Sample Spaces and Uniform Probability

Gregory M. Shinault

Goal for This Lesson

We want to describe the basic setup of a probability model for a random experiment.

This material roughly corresponds to Section 1.1 of the textbook. We will cover Appendices B and C (set theory and counting), before fully tackling Section 1.1.

Introduction

What is Probability Theory?

- The mathematical formalism used to quantify uncertainty or randomness, usually in a scientific or social experiment
- Probability is NOT statistics, nor is it a subset of statistics
- Probability is a mathematical theory

Relationship Between Probability and Statistics

So why is this course required to understand statistics?

- The mean, mode, etc. of a data set are *descriptive statistics*
- Want to use descriptive statistics to make predictions for the future (*inferential statistics*)
- Probability theory is the mathematical tool we use to bridge this gap

Warning: We will never analyze a data set in this course, that is for statistics courses. This is not a statistics course.

The Sample Space

Starting Point for this Course: Sample Space

Question: You have an experiment with some form of randomness in it. What is the first thing you should do to describe this experiment?

- **Answer:** Identify all possible outcomes.

- **Definition:** For a random experiment the set of all basic outcomes is called the *sample space*. It is denoted by Ω.
- *Comment:* The sample space Ω is the fundamental "number system" for probability.

Examples

Identify the sample space in the following random experiments.

1. We roll a six-sided die.
2. We conduct a medical drug trial on 73 patients and count the number of patients for whom the drug is effective.
3. We count the number of car crashes in Madison today.
4. We identify the location of the first lightning strike in Dane county today.
5. We roll a six-sided die twice.

Nuances

The sample spaces can be classified by the number of outcomes in them.

1. Discrete sample space, finite
2. Discrete sample space, finite
3. Discrete sample space, (countably) infinite
4. Continuous sample space
5. Discrete sample space, finite

We will later see that the theory requires different tools for each classification.

Events

Definition

Motivation: Are we always interested in an exact outcome? No, sometimes we care about a range of outcomes.

Definition: An *event* E is a subset of the *sample space*.

Comment: We want to assign probabilities to events, not necessarily outcomes.

Examples

Identify the sample space in the following random experiments and the event described.

1. We roll a six-sided die twice and would like to know the probability the sum is 7.
2. In a family with two children we would like to know if the older child is a boy.
3. We measure the temperature of this room and would like to know the probability that this temperature is between 67F and 75F.

Probability Measure

Main Idea

Starting point: Assume all outcomes are equally likely.
Probability was studied primarily with this assumption starting with Gerolamo Cardano in ~1564 continuing through Pierre-Simon, marquis de Laplace in 1812.

Warning: This assumption (equal likelihood) is not always justified. We will address the more general situation soon.

Formal Definition

The *uniform probability* on a finite sample space gives the probability of an event E by

$$\mathbb{P}(E) = \frac{\#E}{\#\Omega} = \frac{\text{Number of outcomes in } E}{\text{Number of outcomes in } \Omega}.$$

Examples

Compute the probability of the stated event under the equal likelihood assumption where appropriate.

1. We roll a six-sided die twice and would like to know the probability the sum is 7.
2. In a family with two children we would like to know if the older child is a boy.
3. We measure the temperature of this room and would like to know the probability that this temperature is between 67F and 75F.

The Wrap Up

Summary

We have learned three ideas:

1. The *sample space* Ω is the set of all outcomes.
2. An *event* is a subset of the the sample space. These are what we compute probabilities of.
3. If all outcomes are equally likely we assign the *uniform probability measure* to the events.

Next Steps

Let's learn just enough set theory, so that we can formally state the axioms of probability measures needed to move beyond the uniform probability measure.