

# ASSIGNMENT 3

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Download all python codes from

<https://github.com/Ananthoju-Pranav-Sai/EE3900/blob/main/Assignment-3/codes/Assignment-3.py>

and latex-tikz codes from

<https://github.com/Ananthoju-Pranav-Sai/EE3900/tree/main/Assignment-3/Assignment-3.tex>

## 1 CONSTRUCTION 2.11

Construct PLAN where PL = 4, LA = 6.5,  $\angle P = 90^\circ$ ,  $\angle A = 110^\circ$  and  $\angle N = 85^\circ$ ,

## 2 SOLUTION

**Lemma 2.1.** Let ABCD be a quadrilateral with

$$\|B - A\| = a \quad (2.0.1)$$

$$\|C - B\| = b \quad (2.0.2)$$

$$\angle A = \theta \quad (2.0.3)$$

$$\angle C = \beta \quad (2.0.4)$$

$$\angle D = \gamma \quad (2.0.5)$$

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.6)$$

$$\mathbf{B} = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad (2.0.7)$$

then the remaining vectors can be found using

$$\mathbf{C} = \mathbf{B} + b \begin{pmatrix} \cos(180 - \alpha) \\ \sin(180 - \alpha) \end{pmatrix} \quad (2.0.8)$$

where  $\alpha = 360 - (\theta + \beta + \gamma)$

$$\mathbf{D} = d \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \quad (2.0.9)$$

where

$$d = \|A - D\| = e \times \left( \frac{\sin(\beta - \sin^{-1}(\frac{a \sin \alpha}{e}))}{\sin \gamma} \right) \quad (2.0.10)$$

$$e = \|C - A\| = \sqrt{a^2 + b^2 - 2ab \cos \alpha} \quad (2.0.11)$$

*Proof.* Let,

$$\angle ACB = \beta_1 \quad (2.0.12)$$

$$\angle ACD = \beta_2 \quad (2.0.13)$$

$$\Rightarrow \beta_1 + \beta_2 = \beta \quad (2.0.14)$$

Now in  $\triangle ABC$  applying cosine rule gives,

$$e = \sqrt{a^2 + b^2 - 2ab \cos \alpha} \quad (2.0.15)$$

and in  $\triangle ABC$  applying sine rule gives,

$$\frac{\sin \angle ACB}{AB} = \frac{\sin B}{AC} \quad (2.0.16)$$

$$\Rightarrow \frac{\sin \beta_1}{a} = \frac{\sin \alpha}{e} \quad (2.0.17)$$

$$\Rightarrow \beta_1 = \sin^{-1} \left( \frac{a \sin \alpha}{e} \right) \quad (2.0.18)$$

and in  $\triangle ACD$  applying sine rule gives,

$$\frac{\sin \angle ACD}{AD} = \frac{\sin D}{AC} \quad (2.0.19)$$

$$\Rightarrow \frac{\sin \beta_2}{d} = \frac{\sin \gamma}{e} \quad (2.0.20)$$

$$\Rightarrow d = e \times \left( \frac{\sin(\beta - \sin^{-1}(\frac{a \sin \alpha}{e}))}{\sin \gamma} \right) \quad (2.0.21)$$

□

Given,

$$\angle P = 90^\circ = \theta \quad (2.0.22)$$

$$\angle A = 110^\circ = \beta \quad (2.0.23)$$

$$\angle N = 85^\circ = \gamma \quad (2.0.24)$$

$$\Rightarrow \angle L = 75^\circ = \alpha \quad (2.0.25)$$

$$\|\mathbf{L} - \mathbf{P}\| = 4 = a \quad (2.0.26)$$

$$\|\mathbf{A} - \mathbf{L}\| = 6.5 = b \quad (2.0.27)$$

$$\mathbf{P} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.28)$$

$$\mathbf{L} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad (2.0.29)$$

Let,

$$\theta = \angle L \quad (2.0.30)$$

$$\|\mathbf{A} - \mathbf{N}\| = c \quad (2.0.31)$$

$$\|\mathbf{N} - \mathbf{P}\| = d \quad (2.0.32)$$

$$\|\mathbf{A} - \mathbf{P}\| = e \quad (2.0.33)$$

We know that,

$$d = e \times \left( \frac{\sin \left( \beta - \sin^{-1} \left( \frac{a \sin \alpha}{e} \right) \right)}{\sin \gamma} \right) \quad (2.0.34)$$

$$e = \sqrt{a^2 + b^2 - 2ab \cos \alpha} \quad (2.0.35)$$

$$\Rightarrow e = 6.7 \quad (2.0.36)$$

using (2.0.36) in (2.0.34) we get

$$d = 6.49 \quad (2.0.37)$$

then for  $\mathbf{A}$  we have,

$$\mathbf{A} = \mathbf{L} + b \begin{pmatrix} \cos(180 - \alpha) \\ \sin(180 - \alpha) \end{pmatrix} \quad (2.0.38)$$

$$\Rightarrow \mathbf{A} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + 6.5 \begin{pmatrix} \cos 105 \\ \sin 105 \end{pmatrix} \quad (2.0.39)$$

$$\Rightarrow \mathbf{A} = \begin{pmatrix} 2.318 \\ 6.279 \end{pmatrix} \quad (2.0.40)$$

and for  $\mathbf{N}$  we have,

$$\mathbf{N} = d \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \quad (2.0.41)$$

$$\Rightarrow \mathbf{N} = \begin{pmatrix} 0 \\ 6.49 \end{pmatrix} \quad (2.0.42)$$

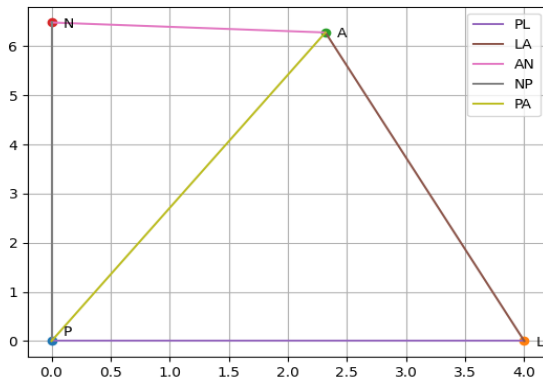


Fig. 0: Quadrilateral PLAN