Data & Analysis of Algorithm

1. Asymphotic notations are used to find the complexity of an algorithm when input is very large

* **Big O(O) :-** $f(n) = O g(n)$

  iff $f(n) \le c g(n)$

  $\forall$ $n \ge n_0$

  for some ~~the~~ constant $c > 0$

  $g(n)$ is "tight upper bound" of $f(n)$

* **Big omega(Ω) :-**

  $f(n) = \Omega(g(n))$

  iff $f(n) \ge c g(n)$

  $\forall$ $n \ge n_0$

  for some constant $c > 0$

  $g(n)$ is "tight lower bound" of $f(n)$

* **Theta(θ) :-**

  $f(n) = \theta(g(n))$

  iff $c_1 g(n) \le f(n) \le c_2 g(n)$

  $\forall$ $n \ge \max(n_1, n_2)$

  for some constant $c_1 > 0$ & $c_2 > 0$

  $g(n)$ is both "tight upper bound &

lower bound of $f(n)$"

Que 2 for ($i = 1$ to n), { $P = P \times 2i$ }

~~1+2, 4+8~~ 1, 2, 4, 8 ... n

let k^{th} term "

$n = 1 \cdot (2^{k-1})$

Taking log on both sides

$\log n = k-1 \log_2 2$

$k = 1 + \log_2 n$

$O(1 + \log n)$

Ans $O(\log n)$

Ques 3 $T(n) = 3T(n-1)$ — ①

$n = n-1$ in ①

$T(n-1) = 3T(n-2)$ — ②

put (2) in ①

$T(n) = 9T(n-2)$

$n = n-2$ in ①

$T(n-2) = 3T(n-3)$ — ③

$T(n) = 27\, T(n-3)$

$T(n) = 3^k T(n-k)$

$n - k = 0$

$n = k$

$T(n) = 3^n T(n-n)$

$= 3^n T(0)$
$= 3^n$

Ans $O(3^n)$

Ques4 $T(n) = 2T(n-1)$ — (1)

$n = n-1$ in eq (1)

$T(n-1) = 2T(n-2)$ — (2)

$T(n) = \cancel{n-2}\ 4T(n-2)$ — (3)

$n = n-2$ in (1)

$T(n-2) = 2T(n-3)$ — (4)

$T(n) = 8T(n-3)$

$T(n) = 2^K T(n-K)$

$n - K = 0$

$K = n$

$T(n) = 2^K T(n-n)$

$= 2^n T(0)$

$= 2^n$

Ans $O(2^n)$

Ques6

```
Void function (int n)
{
  int i, count = 0;
  for (i=1; i*i <= n; i++)
    count ++;
}
```

$O(1+\sqrt{n}+\sqrt{n}+\sqrt{n})$

$O(1+3\sqrt{n})$

$O(3\sqrt{n})$

$O(\sqrt{n})$

Ans $O(n^{1/2})$

Ques 7

```
void function(int n)
{
  int i, j, k, count=0;
  for (i=n/2; i<=n; i++)
   for (j=1; j<=n; j=j*2)
    for (k=1; k<=n; k=k*2)
      count++;
}
```

i	j	k
n/2	1	1, 2, 4, ..., n
n/2	log n	log n

$O(n/2 \times \log n \times \log n)$

$O(n(\log n)^2)$

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Ques 8

```
function (int n)
{
  if (n==1)
   return;
  for (i=1 to n)
  {
     for (j=1 to n)
     {
        printf("K");
     }
  }
  function (n-3);
}
```

n	i	j
n	1	1
	2	n
n-3	n	n
n-6		n

$1+4+7+\dots n$

$n = 1+3(k-1)$

$= 3k-2$

$k = n+2/3$

no. of terms

$$\frac{n+2}{6}\left[2+\left[\frac{n-1}{3}\right]\times 3\right]$$

$$\left[\frac{n+2}{6}\left[n+1\right]\right]\times n^2$$

$$O\left[\frac{(n^2+3n+2)}{6}\times n^2\right]$$

Ans $O[n^4]$

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Ques 9 void function (int n)

```
{
  for (i=1 to n)
  {
    for (j=1 ; j<=n; j=j+1)
      printf("*");   n^2
  }
}
```

$O(n+n^2+n^2+n^2)$

$O(3n^2+n)$

$O(n^2)$ Ans

Ques 10 As given n^k & c^n

relation b/w n^k & c^n is

$n^k = O(c^n)$

$n^k \le a(c^n)$

$\forall$ $n \ge n_0$ &

constant, $a > 0$

for $n_0 = 1$

$c = 2$

$\rightarrow 1^k \le a^{2 \cdot 1}$

$\rightarrow n_0 = 1$ & $c = 2$