ASSIGNMENT 5

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Download all python codes from

https://github.com/Ananthoju−Pranav−Sai/EE3900/blob/main/Assignment−5/codes/Assignment−5.py

and latex-tikz codes from

https://github.com/Ananthoju−Pranav−Sai/EE3900/tree/main/Assignment−5/Assignment−5.tex

1 Quadratic Forms Q.2.62

Find a point on the curve $y = (x-2)^2$ at which the tangent is parallel to the chord joining the points $\begin{pmatrix}2\\0\end{pmatrix}$ and $\begin{pmatrix}4\\4\end{pmatrix}$.

2 Solution

Equation of the given conic in vector form is

$$\mathbf{x}^T\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}\mathbf{x} + 2\begin{pmatrix}-2 & \frac{-1}{2}\end{pmatrix}\mathbf{x} + 4 = 0 \tag{2.0.1}$$

Therefore we have

$$\mathbf{V} = \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix} \tag{2.0.2}$$

$$\mathbf{u} = \begin{pmatrix}-2\\\frac{-1}{2}\end{pmatrix} \tag{2.0.3}$$

$$f = 4 \tag{2.0.4}$$

Given the tangent is parallel to the chord joining the points $\begin{pmatrix}2\\0\end{pmatrix}$ and $\begin{pmatrix}4\\4\end{pmatrix}$. So,

$$\mathbf{m} = \begin{pmatrix}4\\4\end{pmatrix} - \begin{pmatrix}2\\0\end{pmatrix} \tag{2.0.5}$$

$$\mathbf{m} = \begin{pmatrix}2\\4\end{pmatrix} \tag{2.0.6}$$

$$\implies \mathbf{n} = \begin{pmatrix}-4\\2\end{pmatrix} \tag{2.0.7}$$

Lemma 2.1. *If* $\mathbf{V}$ *is not invertible, given the normal vector* $\mathbf{n}$*, the point of contact is given by the matrix equation*

$$\begin{pmatrix}\mathbf{u}^T + \kappa\mathbf{n}^T\\\mathbf{V}\end{pmatrix}\mathbf{q} = \begin{pmatrix}-f\\\kappa\mathbf{n} - \mathbf{u}\end{pmatrix} \tag{2.0.8}$$

where

$$\kappa = \frac{\mathbf{p_1}^T u}{\mathbf{p_1}^T n} \tag{2.0.9}$$

$$\mathbf{V}\mathbf{p_1} = 0 \tag{2.0.10}$$

So let $\mathbf{p_1} = \begin{pmatrix}0\\2\end{pmatrix}$ as it satisfies $\mathbf{V}\mathbf{p_1} = 0$ then

$$\kappa = \frac{\mathbf{p_1}^T u}{\mathbf{p_1}^T n} \tag{2.0.11}$$

$$\kappa = \frac{\begin{pmatrix}0 & 2\end{pmatrix}\begin{pmatrix}-2\\\frac{-1}{2}\end{pmatrix}}{\begin{pmatrix}0 & 2\end{pmatrix}\begin{pmatrix}-4\\2\end{pmatrix}} \tag{2.0.12}$$

$$\kappa = \frac{-1}{4} \tag{2.0.13}$$

Now the matrix equation

$$\begin{pmatrix}\mathbf{u}^T + \kappa\mathbf{n}^T\\\mathbf{V}\end{pmatrix}\mathbf{q} = \begin{pmatrix}-f\\\kappa\mathbf{n} - \mathbf{u}\end{pmatrix} \tag{2.0.14}$$

$$\begin{pmatrix}-1 & -1\\1 & 0\\0 & 0\end{pmatrix}\mathbf{q} = \begin{pmatrix}-4\\3\\0\end{pmatrix} \tag{2.0.15}$$

$$\mathbf{q} = \begin{pmatrix}3\\1\end{pmatrix} \tag{2.0.16}$$

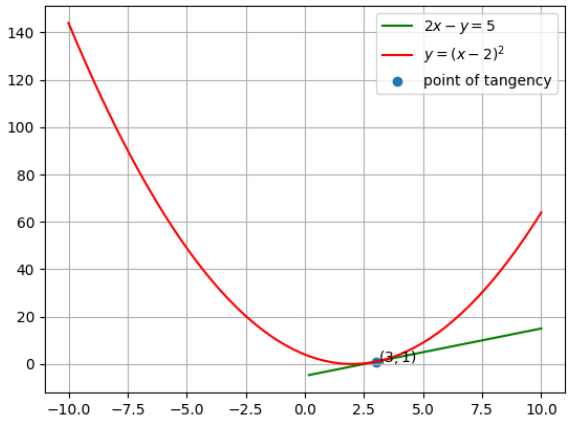

Fig. 0: Plot of the tangent and parabola