ASSIGNMENT 5

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AI20BTECH11004

Download all python codes from

```
https://github.com/Ananthoju-Pranav-Sai/EE3900/blob/main/Assignment-5/codes/Assignment-5.py
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and latex-tikz codes from

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https://github.com/Ananthoju-Pranav-Sai/EE3900/tree/main/Assignment-5/Assignment-5.tex
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1 Quadratic Forms Q.2.62

Find a point on the curve $y = (x-2)^2$ at which the tangent is parallel to the chord joining the points $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$.

2 Solution

Equation of the given conic in vector form is

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -2 & \frac{-1}{2} \end{pmatrix} \mathbf{x} + 4 = 0 \tag{2.0.1}$$

Therefore we have

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{u} = \begin{pmatrix} -2 \\ \frac{-1}{2} \end{pmatrix} \tag{2.0.3}$$

Given the tangent is parallel to the chord joining the points $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$ then the tangent is given by

$$\mathbf{x} = \mathbf{q} + \mu \mathbf{m} \tag{2.0.4}$$

where,

$$\mathbf{m} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{2.0.5}$$

$$\mathbf{m} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \tag{2.0.6}$$

and $\mathbf{q}$ is the point of contact of tangent on the curve.

Lemma 2.1. *If the line*

$$L : \mathbf{x} = \mathbf{q} + \mu \mathbf{m} \tag{2.0.7}$$

is tangent to the conic

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \tag{2.0.8}$$

then

$$\mathbf{m}^T (\mathbf{V}\mathbf{q} + \mathbf{u}) = 0 \tag{2.0.9}$$

So from lemma we have

$$\begin{pmatrix} 2 & 4 \end{pmatrix} \left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{q} + \begin{pmatrix} -2 \\ \frac{-1}{2} \end{pmatrix} \right) = 0 \tag{2.0.10}$$

$$\implies \begin{pmatrix} 2 & 0 \end{pmatrix} \mathbf{q} = 6 \tag{2.0.11}$$

As $\mathbf{q}$ also lies on conic from (2.0.1) we have

$$\mathbf{q}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{q} + 2 \begin{pmatrix} -2 & \frac{-1}{2} \end{pmatrix} \mathbf{q} + 4 = 0 \tag{2.0.12}$$

solving (2.0.11) and (2.0.12) we get

$$\mathbf{q} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \tag{2.0.13}$$

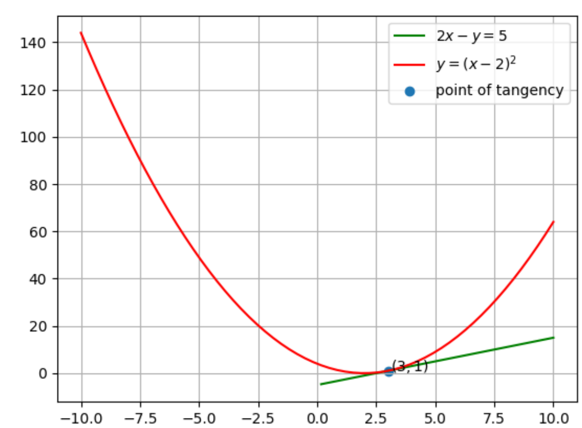

Fig. 0: Plot of the tangent and parabola