Time: 3 Hour **Maximum Marks: 50**

Section-A **7x 5 = 35 Marks**

1. Prove that the relation R = {(x, y) | x – y is an even integer for all x, y ϵ Z} is an equivalence relation.
2. Attempt all:
   a. Prove the following:

   $$Y - X' = Y \cap X$$

   b. Out of 80 students in a class, 60 play football, 53 play hockey and 35 play both the games. How many students
      i. Do not play any of these games?
      ii. Play only hockey but not football?
3. Attempt all:
   a. Solve the following recurrence relation:

   $$u_n = u_{n-1} + 2u_{n-2}, n \geq 2, u_0 = 3, u_1 = 7$$

   b. Find the generating function for the following sequence:

   $$1, 2, 2^2, 2^3, 2^4, \ldots\ldots\ldots$$

4. Attempt all:
   a. By using mathematical induction prove that the given equation is true for all positive integers.

   $$2 + 4 + 6 + \cdots + 2n = n\,(n + 1)$$

   b. Three electric bulbs are chosen at random from 15 bulbs of which 5 are defective. Find the probability that none of them is defective.

Open with Google Docs

5. Attempt all:

a. Show that s is a valid conclusion from the given premises:

$$p \rightarrow \neg q, \qquad q \vee r, \qquad \neg s \rightarrow p, \qquad \neg r.$$

b. Translate following using predicate logic and also find its negation:

'All men are Mortal'

6. Show that (Z, +, .) is a ring where Z denotes the set of integers with operations addition and multiplication.

7. Attempt all:

a. Find the order of all elements in the set $\{a, a^2, a^3, a^4, a^5, a^6\}$ with respect to multiplication operation and identity element a^6.

b. State and prove handshaking theorem on graphs.

Section-B

(a) Attempt all questions: **3 x 2 =6 Marks**

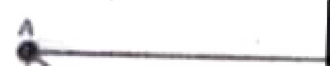
1. State and prove Lagrange's theorem on subgroups.
2. Discuss the concept of bipartite graphs with example.
3. Show that the set {1, 2, 3, 4} is a finite abelian group under multiplication modulo 5.

(b) Attempt all questions: **3 x 3 =9 Marks**

1. What do you understand by homomorphism of groups? Give example.
2. Prove that for any elements a and b in a group (G, *)

a. $(a^{-1})^{-1} = a$ b. $(ab)^{-1} = b^{-1}a^{-1}$

3. Find a Euler and Hamiltonian path/circuit in the graph given below: (if exists)



Page 78 / 91

Section-B

(a) Attempt all questions: **3 x 2 =6 Marks**

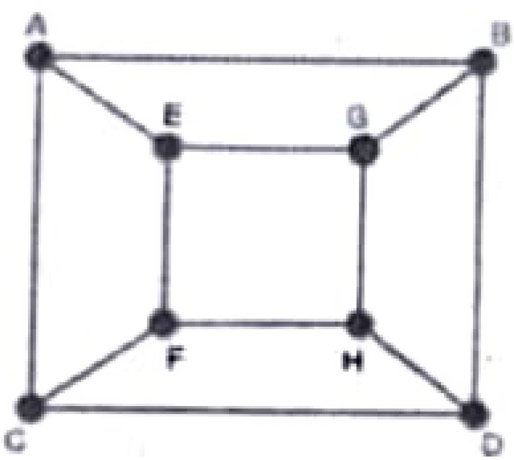
1. State and prove Lagrange's theorem on subgroups.
2. Discuss the concept of bipartite graphs with example.
3. Show that the set {1, 2, 3, 4} is a finite abelian group under multiplication modulo 5.

(b) Attempt all questions: **3 x 3 =9 Marks**

1. What do you understand by homomorphism of groups? Give example.
2. Prove that for any elements a and b in a group (G, *)

   a. $(a^{-1})^{-1} = a$ b. $(ab)^{-1} = b^{-1}a^{-1}$

3. Find a Euler and Hamiltonian path/circuit in the graph given below: (if exists)



1. **Attempt any two:** **35 Marks**

a. Let $A=\{1,2,3,4,5\}\times\{1,2,3,4,5\}$. Let us define R on A by $(x1,y1)R(x2,y2)$ if $x1+y1=x2+y2$. Prove that the given relation is an equivalence relation.

b. Each student in a class of 40 plays at least one indoor game chess, carrom and scrabble. 18 play chess, 20 play scrabble and 27 play carrom. 7 play chess and scrabble, 12 play scrabble and carrom and 4 play chess, carrom and scrabble. Find the number of students who play chess and carrom but not scrabble.

c. Let $A = \{0, 1, 2, 3, 4\}$ and $A - \{0\} = \{1, 2, 3, 4\}$. Can you define a function $f : A - \{0\} \rightarrow Z$ as follows: For every $x \in A - \{0\}$, $f(x)$ is an integer y such that $(xy)\bmod 5 = 1$. Construct an example of such f.

2. **Attempt all:**

a. In how many ways can we write 20 as a sum of 3 non-negative integers?

End Term.pdf
Open with Google Docs

b. In how many ways can the letters of the word 'MILLIMICRON' be arranged?

c. How many integers from 100 to 999 must be picked in order to be sure that atleast 2 of them have a digit in common?

3. Attempt all:

a. A sequence of numbers $u_1, u_2, u_3, \ldots$ is defined as $u_{n+1} = 4u_n + 2$, where $u_1 = 2$. Find the solution of this recurrence relation.

b. Find the closed form of generating function for the following sequence:

1,0,-1,0,1,0,-1,0,1,0,-1,0,........

4. Attempt all:

a. Prove that $\sqrt{5}$ is an irrational number.

b. If from a lottery of 30 tickets, marked 1 to 30, four tickets are to be drawn. What is the probability that those marked 1 and 2 are among them?

Open with Google Docs

5. Attempt any one:

a. Show that the premises "If I study then I will not fail in Mathematics". "If I do not hang out with friends, then I will study". "But I failed in mathematics" imply the conclusion, "I must have hung out with friends." Using inference theory/rules on propositions.

b. Show that the premises "Everyone in this class is happy" and "James is a student in this class" imply the conclusion

"James is happy." Using predicates and inference rules.

6. Show that the set of integers with composition o and *, represented as (Z, o, *) is a ring where the composition o and * is defined by

End Term.pdf Open with Google Docs

"James is happy." Using predicates and inference rules.

6. Show that the set of integers with composition o and *, represented as (Z, o, *) is a ring where the composition o and * is defined by

   $a \circ b = a + b + 1$ and $a * b = ab + a + b$

7. Attempt any two:

   a. Which of the graphs K4 and Q3 is/are planar? Give reasons for your answer.

   b. Let G be a graph with n vertices and G' be the complement of G, then find no of edges in E(G) + E(G').

   c. State and prove handshaking theorem on graphs.

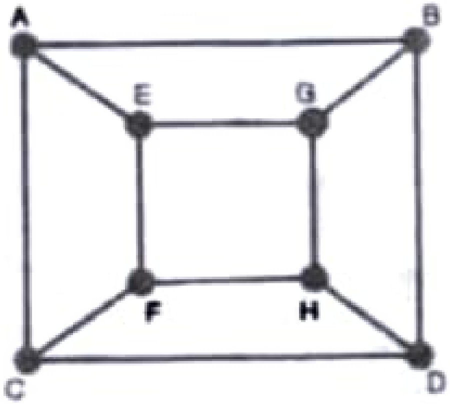
Section-B

(a) Attempt all questions: **(3x2=6)**

1. State and prove Lagrange's theorem on subgroups.
2. Find order of each element 1, 2, 3, and 4 in the group $(G,+_5)$ where $G = \{0,1,2,3,4\}$ and $a +_5 b = (a+b) \bmod 5$.
3. Find out whether the graph given below is bipartite or not? If yes, Give two distinct set of vertices.

End Term.pdf Open with Google Docs

3. Find out whether t... bipartite or not? If yes, Give two distinct set of vertices.



(b) **Attempt all questions:** **(3x3=9)**

1. If the binary operation * is defined as a*b = a+b-2, prove that operation * has an identity. Also find the inverse of an element a in R with respect to this binary operation *.

2. Given the

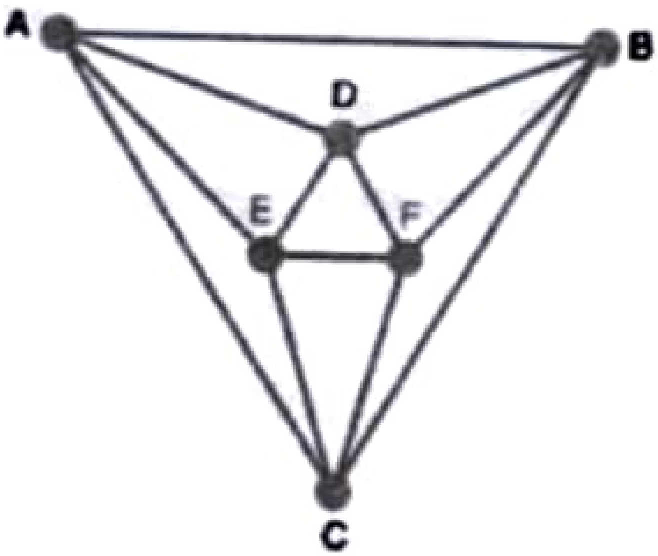
nd Term.pdf Open with Google Docs

1. If the binary ... a*b = a+b-2 ... Prove that operation * has an identity. Also find the inverse of an element a in R with respect to this binary operation *.

2. Given the graph below, find

   a. Euler path and circuit

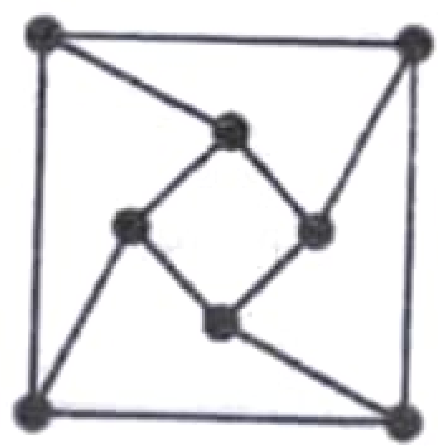
   b. Hamiltonian path and circuit



3. Consider the following three graphs. Find out which of these are isomorphic to each other? Also, give the corresponding mapping of vertices.

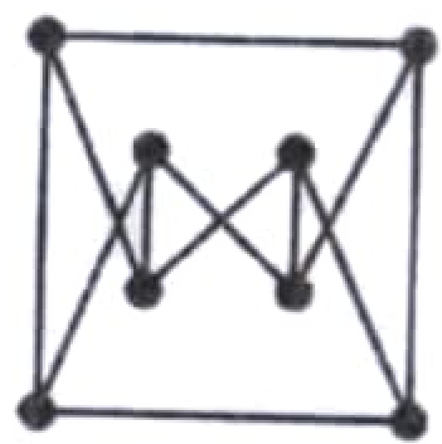
C

3. Consider the following three graphs. Find out which of these are isomorphic to each other? Also, give the corresponding mapping of vertices.



G

G_1



G_2

Section- A

Note: Attempt All Questions. 1 x 16 = 16 marks

I. Write the following sets in set builder form:

$$A = \left\{\frac{1}{5}, -\frac{2}{10}, \frac{3}{17}, -\frac{4}{26}, \ldots\right\}$$

II. In a survey of 85 people it is found that 31 like to drink milk 43 like coffee and 39 like tea. Also 13 like both milk and tea, 15 like milk and coffee, 20 like tea and coffee and12 like none of the three drinks. Find the number of people who like all the three drinks. Display the answer using Venn diagram.

III. If $R = \{(1,2), (4,3), (2,2), (2,1), (3,1)\}$ be a relation on asset $A = \{1,2,3,4\}$. Find:
(a) reflexive closure (b) symmetric closure (c) transitive closure of R.

IV. A bag contains 10 red marbles, 10 white marbles, and 10 blue marbles. What is the minimum no. of marbles you have to choose randomly from the bag to ensure that we get 4 marbles of same color?

V. Show that $\frac{1}{1.2} + \frac{1}{2.3} + \cdots + \frac{1}{n(n+1)} = -\frac{n}{n+1}$

VI. What is meant by a recursively defined function? Give the recursive of factorial function.

VII. Find the negation of the following statement: $(\forall x.P(x)) \vee (\exists y.P(y))$

VIII. Show that $\forall x(P(x) \wedge Q(x))$ is logically equivalent to $\forall x P(x) \wedge \forall x Q(x)$, where quantifiers have the same nonempty domain.

IX. In a class, there are 15 boys and 10 girls. Three students are selected at random. What is the probability that 1 girl and 2 boys are selected?

X. Proof by contrapositive "If a and b are consecutive integers, then the sum a + b is odd".

XI. Determine whether the conclusion C is valid in the following premises:

$$H1: P \Rightarrow (Q \Rightarrow R)$$

$$H2: P \wedge Q$$

$$C: R$$

XII. Determine whether the group $(Z_n, \otimes_n)$ is cyclic for n=5 and n=8.

XIII. Show that the 3-regular graph must have an even number of vertices.

XIV. With the help of appropriate example differentiate between path and circuit.

XV. Suppose G is a group of order n, and k $\in$ N is relatively prime to n. Show that $g: G \to G$ defined by $g(x)=g^k$ is one –one . If G is abelian show that G is auto morphism.

XVI. State Lagrange's theorem.

Section- B

Note: Attempt Any Four Questions. ***3 x 4 = 12 marks***

I. Prove that $(Z_6, \otimes_6)$ is an abelian group of order 6.

II. Let G be a group and Let a, b $\in$ G be any elements. Then show that

(a) $(a^{-1})^{-1}=1$ (b) $(a*b)^{-1}=b^{-1}*a^{-1}$

III. A subset $H \neq \emptyset$ of G is a subgroup of G if and only if for all a ,b $\in$ H, *Note: Attempt Any Four Questions.* *3 x 4 = 12 marks*

I. Prove that $(Z_6, \otimes_6)$ is an abelian group of order 6.

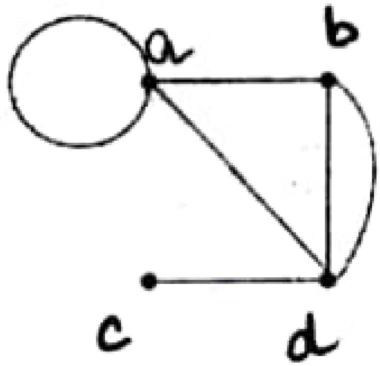
II. Let G be a group and Let a, b $\in$ G be any elements. Then show that

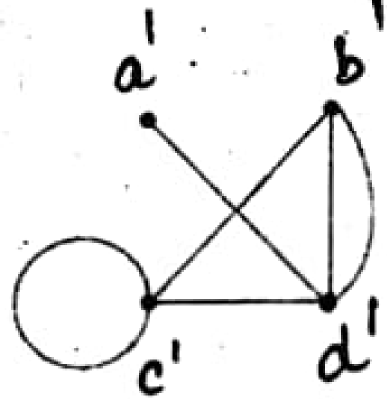
(a) $(a^{-1})^{-1}=1$ (b) $(a*b)^{-1}=b^{-1}*a^{-1}$

III. A subset $H \neq \emptyset$ of G is a subgroup of G if and only if for all a ,b $\in$ H, $a*b^{-1} \in H$.

IV. Define Hamiltonian graph. Prove that complete graphs of three or more vertices have a Hamilton cycle.

V. When two graphs are said to be isomorphic. Whether graph G and G' are isomorphic to each other or not. If, yes list the corresponding Vertices.

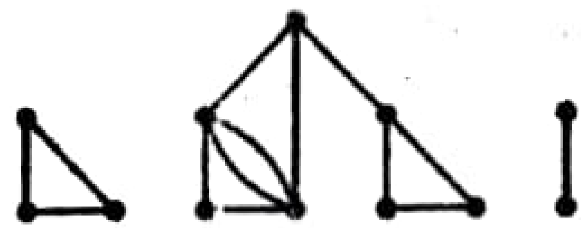




Section- C

Note: Attempt Any Three Questions. ***4 x 3 = 12 marks***

I. Answer the following questions about the planar graph below:



(i) Find the degree of each vertex.
(ii) Find the sum of the degrees of the vertices.
(iii) Does ANY component have an Euler circuit? Why or why not?
(iv) Does ANY component have an Euler path? Why or why not?

II. Model the following situations as (possibly weighted, possibly directed) graphs. Draw each graph, and give the corresponding adjacency matrices.

(a) Amit and Bharti are friends. Amit is also friends with Chetan and David.

II. Model the following situations as (possibly weighted, possibly directed) graphs. Dra[illegible]sponding adjacency matrices.

Open with Google Docs

(a) Amit and Bharti are friends. Amit is also friends with Chetan and David. Bharti, Chetan and Esha are all friends of each other.

(b) Wikipedia has five particularly interesting articles: Animal, Burrow, Chile, Desert, and Elephant. Some of them even link to each other!

(c) It is well-known that in the India, there is a 2-lane highway from Ahamdabad to Baroda, another 2-lane highway from Ahamdabad to

Chandigarh, a 3-lane highway from Baroda to Dehradun, a 1-lane road from Baroda to Eluru and another one from Dedradun to Eluru, and a 5-lane superhighway from Chandigarh to Eluru.

Chandigarh, a 3-lane highway from Baroda to Dehradun, a 1-lane road from Baroda to Eluru and another one from Dedradun to Eluru, and a 5-lane superhighway from Chandigarh to Eluru.

III. Give a simplest possible example of a non-null undirected graph:

(a) Having no vertices of odd degree;

(b) Having no vertices of even degree:

(c) Having exactly one vertices of odd degree;

IV. Consider the set of 2x2 matrices with integer entries and let us denote it by $M_2(Z)$. Then $M_2(Z)$ is a ring under the binary operation of matrix addition and multiplication.