

Assignment 7

Ananthoju Pranav Sai - AI20BTECH11004

Download latex codes from Download all python codes from

https://github.com/Ananthoju-Pranav-Sai/AI1103/tree/main/Assignment_7_1/Codes

and latex codes from

https://github.com/Ananthoju-Pranav-Sai/AI1103/blob/main/Assignment_7_1/main.tex

STATS P1 IESSIS19 Q 31

Let X be a random variable with p.d.f

$$f_X(x) = \begin{cases} \frac{2x}{\pi^2} & 0 < x < \pi \\ 0 & \text{otherwise} \end{cases} \quad (0.0.1)$$

Let $Y = \sin X$, then for $0 < y < 1$, the p.d.f of Y is given by,

- (A) $\frac{2\pi}{\sqrt{1-y^2}}$
- (B) $\frac{\pi}{2} \sqrt{1-y^2}$
- (C) $\frac{2}{\pi} \sqrt{1-y^2}$
- (D) $\frac{2}{\pi \sqrt{1-y^2}}$

SOLUTION

Given p.d.f of X as

$$f_X(x) = \begin{cases} \frac{2x}{\pi^2} & 0 < x < \pi \\ 0 & \text{otherwise} \end{cases} \quad (0.0.2)$$

$$F_X(x) = \Pr(X \leq x) \quad (0.0.3)$$

$$\Rightarrow F_X(x) = \int_{-\infty}^x f_X(t) dt \quad (0.0.4)$$

which can be written as

$$F_X(x) = \begin{cases} 0 & x \leq 0 \\ \frac{x^2}{\pi^2} & 0 < x < \pi \\ 1 & x \geq \pi \end{cases} \quad (0.0.5)$$

Now c.d.f of Y can be written as

$$F_Y(y) = \Pr(Y \leq y) \quad (0.0.6)$$

$$\Rightarrow F_Y(y) = \Pr(\sin X \leq y) \quad (0.0.7)$$

As $X \in (0, \pi)$ $\sin X \leq 1$ has two solutions i.e., either $X \leq \sin^{-1} y$ or $X \geq \pi - \sin^{-1} y$

$$\Rightarrow F_Y(y) = \Pr(X \leq \sin^{-1} y) + \Pr(X \geq \pi - \sin^{-1} y) \quad (0.0.8)$$

$$\Rightarrow F_Y(y) = F_X(\sin^{-1} y) + 1 - \Pr(X \leq \pi - \sin^{-1} y) \quad (0.0.9)$$

$$\Rightarrow F_Y(y) = F_X(\sin^{-1} y) + 1 - F_X(\pi - \sin^{-1} y) \quad (0.0.10)$$

using (0.0.5) in (0.0.10)

$$\Rightarrow F_Y(y) = \frac{(\sin^{-1} y)^2}{\pi^2} + 1 - \frac{(\pi - \sin^{-1} y)^2}{\pi^2} \quad (0.0.11)$$

$$\Rightarrow F_Y(y) = \frac{2 \sin^{-1} y}{\pi} \quad (0.0.12)$$

$$\therefore f_Y(y) = \frac{dF_Y(y)}{dy} \quad (0.0.13)$$

$$\Rightarrow f_Y(y) = \frac{2 d(\sin^{-1} y)}{\pi dy} \quad (0.0.14)$$

$$\Rightarrow f_Y(y) = \frac{2}{\pi \sqrt{1-y^2}} \quad (0.0.15)$$

Hence, option(D) is correct.