UCR-P231-FLIP-F2020

Clever Paper Title

Flip Tanedo
flip.tanedo@ucr.edu
Department of Physics & Astronomy, University of California, Riverside, CA 92521

Abstract

This is a crash course on mathematical methods necessary to succeed in the first-year physics graduate curriculum at UC Riverside. The focus is how to solve differential equations using Green's functions.

Contents

1 Introduction: Why mathematical methods?

Physics 231: Methods of Theoretical Physics is a course for first-year physics and astronomy graduate students. It is a 'crash course' in mathematical methods necessary for graduate courses in electrodynamics, quantum mechanics, and statistical mechanics. It is a *boot camp* rather than a rigorous theorem–proof mathematics class. Where possible, the emphasis is on physical intuition rather than mathematical precision. lec 01

1.1 Green's functions

Our primary goal is to solve linear differential equations:

$$\mathcal{O}f(x) = s(x) \ . \tag{1.1}$$

In this equation, $\mathcal{O}$ is a *differential operator* that encodes some kind of physical dynamics, say $\mathcal{O} = (d/dx)^2 + 3x\,(d/dx)$. $s(x)$ is the *source* of those dynamics. Finally, $f(x)$ is the system's physical *response* that we would like to determine. The solution to this equation is:

$$f(x) = \mathcal{O}^{-1}s(x) \ . \tag{1.2}$$

Simply writing that is deeply unsatisfying! In this course, we think carefully about what $\mathcal{O}^{-1}$ actually *means* and how we can calculate it. As you may have guessed, $\mathcal{O}^{-1}$ is the **Green's function** for the differential operator $\mathcal{O}$.

We will approach this problem by analogy to linear algebra, where a linear transformation A acting on a vector space can give equations like:

$$A\mathbf{v} = \mathbf{w} \ , \tag{1.3}$$

whose solution is

$$\mathbf{v} = A^{-1}\mathbf{w} \ . \tag{1.4}$$

We will connect the notion of a linear differential operator to a matrix in infinite dimensional space to give a working definition of $\mathcal{O}^{-1}$. We will then pull out a bag of tricks from complex analysis to formally solve $\mathcal{O}^{-1}s(x)$ given $\mathcal{O}$ and $s(x)$.

Acknowledgements

P.T. the students of Physics 231 for their patience and feedback on on this course.