HCAI5DS02_AnanyaDahal_2408840_Week-5,6.ipynb

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```python
#Ananya Dahal
#2408840


import scipy.stats as stats
import numpy as np

# Given data
n = 50
sample_mean = 4.2
sample_std = 1.1
confidence = 0.95

# Step 1: compute the standard error
se = sample_std / np.sqrt(n)

# Step 2: get the critical t value (two-tailed)
alpha = 1 - confidence
t_crit = stats.t.ppf(1 - alpha/2, df=n - 1)

# Step 3: compute the margin of error
moe = t_crit * se

# Step 4: build the confidence interval
ci_lower = sample_mean - moe
ci_upper = sample_mean + moe
ci = (ci_lower, ci_upper)

print("t-critical:", t_crit)
print("Standard error:", se)
print("Margin of error:", moe)
print("95% CI for mean delivery time:", ci)
```

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```python
[20]
alpha = 1 - confidence
t_crit = stats.t.ppf(1 - alpha/2, df=n - 1)

# Step 3: compute the margin of error
moe = t_crit * se

# Step 4: build the confidence interval
ci_lower = sample_mean - moe
ci_upper = sample_mean + moe
ci = (ci_lower, ci_upper)

print("t-critical:", t_crit)
print("Standard error:", se)
print("Margin of error:", moe)
print("95% CI for mean delivery time:", ci)
```

```
t-critical: 2.0095752371292397
Standard error: 0.15556349186104046
Margin of error: 0.31261654104530295
95% CI for mean delivery time: (np.float64(3.887383458954697), np.float64(4.512616541045303))
```

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```
95% CI for mean delivery time: (np.float64(3.887383458954697), np.float64(4.512616541045303))
```

```python
import scipy.stats as stats
import numpy as np

# Given data
n = 400
x = 128
confidence = 0.90

# Step 1: compute sample proportion
p_hat = x / n

# Step 2: compute standard error for proportion
se_proportion = np.sqrt(p_hat * (1 - p_hat) / n)

# Step 3: find the z critical value
alpha = 1 - confidence
z_crit = stats.norm.ppf(1 - alpha/2)

# Step 4: compute confidence interval
ci_lower_proportion = p_hat - z_crit * se_proportion
ci_upper_proportion = p_hat + z_crit * se_proportion
ci_proportion = (ci_lower_proportion, ci_upper_proportion)

print("Sample proportion:", p_hat)
print("Standard error (proportion):", se_proportion)
print("z-critical:", z_crit)
print("90% CI for proportion:", ci_proportion)
```

```
Sample proportion: 0.32
Standard error (proportion): 0.0233238075793812
z-critical: 1.6448536269514722
90% CI for proportion: (np.float64(0.28163575050873657), np.float64(0.35836424949126344))
```

```python
import scipy.stats as stats
import numpy as np

# Given data
n1, mean1, std1 = 40, 5200, 610
n2, mean2, std2 = 35, 4900, 580
confidence = 0.95

# Step 1: compute standard error for the difference
se_diff = np.sqrt(std1**2 / n1 + std2**2 / n2)

# Step 2: degrees of freedom (Welch's approximation)
dof = (std1**2/n1 + std2**2/n2)**2 / ((std1**2/n1)**2 / (n1 - 1) + (std2**2/n2)**2 / (n2 - 1))

# Step 3: get critical t value
alpha = 1 - confidence
t_crit_diff = stats.t.ppf(1 - alpha/2, df=dof)

# Step 4: compute confidence interval for the difference
ci_lower_diff = (mean1 - mean2) - t_crit_diff * se_diff
ci_upper_diff = (mean1 - mean2) + t_crit_diff * se_diff
ci_diff = (ci_lower_diff, ci_upper_diff)

print("Standard error (difference):", se_diff)
print("Degrees of freedom:", dof)
print("t-critical (difference):", t_crit_diff)
print("95% CI for difference in means:", ci_diff)
```

```
Standard error (difference): 137.52791924343424
Degrees of freedom: 72.47603755653496
t-critical (difference): 1.993239891612047
95% CI for difference in means: (np.float64(25.87386515358679), np.float64(574.1261348464132))
```

Task

Simulate 100 samples of size 30 from a normal distribution with mean 50 and standard deviation 10. For each sample, compute a 95% confidence interval for the mean. Plot the confidence intervals, coloring them based on whether they contain the true mean.

Set up simulation parameters

Subtask:

Define the true mean and standard deviation of the population, the sample size, the number of simulations, and the confidence level.

Reasoning: The subtask requires defining several variables with specific values. I will define these variables in a single code block.

[16]
```
true_mean = 50
true_std = 10
sample_size = 30
num_simulations = 100
confidence_level = 0.95
```

Simulate samples and calculate confidence intervals

Subtask:

Generate multiple random samples from the population and calculate the confidence interval for the mean for each sample.

Reasoning: Generate multiple random samples, calculate the confidence interval for each sample, and check if the true mean is within the interval.

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```python
confidence_intervals = []
contains_true_mean = []

for _ in range(num_simulations):
    # Generate a random sample
    sample = np.random.normal(loc=true_mean, scale=true_std, size=sample_size)

    # Calculate sample statistics
    sample_mean = np.mean(sample)
    sample_std = np.std(sample, ddof=1) # Use ddof=1 for sample standard deviation

    # Calculate standard error
    se = sample_std / np.sqrt(sample_size)

    # Calculate critical t-value
    alpha = 1 - confidence_level
    t_crit = stats.t.ppf(1 - alpha/2, df=sample_size - 1)

    # Calculate margin of error
    moe = t_crit * se

    # Calculate confidence interval
    ci_lower = sample_mean - moe
    ci_upper = sample_mean + moe
    confidence_intervals.append((ci_lower, ci_upper))

    # Check if the true mean is within the interval
    contains_true_mean.append(ci_lower <= true_mean <= ci_upper)

print("Number of confidence intervals calculated:", len(confidence_intervals))
print("Number of intervals containing the true mean:", sum(contains_true_mean))
```

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```
[17]    # check if the true mean is within the interval
        contains_true_mean.append(ci_lower <= true_mean <= ci_upper)

    print("Number of confidence intervals calculated:", len(confidence_intervals))
    print("Number of intervals containing the true mean:", sum(contains_true_mean))
```

```
Number of confidence intervals calculated: 100
Number of intervals containing the true mean: 98
```

Visualize the confidence intervals

Subtask:

Plot the calculated confidence intervals, color-coding them based on whether they contain the true mean.

Reasoning: Plot the calculated confidence intervals, color-coding them based on whether they contain the true mean, and add a vertical line for the true mean.

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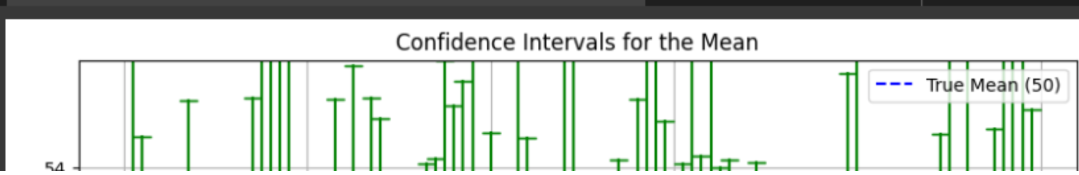
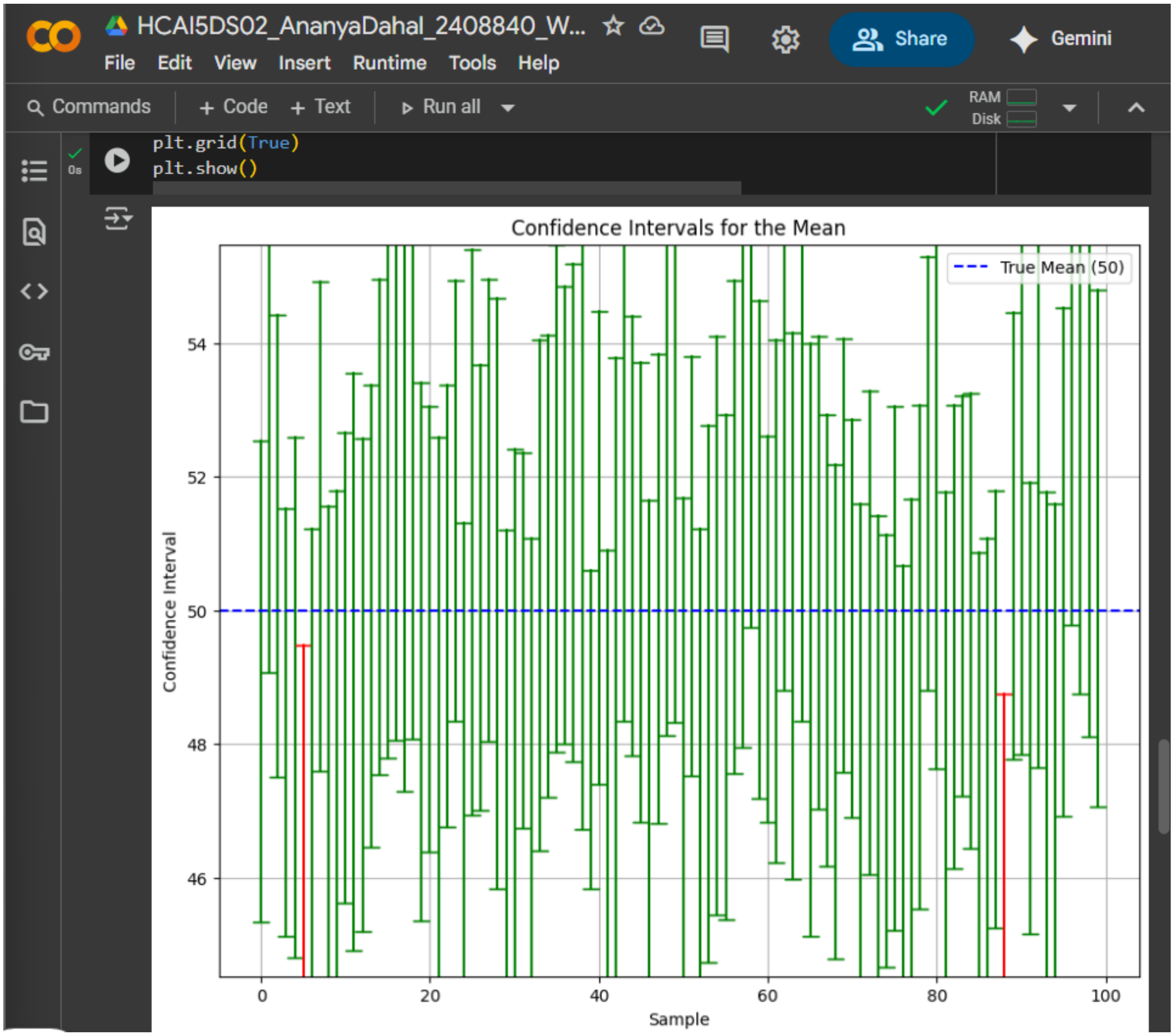
```
import matplotlib.pyplot as plt

plt.figure(figsize=(10, 8))

for i, ci in enumerate(confidence_intervals):
    color = 'green' if contains_true_mean[i] else 'red'
    plt.plot([i, i], [ci[0], ci[1]], marker='_', color=color, markersize=10)
    plt.plot(i, ci[0], marker='_', color=color, markersize=10)
    plt.plot(i, ci[1], marker='_', color=color, markersize=10)


plt.axhline(y=true_mean, color='blue', linestyle='--', label=f'True Mean ({true_mean})')

plt.ylim(true_mean - 3 * true_std / np.sqrt(sample_size), true_mean + 3 * true_std / np.sqrt(s
plt.xlabel('Sample')
plt.ylabel('Confidence Interval')
plt.title('Confidence Intervals for the Mean')
plt.legend()
plt.grid(True)
plt.show()
```

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plt.grid(True)
plt.show()
Confidence Intervals for the Mean
True Mean (50)
Confidence Interval
Sample
46
48
50
52
54
0
20
40
60
80
100

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Analyze and interpret the results

Subtask:

Examine the plot to see the proportion of confidence intervals that contain the true mean and relate this to the chosen confidence level.

Reasoning: Calculate the proportion of confidence intervals that contain the true mean and compare it to the confidence level.

```
# Count the number of intervals containing the true mean (green lines)
num_containing_true_mean = sum(contains_true_mean)

# Calculate the proportion
proportion_containing_true_mean = num_containing_true_mean / num_simulations

# Compare the proportion to the confidence level
print(f"Number of confidence intervals containing the true mean: {num_containing_true_mean}")
print(f"Proportion of intervals containing the true mean: {proportion_containing_true_mean:.4f
print(f"Chosen confidence level: {confidence_level}")

# Analysis
print("\nAnalysis:")
print(f"The observed proportion ({proportion_containing_true_mean:.4f}) of confidence interval
print(f"is close to the chosen confidence level ({confidence_level}). This is expected, as a 9
print("confidence level implies that, in the long run, approximately 95% of the confidence int
print("constructed from random samples will contain the true population parameter.")
```

```
Number of confidence intervals containing the true mean: 98
Proportion of intervals containing the true mean: 0.9800
```

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```
print(f" is close to the chosen confidence level ({confidence_level}). This is expected, as a 9
print("confidence level implies that, in the long run, approximately 95% of the confidence int
print("constructed from random samples will contain the true population parameter.")
```

```
Number of confidence intervals containing the true mean: 98
Proportion of intervals containing the true mean: 0.9800
Chosen confidence level: 0.95

Analysis:
The observed proportion (0.9800) of confidence intervals that contain the true mean
is close to the chosen confidence level (0.95). This is expected, as a 95%
confidence level implies that, in the long run, approximately 95% of the confidence intervals
constructed from random samples will contain the true population parameter.
```

Summary:

Data Analysis Key Findings

- Out of 100 simulated samples, 98 of the 95% confidence intervals calculated for the mean contained the true mean of 50.
- The proportion of confidence intervals containing the true mean was 0.9800.

Insights or Next Steps

- The observed proportion of confidence intervals containing the true mean (0.9800) is close to the theoretical 95% confidence level, which is expected.
- This simulation visually demonstrates the meaning of a confidence interval: it represents the long-run probability that an interval constructed in this manner will contain the true population parameter.

1) a) $n = 19$ $\quad CI = 95\% = 0.95$

$df = 19 - 1 = 18$

$\alpha = 1 - CI$

$= 1 - 0.95 = 0.05$

$\alpha/2 = 0.025$ (since 2 tailed)

from t - table

t. critical $= 2.101$

b) $CI = 90\% = 0.90$

$n = 27$

$df = n - 1 = 27 - 1 = 26$

$\alpha = 1 - 0.90 = 0.1$

$\alpha/2 = 0.05$

t critical $= 1.706$

c) $CI = 80\% = 0.80$

$n = 7$

$df = n - 1 = 7 - 1 = 6$

$\alpha = 1 - 0.8 = 0.2 \quad \alpha/2 = 0.1$

t critical value $= 1.440$.

2) Sol^n

$CI = 90\% = 0.9$

$\alpha = 1 - 0.90 = 0.1$

$\alpha/2 = 0.05$

$MI = \frac{MW}{2} = \frac{10}{2} = 5$.

z table.

$z_{0.05} = 1.6$

$n = \left(\frac{z_{\alpha/2} \cdot 6}{ME}\right)^2$

$= \left(\frac{1.6 \times 4}{5}\right)^2$

$= 1.638$

$n = [1.638] = 2$.

2) Sol^n.

$\sigma = 12$

$CI = 98\%$

$MW = 15$

$\alpha = 1 - 0.98$

$= 0.02$

$\alpha/2 = 0.01$

$ME = \frac{15}{2} = 7.5$

$z_{0.01} = 2.4$

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{ME} \right)^2$$

$$= \left(\frac{2.4 \times 12}{7.5} \right)^2$$

$= 13.9$

$n = 14$

3) Sol^n;

$\sigma = 2$

CI = 92%.

ME = 5

$\alpha = 1 - 0.92 = 0.08$

$\alpha/2 = 0.04$

from z-table

$z_{0.04} = 1.8$

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{ME}\right)^2$$

$$= \left(\frac{1.8 \times 2}{.5}\right)^2$$

$= 0.518$

$n = 1.$

4) Sol^n

$\sigma = 10$

$CI = 99\%$

$ME = 3.$

$\alpha = 1 - 0.99 = 0.01$

$\alpha/2 = 0.005$

from z table

$z_{0.005} = 2.5$

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{ME}\right)^2$$

$$= \left(\frac{2.5 \times 10}{3}\right)^2$$

$$= 69.44$$

$n = 70$

(4) Sample size (n) = 40

Sample mean ($\bar{x}$) = 12%

Sample Standard deviation (S) = 3%

Confidence level : 95% = 0.95

Using Z distribution

$$1 - 0.95 = 0.05$$

$$0.05 = 1.96$$

$$M.E = Z \times SE = 1.96 \times \frac{3}{\sqrt{40}} \left(\frac{s}{\sqrt{n}}\right)$$

$$12 = 0.929$$

$$12 \pm 0.929$$

+ = 11.07% | - = 12.93%

level of confidence = 95% = 0.95

6) Sample size $(n) = 300$

Number of successes $(x) = 240$

Sample proportion $(\hat{p}) = 240/300 = 0.8$

$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$= \sqrt{\frac{0.8(1-0.8)}{300}} = 0.023$

$ME = z \times SE$ (0.95 level of confidence value in z table is 1.96)

$= 1.96 \times 0.023\cancel{0940107}$

$= 0.045\cancel{2642611}$

$CI = \hat{p} \pm ME$

$= 0.8 \pm 0.045$

+ve $= 0.845$ | -ve 0.755

(z-distribution as $n > 30$)

5) $n = 36$

$(\bar{x})$ mean $= 14$

$S = 2$

Confidence level $= 90\% = 0.90$

$\alpha = 1 - 0.90$

$\alpha = 0.10$

$\alpha/2 = 0.10/2 = 0.05$

Now, from z table

$z_{0.05} = 1.645$

Now, $SE = \frac{s}{\sqrt{n}} = \frac{2}{\sqrt{36}} = \frac{2}{6} = 0.3333$

Now, $ME = (z_{0.05} \times SE)$ $\cancel{0.05 \times 0.333}$ $1.645 \times 0.3333)$

$= 0.548$

Now, Confidence interval $(CI) = \bar{x} \pm ME$

$= 14 \pm 0.548$

+ve $= 14.548$ | -v 13.4

7) $n = 25$

mean $= 3400$

$S = 600$

Est $= 99\% = 0.99$

$1 - 0.99 = 0.01$

$$M.E = t \times SE = 248.5 \times \frac{600}{\sqrt{25}}$$

$$= \frac{248.5 \times 6}{5}$$

$$= 298.2$$

$$CI = 3400 \pm 298.2.$$

+ve $= 3698.2$, -ve 3101.8.

ME = Margin of error

SE = Standard Error

CI = Confidence Interval.

5% for t table

95% 5% 2.5% 50

→ 2.5% in upper
→ 2.5% in lower

8) Sample size $(n) = 20$

$S = 13.7$

mean $= 107.3$

100% − 2.5% = 97.5%

$= 0.975$

a) t value as $n < 30$.

$df = n - 1 = 20 - 1 = 19$.

b) SE (Standard Error) $= \frac{s}{\sqrt{n}} = \frac{13.7}{\sqrt{20}} = \frac{13.7}{4.472} = 3.0$

* t-critical value

$df = 19$.

Confidence $= 95\% = 0.95$

$\alpha = 1 - 0.95 = 0.05$

$\alpha/2 = \frac{0.05}{2} = 0.025$

Now,

$t_{0.975}$ where $df = 19 = 2.093$.

Now, $ME = t_{0.975} \times SE = 2.093 \times 3.063$

$= 6.410859$

Now, $CI = \bar{x} \pm ME$

$= 107.3 \pm 6.41$

$= +ve = 113.71$ $-ve = 100.89$

c) Confidence $= 98\% = 0.98$

degree of freedom $df = 19$.

significance level $\alpha = 1 - 0.98 = 0.02$

$\therefore t_{0.02} = 2.539$.

$ME = t_{0.02} \times SE$

$= 2.539 \times 3.063$

$= 7.776$

$CI = 107.3 \pm 7.776$

$+ve = 115.076$ $\quad -ve = 99.524$

d) $\mu = 105$. (True mean)

$CI = 95\%$ $(113.701, 100.89)$

$CI = 98\%$ $(115.076, 99.524)$

as 105 falls under the interval

it did a good job $\therefore$

... ko interval ma aaucha.

q) $n = 32$ (Sample size)

$s = 18$ (standard deviation)

mean $(\bar{x}) = 31$

$t = 95\%$ (confidence level)

a) ⇒ t distribution ~~betwe~~ because sample size is greater than 30.

b) now,

$n = 31 - 1 = 30$ (value lagi lina ko)

$\alpha = 1 - 95\%$

$= 1 - 0.05$

$= 0.95$

$\alpha/2 = \frac{0.95}{2} = 0.025$

$t = 2.042$

$ME = t - \frac{s}{\sqrt{n}} = 6.49$

31 ± 6.49
+ve
37.49
−ve
24.5

c) 90% confidence interval for mean.

critical value :-

$t_{0.05, 31} = 1.696$

$\therefore ME = 1.696 \times 3.183 = 5.40$

$CI = 31 \pm 5.40$ $\quad$ $\therefore +ve = 36.40$

$-ve = 25.60$