GATE ASSIGNMENT 4

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AI20BTECH11004

Download all python codes from

https://github.com/Ananthoju−Pranav−Sai/EE3900/blob/main/Gate_Assignment_4/codes

and latex-tikz codes from

https://github.com/Ananthoju−Pranav−Sai/EE3900/tree/main/Gate_Assignment_4/Gate_Assignment_4.tex

1 GATE EC 2000 Q.2.31

Let u(t) be the step function. Plot the wave form corresponding to the convolution of $u(t) - u(t-1)$ with $u(t) - u(t-2)$.

2 Solution

We define unit step function as follows

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases} \tag{2.0.1}$$

Now let $f(t) = u(t) - u(t-1)$ and $g(t) = u(t) - u(t-2)$ then,

$$f(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 \leq t < 1 \\ 0 & t \geq 1 \end{cases} \tag{2.0.2}$$

$$\implies f(t) = rect\left(t - \frac{1}{2}\right) \tag{2.0.3}$$

$$g(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 \leq t < 2 \\ 0 & t \geq 2 \end{cases} \tag{2.0.4}$$

$$\implies g(t) = rect\left(\frac{t-1}{2}\right) \tag{2.0.5}$$

Let $y(t)$ be convolution of $f(t)$ and $g(t)$ So we have,

$$y(t) = rect\left(t - \frac{1}{2}\right) * rect\left(\frac{t-1}{2}\right) \tag{2.0.6}$$

$$y(t) = rect\left(t - \frac{1}{2}\right) * \left(rect\left(t - \frac{1}{2}\right) + rect\left(t - \frac{3}{2}\right)\right) \tag{2.0.7}$$

$$y(t) = rect\left(t - \frac{1}{2}\right) * rect\left(t - \frac{1}{2}\right) + rect\left(t - \frac{1}{2}\right) * rect\left(t - \frac{3}{2}\right)$$

We know that

$$\triangle(t) = rect(t) * rect(t) \tag{2.0.8}$$

So by using the following property of convolution

$$x_1(t) * x_2(t) = y(t) \tag{2.0.9}$$

$$x_1(t - t_0) * x_2(t - t_1) = y(t - t_0 - t_1) \tag{2.0.10}$$

we get

$$y(t) = \triangle\left(t - \frac{1}{2} - \frac{1}{2}\right) + \triangle\left(t - \frac{1}{2} - \frac{3}{2}\right) \tag{2.0.11}$$

$$\implies y(t) = \triangle(t-1) + \triangle(t-2) \tag{2.0.12}$$

Fourier Transform of the output

$$Y(f) = F(f)G(f) \tag{2.0.13}$$

We know that the fourier transform

$$rect\left(\frac{t}{\tau}\right) \iff \tau sinc\left(f\tau\right) \tag{2.0.14}$$

and using the properties of fourier transform

$$x(t \pm t_0) \iff X(f)e^{\pm 2\pi f t_0} \tag{2.0.15}$$

$$x(\alpha t) \iff \frac{1}{|\alpha|}X\left(\frac{f}{\alpha}\right) \tag{2.0.16}$$

We get

$$F() = sinc\left(f\right) e^{(-i\pi f)} \tag{2.0.17}$$

$$G(\omega) = 2sinc\left(f\right) e^{-2i\pi f} \tag{2.0.18}$$

Using (2.0.13),(2.0.17) and (2.0.18)

$$\implies Y(\omega) = 2sinc\left(2f\right) sinc\left(f\right) e^{(-3i\pi f)} \tag{2.0.19}$$

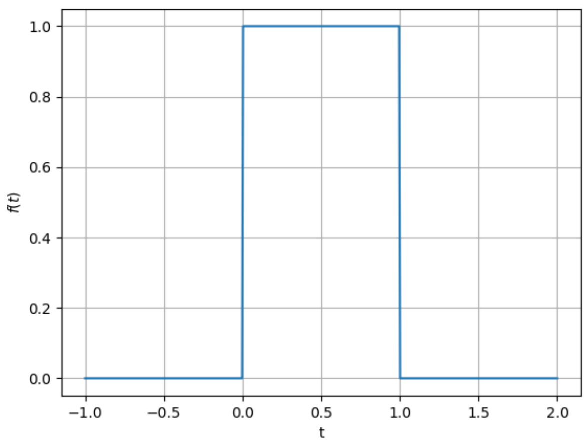

Fig. 0: Plot of f(t)

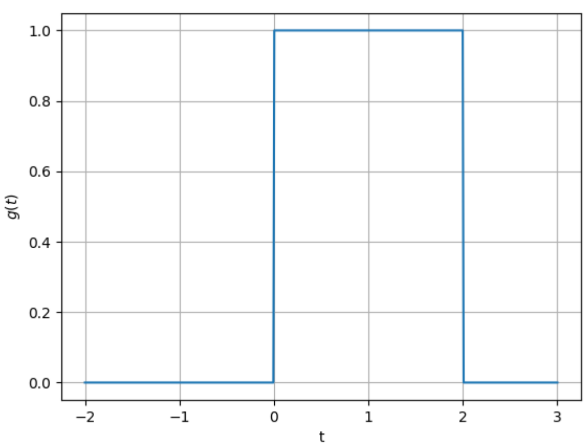

Fig. 0: Plot of g(t)

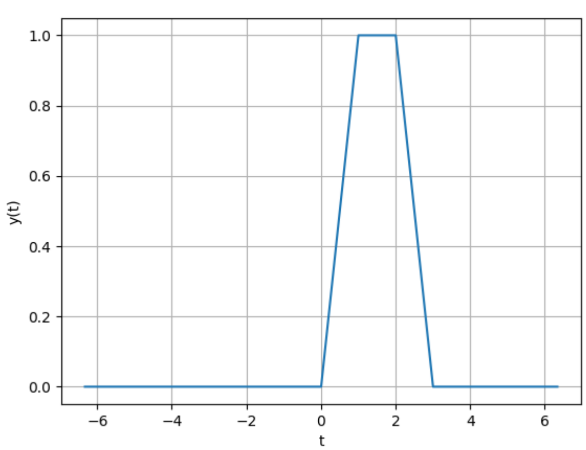

Fig. 0: Simulated plot of output signal y(t)

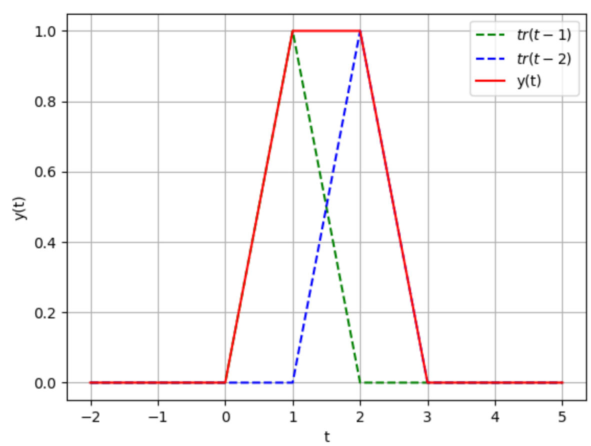

Fig. 0: y(t) as sum of 2 shifted triangles