Warm Up Exercises

Let

$$A = \begin{bmatrix} -1 & 2 & -5 \\ 2 & -1 & 6 \\ 2 & -2 & 7 \end{bmatrix}, B = \begin{bmatrix} 1 & -2 & 5 \\ 0 & 3 & -4 \\ 0 & 0 & -1 \end{bmatrix}$$

1. Show that A and B are row equivalent. What sequence of row operations did you use?
2. What is the reduced row echelon form of A (and B)?
3. Explain why A (and B) must be invertible.

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   Here the E_i are all elementary row operation matrices (which are invertible).

More Exercises!

Now consider the matrix:

$$A = \begin{bmatrix} 2 & 5 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

1. Let B be the RREF of A. Find B.
2. Interpret both A and B as an augmented matrix and write down the corresponding system of equations.
3. Show that the solution to the system of equations corresponding to B is also the solution to the system of equations corresponding to A.

Results

We have:

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This gives the two systems:

$$\begin{array}{rcl} 2x + 5y & = & 1 \\ -x - y & = & 0 \end{array} \quad \text{and} \quad \begin{array}{rcl} x & = & -1/3 \\ y & = & 1/3 \end{array}$$

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Matrices which are row equivalent differ by a finite sequence of row operations. When interepreted as systems of equations each row operation corresponds to an elemination operation. Systems of equations which differ by a finite collection of elimination operations have the same solution set. $\square$

Gaussian Elimination (aka the Gauss-Jordan Procedure)

We can discuss the process of finding RREF for a matrix as an algorithm.

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Definition

A **pivot** is a non-zero matrix entry with only zero entries to the left of it.

Algorithm Phase 1 (Forward elimination)

Let $k, \ell = 1$.

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1. While $x_{ij} = 0$ for $i \geq k$ and $j \geq \ell$ set $\ell = \ell + 1$.
2. Working from row k to the bottom find the first pivot in the ℓ-th column.
3. Swap rows to place the row with the first pivot into the the k-th row.
4. Make the pivot value a one.
5. Make all values below the pivot a 0.
6. Set $k = k + 1$.
7. If there are more columns go back to (1). If not stop.

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At this point the matrix will be in REF.

Algorithm phase 2 (Back Substitution)

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Now the matrix will be in RREF.

Exercises

Solve the following systems of equations:

$$\begin{gathered} x + 2y + 3z = 9 \\ 2x - y + z = 8 \\ 3x - z = 3 \end{gathered}$$

$$\begin{gathered} x + y + z + w = 0 \\ x + w = 0 \\ x + 2y + z = 0 \end{gathered}$$

$$\begin{gathered} x + 2y + 3z + 4w = 5 \\ y + 2z + 3w = 6 \\ x + 3y + 5z + 7w = 11 \end{gathered}$$