ASSIGNMENT 3

Ananthoju Pranav Sai
AI20BTECH11004

Download all python codes from

https://github.com/Ananthoju−Pranav−Sai/EE3900/blob/main/Assignment−1/codes/Assignment−3.py

and latex-tikz codes from

https://github.com/Ananthoju−Pranav−Sai/EE3900/tree/main/Assignment−1/Assignment−3.tex

1 Construction 2.11

Construct PLAN where PL = 4, LA = 6.5, ∠P = 90°,∠A = 110° and ∠N = 85°,

2 Solution

Lemma 2.1. *Let ABCD be a quadrilateral with*

$$\|B-A\| = a \tag{2.0.1}$$
$$\|C-B\| = b \tag{2.0.2}$$
$$\angle A = \theta \tag{2.0.3}$$
$$\angle C = \beta \tag{2.0.4}$$
$$\angle D = \gamma \tag{2.0.5}$$
$$\mathbf{A} = \begin{pmatrix}0\\0\end{pmatrix} \tag{2.0.6}$$
$$\mathbf{B} = \begin{pmatrix}a\\0\end{pmatrix} \tag{2.0.7}$$

then the remaining vectors can be found using

$$\mathbf{C} = \mathbf{B} + b\begin{pmatrix}\cos(180-\alpha)\\ \sin(180-\alpha)\end{pmatrix} \tag{2.0.8}$$

where $\alpha = 360 - (\theta+\beta+\gamma)$

$$\mathbf{D} = d\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix} \tag{2.0.9}$$

where

$$d = \|A-D\| = e\times\left(\frac{\sin\left(\beta-\sin^{-1}\left(\frac{a\sin\alpha}{e}\right)\right)}{\sin\gamma}\right) \tag{2.0.10}$$

$$e = \|C-A\| = \sqrt{a^2+b^2-2ab\cos\alpha} \tag{2.0.11}$$

Proof. Let,

$$\angle ACB = \beta_1 \tag{2.0.12}$$
$$\angle ACD = \beta_2 \tag{2.0.13}$$
$$\implies \beta_1+\beta_2 = \beta \tag{2.0.14}$$

Now in △ ABC applying cosine rule gives,

$$e = \sqrt{a^2+b^2-2ab\cos\alpha} \tag{2.0.15}$$

and in △ ABC applying sine rule gives,

$$\frac{\sin\angle ACB}{AB} = \frac{\sin B}{AC} \tag{2.0.16}$$
$$\implies \frac{\sin\beta_1}{a} = \frac{\sin\alpha}{e} \tag{2.0.17}$$
$$\implies \beta_1 = \sin^{-1}\left(\frac{a\sin\alpha}{e}\right) \tag{2.0.18}$$

and in △ ACD applying sine rule gives,

$$\frac{\sin\angle ACD}{AD} = \frac{\sin D}{AC} \tag{2.0.19}$$
$$\implies \frac{\sin\beta_2}{d} = \frac{\sin\gamma}{e} \tag{2.0.20}$$
$$\implies d = e\times\left(\frac{\sin\left(\beta-\sin^{-1}\left(\frac{a\sin\alpha}{e}\right)\right)}{\sin\gamma}\right) \tag{2.0.21}$$

□

Given,

$$\angle P = 90^\circ = \theta \tag{2.0.22}$$
$$\angle A = 110^\circ = \beta \tag{2.0.23}$$
$$\angle N = 85^\circ = \gamma \tag{2.0.24}$$
$$\implies \angle L = 75^\circ = \alpha \tag{2.0.25}$$
$$\|\mathbf{L}-\mathbf{P}\| = 4 = a \tag{2.0.26}$$
$$\|\mathbf{A}-\mathbf{L}\| = 6.5 = b \tag{2.0.27}$$
$$\mathbf{P} = \begin{pmatrix}0\\0\end{pmatrix} \tag{2.0.28}$$
$$\mathbf{L} = \begin{pmatrix}4\\0\end{pmatrix} \tag{2.0.29}$$

Let,

$$\theta = \angle L \tag{2.0.30}$$
$$\|\mathbf{A} - \mathbf{N}\| = c \tag{2.0.31}$$
$$\|\mathbf{N} - \mathbf{P}\| = d \tag{2.0.32}$$
$$\|\mathbf{A} - \mathbf{P}\| = e \tag{2.0.33}$$

We know that,

$$d = e \times \left(\frac{\sin\left(\beta - \sin^{-1}\left(\frac{a\sin\alpha}{e}\right)\right)}{\sin\gamma} \right) \tag{2.0.34}$$
$$e = \sqrt{a^2 + b^2 - 2ab\cos\alpha} \tag{2.0.35}$$
$$\implies e = 6.7 \tag{2.0.36}$$

using (2.0.36) in (2.0.34) we get

$$d = 6.49 \tag{2.0.37}$$

then for $\mathbf{A}$ we have,

$$\mathbf{A} = \mathbf{L} + b\begin{pmatrix}\cos(180-\alpha)\\ \sin(180-\alpha)\end{pmatrix} \tag{2.0.38}$$
$$\implies \mathbf{A} = \begin{pmatrix}4\\0\end{pmatrix} + 6.5\begin{pmatrix}\cos 105\\ \sin 105\end{pmatrix} \tag{2.0.39}$$
$$\implies \mathbf{A} = \begin{pmatrix}2.318\\6.279\end{pmatrix} \tag{2.0.40}$$

and for $\mathbf{N}$ we have,

$$\mathbf{N} = d\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix} \tag{2.0.41}$$
$$\implies \mathbf{N} = \begin{pmatrix}0\\6.49\end{pmatrix} \tag{2.0.42}$$

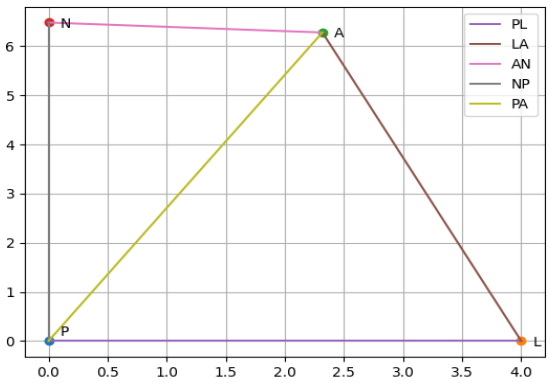

Fig. 0: Quadrilateral PLAN