Discrete Mathematics
BCSC 0010

Module 2
Propositional logic

Introduction

- A *proposition* is a declarative statement which is either true or false, but not both.

Examples

- All the following declarative sentences are propositions.

1. Delhi is the capital of India.
2. Toronto is the capital of Canada.
3. 1 + 1 = 2.
4. 2+2 = 3.

- Propositions 1 and 3 are true, whereas 2 and 4 are false

Consider the following sentences

1. What time is it?
2. Read this carefully.
3. $x + 1 = 2$.
4. $x + y = z$.

- Sentences 1 and 2 are not propositions because they are not declarative sentences.
- Sentences 3 and 4 are not propositions because they are neither true nor false

Examples

- (i) Ice floats in water.
- (ii) China is in Europe.
- (iii) $2 + 2 = 4$
- (iv) $2 + 2 = 5$
- (v) Where are you going?
- (vi) Do your homework.

- The first four are propositions, the last two are not.
- Also, (i) and (iii) are true, but (ii) and (iv) are false.
- These are called primitive propositions

Propositional variables

- We use letters to denote **propositions**

- These letters are called **propositional variables** (or **statement** variables)

- The conventional letters used for propositional variables are $p, q, r, s \ldots$ or $P, Q, R, S, \ldots\ldots$

- Example : P=Lucknow is the capital of U.P.

- The **truth value** of a proposition is denoted by T (True) or F (False)

- The area of logic that deals with propositions is called the **propositional calculus or propositional logic.**

Compound Propositions

- Many mathematical statements are constructed by combining one or more propositions.

- New propositions, called **compound propositions,** are formed from existing propositions using logical operators(connectives).

Compound Propositions

- A proposition is said to be ***primitive*** if it cannot be broken down into simpler propositions.

- A propositions is said to be ***composite*** *if it is* composed of ***subpropositions*** which are connected with the help of various connectives .

- Such composite propositions are called *compound propositions*.

Examples

- “Roses are red and violets are blue.”
- “John is smart or he studies every night.”

Logical Connectives/Sentence Connectives

Words or symbols used to combine two sentences to form compound sentences

Sr. No.	Connective	Symbol	Compound statement
1	AND	$\wedge$	Conjuction
2	OR	$\vee$	Disjunction
3	NOT	$\neg$	Negation
4	If……..then	$\rightarrow$	Conditional or implication
5	If and only if (iff)	$\leftrightarrow$	Biconditional or equivalence

Conjunction, $p \wedge q$

- Any two propositions can be combined by the word **"and"** to form a compound proposition called the *conjunction* of the original propositions.
- Symbolically, $p \wedge q$ read "p and q,"
- If p and q *both* are true, then $p \wedge q$ is true; otherwise $p \wedge q$ is false.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

(a) "p and q"

Example

- *If* p is the proposition "Today is Friday"
- and q is the proposition "It is raining today"
- Then $p \land q$ *will be*
- Today is Friday and it is raining today

Disjunction, $p \vee q$

- Any two propositions can be combined by the word **"or"** to form a compound proposition called the *disjunction* of the original propositions.
- Symbolically, $p \vee q$ read "p or q"
- If p and q *both* are false, then $p \vee q$ is false; otherwise $p \vee q$ is true.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

(b) "p or q"

Example

- *If p* is the proposition “Today is Friday”
- and *q* is the proposition “It is raining today”
- Then $p \vee q$ *will be*
- Today is Friday or it is raining today

Negation, $\neg p$

- Given any proposition p, another proposition, called the ***negation*** of p, can be formed by writing "It is not true that . . ." or "It is false that . . ." before p or by inserting in p the word **"not."**
- Symbolically, the negation of p, read "not p," is denoted by $\neg p$
- If p is true, then $\neg p$ is false; and if p is false, then $\neg p$ is true.

p	$\neg p$
T	F
F	T

Example

- If p is the proposition “Today is Friday”

  then $\neg p$ *will be* “*Today is not Friday*”

  or “*It is not Friday today*”

- The logical notation for the connectives “and,” “or,” and “not” is not completely standardized.

- For example, some texts use:
- p & q, $p \cdot q$ or pq for $p \land q$
- $p + q$ for $p \lor q$
- p', $\bar{p}$ or $\sim p$ for $\neg p$

Next topic

- Conditional Statement