

Deep Learning Fundamentals

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Chapter 3

Recurrent Neural Network

1. What is RNN?

Analyzing Longitudinal Trends



*I want to learn deep
learning because I am...*

Recurrent Neural Network (RNN)

- A neural network with a recursive structure.

“Recursive”: A thing's definition or description contains the thing itself.

- The recursive structure enables RNNs to handle longitudinal data.

e.g., Stock price data

Time-lagged data in psychological research

Longitudinal Data

Employee Motivation Data

Not Longitudinal

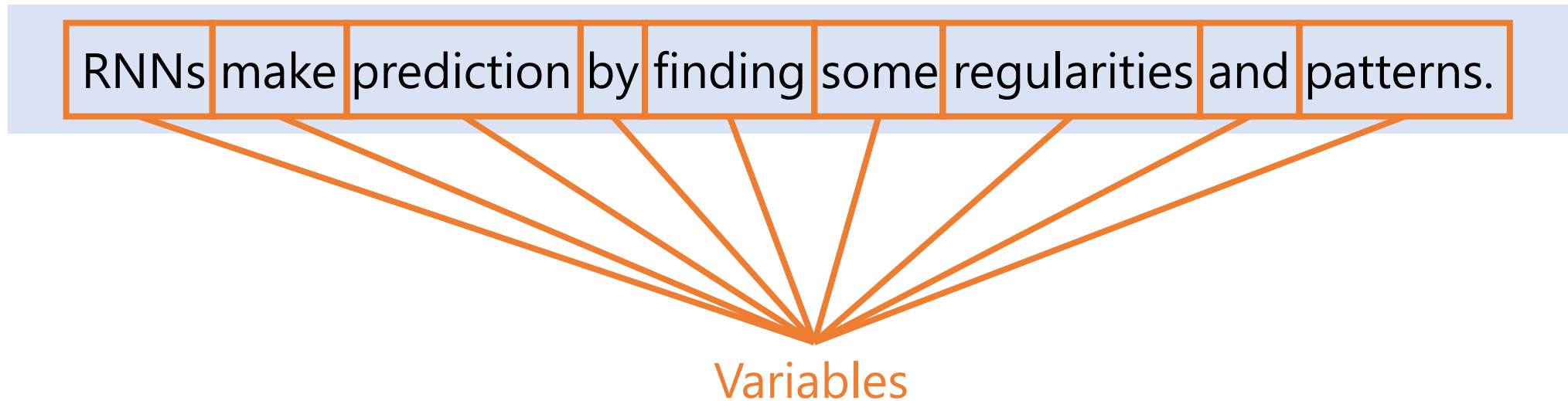
Employee ID	Motivation
Emp1	4
Emp2	5
Emp3	3
Emp4	3
Emp5	2
Emp5	5
emp6	4

Longitudinal Data

Employee ID	Time 1	Time 2	Time 3	Time 4	Time 5
Emp1	4	3	5	4	3
Emp2	5	5	4	5	4
Emp3	3	3	2	3	3
Emp4	3	2	4	3	2
Emp5	2	2	1	2	1
Emp5	5	5	4	4	5
emp6	4	3	2	3	4

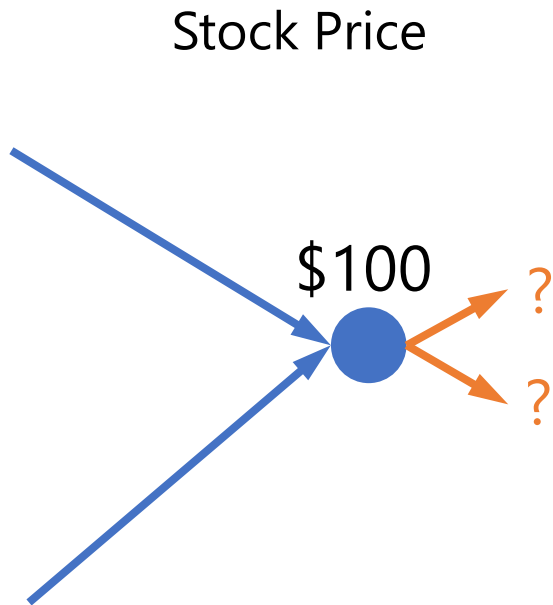
Analyzing Longitudinal Data by RNN

- RNNs make prediction by finding some regularities or patterns in the variation of sequenced variables.



Importance of Past State

- In a longitudinal analysis, the model needs to find regularities and pattern in a sequence.

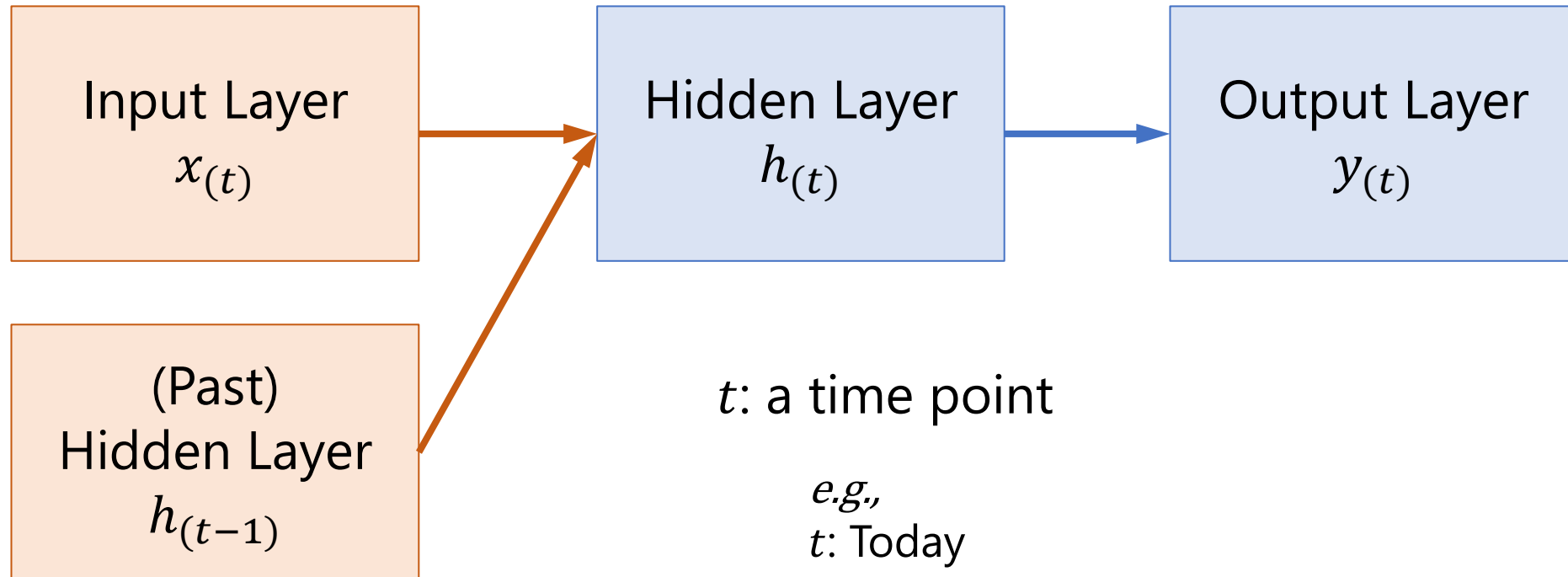


- You may do it well.
- You may well do it.

- There is a cloud in the sky.
- There is a stone in the ____.

2. Structure of RNN?

Basic Structure of RNN



t : a time point

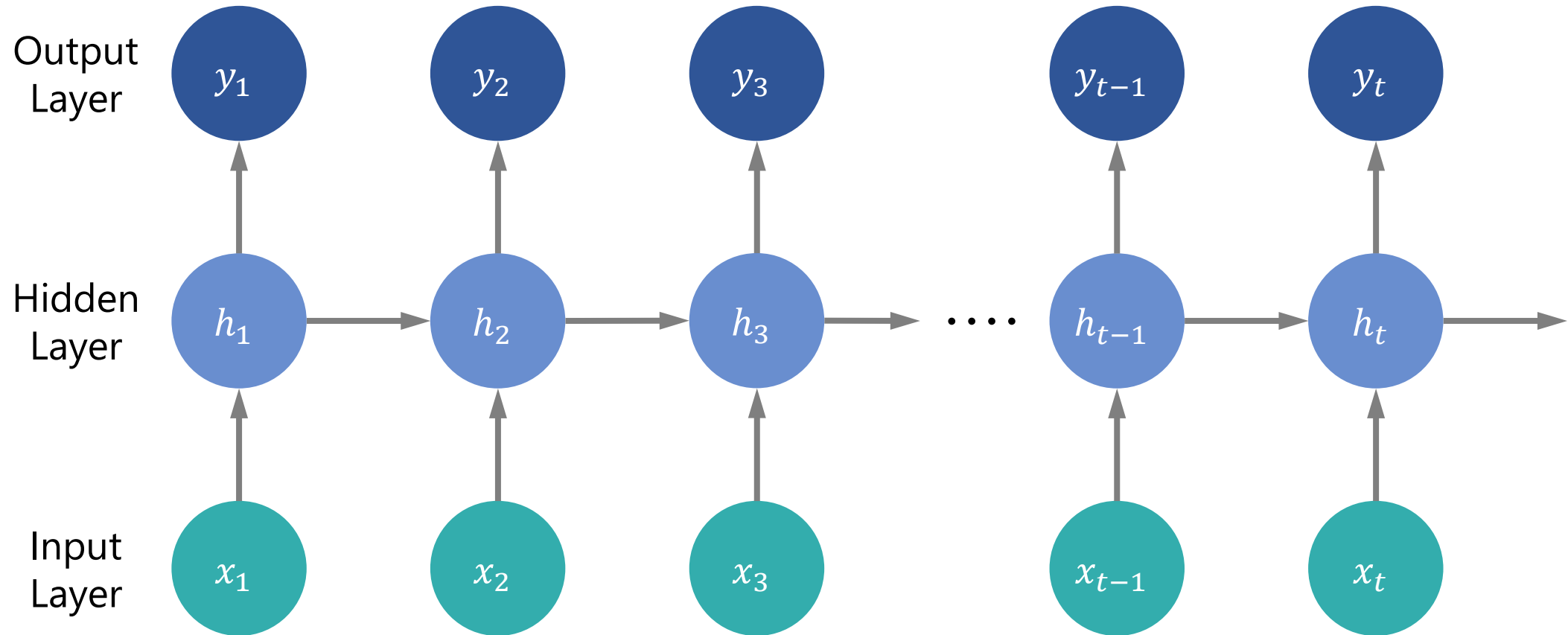
e.g.,

t : Today

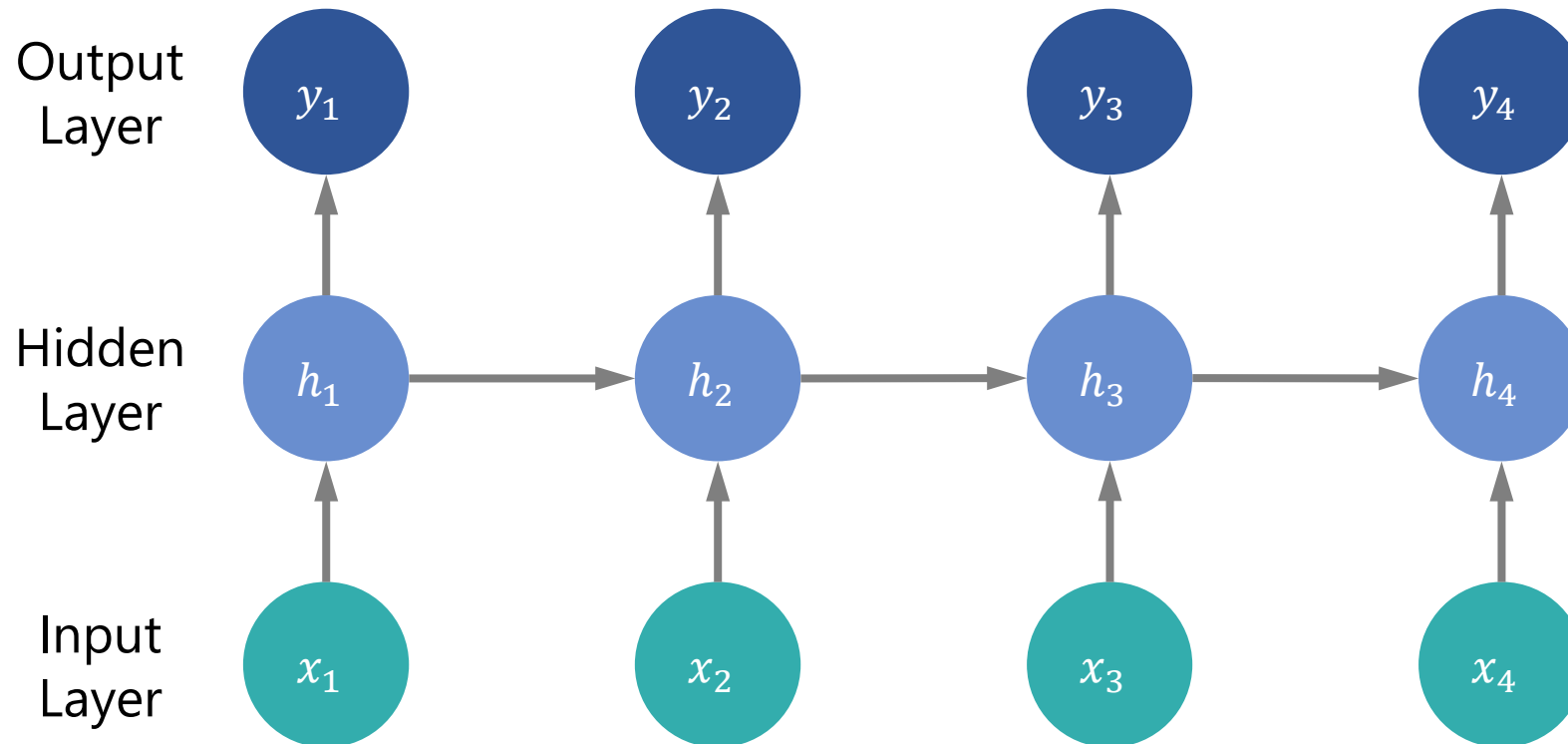
$t - 1$: Yesterday

$t + 1$: Tomorrow

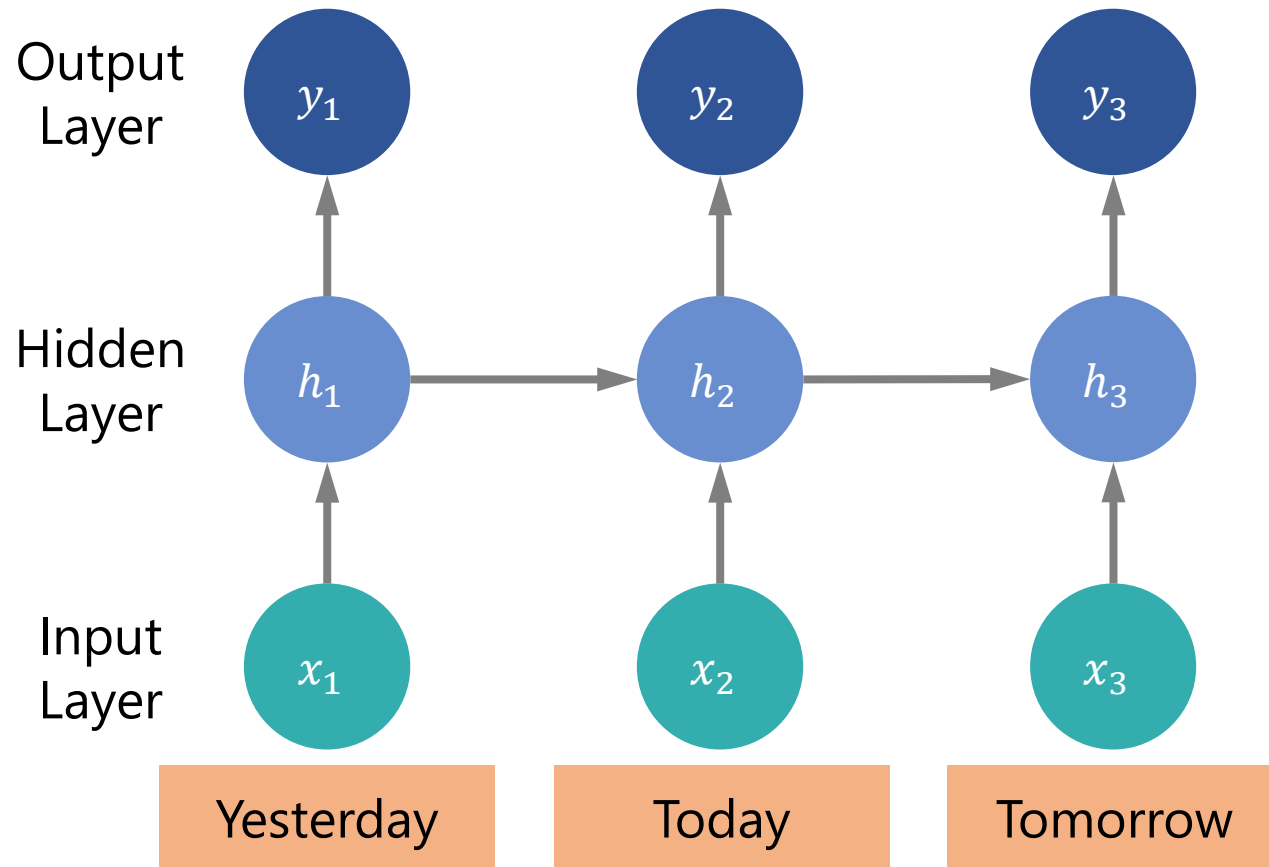
Bigger Picture of RNN



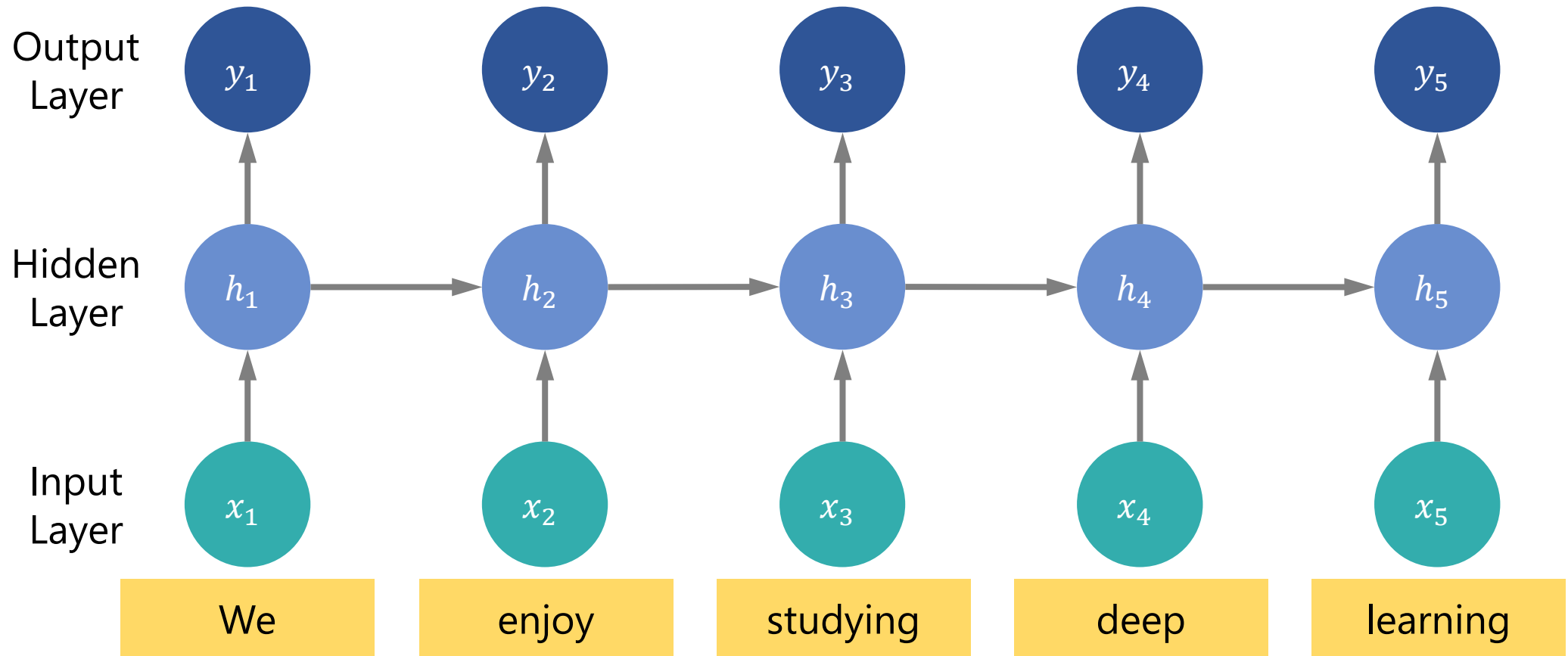
Input Data Size and Repetition



Input Data Size and Repetition

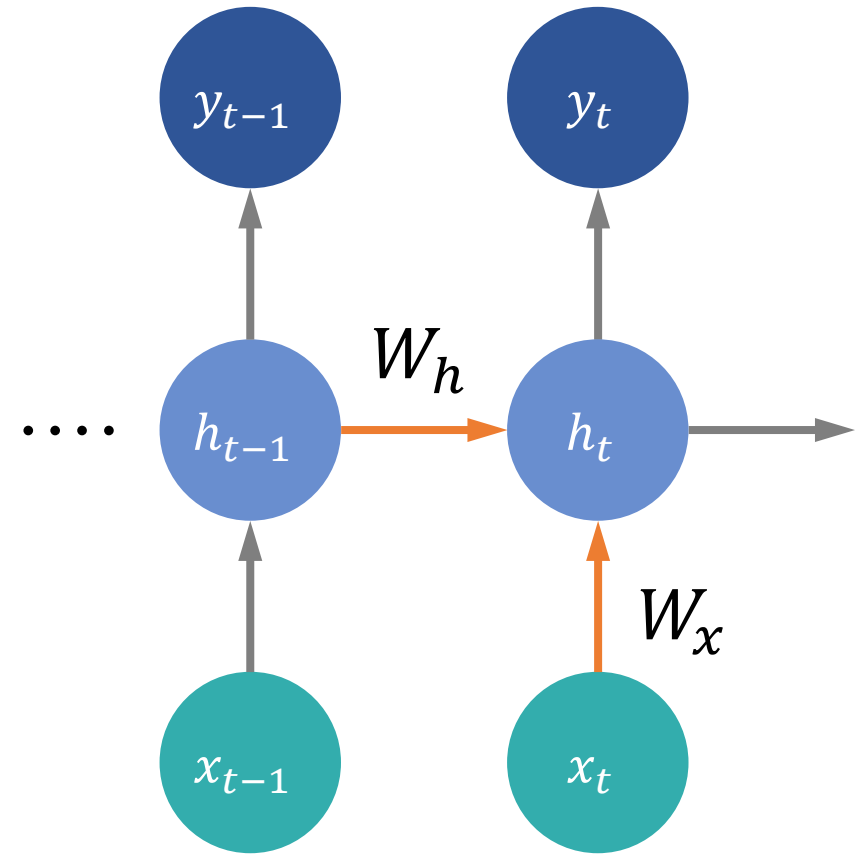


Input Data Size and Repetition

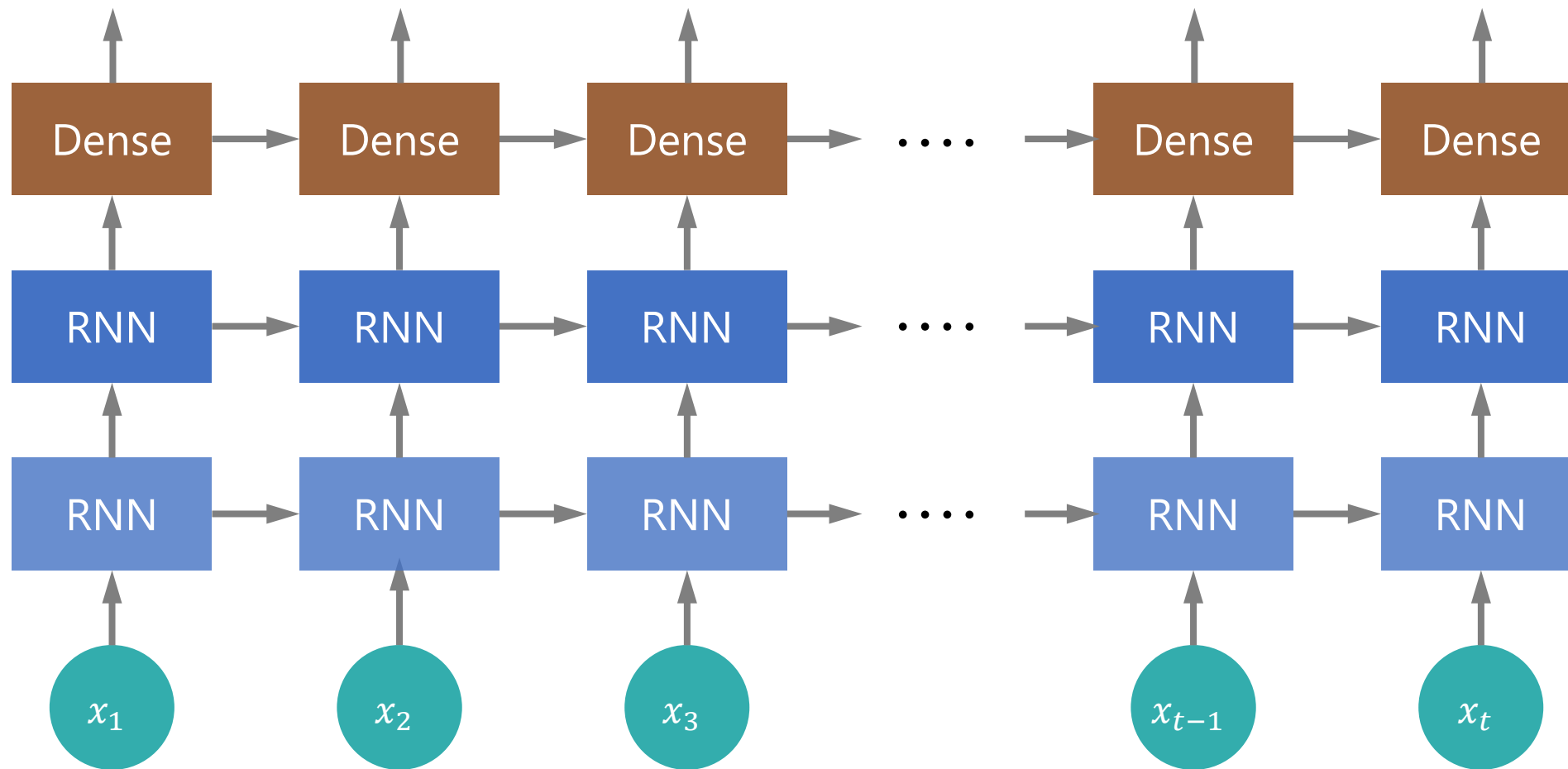


Output of the Hidden Layer

$$h_t = \tanh(h_{t-1}W_h + x_tW_x + b)$$

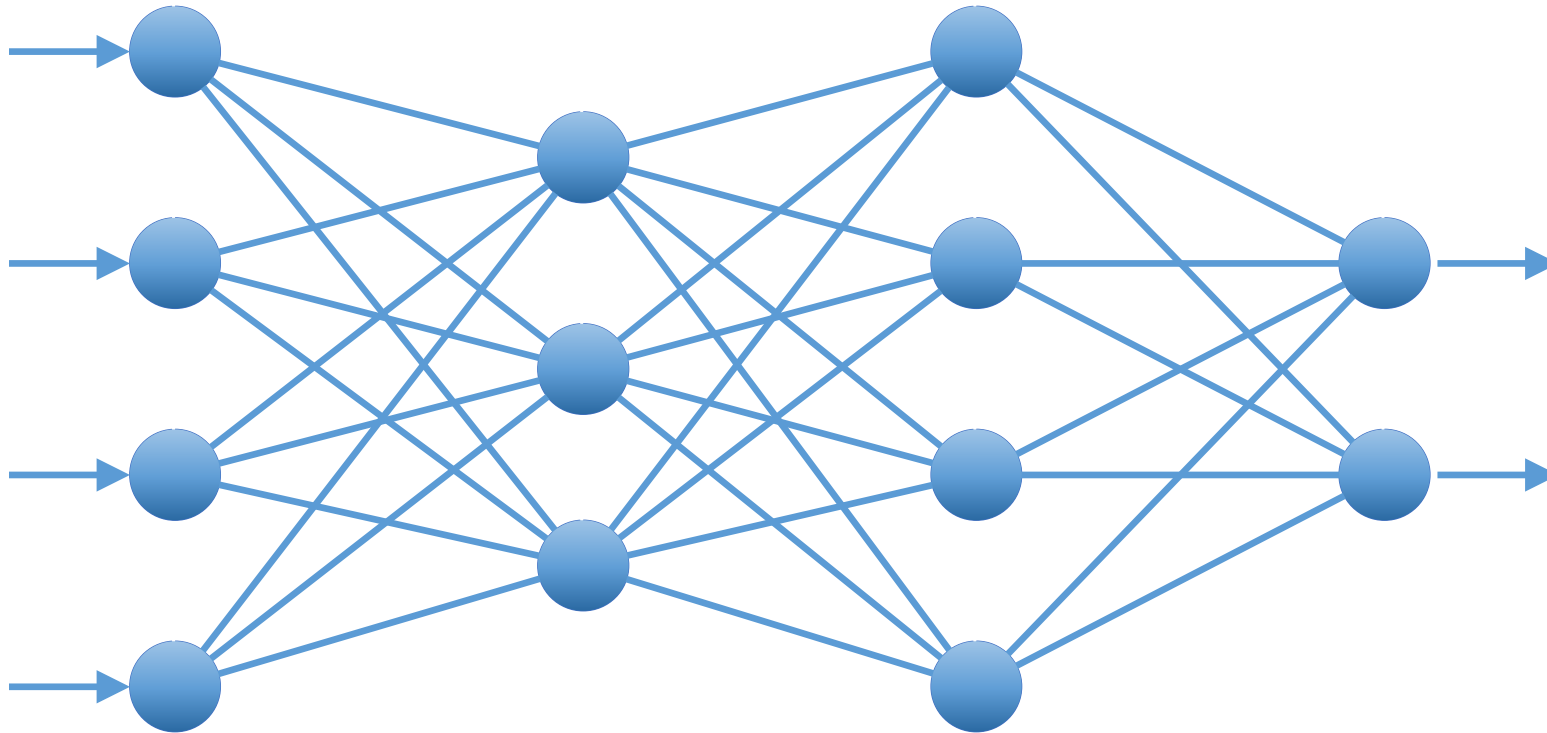


More Complicated Structure



3. Variable-Length Input

Case: Ordinary ANN



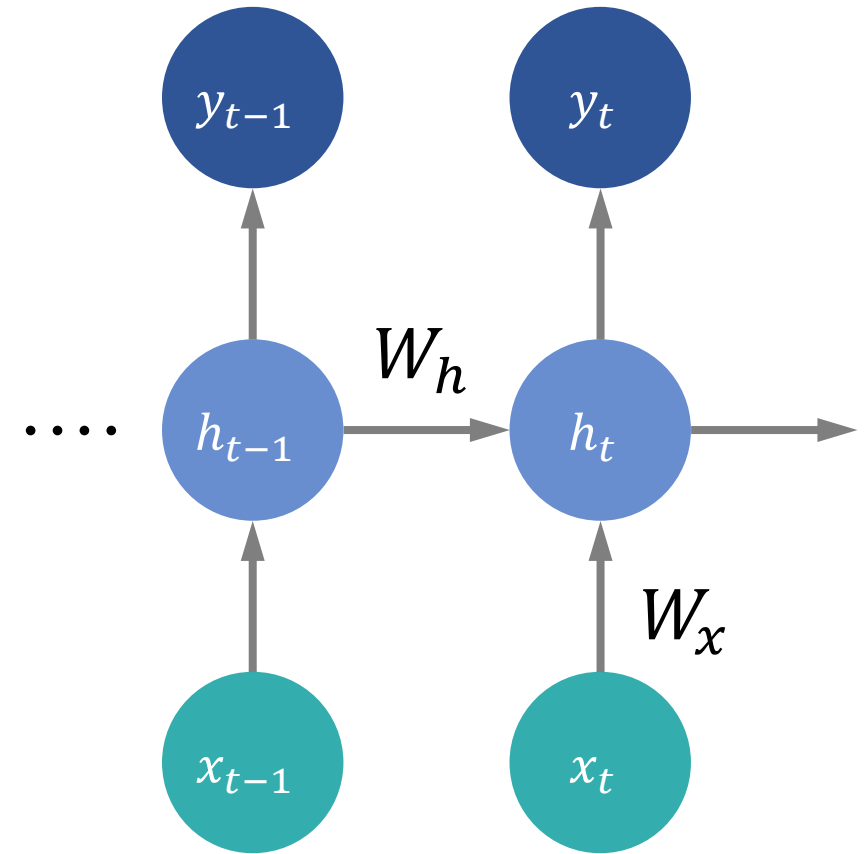
Variable-Length Input

- We can use different sized datasets for training and testing.
- RNN is effective for time-series analysis.
- RNN is also effective for analyzing text data.

4. Weight & Bias

Recap: Structure of RNN

$$h_t = \tanh(h_{t-1}W_h + x_tW_x + b)$$



Weights and Biases

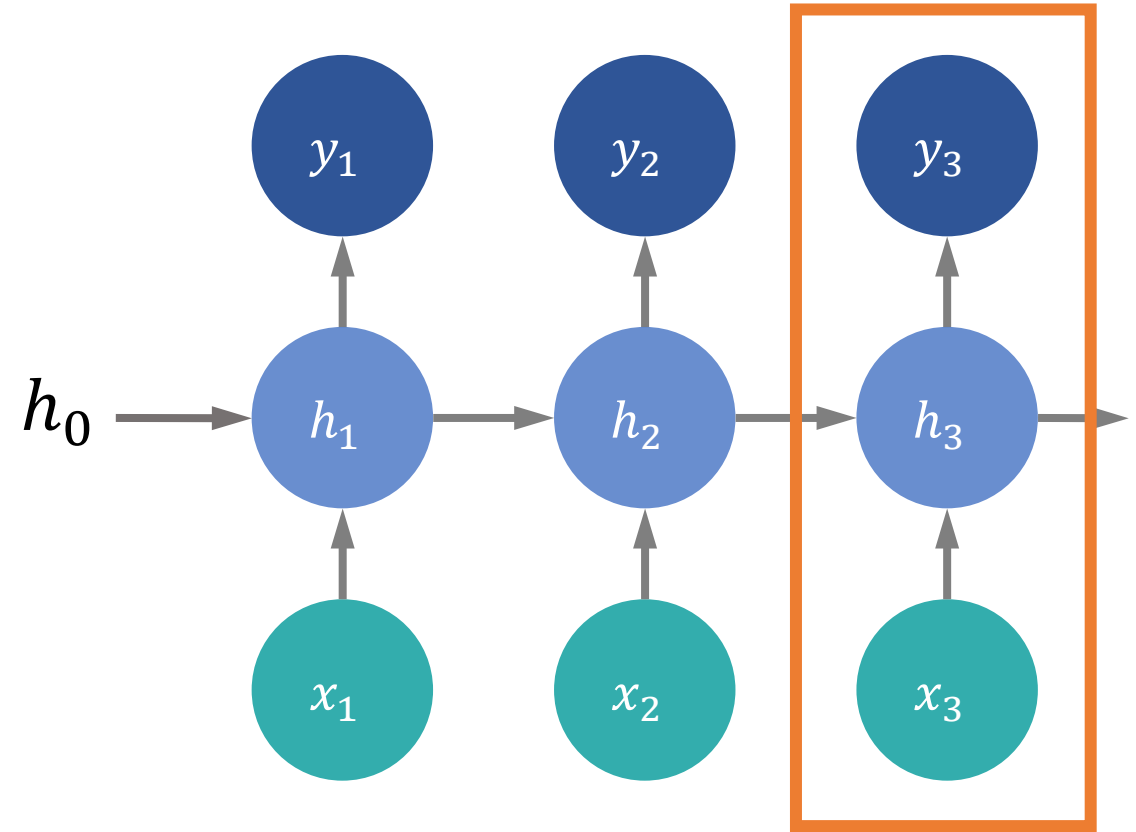
$$h_t = \tanh(h_{t-1}W_h + x_tW_x + b)$$

$$h_1 = \tanh(h_0W_h + x_1W_x + b)$$

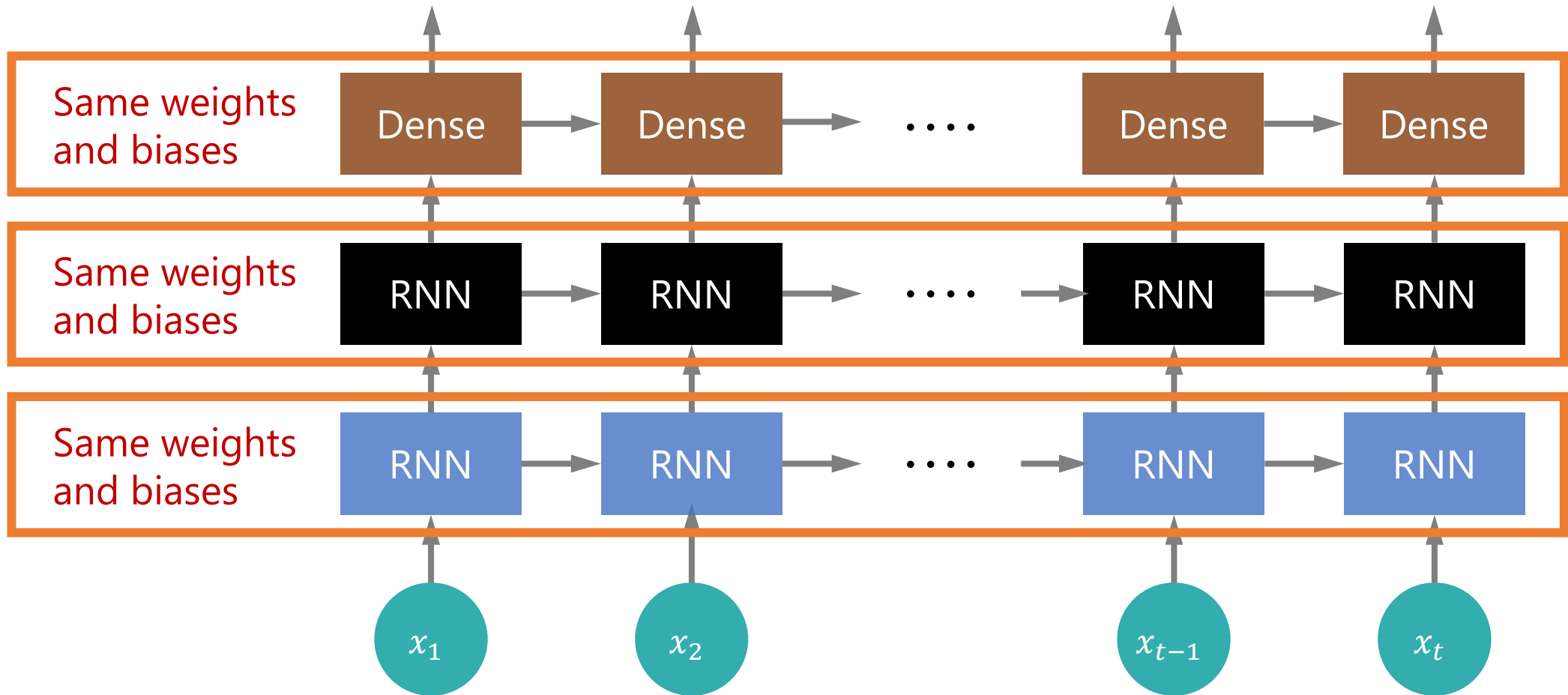
$$h_2 = \tanh(h_1W_h + x_2W_x + b)$$

$$h_3 = \tanh(h_2W_h + x_3W_x + b)$$

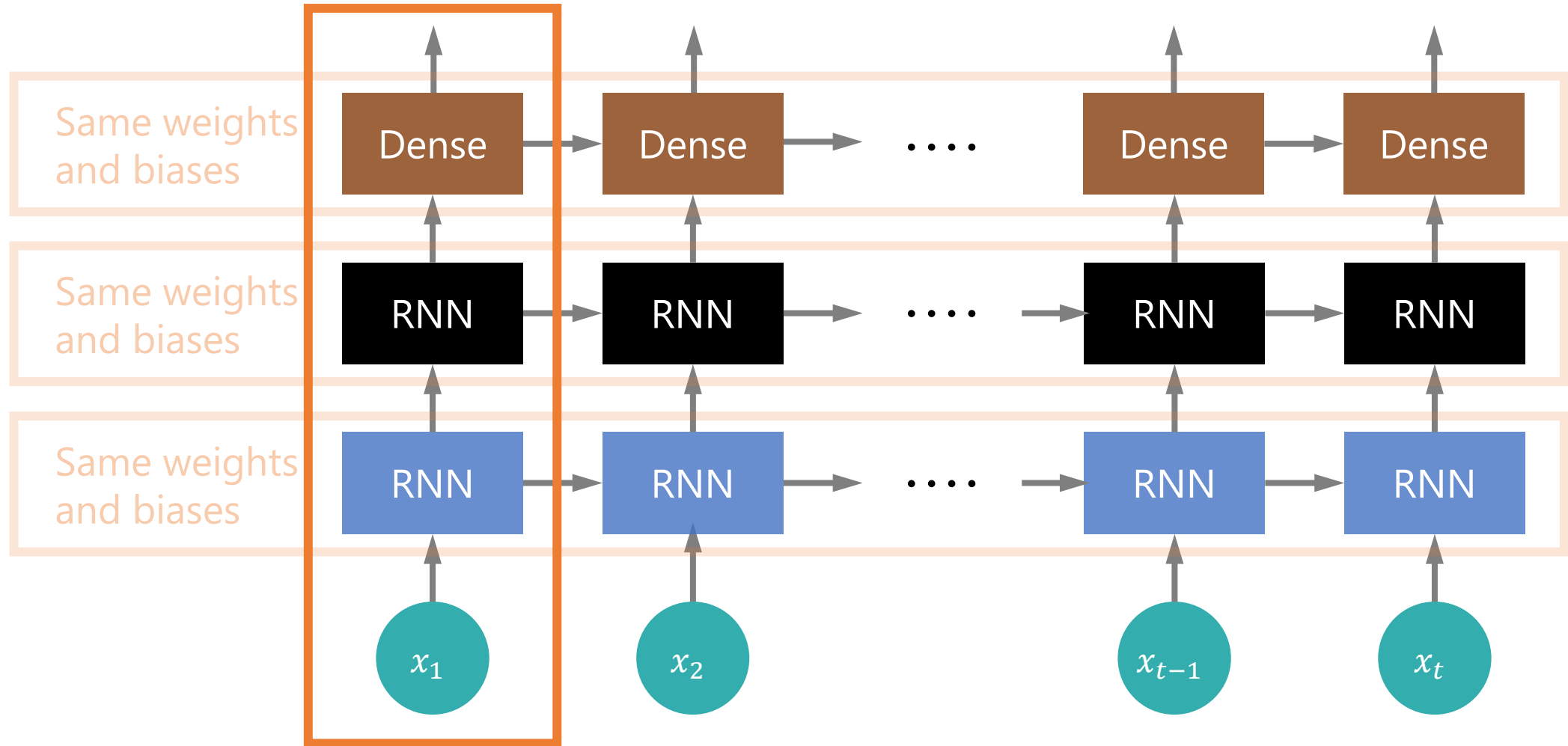
$$h_4 = \tanh(h_3W_h + x_4W_x + b)$$



Weights and Biases in More Complicated Structure



Weights and Biases in More Complicated Structure



5. Types of RNN

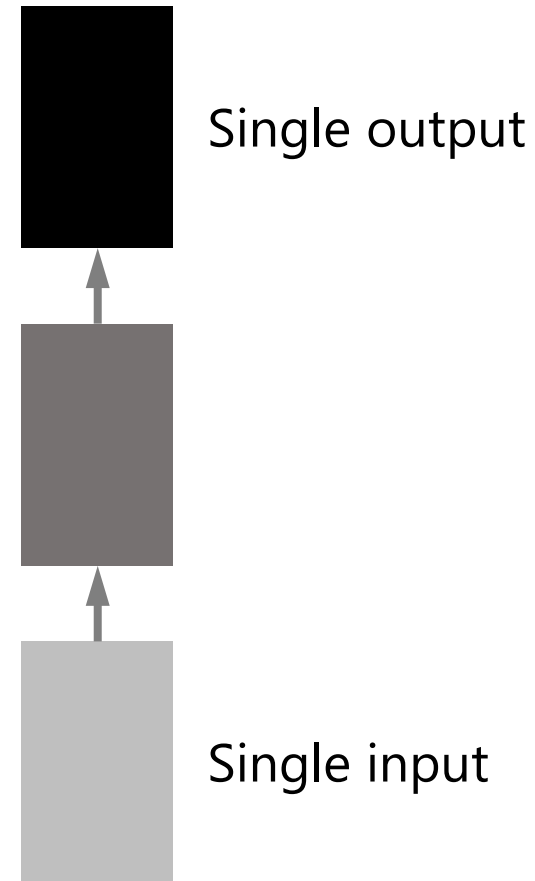
Types of RNN

We can change the input and output layers' sizes and types.

- One to One
- One to Many
- Many to One
- Many to Many

One-to-One RNN

- General machine learning problems



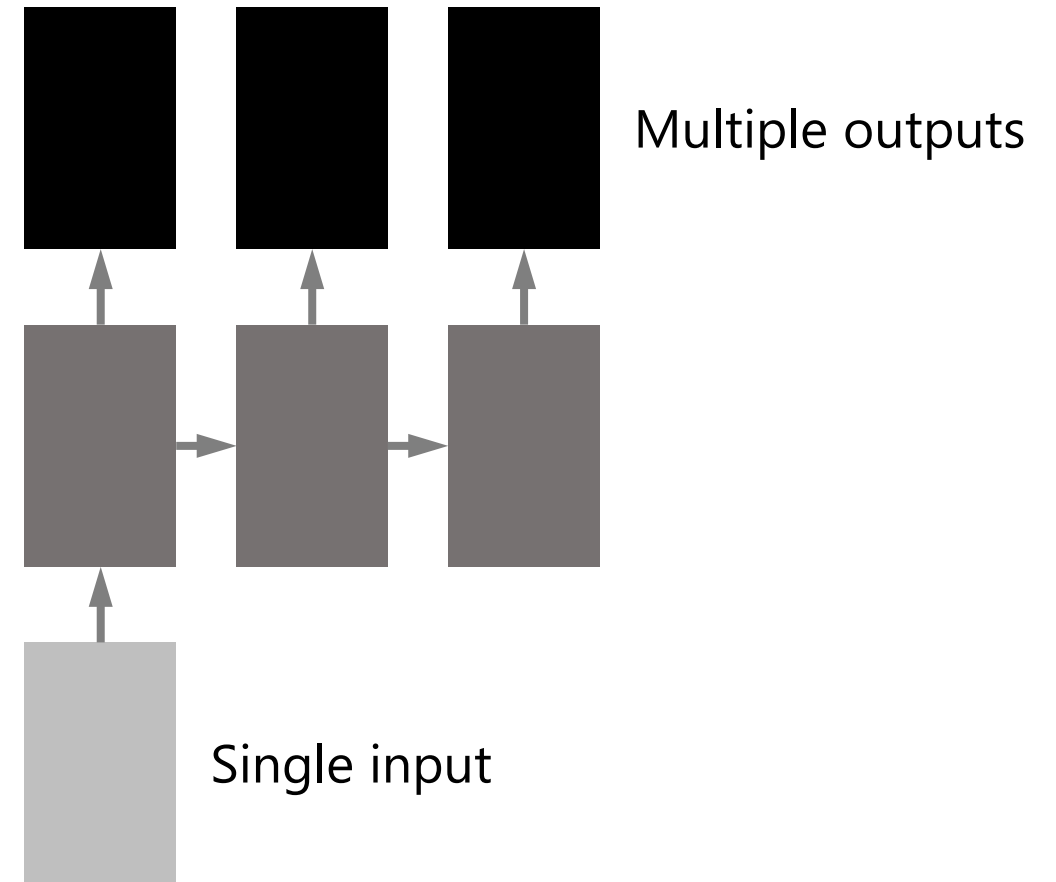
One-to-Many RNN

- The input data is not a sequence, but the output is a sequence.

A One-to-Many RNN generates multiple outputs from a single input.

e.g., Image captioning

Boys are playing soccer.



Many-to-One RNN

- The input data is a sequence, but the output is a single element.

A Many-to-One RNN uses multiple inputs to generate a single output.

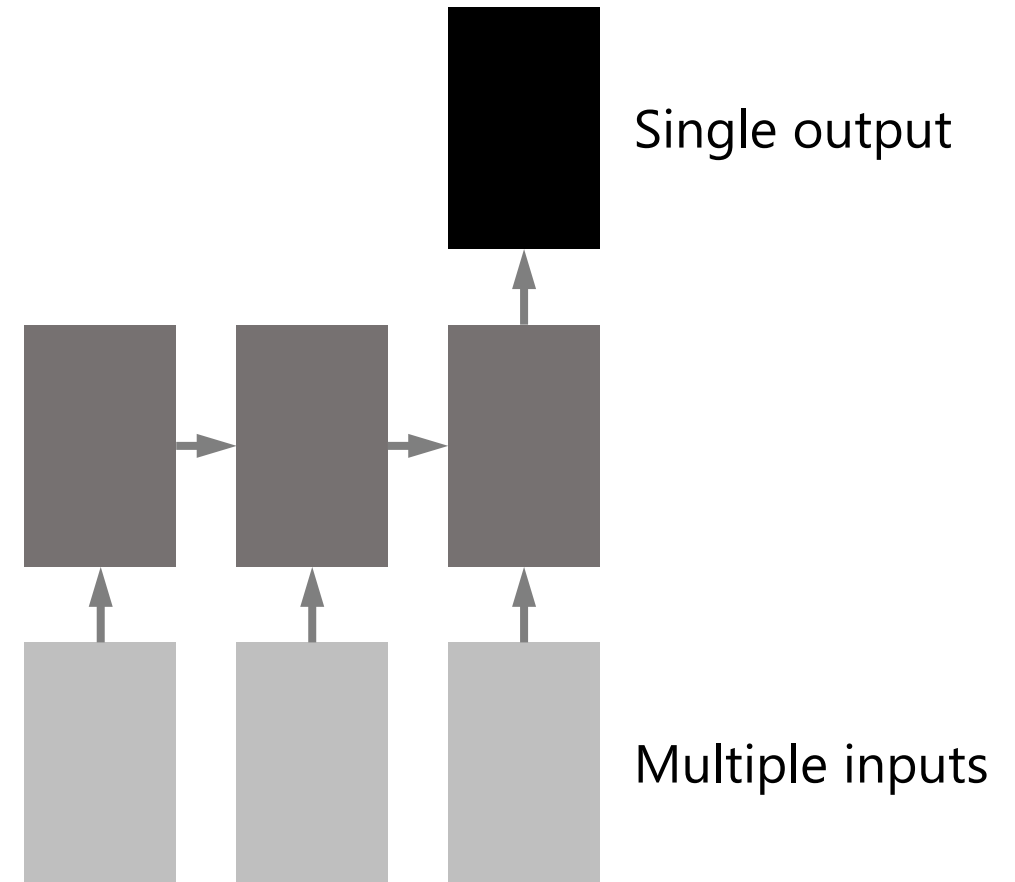
e.g., Sentiment analysis

I liked this course since
it provided me with a
concise guide of RNN. → Score
4/5

e.g., Video classification



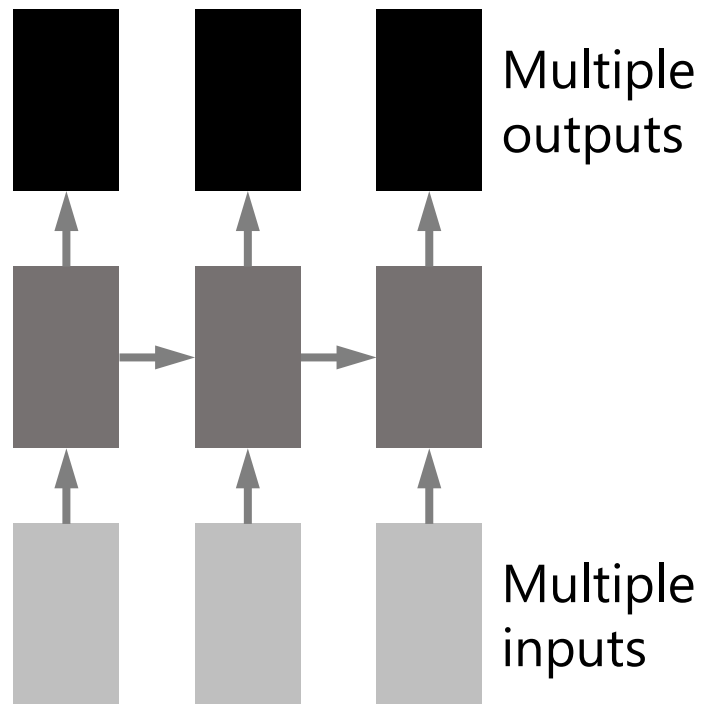
Soccer Match



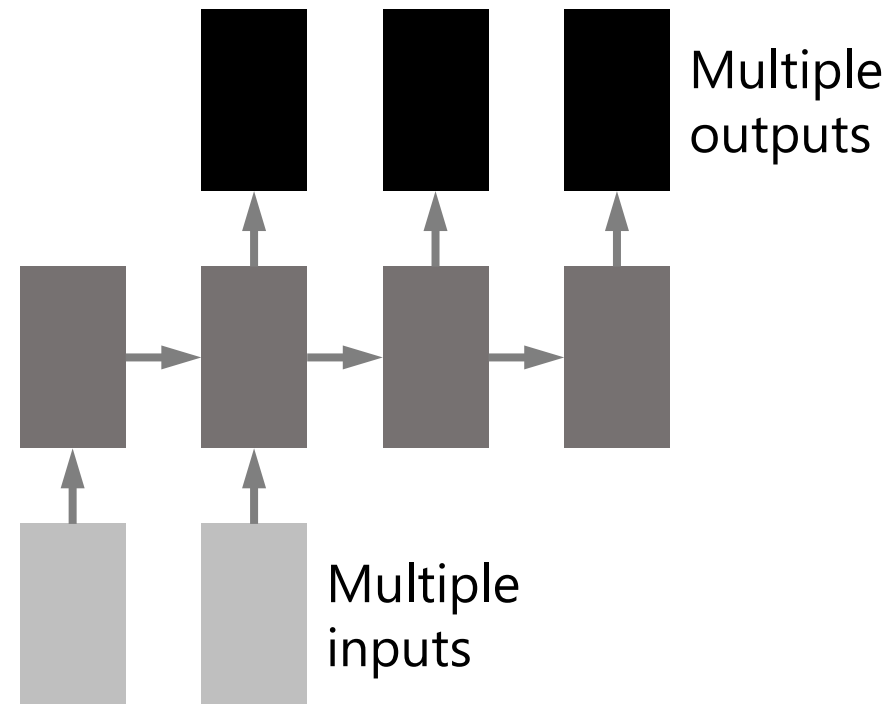
Many-to-Many RNN

- Both input and output are sequences.

A Many-to-Many RNN uses a sequence to generate a sequence.



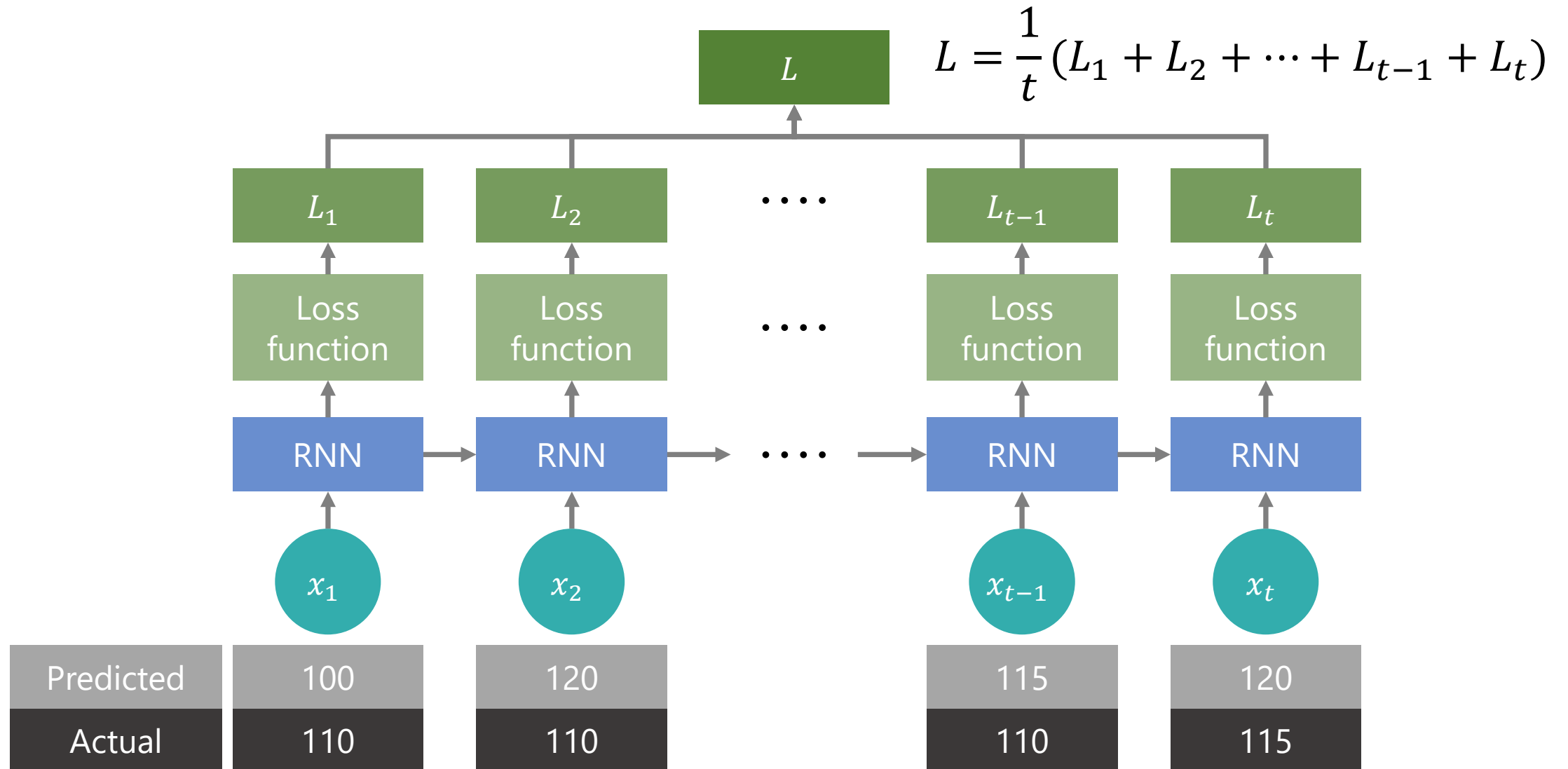
e.g., Video captioning



e.g., Machine translation

6. BPTT

Loss Function in RNN



BPTT

$$W = W - \eta \frac{\partial L}{\partial W} \quad \eta : \text{Learning rate}$$

$$\frac{\partial L}{\partial W} = \sum_{t=1}^T \frac{\partial L_t}{\partial W}$$

$$\frac{\partial L_T}{\partial W} = \frac{\partial L_T}{\partial h_T} \frac{\partial h_T}{\partial h_{T-1}} \frac{\partial h_{T-1}}{\partial h_{T-2}} \dots \frac{\partial h_3}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W}$$

$$= \frac{\partial L_t}{\partial h_T} \left(\prod_{t=2}^T \frac{\partial h_t}{\partial h_{t-1}} \right) \frac{\partial h_1}{\partial W}$$

BPTT (2)

$$\frac{\partial L_T}{\partial W} = \frac{\partial L_t}{\partial h_T} \left(\prod_{t=2}^T \frac{\partial h_t}{\partial h_{t-1}} \right) \frac{\partial h_1}{\partial W}$$

$$h_t = \tanh(h_{t-1}W_h + x_tW_x + b)$$

$$\frac{\partial h_t}{\partial h_{t-1}} = \tanh'(h_{t-1}W_h + x_tW_x + b) \cdot \frac{\partial(h_{t-1}W_h + x_tW_x + b)}{\partial h_{t-1}}$$

$$= \tanh'(h_{t-1}W_h + x_tW_x + b) \cdot W_h$$

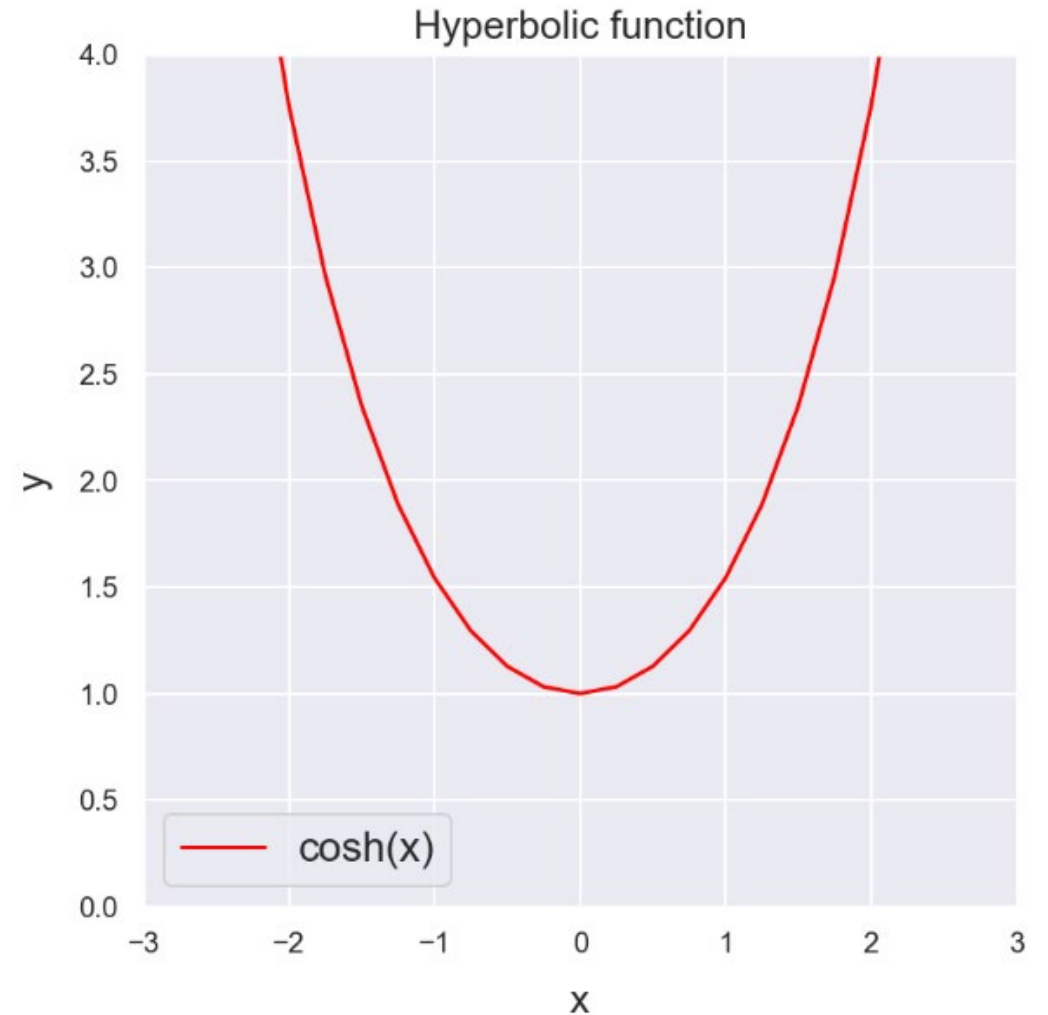
$$= \frac{\partial L_t}{\partial h_T} \left(\prod_{t=2}^T \tanh'(h_{t-1}W_h + x_tW_x + b) \cdot W_h \right) \frac{\partial h_1}{\partial W}$$

Vanishing and Exploding Gradients

$$\frac{\partial L_T}{\partial W} = \frac{\partial L_t}{\partial h_T} \left(\prod_{t=2}^T \tanh'(h_{t-1}W_h + x_tW_x + b) \cdot W_h \right) \frac{\partial h_1}{\partial W}$$

$$\tanh x' = \frac{1}{\cosh^2 x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$



7. LSTM

Long-Term Dependencies

Since I have lived in Japan for 10 years, I am good at speaking ().

Long-Term Dependencies

Since I have lived in Japan for 10 years, I am good at speaking (Japanese).

Long-Term Dependencies (Continued)

I lived in Japan when I was a high school student. In my high school days, I spent a lot of time playing baseball, and it was enjoyable. After graduating from high school, I came back to India and started working as a salesperson in a bookstore. Then, more than ten years passed. In high school, I met and played with my friends every day. However, now, there is no correspondence between us. So, now, I can no longer speak ().

Long-Term Dependencies (Continued)

I lived in Japan when I was a high school student. In my high school days, I spent a lot of time playing baseball, and it was enjoyable. After graduating from high school, I came back to India and started working as a salesperson in a bookstore. Then, more than ten years passed. In high school, I met and played with my friends every day. However, now, there is no correspondence between us. So, now, I can no longer speak (Japanese).

The information depends on far past information.

→ The training data must be a long sequence that RNN cannot handle.

LSTM Networks

- Long Short-Term Memory networks

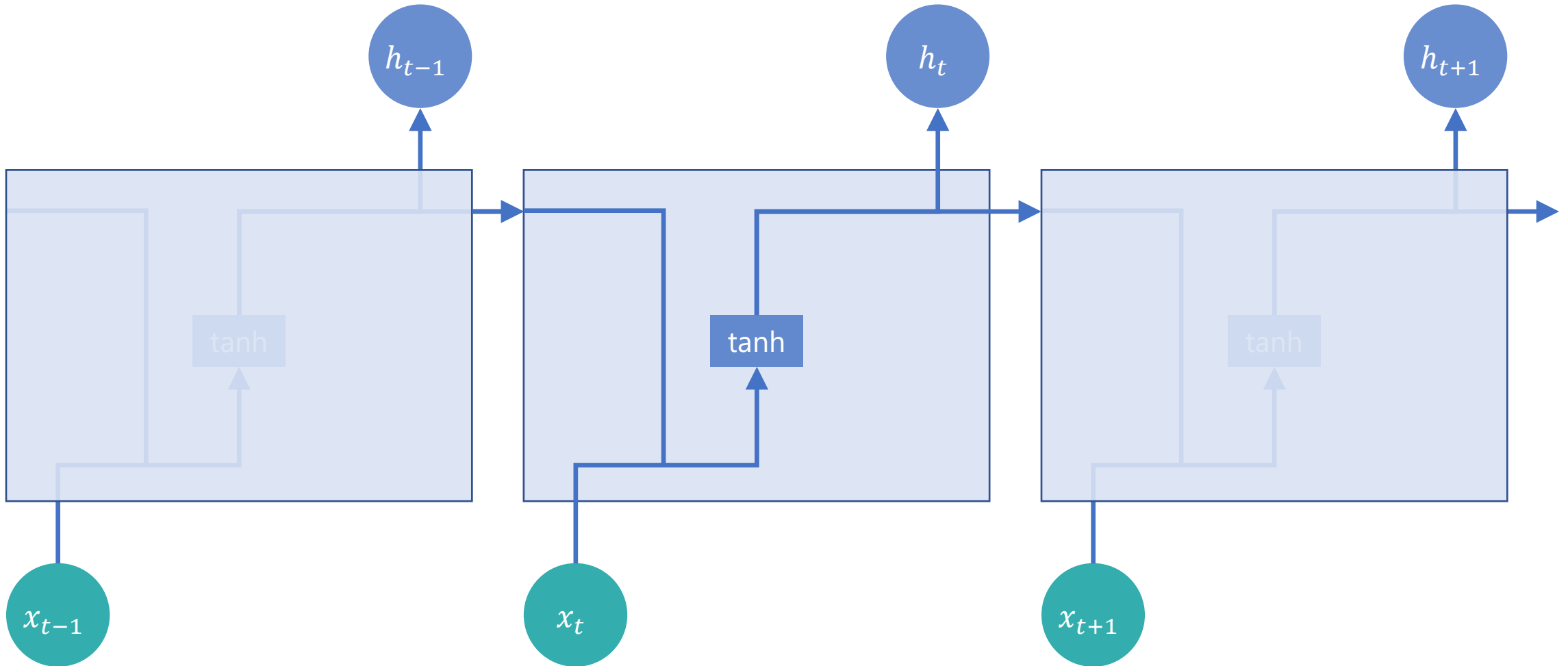
*Hochreiter, S., & Schmidhuber, J. (1997). Long short-term memory. Neural computation, 9(8), 1735-1780.

- Central idea: "Learning to forget."

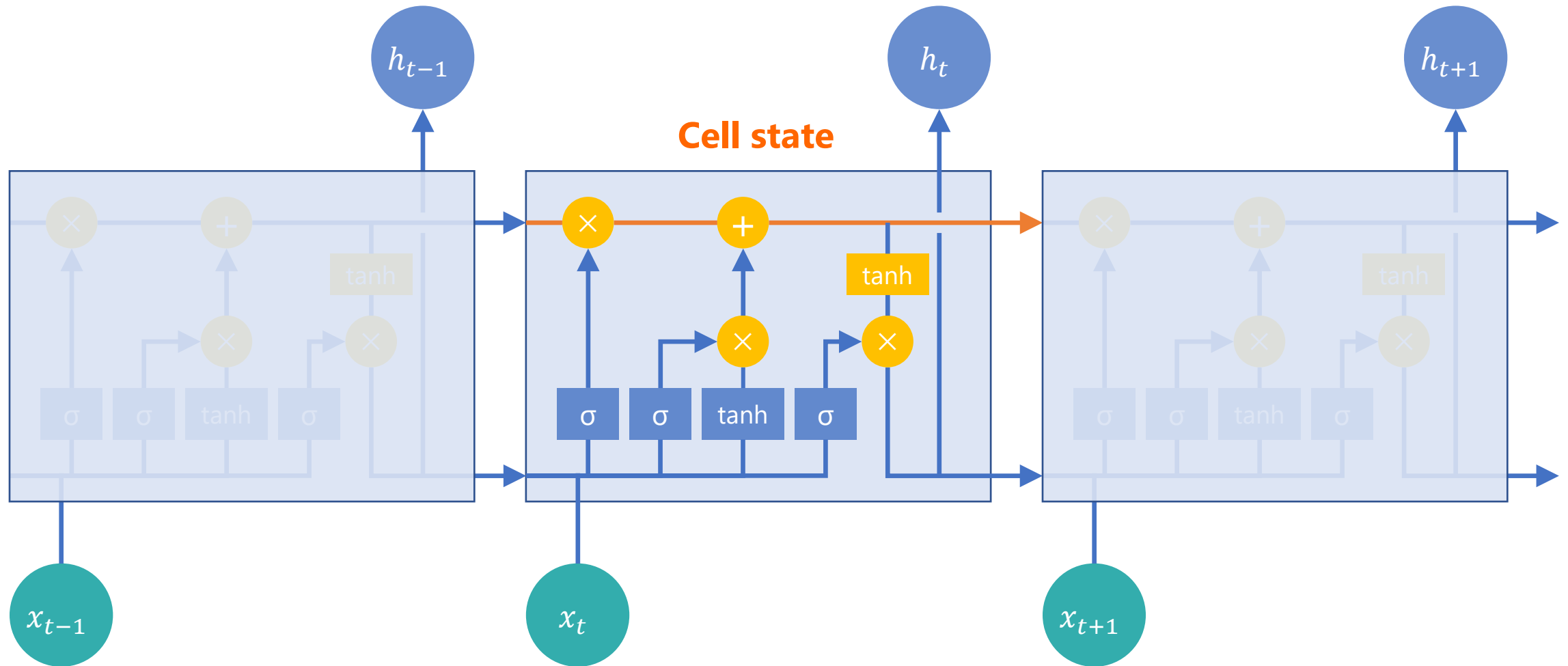
LSTM erases unnecessary memories.

→ It can prevent the vanishing gradient problem.

Structure of RNN



LSTM Block

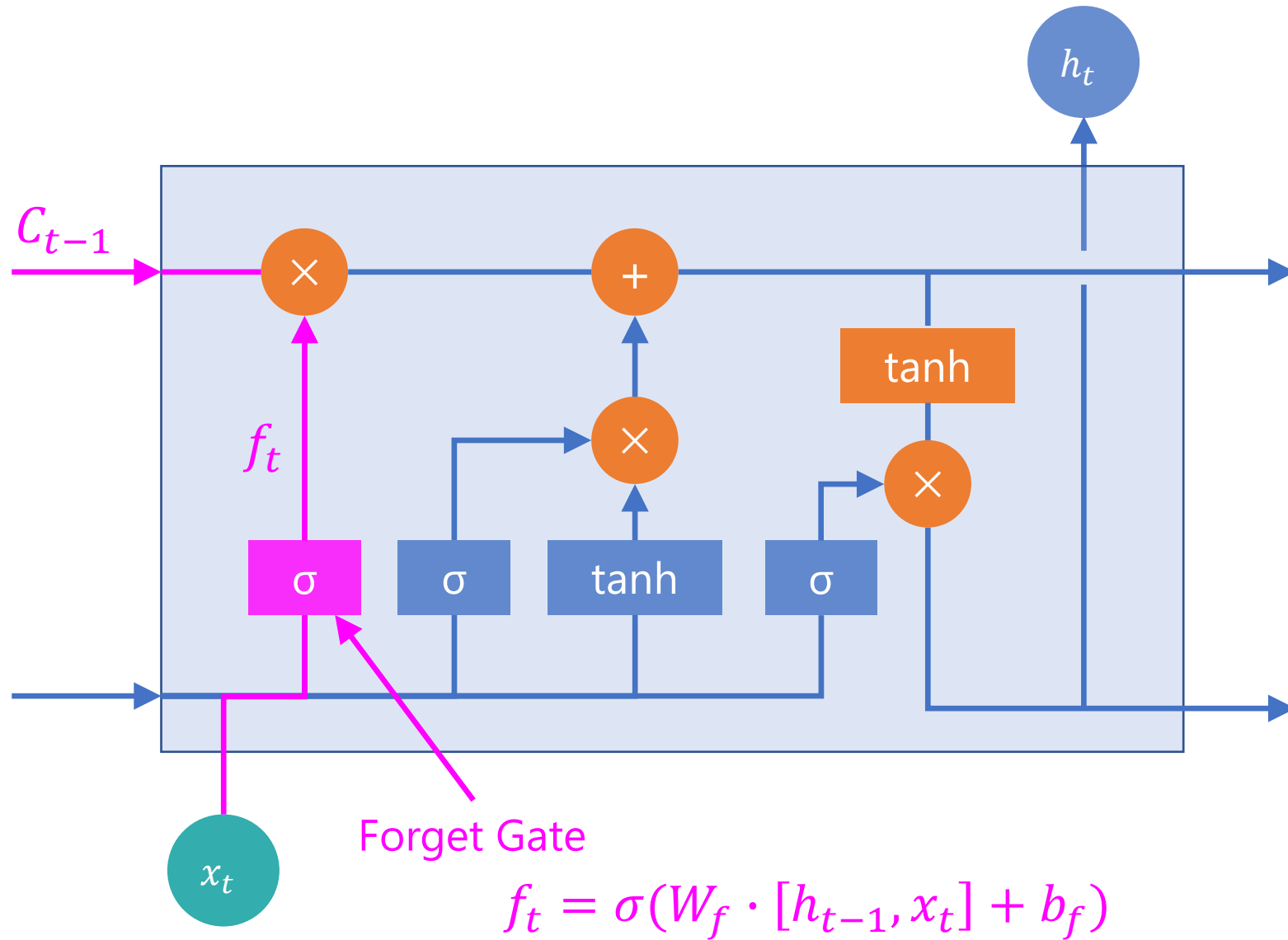


8. How does LSTM work?

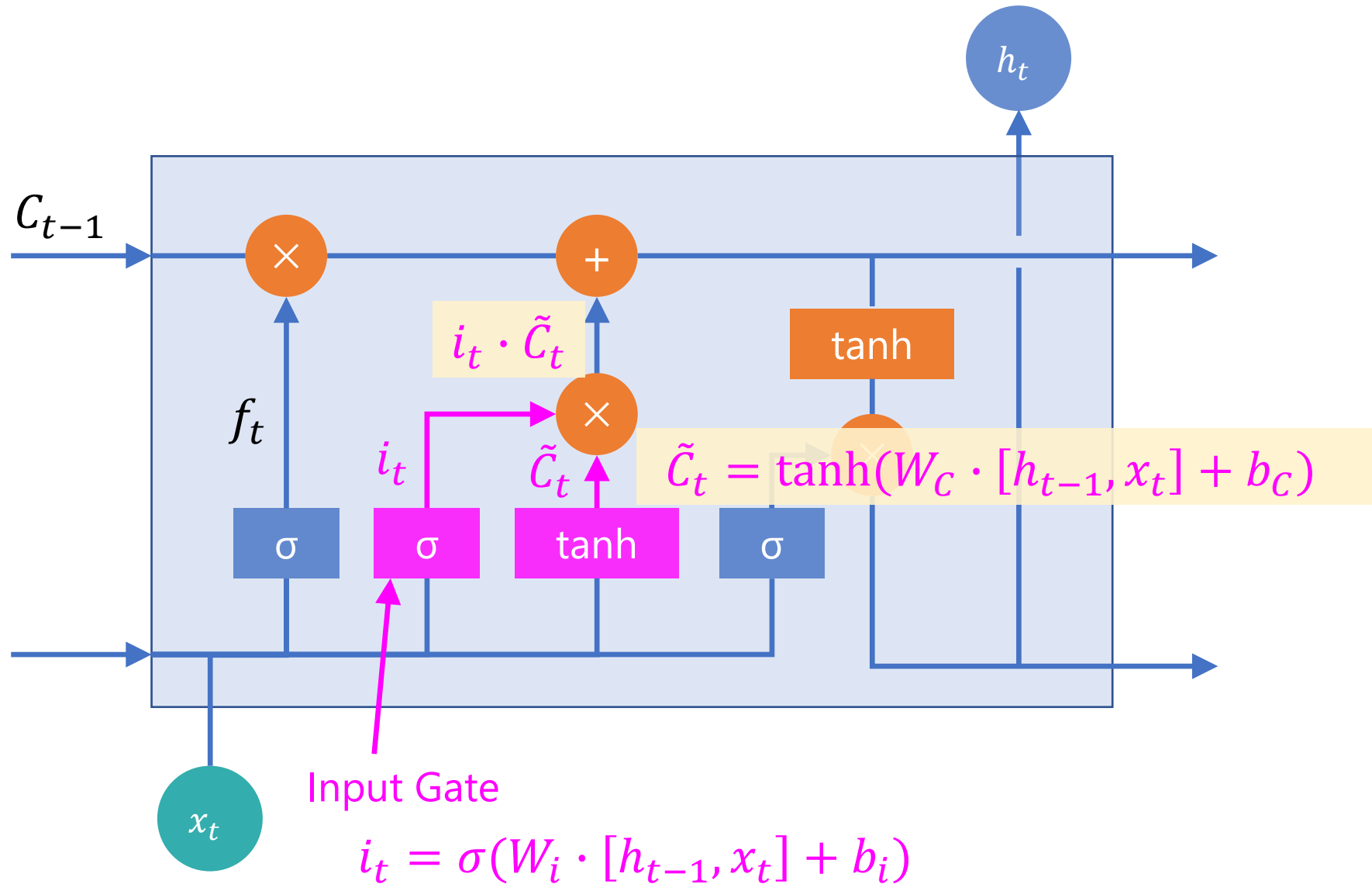
Gates

- Forget gate
- Input gate
- Output gate

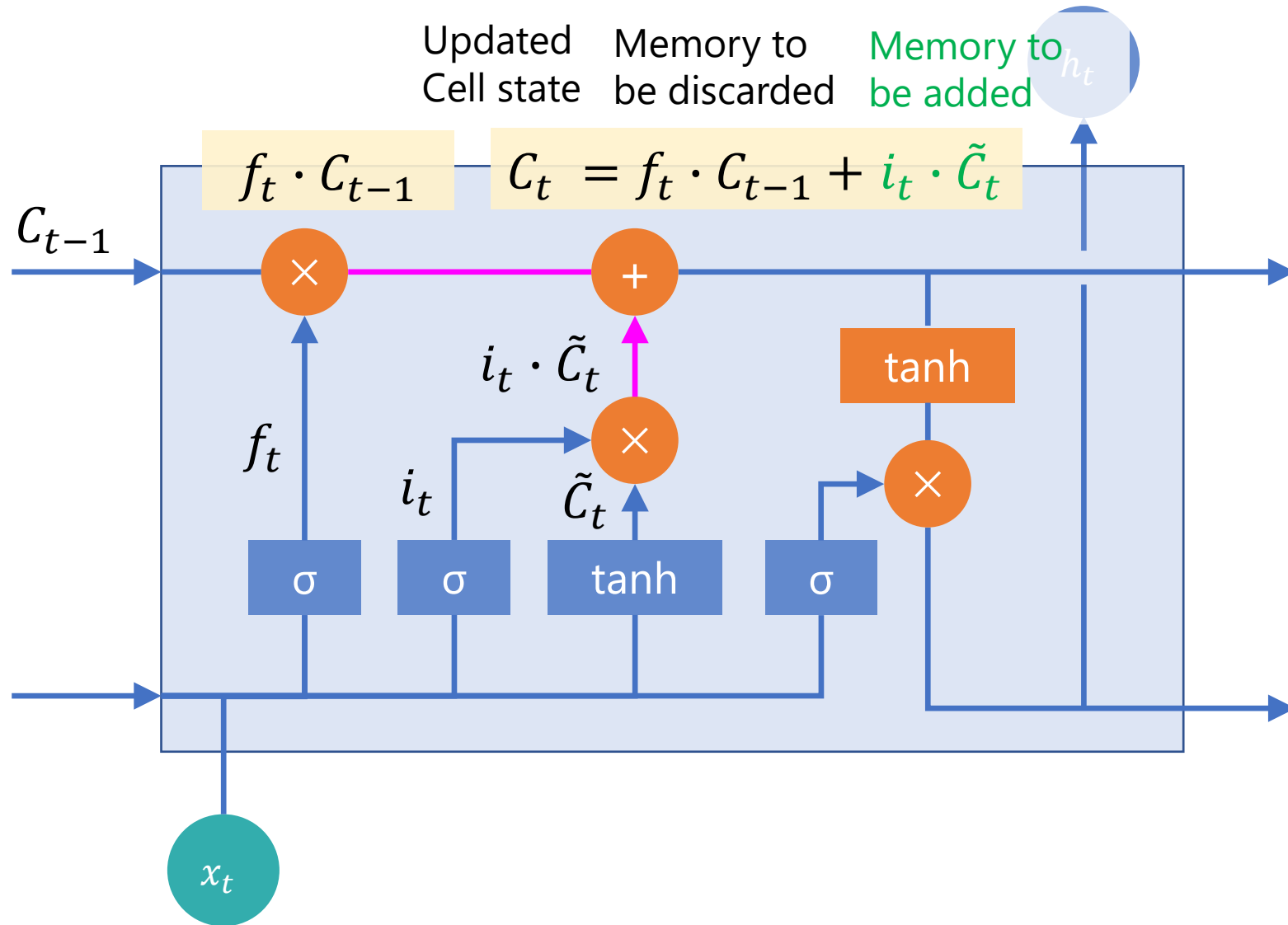
Forget Gate



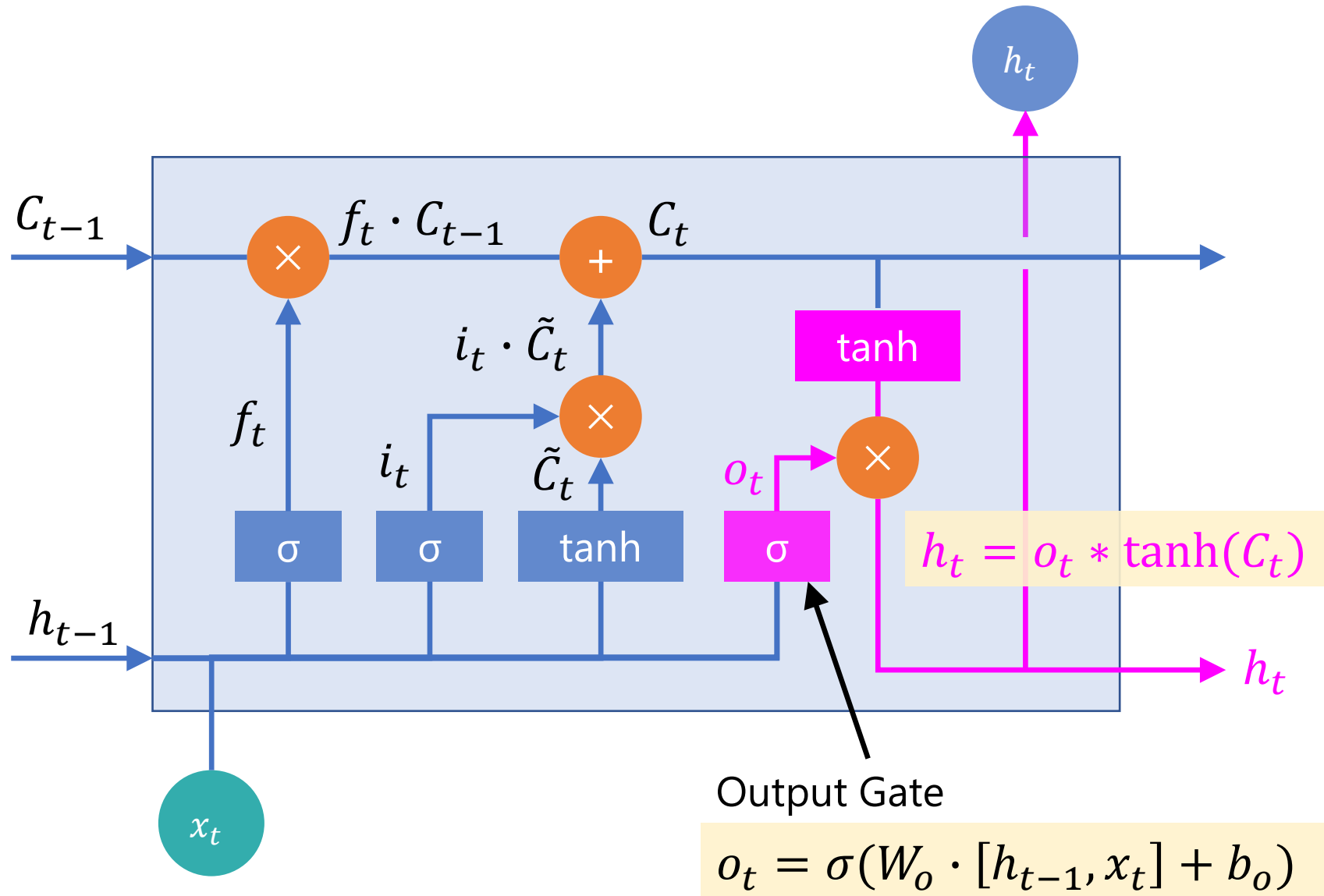
Input Gate



Input Gate (Continued)



Output Gate

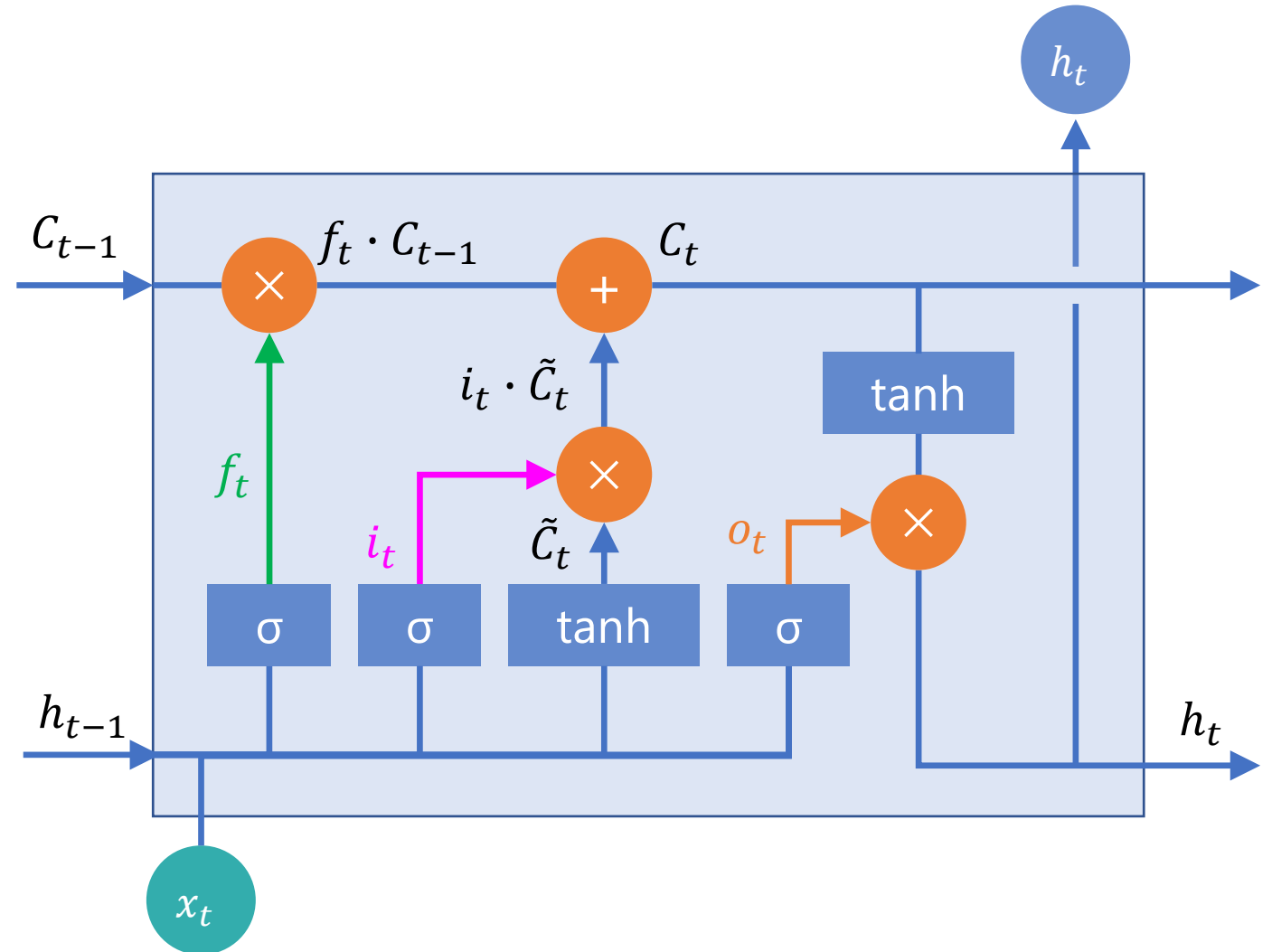


Peephole Architecture

$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f)$$

$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i)$$

$$o_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o)$$



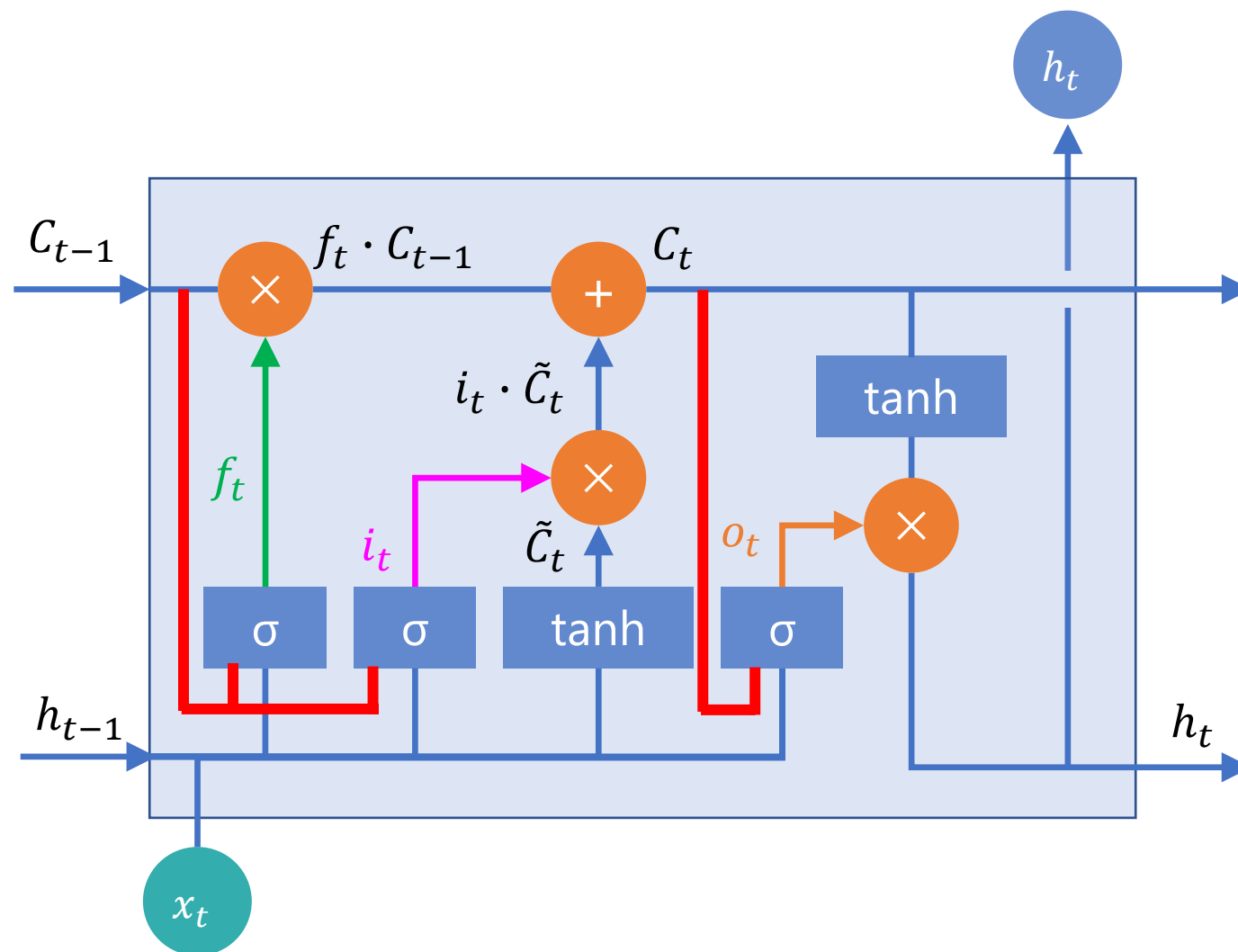
Peephole Architecture (Continued)

$$f_t = \sigma(W_f \cdot [C_{t-1}, h_{t-1}, x_t] + b_f)$$

$$i_t = \sigma(W_i \cdot [C_{t-1}, h_{t-1}, x_t] + b_i)$$

$$o_t = \sigma(W_o \cdot [C_{t-1}, h_{t-1}, x_t] + b_o)$$

Gers, F. A., & Schmidhuber, J. (2000, July).
Recurrent nets that time and count.
In *Proceedings of the IEEE-INNS-ENNS
International Joint Conference on Neural
Networks. IJCNN 2000. Neural Computing:
New Challenges and Perspectives for the
New Millennium* (Vol. 3, pp. 189-194). IEEE.



9. BPTT in LSTM

Gradient of Loss in LSTM


- LSTM can use gradients to update the parameters for optimization.

- The gradient of the loss:

$$\frac{\partial L}{\partial W} = \sum_{t=1}^T \frac{\partial L_t}{\partial W}$$

Gradient of Loss in LSTM

$$\frac{\partial L_T}{\partial W} = \frac{\partial L_T}{\partial h_T} \frac{\partial h_T}{\partial c_T} \frac{\partial c_T}{\partial c_{T-1}} \cdots \frac{\partial c_3}{\partial c_2} \frac{\partial c_2}{\partial c_1} \frac{\partial c_1}{\partial W}$$

$$= \frac{\partial L_T}{\partial h_T} \frac{\partial h_T}{\partial c_T} \left(\prod_{t=2}^T \frac{\partial c_t}{\partial c_{t-1}} \right) \frac{\partial c_1}{\partial W}$$


Cell State in LSTM

$$c_t = f_t \cdot c_{t-1} + i_t \cdot \tilde{c}_t$$

$$\frac{\partial c_t}{\partial c_{t-1}} = \frac{\partial (f_t \cdot c_{t-1} + i_t \cdot \tilde{c}_t)}{\partial c_{t-1}}$$

$$= \frac{\partial (f_t \cdot c_{t-1})}{\partial c_{t-1}} + \frac{\partial (i_t \cdot \tilde{c}_t)}{\partial c_{t-1}}$$

$$= \frac{\partial f_t}{\partial c_{t-1}} \cdot c_{t-1} + \frac{\partial c_{t-1}}{\partial c_{t-1}} \cdot f_t + \frac{\partial i_t}{\partial c_{t-1}} \cdot \tilde{c}_t + \frac{\partial \tilde{c}_t}{\partial c_{t-1}} \cdot i_t$$

Derivative of Cell State

$$\frac{\partial L_T}{\partial W} = \frac{\partial L_T}{\partial h_T} \frac{\partial h_T}{\partial c_T} \left(\prod_{t=2}^T \frac{\partial c_t}{\partial c_{t-1}} \right) \frac{\partial c_1}{\partial W}$$

$$\frac{\partial c_t}{\partial c_{t-1}} = \frac{\partial f_t}{\partial c_{t-1}} \cdot c_{t-1} + \frac{\partial c_{t-1}}{\partial c_{t-1}} \cdot f_t + \frac{\partial i_t}{\partial c_{t-1}} \cdot \tilde{c}_t + \frac{\partial \tilde{c}_t}{\partial c_{t-1}} \cdot i_t$$

$$\prod_{t=2}^T \left(\frac{\partial f_t}{\partial c_{t-1}} \cdot c_{t-1} + \frac{\partial c_{t-1}}{\partial c_{t-1}} \cdot f_t + \frac{\partial i_t}{\partial c_{t-1}} \cdot \tilde{c}_t + \frac{\partial \tilde{c}_t}{\partial c_{t-1}} \cdot i_t \right)$$

$$\prod_{t=2}^T (0.10 + 0.15 + 0.20 + 0.25)$$

$$\prod_{t=2}^T (0.10 \times 0.15 \times 0.20 \times 0.25)$$

10. GRU

(Gated Recurrent Unit)

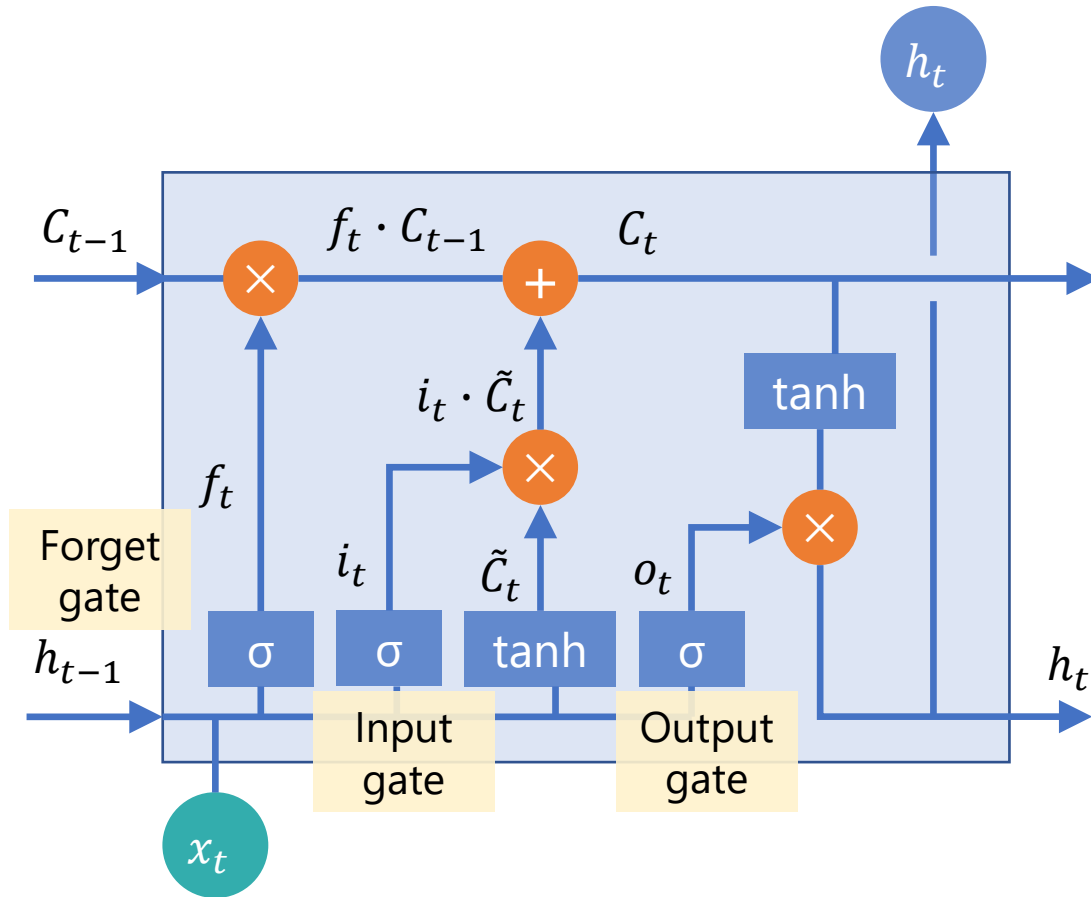
Weakness of LSTM

- LSTM consists of several gates, and thus, contains many parameters.
 - Its computational cost is high.
 - LSTM takes much time for model training.
- GRU (gated recurrent unit) was introduced to address this problem (Cho et al. 2014).

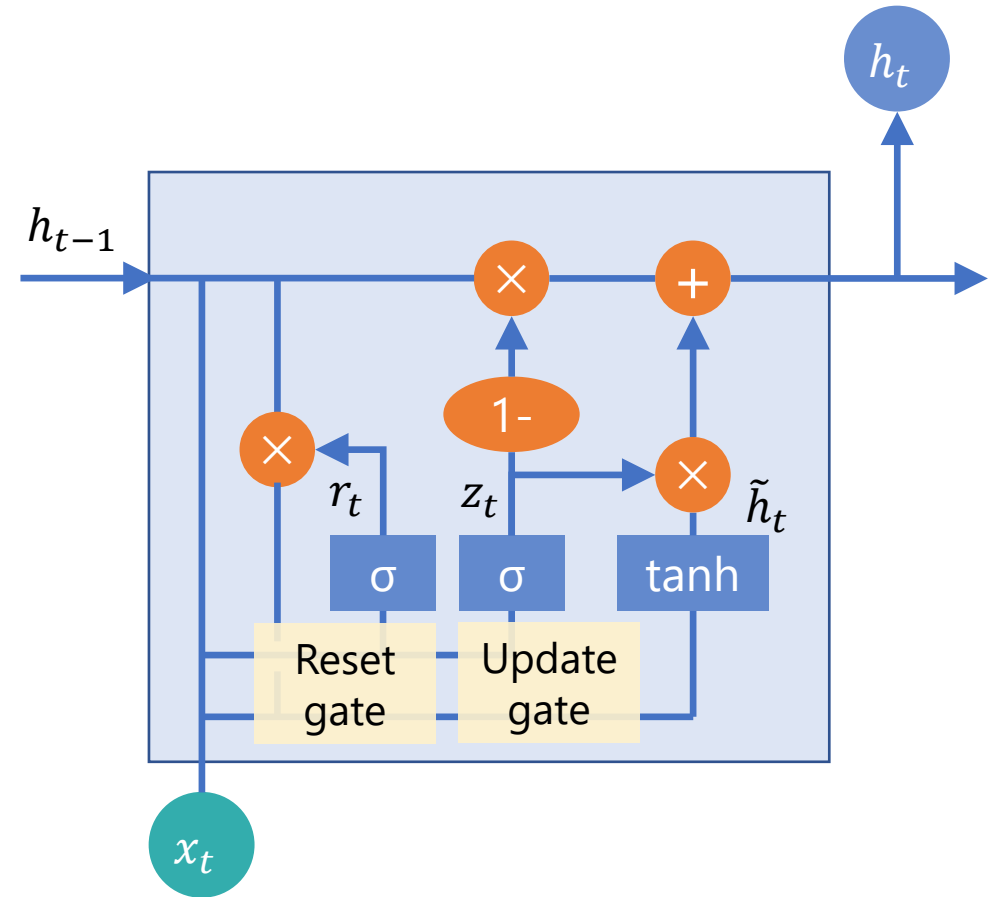
Cho, K., Van Merriënboer, B., Gulcehre, C., Bahdanau, D., Bougares, F., Schwenk, H., & Bengio, Y. (2014). Learning phrase representations using RNN encoder-decoder for statistical machine translation. arXiv preprint arXiv:1406.1078.

GRU

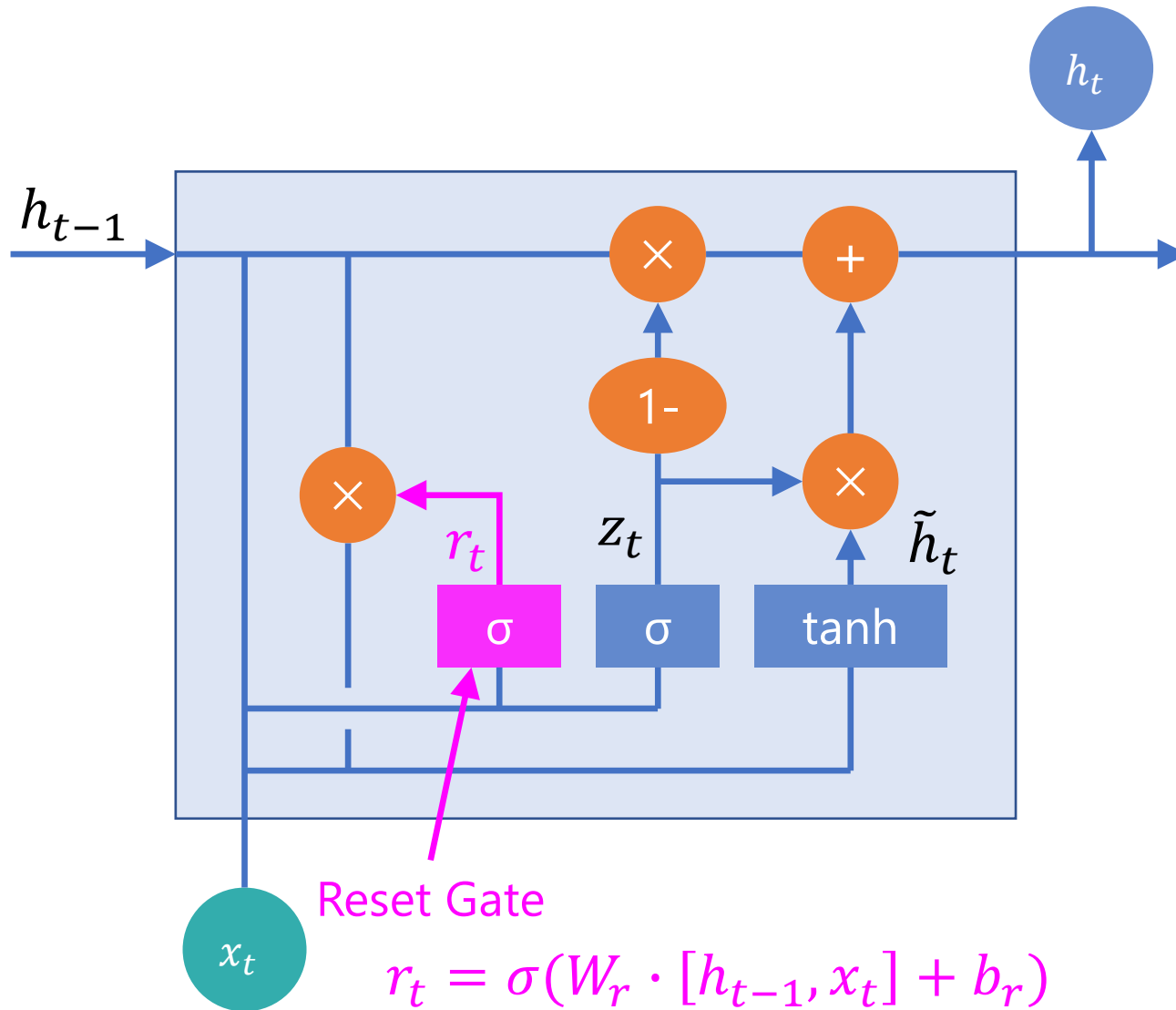
LSTM



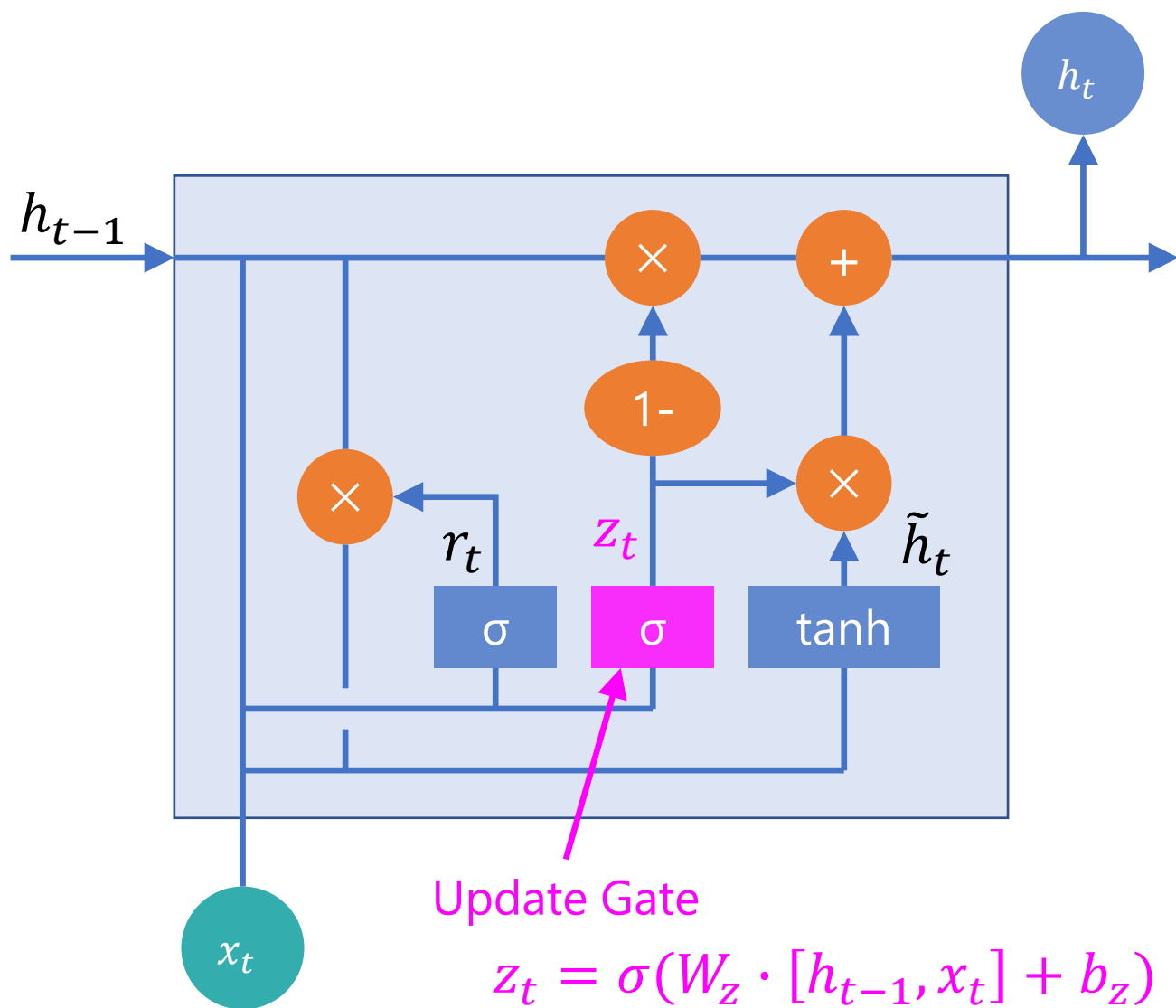
GRU



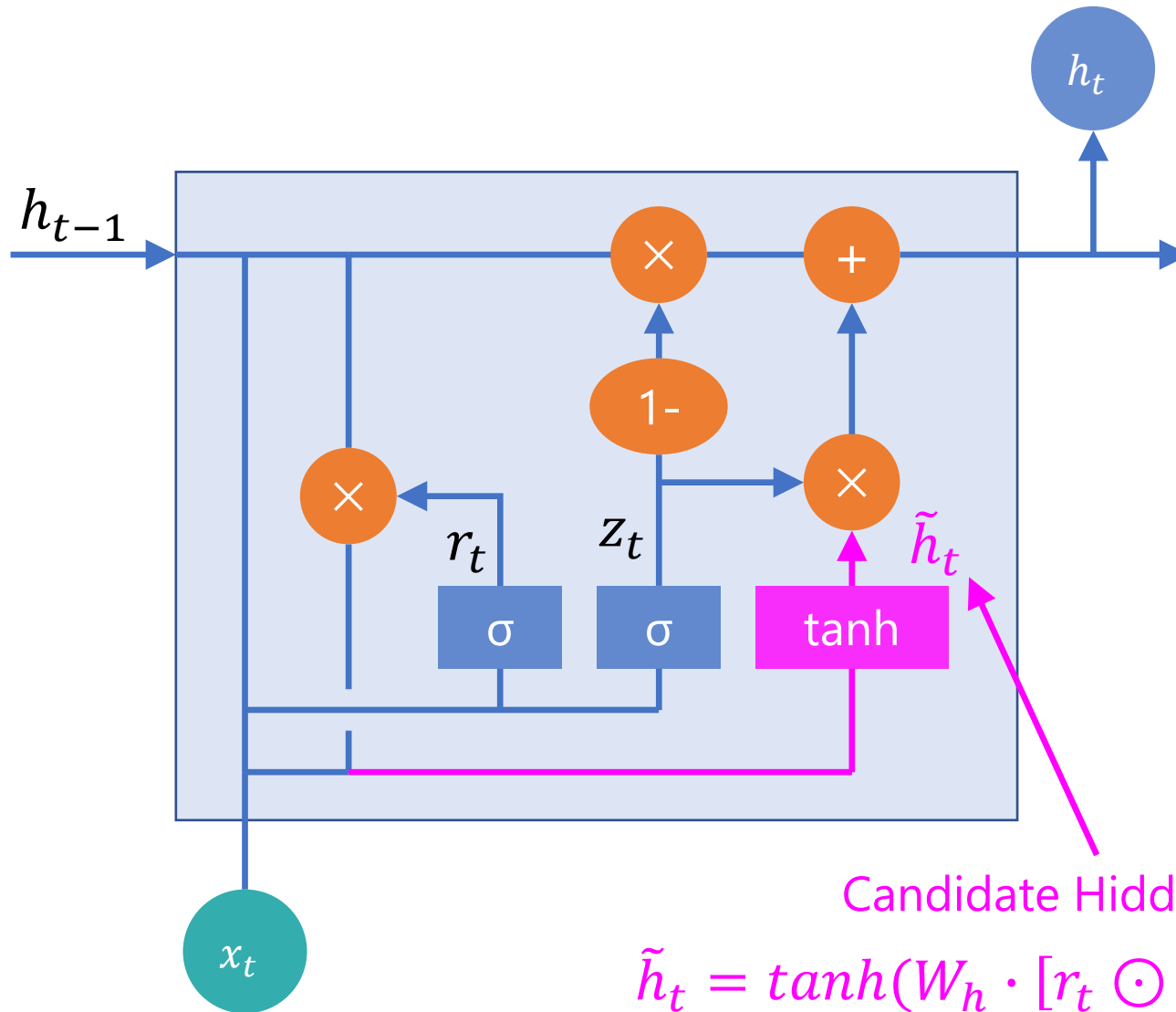
Reset Gate



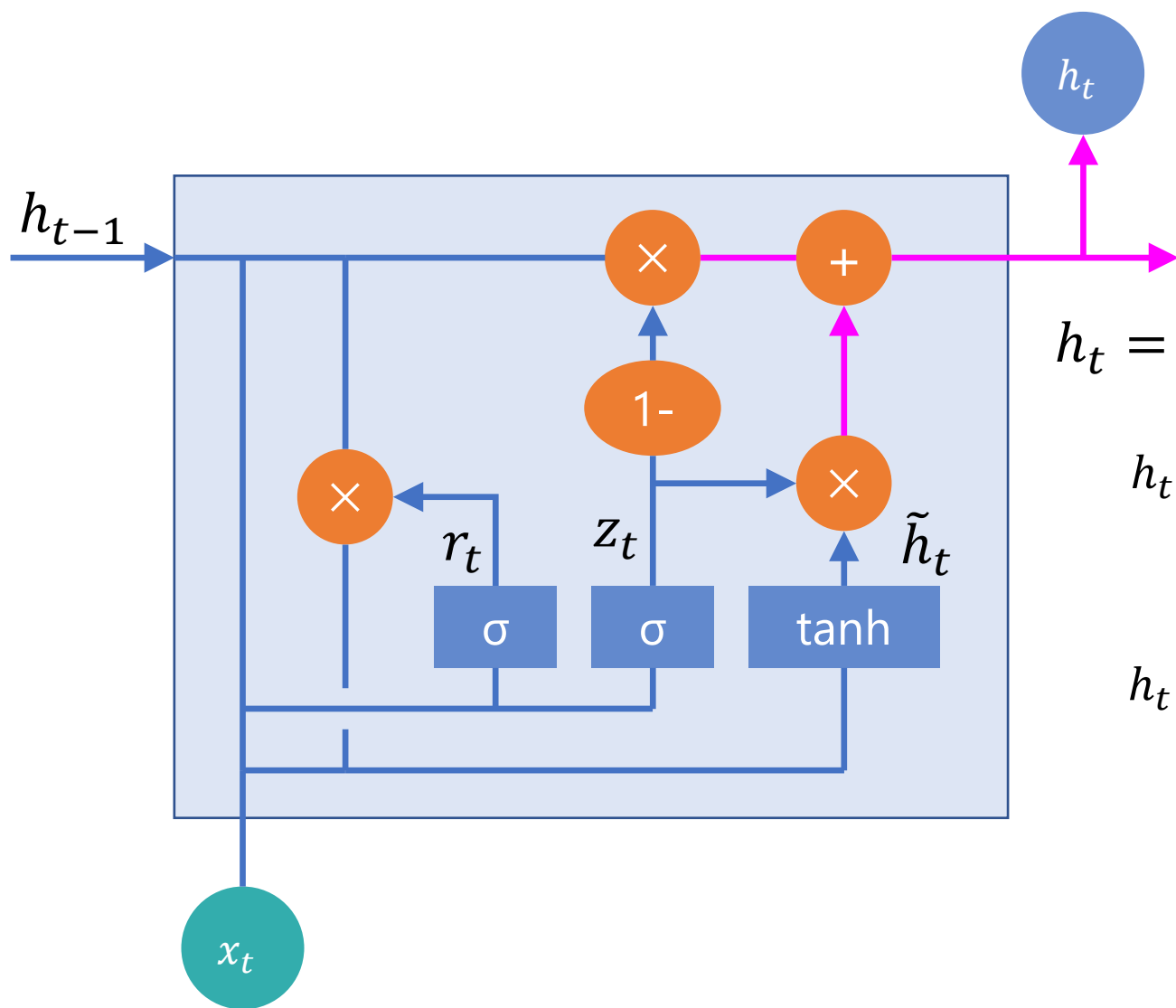
Update Gate



Candidate Hidden State



Hidden State



$$h_t = (1 - z_t) \odot h_{t-1} + z_t \odot \tilde{h}_t$$

$$\begin{aligned} h_t &= (1 - 0) \odot h_{t-1} + 0 \odot \tilde{h}_t \\ &= h_{t-1} \end{aligned}$$

$$\begin{aligned} h_t &= (1 - 1) \odot h_{t-1} + 1 \odot \tilde{h}_t \\ &= \tilde{h}_t \end{aligned}$$

Summary: GRU

- Simpler structure than LSTM
 - Reset gate: Short-term dependencies
 - Update gate: Long-term dependencies
- A simpler structure enables GRU to learn the long-term dependencies with a smaller number of parameters.
 - Lower computational cost.

11. RNN, LSTM and GRU with Python

Data: Nikkei Stock Average (Nikkei 225)

- Stock market index of the First Section of the Tokyo Stock Exchange (TSE), Japan.
- Nikkei Stock Average consists of 225 representative companies.
- Data source: Yahoo! finance.



Import Libraries

```
# Import libraries
```

```
import numpy as np
```

```
import pandas as pd
```

```
import seaborn as sns
```

```
import matplotlib.pyplot as plt
```

```
%matplotlib inline
```

```
from sklearn.preprocessing import MinMaxScaler
```

```
import math
```

```
from tensorflow.keras.models import Sequential
```

```
from tensorflow.keras.layers import Dense, Activation
```

```
from tensorflow.keras.layers import SimpleRNN, LSTM, GRU
```

```
from tensorflow.keras.optimizers import Adam
```

```
from tensorflow.keras.callbacks import EarlyStopping
```

Data Preparation

Import and show dataset

```
data = pd.read_csv("nikkei_stock_price.csv")  
print("Shape of Data: ", df.shape)  
data.head()
```

Shape of Data: (244, 2)

	date	closed_price
0	2021/5/6	29,331.37
1	2021/5/7	29,357.82
2	2021/5/10	29,518.34
3	2021/5/11	28,608.59
4	2021/5/12	28,147.51

Data information

```
data.info()
```

```
<class 'pandas.core.frame.DataFrame'>  
RangeIndex: 244 entries, 0 to 243  
Data columns (total 2 columns):  
date                244 non-null object  
closed_price        244 non-null object  
dtypes: object(2)  
memory usage: 3.9+ KB
```

Data Preprocessing (1)

Delete the commas from closed price

```
data['closed_price'] = data['closed_price'].str.replace(',', '')  
data.head()
```

	date	closed_price
0	2021/5/6	29331.37
1	2021/5/7	29357.82
2	2021/5/10	29518.34
3	2021/5/11	28608.59
4	2021/5/12	28147.51

Data Preprocessing (2)

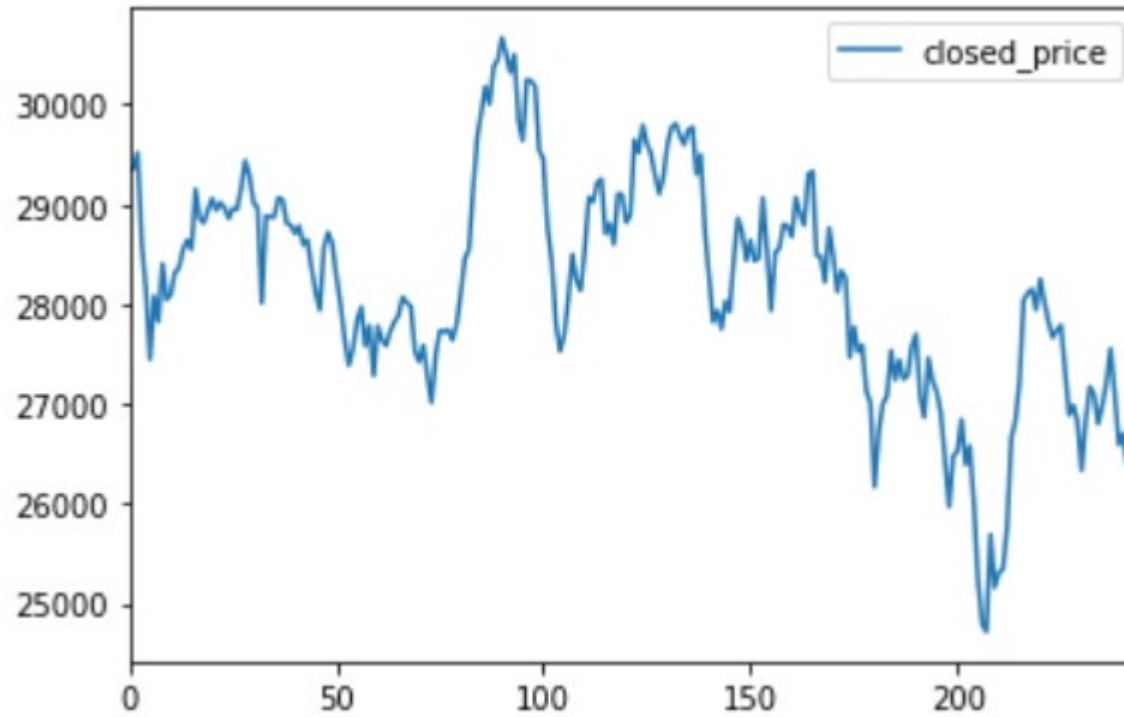
Convert closed price to numerical data.

```
data['closed_price'] = pd.to_numeric(data['closed_price'], errors = 'coerce')  
print(data.info())
```

```
<class 'pandas.core.frame.DataFrame'>  
RangeIndex: 244 entries, 0 to 243  
Data columns (total 2 columns):  
date                244 non-null object  
closed_price        244 non-null float64  
dtypes: float64(1), object(1)  
memory usage: 3.9+ KB  
None
```

Data Visualization

```
# Plot data  
data.plot()
```



Data Preprocessing (3)

Create a dataframe with only closed price

```
df=data.filter(['closed_price'])
```

Convert the dataframe to a NumPy array

```
df=df.values
```

```
print(df[:5])
```

```
[[29331.37]  
 [29357.82]  
 [29518.34]  
 [28608.59]  
 [28147.51]]
```


Data Preprocessing (4)

Normalize the data to make it applicable for RN

```
scaler=MinMaxScaler(feature_range=(0,1))
```

```
df_scaled=scaler.fit_transform(df)
```

```
df_scaled[:5]
```

```
array([[0.7751005 ],  
       [0.77954396],  
       [0.80651047],  
       [0.65367732],  
       [0.57621834]])
```

Data Preprocessing (5)

Split data into predictors and outcomes

Predict the present stock price by the past 5 days' stock prices

X=[]

y=[]

sequence=5

for i in range(len(df_scaled) - sequence):

 X.append(df_scaled[i:(i + sequence), 0])

 y.append(df_scaled[i + sequence, 0])

X, y = np.array(X), np.array(y)

Data Preprocessing (6)

Show the predictors and outcomes

```
print("Predictors")  
print(X[:5])  
print("Outcomes")  
print(y[:5])
```

Predictors

```
[[0.7751005  0.77954396 0.80651047 0.65367732 0.57621834]  
 [0.77954396 0.80651047 0.65367732 0.57621834 0.45870607]  
 [0.80651047 0.65367732 0.57621834 0.45870607 0.56562796]  
 [0.65367732 0.57621834 0.45870607 0.56562796 0.52200982]  
 [0.57621834 0.45870607 0.56562796 0.52200982 0.6197844  ]]
```

Outcomes

```
[0.45870607 0.56562796 0.52200982 0.6197844  0.55890481]
```

Data Preprocessing (7)

Reshape the predictor

So that it can be handled by RNN

```
X = np.reshape(X, (X.shape[0], X.shape[1], 1))
```

```
X.shape
```

```
(239, 5, 1)
```

Data Preprocessing (8)

Split data into training and test sets

Set the size of training and test data

Use 75% of the data for training

`train_size = math.ceil(len(X) * 0.75)`

Data Preprocessing (8)

Split data into training and test sets

Set the size of training and test data

Use 75% of the data for training

```
train_size = math.ceil(len(X) * 0.75)
```

Split X and y into training and test sets

```
X_train = X[:train_size, :]
```

```
y_train = y[:train_size]
```

```
X_test = X[train_size:len(X), :]
```

```
y_test = y[train_size:len(y)]
```

Show the size of training and test sets

```
print("X_train: ", X_train.shape)
```

```
print("y_train: ", y_train.shape)
```

```
print("X_test : ", X_test.shape)
```

```
print("y_test : ", y_test.shape)
```

```
X_train: (180, 5, 1)
```

```
y_train: (180,)
```

```
X_test : (59, 5, 1)
```

```
y_test : (59,)
```

Model Building (1) –RNN model

Build Simple RNN model

```
rnn=Sequential()
rnn.add(SimpleRNN(units=32, return_sequences=True, input_shape=(X_train.shape[1],1)))
rnn.add(SimpleRNN(units=32, return_sequences=True))
rnn.add(SimpleRNN(units=32, return_sequences=True))
rnn.add(SimpleRNN(units=32))
rnn.add(Dense(units=1))
```

```
rnn.compile(optimizer='adam',
            loss='mean_squared_error')
```

```
rnn.summary()
```

Layer (type)	Output Shape	Param #
=====	=====	=====
simple_rnn (SimpleRNN)	(None, 5, 32)	1088
simple_rnn_1 (SimpleRNN)	(None, 5, 32)	2080
simple_rnn_2 (SimpleRNN)	(None, 5, 32)	2080
simple_rnn_3 (SimpleRNN)	(None, 32)	2080
dense (Dense)	(None, 1)	33
=====	=====	=====
Total params: 7,361		
Trainable params: 7,361		
Non-trainable params: 0		
=====		

Model Building (2) –LSTM model

Build LSTM model

```
lstm=Sequential()  
lstm.add(LSTM(units=32, return_sequences=True, input_shape=(X_train.shape[1],1)))  
lstm.add(LSTM(units=32, return_sequences=True))  
lstm.add(LSTM(units=32, return_sequences=True))  
lstm.add(LSTM(units=32))  
lstm.add(Dense(units=1))
```

```
lstm.compile(optimizer='adam',  
             loss='mean_squared_error')
```

```
lstm.summary()
```

Layer (type)	Output Shape	Param #
=====	=====	=====
lstm (LSTM)	(None, 5, 32)	4352
lstm_1 (LSTM)	(None, 5, 32)	8320
lstm_2 (LSTM)	(None, 5, 32)	8320
lstm_3 (LSTM)	(None, 32)	8320
dense_1 (Dense)	(None, 1)	33
=====	=====	=====
Total params: 29,345		
Trainable params: 29,345		
Non-trainable params: 0		
=====		

Model Building (3) –GRU model

Build GRU model

```
gru=Sequential()
gru.add(GRU(units=32, return_sequences=True, input_shape=(X_train.shape[1],1)))
gru.add(GRU(units=32, return_sequences=True))
gru.add(GRU(units=32, return_sequences=True))
gru.add(GRU(units=32))
gru.add(Dense(units=1))
```

```
gru.compile(optimizer='adam',
            loss='mean_squared_error')
```

```
gru.summary()
```

Layer (type)	Output Shape	Param #
=====		
gru (GRU)	(None, 5, 32)	3360

gru_1 (GRU)	(None, 5, 32)	6336

gru_2 (GRU)	(None, 5, 32)	6336

gru_3 (GRU)	(None, 32)	6336

dense_2 (Dense)	(None, 1)	33
=====		
Total params: 22,401		
Trainable params: 22,401		
Non-trainable params: 0		

Set Early Stopping

Set Early Stopping

```
early_stop = EarlyStopping(monitor='val_loss', patience=40)
```

Model Training

Fit RNN model and store the history

```
rnn_history = rnn.fit(X_train, y_train,  
                      batch_size=16,  
                      epochs=1000,  
                      validation_split=0.2,  
                      callbacks=[early_stop],  
                      verbose=1)
```

Fit LSTM model and store the history

```
lstm_history = lstm.fit(X_train, y_train,  
                        batch_size=16,  
                        epochs=1000,  
                        validation_split=0.2,  
                        callbacks=[early_stop],  
                        verbose=1)
```

Fit GRU model and store the history

```
gru_history = gru.fit(X_train, y_train,  
                      batch_size=16,  
                      epochs=1000,  
                      validation_split=0.2,  
                      callbacks=[early_stop],  
                      verbose=1)
```

Visualize Training Progress

Visualize Training Progress

Create a list of subplot subtitles

```
titles = ['RNN', 'LSTM', 'GRU']
```

Create a list of prediction models

```
models = [rnn_history, lstm_history, gru_history]
```

Set the plot area

```
fig, ax = plt.subplots(1, 3, figsize=(16,4),  
                        tight_layout=True)
```

Plot learning progress

```
plt.suptitle('Learning History')
```

```
for i in range(0, 3):
```

```
    plt.subplot(1, 3, i+1)
```

```
    plt.title(titles[i], fontsize=10)
```

```
    plt.plot(models[i].history['loss'], label='loss')
```

```
    plt.plot(models[i].history['val_loss'], label='val loss')
```

```
    plt.xlabel('epoch')
```

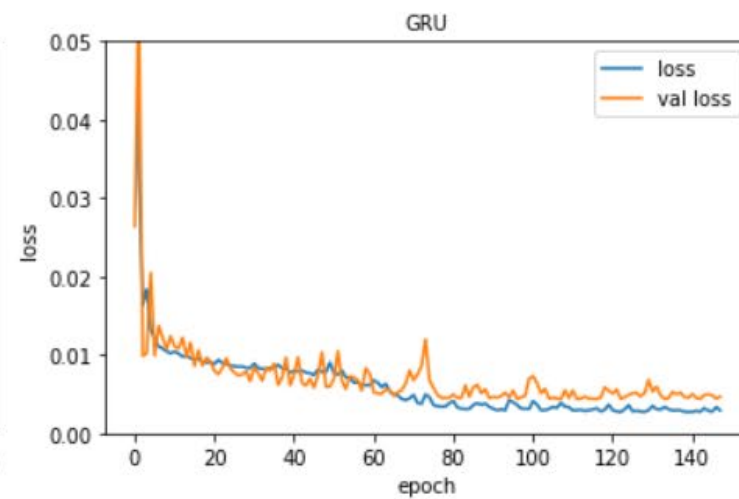
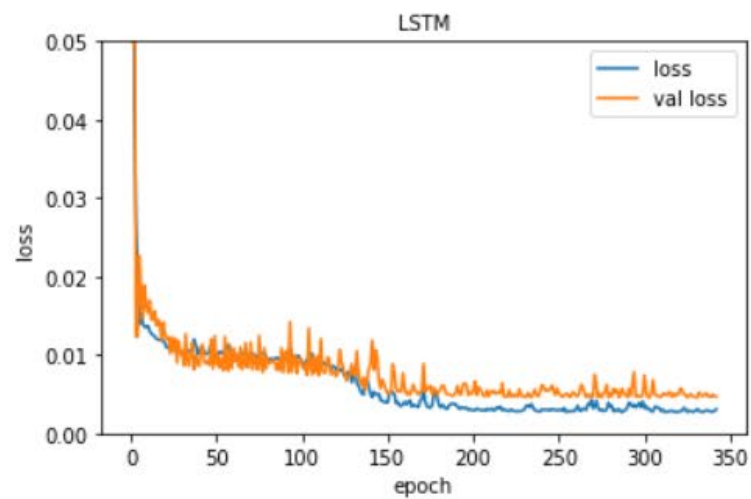
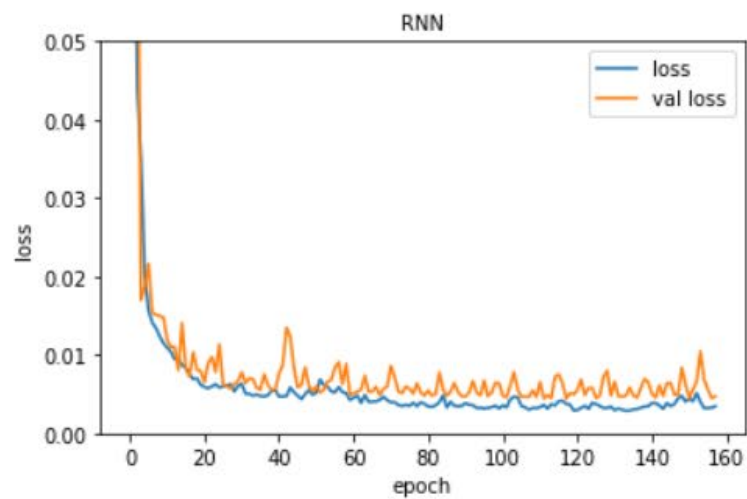
```
    plt.ylabel('loss')
```

```
    plt.legend(loc='best')
```

```
    plt.ylim([0,0.05])
```

Visualize Training Progress

Learning History



Make Predictions

Make predictions and reverse the predicted values to actual values

Predict by RNN model

```
rnn_y_pred=rnn.predict(X_test)
```

```
rnn_y_pred=scaler.inverse_transform(rnn_y_pred)
```

Predict by LSTM model

```
lstm_y_pred=lstm.predict(X_test)
```

```
lstm_y_pred=scaler.inverse_transform(lstm_y_pred)
```

Predict by GRU model

```
gru_y_pred=gru.predict(X_test)
```

```
gru_y_pred=scaler.inverse_transform(gru_y_pred)
```

Reverse test data to actual values

```
y_test=y_test.reshape(y_test.shape[0],1)
```

```
y_test=scaler.inverse_transform(y_test)
```

Visualize Prediction Results

Visualize Prediction Results

Set subplot subtitles

```
titles = ['RNN', 'LSTM', 'GRU']
```

Create a list of prediction models

```
models = [rnn_y_pred, lstm_y_pred, gru_y_pred]
```

Set the plot area

```
fig, ax = plt.subplots(1, 3, figsize=(16,4),  
                        tight_layout=True)
```

Set the title

```
plt.suptitle('Japan Stock Price')
```

Create and show subplots

```
for i in range(0, 3):
```

```
    plt.subplot(1, 3, i+1)
```

```
    plt.title(titles[i], fontsize=10)
```

```
    plt.plot(y_test, label='Real Stock Price')
```

```
    plt.plot(models[i], label=titles[i]+' Prediction')
```

```
    plt.legend()
```

```
    plt.xlabel('Time')
```

```
    plt.ylabel('Stock Price')
```

Visualize Prediction Results

