QUIZ 1

Ananthoju Pranav Sai
AI20BTECH11004

Download all python codes from

https://github.com/Ananthoju−Pranav−Sai/EE3900/blob/main/Quiz_1/codes

and latex-tikz codes from

https://github.com/Ananthoju−Pranav−Sai/EE3900/tree/main/Quiz_1/Quiz_1.tex

1 Discrete Time Signal Processing Q 2.32

Consider an LTI system with frequency response

$$H(e^{jw}) = e^{-j(\omega-\frac{\pi}{4})}\left(\frac{1+e^{-j2\omega}+4e^{-j4\omega}}{1+\frac{1}{2}e^{-j2\omega}}\right) \tag{1.0.1}$$

where $-\pi < \omega \leq \pi$
Determine the output $y[n]$ for all n if the input for all n is

$$x[n] = cos\left(\frac{\pi n}{2}\right) \tag{1.0.2}$$

2 Solution

Given,

$$H(e^{j\omega}) = e^{\left(\frac{j\pi}{4}\right)}e^{-j\omega}\left(\frac{1+e^{-j2\omega}+4e^{-j4\omega}}{1+\frac{1}{2}e^{-j2\omega}}\right) \tag{2.0.1}$$

$$x[n] = \frac{e^{\frac{j\pi n}{2}}+e^{\frac{-j\pi n}{2}}}{2} \tag{2.0.2}$$

Taking fourier transform of $x[n]$

$$X(e^{j\omega}) = \frac{\delta(\omega-\pi/2)+\delta(\omega+\pi/2)}{2} \tag{2.0.3}$$

$$\tag{2.0.4}$$

Now we know that

$$y[n] = x[n] * h[n] \tag{2.0.5}$$

and by taking fourier transforms we get

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) \tag{2.0.6}$$

$$Y(e^{j\omega}) = \frac{H(e^{\frac{j\pi}{2}})\delta(\omega-\pi/2)+H(e^{\frac{-j\pi}{2}})\delta(\omega+\pi/2)}{2} \tag{2.0.7}$$

Because for any ω other than $\pm\pi/2$, $X(e^{j\omega}) = 0$

$$Y(e^{j\omega}) = \frac{8e^{-j\pi/4}\delta(\omega-\pi/2)+8e^{j3\pi/4}\delta(\omega+\pi/2)}{2} \tag{2.0.8}$$

Taking inverse fourier transform of $Y(e^{j\omega})$

$$y[n] = 4e^{-j\pi/4}e^{j\pi n/2}+4e^{j3\pi/4}e^{-j\pi n/2} \tag{2.0.9}$$

$$\implies y[n] = 4e^{j\pi/4}\left(e^{j\pi(n-1)/2}+e^{-j\pi(n-1)/2}\right) \tag{2.0.10}$$

$$\therefore y[n] = 8e^{j\pi/4}cos\left(\frac{\pi(n-1)}{2}\right) \tag{2.0.11}$$

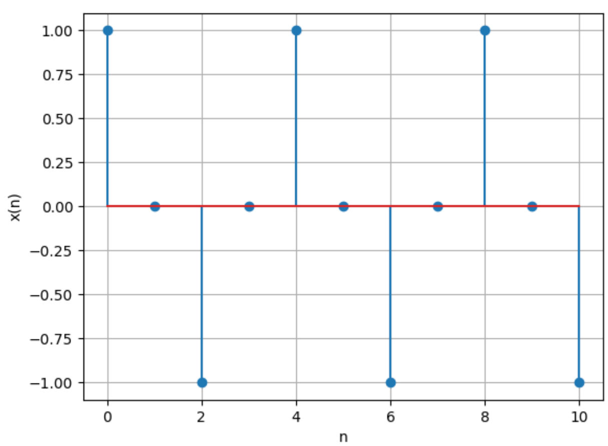

Fig. 0: Input signal x[n]

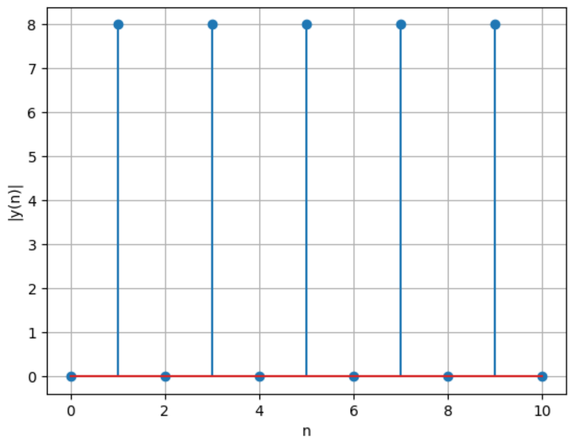

Fig. 0: Amplitude of y[n]

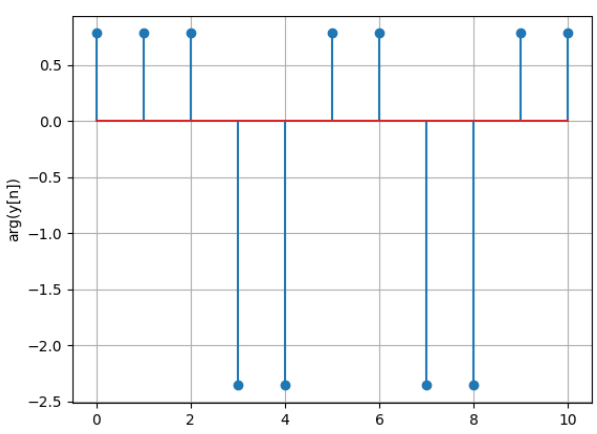

Fig. 0: Phase of y[n]