THEORY OF AUTOMATA AND FORMAL LANGUAGES

BCSC0011

LECTURE: Normalization of CFG

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SIMPLIFICATION OF GRAMMAR

REDUCTION

1. ʌ-Production Elimination
2. Unit Production Elimination
3. Useless Symbol Elimination

NORMALIZATION

1. CNF
2. GNF

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NORMALIZATION of CFG

To provide fix number of terminals and non-terminals we use normalization.

Or

To provide fix format to particular grammar.

1. **Chomsky Normal Form (CNF)**
2. **Greibach Normal Form (GNF)**

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CHOMSKY NORMAL FORM (CNF)

A context-free grammar G is in Chomsky Normal Form if every production is of the form : $A \rightarrow a$

$A \rightarrow BC$

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REDUCTION TO CHOMSKY NORMAL FORM

Step 1: Elimination of null productions and unit productions.

Step 2: Elimination of terminals on R.H.S.

Step 3: Restricting the number of variables on R.H.S.

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REDUCTION TO CHOMSKY NORMAL FORM

Reduce the following grammar G to CNF. G is $S \rightarrow aAD$, $A \rightarrow aB \mid bAB$, $B \rightarrow b$, $D \rightarrow d$.

Solution

As there are no null productions or unit productions, we can proceed to step 2.

Step 2 Let $G_1 = (V'_N, \{a, b, d\}, P_1, S)$, where P_1 and V'_N are constructed as follows:

(i) $B \rightarrow b$, $D \rightarrow d$ are included in P_1.

(ii) $S \rightarrow aAD$ gives rise to $S \rightarrow C_aAD$ and $C_a \rightarrow a$.
$A \rightarrow aB$ gives rise to $A \rightarrow C_aB$.
$A \rightarrow bAB$ gives rise to $A \rightarrow C_bAB$ and $C_b \rightarrow b$.
$V'_N = \{S, A, B, D, C_a, C_b\}$.

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REDUCTION TO CHOMSKY NORMAL FORM

Solution

Step 3 P_1 consists of $S \to C_aAD$, $A \to C_aB \mid C_bAB$, $B \to b$, $D \to d$, $C_a \to a$, $C_b \to b$.

$A \to C_aB$, $B \to b$, $D \to d$, $C_a \to a$, $C_b \to b$ are added to P_2

$S \to C_aAD$ is replaced by $S \to C_aC_1$ and $C_1 \to AD$.

$A \to C_bAB$ is replaced by $A \to C_bC_2$ and $C_2 \to AB$.

Let

$$G_2 = (\{S, A, B, D, C_a, C_b, C_1, C_2\}, \{a, b, d\}, P_2, S)$$

where P_2 consists of $S \to C_aC_1$, $A \to C_aB \mid C_bC_2$, $C_1 \to AD$, $C_2 \to AB$, $B \to b$, $D \to d$, $C_a \to a$, $C_b \to b$. G_2 is in CNF and equivalent to G.

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REDUCTION TO CHOMSKY NORMAL FORM

Find a grammar in Chomsky normal form equivalent to $S \to aAbB$, $A \to aA \mid a$, $B \to bB \mid b$.

Solution

As there are no unit productions or null productions, we need not carry out step 1. We proceed to step 2.

Step 2 Let $G_1 = (V'_N \{a, b\}, P_1, S)$, where P_1 and V'_N are constructed as follows:

(i) $A \to a$, $B \to b$ are added to P_1.

(ii) $S \to aAbB$, $A \to aA$, $B \to bB$ yield $S \to C_aAC_bB$, $A \to C_aA$, $B \to C_bB$, $C_a \to a$, $C_b \to b$.

$$V'_N = \{S, A, B, C_a, C_b\}.$$

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REDUCTION TO CHOMSKY NORMAL FORM

Solution

Step 3 P_1 consists of $S \rightarrow C_aAC_bB$, $A \rightarrow C_aA$, $B \rightarrow C_bB$, $C_a \rightarrow a$, $C_b \rightarrow b$, $A \rightarrow a$, $B \rightarrow b$.

$$S \rightarrow C_aAC_bB \text{ is replaced by } S \rightarrow C_aC_1,\ C_1 \rightarrow AC_2,\ C_2 \rightarrow C_bB$$

The remaining productions in P_1 are added to P_2. Let

$$G_2 = (\{S, A, B, C_a, C_b, C_1, C_2\}, \{a, b\}, P_2, S),$$

where P_2 consists of $S \rightarrow C_aC_1$, $C_1 \rightarrow AC_2$, $C_2 \rightarrow C_bB$, $A \rightarrow C_aA$, $B \rightarrow C_bB$, $C_a \rightarrow a$, $C_b \rightarrow b$, $A \rightarrow a$, and $B \rightarrow b$.

G_2 is in CNF and equivalent to the given grammar.

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GREIBACH NORMAL FORM (GNF)

Definition 6.12 A context-free grammar is in Greibach normal form if every production is of the form $A \to a\alpha$, where $\alpha \in V_N^*$ and $a \in \Sigma(\alpha$ may be $\Lambda)$,

Non-Terminal → One Terminal * Any no. of Non-Terminal

S→aA
S→a

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Conversion into GNF using substitution

$$S \rightarrow aBc$$
$$B \rightarrow b$$

$\Rightarrow$

$$S \rightarrow aBC$$
$$B \rightarrow b$$
$$C \rightarrow c$$

$$S \rightarrow Bc$$
$$B \rightarrow b$$

$\Rightarrow$

$$S \rightarrow bC$$
$$B \rightarrow b$$
$$C \rightarrow c$$

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Conversion into GNF using substitution

S → AB
A → aA | bB | b
B → b

⇒

S → aAB | bBB | bB
A → aA | bB | b
B → b

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GREIBACH NORMAL FORM (GNF)

To convert CFG to GNF

1. The grammar must be in Chomsky Normal Form
2. There must be no Left-Recursive Rules

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LEFT RECURSION

Left Recursive Rules : **$A_k \rightarrow A_k\alpha \mid \beta$**

Language generated by **$A_k \rightarrow A_k\alpha \mid \beta$** ?

$$A_k \rightarrow A_k\alpha \rightarrow \beta\alpha$$
$$A_k \rightarrow A_k\alpha \rightarrow A_k\alpha\alpha \rightarrow \beta\alpha\alpha$$
$$A_k \rightarrow A_k\alpha \rightarrow A_k\alpha\alpha \rightarrow A_k\alpha\alpha\alpha \rightarrow \beta\alpha\alpha\alpha$$

such that β do not start with A_k

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Elimination of Left Recursion

The left recursion rule: $A_k \rightarrow A_k\alpha \mid \beta$ can be replaced by the following rules:

$$A_k \rightarrow \beta \mid \beta Z$$
$$Z \rightarrow \alpha Z \mid \alpha$$

With these two types of rules the left recursion can be eliminated.

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Multiple Left Recursion

$$A_k \rightarrow A_k\alpha_1 \mid A_k\alpha_2 \mid \ldots \mid A_k\alpha_r$$
$$A_k \rightarrow \beta_1 \mid \beta_2 \mid \ldots \mid \beta_s$$

- The set of A_k prod^{n}s in P_1 are

$$A_k \rightarrow \beta_1 \mid \beta_2 \mid \ldots \mid \beta_s$$

$$A_k \rightarrow \beta_1 Z_k \mid \beta_2 Z_k \mid \ldots \mid \beta_s Z_k$$

- The set of Z prod^{n}s in P_1 are

$$Z_k \rightarrow \alpha_1 Z_k \mid \alpha_2 Z_k \mid \ldots \mid \alpha_r Z_k$$

$$Z_k \rightarrow \alpha_1 \mid \alpha_2 \mid \ldots \mid \alpha_r$$

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LOOP

Convert to GNF

$A_1 \rightarrow A_2 A_3$

$A_2 \rightarrow A_3 A_3$

$A_3 \rightarrow a A_3 \mid a$

Using substitution, we can easily convert to GNF

$A_1 \rightarrow a A_3 A_3 A_3 \mid a A_3 A_3$

$A_2 \rightarrow a A_3 A_3 \mid a A_3$

$A_3 \rightarrow a A_3 \mid a$

Convert to GNF

$A_1 \rightarrow A_2 A_3$

$A_2 \rightarrow A_3 A_3$

$A_3 \rightarrow A_2 A_3 \mid a$

There is a LOOP

$A2 \rightarrow A_3 \rightarrow A_2$

Which gives rise to Left recursion when we do substitution

$A_2 \rightarrow A_2 A_3 A_3 \mid a A_3$

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GREIBACH NORMAL FORM (GNF)

Lemma 1:

Let G (V, T, P, S) be CFG

if

$A \rightarrow Ba$

$B \rightarrow \beta_1 \mid \beta_2 \mid \beta_3 \ldots\ldots\ldots\ldots\ldots \mid \beta_n$

then,

$A \rightarrow \beta_1 a \mid \beta_2 a \mid \ldots\ldots\ldots \mid \beta_n a$

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GREIBACH NORMAL FORM (GNF)

Example:

S → Aa

A → aA | bA | aAS | b

Apply Lemma 1.

Solution:

S → aAa | bAa | aASa | ba

A → aA | bA | aAS | b

[According to lemma 1]

Lemma 1:

Let G (V, T, P, S) be CFG

if

$A \rightarrow Ba$

$B \rightarrow \beta_1 \mid \beta_2 \mid \beta_3 \ldots\ldots\ldots\ldots\ldots \mid \beta_n$

then,

$A \rightarrow \beta_1 a \mid \beta_2 a \mid \ldots\ldots\ldots \mid \beta_n a$

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GREIBACH NORMAL FORM (GNF)

Lemma 2: (Elimination of Left Recursion)

Let G (V, T, P, S) be CFG
if

$$A \rightarrow Aa_1 \mid Aa_2 \mid Aa_3 \ldots\ldots \mid Aa_n \mid \beta_1 \mid \beta_2 \mid \beta_3 \ldots\ldots\ldots\ldots \mid \beta_n$$

such that β_i do not start with A then equivalent grammar in GNF form,

$$A \rightarrow \beta_1 \mid \beta_2 \mid \beta_3 \ldots\ldots\ldots\ldots \mid \beta_n$$
$$A \rightarrow \beta_1 Z \mid \beta_2 Z \ldots\ldots\ldots\ldots \mid \beta_n Z$$
$$Z \rightarrow a_1 \mid a_2 \mid a_3 \ldots\ldots\ldots \mid a_n$$
$$Z \rightarrow a_1 Z \mid a_2 Z \mid a_3 Z \ldots\ldots\ldots \mid a_n Z$$

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Thank You