Exercise

Is the vector

$$v = \begin{bmatrix} 6 \\ 6 \\ -8 \end{bmatrix}$$

a linear combination of the vectors

$$\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \quad \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} \quad \begin{bmatrix} -3 \\ -2 \\ 0 \end{bmatrix}?$$

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Solution: Try to solve

$$a \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} + b \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} + c \begin{bmatrix} -3 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ -8 \end{bmatrix}$$

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This becomes

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & -2 \\ -3 & -2 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ -8 \end{bmatrix}.$$

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Solutions 2 and 3

(2) Examine only the system matrix:

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Span

Recall from last class the following

Definition

Let V be a vector space and $W \subseteq V$. Then the **span** of W is the set of all (finite) linear combinations of elements of W:

$$\text{span}(W) := \left\{ \sum_{i=1}^{n} a_i v_i : a_i \in F, v_i \in W, n \in \mathbb{N} \right\}$$

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Some comments:

- We already showed before that $\operatorname{span}(W)$ is a subspace of V.
- The question "Is v a linear combination of elements of W?" can be rephrased as "Is $v \in \operatorname{span}(W)$?"

Exercises

1. Is
$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \in \operatorname{span}\left(\left\{ \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}\right)?$$
2. Is x^2 in the span of $x^2 - 2x - 1$, $x + 1$, 1?
3. Explain why every vector in $\mathbb{R}^3$ is in the span of
$$\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

Solutions

1. No. Row reduce

$$\left[\begin{array}{cc|c} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{array}\right]$$

   and find there is no solution.

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and find there is no solution.
2. Yes.
$$x^2 = (x^2 - 2x + 1) + 2(x + 1) - 3(1).$$
3. Note that
$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$
is not singular.

Lingo

In the last case we have that

$$\mathbb{R}^3 = \operatorname{span}\left(\underbrace{\left\{ \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}}_{W} \right)$$

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We say

“W *spans* $\mathbb{R}^3$.”
or
“$\mathbb{R}^3$ is *spanned* by W.”