ASSIGNMENT 5

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1 QUADRATIC FORMS Q.2.62

Find a point on the curve $y = (x - 2)^2$ at which the tangent is parallel to the chord joining the points $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$.

2 SOLUTION

Equation of the given conic in vector form is

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -2 & \frac{-1}{2} \end{pmatrix} \mathbf{x} + 4 = 0 \qquad (2.0.1)$$

Therefore we have

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad (2.0.2)$$

$$\mathbf{u} = \begin{pmatrix} -2 \\ \frac{-1}{2} \end{pmatrix} \qquad (2.0.3)$$

$$f = 4 \qquad (2.0.4)$$

Given the tangent is parallel to the chord joining the points $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$. So,

$$\mathbf{m} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \end{pmatrix} \qquad (2.0.5)$$

$$\mathbf{m} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \qquad (2.0.6)$$

$$\implies \mathbf{n} = \begin{pmatrix} -4 \\ 2 \end{pmatrix} \qquad (2.0.7)$$

Lemma 2.1. *If* **V** *is not invertible, given the normal vector* **n**, *the point of contact is given by the matrix equation*

$$\begin{pmatrix} \mathbf{u}^T + \kappa \mathbf{n}^T \\ \mathbf{V} \end{pmatrix} \mathbf{q} = \begin{pmatrix} -f \\ \kappa \mathbf{n} - \mathbf{q} \end{pmatrix} \qquad (2.0.8)$$

where

$$\kappa = \frac{\mathbf{p_1}^T u}{\mathbf{p_1}^T n} \qquad (2.0.9)$$

$$\mathbf{V p_1} = 0 \qquad (2.0.10)$$

So let $\mathbf{p_1} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$ as it satisfies $\mathbf{V p_1} = 0$ then

$$\kappa = \frac{\mathbf{p_1}^T u}{\mathbf{p_1}^T n} \qquad (2.0.11)$$

$$\kappa = \frac{\begin{pmatrix} 0 & 2 \end{pmatrix} \begin{pmatrix} -2 \\ \frac{-1}{2} \end{pmatrix}}{\begin{pmatrix} 0 & 2 \end{pmatrix} \begin{pmatrix} -4 \\ 2 \end{pmatrix}} \qquad (2.0.12)$$

$$\kappa = \frac{-1}{4} \qquad (2.0.13)$$

Now the matrix equation

$$\begin{pmatrix} \mathbf{u}^T + \kappa \mathbf{n}^T \\ \mathbf{V} \end{pmatrix} \mathbf{q} = \begin{pmatrix} -f \\ \kappa \mathbf{n} - \mathbf{q} \end{pmatrix} \qquad (2.0.14)$$

$$\begin{pmatrix} -1 & -1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{q} = \begin{pmatrix} -4 \\ 3 \\ 0 \end{pmatrix} \qquad (2.0.15)$$
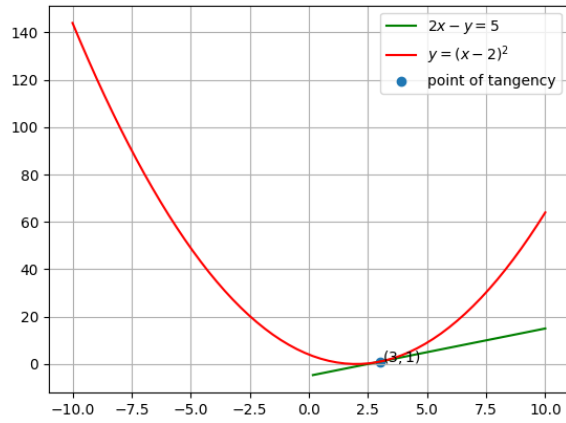
$$\mathbf{q} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \qquad (2.0.16)$$

Fig. 0: Plot of the tangent and parabola