QUIZ 2

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Download all python codes from

```
https://github.com/Ananthoju−Pranav−Sai/EE3900/
blob/main/Quiz_2/codes
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and latex-tikz codes from

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https://github.com/Ananthoju−Pranav−Sai/EE3900/
tree/main/Quiz_2/Quiz_2.tex
```

1 Discrete Time Signal Processing Q 3.21(b)

Consider an linear time-invariant system with impulse response

$$h[n] = \begin{cases} a^n & n \geq 0 \\ 0 & n < 0 \end{cases} \tag{1.0.1}$$

and input

$$x[n] = \begin{cases} 1 & 0 \leq n \leq (N-1) \\ 0 & otherwise \end{cases} \tag{1.0.2}$$

Determine the output $y[n]$ by computing the inverse z-transform of the product of the z-transforms of $x[n]$ and $h[n]$.

2 Solution

Given,

$$h[n] = \begin{cases} a^n & n \geq 0 \\ 0 & n < 0 \end{cases} \tag{2.0.1}$$

applying z-transform

$$H(z) = \mathcal{Z}(h[n]) \tag{2.0.2}$$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n} \tag{2.0.3}$$

$$\implies H(z) = \sum_{n=0}^{\infty} a^n z^{-n} \tag{2.0.4}$$

$$\implies H(z) = \frac{1}{1-az^{-1}} \qquad |z| > |a| \tag{2.0.5}$$

input

$$x[n] = \begin{cases} 1 & 0 \leq n \leq (N-1) \\ 0 & otherwise \end{cases} \tag{2.0.6}$$

applying z-transform

$$X(z) = \mathcal{Z}(x[n]) \tag{2.0.7}$$

$$\implies X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} \tag{2.0.8}$$

$$\implies X(z) = \sum_{n=0}^{N-1} z^{-n} \tag{2.0.9}$$

$$\implies X(z) = \frac{1-z^{-N}}{1-z^{-1}} \tag{2.0.10}$$

We know

$$Y(z) = X(z)H(z) \tag{2.0.11}$$

$$= \frac{1-z^{-N}}{(1-z^{-1})(1-az^{-1})} \qquad |z| > |a| \tag{2.0.12}$$

$$= \frac{1}{(1-z^{-1})(1-az^{-1})} - \frac{z^{-N}}{(1-z^{-1})(1-az^{-1})} \tag{2.0.13}$$

Let,

$$G(z) = \frac{1}{(1-az^{-1})(1-z^{-1})} \tag{2.0.14}$$

$$\implies G(z) = \left(\frac{1}{1-a}\right)\left(\frac{1}{1-z^{-1}} - \frac{a}{1-az^{-1}}\right) \tag{2.0.15}$$

applying inverse z-transform

$$g[n] = \mathcal{Z}^{-1}(G(z)) \tag{2.0.16}$$

$$\implies g[n] = \left(\frac{1}{1-a}\right)(u[n] - a(a^n u[n])) \tag{2.0.17}$$

$$\implies g[n] = \left(\frac{1-a^{n+1}}{1-a}\right)u[n] \tag{2.0.18}$$

from (2.0.13) and (2.0.14) we have

$$Y(z) = G(z) - z^{-N}G(z) \tag{2.0.19}$$

applying inverse z-transform

$$y[n] = \mathcal{Z}^{-1}(Y(z)) \tag{2.0.20}$$

$$\implies y[n] = \mathcal{Z}^{-1}(G(z)) - \mathcal{Z}^{-1}(z^{-N}G(z)) \tag{2.0.21}$$

$$\implies y[n] = g[n] - g[n-N] \tag{2.0.22}$$

from (2.0.18) we have

$$y[n] = \left(\frac{1-a^{n+1}}{1-a}\right)u[n] - \left(\frac{1-a^{n-N+1}}{1-a}\right)u[n-N] \tag{2.0.23}$$

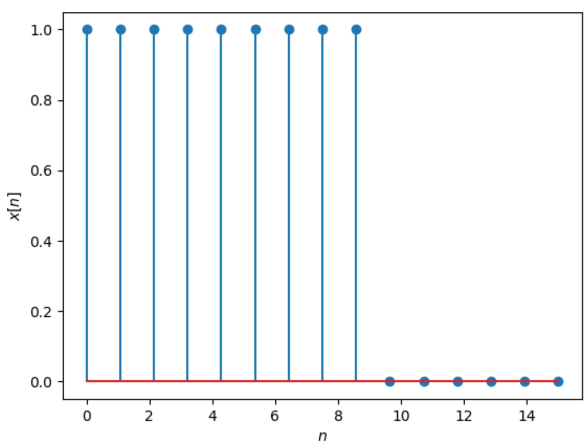

Fig. 0: Plot of input signal x[n] for N=10

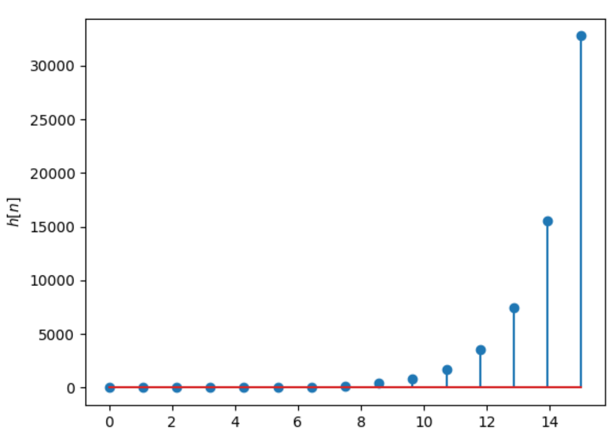

Fig. 0: Plot of impulse response h[n] for a=2

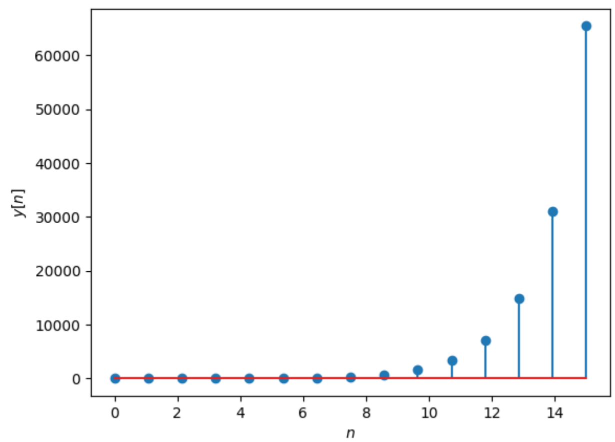

Fig. 0: Plot of output signal y[n] for a=2 and N=10