Assignment 7

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Download all python codes from

https://github.com/Ananthoju−Pranav−Sai/AI1103/tree/main/Assignment_7/Codes

and latex codes from

https://github.com/Ananthoju−Pranav−Sai/AI1103/blob/main/Assignment_7/main.tex

UGC Dec 2018 Math set A Q 114

Suppose that $X_1, X_2, X_3, ..., X_{10}$ are i.i.d, N(0,1). Which of the following statements are correct ?

(A) $\Pr(X_1 > X_2 + X_3 + ... + X_{10}) = \frac{1}{2}$
(B) $\Pr(X_1 > X_2X_3...X_{10}) = \frac{1}{2}$
(C) $\Pr(\sin X_1 > \sin X_2 + \sin X_3 + ... + \sin X_{10}) = \frac{1}{2}$
(D) $\Pr(\sin X_1 > \sin X_2 \sin X_3 ... \sin X_{10}) = \frac{1}{2}$

Solution

Given, $X_1, X_2, X_3, ..., X_{10}$ are identically independent events which follow normal distribution with mean 0 and variance 1.

(A) Now let Y be a random variable which is defined as follows

$$Y = X_2 + X_3 + ... + X_{10} - X_1 \tag{0.0.1}$$

As all X_i's follow normal distribution, Y will also follow normal distribution.
Then $\Pr(X_1 > X_2 + X_3 + ... + X_{10})$ can be written as $\Pr(Y < 0)$

$$E(Y) = E(X_2)+E(X_3)+E(X_4)+E(X_5)+E(X_6) + E(X_7) + E(X_8) + E(X_9) + E(X_{10}) - E(X_1) \tag{0.0.2}$$

$$\implies E(Y) = 0 \tag{0.0.3}$$

$$\implies \mu_Y = 0 \tag{0.0.4}$$

As all X_i's are independent variance of Y can be written as

$$\sigma_Y^2 = \sigma_{X_2}^2 + \sigma_{X_3}^2 + \sigma_{X_4}^2 + \sigma_{X_5}^2 + \sigma_{X_6}^2 + \sigma_{X_7}^2 + \sigma_{X_8}^2 + \sigma_{X_9}^2 + \sigma_{X_{10}}^2 + \sigma_{X_1}^2 \tag{0.0.5}$$

$$\implies \sigma_Y^2 = 10 \tag{0.0.6}$$

We know that for normal distribution pdf would look like a So, now we have pdf of Y as

$$f_Y(y) = \frac{1}{\sqrt{20\pi}} e^{-\frac{y^2}{20}} \tag{0.0.7}$$

Now,

$$\Pr(Y < 0) = \int_{-\infty}^{0} f_Y(y)\, dy \tag{0.0.8}$$

$$\Pr(Y < 0) = \frac{2\int_{-\infty}^{0} f_Y(y)\, dy}{2} \tag{0.0.9}$$

$$\Pr(Y < 0) = \frac{\int_{-\infty}^{\infty} f_Y(y)\, dy}{2} \tag{0.0.10}$$

$$\implies \Pr(Y < 0) = \frac{1}{2} \tag{0.0.11}$$

$$\therefore \Pr(X_1 > X_2 + X_3 + ... + X_{10}) = \frac{1}{2} \tag{0.0.12}$$

(B) We will consider only three variables first and proceed to generalise the method.

$$\Pr(X_1 > X_2X_3) \tag{0.0.13}$$

$$= \sum_z (\Pr(X_1 > z))(\Pr(X_2X_3 = z)) \tag{0.0.14}$$

$$= \int_{-\infty}^{\infty} (1 - F_{X_1}(z))(\Pr(X_2X_3 = x))\, dz \tag{0.0.15}$$

$$= \int_{-\infty}^{\infty} (1 - F_{X_1}(z)) \left(\sum_k (\Pr(X_2 = z/k) \Pr(X_3 = k)) \right) dz \tag{0.0.16}$$

$$= \int_{-\infty}^{\infty} (1 - F_{X_1}(z)) \left(\int_{-\infty}^{\infty} f_{X_2}(z/k) f_{X_3}(k)\, dk \right) dz \tag{0.0.17}$$

$$= \int_{-\infty}^{\infty} (1 - F_{X_1}(z)) \left(\frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-\left(\frac{z^2+k^4}{k^2}\right)} dk \right) dz \tag{0.0.18}$$