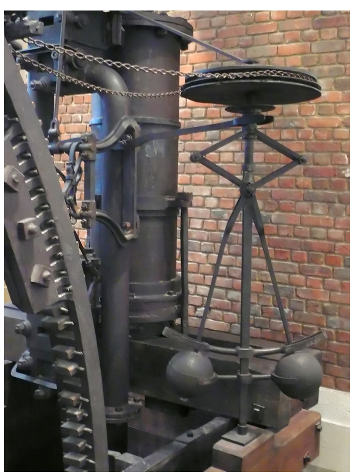
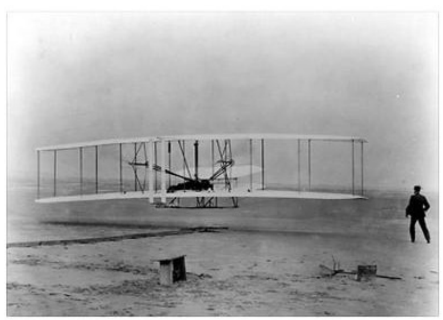
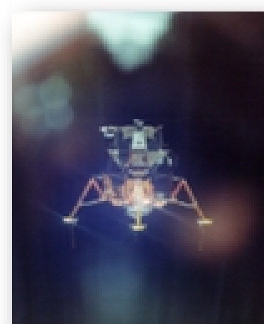
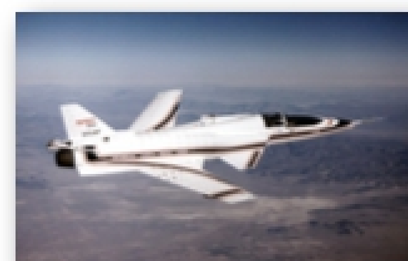
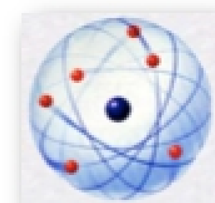
- A naturally cross-disciplinary discipline with a fascinating history...

1788 | 1903 | 1968 | ~1980 | 2000+

Classical control systems

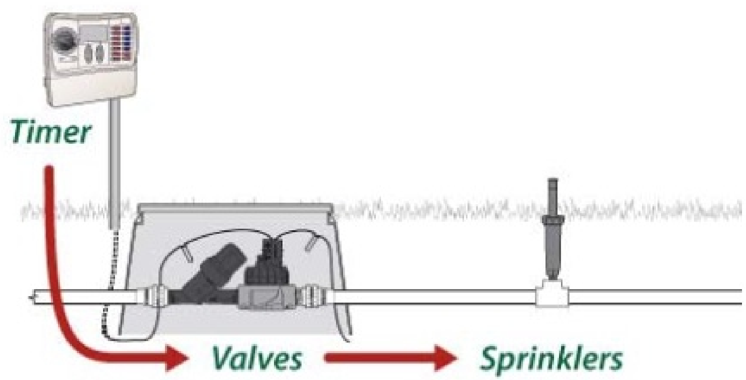
- Classical control systems are all around us:
  - ➞ Temperature control... Cruise control... Flight and motion planning control...
  - ➞ Traffic and network control... Electrical grid control... Population dynamics and epidemics (!)...
  - ➞ Robotics and artificial intelligence...

  The role of classical control is *so pervasive* that it almost goes unnoticed!...

- Basic types of control:

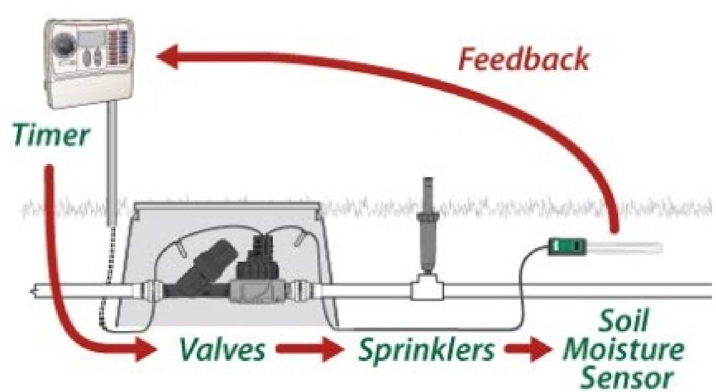
  (1) 'Open-loop' [One-way] – Control actions are predetermined

  - ➢ Control variables have no dependence upon underlying state of S.



  (2) 'Closed-loop' [Feedback] – Control actions depend on information gained as S operates

  - ➢ Control variables may depend upon past values of state of S and control variables.



Closed quantum control systems

- The free evolution of the system S is unitary, generated by Hamiltonian operator.
  Simplest control setting: Semiclassical open-loop control

$$\frac{d}{dt}U(t) = -\frac{i}{\hbar}\Big[\underbrace{H}_{\text{Drift}} + \sum_{\ell=1}^{m} \underbrace{u_\ell(t)}_{\text{Control inputs}} H_\ell\Big]U(t), \quad U(0) = I$$

➞ 'Bilinear' control system on a Lie group ⇒ Borrow from classical 'geometric' control...

Jurdjevic & Sussman, J. Diff. Eqs. **12** (1972).

➞ Coherently driven system, *purity- and spectrum-preserving:* A pure initial state remains pure... Only unitarily equivalent density operators can be reached from initial one...

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- Key questions and control tasks:

  (1) State and operator controllability – Given arbitrary initial state, can control drive S to *any* desired final state? Can *every* desired unitary transformation be enacted?

  - 'Easy' in finite-dim, d: Yes iff Lie algebra generated by $iH, iH_1, \ldots, iH_m$ is full $\mathfrak{su}(d)$

  (2) Synthesis and optimality – How to *construct* a control that achieves the objective? Can we do so with *minimum* expenditure of time and resources?

  - Hard in general ⇒ [Numerical] quantum optimal control

Tannor & Rice, J. Chem. Phys. **83** (1985), Peirce, Dahleh & Rabitz, Phys. Rev. A **37** (1988).

Open quantum systems

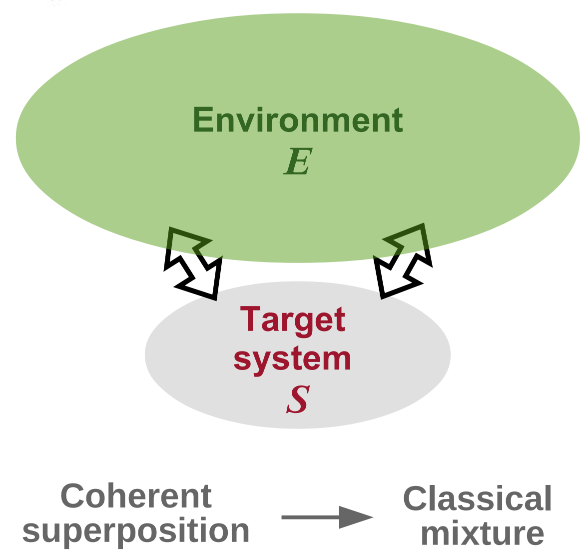
<u>Fact</u>: Real-world dynamical systems are (to a lesser or greater extent) *open*...

- Open system S = *Subsystem* of a composite closed system Joint Hamiltonian evolution

$$H = H_S \otimes \mathbf{I}_E + \mathbf{I}_\mathbf{S} \otimes H_E + H_{SE}, \quad H_{SE} = \sum_a S_a \otimes E_a$$

➞ State of S described by *reduced density operator*

$$\rho_S(t) = \mathrm{Tr}_E\{\rho_{SE}(t)\}$$
$$= \mathrm{Tr}_E\{U_{SE}(t)\rho_S(0) \otimes \rho_E(0) U_{SE}(t)^\dagger\}$$



➞ Evolution of S is no longer unitary – no longer reversible!

- ➢ Discrete time: Quantum CPTP maps or 'super-operators'...
- ➢ Continuous time: Quantum master equations...

- Open quantum dynamics encompasses an enormous range of physical phenomena:

➞ Quantum measurement (POVM and beyond) and quantum feedback...

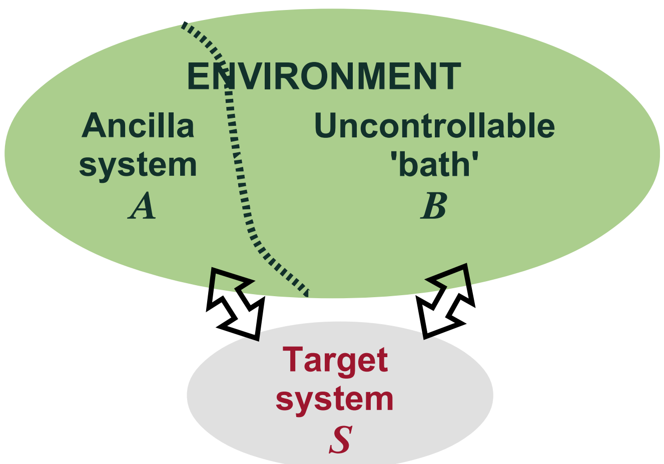
➞ Quantum irreversibility: Noise and fluctuations, approach to equilibrium, cooling...

➞ Quantum decoherence, quantum-to-classical transition...

⋮

Open quantum control systems

'Openness' of the target system can be both
a blessing and a curse for control...



- Two prevailing complementary control philosophies:

  (1) 'Environment as a resource'

  - Access to a *controllable auxiliary quantum system* enables *engineering open dynamics* via open-loop control plus measurement or 'unitary design'...

PHYSICAL REVIEW A, VOLUME 65, 010101(R)

Engineering quantum dynamics

Seth Lloyd* and Lorenza Viola†
d'Arbeloff Laboratory for Information Systems and Technology, Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 25 August 2000; published 10 December 2001)

Quantum Sci. Technol. **2** (2017) 034001

Quantum Science and Technology

PAPER

Quantum and classical resources for unitary design of open-system evolutions

Francesco Ticozzi[1,2] and Lorenza Viola[2,3]

Open quantum control systems

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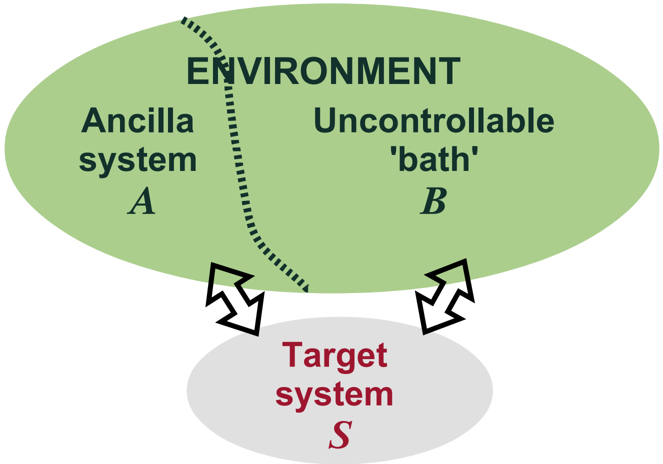


- Two prevailing complementary control philosophies:

  (1) 'Environment as a resource'

  - Access to a *controllable auxiliary quantum system* enables *engineering open dynamics* via open-loop control plus measurement or 'unitary design'...
  - Control for 'quantum reservoir engineering' ⇒ *Dissipation-assisted* or *dissipation-enabled* quantum tasks (purification/'qubit refresh', cooling, stabilization...)

  (2) 'Environment as an enemy'

  - Coupling to quantum or classical *uncontrollable degrees of freedom* induces unwanted non-unitary effects
  - Control for 'quantum noise mitigation' ⇒ *Noise characterization and rejection...*

- Focus: Hamiltonian description of S+E together, with E uncontrollable ('bath') and open-loop Hamiltonian control acting non-trivially on S alone.

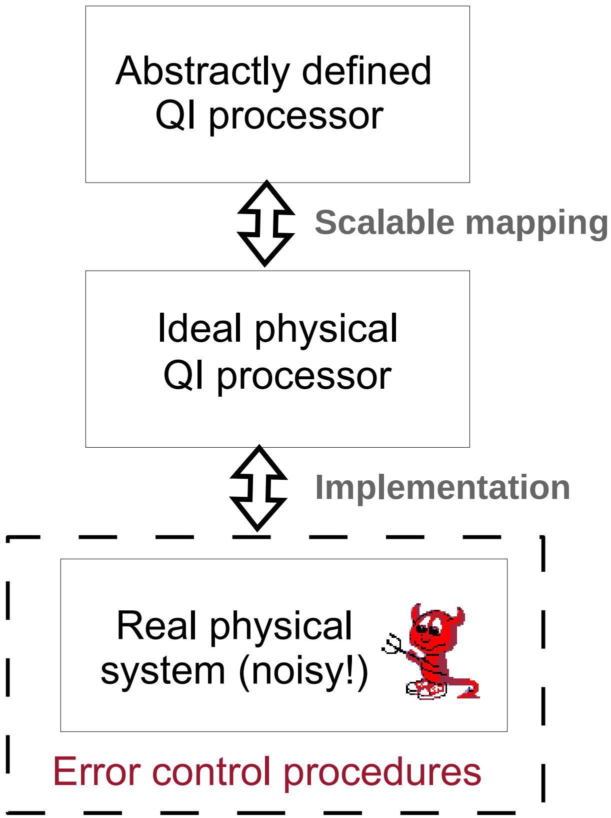
From open quantum systems to noisy qubits

QIP needs to be physically realized in order to be practically useful...

- Real-world qubit devices are inevitably
  (1) Imperfectly isolated
  - Decoherence, dissipation, depolarization, leakage...

  (2) Imperfectly characterized and controllable
  - Modeling errors, parameter uncertainties...
  - Systematic and random rotations, faulty readouts...

  ➞ The combined effect of *environmental and control noise* inevitably results in logical qubit error...



- Viewed as a fundamental issue in the early days...
  "Advocates of quantum parallelism readily admit that a totally coherent evolution may be hard to obtain in practice. But if error correction is needed, *this is inevitably dissipative and incoherent*, and prevents the quantum parallelism sought by Deutsch..."

R. Landauer, Is quantum mechanics useful? (1995).

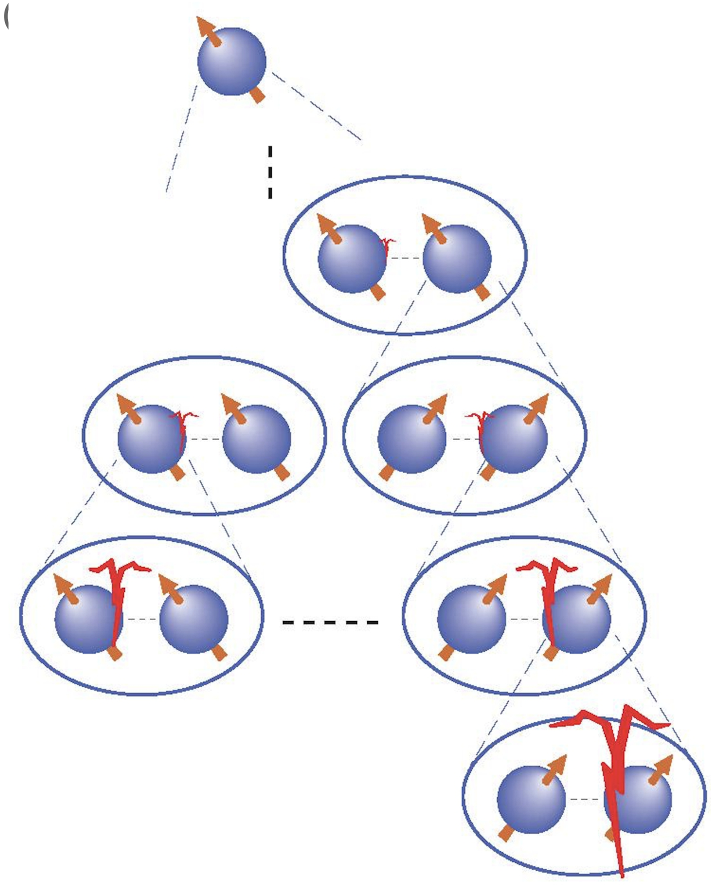
The quest for quantum fault-tolerance – Theory

Scalable QIP requires that information is realized fault-tolerantly...

- Central insight: Quantum information can be *logically represented* ('encoded') in *subsystems* that may be less exposed to noise.

  Knill, Laflamme & LV, Phys. Rev. Lett. **84** (

  ➞ Scaling up in complexity demands that methods for protecting QI *remove more noise than they introduce*...



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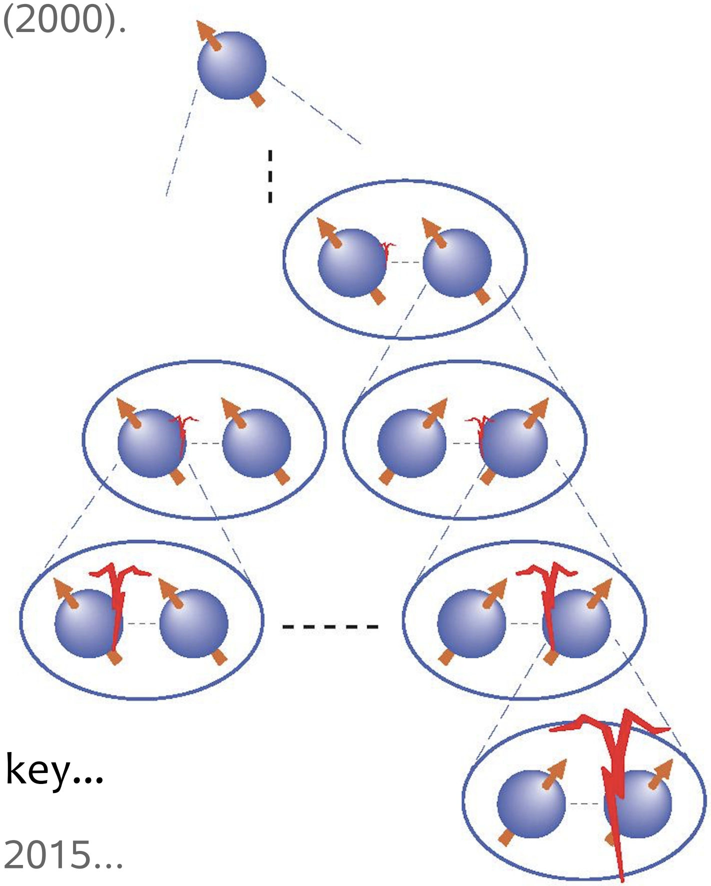


- Accuracy threshold theorem(s): Scalable QIP *is* possible if 'error probability per gate' (EPG) is below a threshold

  Shor 1996; Kitaev 1997; Knill *et al* 1998; Aharonov & Ben-Or 1998; Preskill 1998; Steane 1999; Knill 2005...

$$\mathrm{EPG} < \mathrm{EPG}_{\mathrm{threshold}} \approx 10^{-6} \text{ to } 10^{-2}$$

  ➞ Landmark discovery – although, derived under idealized error-model assumptions...

  - Most calculations done for *local Markovian Pauli* errors
  - For non-Markovian noise, *'well-behaved' correlations* are key...

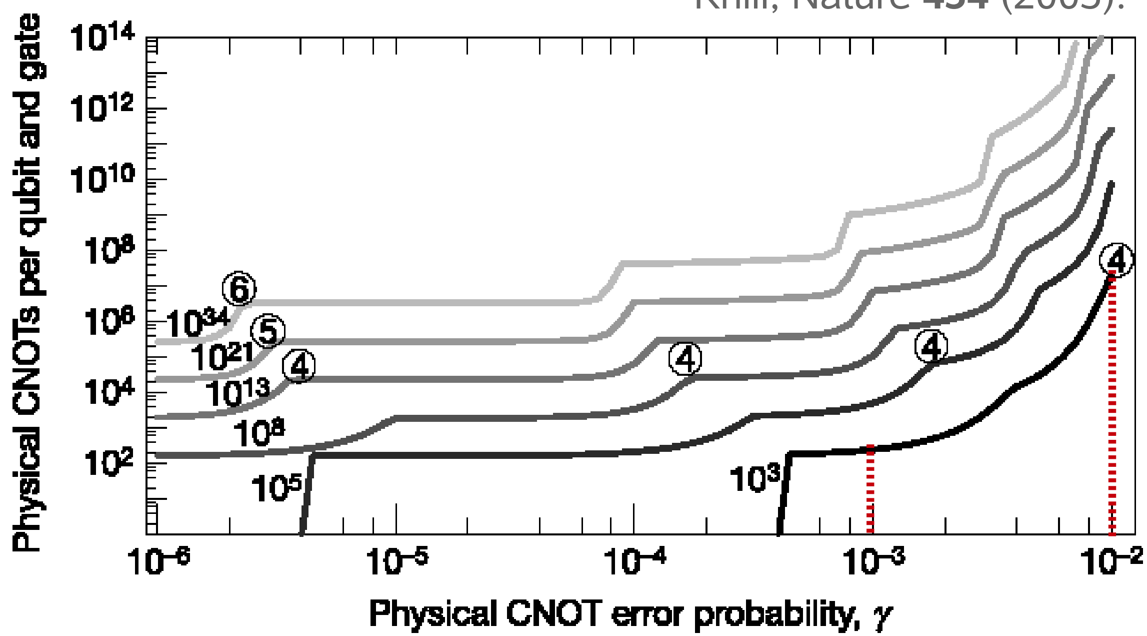
  Ng & Preskill 2009; Preskill 2015...

Noise is no longer a fundamental obstacle – in principle...

The quest for quantum fault-tolerance – Practice

- Fault-tolerant architectures realize *low-error logical qubits and gates by using quantum error-correcting codes* as a means to effectively pump out entropy:
  - ➡ 'Concatenated' quantum codes – hierarchical structure, 'recursive' error protection
  - ➡ 'Surface' quantum codes – 'topological' error protection

Knill, Nature **434** (2005).



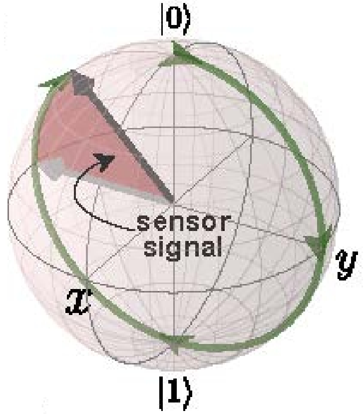
- Regardless of the specifics, the resource requirements of fault-tolerant QC are massive...
  - ➡ 'Overhead factor' = Number of physical qubits per logical qubits $\approx 10^3 - 10^4$ ☹
  - ➡ Overhead requirements decrease rapidly with physical EPG probability... ☺

Recap & (some) outstanding challenges

- In order to bring scalable, large-scale QIP closer to reality, can control theory help to develop tools to significantly *reduce physical error rates* and resource requirements?...
  - In the NISQ era, given access to 'pre-threshold' devices, can the quality of control be boosted enough to ensure *reliable execution* of useful quantum-enhanced algorithms?...

- To what extent (how) can we turn the sensitivity of qubits to their environment into a boon to *quantum system identification* and enhanced sensing?...
  - Can we develop estimation tools to precisely *characterize complex noise environments* in realistic open quantum systems?...
  - Can we aim to *resource-efficient, noise-optimized* quantum control performance?...
  - Can we validate (or disprove!) *quantum fault-tolerance assumptions*?...

R.J. Schoelkopf, A.A. Clerk, S.M. Girvin, K.W. Lehnert, and M.H. Devoret, *Qubits as spectrometers of quantum noise,* Nato Science Series **97** (2002).



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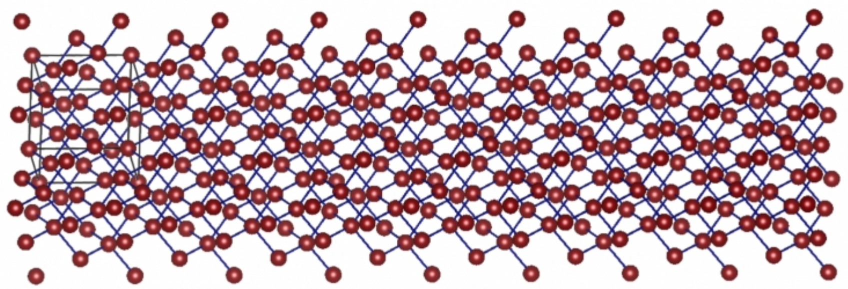


December 14 – 16 2020

An Introduction to Quantum Computing and Materials

Part II.

Open-Loop Hamiltonian Engineering

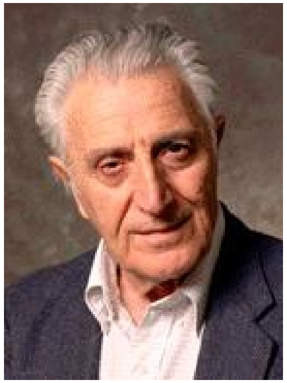
for noise characterization and suppression

Open-loop control to the rescue

- Key physical principle: 'Coherent averaging' of interactions

Paradigmatic example: Spin echo $\Leftrightarrow$ Effective time-reversal

Hahn, Phys. Rev. **80** (1950);
Haeberlen & Waugh, Phys. Rev. **175** (1968).



➡ Unwanted evolution due to a *static* dephasing environment may be 'refocused', so that 'phase coherence' is restored...

➡ The system effectively behaves [stroboscopically] *as if* the unwanted interaction has been *averaged out* by the external control ⇒ *Dynamical decoupling (DD)*

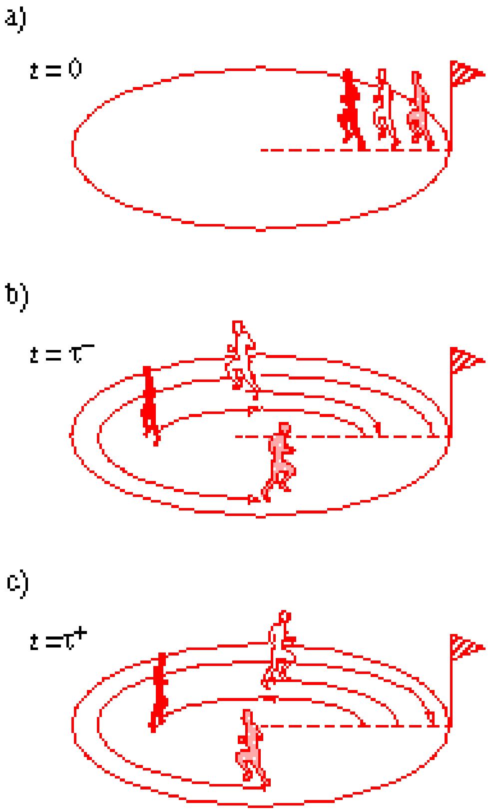
$$H = \omega_1 \sigma_z^1 + \omega_2 \sigma_z^2 \quad \rightarrow \quad H_{eff} = 0$$

Non-selective

$$H = \omega_1 \sigma_z^1 + \omega_2 \sigma_z^2 \quad \rightarrow \quad H_{eff} = \omega_1 \sigma_z^1$$

$$H = \omega_1 \sigma_z^1 + \omega_2 \sigma_z^2 + J\, \sigma_z^1 \sigma_z^2 \quad \rightarrow \quad H_{eff} = \omega_1 \sigma_z^1$$

Selective



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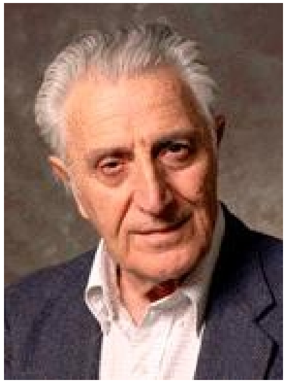
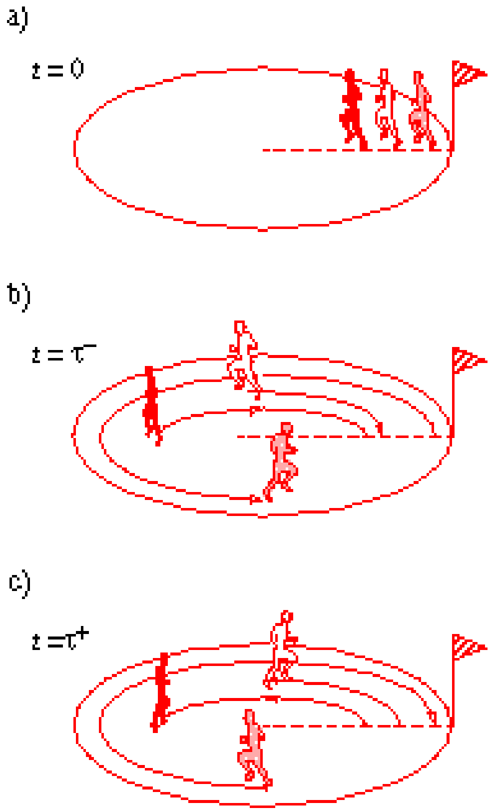
Selective

- Key difference: Open quantum system dynamics
  - Environment (bath) is *not controllable* nor precisely known...
  - Environment is both *non-static* and *non-classical* in general...





A paradigmatic example: DD of qubit dephasing

LV & Lloyd, Phys. Rev. A **58**, 2733 (1998).

- Dephasing spin-boson model:

$$H = H_S \otimes \mathbf{I}_B + \mathbf{I}_S \otimes H_B + H_{SB} = \omega_0 \sigma_z \otimes \mathbf{I}_B + \mathbf{I}_S \otimes \sum_k \Omega_k b_k^\dagger b_k + \sigma_z \otimes \sum_k g_k (b_k^\dagger + b_k)$$

➡ Preferred basis: $[H_S, H_{SB}] = 0$ – But, genuinely dynamical quantum bath: $[H_B, H_{SB}] \neq 0$

➡ Exact solution for free qubit coherence dynamics (thermal bath):

Decay function

$$\rho_{01}(t) \equiv \langle 0|\rho_S(t)|1\rangle = \rho_{01}(0) e^{2i\omega_0 t} e^{-\chi(t)}$$

$$\chi(t) = \int_0^\infty d\omega I(\omega)[2\coth(\beta\omega/2)] \frac{1-\cos(\omega t)}{\omega^2} = \int_0^\infty d\omega S(\omega) \frac{1-\cos(\omega t)}{\omega^2}$$

Noise power spectrum (PSD)

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$$\chi(t) = \int_0^\infty d\omega I(\omega)[2\coth(\beta\omega/2)] \frac{1-\cos(\omega t)}{\omega^2} = \int_0^\infty d\omega S(\omega) \frac{1-\cos(\omega t)}{\omega^2}$$

Noise power spectrum (PSD)

- Control protocol on qubit: A train of resonant, instantaneous, equally spaced, identical π pulses. Elementary control period $T_p = 2\Delta t$:

$$U(T_p) = PU(\Delta t)PU(\Delta t) = e^{-iH'\Delta t} e^{-iH\Delta t} \equiv e^{-iH_{\rm eff} T_p}, \qquad H' = -H_S - H_{SB} + H_B$$

$$H_{\rm eff} = H' + H + \mathcal{O}(\Delta t) \approx \mathbf{I}_S \otimes H_B$$

Approximate time reversal if bath is 'frozen'!

A paradigmatic example: DD of qubit dephasing

LV & Lloyd, Phys. Rev. A **58**, 2733 (1998).

- Controlled reduced dynamics may still be exactly computed:

$$\chi(t) \mapsto \chi_c(t) = \int_0^\infty d\omega S(\omega) \frac{F(\omega, \Delta t)}{\omega^2}, \quad F(\omega, \Delta t) = \tan^2(\omega \Delta t/2)$$

DD 'f lter function'

➡ Exact DD is achievable for *arbitrarily fast* control:

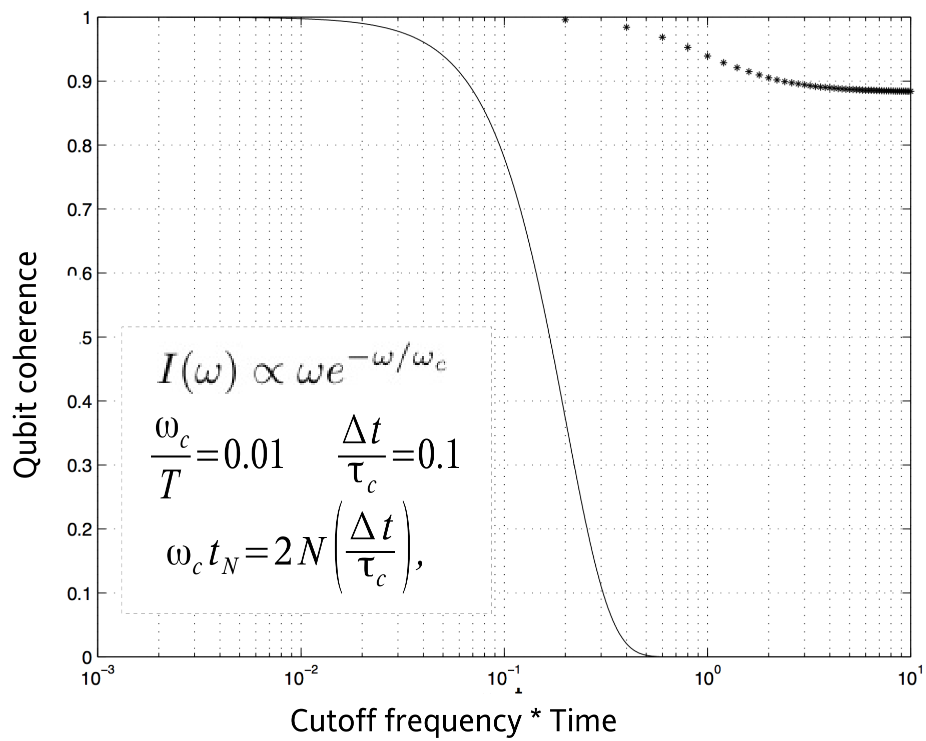
$$S(\omega) \propto \omega^s f(\omega, \omega_c)$$

$$\lim_{T_p \to 0, N \to \infty} \rho_{01}(t_N = NT_p) = \rho_{01}(0)$$

High-frequency cutoff

- System-bath interaction may be approximately *averaged out* if the control is *fast* relative to shortest characteristic timescale of the bath:

$$\frac{\tau_{\text{ctrl}}}{\tau_{\text{corr}}} \approx \omega_c \Delta t \quad small$$



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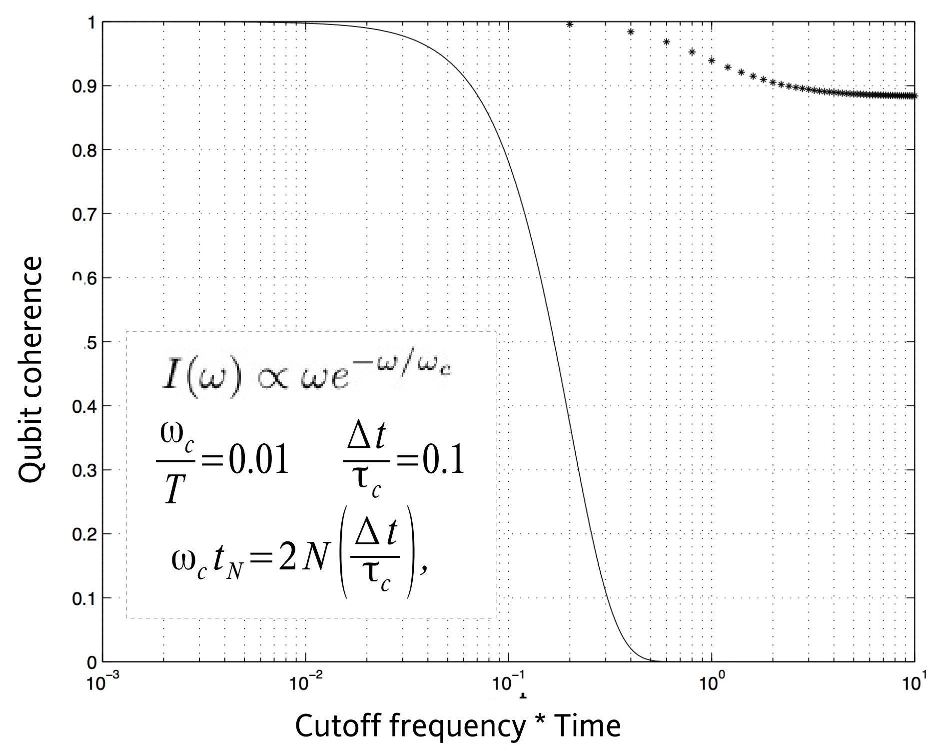
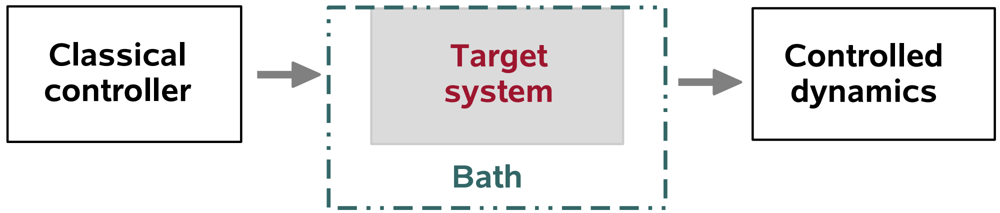


How small for good DD in practice?...
What about general noise?... multiple qubits?...
realistic, bounded controls?...

Noise suppression via DD: Control framework



- Target system S (finite-dim, e.g., n qubit) coupled to *quantum or classical* bath *B*:

$$H(t) = H_S \otimes \mathbb{I}_B + H_{SB}(t), \qquad H_{SB} = \sum_a S_a \otimes B_a$$

with respect to interaction picture defined by $\mathbb{I}_S \otimes H_B$.

➞ Classical noise formally recovered for $H(t) \equiv H_S(t)$ [stochastic time-dependence]

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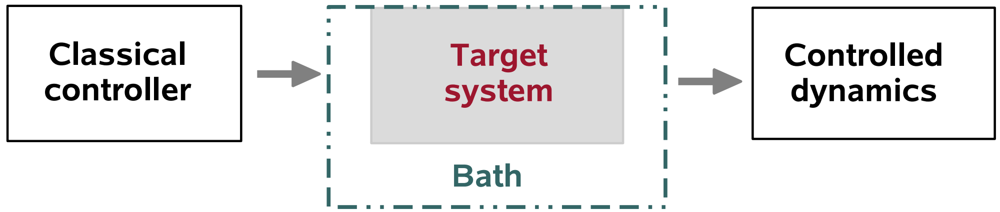

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➞ Classical noise formally recovered for $H(t) \equiv H_S(t)$ [stochastic time-dependence]

- Noise-inducing bath *B* is uncontrollable ⇒ Add *open-loop* controller, acting *only* on *S*:

$$H_{\rm tot}(t) = H(t) + H_c(t) \otimes \mathbf{I}_B \equiv H_{\rm err}(t) + H_c(t)$$

$$U_{SB}(T) \equiv U_c(T)e^{-i\Phi(T)} = \mathcal{T}\exp\left\{-i\int_0^T [U_c(t)^\dagger H_{\rm err}(t) U_c(t)]\, dt\right\} = \mathbf{I}_S \otimes V_B(T) + \ldots$$

Error action operator

➞ DD problem amounts to reducing the norm of the error operator, 'up to pure-bath terms'
⇒ *Perturbative (Magnus-series) cancellation* $||\ldots|| = ||T H_{SB}^{\rm eff}(T)|| = \mathcal{O}(T^{\alpha+1})$

DD theory: Selected achievements

- The open-loop Hamiltonian engineering framework has allowed to substantially expand applicability of DD *beyond its original setting and assumptions:*
  - ➞ Target system need not be a single qubit ⇒ Arbitrary [finite-dim] system
  - ➞ Control objective need not be a simple averaging ⇒ High-order suppression
  - ➞ Noise need not be pure dephasing ⇒ Arbitrary (multi-axis) decoherence and relaxation
    - ➢ *Universal* two-axis DD: first-order (symmetry-based), *high-order* ('concatenated')

      LV, Knill & Lloyd, PRL 1999; Khodjasteh & Lidar, *ibid.* 2005...
  - ➞ Control need not require arbitrarily fast, uniformly-spaced, instantaneous, perfect pulses
    - ➢ Instantaneous pulses subject to *min separation constraint* Khodjasteh & LV, PRA 2011...
    - ➢ Instantaneous pulses with *optimal* (Uhrig DD) vs. *digital timings* (Walsh DD)

      Uhrig, PRL 2009; Quiroz & Lidar, PRA 2011; Wang & Liu, *ibid.* 2011; Hayes et al, *ibid.* 2011...
    - ➢ *Bounded* control: DD with finite pulses ('Eulerian'), continuous(-wave) DD

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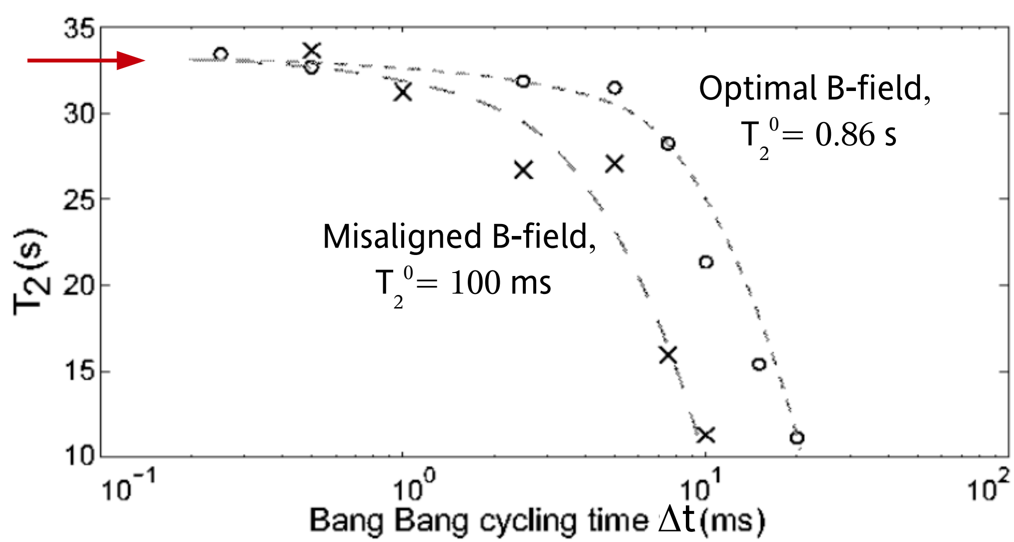
    ⋮

- Key physical requirement: *Temporally correlated, non-Markovian noise* ('colored spectrum')
- Key challenges: (1) Reduce complexity; (2) Optimally tailor to a given noise environment

DD experiment: Selected achievements

- DD schemes for (single-)qubit storage have been experimentally validated across a variety of device technologies – under different noise environments and control imperfections...

Fraval *et al*, *Dynamic decoherence control of a solid-state nuclear-quadrupole qubit*, Phys. Rev. Lett. **95** (2005).

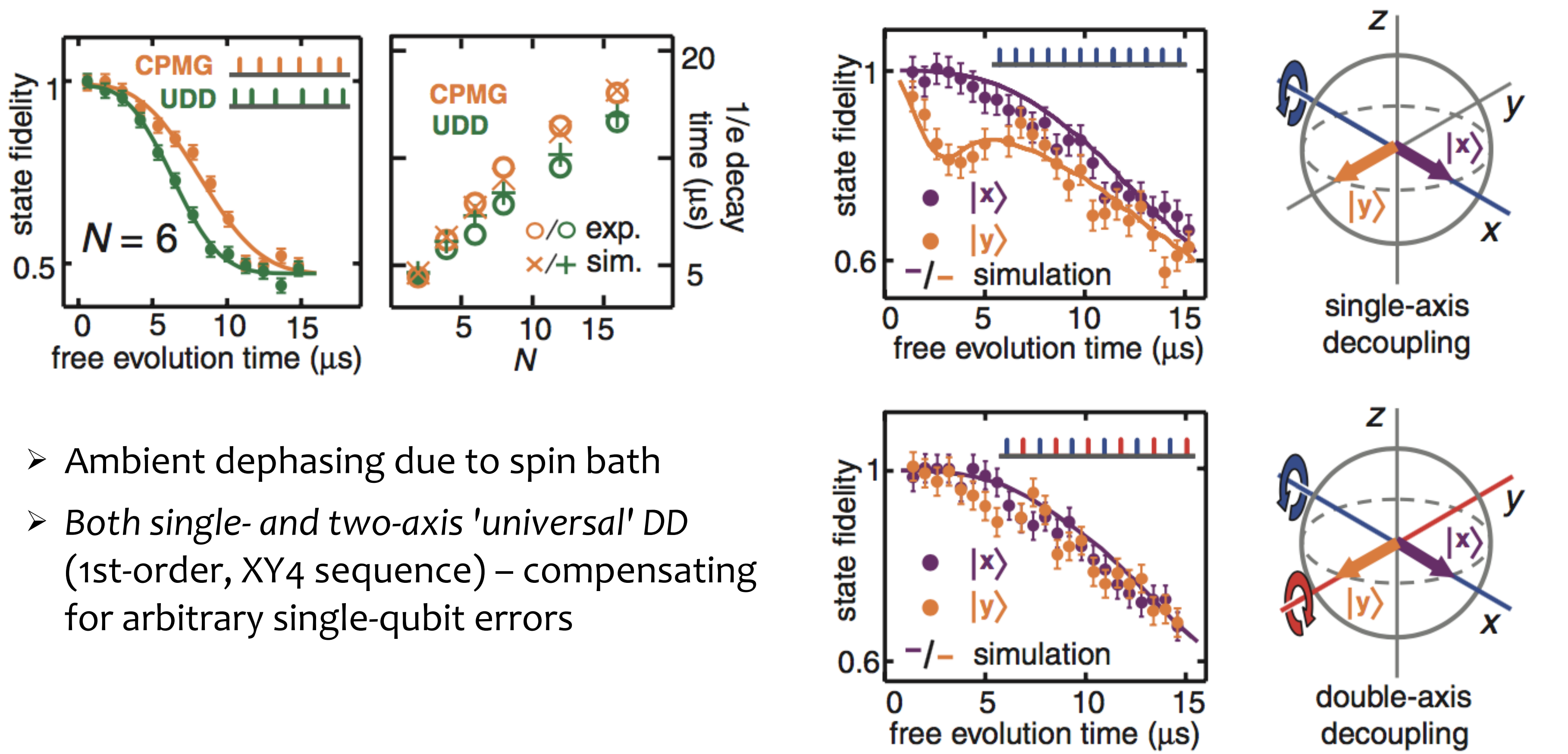


- ➢ Ambient dephasing due to fluctuating magnetic fields
- ➢ Repeated spin-echo protocol (with phase alternation) – *Single-axis, 1st-order periodic DD* (up to 1000 DD cycles)

DD experiment: Selected achievements

- DD schemes for (single-)qubit storage have been experimentally validated across a variety of device technologies – under different noise environments and control imperfections...

de Lange *et al*, *Universal dynamical decoupling of a single solid-state spin from a spin bath*, Science **330** (2010).

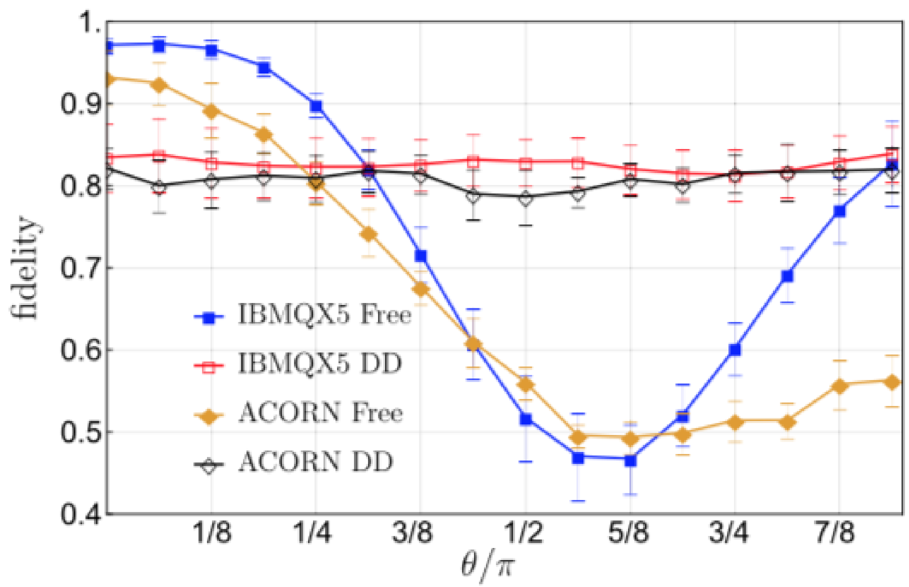


- Ambient dephasing due to spin bath
- *Both single- and two-axis 'universal' DD* (1st-order, XY4 sequence) – compensating for arbitrary single-qubit errors

DD experiment: Selected achievements

- DD schemes (XY4, CPMG) are *already* giving substantial benefit on NISQ machines…

Pokharel *et al*, *Demonstration of Fidelity Improvement Using Dynamical Decoupling with Superconducting Qubits*, Phys. Rev. Lett. **121** (2018).



- ➢ Substantial improvement expected if DD could be further tailored to the specific noise environment

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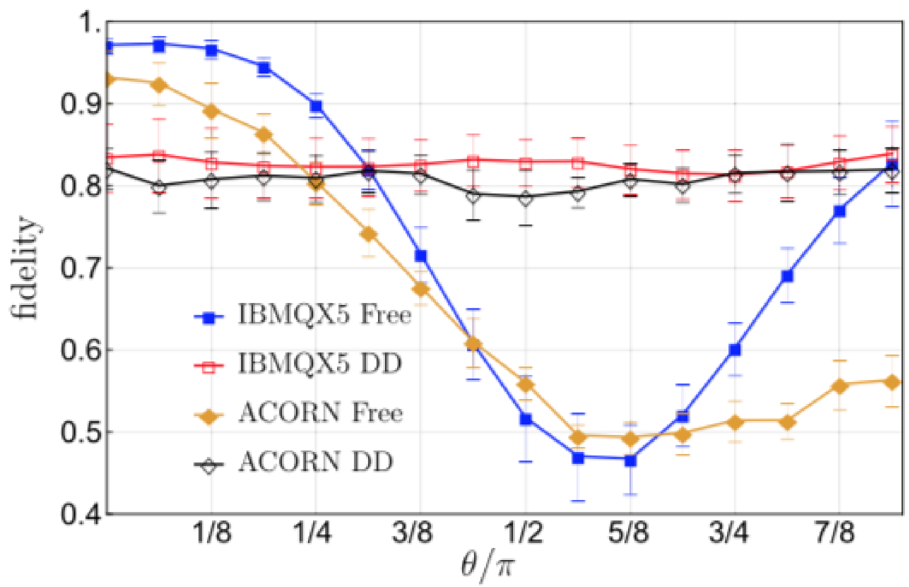
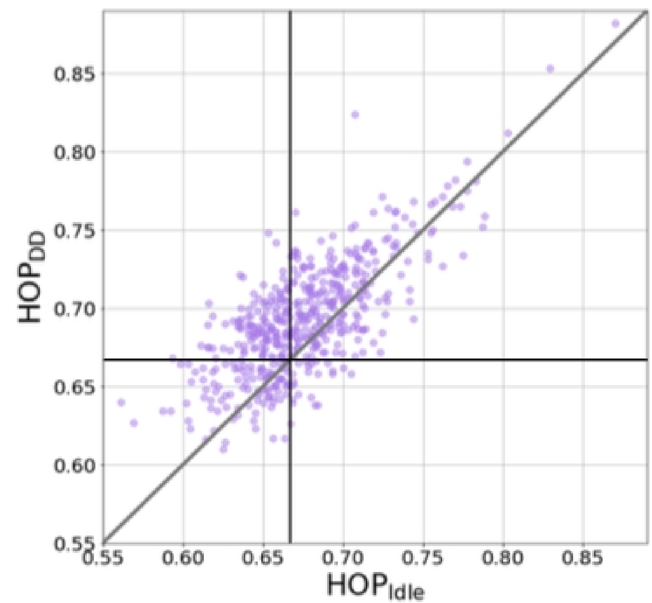


- ➢ Substantial improvement expected if DD could be further tailored to the specific noise environment

- ➢ Use of DD improves 72.8% of all random 6-qubit circuits tested (on 27-qubit IBMQ)

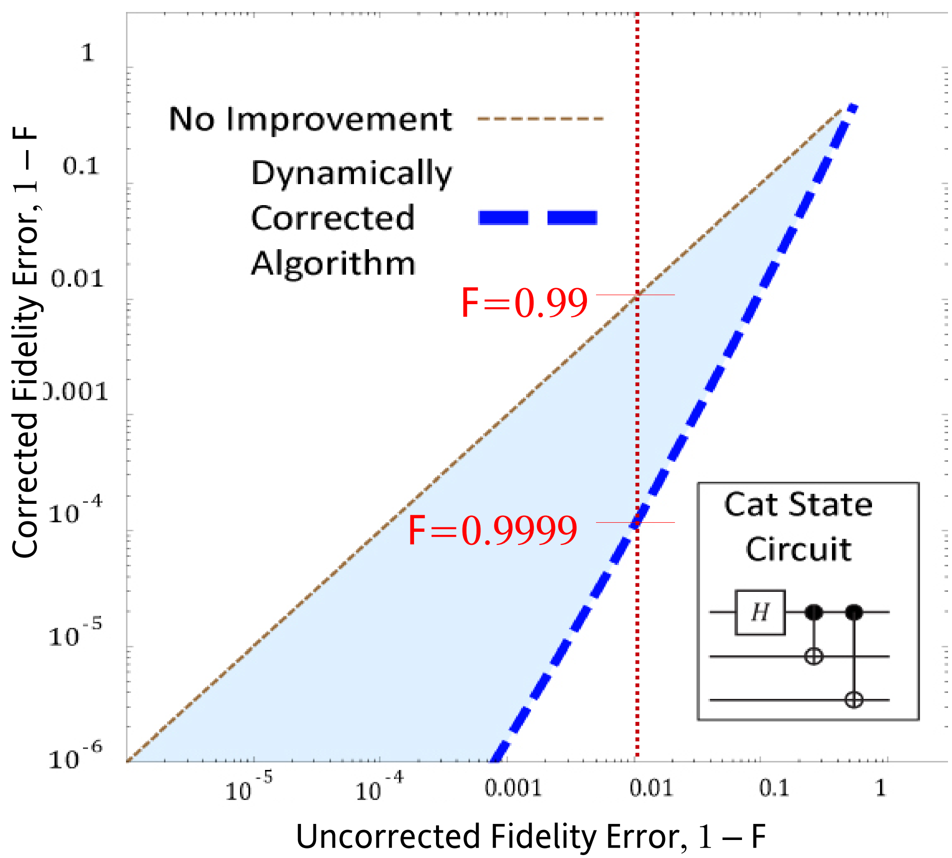
Jurcevic *et al*, *Demonstration of quantum volume 64 on a superconducting quantum computing system*, arXiv:2008.08571



Open-loop noise suppression beyond quantum memory

Khodjasteh & LV, *Dynamically error-corrected gates for universal quantum computation*, Phys. Rev. Lett. **102** (2009); *ibid.* **104** (2010).

- Noise suppression may be extended to single- and two-qubit *quantum gates* by designing the control propagator according to continuous paths that obey appropriate symmetries [modified 'Eulerian paths on Cayley graph'] ⇒ Dynamically Corrected Gates (DCGs)



➞ DCG implementation consists of [2 + 2 x 6] x 16 = 256 'bare' gates

➞ Performance indicator: Change in error-corrected slope

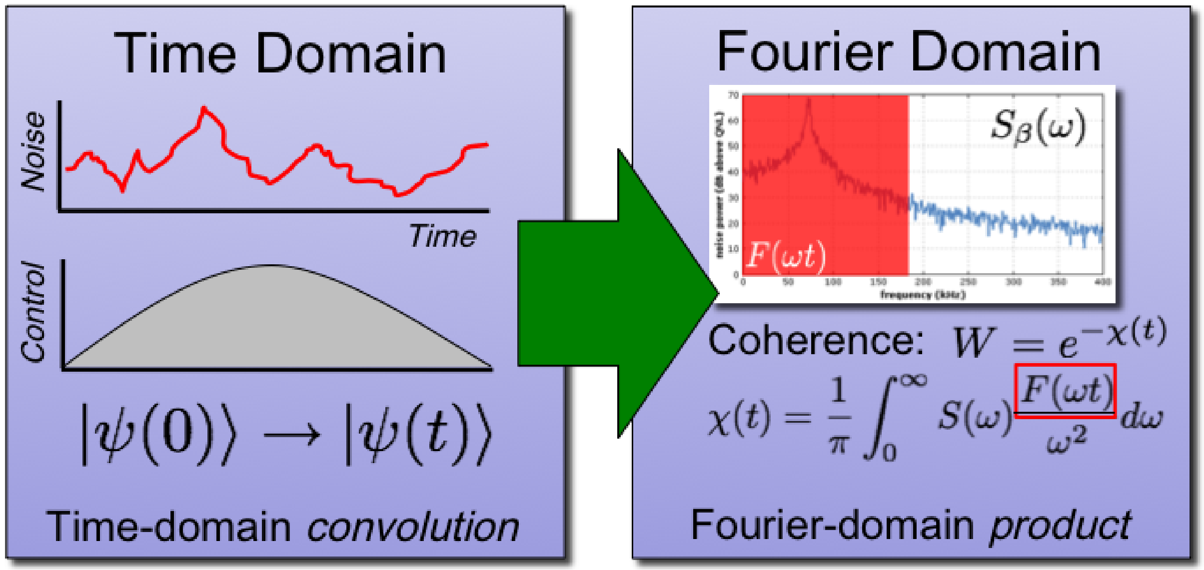
$$\mathrm{EPG} = \mathcal{O}\left[\tau_{\min}||H_{\mathrm{error}}||\right] \mathrm{EPG}_{\mathrm{phys}}$$

- Suppression may be pushed to high order by using concatenated design:

$$\mathrm{EPG}^{[m]} = \mathcal{O}\left[(\tau_{\min}||H_{\mathrm{error}}||)^{[m+1]}\right]$$

Unified picture: Filter design

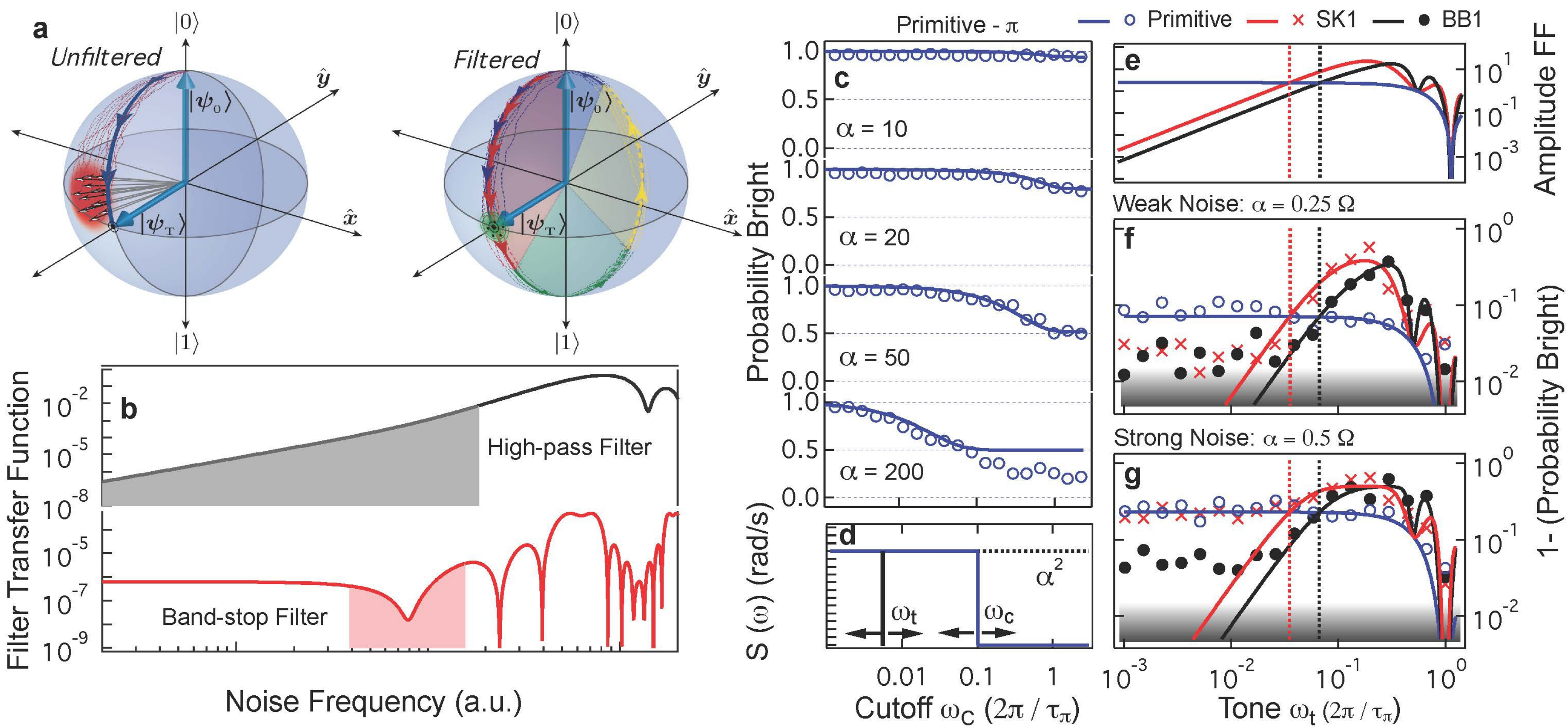
Kofman & Kurizki, Phys. Rev. Lett. **87** (2001), Cywinski *et al*, Phys. Rev. B **77** (2008); Green *et al*, Phys. Rev. Lett. **109** (2012); Paz-Silva & LV, *ibid*. **113** (2014)...



- Picture the control modulation as enacting a filter in the frequency domain: The filter may be designed to have a *specific frequency response*, such that certain noise components are suppressed [⇒ DD] or let through and sensed [⇒ *quantum noise spectroscopy*, QNS]...
  - ➞ Simplest case: Single qubit under *classical Gaussian dephasing*, and perfect π pulses. A *single* filter function (FF) suffices to describe controlled decoherence dynamics.
  - ➞ Arbitrary high-order FFs share a *common structure*, and may be explicitly, *non-recursively* computed as linear combinations of products of *fundamental FFs* of same and lower order.

Filter-function formalism: Experimental validation

Soare *et al*, *Experimental noise filtering by quantum control*, Nat. Phys. **10** (2014).



- ➞ Control objective: Noise-suppressed single-qubit π rotations under (temporally correlated, engineered) amplitude control noise.
- ➞ Control protocols: Both DCGs and NMR-inspired composite-pulse sequences.
- ➞ Quantitative agreement with analytical FF predictions observed in the weak-noise limit.

Filter shaping for noise characterization

Key insight: Qubits driven by external control may be used as *quantum sensors* for noise.

- Simplest setting: Single qubit sensor, *purely dephasing* noise, *stationary* and *Gaussian*

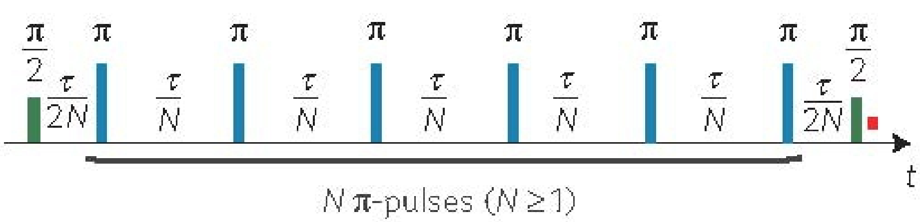
$$S(\omega) = \int d\tau\, e^{-i\omega\tau} \langle B(t+\tau)B(t)\rangle$$

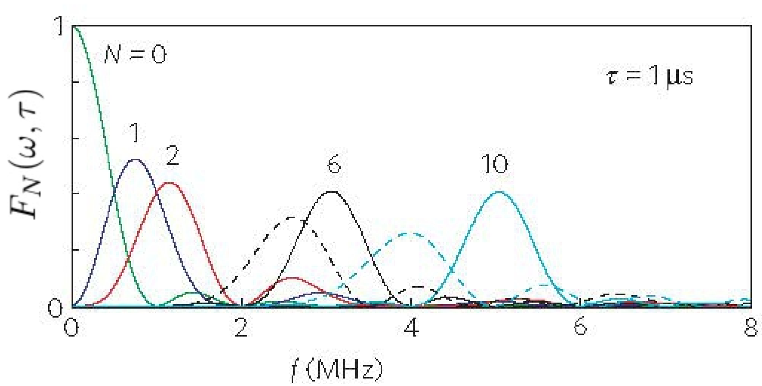
Noise power spectrum (PSD)

***Filter shaping in Fourier space*: Applied control modifies sensor's spectral response to noise**

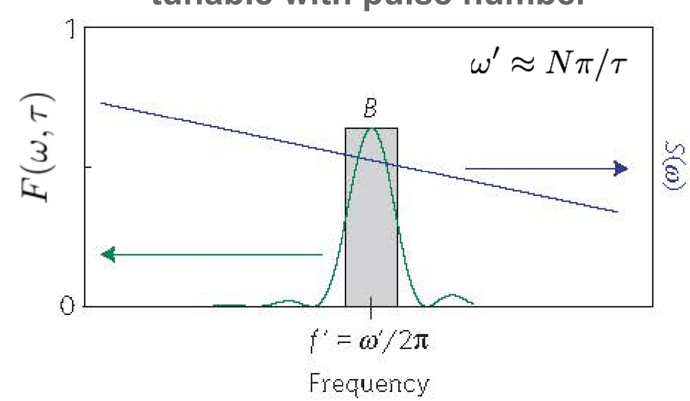
$$\mathcal{M}(\tau) \approx S(\omega=\omega') \int_{\omega'-B/2}^{\omega'+B/2} \mathrm{d}\omega\, F(\omega,\tau)$$

CP/CPMG





Approximate *pass-band filter*, tunable with pulse number



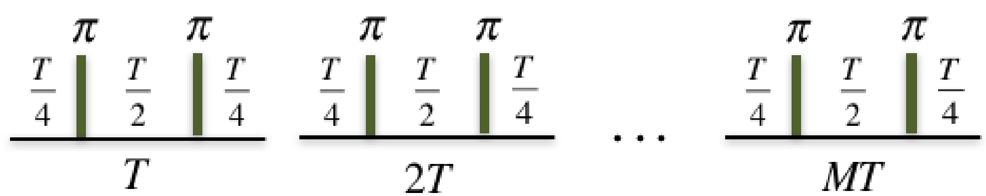
Noise characterization via DD

G. A. Alvarez and D. Suter, PRL **107**, 230501 (2011).

- Simplest setting: Single qubit sensor, *purely dephasing* noise, *stationary* and *Gaussian*
  - ➞ Recall that decay parameter for qubit coherence is determined by overlap integral

$$\chi(T) = \frac{1}{\pi}\int_{-\infty}^{\infty} d\omega\, F(\omega, T) S(\omega)$$

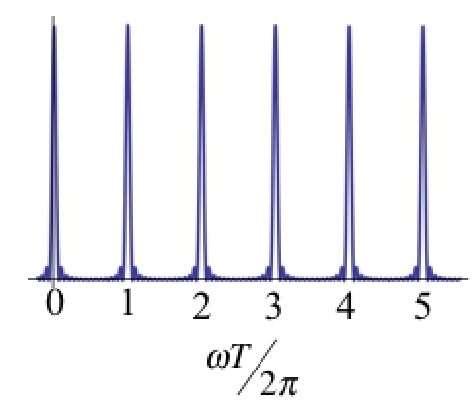
- Control protocol on qubit: Iterate a 'base' DD sequence (CPMG) M times



- ➞ FFs compose in a simple way under sequence repetition

$$\chi(MT) = \frac{1}{\pi}\int_{-\infty}^{\infty} d\omega\, \frac{\sin^2(M\omega T/2)}{\sin^2(\omega T/2)} F(\omega, T) S(\omega)$$
$$\approx \frac{2M}{T}\sum_{k=-\infty}^{\infty} F\left(\frac{2\pi k}{T}\right) S\left(\frac{2k\pi}{T}\right)$$

$$\frac{\sin^2(M\omega T/2)}{\sin^2(\omega T/2)} \approx \frac{2M}{T}\sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$$



- ➞ Emergence of *frequency comb enables sampling* of the target PSD at the harmonic frequencies.

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- Control protocol on qubit: Iterate a 'base' DD sequence (CPMG) M times
  - ➡ Use CPMG sequence with period T, $T/2$, ... T/n and truncate each series to nth harmonics

$$\underbrace{\begin{pmatrix} \chi_1(MT) \\ \chi_2(MT) \\ \vdots \\ \chi_n(MT) \end{pmatrix}}_{\text{decay parameters}} = \underbrace{\begin{pmatrix} A_1 & A_2 & \cdots & A_n \\ 0 & A_2 & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & A_n \end{pmatrix}}_{n\times n \text{ matrix}} \underbrace{\begin{pmatrix} S\left(\frac{2\pi}{T}\right) \\ S\left(\frac{4\pi}{T}\right) \\ \vdots \\ S\left(\frac{2n\pi}{T}\right) \end{pmatrix}}_{\text{spectrum}}$$

$$A_k \propto F\left(\frac{2\pi k}{T}\right)$$

➡ An estimate of the sampled spectrum may be obtained by matrix inversion.

Noise characterization by qubit sensors: Basic taxonomy

Classical controller → Qubit-based sensing system / Environment → Measurement of sensor's observables → 'Deconvolution'/ spectral estimation

- Different QNS approaches have been envisioned and pursued – based on

→ *Complexity of target noise environment: Role of prior assumptions*
Classical vs. quantum, [weak] dephasing vs. relaxation, Gaussian vs. non-Gaussian...

→ *Control modalities: Pulsed vs. continuous open-loop modulation, e.g.,*

- DD QNS: Narrow π pulses, either single (e.g., N-pulse CPMG) or repeated DD sequences
- CW QNS: Evolution under a continuous, 'spin-locking' coherent drive ($T_{1\rho}$ relaxometry)
- Optimally band-limited QNS: Control modulation by 'Slepian' waveforms

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- Optimally band-limited QNS: Control modulation by 'Slepian' waveforms

→ *Sensor's complexity:*
Single- vs. multiple qubits
Estimation of both self- and *cross- correlation* spectra...

PRX QUANTUM **1**, 010305 (2020)

Two-Qubit Spectroscopy of Spatiotemporally Correlated Quantum Noise in Superconducting Qubits

Uwe von Lüpke,[1,†] Félix Beaudoin,[2,3] Leigh M. Norris,[2] Youngkyu Sung,[1] Roni Winik,[1] Jack Y. Qiu,[1] Morten Kjaergaard,[1] David Kim,[4] Jonilyn Yoder,[4] Simon Gustavsson,[1] Lorenza Viola,[2] and William D. Oliver[1,4,5,*]

Concluding remarks: Where to next?

- Challenges that NISQ-era quantum devices pose [at all stages of control] also present new opportunities for *new paradigms* to emerge – by cross-fertilization between different fields...
  - ➞ Classical automatic and robotic control, large-scale control-system techniques...
  - ➞ Statistical learning and AI techniques...

- Quantum control is poised to have as *transformative and widespread* a role across quantum science and technologies as classical control has had since its inception – and it may well be a *key enabler of new physics explorations*...
  - ➞ Engineering of *non-equilibrium phases* of matter and novel materials?...
  - ➞ Certification protocols for *open and/or many-body quantum simulation*?...
  - ➞ Optimal control for *quantum-enhanced sensing and metrology*?...
  - ⋮

"Now, you might say – Who should do this and why should they do it?... Well, I pointed out a few of the applications, but I know that the reason you would do it might be just for fun. But have some fun!'

R.P. Feynman (1959).