

# Assignment 8

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Download the latex code from

[https://github.com/Ananthoju-Pranav-Sai/AI1103/blob/main/Assignment\\_8/main.tex](https://github.com/Ananthoju-Pranav-Sai/AI1103/blob/main/Assignment_8/main.tex)

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Suppose  $(X_1, X_2)$  follows a bivariate normal distribution with

$$E(X_1) = E(X_2) = 0 \quad (0.0.1)$$

$$V(X_1) = V(X_2) = 2 \quad (0.0.2)$$

$$\text{Cov}(X_1, X_2) = -1 \quad (0.0.3)$$

If  $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{y^2}{2}} dy$ , then  $\Pr(X_1 - X_2 > 6) = ?$

- 1)  $\Phi(-1)$
- 2)  $\Phi(-3)$
- 3)  $\Phi(\sqrt{6})$
- 4)  $\Phi(-\sqrt{6})$

SOLUTION

Given,  $(X_1, X_2)$  follows a bivariate normal distribution. So any random variable  $Z = aX_1 + bX_2$  is normal.

So consider a random variable  $Z$  defined as follows

$$Z = X_2 - X_1 \quad (0.0.4)$$

Then  $\Pr(X_1 - X_2 > 6)$  can be written as  $\Pr(Z < -6)$

As  $Z = X_2 - X_1$  mean of  $Z$  is,

$$\mu_Z = E(Z) \quad (0.0.5)$$

$$\Rightarrow \mu_Z = E(X_2) + E(X_1) \quad (0.0.6)$$

$$\Rightarrow \mu_Z = 0 \quad (0.0.7)$$

and variance of  $Z$ ,

$$\sigma_Z^2 = \text{Var}(Z) \quad (0.0.8)$$

$$\Rightarrow \sigma_Z^2 = \text{Var}(X_2 - X_1) \quad (0.0.9)$$

$$\Rightarrow \sigma_Z^2 = \text{Var}(X_2) + \text{Var}(X_1) - 2\text{Cov}(X_1, X_2) \quad (0.0.10)$$

$$\Rightarrow \sigma_Z^2 = 6 \quad (0.0.11)$$

As  $Z$  follows normal distribution, pdf of  $Z$  can be written as

$$f_Z(z) = \frac{1}{\sigma_Z \sqrt{2\pi}} e^{-\frac{(z-\mu_Z)^2}{2\sigma_Z^2}} \quad (0.0.12)$$

$$\Rightarrow f_Z(z) = \frac{1}{\sqrt{12\pi}} e^{-\frac{z^2}{12}} \quad (0.0.13)$$

Now for  $\Pr(Z < -6)$

$$\Pr(Z < -6) = \int_{-\infty}^{-6} f_Z(z) dz \quad (0.0.14)$$

$$= \int_{-\infty}^{-6} \frac{1}{\sqrt{12\pi}} e^{-\frac{z^2}{12}} dz \quad (0.0.15)$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-6} \frac{1}{\sqrt{6}} e^{-\frac{z^2}{12}} dz \quad (0.0.16)$$

let  $y = \frac{z}{\sqrt{6}}$  then (0.0.16) can be written as

$$\Pr(Z < -6) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-\sqrt{6}} e^{-\frac{y^2}{2}} dy \quad (0.0.17)$$

$$\Rightarrow \Pr(Z < -6) = \Phi(-\sqrt{6}) \quad (0.0.18)$$

$$\therefore \Pr(X_1 - X_2 > 6) = \Phi(-\sqrt{6}) \quad (0.0.19)$$