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Assignment 7

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Download latex codes from Download all python codes from

https://github.com/Ananthoju-Pranav-Sai/AI1103/ tree/main/Assignment 7 1/Codes

and latex codes from

https://github.com/Ananthoju-Pranav-Sai/AI1103/blob/main/Assignment_7_1/main.tex

STATS P1 IESISS19 Q 31

Let X be a random variable with p.d.f

$$f_X(x) = \begin{cases} \frac{2x}{\pi^2} & 0 < x < \pi \\ 0 & \text{otherwise} \end{cases}$$
 (0.0.1)

Let $Y = \sin X$, then for 0 < y < 1, the p.d.f of Y is given by,

(A)
$$\frac{2\pi}{\sqrt{1-v^2}}$$

(B)
$$\frac{\pi}{2} \sqrt{1 - y^2}$$

(C)
$$\frac{2}{\pi} \sqrt{1 - y^2}$$

(D)
$$\frac{2}{\pi \sqrt{1-y^2}}$$

SOLUTION

Given p.d.f of X as

$$f_X(x) = \begin{cases} \frac{2x}{\pi^2} & 0 < x < \pi \\ 0 & \text{otherwise} \end{cases}$$
 (0.0.2)

$$F_X(x) = \Pr\left(X \le x\right) \tag{0.0.3}$$

$$\implies F_X(x) = \int_{-\infty}^x f_X(t) \, dt \tag{0.0.4}$$

which can be written as

$$F_X(x) = \begin{cases} 0 & x \le 0\\ \frac{x^2}{\pi^2} & 0 < x < \pi\\ 1 & x \ge \pi \end{cases}$$
 (0.0.5)

Now c.d.f of Y can be written as

$$F_Y(y) = \Pr(Y \le y) \tag{0.0.6}$$

$$\implies F_Y(y) = \Pr(\sin X \le y)$$
 (0.0.7)

As $X \in (0, \pi)$, $\sin X \le y$ has two solutions i.e., either $X \le \sin^{-1} y$ or $X \ge \pi - \sin^{-1} y$ as shown in 4

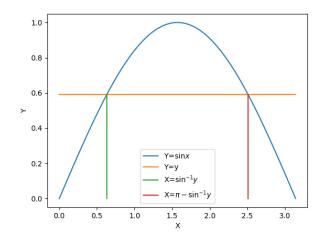


Fig. 4: $Y = \sin X$ plot

$$\implies F_Y(y) = \Pr\left(X \le \sin^{-1} y\right) + \Pr\left(X \ge \pi - \sin^{-1} y\right)$$
(0.0.8)

$$\implies F_Y(y) = F_X(\sin^{-1} y) + 1 - \Pr(X \le \pi - \sin^{-1} y)$$
(0.0.9)

$$\implies F_Y(y) = F_X(\sin^{-1} y) + 1 - F_X(\pi - \sin^{-1} y)$$
(0.0.10)

using (0.0.5) in (0.0.10)

$$\implies F_Y(y) = \frac{\left(\sin^{-1} y\right)^2}{\pi^2} + 1 - \frac{\left(\pi - \sin^{-1} y\right)^2}{\pi^2}$$
(0.0.11)

$$\implies F_Y(y) = \frac{2\sin^{-1}y}{\pi} \tag{0.0.12}$$

Now pdf of Y can be written as

$$f_Y(y) = \frac{\mathrm{d}F_Y(y)}{\mathrm{d}y} \tag{0.0.13}$$

$$f_Y(y) = \frac{dF_Y(y)}{dy} \qquad (0.0.13)$$

$$\implies f_Y(y) = \frac{2}{\pi} \frac{d\left(\sin^{-1} y\right)}{dy} \qquad (0.0.14)$$

$$\implies f_Y(y) = \frac{2}{\pi} \sqrt{1 - y^2} \qquad (0.0.15)$$

$$\implies f_Y(y) = \frac{2}{\pi \sqrt{1 - y^2}} \tag{0.0.15}$$

Hence, option(D) is correct.