When to use random effects? Ben Bolker says:

Philosophically: We are not interested in the estimates of the random effects. In frequentist settings, we don't even call these "estimates", but rather "predictions", as in BLUP (best linear unbiased predictor). We want random effect levels to be drawn from a larger population, to be "exchangeable" (i.e. we could relabel/swap around the levels without any change in meaning) and their estimates are a random variable.

Pragmatically: We have lots of levels, with not much information about each individual level, and possibly unbalanced amounts of information. We don't want to use up the degrees of freedom associated with fitting one parameter for each level; automatically adjust between "completely pooled" (no effect) and "completely separate" (fixed effect). We have enough levels that it is practical to estimate a variance (i.e. at least 5-6, preferably more than that).

To Bayesians, the difference between fixed and random is much simpler and more pragmatic: do we add a hyperparameter to the model to characterize the estimates (= random effect = we estimate a variance), or estimate them separately (= fixed effect = we use an infinite variance)?

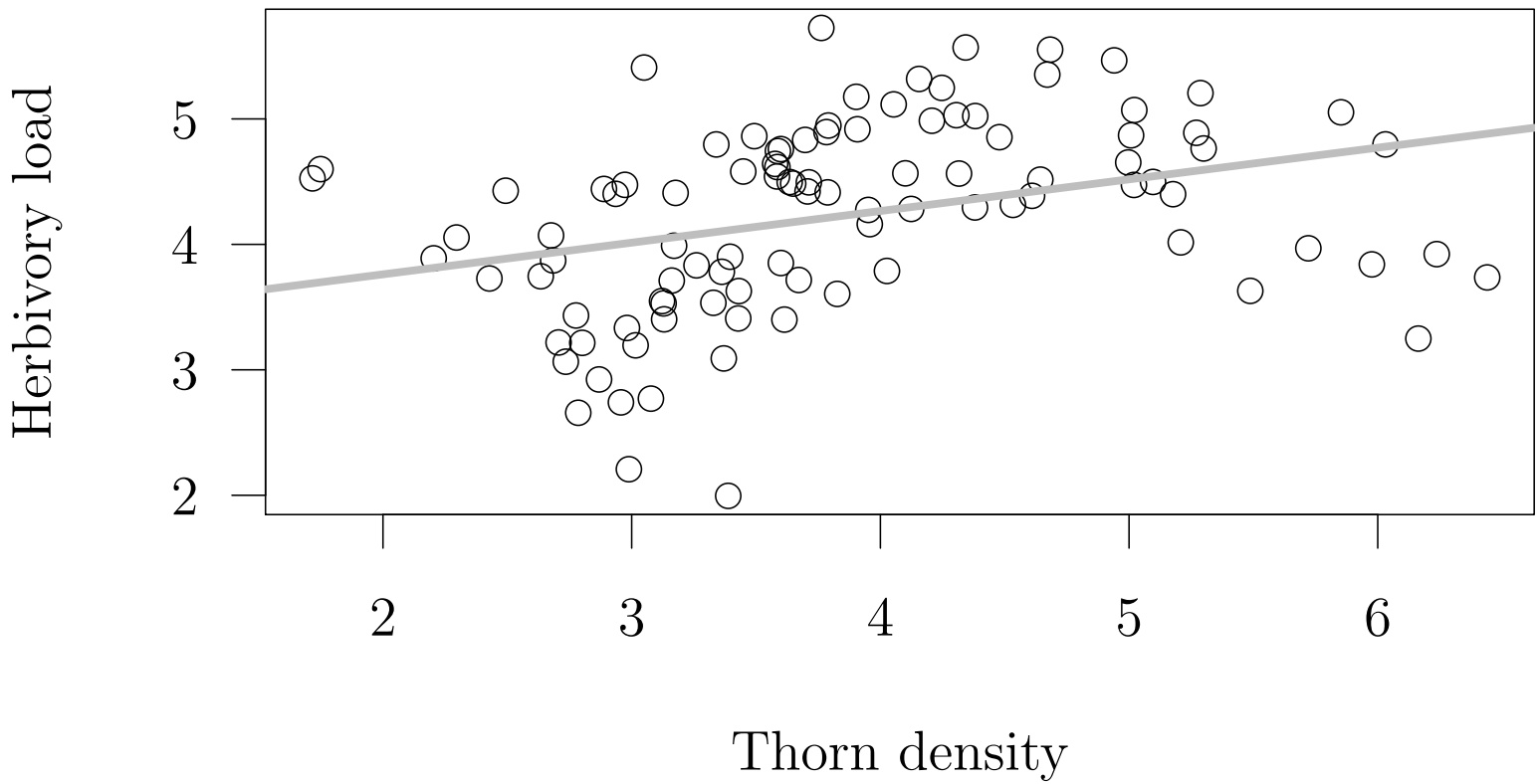
Linear mixed models 2

Uncertainty in random effects / random interactions

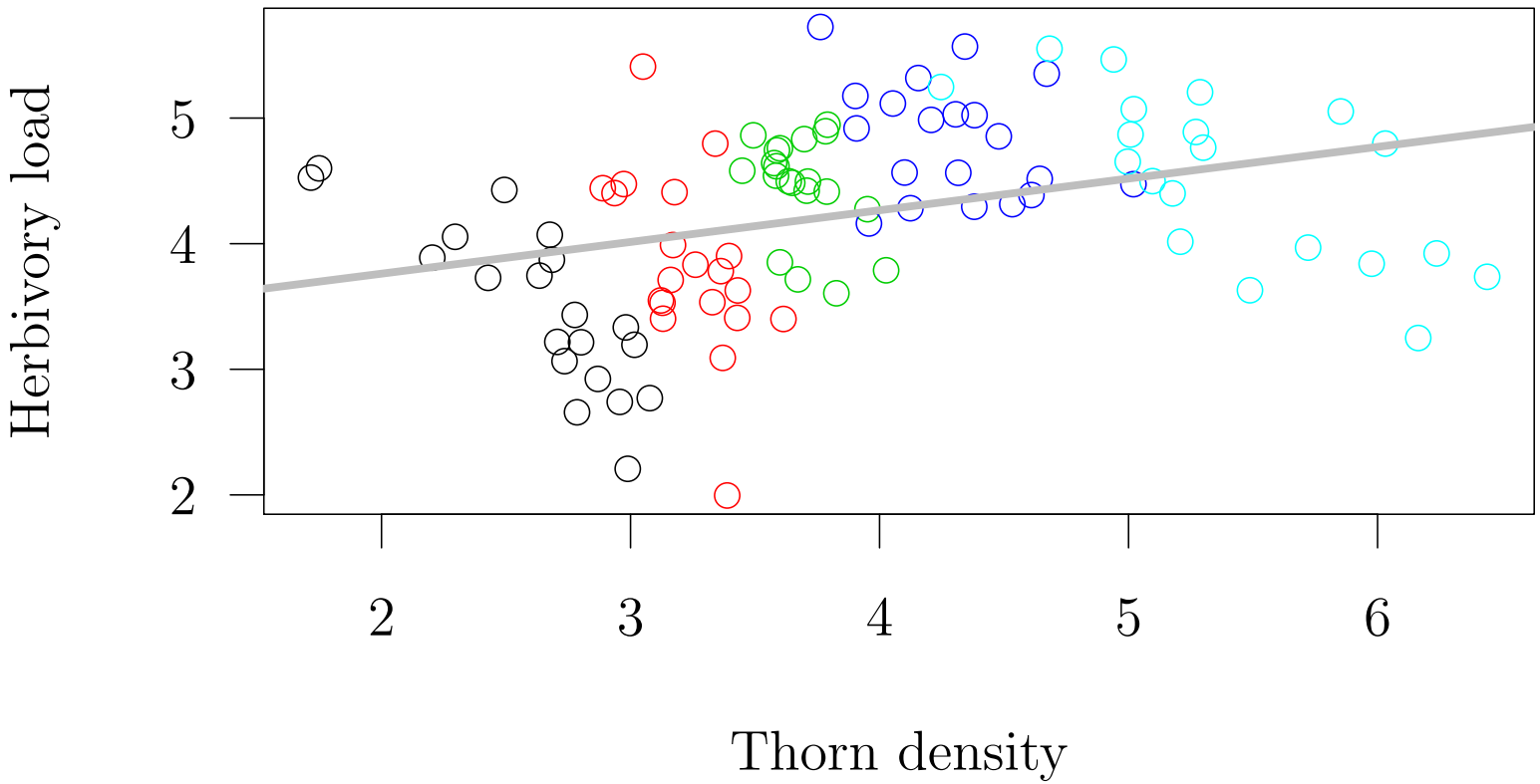
Timothée Bonnet

February 21, 2019

Mixed models reminder



Mixed models reminder



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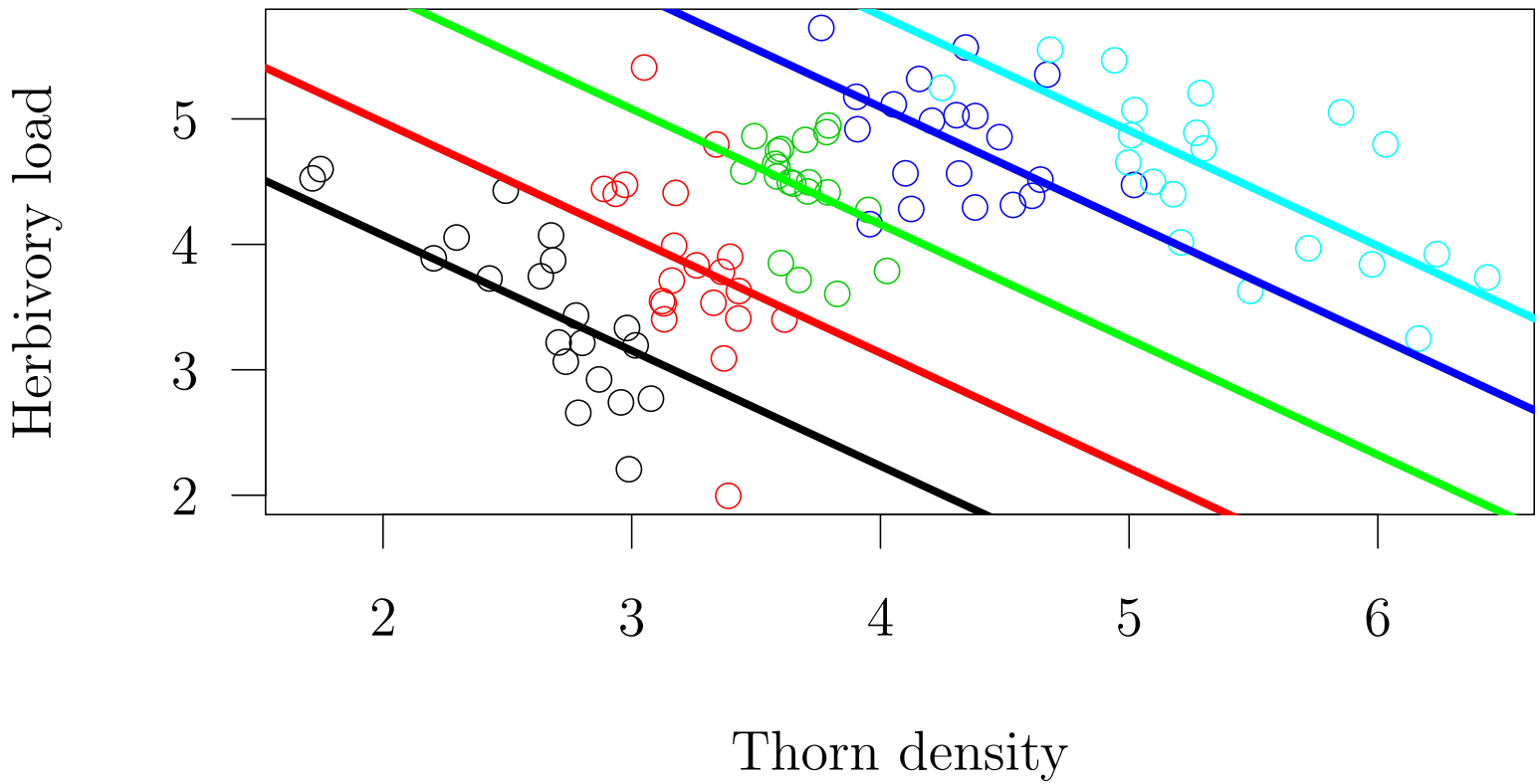
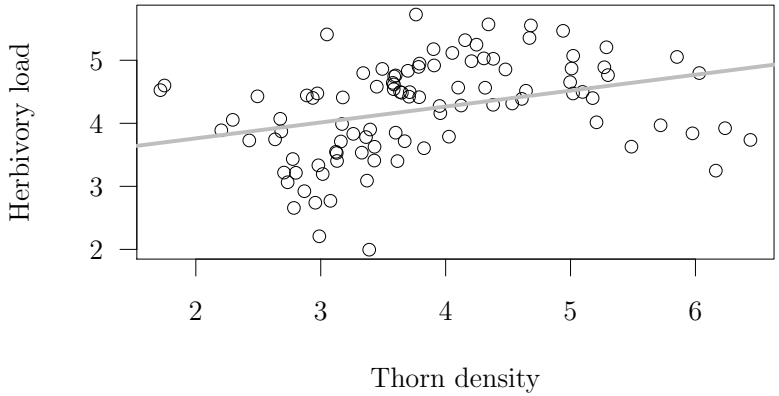
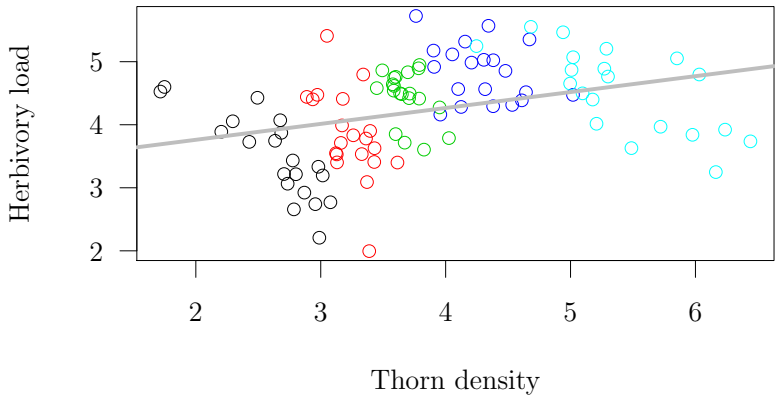


Mixed models reminder

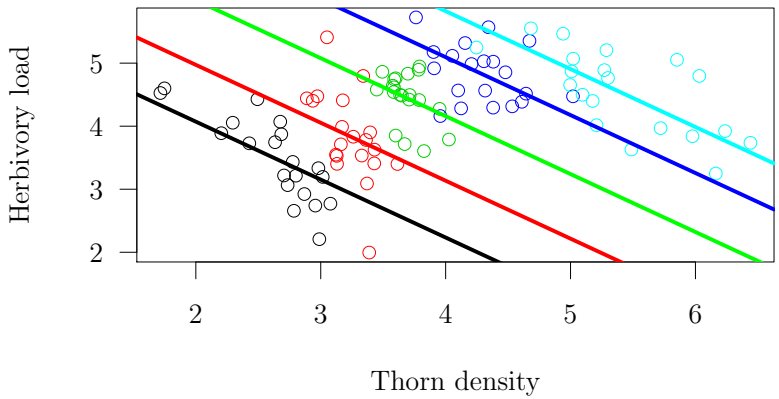
1. First model assumes residuals are independent



2. But they are not. Data come from five different places



3. Adding random effect "place" gets correct slope. Residuals are now really independent



Mixed models reminder

Add a random effect to a basic lm:

```
lm(response ~ predictor, data=thorns)
library(lme4)
lmer(response ~ predictor + (1|block), data=thorns)
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Demo in R: Excercise 1

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Questions you may have:

- Is the random effect "significant"?
- Should I include as many random effects as possible?

Mixed models today

1. Quantify uncertainty in random effects
   - p-values and tests (*R-coding!*)
   - confidence intervals
   - BLUPs = random effect levels
2. Beyond random intercepts
   - Random interactions

1. Uncertainty in random effects
2. Beyond random intercepts

Uncertainty in random effects

```
lmm1 <- lmer(herbivory ~ thorndensity + (1|site),
data=thorns)
summary(lmm1)
```

Does not measure uncertainty in site variance. Does "site" matter?

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How to?

- Confidence interval: far from zero? How does it compare to residual variance or total variance?
- Null-hypothesis testing: p-value.

Exercises 2 and 3

Issue with null-hypothesis testing for random effects

What is a p-value?

Issue with null-hypothesis testing for random effects

What is a p-value?
How do we know whether a test works?

Issue with null-hypothesis testing for random effects

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How do we know whether a test works?

Let's check with Exercise 4

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Why??

A variance cannot be negative. Anova doesn't know.

P-value computed as `1-pchisq(LRT0$Chisq[2],df=1)` but there is LESS than one degree of freedom.

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Solutions, from worse to best:

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NB: same problem with AIC, a random intercept should maybe count as 0.5 parameter, but R always count 1 parameter. Actually it is more complicated and there is no definitive answer yet!

Example of application: repeatability

Do kangaroos have personality = repeatable behaviour?
(inspired by Weliton's work)

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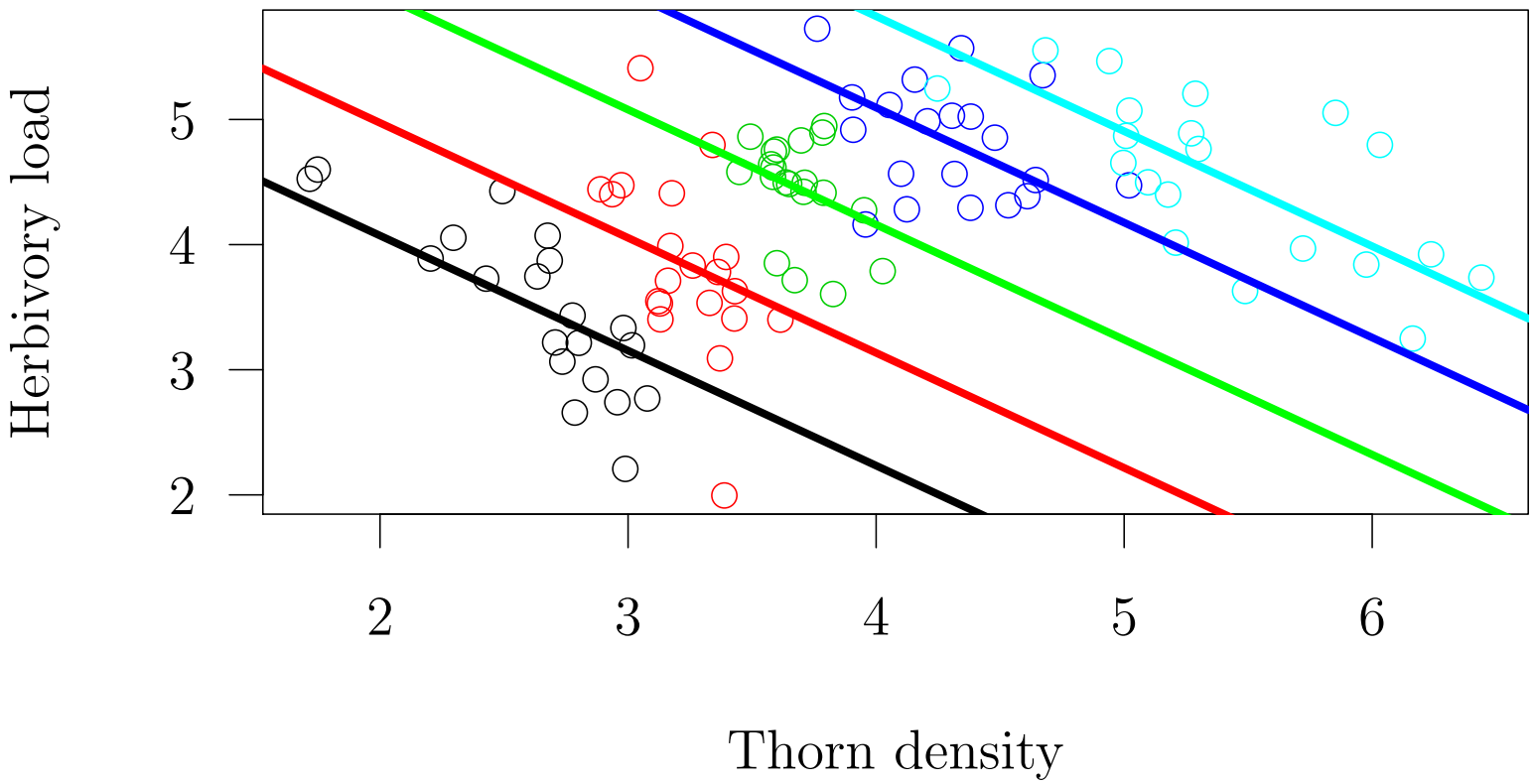
Remove non-significant?

- No if clearly part of experimental design
- But doesn't matter if estimated variance is zero
- Maybe remove if model too complex (i.e., difficult to interpret or convergence issues) but acknowledge assumptions!

1. Uncertainty in random effects
2. Beyond random intercepts

Not only intercept vary!

Assume parallel slopes:



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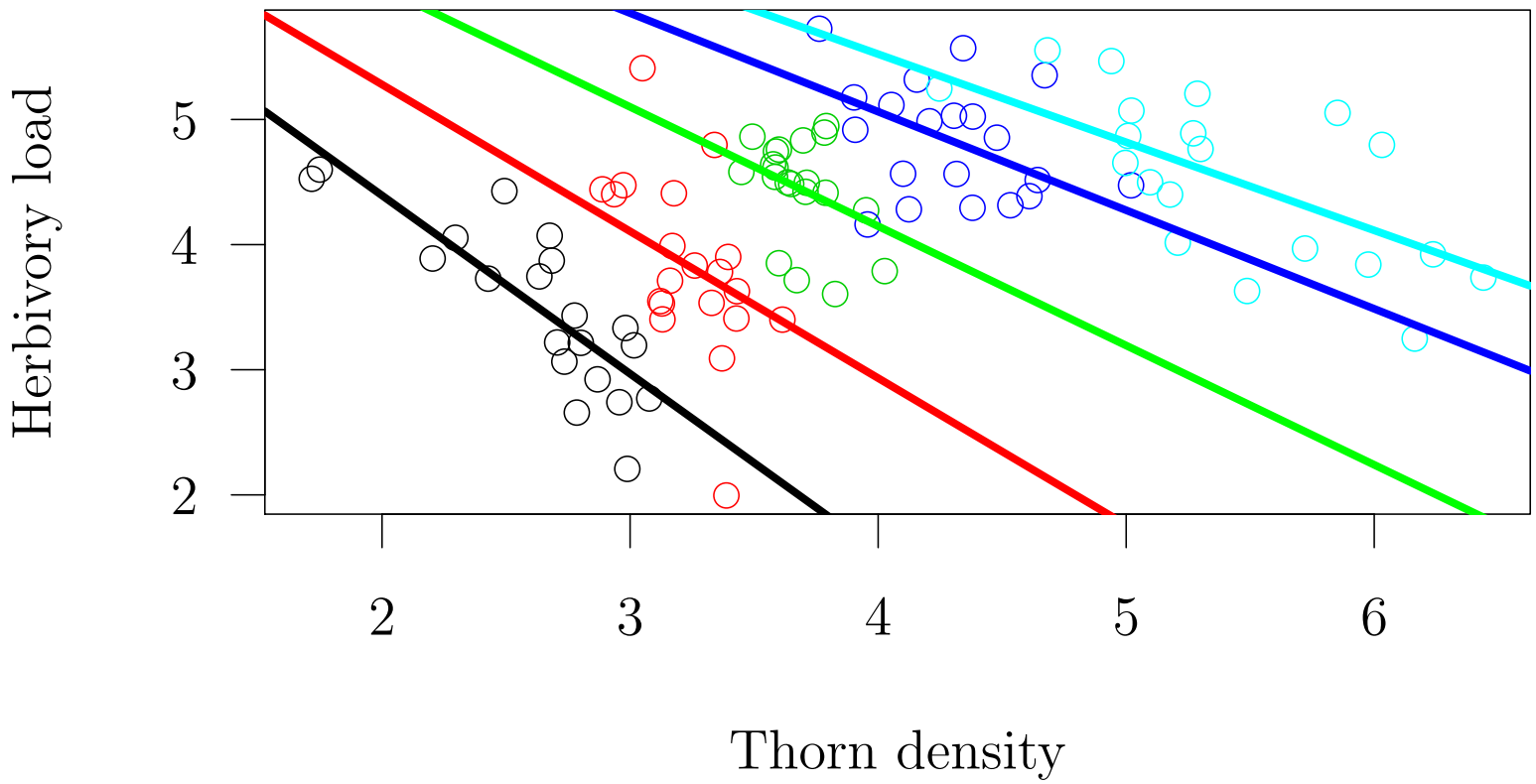
Allow slopes to vary:



How to?

Right-hand side = what groups observations

Nested, crossed et al. on the right hand side of the `|`: `(1|something)`
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How are data related to each other, what groups them

Left-hand side = what varies according to grouping

The 1 stands for **intercept**
But many things can go to the left hand side.
Random interactions, random regressions, random slopes. . . e.g.,
$y \sim 1 + x + (1 + x|something)$

Everything you need to know about mixed models

- `http://bbolker.github.io/mixedmodels-misc/glmmFAQ.html`
- Subscribe to mailing-list:
  `https://stat.ethz.ch/mailman/listinfo/r-sig-mixed-models`