Discrete Mathematics
BCSC 0010

Module 2
Propositional Logic
(Logical Connectives and Truth Tables)

Conditional or Implication

- Let *p* and *q* be propositions.
- The *conditional statement p* □ *q* is the proposition "if *p*, then *q*."
- The conditional statement *p* □ *q* is false when *p* is true and q is false, and true otherwise.
- In the conditional statement *p* □ *q*, *p* is called the *hypothesis* (or *antecedent* or *premise)* and *q* is called the *conclusion* (or *consequence)*.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Variety of terminology is used to express $p \rightarrow q$

- "if p, then q"
- "if p, q"
- "q if p"
- "q when p"
- "q unless $\sim p$"
- "p implies q"
- "p only if q"
- "q whenever p"
- "q follows from p"

Example

- Let p be the statement "Maria learns discrete mathematics" and q the statement "Maria will find a good job." Express the statement p □ q as a statement in English.

- "If Maria learns discrete mathematics, then she will find a good job."

or

- "Maria will find a good job when she learns discrete mathematics."

CONVERSE, CONTRAPOSITIVE, AND INVERSE

- We can form some new conditional statements starting with a conditional statement p □ q.

- The converse of p □ q is the proposition q □ p.

- The contrapositive of p □ q is the proposition ~ q □ ~ p.

- The inverse of p □ q *is the* proposition ~ p □ ~ q.

- The contrapositive always has the same truth value as $p \rightarrow q$.

- When two compound propositions always have the same truth value we call them **equivalent**, so conditional statement and its contrapositive are equivalent.

- The converse and the inverse of a conditional statement are also equivalent

What are the contrapositive, the converse, and the inverse of the conditional statement "**The home team wins whenever it is raining**."?

- ***Solution:***
- Because "*q* whenever *p*" is one of the ways to express the conditional statement p □ q, the original statement can be rewritten as
- "If it is raining, then the home team wins."
- **Consequently,**
- contrapositive : "If the home team does not win, then it is not raining."
- converse: "If the home team wins, then it is raining."
- Inverse: "If it is not raining, then the home team does not win."

Biconditional or Equivalence

- We now introduce another way to combine propositions that expresses that two propositions have the same truth value.
- Let p and q be propositions. The *biconditional statement* $p \leftrightarrow q$ is the proposition *"p* if and only if *q."*
- The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Biconditional statements are also called *bi-implications.*

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Biconditional or Equivalence

- There are some other common ways to express $p \leftrightarrow q$

- "*p* is necessary and sufficient for *q*"
- "if *p* then *q*, and conversely"
- "*p* iff *q*"

***p* $\leftrightarrow$ *q* has exactly the same truth value as *(p* □ *q)* $\land$ *(q* □ *p)*.**

Example

- Let p be the statement "You can take the flight"
- and let q be the statement "You buy a ticket."

- Then $p \leftrightarrow q$ is the statement :
- "You can take the flight if and only if you buy a ticket."

Precedence of connectives

Operator	Precedence
$\neg$	1
$\wedge$ $\vee$	2 3
$\rightarrow$ $\leftrightarrow$	4 5

For example, $\neg p \wedge q$ means $(\neg p) \wedge q$
not $\neg(p \wedge q)$

Exclusive OR (XOR)

- Let p and q be propositions.
- The *exclusive or* of p and q, denoted by $p \oplus q$ is the proposition that is true when exactly one of p and q is true and is false otherwise.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Truth Tables of Compound Propositions

Construct a truth table for the proposition $\neg(p \land \neg q)$

p	q	$\neg q$	$p \land \neg q$	$\neg(p \land \neg q)$
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

Construct a truth table for the proposition

$(p \vee \sim q) \square (p \wedge q)$

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Next Topic

- How to Translate English Sentences into expressions