Assignment 7

Ananthoju Pranav Sai - AI20BTECH11004

Download all python codes from

https://github.com/Ananthoju−Pranav−Sai/AI1103/tree/main/Assignment_7/Codes

and latex codes from

https://github.com/Ananthoju−Pranav−Sai/AI1103/blob/main/Assignment_7/main.tex

STATS P1 IESISS19 Q 31

Let X be a random variable with p.d.f

$$f_X(x) = \begin{cases} \frac{2x}{\pi^2} & 0 < x < \pi \\ 0 & otherwise \end{cases} \tag{0.0.1}$$

Let Y = $\sin X$, then for $0 < y < 1$, the p.d.f of Y is given by,

(A) $\frac{2\pi}{\sqrt{1-y^2}}$

(B) $\frac{\pi}{2}\sqrt{1-y^2}$

(C) $\frac{2}{\pi}\sqrt{1-y^2}$

(D) $\frac{2}{\pi\sqrt{1-y^2}}$

Solution

Given p.d.f of X as

$$f_X(x) = \begin{cases} \frac{2x}{\pi^2} & 0 < x < \pi \\ 0 & otherwise \end{cases} \tag{0.0.2}$$

We can notice that if $0 < x < \pi$ then $0 < \sin x < 1$
The c.d.f of Y can be written as

$$F_Y(y) = \Pr(Y \le y) \tag{0.0.3}$$

$$\implies F_Y(y) = \Pr(\sin X \le y) \tag{0.0.4}$$

Now for $\sin X \le y$ we have two solutions i.e, either $X \le \sin^{-1} y$ or $X \ge \pi - \sin^{-1} y$ as $X \in (0, \pi)$

$$\implies F_Y(y) = \Pr\left(X \le \sin^{-1} y\right) + \Pr\left(X \ge \pi - \sin^{-1} y\right) \tag{0.0.5}$$

$$\implies F_Y(y) = \int_{-\infty}^{\sin^{-} 1 y} f_X(x)\,dx + \int_{\pi-\sin^{-1} y}^{\infty} f_X(x)\,dx \tag{0.0.6}$$

$$\implies F_Y(y) = \int_{0}^{\sin^{-} 1 y} f_X(x)\,dx + \int_{\pi-\sin^{-1} y}^{\pi} f_X(x)\,dx \tag{0.0.7}$$

$$\implies F_Y(y) = \int_{0}^{\sin^{-} 1 y} \frac{2x}{\pi^2}\,dx + \int_{\pi-\sin^{-1} y}^{\pi} \frac{2x}{\pi^2}\,dx \tag{0.0.8}$$

$$\implies F_Y(y) = \frac{x^2}{\pi^2}\bigg]_0^{\sin^{-1} y} + \frac{x^2}{\pi^2}\bigg]_{\pi-\sin^{-1} y}^{\pi} \tag{0.0.9}$$

$$\implies F_Y(y) = \frac{\left(sin^{-1}y\right)^2}{\pi^2} + 1 - \frac{\left(\pi - sin^{-1}y\right)^2}{\pi^2} \tag{0.0.10}$$

$$\implies F_Y(y) = \frac{2sin^{-1}y}{\pi} \tag{0.0.11}$$

Now for the p.d.f of Y

$$f_Y(y) = \frac{\mathrm{d}F_Y(y)}{\mathrm{d}y} \tag{0.0.12}$$

$$\implies f_Y(y) = \frac{2}{\pi}\frac{\mathrm{d}\left(sin^{-1}y\right)}{\mathrm{d}y} \tag{0.0.13}$$

$$\implies f_Y(y) = \frac{2}{\pi\sqrt{1-y^2}} \tag{0.0.14}$$

Hence option (D) is correct.