Topological quantum materials

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2020 Quantum Winter School

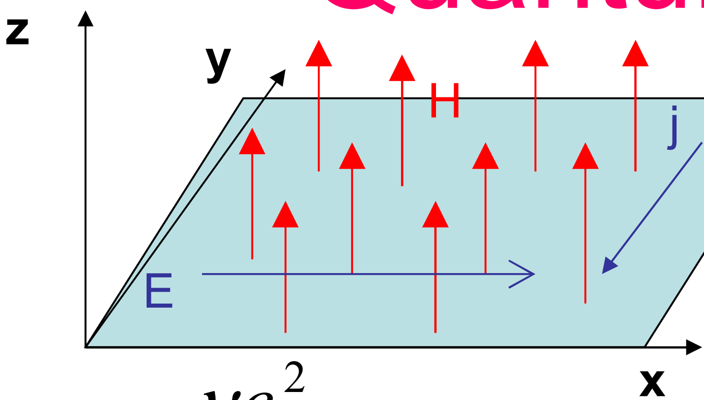
An Introduction to Quantum Computing and Materials

December 16, 2020

Outline

- Key Ideas
  - topological properties
  - edge and surface states
  - role of interactions
  - fractionalization
  - symmetries
- Topological quantum computing
- Non-Abelian quantum matter

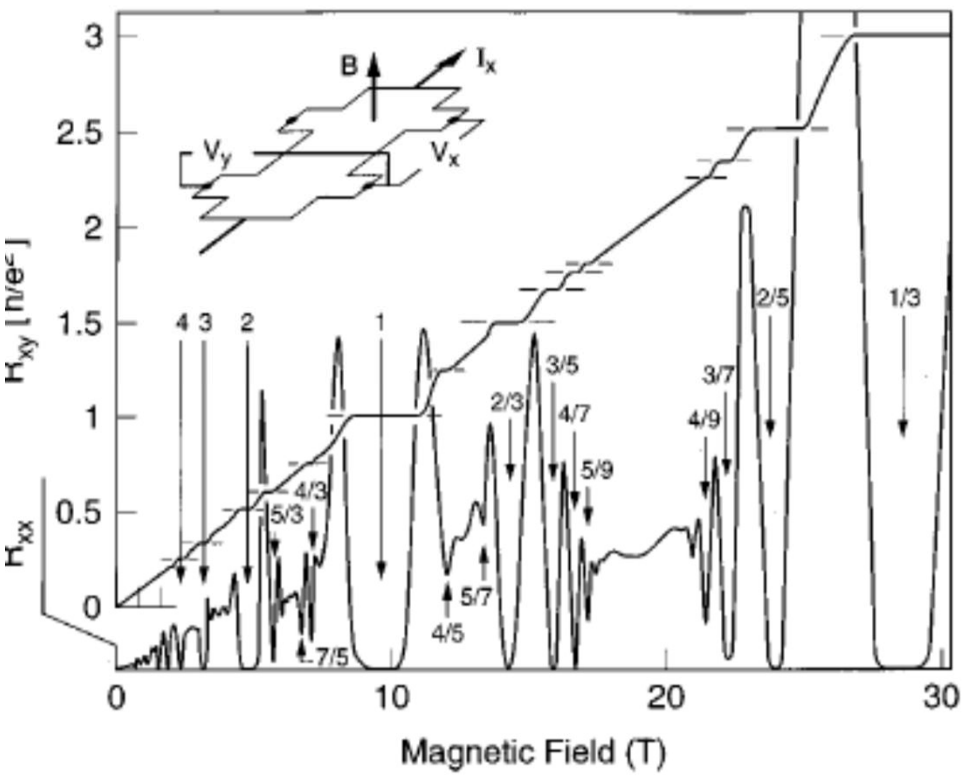
Quantum Hall Effect



$$I_x = \sigma_{xx} V = 0$$

$$I_y = \sigma_{xy} V$$

$$I = \frac{\nu e^2}{h} V; \quad \nu \text{ - rational number (filling factor)}$$

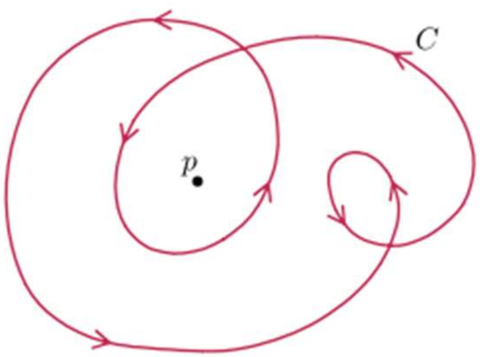


$$\frac{e^2}{h} = (25.8k\Omega)^{-1}$$

$$= 1\,\text{Klitzing}$$

H.L. Stormer *et al*., RMP S298 (1999)

Idea I: Topological properties

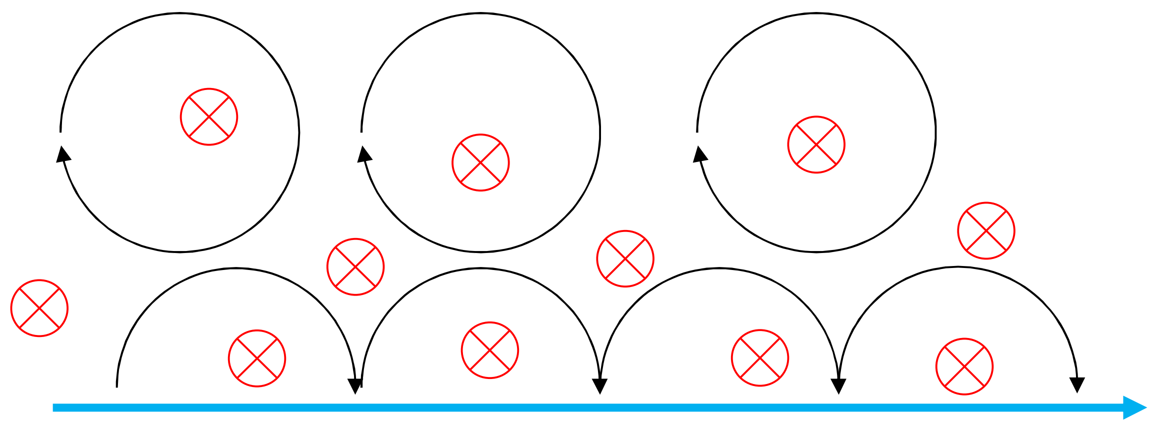


Winding number: how many times a curve winds around a point. This *integer number* is *a topological invariant* and does not change at small deformations of the curve.

Quantum Hall resistance $R_H = \frac{h}{ne^2}$

n **is an integer *topological invariant* that does not change at small changes of the Hamiltonian. It is assumed that the system is an insulator, that is, the ground state is separated by a finite gap from all excitations.**

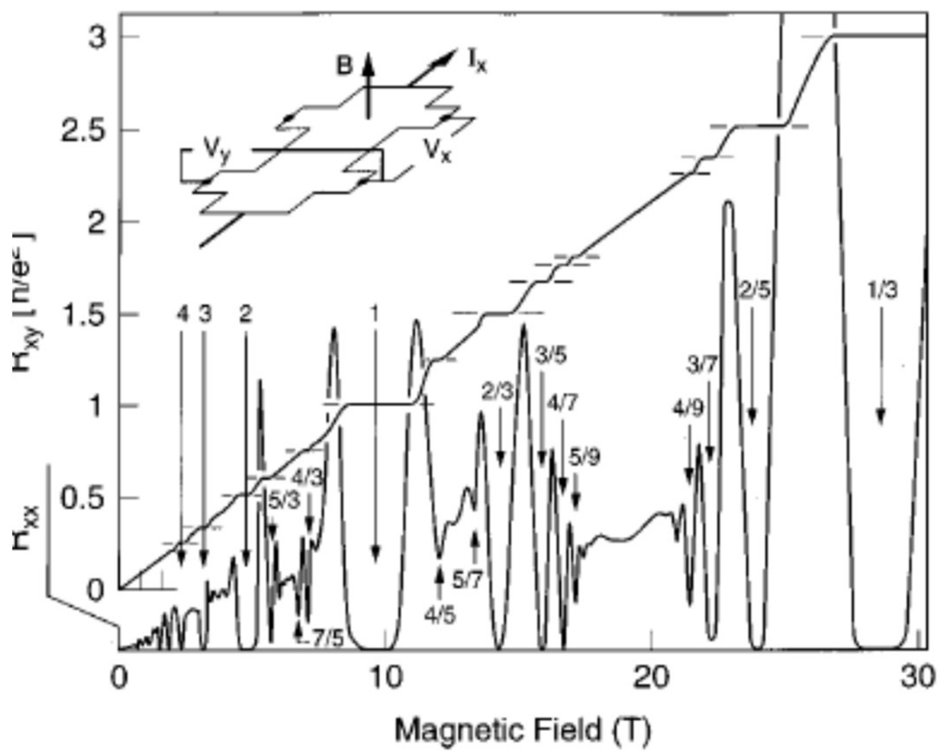
Idea II: Edge states



Insulator in the bulk. No gap on the edges.

Transport on the edges.

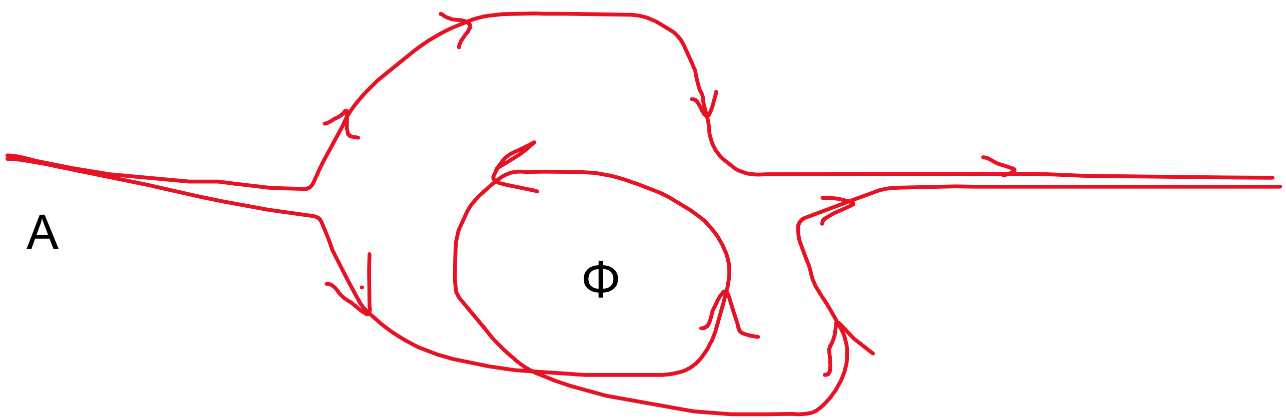
Idea III: Role of interaction



$$\Psi = \prod (z_i - z_j)^n \exp\left(-\sum \frac{|z_i|^2}{4l^2}\right) \qquad z_k = x_k + i y_k$$

Idea IV: Fractionalization

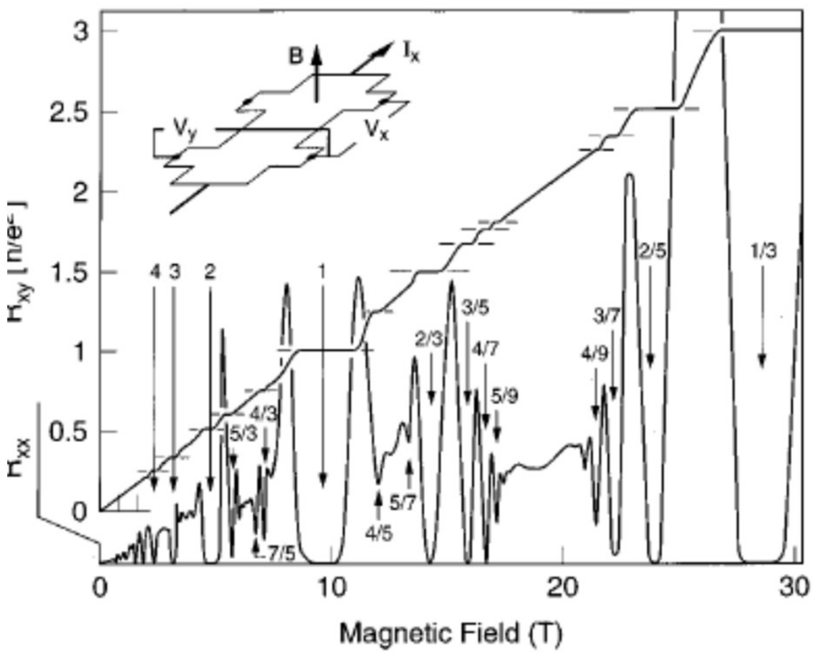
Aharonov-Bohm Effect



Phase difference $\frac{2\pi\Delta L}{\lambda} + \frac{2\pi q\Phi}{hc}$

Invisible *flux quantum* $\frac{hc}{q}$

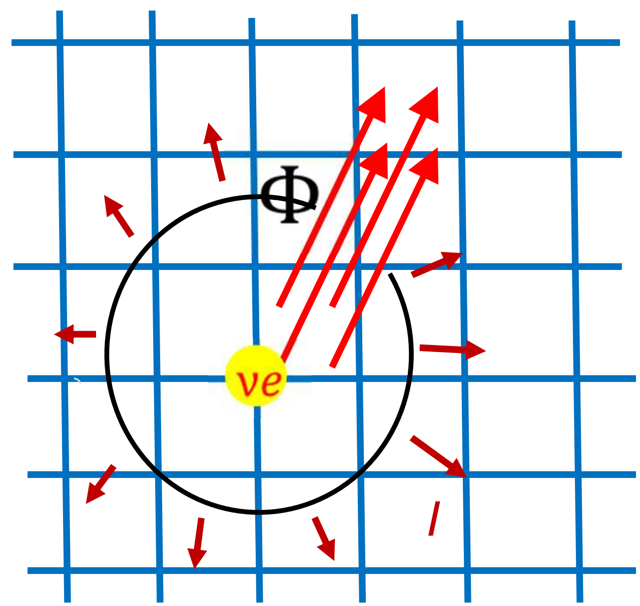
Idea IV: Fractionalization



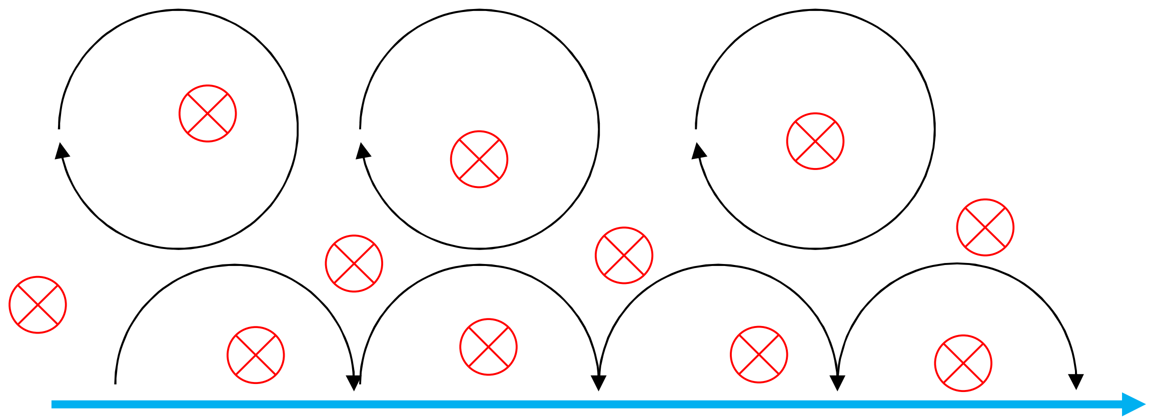
Fractional quantum Hall effect:
$R_H = \frac{h}{\nu e^2}$, **where ν is a fraction**

$$\oint E dl = -\frac{1}{c}\frac{\partial \Phi}{\partial t}; \;\; I = \frac{\nu e^2}{h}\oint E dl$$

$$Q = \int I dt = -\frac{\nu e^2}{ch}\Delta\Phi = -\nu e$$

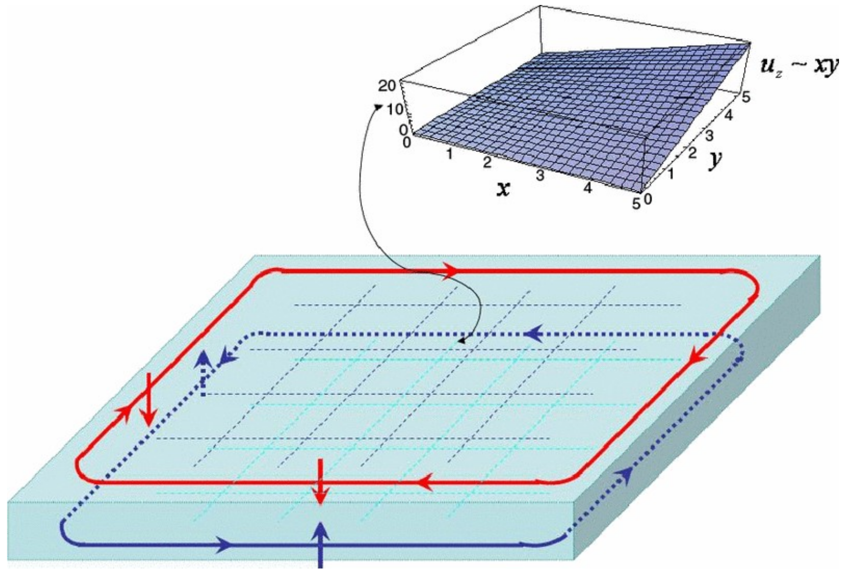


No symmetries in quantum Hall effect

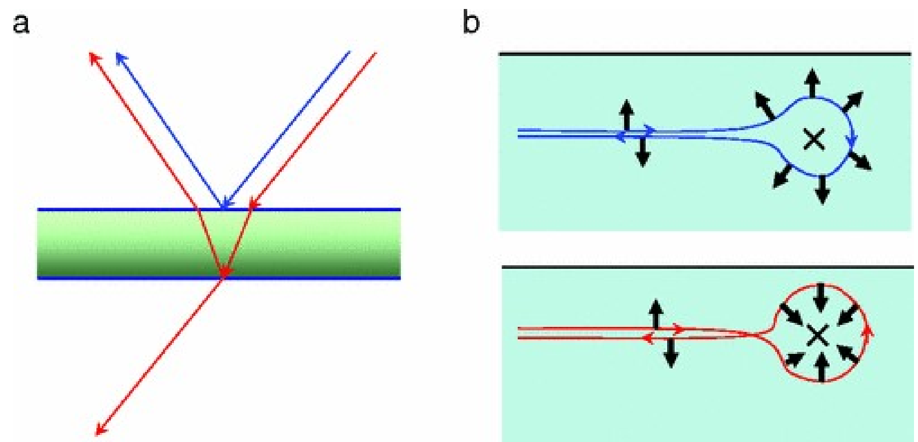


Time-reversal symmetry destroyed by magnetic field: current flows in one direction only.
Other symmetries destroyed by impurities.

Idea V: Role of symmetry



[B. A. Bernevig and S.-C. Zhang, Phys. Rev. Lett. **96**, 106802 (2006).]



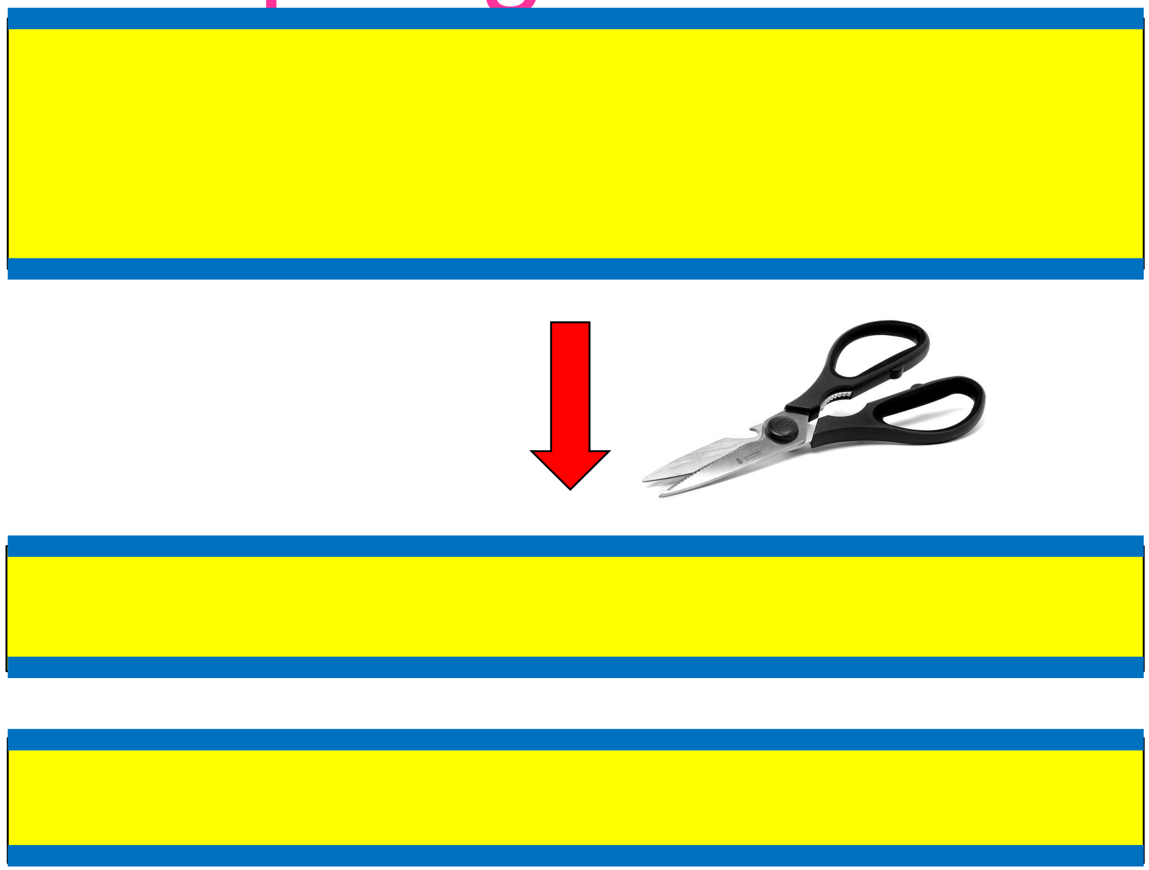
Disorder does not destroy edge states in the absence of a magnetic field due to quantum interference.

No magnetic field = time-reversal symmetry

[X.-L. Qi and S.-C. Zhang, Rev. Mod. Phys. **83**, 1057 (2011).]

Symmetry-protected topological states in 2D topological insulators

3D topological insulators



Insulator with topologically protected metal on the surface

Numerous experimental realizations: Bi_2Se_3, SmB_6, $Bi_{14}Rh_3I_9$, etc.

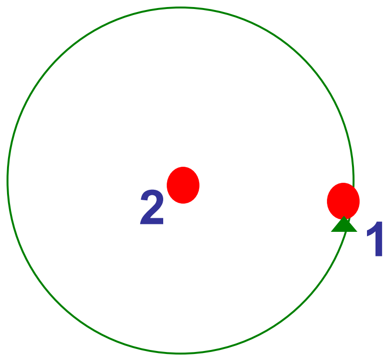
Limited protection:

M. McGinley and N. R. Cooper. Nature Phys. **16**, 1181 (2020)

Fractional Statistics

Bosons: $\psi(x_1, x_2) = \psi(x_2, x_1)$

Fermions: $\psi(x_1, x_2) = -\psi(x_2, x_1)$

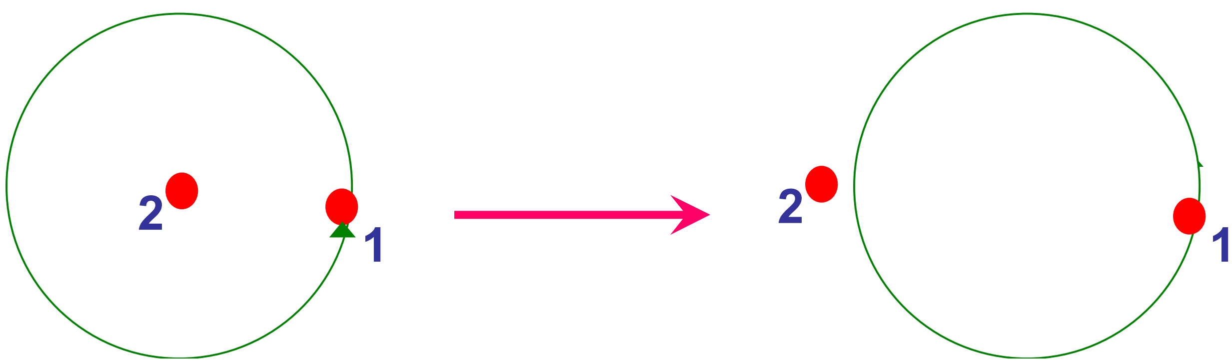


Statistical phase:

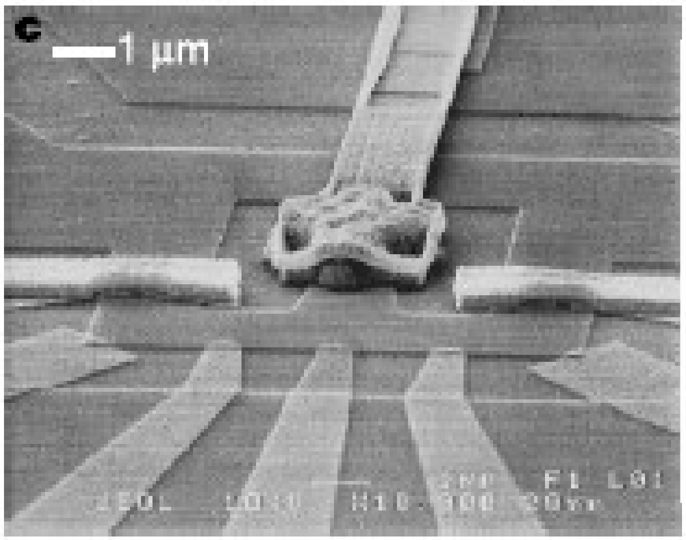
$$\psi \rightarrow \exp(i\theta)\psi$$

Anyons: $\theta \neq 2\pi n$

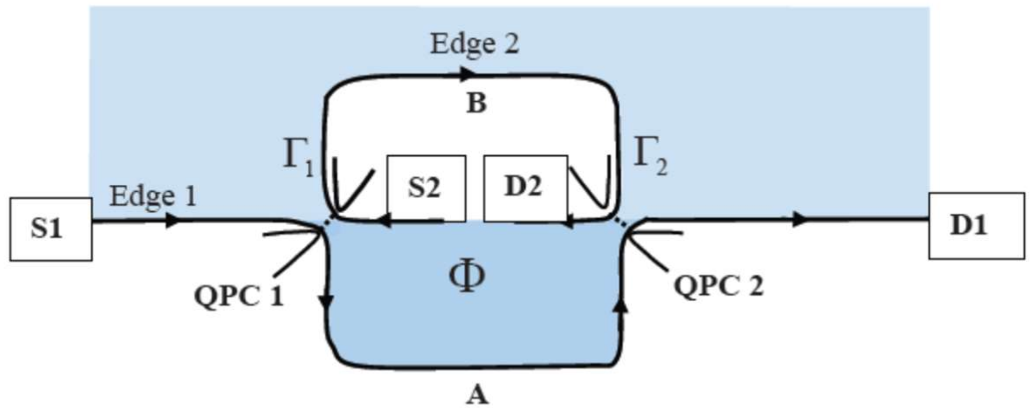
No anyons in 3D



Electronic Mach-Zehnder Interferometer



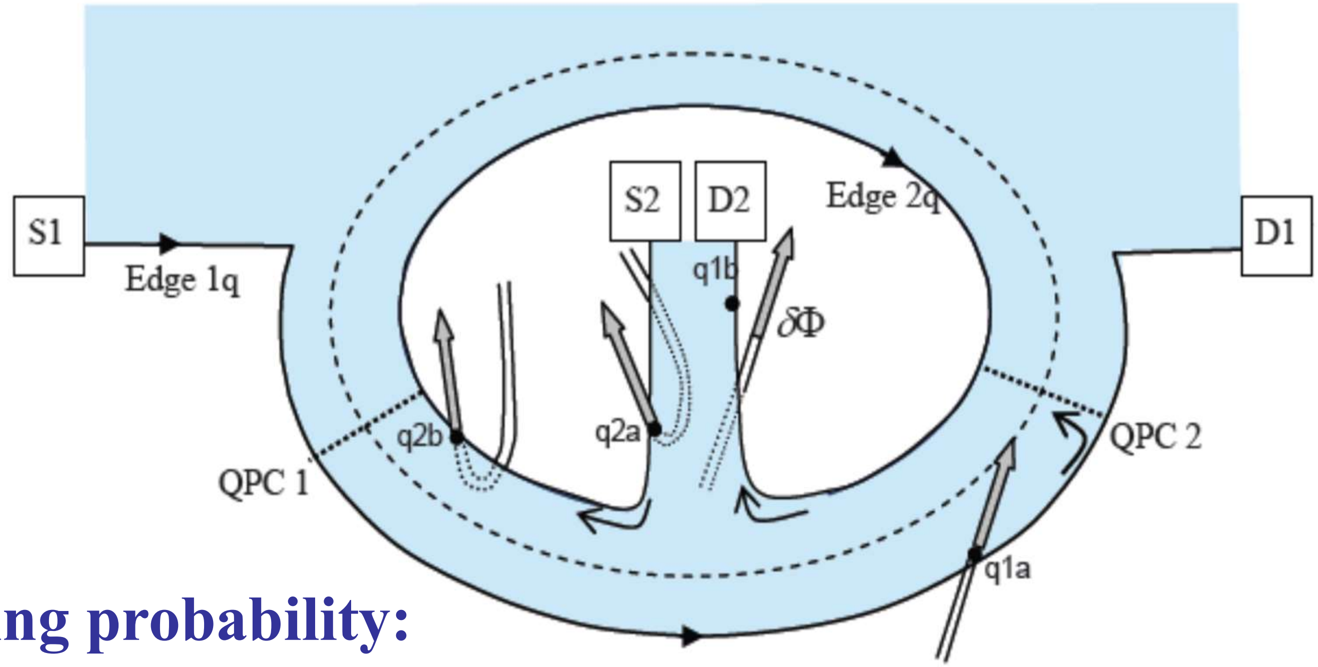
Y. Ji et al. Nature 422, 415 (2003)



K. T. Law, D. E. Feldman, and Y. Gefen
Phys. Rev. B 74, 045319 (2006)

What is the period: *hc*/*e* or *hc*/*q* ?

Solution of the puzzle



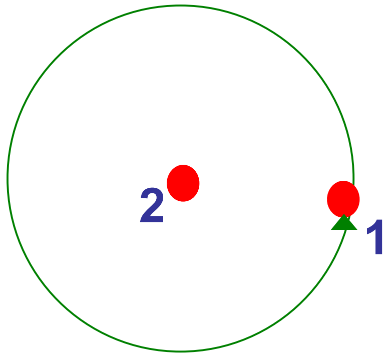
Tunneling probability:

$$p(\Phi) = c_1[|\Gamma_1|^2 + |\Gamma_2|^2] + c_2\Gamma_1\Gamma_2^* \exp(-2\pi\nu i\Phi/\Phi_0) + c.c.$$

$$p_1 = p(\Phi+\Phi_0); p_2 = p(\Phi+2\Phi_0);...; p_{1/\nu} = p(\Phi)$$

$$t_i = 1/p_i; t = \sum 1/p_i; I = \frac{\nu e \times 1/\nu}{t} = \frac{1/\nu}{\sum 1/I(\Phi+n\Phi_0)}$$

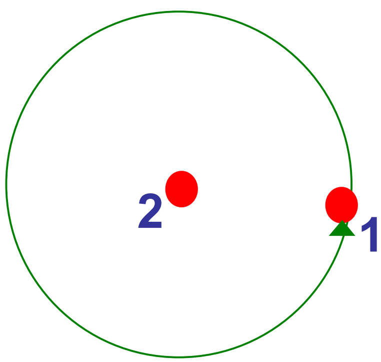
Non-Abelian statistics



$$\left|\psi_f\right\rangle \neq \exp(i\theta)\left|\psi_i\right\rangle$$

Several states at given quasiparticle positions

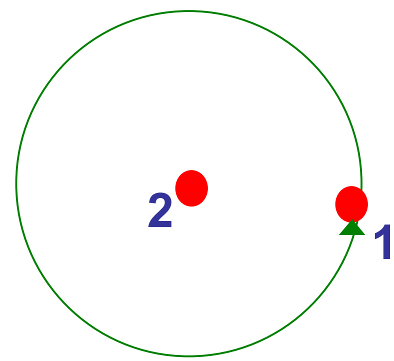
Vacuum superselection sector $\left|1\right\rangle$



$$\psi \to \psi$$

$$\theta = 0$$

Fermion sector $\left|\varepsilon\right\rangle$

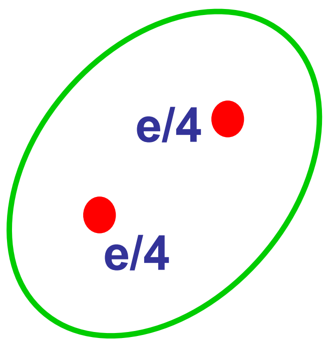


$$\psi \to -\psi$$

$$\theta = \pi$$

$$\alpha\left|1\right\rangle\left|\quad\right\rangle + \beta\left|\varepsilon\right\rangle\left|\quad\right\rangle \to \alpha\left|1\right\rangle\left|\quad\right\rangle - \beta\left|\varepsilon\right\rangle\left|\quad\right\rangle$$

Non-Abelian qubit and quantum computing



Vacuum state = 1

Fermion state = 0

Stable with respect to local perturbations!

Operations ⟶ Quasiparticle braiding

Universal quantum computation:

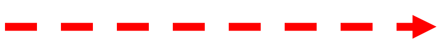
12/5 (Read-Rezayi) states etc.

Thermal transport

Edge point of view on topological order:

gapless edge modes

Thermal conductance of a bosonic channel $\kappa = 1$ in units of $\pi^2 k^2 T/3h$

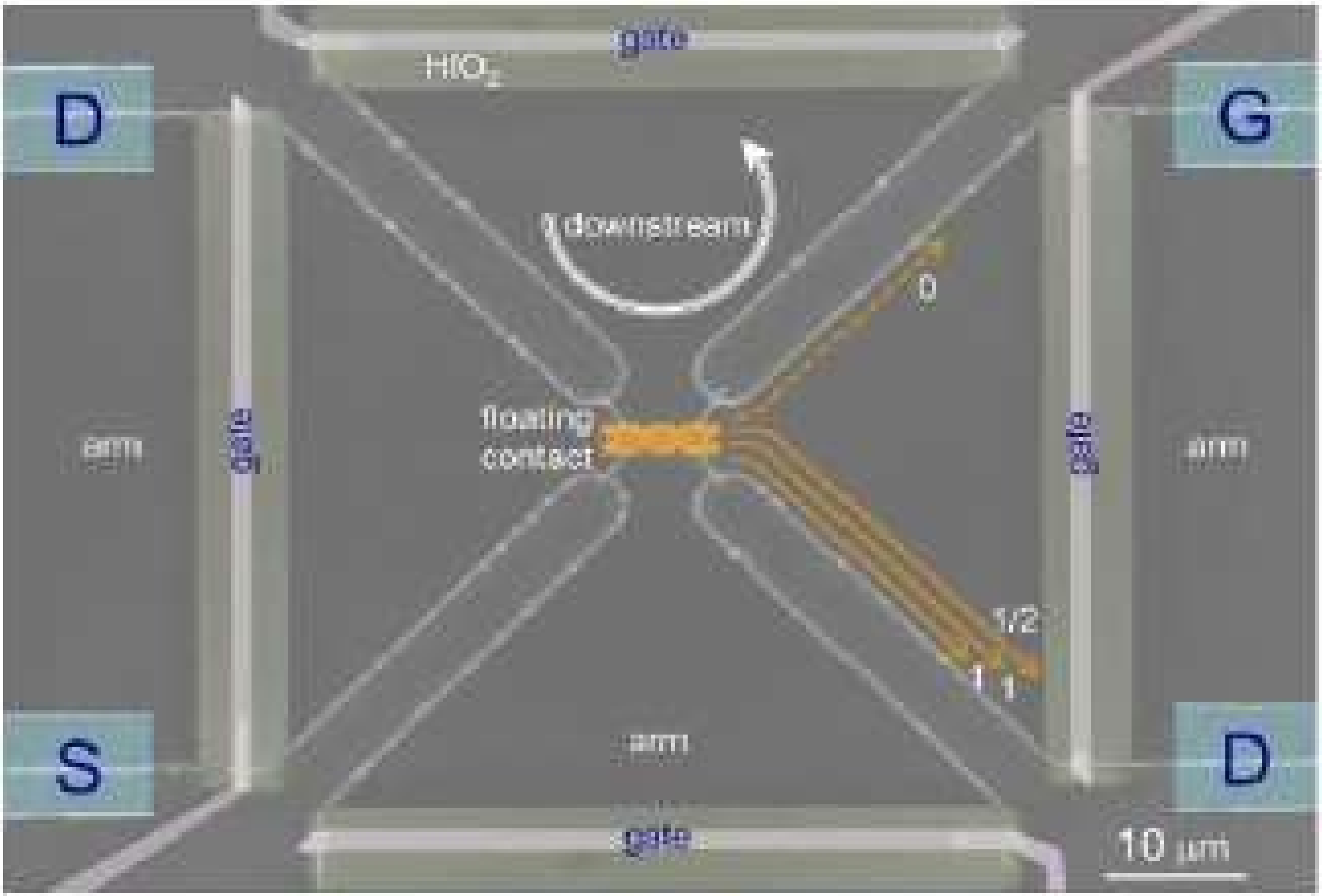


Thermal conductance of a Majorana channel $\kappa = 1/2$

Thermal conductances of co-propagating modes add: $\kappa = 3$ at $f = 7/3$

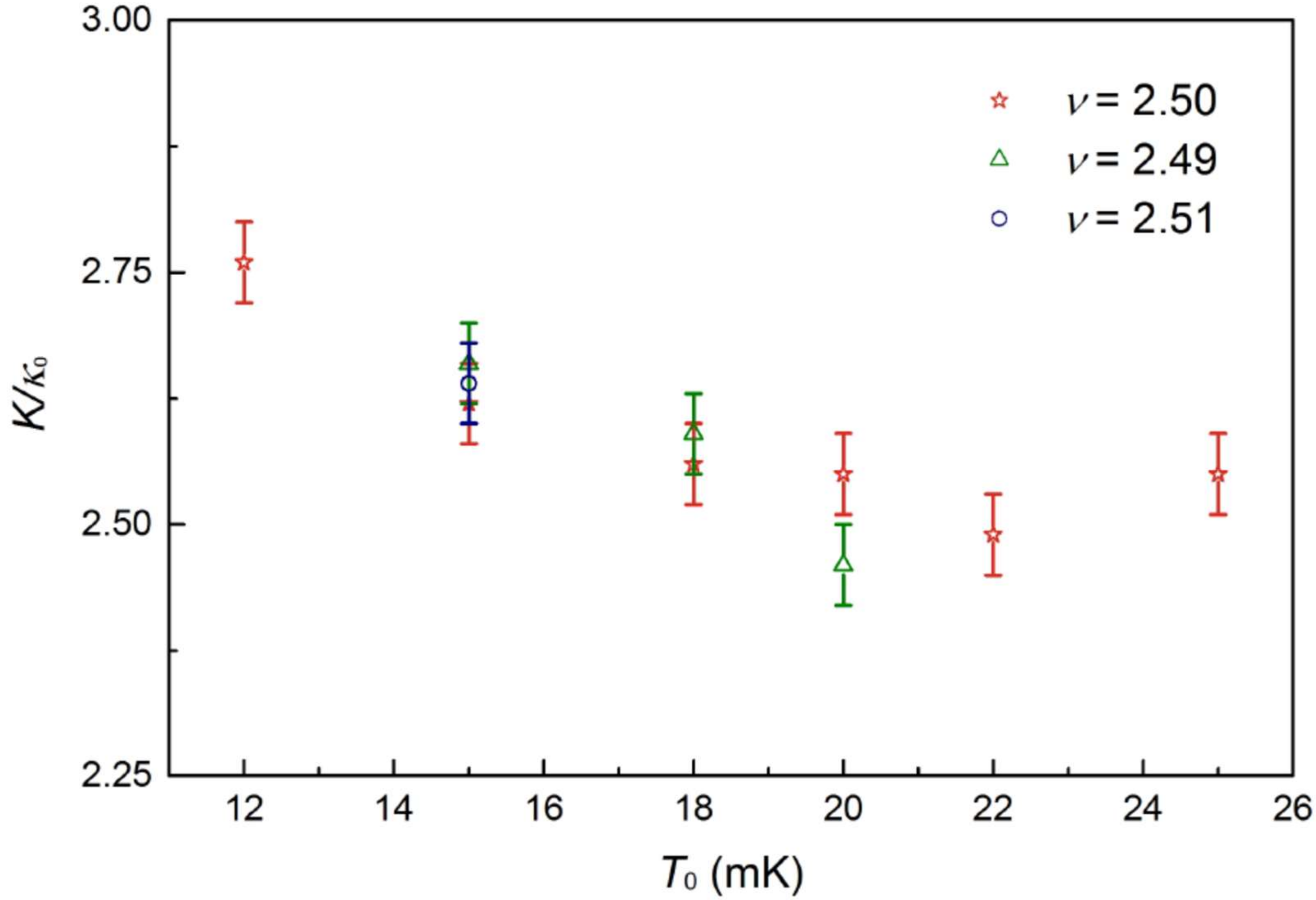
Contributions of upstream modes are subtracted: $\kappa = 2$ at $f = 4/7$

Thermal conductance setup



Thermal conductance at *f*=5/2

M. Banerjee, M. Heiblum, V. Umansky, D. E. Feldman, Y. Oreg, and A. Stern, *Nature* **559**, 205 (2018)



Less relevant operators are responsible for equilibration in the PH-Pfaffian state than in the other states: breakdown of equilibration at low *T*.

Summary

- Five key ideas
- Topological matter is robust
- Non-Abelian statistics
- Topological quantum computing
- Thermal conductance gives evidence of non-Abelian topological matter