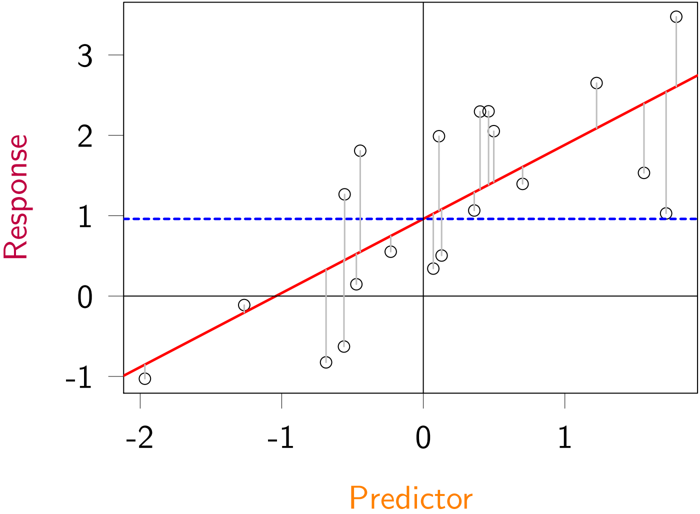
Multiple regressions and interactions

April 6, 2018

1. Linear model, reminder
2. Multiple regression
3. Interaction

A simple linear model

Response = **Intercept** + **Slope** × **Predictor** + **Error**



A multiple linear model

Response = **Intercept** + **Slope1** × **Predictor1** + **Slope2** × **Predictor2** + **Error**

In R:

```r
lm(response ~ 1 + predictor1 + predictor2, data=data)
```

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Sequential regression

We want to explain a response by three predictors

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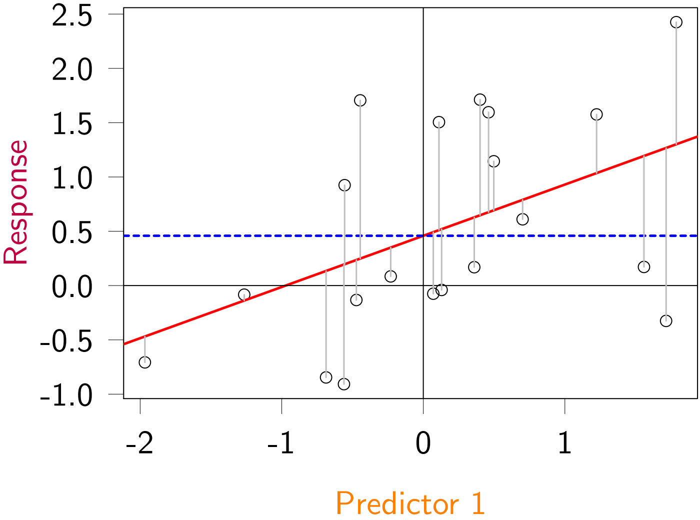
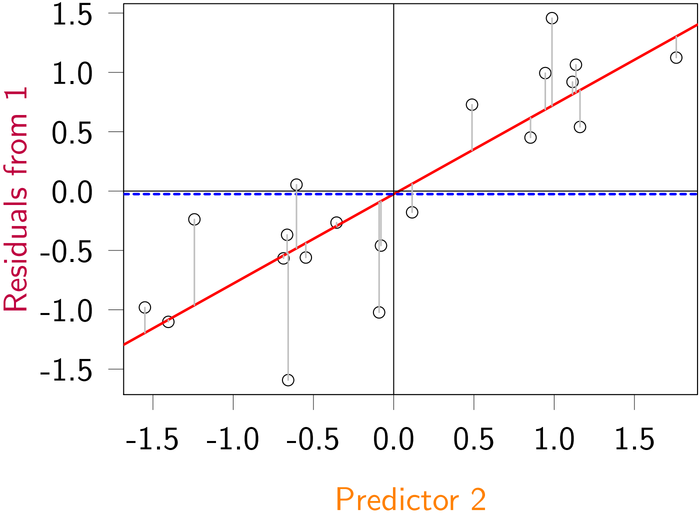


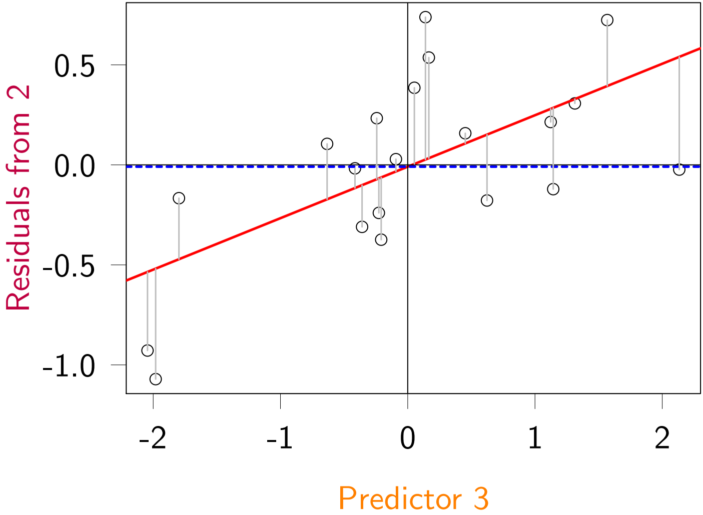
Sequential regression

We want to explain a response by three predictors



Sequential regression

We want to explain a response by three predictors



Sequential regression

```r
m1 <- lm(y ~ x1)
m2 <- lm(m1$residuals ~ x2)
m3 <- lm(m2$residuals ~ x3)
```

Sequential regression

But estimates in

```
  m1 <- lm(y ~ x1)
  m2 <- lm(m1$residuals ~ x2)
  m3 <- lm(m2$residuals ~ x3)

  c(coefficients(m1)[2], coefficients(m2)[2], coefficients(m3)[2])

       x1        x2        x3
0.4715738 0.7542078 0.2573059
```

are different from

```
    m1 <- lm(y ~ x3)
    m2 <- lm(m1$residuals ~ x2)
    m3 <- lm(m2$residuals ~ x1)

    c(coefficients(m1)[2], coefficients(m2)[2], coefficients(m3)[2])

        x3         x2         x1
-0.1036939  0.9753419 -0.1019184
```

Sequential regression

Also what happens with classical ANOVA (aov in R)

```
  summary(aov(y ~ x1 + x2 + x3))

            Df Sum Sq Mean Sq F value   Pr(>F)    
x1           1  3.997   3.997  394.05 1.07e-12 ***
x2           1 13.998  13.998 1379.87  < 2e-16 ***
x3           1  0.120   0.120   11.82  0.00338 ** 
Residuals   16  0.162   0.010                     
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

  summary(aov(y ~ x2 + x3 + x1))

            Df Sum Sq Mean Sq  F value  Pr(>F)    
x2           1 17.931  17.931 1767.562 < 2e-16 ***
x3           1  0.183   0.183   18.003 0.00062 ***
x1           1  0.002   0.002    0.176 0.68076    
Residuals   16  0.162   0.010                     
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Multiple regression

In contrast lm() optimizes relationships simultaneously
Order does **not** matter:

```
  coefficients(lm(y ~ x1 + x2 + x3))

(Intercept)          x1          x2          x3
 0.48948612 -0.01357404  1.03015700  0.08395938

  coefficients(lm(y ~ x2 + x3 + x1))

(Intercept)          x2          x3          x1
 0.48948612  1.03015700  0.08395938 -0.01357404
```

Multiple regression

BUT estimates may change with extra covariates

```
    coefficients(lm(y ~ x1 + x2 ))

(Intercept)          x1          x2
 0.50022999 -0.07029467  1.03858671

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```

??

- That is a good thing
- Estimates are independent effects, conditional on the other parameters

Conditional estimation

Exercise

1. load jumpingdistance.csv
2. Use plots and lm() to test whether mass increases jumping distance

```r
jumping <- read.csv(file = "jumpingdistance.csv")
```

A first approach suggests mass increases jumping distance:

```r
summary(lm(jump ~ mass, data=jumping))
plot(mass, jump)
```

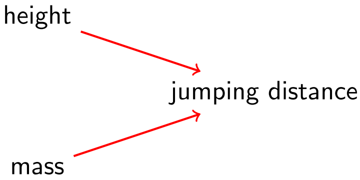
But that is incorrect and due to the correlation between mass and height:

```r
summary(lm(jump ~ mass + height, data=jumping))
```

The direct (causal) effect of mass is negative, as revealed by a multiple regression

Conditional estimation

Total / marginal effects

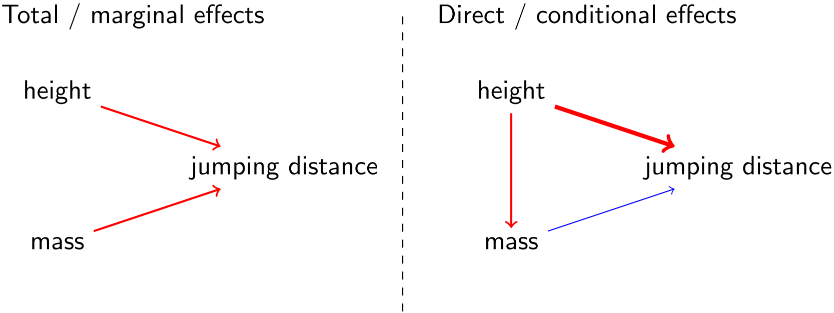


height

jumping distance

mass

Conditional estimation



- Marginal effects ≈ raw correlations, sum of direct and indirect effects
- Multiple regression estimates direct effects (conditional on other predictors) → may reveal causal relationships

Conditional estimation

Exercise

1. Load babies.csv
2. What drives change in number of babies born?

```
babies <- read.csv("babies.csv")
```

Marginal (=total) effects of storks and temperature

```
  plot(babies)
summary(lm(babies_born ~ number_of_storks, data=babies))
summary(lm(babies_born ~ mean_temperature, data=babies))
```

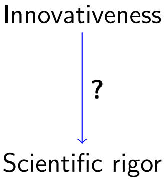
But this is a spurious effect due to time covariation. Adding year as a covariate shows that there is no evidence of an effect of storks and temperatures on number of births.

```
  summary(lm(babies_born ~ mean_temperature + year, data=babies))
  summary(lm(babies_born ~ number_of_storks + year, data=babies))
```

Conditional estimation final warning: more is not always better

Are more innovative papers less rigorous?

Research question



Innovativeness

?

Scientific rigor

Should you correct for publication impact?

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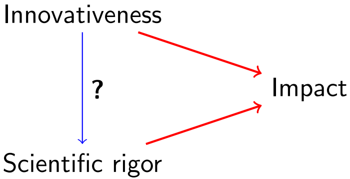
Are more innovative papers less rigorous?

Research question


Innovativeness

?

Impact

Scientific rigor

Should you correct for publication impact?

Conditional estimation final warning: more is not always better

Should you include publication impact?

```
    summary(lm(rigor ~ innovativeness + impact))$coefficients

                 Estimate Std. Error    t value      Pr(>|t|)
(Intercept)     0.0301366 0.02188752   1.376885  1.688569e-01
innovativeness -0.3150363 0.03051417 -10.324262  8.238502e-24
impact          0.5135830 0.01538756  33.376503 1.361378e-164
```

Apparent negative effect of innovativeness ?

```
    summary(lm(rigor ~ innovativeness))$coefficients

                 Estimate Std. Error   t value     Pr(>|t|)
(Intercept)    0.04104524 0.03182923  1.289545 1.975073e-01
innovativeness 0.38804729 0.03210760 12.085841 1.758144e-31
```

Apparent positive effect of innovativeness ?

Conditional estimation final warning: more is not always better

Should you include publication impact?

Conditional estimation final warning: more is not always better

Should you include publication impact?
Data simulated with positive effect of innovativeness on rigor (simulated slope 0.3)

Conditional estimation final warning: more is not always better

Should you include publication impact?
Data simulated with positive effect of innovativeness on rigor (simulated slope 0.3)
You should NOT correct for impact

Conditional estimation final warning: more is not always better

Should you include publication impact?
Data simulated with positive effect of innovativeness on rigor (simulated slope 0.3)
You should NOT correct for impact

Rule of Thumb: Do not correct for variables influenced by your predictor outside the causal path of interest

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Warnings

Vocabulary warning!

- **correlation**: linear association between two variables *"how well does x explain y ?"*

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- **interaction**: non-additive effect of two or more variables *"does the effect of* x_1 *on y change as a function of* x_2*?"*. Adds a predictor (or several) to a model.

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Warnings

Vocabary warning!

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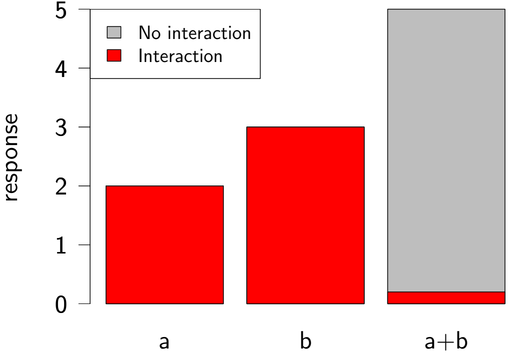

Fitting an interaction

```
lm(y ~ 1 + x1 * x2)
lm(y ~ 1 + x1 + x2 + x1:x2)
```

Fitting an interaction

```
lm(y ~ 1 + x1 * x2)
lm(y ~ 1 + x1 + x2 + x1:x2)
```

```
summary(lm(y~ 1 + x1*x2))
```

```


Call:
lm(formula = y ~ 1 + x1 * x2)

Residuals:
    Min      1Q  Median      3Q     Max 
-1.8719 -0.6777 -0.1086  0.5897  2.3166 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  1.14098    0.09578  11.913  < 2e-16 ***
x1          -0.49281    0.10834  -4.549 1.58e-05 ***
x2           0.53434    0.09881   5.408 4.67e-07 ***
x1:x2        0.35911    0.11449   3.137  0.00227 ** 
---
```

Fitting an interaction

Why the multiplication sign?

Fitting an interaction

Why the multiplication sign?

```
x1Xx2 <- x1*x2
```

Fitting an interaction

Why the multiplication sign?

```
x1Xx2 <- x1*x2
```

```
    summary(lm(y~ 1 + x1 + x2 + x1Xx2))


Call:
lm(formula = y ~ 1 + x1 + x2 + x1Xx2)

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---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Warnings

Modeling warning!

- ~~DO NOT COMPARE P-VALUES OF TWO MODELS TO TEST FOR AN INTERACTION~~

Exercise

1. Load the data masssex.csv
2. Fit a simple regression explaining movement by mass for each sex separately. Is the relationship different between sexes?
3. Fit the multiple regression explaining movement by mass, sex, and mass:sex, using the full dataset. Is the relationship different between sexes?
4. Try to understand the discreapancy by plotting the data

Warnings

1.

```r
masssex <- read.csv(file="masssex.csv")
```

Warnings

1.

```
masssex <- read.csv(file="masssex.csv")
```

2.

```
summary(lm(movement ~ mass, data=masssex[masssex$sex==0,]))
summary(lm(movement ~ mass, data=masssex[masssex$sex==1,]))
```

Warnings

1.

```
masssex <- read.csv(file="masssex.csv")
```

2.

```
summary(lm(movement ~ mass, data=masssex[masssex$sex==0,]))
summary(lm(movement ~ mass, data=masssex[masssex$sex==1,]))
```

3.

```
summary(lm(movement ~ mass*sex, data=masssex))
```

Warnings

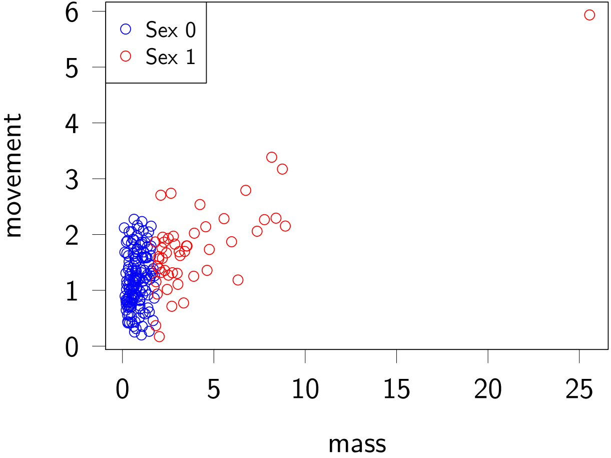
4. Visualize the problem (result on next slide):

```
plot(masssex[masssex$sex==0,"mass"],masssex[masssex$sex==0,"movement"],
 col="blue", xlim=range(masssex$mass), ylim=range(masssex$movement),
   xlab="mass", ylab="movement")
points(masssex[masssex$sex==1,"mass"],
       masssex[masssex$sex==1,"movement"], col="red")
legend(x="topleft", col=c("blue", "red"),
       legend = c("Sex 0", "Sex 1"), pch=1)
```

The slope of movement on mass is the same for both sexes, but the range of values is much smaller for sex 0, so that there is no power to detect a significant effect. Analysing sexes separately is unsound. You must fit an interaction in a model with both sexes to test for an interaction.

Warnings

4.



Prediction

Exercise

1. Load plantsize.csv and plot the data
2. Fit an additive model explaining plant size by x and y coordinates

```r
plantsize <- read.csv("plantsize.csv")
m0 <- lm(plantsize ~ x_location + y_location, data=plantsize)
```

Prediction

Exercise

1. Load plantsize.csv and plot the data
2. Fit an additive model explaining plant size by x and y coordinates
3. Create a prediction for plant size as a function of x for two values of y

```r
plantsize <- read.csv("plantsize.csv")
m0 <- lm(plantsize ~ x_location + y_location, data=plantsize)
```

Prediction

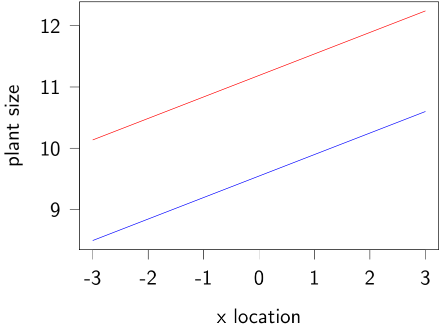
3.1. Predict

```r
newdata <- data.frame(x_location = rep(seq(-3,3, length.out = 100),2),
                      y_location = c(rep(-3, 100), rep(4,100)))
newdata$prediction <- predict(m0, newdata = newdata)
```

Prediction

3.2 Visualize

```
plot(newdata$x_location[newdata$y_location==-3],
     newdata$prediction[newdata$y_location==-3],
     xlab="x location", ylab="plant size", type="l",
     ylim = range(newdata$prediction), col="blue")
lines(newdata$x_location[newdata$y_location==4],
      newdata$prediction[newdata$y_location==4], col="red")
```

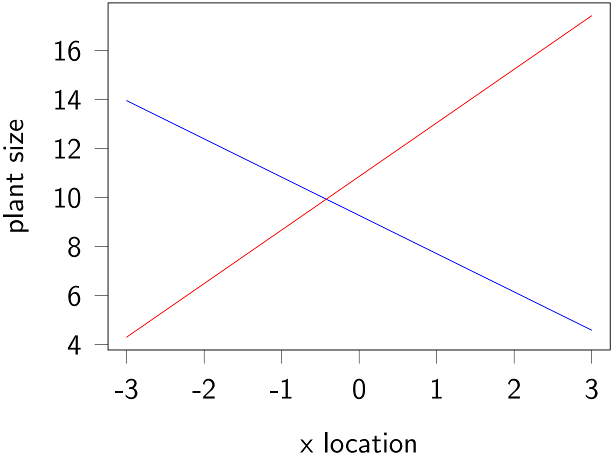


Prediction with interaction

Exercise

1. Load plantsize.csv and plot the data
2. Fit an additive model explaining plant size by x and y coordinates
3. Create a prediction for plant size as a function of x for two values of y and plot it
4. Fit an interaction between x and y coordinates
5. Create a new prediction with interaction, and plot it

Prediction with interaction



Prediction with interaction

Exercise

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5. Create a new prediction with interaction, and plot it
6. Compare estimates and p-values across models. Do you think x location has an effect or not?

Prediction with interaction

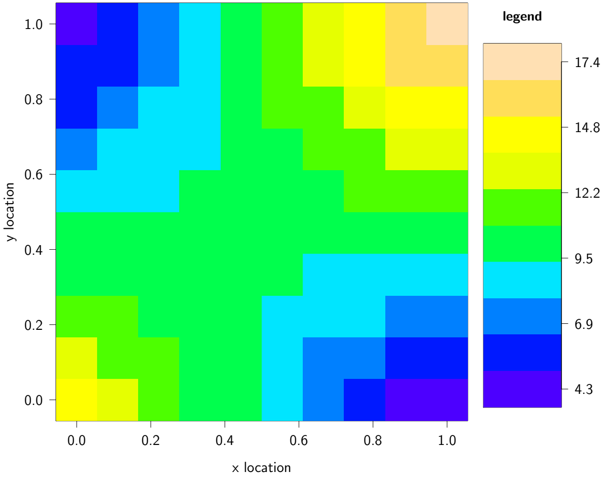
```
m1 <- lm(plantsize ~ x_location * y_location, data=plantsize)
newdata <- data.frame(x_location = rep(seq(-3,3, length.out = 10),10),
                 y_location = rep(seq(-3,4, length.out = 10), each=10))
newdata$prediction <- predict(m1, newdata = newdata)
plot(newdata$x_location[newdata$y_location==-3],
     newdata$prediction[newdata$y_location==-3],
       xlab="x location", ylab="plant size", type="l",
     ylim = range(newdata$prediction), col="blue")
lines(newdata$x_location[newdata$y_location==4],
      newdata$prediction[newdata$y_location==4], col="red")
```

Prediction with interaction

```
   library(reshape2)
  matpred <- acast(newdata, x_location~y_location, value.var="prediction")
  layout(mat = matrix(data = c(1,2),nrow = 1), widths = c(3,1) )
  image(t(matpred), col = topo.colors(10), xlab = "x location",
        ylab = "y location")

  par(mar=c(5, 0,4, 6)+0.1)
  image(matrix(data = seq(min(newdata$prediction),
              max(newdata$prediction), length.out = 10), nrow= 1 ),
        col=topo.colors(10), xaxt = "n", yaxt="n", main="legend")
  axis(side = 4, at = c(0,0.2,0.4,0.6,0.8,1),
labels = round(seq(min(newdata$prediction), max(newdata$prediction),
                   length.out =6),1))
```

Prediction with interaction



Next times

- April 20th Kevin on ggplot
- May 4th Nina on Structural Equation Modeling
- then, mixed models and GLM
- **Other requests?**