HOW TO FIND UNIT DIGITS
OF VARIOUS NUMBER FORMATS ?

① Find the unit digit of $(2153)^{167} \times (8267)^{153}$

a	1
b	3
c	7
d	9

$(3)^{167} \times (7)^{153}$ ↑ ↑   2, 3, 7, 8

⟹ next page.

eg: $1036 \times 5412 \times 513 \times 2036 = $ _____

$6 \times 2 \times 3 \times 6 = 36 = 6.$

12 ⤶ 6 ⤴

| 0 | = | 0 | ⎱ cyclicity |
| 1 | = | 1 | ⎰ |

2
3
4 ___Family✔___
5
6   ⟹ 0, 1, 5, 6 | 2, 3, 7, 8 | 4, 9
7
8   0, 1, 5, 6 | 4 | $4^{odd} = 4$
9                                      $4^{even} = 6$
                                       $9^{odd} = 9$
                                       $9^{even} = 1$

$2^1 = \boxed{2}$ $2^2 = 4$ $2^3 = 8$ $2^4 = 16$ $2^5 = 3\boxed{2}$
   ①   ②   ③   ④

① Find the unit digit of $(2153)^{167} \times (8267)^{153}$

$a = 1$
$b = 3$
$c = 7$
$\boxed{d = 9}$

$$(3)^{167} \times (7)^{153}$$

2, 3, 7, 8

$\Rightarrow (3)^3 \times (7)^1$

$\Rightarrow \underline{27} \times \underline{7}$

$\Rightarrow 4\,\boxed{9}$

$\Rightarrow \boxed{\text{unit digit}}$

$$\dfrac{\text{Power}}{4} \quad \begin{array}{l}\text{Remain} = 0\,④ \\ \text{Remain} \neq 0\end{array}$$

$$\begin{array}{r} 38 \\ 4\,\overline{)153} \\ 12 \\ \hline 33 \\ 32 \\ \hline 1 \quad \boxed{R=1} \end{array} \qquad \begin{array}{r} 4\,\overline{)167}\,(41 \\ 16 \\ \hline 7 \\ 4 \\ \hline 3 \quad \boxed{R=3} \end{array}$$

> * Suppose $\boxed{R=0}$ you should retain 4 in the place of power.

② Find the unit digit of $(432)^{412} \times (499)^{431}$

$\uparrow$ $\uparrow$ odd

$9 = 9$

$a = 2$
$\boxed{b = 4}$
$c = 6$
$d = 8$

$\Rightarrow (2)^{412} \times (9)^{431}$
$= (2)^4 \times 9$
$= \underline{16} \times 9$
$= 5\,\boxed{4}$

$$\begin{array}{cccc} 2 & 3 & 7 & 8 \end{array}$$
$$\dfrac{\text{Power}}{4} \quad R = ?$$

$$\begin{array}{r} 4\,\overline{)412}\,(13 \\ 4 \\ \hline 012 \\ 12 \\ \hline 00 \quad \boxed{R=0} \end{array}$$

③ Find the unit digit of $(217)^{413} \times (819)^{547} \times (414)^{624} \times (342)^8$

$a = 2$
$b = 4$
$c = 6$
$\boxed{d = 8}$

$= (7)^{413} \times (9)^{547} \times (4)^{624} \times (2)^8$
$= (7)^1 \times 9^{\text{odd}} \times (4)^4 \times (2)^4$
$= 7 \times 9 \times \underline{16} \times \underline{16}$
$= 8$

$$\begin{array}{r} 13 \\ 4\,\overline{)413} \\ 4\,3 \\ \hline 53 \\ 4 \\ \hline 4 \end{array} \qquad \boxed{R=1}$$

$$4\overline{)8}$$
$$\underline{8}$$
$$0$$

$$4\overline{)624}(156$$
$$\underline{4\downarrow}$$
$$22$$
$$\underline{20\downarrow}$$
$$24$$
$$\underline{24}$$
$$00$$

④ If $X = (164)^{169} + (333)^{377} - (727)^{726}$, then what is the unit digit of X

$a = 7$
$b = 9$
$\boxed{c = 8}$
$d = 5$

$X = (4)^{169} + (3)^{337} - (7)^{726}$
$= 4^{odd} + (3)^{1} - (7)^{2}$
$= 4 + 3 - 49$
$= 7 - 49$
$= -2 \;(+10)\; \text{carry.}$
$= \underline{\underline{8}}$

$$4\overline{)726}$$
$$\underline{4}$$
$$32$$
$$\underline{32\downarrow}$$
$$6$$
$$4$$
$$2 \quad \boxed{R=1}$$

$$181$$

$$4\overline{)337}(84$$
$$\underline{32\downarrow}$$
$$17$$
$$\underline{16}$$
$$\boxed{R=1}$$

⑤ Find the unit digit of $\left((137)^{13}\right)^{47}$

$a = 7$
$b = 9$
$c = 8$
$\boxed{d = 3}$

$= \left((137)^{13}\right)^{47} = (137)^{13 \times 47}$

$\qquad (a^m)^n = a^{m \times n}$

$= (7)^{\frac{13}{4} \times \frac{47}{4}}$

$= (7)^{(1) \times (3)}$

$= (7)$

$= 6\boxed{3}$

$$\begin{array}{c} 3 \\ 4\overline{)13} \\ \underline{12} \\ \boxed{R=1} \end{array} \qquad \begin{array}{c} 11 \\ 4\overline{)47} \\ \underline{44} \\ \boxed{R=03} \end{array}$$

⑥ If $N = 7^{95} - 3^{58}$, then the digit at the unit place of N is:

$$7 \times 7$$
$$\underline{49 \times 7}$$
$$6\underline{3}$$

$a = 3$
$\boxed{b = 4}$
$c = 6$
$d = 7$

$N = (7)^{95} - (3)^{58}$

$= (7)^{3} - (3)^{2}$

$= 63 - 9 \quad = 4$

$$\begin{array}{c} 14 \\ 4\overline{)58} \\ \underline{1} \\ 18 \\ \underline{16} \\ 2 \end{array} \qquad \begin{array}{c} 23 \\ 4\overline{)95} \\ \underline{8\downarrow} \\ 15 \\ \underline{12} \\ 3 \end{array}$$

⑦

Find the unit digit of the product of all the prime numbers

a = 0

b = 1

c = 2

d = 5

* The no, which can be divide by 1 and itself.

$\checkmark$ 2 $<$ 1, 2

$\checkmark$ 3 $<$ 1, 3

$\checkmark$ 5 $<$ 1, 5

1 $<$ neither prime, Nor composite

$\Rightarrow 2 \times 3 \times 5 \times 7 \times \cdots \cdots$

$\boxed{6}$ $3\boxed{0}$ $= 0$

composite number $\times$ 6 $<$ 1, 2, 3, 6 $\times$

②

Find the last digit of $(32)^{32 \times 32}$, then the digit at the unit place of N is:

a = 2

b = 4

$\boxed{c = 6}$

d = 8

$= \left((2)^{32} \right)^{32} = (2)^{\frac{32}{4} \times \frac{32}{4} \times \cdots \cdots \times \frac{32}{4}}$

$= (2)^4 = 1\boxed{6}$

$4)\overline{32}(8$
$\underline{32}$
$\overline{0\ 0}$
$\boxed{R = 4}$