Some Special Matrices

An $n \times n$ matrix A is **diagonal** if $a_{ij} = 0$ when $i \neq j$.

A matrix S is **scalar** if it is diagonal and all of its diagonal elements are equal.

The $n \times n$ **identity** matrix I_n is a diagonal matrix with only 1s on the diagonal.

An $n \times n$ matrix U is **upper triangular** if $u_{ij} = 0$ when $i > j$.

An $n \times n$ matrix L is **lower triangular** if $\ell_{ij} = 0$ when $i < j$.

Give an example of each type of matrix in $\mathbb{R}^{3\times 3}$.

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Summary: Matrix Addition is pretty normal.

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The reason, again, is that the component-wise definition of scalar multiplication agrees with the standard multiplicative properties of the real numbers.

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These last two properties show that transposition is a **linear** operation.

Exercise Break

Compute the following:

1.
$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 4 & -6 \\ -2 & 3 \end{bmatrix}$$

2.
$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 2 & 2 \\ 3 & -1 \end{bmatrix}$$

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$$\begin{bmatrix} 0 & 2 & 3 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 2 & 3 \end{bmatrix}$$

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- $(A + B)C = AC + BC$ and $C(A + B) = CA + CB$. Distributive.

But why do we have two distributive laws?

Because there are also some "not nice" properties:

- AB need not be equal to BA!
- $AB = 0$ does not mean that one of A or B is 0!
- $AB = AC$ even if $B \neq C$!

The last two properties are a little troublesome! In general we cannot cancel/divide/etc.!

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So *if* we could cancel then we would know that $x = v$. That is: a solution *exists* and is *unique*!

No cancellation $\Leftrightarrow$ bad solution sets

The lack of a cancellation law is exactly the reason why linear systems can have no solution or an infinite number of solutions! (And vice versa.)

Summary

- Matrix addition and scalar multiplication: Behaves well. "Vector space".
- Transposition: Is novel. "Linear".
- Multiplication: Has some issues. Be aware of them! No cancellation law! Makes "solving" an issue.
- Proving properties is not too rough. Reduce operation to the component definitions.