Statistical inference and linear models

February 20, 2018

If you get bored

- Go to the last slide for bonus exercises
- Work on code for your research and ask question during exercise time
- But try and keep an eye out, just in case

1. Statistical inference

2. t-test, ANOVA, regression: all is one, one is all

General approach

1. Scientific question

General approach

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$\downarrow$

2. Model and Statistical question

General approach

1. Scientific question

$\downarrow$

2. Model and Statistical question $\longleftrightarrow$ 3. Data collection

General approach

$$
\begin{array}{c}
\text{1. Scientific question} \\
\downarrow \\
\text{2. Model and Statistical question} \longleftrightarrow \text{3. Data collection} \\
\downarrow \\
\text{4. Estimation} \\
\text{4.b Uncertainty and statistical significance}
\end{array}
$$

General approach

1. Scientific question

2. Model and Statistical question

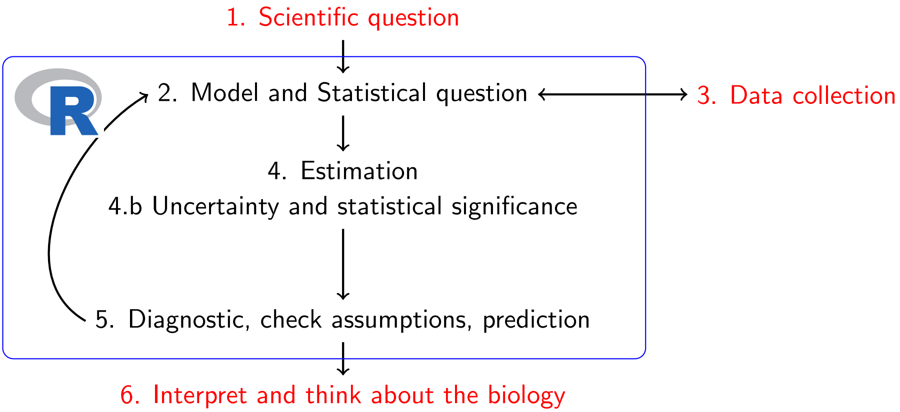
3. Data collection

4. Estimation

4.b Uncertainty and statistical significance

5. Diagnostic, check assumptions, prediction

General approach



Reminder t.test

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```r
data("iris")
```

Reminder t.test

```
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```

One t-test for sepal length between *setosa* and *versicolor*:

```
t.test(x = iris$Sepal.Length[iris$Species == "setosa"],
       y = iris$Sepal.Length[iris$Species == "versicolor"])
```

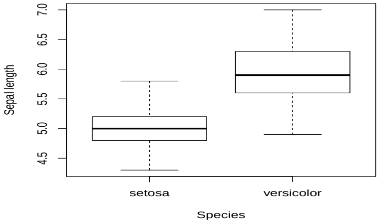
```


Welch Two Sample t-test

data:  iris$Sepal.Length[iris$Species == "setosa"] and iris$Sepal.Le
t = -10.521, df = 86.538, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -1.1057074 -0.7542926
sample estimates:
mean of x mean of y
    5.006     5.936
```

Reminder t.test: Are they different?

```
boxplot(Sepal.Length ~ Species,
        data = iris[iris$Species %in% c("setosa","versicolor"),],
        drop = TRUE, ylab="Sepal length", xlab="Species")
```



- Means: 5.006 vs. 5.936
- Standard deviation: 0.35 and 0.52
- Standard error (SD/ $\sqrt{n}$): 0.05 and 0.07

When do we know it is different?

Estimation

Estimated difference = $\color{red}{\text{Mean}_1 - \text{Mean}_2}$

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Quantify uncertainty / Statistical significance

t-statistic unlikely to be *large* when estimated means are equal

$$t = \frac{\text{Mean}_1 - \text{Mean}_2}{\text{Variation}} \frac{\sqrt{\text{Sample Size}}}{\sqrt{2}}$$

1. Larger absolute difference
2. Smaller variability
3. Larger sample size

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Same for every statistical model

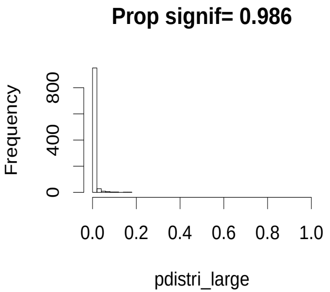
When do we know it is different?

1. Larger absolute difference

```
nbsim <- 1000
pdistri_large <- vector(length = nbsim)
pdistri_small <- vector(length = nbsim)
for (i in 1:nbsim)
  {
  x1 <- rnorm(n = 10, mean = 2, sd = 1)
  x2 <- rnorm(n = 10, mean = 4, sd = 1) #large diff
  x3 <- rnorm(n = 10, mean = 2.5, sd = 1) #small diff
  out_large <- t.test(x1, x2)
  out_small <- t.test(x1, x3)
  pdistri_large[i]<-out_large$p.value
  pdistri_small[i]<-out_small$p.value
}
```

When do we know it is different?

```
par(mfrow=c(1,2), cex=2)
hist(pdistri_large, xlim=c(0,1),
     main=paste("Prop signif=",mean(pdistri_large<0.05)))
hist(pdistri_small, xlim=c(0,1),
     main=paste("Prop signi=",mean(pdistri_small<0.05)))
```




```
par(mfrow=c(1,1))
```

When do we know it is different? Try it!

Exercise

Check the effect of smaller variability and/or larger sample size.

By the way, what are these p-values?

Blabla

Properties

- Reference to a null-model (H_0) with assumptions
- Uniform distribution under H_0 ...
- ... hence proportion(significance under H_0) $=$ significance threshold

T-test exercise

```
t.test(x = ..., y=...., var.equal = TRUE)
t.test(x = ..., y=...., var.equal = FALSE)
```

What if variance are different by chance only?

```
set.seed(1234)
var(rnorm(20, mean = 0, sd = 1))

[1] 1.027806

var(rnorm(20, mean = 0, sd = 1))

[1] 0.6265501
```

Exercise

What option is more correct for var.equal?

A small example

Animal behavior in response to weather
Load data:

```
getwd()
setwd()
```

```
dat.behav <- read.csv(file = "datbehav.csv") # path to file
```

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STEP 1: have a look at your data

```
str(dat.behav)
summary(dat.behav)
plot(dat.behav)
```

t-test

```
fitstudent <- t.test(x = dat.behav$activity[dat.behav$weather==
                                                "rainy"],
                     y = dat.behav$activity[dat.behav$weather==
                                                "sunny"],
                     var.equal = TRUE)
print(fitstudent)


Two Sample t-test

data:  dat.behav$activity[dat.behav$weather == "rainy"] and dat.beha
t = 3.2752, df = 33, p-value = 0.002485
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 0.6138373 2.6270325
sample estimates:
mean of x mean of y
 6.781476  5.161041
```

ANOVA

```
fitanova <- aov(data = dat.behav, formula = activity ~ weather)
summary(fitanova)

            Df Sum Sq Mean Sq F value  Pr(>F)
weather      1  11.25  11.253   10.73 0.00248 **
Residuals   33  34.62   1.049
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Linear regression

```
fitlm <- lm(data = dat.behav, formula = activity ~ weather)
summary(fitlm)


Call:
lm(formula = activity ~ weather, data = dat.behav)

Residuals:
    Min      1Q  Median      3Q     Max
-2.3547 -0.6028  0.2346  0.6419  1.6534

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)    6.7815     0.4581  14.805 3.94e-16 ***
weathersunny  -1.6204     0.4948  -3.275  0.00248 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.024 on 33 degrees of freedom
```

NB: aov() vs. anova()

```
aov(data = dat.behav, formula = activity ~ weather)
anova(fitlm)
```