Multiple regressions and interactions

April 5, 2018
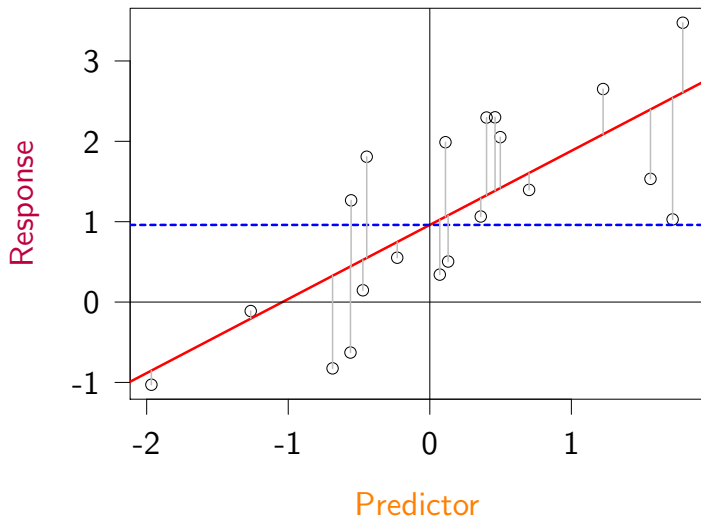
A simple linear model

Response = **Intercept** + **Slope** $\times$ **Predictor** + Error

A multiple linear model

Response = Intercept + Slope1 × Predictor1 + Slope2 × Predictor2 + Error

In R:

```
lm(response ~ 1 + predictor1 + predictor2, data=data)
```

Sequential regression

We want to explain a response by three predictors
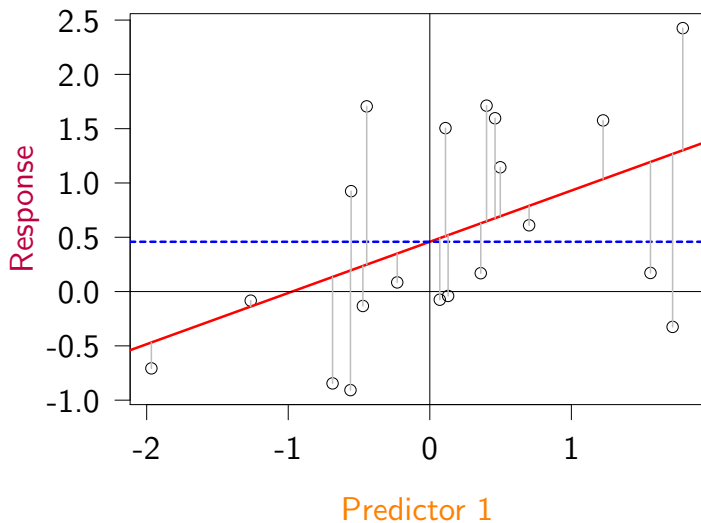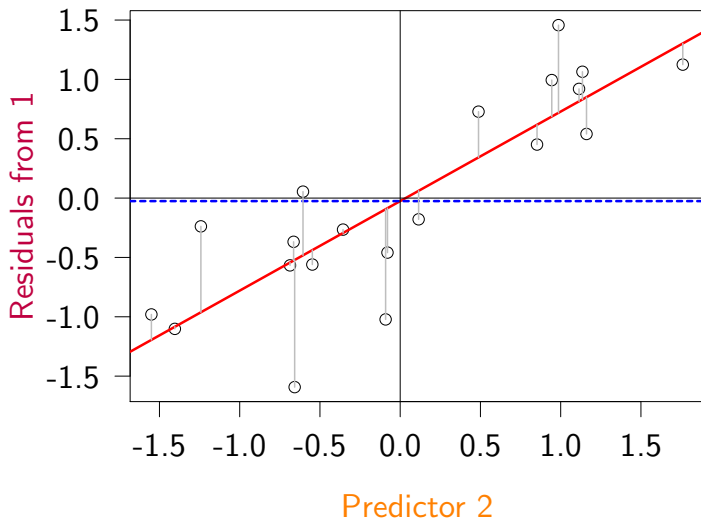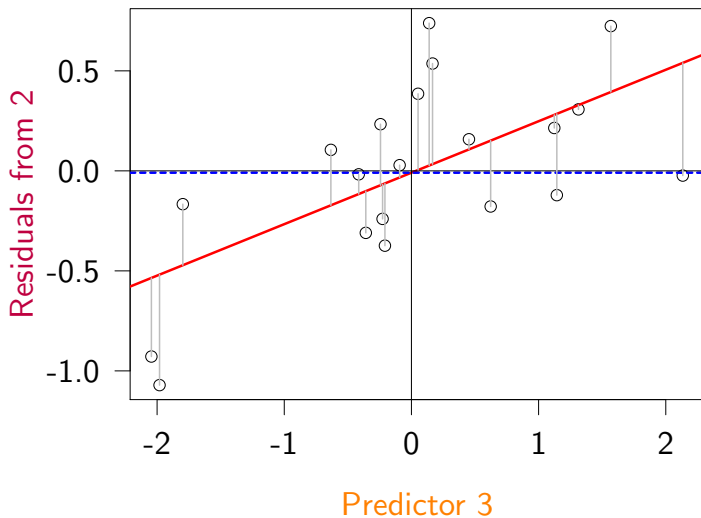
Sequential regression

We want to explain a response by three predictors

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Sequential regression

```
m1 <- lm(y ~ x1)
m2 <- lm(m1$residuals ~ x2)
m3 <- lm(m2$residuals ~ x3)
```

Sequential regression

But,

```
m1 <- lm(y ~ x1)
m2 <- lm(m1$residuals ~ x2)
m3 <- lm(m2$residuals ~ x3)

coefficients(m1); coefficients(m2); coefficients(m3)
```

```
(Intercept)          x1
  0.4585067    0.4715738
(Intercept)          x2
-0.02542972   0.75420781
 (Intercept)          x3
-0.009178959  0.257305942
```

is different from

```
m1 <- lm(y ~ x3)
m2 <- lm(m1$residuals ~ x2)
m3 <- lm(m2$residuals ~ x1)

coefficients(m1); coefficients(m2); coefficients(m3)
```

Sequential regression

Also what happens with classical ANOVA (aov in R)

```
summary(aov(y ~ x1 + x2 + x3))

          Df Sum Sq Mean Sq F value  Pr(>F)
x1         1  3.997   3.997  394.05 1.07e-12 ***
x2         1 13.998  13.998 1379.87  < 2e-16 ***
x3         1  0.120   0.120   11.82  0.00338 **
Residuals 16  0.162   0.010
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

summary(aov(y ~ x2 + x3 + x1))

          Df Sum Sq Mean Sq F value  Pr(>F)
x2         1 17.931  17.931 1767.562 < 2e-16 ***
x3         1  0.183   0.183   18.003 0.00062 ***
x1         1  0.002   0.002    0.176 0.68076
Residuals 16  0.162   0.010
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Multiple regression

In contrast lm() optimizes relationships simultaneously
Order does **not** matter:

```
  coefficients(lm(y ~ x1 + x2 + x3))

(Intercept)          x1          x2          x3
 0.48948612 -0.01357404  1.03015700  0.08395938

  coefficients(lm(y ~ x2 + x3 + x1))

(Intercept)          x2          x3          x1
 0.48948612  1.03015700  0.08395938 -0.01357404
```

Multiple regression

BUT estimates may change with extra covariates

```
    coefficients(lm(y ~ x1 + x2 ))

(Intercept)          x1          x2
 0.50022999 -0.07029467  1.03858671

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```

??

- That is a good thing
- Estimates are independent effects, conditional on the other parameters
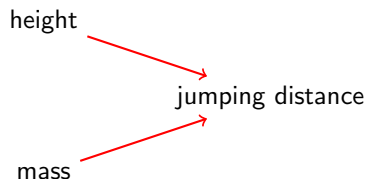
Conditional estimation

Exercise

1. load jumpingdistance.csv
2. Use plots and lm() to test whether mass increases jumping distance

```
jumping <- read.csv(file = "jumpingdistance.csv")
```
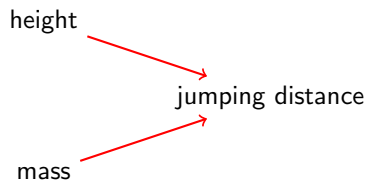
Conditional estimation

Total / marginal effects

- Marginal effects $\approx$ raw correlations, sum of direct and indirect effects
- Multiple regression estimates direct effects may reveal causal relationships

Conditional estimation

Total / marginal effects

height

jumping distance

mass

Direct / conditional effects

height

jumping distance

mass

Conditional estimation

Total / marginal effects

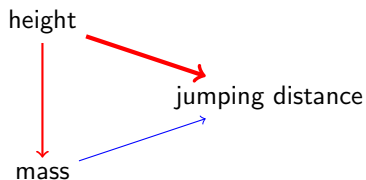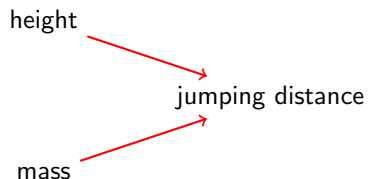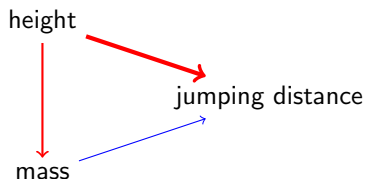
Direct / conditional effects

- Marginal effects $\approx$ raw correlations, sum of direct and indirect effects
- Multiple regression estimates direct effects may reveal causal relationships

Conditional estimation

Conditional estimation final warning: more is not always better

Are more innovative papers less rigorous?

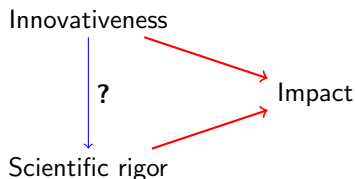Research question

Innovativeness

?

Scientific rigor

Should you correct for publication impact?

Conditional estimation final warning: more is not always better

Are more innovative papers less rigorous?

Research question

Should you correct for publication impact?

Conditional estimation final warning: more is not always better

Should you include publication impact?

```
    summary(lm(rigor ~ innovativeness + impact))$coefficients
```

```
                Estimate Std. Error    t value       Pr(>|t|)
(Intercept)      0.0301366 0.02188752   1.376885   1.688569e-01
innovativeness  -0.3150363 0.03051417 -10.324262   8.238502e-24
impact           0.5135830 0.01538756  33.376503 1.361378e-164
```

Apparent negative effect of innovativeness ?

```
    summary(lm(rigor ~ innovativeness))$coefficients
```

```
                Estimate Std. Error   t value      Pr(>|t|)
(Intercept)      0.04104524 0.03182923  1.289545 1.975073e-01
innovativeness   0.38804729 0.03210760 12.085841 1.758144e-31
```

Apparent positive effect of innovativeness ?

Conditional estimation final warning: more is not always better

Should you include publication impact?

Conditional estimation final warning: more is not always better

Should you include publication impact?
Data simulated with positive effect of innovativeness on rigor (simulated slope 0.3)

Conditional estimation final warning: more is not always better

Should you include publication impact?
Data simulated with positive effect of innovativeness on rigor (simulated slope 0.3)
You should NOT correct for impact

Conditional estimation final warning: more is not always better

Should you include publication impact?
Data simulated with positive effect of innovativeness on rigor (simulated slope 0.3)
You should NOT correct for impact

Rule of Thumb: Do not correct for variables outside the causal path of interest

Conditional estimation final warning: more is not always better

Should you include publication impact?
Data simulated with positive effect of innovativeness on rigor (simulated slope 0.3)
You should NOT correct for impact

Rule of Thumb: Do not correct for variables outside the causal path of interestRule: Think twice about the meaning of conditional parameters (?? effect of innovativeness on rigor, conditional on publication impact ??)

Warnings

Vocabulary warning!

- **correlation**: linear association between two variables *"how well does x explain y ?"*

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- **interaction**: non-additive effect of two or more variables *"does the effect of x_1 on y change as a function of x_2?"*. Adds a predictor (or several) to a model.
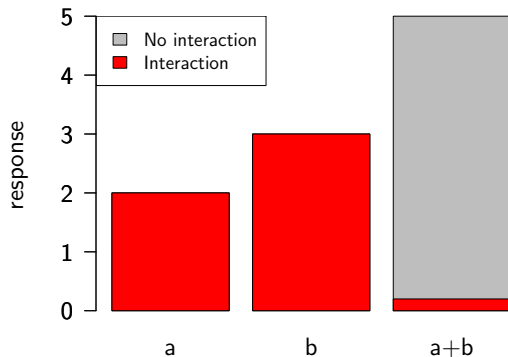
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Fitting an interaction

```
lm(y ~ 1 + x1 * x2)
lm(y ~ 1 + x1 + x2 + x1:x2)
```

Fitting an interaction

```
lm(y ~ 1 + x1 * x2)
lm(y ~ 1 + x1 + x2 + x1:x2)
```

```
summary(lm(y~ 1 + x1*x2))
```

```
Call:
lm(formula = y ~ 1 + x1 * x2)

Residuals:
    Min      1Q  Median      3Q     Max
-1.8719 -0.6777 -0.1086  0.5897  2.3166

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.14098    0.09578  11.913  < 2e-16 ***
x1          -0.49281    0.10834  -4.549 1.58e-05 ***
x2           0.53434    0.09881   5.408 4.67e-07 ***
x1:x2        0.35911    0.11449   3.137  0.00227 **
---
```

Fitting an interaction

Why the multiplication sign?

Fitting an interaction

Why the multiplication sign?

```
x1Xx2 <- x1*x2
```

Fitting an interaction

Why the multiplication sign?

```
x1Xx2 <- x1*x2
```

```
summary(lm(y~ 1 + x1 + x2 + x1Xx2))
```

```
Call:
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x1Xx2        0.35911    0.11449   3.137 0.00227 **
---
```

Warnings

Modeling warning!

- ~~DO NOT COMPARE P-VALUES OF TWO MODELS TO TEST FOR AN INTERACTION~~

Exercise

1. Load the data masssex.csv
2. Fit a simple regression explaining movement by mass for each sex separately. Is the relationship different between sexes?
3. Fit the multiple regression explaining movement by mass, sex, and mass:sex, using the full dataset. Is the relationship different between sexes?
4. Try to understand the discreapancy by plotting the data

Warnings

1.

```
masssex <- read.csv(file="masssex.csv")
```

Warnings

1.

```
masssex <- read.csv(file="masssex.csv")
```

2.

```
summary(lm(movement ~ mass, data=masssex[masssex$sex==0,]))
summary(lm(movement ~ mass, data=masssex[masssex$sex==1,]))
```

Warnings

1.

```
masssex <- read.csv(file="masssex.csv")
```
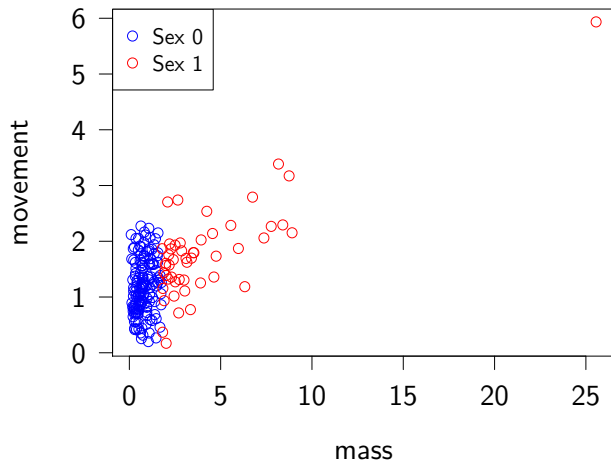
2.

```
summary(lm(movement ~ mass, data=masssex[masssex$sex==0,]))
summary(lm(movement ~ mass, data=masssex[masssex$sex==1,]))
```

3.

```
summary(lm(movement ~ mass*sex, data=masssex))
```

Warnings

4.

Prediction

Exercise

1. Load plantsize.csv and plot the data
2. Fit an additive model explaining plant size by x and y coordinates

```
plantsize <- read.csv("plantsize.csv")
m0 <- lm(plantsize ~ x_location + y_location, data=plantsize)
```

Prediction

```
plantsize <- read.csv("plantsize.csv")
m0 <- lm(plantsize ~ x_location + y_location, data=plantsize)
```

Prediction
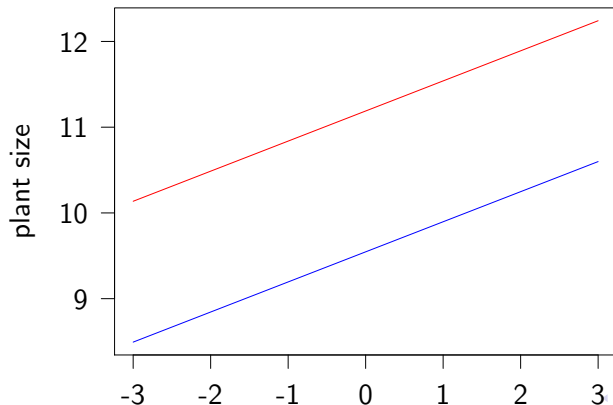
3.1. Predict

```
newdata <- data.frame(x_location = rep(seq(-3,3, length.out = 100),2),
                      y_location = c(rep(-3, 100), rep(4,100)))
newdata$prediction <- predict(m0, newdata = newdata)
```

Prediction

3.2 Visualize

```
setPar()
plot(newdata$x_location[newdata$y_location==-3], newdata$prediction[newda
    xlab="x location", ylab="plant size", type="l", ylim = range(newdata
lines(newdata$x_location[newdata$y_location==4], newdata$prediction[newda
```
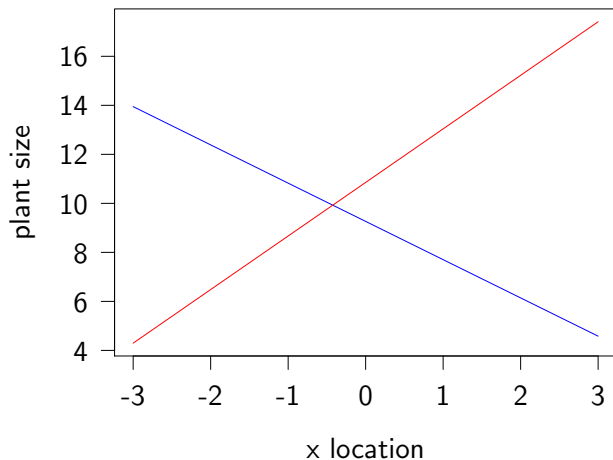
Prediction with interaction

Exercise

1. Load plantsize.csv and plot the data
2. Fit an additive model explaining plant size by x and y coordinates
3. Create a prediction for plant size as a function of x for two values of y and plot it
4. Fit an interaction between x and y coordinates
5. Create a new prediction with interaction, and plot it

Prediction with interaction

Prediction with interaction

Exercise

1. Load plantsize.csv and plot the data
2. Fit an additive model explaining plant size by x and y coordinates
3. Create a prediction for plant size as a function of x for two values of y and plot it
4. Fit an interaction between x and y coordinates
5. Create a new prediction with interaction, and plot it
6. Compare estimates and p-values across models. Do you think x location has an effect or not?

Prediction with interaction
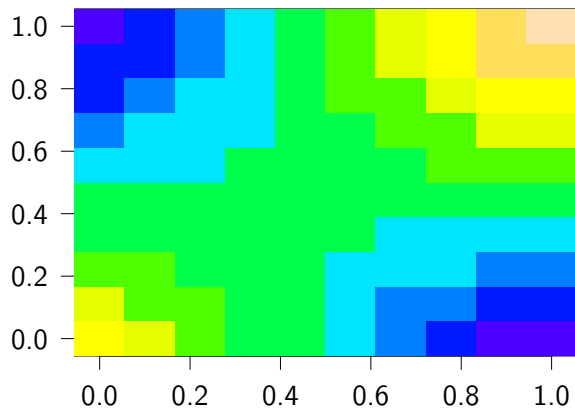
Next times

- April 20th Kevin on ggplot
- May 4th Nina on Structural Equation Modeling
- then, mixed models and GLM
- **Other requests?**