

# GATE ASSIGNMENT 4

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AI20BTECH11004

Download all python codes from

[https://github.com/Ananthoju-Pranav-Sai/EE3900/blob/main/Gate\\_Assignment\\_4/codes](https://github.com/Ananthoju-Pranav-Sai/EE3900/blob/main/Gate_Assignment_4/codes)

and latex-tikz codes from

[https://github.com/Ananthoju-Pranav-Sai/EE3900/tree/main/Gate\\_Assignment\\_4/Gate\\_Assignment\\_4.tex](https://github.com/Ananthoju-Pranav-Sai/EE3900/tree/main/Gate_Assignment_4/Gate_Assignment_4.tex)

## 1 GATE EC 2000 Q.2.31

Let  $u(t)$  be the step function. Plot the wave form corresponding to the convolution of  $u(t) - u(t - 1)$  with  $u(t) - u(t - 1)$ .

### 2 SOLUTION

We define unit step function as follows

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases} \quad (2.0.1)$$

Now let  $f(t) = u(t) - u(t - 1)$  and  $g(t) = u(t) - u(t - 2)$  then,

$$f(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 \leq t < 1 \\ 0 & t \geq 1 \end{cases} \quad (2.0.2)$$

$$\Rightarrow f(t) = \text{rect}\left(t - \frac{1}{2}\right) \quad (2.0.3)$$

$$g(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 \leq t < 2 \\ 0 & t \geq 2 \end{cases} \quad (2.0.4)$$

$$\Rightarrow g(t) = \text{rect}\left(\frac{t-1}{2}\right) \quad (2.0.5)$$

Let  $y(t)$  be convolution of  $f(t)$  and  $g(t)$  So we have,

$$y(t) = \int_{-\infty}^{\infty} f(\tau)g(t - \tau) d\tau \quad (2.0.6)$$

$$\Rightarrow y(t) = \int_0^1 g(t - \tau) d\tau \quad (2.0.7)$$

$$(2.0.8)$$

$$y(t) = \begin{cases} 0 & t < 0 \\ \int_0^t 1 d\tau & 0 \leq t < 1 \\ \int_0^1 1 d\tau & 1 \leq t < 2 \\ \int_{t-2}^1 1 d\tau & 2 \leq t < 3 \\ 0 & t \geq 3 \end{cases} \quad (2.0.9)$$

So we get  $y(t)$  as follows

$$y(t) = \begin{cases} 0 & t < 0 \\ t & 0 \leq t < 1 \\ 1 & 1 \leq t < 2 \\ 3 - t & 2 \leq t < 3 \\ 0 & t \geq 3 \end{cases} \quad (2.0.10)$$

Fourier Transform of the output

$$Y(\omega) = F(\omega)G(\omega) \quad (2.0.11)$$

We know that the fourier transform

$$\text{rect}\left(\frac{t}{\tau}\right) \Longleftrightarrow \tau \text{sinc}\left(\frac{\omega\tau}{2}\right) \quad (2.0.12)$$

and using the properties of fourier transform

$$x(t \pm t_0) \Longleftrightarrow X(\omega)e^{\pm i\omega t_0} \quad (2.0.13)$$

$$x(\alpha t) \Longleftrightarrow \frac{1}{|\alpha|} X\left(\frac{\omega}{\alpha}\right) \quad (2.0.14)$$

We get

$$F(\omega) = \text{sinc}\left(\frac{\omega}{2}\right)e^{(-\frac{i\omega}{2})} \quad (2.0.15)$$

$$G(\omega) = 2\text{sinc}(\omega)e^{-i\omega} \quad (2.0.16)$$

$$\Rightarrow Y(\omega) = 2\text{sinc}(\omega) \text{sinc}\left(\frac{\omega}{2}\right)e^{(-\frac{3i\omega}{2})} \quad (2.0.17)$$

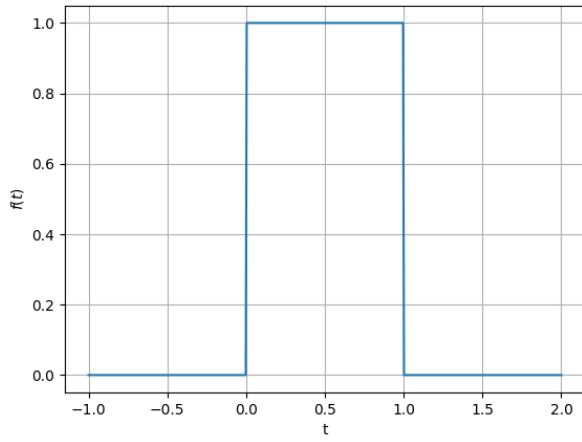


Fig. 0: Plot of  $f(t)$

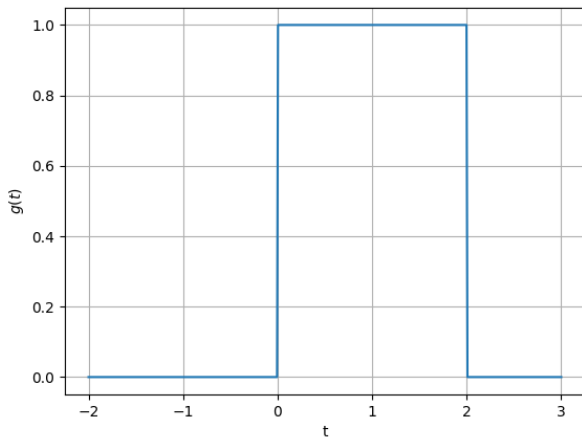


Fig. 0: Plot of  $g(t)$

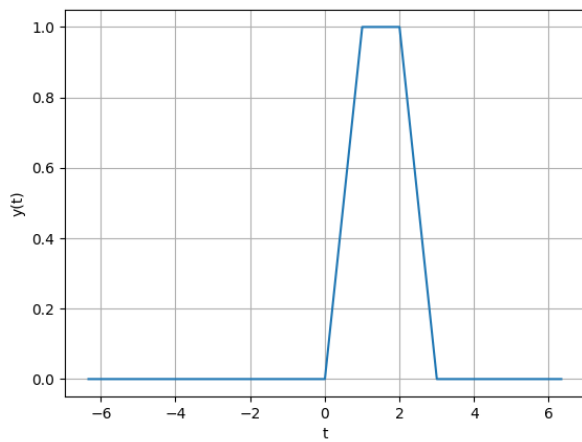


Fig. 0: Simulated plot of output signal  $y(t)$