GATE ASSIGNMENT 1

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October 3, 2021

Question

GATE EC 2021 Q.39

The exponential Fourier series representation of a continuous-time periodic signal x(t) is defined as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{ik\omega_0 t} \tag{1}$$

where ω_0 is the fundamental angular frequency of $x(t)$ and the coefficients of the series are a_k. The following information is given about $x(t)$ and a_k.

I $x(t)$ is real and even, having a fundamental period of 6

II The average value of $x(t)$ is 2

III $a_k = \begin{cases} k & 1 \leq k \leq 3 \\ 0 & k > 3 \end{cases}$

The average power of the signal $x(t)$ (rounded off to one decimal place) is

Parseval's Theorem

Theorem (Parseval's power theorem)

If a exponential Fourier series representation of a continuous-time periodic signal x(t) is defined by

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{in\omega_0 t} \tag{2}$$

where C_n is given by

$$C_n = \frac{1}{T} \int_0^T x(t) e^{-jn\omega_0 t}\, dt \tag{3}$$

then the average power of signal $x(t)$ is given by

$$\frac{1}{T} \int_0^T |x(t)|^2\, dt = \sum_{n=-\infty}^{\infty} |C_n|^2 \tag{4}$$

Proof.

Given,

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{in\omega_0 t} \tag{5}$$

Its complex conjugate can be written as

$$x(t)^* = \sum_{n=-\infty}^{\infty} C_n^* e^{-in\omega_0 t} \tag{6}$$

Now we know that the average power of signal $x(t)$ is given by

$$P_{x(t)} = \frac{1}{T}\int_0^T |x(t)|^2 \, dt \tag{7}$$

$$\implies P_{x(t)} = \frac{1}{T}\int_0^T x(t)x(t)^* \, dt \tag{8}$$

□

Proof Contd..

$$\implies P_{x(t)} = \frac{1}{T}\int_0^T x(t)\left(\sum_{n=-\infty}^{\infty} C_n^* e^{-in\omega_0 t}\right) dt \tag{9}$$

$$\implies P_{x(t)} = \sum_{n=-\infty}^{\infty} C_n^* \left(\frac{1}{T}\int_0^T x(t)e^{-jn\omega_0 t}\, dt\right) \tag{10}$$

$$\implies P_{x(t)} = \sum_{n=-\infty}^{\infty} C_n^* C_n \tag{11}$$

$$\implies P_{x(t)} = \sum_{n=-\infty}^{\infty} |C_n|^2 \tag{12}$$

Solution

The given signal is defined as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{ik\omega_0 t} \tag{13}$$

where a_k is Fourier Series Coefficient. Given that the $x(t)$ is real and even. So the spectrum is also real and even. $a_k = \begin{cases} k & 1 \leq k \leq 3 \\ 0 & k > 3 \end{cases}$

$\therefore$ Average power is :

$$\frac{1}{T}\int_0^T |x(t)|^2 \, dt = \sum_{k=-3}^{3} |a_k|^2 \tag{14}$$

$$P_{avg} = 9 + 4 + 1 + 4 + 1 + 4 + 9 \tag{15}$$

$$P_{avg} = 32 \tag{16}$$

Solution contd..

Discrete frequency domain signals are called $X[k]$ and is given by

$$X[k] = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_0 t}\,dt \tag{17}$$

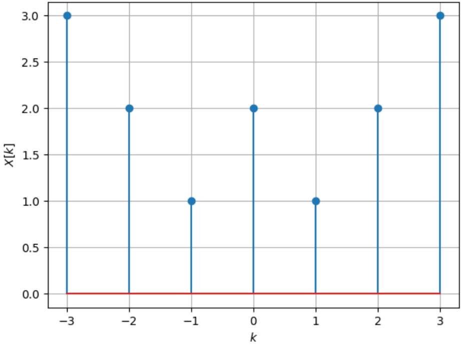

Figure: Plot of $X[k]$

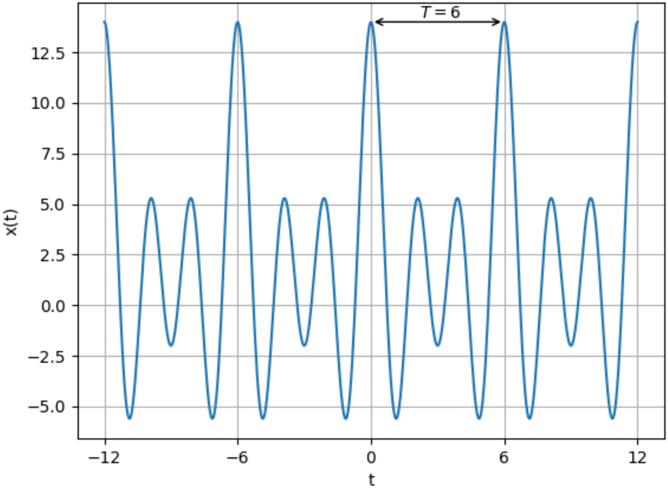

Figure: Plot of $x(t)$