ASSIGNMENT 3

ANANTHOJU PRANAV SAI - AI20BTECH11004

September 14, 2021

Question

Construction 2.11

Construct PLAN where $PL = 4$, $LA = 6.5$, $\angle P = 90^\circ$,$\angle A = 110^\circ$ and $\angle N{=}85^\circ$

Lemma

Let ABCD be a quadrilateral with

$$\|B - A\| = a \tag{1}$$

$$\|C - B\| = b \tag{2}$$

$$\angle A = \theta \tag{3}$$

$$\angle C = \beta \tag{4}$$

$$\angle D = \gamma \tag{5}$$

$$\mathrm{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{6}$$

$$\mathrm{B} = \begin{pmatrix} a \\ 0 \end{pmatrix} \tag{7}$$

Lemma contd..

then the remaining vectors can be found using

$$\mathrm{C} = \mathrm{B} + b\begin{pmatrix}\cos(180-\alpha)\\ \sin(180-\alpha)\end{pmatrix} \tag{8}$$

where $\alpha = 360 - (\theta + \beta + \gamma)$

$$\mathrm{D} = d\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix} \tag{9}$$

where

$$d = \|A - D\| = e \times \left(\frac{\sin\left(\beta - \sin^{-1}\left(\frac{a\sin\alpha}{e}\right)\right)}{\sin\gamma}\right) \tag{10}$$

$$e = \|C - A\| = \sqrt{a^2 + b^2 - 2ab\cos\alpha} \tag{11}$$

Proof.

Let,

$$\angle ACB = \beta_1 \tag{12}$$

$$\angle ACD = \beta_2 \tag{13}$$

$$\implies \beta_1 + \beta_2 = \beta \tag{14}$$

Now in $\triangle$ ABC applying cosine rule gives,

$$e = \sqrt{a^2 + b^2 - 2ab\cos\alpha} \tag{15}$$

and in $\triangle$ ABC applying sine rule gives,

$$\frac{\sin\angle ACB}{AB} = \frac{\sin B}{AC} \tag{16}$$

$$\implies \frac{\sin\beta_1}{a} = \frac{\sin\alpha}{e} \tag{17}$$

$$\implies \beta_1 = \sin^{-1}\left(\frac{a\sin\alpha}{e}\right) \tag{18}$$

Proof cont..d

and in $\triangle$ ACD applying sine rule gives,

$$\frac{\sin \angle ACD}{AD} = \frac{\sin D}{AC} \tag{19}$$

$$\implies \frac{\sin \beta_2}{d} = \frac{\sin \gamma}{e} \tag{20}$$

$$\implies d = e \times \left(\frac{\sin \left(\beta - \sin^{-1} \left(\frac{a \sin \alpha}{e} \right) \right)}{\sin \gamma} \right) \tag{21}$$

Solution

Given,

$$\angle P = 90^\circ = \theta \tag{22}$$
$$\angle A = 110^\circ = \beta \tag{23}$$
$$\angle N = 85^\circ = \gamma \tag{24}$$
$$\implies \angle L = 75^\circ = \alpha \tag{25}$$
$$\|\mathrm{L} - \mathrm{P}\| = 4 = a \tag{26}$$
$$\|\mathrm{A} - \mathrm{L}\| = 6.5 = b \tag{27}$$
$$\mathrm{P} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{28}$$
$$\mathrm{L} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{29}$$

Solution contd..

Let,

$$\theta = \angle L \tag{30}$$
$$\|\mathbf{A} - \mathbf{N}\| = c \tag{31}$$
$$\|\mathbf{N} - \mathbf{P}\| = d \tag{32}$$
$$\|\mathbf{A} - \mathbf{P}\| = e \tag{33}$$

We know that,

$$d = e \times \left(\frac{\sin\left(\beta - \sin^{-1}\left(\frac{a \sin\alpha}{e}\right)\right)}{\sin\gamma} \right) \tag{34}$$
$$e = \sqrt{a^2 + b^2 - 2ab\cos\alpha} \tag{35}$$
$$\implies e = 6.7 \tag{36}$$

Solution contd..

using (36) in (34) we get

$$d = 6.49 \tag{37}$$

then for A we have,

$$\mathrm{A} = \mathrm{L} + b\begin{pmatrix} \cos(180-\alpha) \\ \sin(180-\alpha) \end{pmatrix} \tag{38}$$

$$\implies \mathrm{A} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + 6.5\begin{pmatrix} \cos 105 \\ \sin 105 \end{pmatrix} \tag{39}$$

$$\implies \mathrm{A} = \begin{pmatrix} 2.318 \\ 6.279 \end{pmatrix} \tag{40}$$

and for N we have,

$$\mathrm{N} = d\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{41}$$

$$\implies \mathrm{N} = \begin{pmatrix} 0 \\ 6.49 \end{pmatrix} \tag{42}$$

Plot of Quadrilateral PLAN

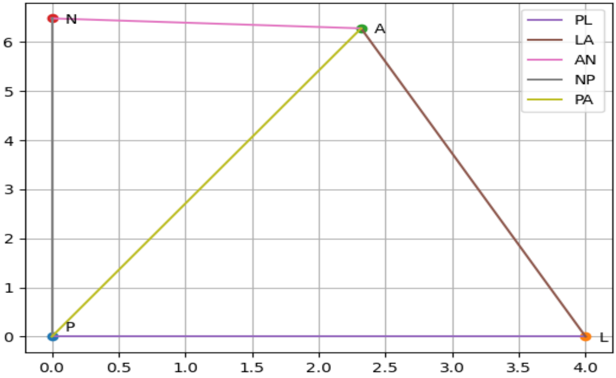

Figure: Quadrilateral PLAN