# Discrete Mathematics
# BCSC 0010
# Module 2
# Predicate logic

(Translating from English into Logical Expressions)

# Translating from English into Logical Expressions

Express the statement "Every student in this class has studied calculus" using predicates and quantifiers.

*Solution:* First, we rewrite the statement so that we can clearly identify the appropriate quantifiers to use. Doing so, we obtain:

"For every student in this class, that student has studied calculus."

Next, we introduce a variable $x$ so that our statement becomes

"For every student $x$ in this class, $x$ has studied calculus."

Continuing, we introduce $C(x)$, which is the statement "$x$ has studied calculus." Consequently, if the domain for $x$ consists of the students in the class, we can translate our statement as $\forall x C(x)$.

# Contd..

If we change the domain to consist of all people, we will need to express our statement as

"For every person $x$, if person $x$ is a student in this class then $x$ has studied calculus."

If $S(x)$ represents the statement that person $x$ is in this class, we see that our statement can be expressed as $\forall x(S(x) \rightarrow C(x))$. [*Caution!* Our statement *cannot* be expressed as $\forall x(S(x) \wedge C(x))$ because this statement says that all people are students in this class and have studied calculus!]

## Example

Express the statements "Some student in this class has visited Mexico" and "Every student in this class has visited either Canada or Mexico" using predicates and quantifiers.

*Solution:* The statement "Some student in this class has visited Mexico" means that

"There is a student in this class with the property that the student has visited Mexico."

We can introduce a variable $x$, so that our statement becomes

"There is a student $x$ in this class having the property that $x$ has visited Mexico."

We introduce $M(x)$, which is the statement "$x$ has visited Mexico." If the domain for $x$ consists of the students in this class, we can translate this first statement as $\exists x\, M(x)$.

# Contd..



However, if we are interested in people other than those in this class, we look at the statement a little differently. Our statement can be expressed as

"There is a person $x$ having the properties that $x$ is a student in this class and $x$ has visited Mexico."

In this case, the domain for the variable $x$ consists of all people. We introduce $S(x)$ to represent "$x$ is a student in this class." Our solution becomes $\exists x(S(x) \wedge M(x))$ because the statement is that there is a person $x$ who is a student in this class and who has visited Mexico. [*Caution!* Our statement cannot be expressed as $\exists x(S(x) \rightarrow M(x))$, which is true when there is someone not in the class ]

Contd..

Similarly, the second statement can be expressed as

"For every $x$ in this class, $x$ has the property that $x$ has visited Mexico or $x$ has visited Canada."

We let $C(x)$ be "$x$ has visited Canada."

Following our earlier reasoning, we see that if the domain for $x$ consists of the students in this class, this second statement can be expressed as $\forall x(C(x) \vee M(x))$.

# Contd..

However, if the domain for $x$ consists of all people, our statement can be expressed as

"For every person $x$, if $x$ is a student in this class, then $x$ has visited Mexico or $x$ has visited Canada."

In this case, the statement can be expressed as $\forall x(S(x) \rightarrow (C(x) \vee M(x)))$.