

GATE ASSIGNMENT 1

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Question

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The exponential Fourier series representation of a continuous-time periodic signal $x(t)$ is defined as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{ik\omega_0 t} \quad (1)$$

where ω_0 is the fundamental angular frequency of $x(t)$ and the coefficients of the series are a_k . The following information is given about $x(t)$ and a_k .

- I $x(t)$ is real and even, having a fundamental period of 6
- II The average value of $x(t)$ is 2
- III $a_k = \begin{cases} k & 1 \leq k \leq 3 \\ 0 & k > 3 \end{cases}$

The average power of the signal $x(t)$ (rounded off to one decimal place) is

Parseval's Theorem

Theorem (Parseval's power theorem)

If a exponential Fourier series representation of a continuous-time periodic signal $x(t)$ is defined by

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \quad (2)$$

where C_n is given by

$$C_n = \frac{1}{T} \int_0^T x(t) e^{-jn\omega_0 t} dt \quad (3)$$

then the average power of signal $x(t)$ is given by

$$\frac{1}{T} \int_0^T |x(t)|^2 dt = \sum_{n=-\infty}^{\infty} |C_n|^2 \quad (4)$$

Proof.

Given,

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{in\omega_0 t} \quad (5)$$

Its complex conjugate can be written as

$$x(t)^* = \sum_{n=-\infty}^{\infty} C_n^* e^{-in\omega_0 t} \quad (6)$$

Now we know that the average power of signal $x(t)$ is given by

$$P_{x(t)} = \frac{1}{T} \int_0^T |x(t)|^2 dt \quad (7)$$

$$\Rightarrow P_{x(t)} = \frac{1}{T} \int_0^T x(t)x(t)^* dt \quad (8)$$



Proof Contd..

$$\Rightarrow P_{x(t)} = \frac{1}{T} \int_0^T x(t) \left(\sum_{n=-\infty}^{\infty} C_n^* e^{-jn\omega_0 t} \right) dt \quad (9)$$

$$\Rightarrow P_{x(t)} = \sum_{n=-\infty}^{\infty} C_n^* \left(\frac{1}{T} \int_0^T x(t) e^{-jn\omega_0 t} dt \right) \quad (10)$$

$$\Rightarrow P_{x(t)} = \sum_{n=-\infty}^{\infty} C_n^* C_n \quad (11)$$

$$\Rightarrow P_{x(t)} = \sum_{n=-\infty}^{\infty} |C_n|^2 \quad (12)$$

Solution

The given signal is defined as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{ik\omega_0 t} \quad (13)$$

where a_k is Fourier Series Coefficient. Given that the $x(t)$ is real and even.

So the spectrum is also real and even. $a_k = \begin{cases} k & 1 \leq k \leq 3 \\ 0 & k > 3 \end{cases}$

\therefore Average power is :

$$\frac{1}{T} \int_0^T |x(t)|^2 dt = \sum_{k=-3}^3 |a_k|^2 \quad (14)$$

$$P_{avg} = 9 + 4 + 1 + 4 + 1 + 4 + 9 \quad (15)$$

$$P_{avg} = 32 \quad (16)$$

Solution contd..

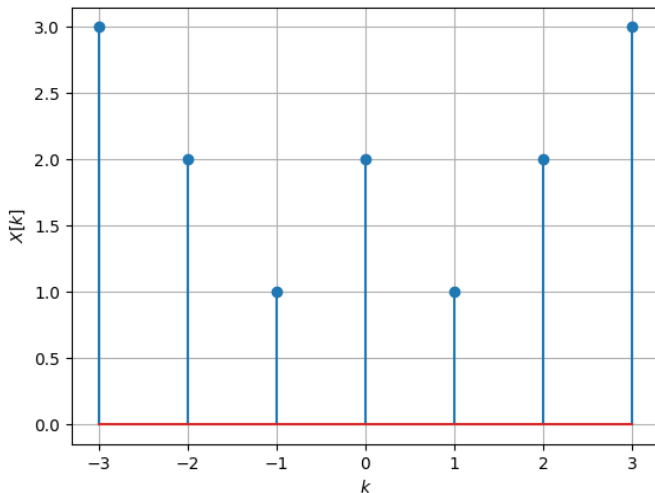


Figure: Plot of $X[k]$