Discrete Mathematics
BCSC 0010

Module 2
Normal Forms

Normal forms

- If the number of variables , involved in a given statement are more, then the construction of truth tables may not be practical, therefore, we consider other method known as reduction to **normal form.**

- In this method, we use the word
- "product" in place of "**conjunction**"
- "sum" in place of "**disjunction**"

Some Basic terms Related to Normal Form

- **Elementary Product:** A product of the variables and their negations is called an elementary product.
- Eg: Let p and q be any two variables, then p, q, $p \wedge q$, $\sim p \wedge \sim q$, $p \wedge \sim q$, $\sim p \wedge q$..... etc. are elementary products.

- **Elementary Sum:** A Sum of the variables and their negations is called an elementary sum.
- Eg: p, q, $p \vee q$, $p \vee \sim q$, $\sim p \vee \sim q$, $\sim p \vee q$..... etc. are elementary sums.

Types of Normal Form

- Disjunctive Normal Form(DNF)
- Conjunctive Normal Form(CNF)

Disjunctive Normal Form(DNF)

- A statement which consists of a sum of elementary products of propositional variables and is equivalent to the given compound statement, is called a disjunctive normal form of the given statement.

- This form is not unique for the given statement.

Method to obtain disjunctive normal form

- **Step1.** replace → and ↔ by Λ, V, and ~ in the statement.

- **Step 2.** Manipulate to get an equivalent form which is the sum of elementary product terms.

Example

- Obtain DNF of (p□q)^~q

- **Solution:**
- (p□q)^~q
- (~pvq)^~q   By logical equivalence
- (~p^~q)v(q^~q)   By Distributive law

- This is the required DNF

Conjunctive Normal Form(CNF)

- A statement which consists of a product of elementary sums of propositional variables and is equivalent to the given compound statement, is called a Conjunctive normal form of the given statement.

- This form is not unique for the given statement.

Method to obtain conjunctive normal form

- **Step1.** replace → and ↔ by ∧, ∨, and ~ in the statement.

- **Step 2.** Manipulate to get an equivalent form which is the product of elementary sum terms.

Example

- Obtain CNF of (p□q)^~q

- **Solution:**
- (p□q)^~q
- (~pvq)^~q By logical equivalence

- This is the required CNF

Example

- Obtain DNF of pv(~p□(qv(q□~r)))

- pv(~p□(qv(q□~r)))
- pv(~p□(qv(~q v ~r)))   By logical equivalence
- pv (~p□(qv~q)v~r)))   Rearranged
- p v p v q v ~q v ~r   By logical equivalence

This is the required DNF

Minterms

- Let p and q be two propositional variables.
- All possible formulas which consist of product of p or its negation and product of q or its negation, but should not contain both the variable and its negation in any one of the formula are called minterms of p and q.
- Eg: For two variables p and q, there are $2^2 = 4$ minterms, namely,

$p \wedge q$, $p \wedge \sim q$, $\sim p \wedge q$, $\sim p \wedge \sim q$

- **If there are n variables then number of minterms = 2^n**

Maxterms

- Let p and q be two propositional variables.

- All possible formulas which consist of sum of p or its negation and sum of q or its negation, but should not contain both the variable and its negation in any one of the formula are called maxterm of p and q.

- Eg: For two variables p and q, there are $2^2 = 4$ maxterms, namely, $p \vee q$, $p \vee \sim q$, $\sim p \vee q$, $\sim p \vee \sim q$.

- **If there are n variables then number of maxterms = 2^n**

Principal disjunctive normal form(PDNF)

- For a given formula, an equivalent formula consisting of **disjunction of minterms** only is known as its Principal disjunctive normal form (PDNF) or Sum of products canonical form.

- Each minterm has truth value T for exactly one combination of truth values

- A formula which is tautology will have all **minterms**

- A formula which is contradiction will have no **minterms**

Method to find PDNF

- 1. Construct truth table for the given formula.
- 2. Identify the rows in which the formula has true – **as truth value.**
- 3. Construct minterms from each such rows by taking
- (i) the variable with true – as variable itself and
- (ii) the variable with false – as negated variable.
- 4. **Sum of these minterms will be the required PDNF.**

Example

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

PDNF for p□q will be (pˆq)v(~pˆq)v(~pˆ~q)
PDNF for p ↔q will be (pˆq)v(~pˆ~q)

Obtain PDNF of (p^q)v(~p^r)v(q^r)

p	q	r	~p	p^q	~p^r	q^r	(p^q)v(~p^r)	(p^q)v(~p^r)v(q^r)
T	T	T	F	T	F	T	T	T
T	T	F	F	T	F	F	T	T
T	F	T	F	F	F	F	F	F
T	F	F	F	F	F	F	F	F
F	T	T	T	F	T	T	T	T
F	T	F	T	F	F	F	F	F
F	F	T	T	F	T	F	T	T
F	F	F	T	F	F	F	F	F

PDNF: (p ^q^r)v(p^q^~r)v(~p^q^r)v(~p^~q^r)

Principal conjunctive normal form(PCNF)

- For a given formula, an equivalent formula consisting of **conjunction of maxterms** only is known as its Principal disjunctive normal form(PCNF) or product of sums canonical form.

- Each maxterm has truth value F for exactly one combination of truth values

- A formula which is tautology will have no **maxterms**

- A formula which is contradiction will have all **maxterms**

Method to find PCNF

- 1. Construct truth table for the given formula.
- 2. Identify the rows in which the formula has **false – as truth value.**
- 3. Construct maxterm from each such row by taking
- (i) the variable with true -as negated variable and
- (ii) the variable with false- as the variable itself.
- 4. **Product of these maxterms will be the required PCNF.**

Example

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

PCNF for p□q will be (~pvq)

PCNF for p ↔q will be (~p vq)^(p v ~q)

Obtain PCNF of (p^q)v(~p^r)v(q^r)

p	q	r	~p	p^q	~p^r	q^r	(p^q)v(~p^r)	(p^q)v(~p^r)v(q^r)
T	T	T	F	T	F	T	T	T
T	T	F	F	T	F	F	T	T
T	F	T	F	F	F	F	F	F
T	F	F	F	F	F	F	F	F
F	T	T	T	F	T	T	T	T
F	T	F	T	F	F	F	F	F
F	F	T	T	F	T	F	T	T
F	F	F	T	F	F	F	F	F

PCNF: (~pvqv~r)^(~pvqvr)^(pv~qvr)^(pvqvr)

Example

- Obtain PCNF of (pvq)^~p□~q
- Obtain CNF of ~(pvq) $\leftrightarrow$ (p$\wedge$q)
- Obtain DNF of ~(pvq) $\leftrightarrow$ (p$\wedge$q)

Next Topic

- **Predicate and Quantifiers**