ASSIGNMENT 4

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Download all python codes from

https://github.com/Ananthoju−Pranav−Sai/EE3900/blob/main/Assignment−4/codes/Assignment−4.py

and latex-tikz codes from

https://github.com/Ananthoju−Pranav−Sai/EE3900/tree/main/Assignment−4/Assignment−4.tex

1 Linear Forms 2.37

Find the coordinates of the point where the line through $\begin{pmatrix}5\\1\\6\end{pmatrix}$ and $\begin{pmatrix}3\\4\\1\end{pmatrix}$ crosses the ZX-plane.

2 Solution

The equation of line joining A and B is given by

$$\mathbf{r} = \mathbf{A} + \lambda(\mathbf{B} - \mathbf{A}) \tag{2.0.1}$$

General equation of plane is given by

$$\mathbf{n}^T \mathbf{r} = c \tag{2.0.2}$$

where $\mathbf{n}$ is normal vector to the plane

$$\mathbf{n} = \begin{pmatrix}n_1\\n_2\\n_3\end{pmatrix} \tag{2.0.3}$$

Lemma 2.1. *The point of intersection of line*

$$\mathbf{r} = \mathbf{A} + \lambda(\mathbf{B} - \mathbf{A}) \tag{2.0.4}$$

and plane

$$\mathbf{n}^T \mathbf{r} = c \tag{2.0.5}$$

is given by

$$\mathbf{r_0} = \mathbf{A} + \left(\frac{c - \mathbf{n}^T\mathbf{A}}{\mathbf{n}^T(\mathbf{B} - \mathbf{A})}\right)(\mathbf{B} - \mathbf{A}) \tag{2.0.6}$$

Proof. Let r_0 be the point of intersection of the line and the plane then the point lies on both line and plane so,

$$\mathbf{r_0} = \mathbf{A} + \lambda(\mathbf{B} - \mathbf{A}) \tag{2.0.7}$$

As r_0 also lies on plane

$$\mathbf{n}^T \mathbf{r_0} = c \tag{2.0.8}$$

$$\implies \mathbf{n}^T(\mathbf{A} + \lambda(\mathbf{B} - \mathbf{A})) = c \tag{2.0.9}$$

$$\implies \lambda\mathbf{n}^T(\mathbf{B} - \mathbf{A}) = c - \mathbf{n}^T\mathbf{A} \tag{2.0.10}$$

$$\implies \lambda = \left(\frac{c - \mathbf{n}^T\mathbf{A}}{\mathbf{n}^T(\mathbf{B} - \mathbf{A})}\right) \tag{2.0.11}$$

Therefore,

$$\mathbf{r_0} = \mathbf{A} + \left(\frac{c - \mathbf{n}^T\mathbf{A}}{\mathbf{n}^T(\mathbf{B} - \mathbf{A})}\right)(\mathbf{B} - \mathbf{A}) \tag{2.0.12}$$

□

Given,

$$\mathbf{A} = \begin{pmatrix}5\\1\\6\end{pmatrix} \tag{2.0.13}$$

$$\mathbf{B} = \begin{pmatrix}3\\4\\1\end{pmatrix} \tag{2.0.14}$$

Equation of line joining A and B is given by

$$\mathbf{r} = \mathbf{A} + \lambda(\mathbf{B} - \mathbf{A}) \tag{2.0.15}$$

$$\tag{2.0.16}$$

Equation of ZX-plane is given by

$$y = 0 \tag{2.0.17}$$

$$\implies \mathbf{n} = \begin{pmatrix}0\\1\\0\end{pmatrix}, c = 0 \tag{2.0.18}$$

Therefore coordinates of the intersection point are

$$\mathbf{r_0} = \mathbf{A} + \left(\frac{c - \mathbf{n}^T\mathbf{A}}{\mathbf{n}^T(\mathbf{B} - \mathbf{A})}\right)(\mathbf{B} - \mathbf{A}) \tag{2.0.19}$$

substituting all the vectors gives,

$$\mathbf{r_0} = \frac{1}{3}\begin{pmatrix} 17 \\ 0 \\ 23 \end{pmatrix} \tag{2.0.20}$$

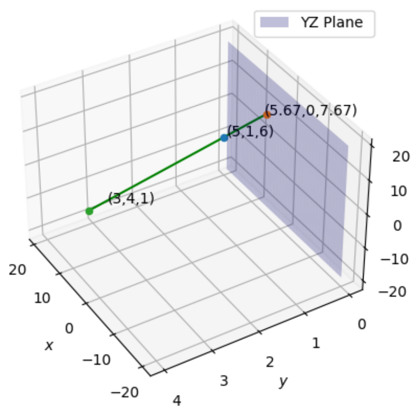

Fig. 0: Line and point of intersection