

Discrete Mathematics BCSC 0010

Module 2 Propositional logic



Introduction

•A *proposition* is a declarative statement which is either true or false, but not both.

Examples



•All the following declarative sentences are propositions.

- 1. Delhi is the capital of India.
- 2. Toronto is the capital of Canada.
- 3.1 + 1 = 2.
- 4.2+2=3.

• Propositions 1 and 3 are true, whereas 2 and 4 are false



Consider the following sentences

- 1. What time is it?
- 2. Read this carefully.

$$3. x + 1 = 2.$$

4.
$$x + y = z$$
.

- Sentences 1 and 2 are not propositions because they are not declarative sentences.
- Sentences 3 and 4 are not propositions because they are neither true nor false

Examples

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- (i) Ice floats in water.
- (ii) China is in Europe.
- (iii) 2 + 2 = 4
- (iv) 2 + 2 = 5
- (v) Where are you going?
- (vi) Do your homework.

- The first four are propositions, the last two are not.
- Also, (i) and (iii) are true, but (ii) and (iv) are false.
- These are called primitive propositions



Propositional variables

• We use letters to denote **propositions**

• These letters are called **propositional variables** (or **statement** variables)

• The conventional letters used for propositional variables are p,q,r,s... or P,Q,R,S,...

• Example: P=Lucknow is the capital of U.P.



• The **truth value** of a proposition is denoted by T (True) or F (False)

• The area of logic that deals with propositions is called the propositional calculus or propositional logic.



Compound Propositions

• Many mathematical statements are constructed by combining one or more propositions.

• New propositions, called **compound propositions**, are formed from existing propositions using logical operators(connectives).



Compound Propositions

• A proposition is said to be *primitive* if it cannot be broken down into simpler propositions.

• A propositions is said to be *composite* if it is composed of *subpropositions* which are connected with the help of various connectives.

• Such composite propositions are called *compound propositions*.



Examples

- "Roses are red and violets are blue."
- "John is smart or he studies every night."



Logical Connectives/Sentence Connectives

Words or symbols used to combine two sentences to form compound sentences

| Sr. No. | Connective | Symbol | Compound statement |
|---------|----------------------|-------------------|------------------------------|
| 1 | AND | ^ | Conjuction |
| 2 | OR | V | Disjunction |
| 3 | NOT | _ | Negation |
| 4 | Ifthen | \rightarrow | Conditional or implication |
| 5 | If and only if (iff) | \leftrightarrow | Biconditional or equivalence |



Conjunction, $p \land q$

- Any two propositions can be combined by the word "and" to form a compound proposition called the *conjunction* of the original propositions.
- Symbolically, $p \land q$ read "p and q,"
- If p and q both are true, then $p \land q$ is true; otherwise $p \land q$ is false.

| Т | Т | Т |
|---|---|---|
| Т | F | F |
| F | Т | F |
| F | F | F |



Example

- If p is the proposition "Today is Friday"
- and q is the proposition "It is raining today"
- Then $p \land q$ will be
- Today is Friday and it is raining today



Disjunction, $p \lor q$

- Any two propositions can be combined by the word "or" to form a compound proposition called the *disjunction* of the original propositions.
- Symbolically, $p \lor q$ read "p or q"
- If p and q both are false, then $p \lor q$ is false; otherwise $p \lor q$ is true.

| p | q | $p \lor q$ |
|---|---|------------|
| T | Т | Т |
| T | F | T |
| F | Т | T |
| F | F | F |



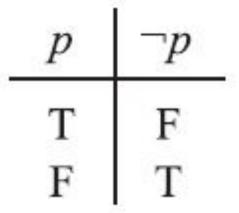
Example

- If p is the proposition "Today is Friday"
- and q is the proposition "It is raining today"
- Then p v q will be
- Today is Friday or it is raining today



Negation, $\neg p$

- Given any proposition p, another proposition, called the *negation* of p, can be formed by writing "It is not true that . . ." or "It is false that . . ." before p or by inserting in p the word "not."
- Symbolically, the negation of p, read "not p," is denoted by $\neg p$
- If p is true, then $\neg p$ is false; and if p is false, then $\neg p$ is true.





Example

• If p is the proposition "Today is Friday" then ¬p will be "Today is not Friday" or "It is not Friday today"

• The logical notation for the connectives "and," "or," and "not" is not completely standardized.

- For example, some texts use:
- p & q, p q or pq for $p \land q$
- p + q for $p \vee q$
- p', \bar{p} or $\sim p$ for $\neg p$



Next topic

Conditional Statement