GATE ASSIGNMENT 4

Ananthoju Pranav Sai
AI20BTECH11004

Download all python codes from

```
https://github.com/Ananthoju-Pranav-Sai/EE3900/
    blob/main/Gate_Assignment_4/codes
```

and latex-tikz codes from

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https://github.com/Ananthoju-Pranav-Sai/EE3900/
    tree/main/Gate_Assignment_4/
    Gate_Assignment_4.tex
```

1 GATE EC 2000 Q.2.31

Let u(t) be the step function. Plot the wave form corresponding to the convolution of $u(t) - u(t-1)$ with $u(t) - u(t-2)$.

2 Solution

We define unit step function as follows

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases} \tag{2.0.1}$$

Now let $f(t) = u(t) - u(t-1)$ and $g(t) = u(t) - u(t-2)$ then,

$$f(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 \leq t < 1 \\ 0 & t \geq 1 \end{cases} \tag{2.0.2}$$

$$\implies f(t) = rect\left(t - \frac{1}{2}\right) \tag{2.0.3}$$

$$g(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 \leq t < 2 \\ 0 & t \geq 2 \end{cases} \tag{2.0.4}$$

$$\implies g(t) = rect\left(\frac{t-1}{2}\right) \tag{2.0.5}$$

Let $y(t)$ be convolution of $f(t)$ and $g(t)$ So we have,

$$y(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)\, d\tau \tag{2.0.6}$$

$$\implies y(t) = \int_{0}^{1} g(t-\tau)\, d\tau \tag{2.0.7}$$

(2.0.8)

$$y(t) = \begin{cases} 0 & t < 0 \\ \int_0^t 1\, d\tau & 0 \leq t < 1 \\ \int_0^1 1\, d\tau & 1 \leq t < 2 \\ \int_{t-2}^1 1\, d\tau & 2 \leq t < 3 \\ 0 & t \geq 3 \end{cases} \tag{2.0.9}$$

So we get $y(t)$ as follows

$$y(t) = \begin{cases} 0 & t < 0 \\ t & 0 \leq t < 1 \\ 1 & 1 \leq t < 2 \\ 3-t & 2 \leq t < 3 \\ 0 & t \geq 3 \end{cases} \tag{2.0.10}$$

Fourier Transform of the output

$$Y(\omega) = F(\omega)G(\omega) \tag{2.0.11}$$

We know that the fourier transform

$$rect\left(\frac{t}{\tau}\right) \Longleftrightarrow \tau sinc\left(\frac{\omega\tau}{2}\right) \tag{2.0.12}$$

and using the properties of fourier transform

$$x(t \pm t_0) \Longleftrightarrow X(\omega)e^{\pm i\omega t_0} \tag{2.0.13}$$

$$x(\alpha t) \Longleftrightarrow \frac{1}{|\alpha|}X\left(\frac{\omega}{\alpha}\right) \tag{2.0.14}$$

We get

$$F(\omega) = sinc\left(\frac{\omega}{2}\right)e^{\left(-\frac{i\omega}{2}\right)} \tag{2.0.15}$$

$$G(\omega) = 2sinc\,(\omega)\, e^{-i\omega} \tag{2.0.16}$$

Using (2.0.11),(2.0.15) and (2.0.16)

$$\implies Y(\omega) = 2sinc\,(\omega)\, sinc\left(\frac{\omega}{2}\right)e^{\left(-\frac{3i\omega}{2}\right)} \tag{2.0.17}$$

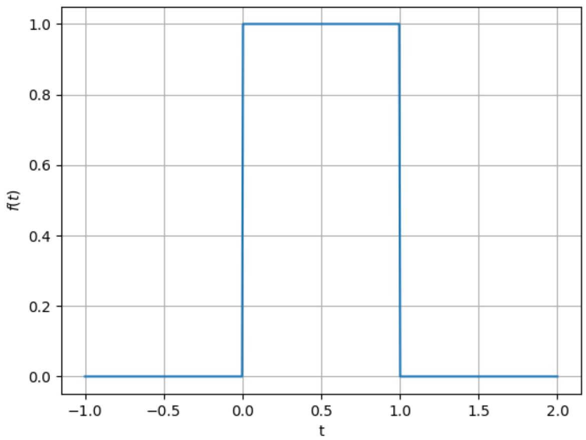

Fig. 0: Plot of f(t)

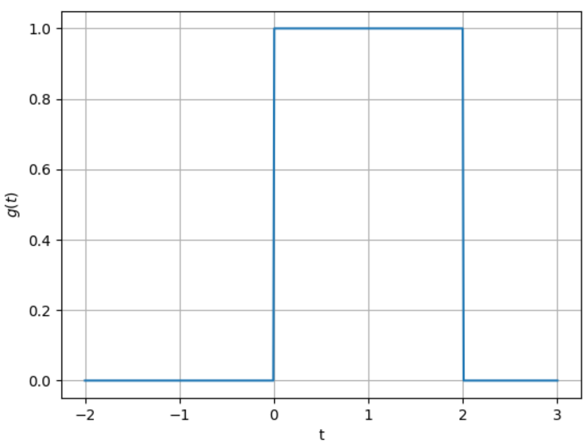

Fig. 0: Plot of g(t)

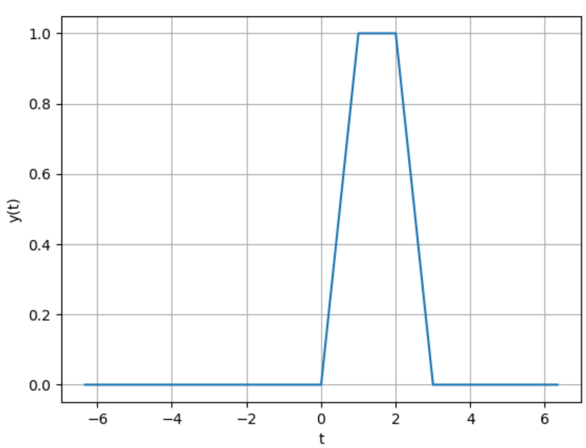

Fig. 0: Simulated plot of output signal y(t)

We can represent the output signal as sum of two shifted triangles as follows. So output signal $y(t)$ can

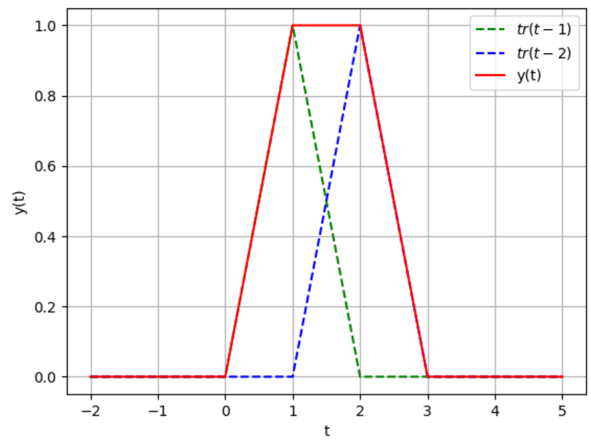

Fig. 0: y(t) as sum of 2 shifted triangles

also be written as

$$y(t) = \triangle(t-1) + \triangle(t-2) \tag{2.0.18}$$

where triangular function $\triangle(t)$ is defined as

$$\triangle(t) = rect(t) * rect(t) \tag{2.0.19}$$