

GATE ASSIGNMENT 4

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AI20BTECH11004

Download all python codes from

https://github.com/Ananthoju-Pranav-Sai/EE3900/blob/main/Gate_Assignment_4/codes

and latex-tikz codes from

https://github.com/Ananthoju-Pranav-Sai/EE3900/tree/main/Gate_Assignment_4/Gate_Assignment_4.tex

1 GATE EC 2000 Q.2.31

Let $u(t)$ be the step function. Plot the wave form corresponding to the convolution of $u(t) - u(t - 1)$ with $u(t) - u(t - 2)$.

2 SOLUTION

We define unit step function as follows

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases} \quad (2.0.1)$$

Now let $f(t) = u(t) - u(t - 1)$ and $g(t) = u(t) - u(t - 2)$ then,

$$f(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 \leq t < 1 \\ 0 & t \geq 1 \end{cases} \quad (2.0.2)$$

$$\Rightarrow f(t) = \text{rect}\left(t - \frac{1}{2}\right) \quad (2.0.3)$$

$$g(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 \leq t < 2 \\ 0 & t \geq 2 \end{cases} \quad (2.0.4)$$

$$\Rightarrow g(t) = \text{rect}\left(\frac{t-1}{2}\right) \quad (2.0.5)$$

Let $y(t)$ be convolution of $f(t)$ and $g(t)$ So we have,

$$y(t) = \text{rect}\left(t - \frac{1}{2}\right) * \text{rect}\left(\frac{t-1}{2}\right) \quad (2.0.6)$$

$$y(t) = \text{rect}\left(t - \frac{1}{2}\right) * \left(\text{rect}\left(t - \frac{1}{2}\right) + \text{rect}\left(t - \frac{3}{2}\right)\right) \quad (2.0.7)$$

$$y(t) = \text{rect}\left(t - \frac{1}{2}\right) * \text{rect}\left(t - \frac{1}{2}\right) + \text{rect}\left(t - \frac{1}{2}\right) * \text{rect}\left(t - \frac{3}{2}\right)$$

We know that

$$\Delta(t) = \text{rect}(t) * \text{rect}(t) \quad (2.0.8)$$

So by using the following property of convolution

$$x_1(t) * x_2(t) = y(t) \quad (2.0.9)$$

$$x_1(t - t_0) * x_2(t - t_1) = y(t - t_0 - t_1) \quad (2.0.10)$$

we get

$$y(t) = \Delta\left(t - \frac{1}{2} - \frac{1}{2}\right) + \Delta\left(t - \frac{1}{2} - \frac{3}{2}\right) \quad (2.0.11)$$

$$\Rightarrow y(t) = \Delta(t - 1) + \Delta(t - 2) \quad (2.0.12)$$

Fourier Transform of the output

$$Y(\omega) = F(\omega)G(\omega) \quad (2.0.13)$$

We know that the fourier transform

$$\text{rect}\left(\frac{t}{\tau}\right) \Longleftrightarrow \tau \text{sinc}\left(\frac{\omega\tau}{2}\right) \quad (2.0.14)$$

and using the properties of fourier transform

$$x(t \pm t_0) \Longleftrightarrow X(\omega)e^{\pm i\omega t_0} \quad (2.0.15)$$

$$x(\alpha t) \Longleftrightarrow \frac{1}{|\alpha|} X\left(\frac{\omega}{\alpha}\right) \quad (2.0.16)$$

We get

$$F(\omega) = \text{sinc}\left(\frac{\omega}{2}\right)e^{(-\frac{i\omega}{2})} \quad (2.0.17)$$

$$G(\omega) = 2\text{sinc}(\omega)e^{-i\omega} \quad (2.0.18)$$

Using (2.0.13), (2.0.17) and (2.0.18)

$$\Rightarrow Y(\omega) = 2\text{sinc}(\omega) \text{sinc}\left(\frac{\omega}{2}\right)e^{(-\frac{3i\omega}{2})} \quad (2.0.19)$$

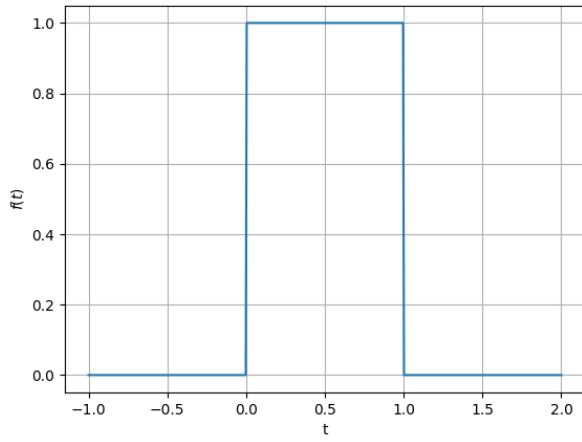


Fig. 0: Plot of $f(t)$

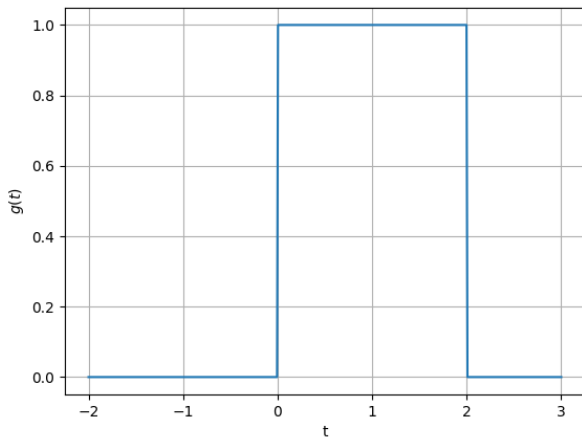


Fig. 0: Plot of $g(t)$

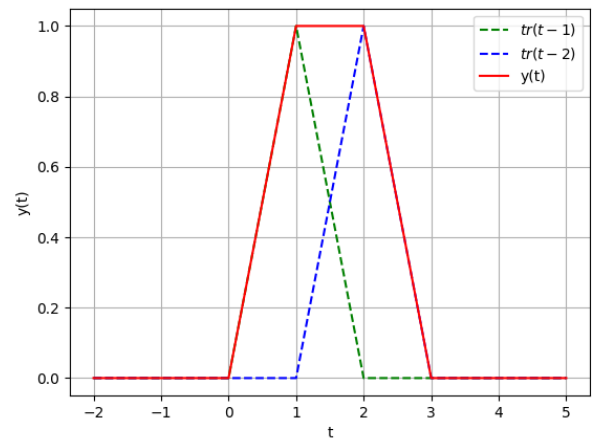


Fig. 0: $y(t)$ as sum of 2 shifted triangles

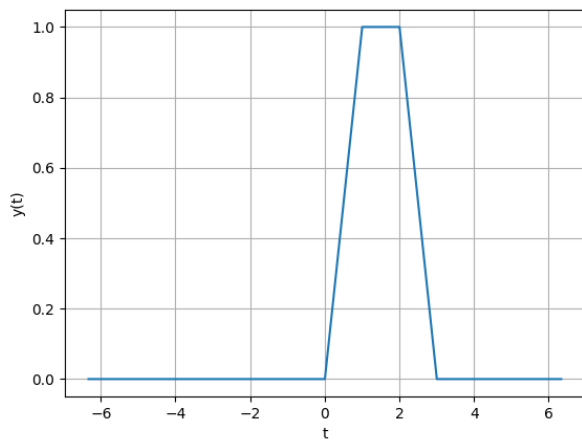


Fig. 0: Simulated plot of output signal $y(t)$