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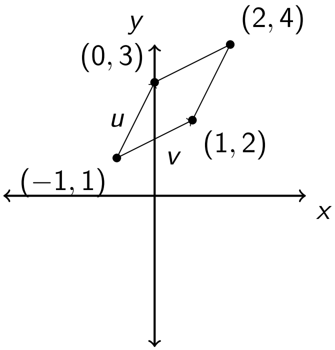

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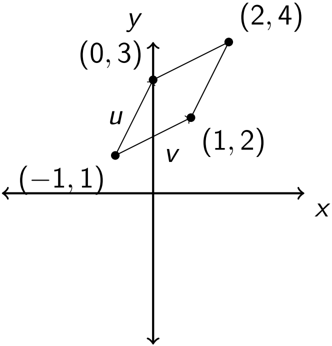

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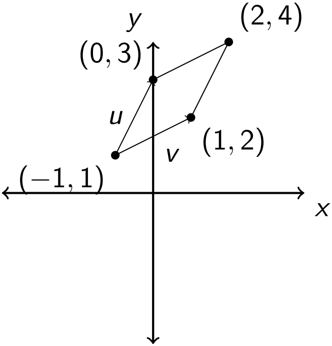

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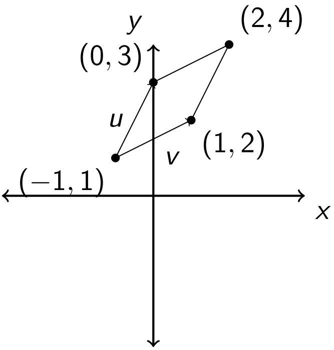

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Compute:

$$\left|\det\left(\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\right)\right| = |1-4| = 3$$

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- The x_i are called the *components* of $\vec{PQ}$.

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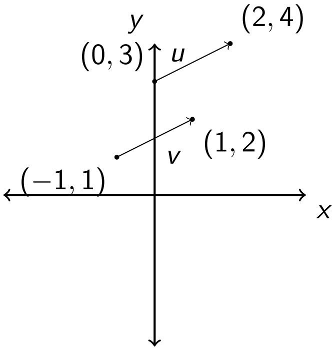

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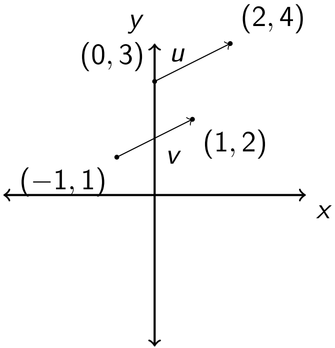

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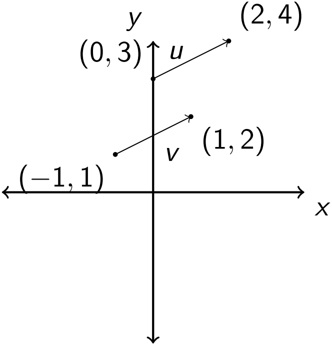

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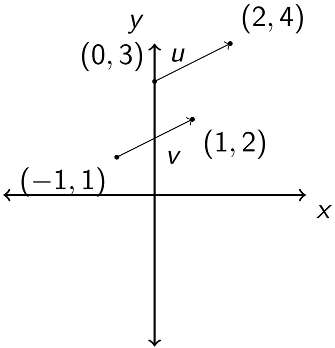

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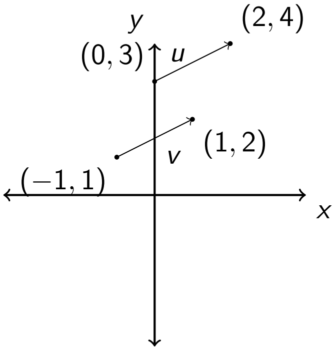

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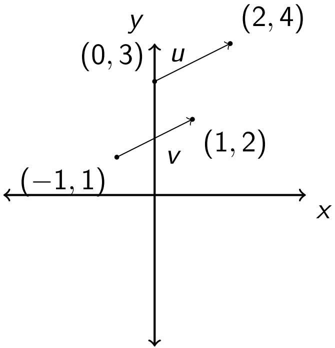

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Vector Operations

Let

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \quad \text{and} \quad y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

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$$x + y = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} x_1 + y_1 \\ \vdots \\ x_n + y_n \end{bmatrix}$$

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- Scalar multiplication:

$$rx = r \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} rx_1 \\ \vdots \\ rx_n \end{bmatrix}$$

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- Graph x, y, $x + y$ and $x - y$ on the same axis. What geometric relations do you find?

Properties of Addition and Scalar Multiplication in $\mathbb{R}^n$:

Let u, v, and w be vectors in $\mathbb{R}^n$ with scalar numbers r and s in $\mathbb{R}$. Then

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Question: What is more important? The operations which produce the properties? Or just the properties themselves?