GATE ASSIGNMENT 1

Ananthoju Pranav Sai
AI20BTECH11004

Download all python codes from

```
https://github.com/Ananthoju-Pranav-Sai/EE3900/
    blob/main/Gate_Assignment-1/codes/
    Gate_Assignment_1.py
```

and latex-tikz codes from

```
https://github.com/Ananthoju-Pranav-Sai/EE3900/
    tree/main/Gate_Assignment-1/
    Gate_Assignment_1.tex
```

1 GATE EC 2021 Q.39

The exponential Fourier series representation of a continuous-time periodic signal x(t) is defined as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{ik\omega_0 t} \tag{1.0.1}$$

where ω_0 is the fundamental angular frequency of $x(t)$ and the coefficients of the series are a_k. The following information is given about $x(t)$ and a_k.

I $x(t)$ is real and even, having a fundamental period of 6

II The average value of $x(t)$ is 2

III $a_k = \begin{cases} k & 1 \le x \le 3 \\ 0 & k > 3 \end{cases}$

The average power of the signal $x(t)$ (rounded off to one decimal place) is

2 Solution

Theorem 2.1 (Parseval's power theorem)**.** *If a exponential Fourier series representation of a continuous-time periodic signal x(t) is defined by*

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{in\omega_0 t} \tag{2.0.1}$$

where C_n is given by

$$C_n = \frac{1}{T}\int_0^T x(t) e^{-jn\omega_0 t}\, dt \tag{2.0.2}$$

then the average power of signal $x(t)$ is given by

$$\frac{1}{T}\int_0^T |x(t)|^2\, dt = \sum_{n=-\infty}^{\infty} |C_n|^2 \tag{2.0.3}$$

Proof. Given,

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{in\omega_0 t} \tag{2.0.4}$$

Its complex conjugate can be written as

$$x(t)^* = \sum_{n=-\infty}^{\infty} C_n^* e^{-in\omega_0 t} \tag{2.0.5}$$

Now we know that the average power of signal $x(t)$ is given by

$$P_{x(t)} = \frac{1}{T}\int_0^T |x(t)|^2\, dt \tag{2.0.6}$$

$$\implies P_{x(t)} = \frac{1}{T}\int_0^T x(t)x(t)^*\, dt \tag{2.0.7}$$

$$\implies P_{x(t)} = \frac{1}{T}\int_0^T x(t)\left(\sum_{n=-\infty}^{\infty} C_n^* e^{-in\omega_0 t}\right) dt \tag{2.0.8}$$

$$\implies P_{x(t)} = \sum_{n=-\infty}^{\infty} C_n^* \left(\frac{1}{T}\int_0^T x(t) e^{-jn\omega_0 t}\, dt\right) \tag{2.0.9}$$

$$\implies P_{x(t)} = \sum_{n=-\infty}^{\infty} C_n^* C_n \tag{2.0.10}$$

$$\implies P_{x(t)} = \sum_{n=-\infty}^{\infty} |C_n|^2 \tag{2.0.11}$$

□

The given signal is defined as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{ik\omega_0 t} \tag{2.0.12}$$

where a_k is Fourier Series Coefficient. Given that the $x(t)$ is real and even. So the spectrum is also real and even. $a_k = \begin{cases} k & 1 \le x \le 3 \\ 0 & k > 3 \end{cases}$

∴ Average power is :

$$\frac{1}{T}\int_0^T |x(t)|^2\, dt = \sum_{k=-3}^{3} |a_k|^2 \tag{2.0.13}$$

$$P_{avg} = 9 + 4 + 1 + 4 + 1 + 4 + 9 \tag{2.0.14}$$

$$P_{avg} = 32 \tag{2.0.15}$$

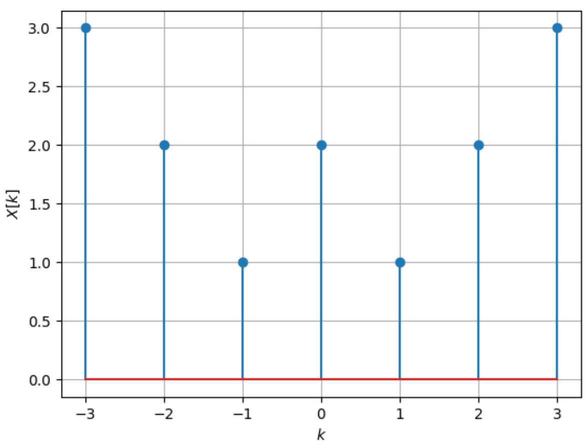

Fig. 3: Plot of $X[k]$

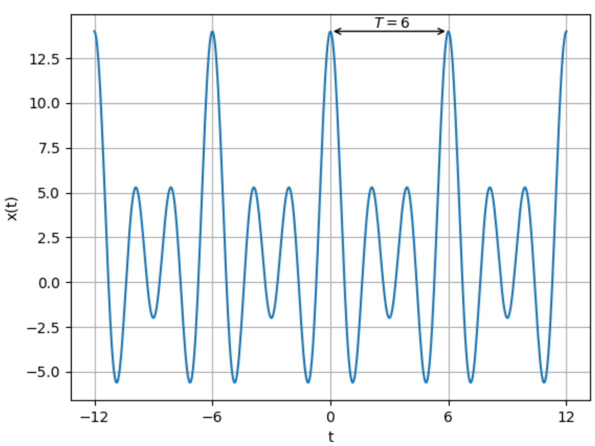

Fig. 3: Plot of $x(t)$