Week- 8:

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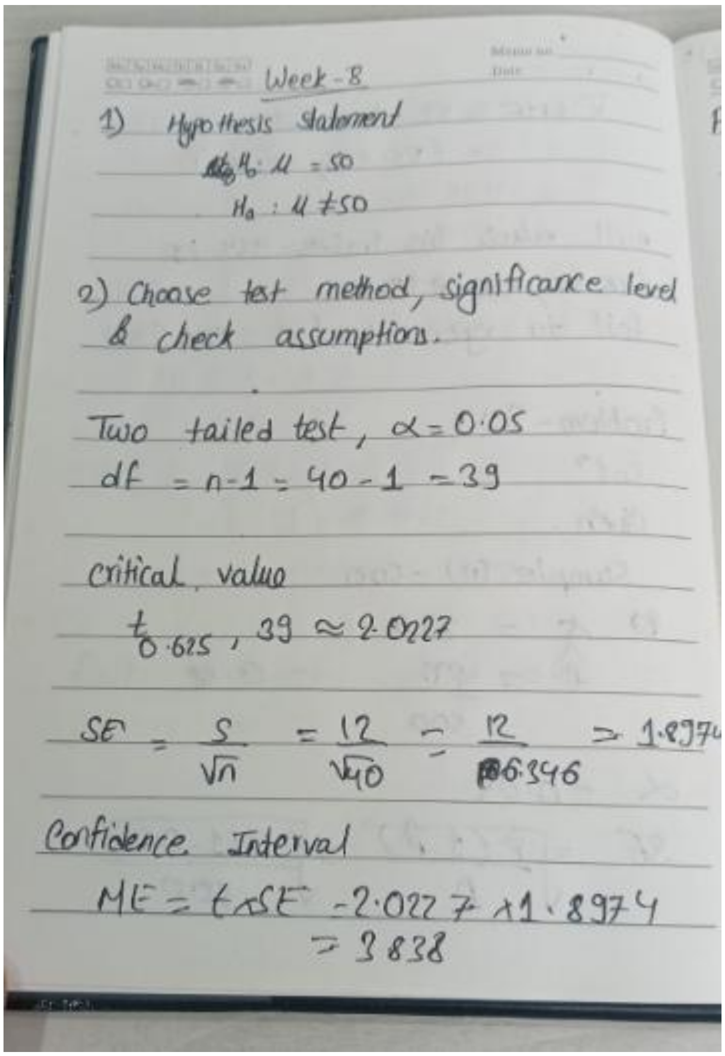
Workshop 8:

AI:



Week-8

1) Hypothesis statement

$H_0: \mu = 50$

$H_a: \mu \neq 50$

2) Choose test method, significance level & check assumptions.

Two tailed test, $\alpha = 0.05$

$df = n-1 = 40-1 = 39$

critical value

$t_{0.025, 39} \approx 2.0227$

$SE = \frac{s}{\sqrt{n}} = \frac{12}{\sqrt{40}} = \frac{12}{6.346} = 1.897$

Confidence Interval

$ME = t \times SE = 2.0227 \times 1.8974$

$= 3.838$

$\bar{x} + ME = 52 \pm 3.838$

$= (55.838, 48.162)$

null value lies inside 95% CI

(48.16, 55.838)

fail to reject H_0 at $\alpha = 0.05$.

Problem - 3:

Sol^n

Given

Sample (n) = 500

$x = 290$

$\hat{p} = \frac{290}{500} = 0.58$

$\alpha = 0.05$

$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.58(1-0.58)}{500}}$

$= \sqrt{\frac{0.2436}{500}} \doteq 0.022$

critical value at 95% is

$z_{0.025} \approx 1.9.$

$ME = z \times SE$

$= 1.9 \times 0.22$

$= 0.418$

$CI = \hat{p} \pm ME$

$= 0.58 \pm 0.418$

$= 0.998, 0.162$

fail to reject at $\alpha = 0.05$.

Problem-2: Delivery time (one-sided)

$H_0 : \mu = 4$ vs $H_a : \mu < 4$

t-distributions,

$\alpha = 0.05$

$df = n - 1 = 25 - 1 = 24$

Critical Value.

$SE = \frac{s}{\sqrt{n}} = \frac{0.9}{\sqrt{25}} = 0.18$

For one-sided test at $\alpha = 0.05$

$t_{0.05}, 24 \approx 1.7109$

$ME = t \times SE = 1.7109 \times 0.18 = 0.3$

$CI = \bar{x} \pm ME = 3.8 \pm 0.307$

$= (4.107, 3.493)$

fail to reject.

Problem-2: Delivery time (one-sided)

$H_0 : \mu = 4$ vs $H_a : \mu < 4$

t-distributions,

$\alpha = 0.05$

$df = n-1 = 25-1 = 24$

Critical Value.

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for one-sided test at $\alpha = 0.05$

$t_{0.05, 24} \approx 1.7109$

$ME = t \times SE = 1.7109 \times 0.18 = 0.3$

$CI = \bar{x} \pm ME = 3.8 \pm 0.307$

$= (4.107, 3.493)$

fail to reject.

3.3. $\bar{x} = 79.8$

$s = 9.6$

$\alpha = 0.05$

$H_0 : \mu = 75$

$H_a : \mu \neq 75$

$df = n - 1 = 12 - 1 = 11$

$t_{0.025\,;\,11} \approx 2.201$

$$SE = \frac{s}{\sqrt{n}} = \frac{9.6}{\sqrt{12}} \approx 2.77$$

$ME = t \times SE = 2.201 \times 2.77 \approx 6.1$

$CI = \bar{x} \pm ME$

$= 79.8 \pm 6.10$

$= (73.7, 85.9)$

$$= \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{79.8 - 75}{2.77} =$$

$|1.732| < 2.201$

Problem B:-

$H_0 : \mu \leq 2.5$

$H_1 : \mu > 2.5$

$n = 16$

$\bar{x} = 2.8$

$s = 0.9$

$\alpha = 0.10$

Compute t- statistics

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{2.8 - 2.5}{0.9/\sqrt{16}} = \frac{0.3}{0.225}$$

$df = 16 - 1 = 15$

$t_{0.9, 15} \approx 1.3406$

$$SE = \frac{s}{\sqrt{n}} = \frac{0.9}{\sqrt{16}} = 0.225$$

Problem B:-

$H_0 : \mu \leq 2.5$

$H_1 : \mu > 2.5$

$n = 16$

$\bar{x} = 2.8$

$s = 0.9$

$\alpha = 0.10$

Compute t- statistics

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{2.8 - 2.5}{0.9/\sqrt{16}} = \frac{0.3}{0.225} = 1.33$$

$df = 16 - 1 = 15$

$t_{0.9, 15} \approx 1.3406$

$$SE = \frac{s}{\sqrt{n}} = \frac{0.9}{\sqrt{16}} = 0.225$$

$ME = t \times SE$

$= 1.3406 \times 0.225$

$= 0.3016.$

$CI = ME \pm \bar{x}$

$= 2.8 \pm 0.3016$

$= 3.1016, 2.4924$

$|t| > t$

$|1.33| < 1.3406$ fail to reject.

Problem C: Email Click-Through

$H_0 : p = 0.08$

$H_1 : p \neq 0.08$

$\alpha = 0.05$

$\hat{p} = \frac{78}{900} = 0.0867$

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.0867 - 0.08}{\sqrt{\frac{0.08 \times 0.92}{900}}}$$

$$= \frac{0.0067}{0.00906} \approx 0.74$$

two - tailed test $\alpha = 0.05$

$z_{0.975} \approx \pm 1.96$

$|z| > 1.96$

$|z| = 0.74 < 1.96$ fail to reject.

Problem D : Delivery Reliability.

$n = 1200$

$x = 1100.$

$\hat{p} = \frac{1100}{1200} = 0.9167$

$\alpha = 0.01$

$H_0 : p = 0.95$

$H_1 : p \neq 0.95$

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.9167 - 0.95}{\sqrt{\frac{0.95 \times 0.05}{1200}}}$$

$$= \frac{-0.0333}{0.00629} \approx -5.29$$

wo tailed test $\alpha = 0.01$

$z_{0.995} \approx \pm 2.576$

$|z| > 2.576.$

Problem E: A/B Test Revenue lift

for grp- Control (A):

$n_1 = 45$; $\bar{x}_1 = 24.50$; $s_1 = 7.2$

for grp - treatment (B)

$n_2 = 50$ $\bar{x}_2 = 27.10$, $s_2 = 8.0$

Hypothesis statement:

$H_0 : \mu_2 = \mu_1$, vs $H_a : \mu_2 > \mu_1$

one right tailed test, with significance level of $\alpha = 0.0$

Compute test statistic (welch statistic

$$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{7.2^2}{45} + \frac{8.0^2}{50}}$$

$$\approx 1.55949.$$

$$df \approx \frac{\left(s_1^2/n + s_2^2/n\right)^2}{\frac{(s_1^2/n_1)^2}{n_1-1} + \frac{(s_2^2/n_2)^2}{n_2-1}}$$

$= 93.0$

CI $t_{0.975},\ 93 \approx 1.661$

$|t| >$ t critical i.e. $1.6672 > 1.661$ so reject.

4.4 Problem: A.

1) Provided Data.

Grp A : $n_1 = 30$, $\bar{x}_1 = 5.2$, $s_1 = 0.9$

Grp B : $n_2 = 30$, $\bar{x}_2 = 5.7$ $s_2 = 1.0$

Grp C : $n_3 = 30$, $\bar{x}_3 = 5.1$ $s_3 = 0.8$

Grp D : $n_4 = 30$, $\bar{x}_4 = 5.9$ $s_4 = 1.1$

2) Hypothesis statement.

$H_0 : \mu_A = \mu_B = \mu_c = \mu_D$ vs H_a at least one mean differs.

3) Compute Grand Mean:

$$\bar{x}_a = \frac{\sum_{i=1}^{4} n_i \bar{x}_i}{\sum_{i=1}^{4} n_i}$$

$$= \frac{30(5.2 + 5.7 + 5.1 + 5.9)}{120} = 5\cdot$$

4)

4) $SSB := \sum_{i=1}^{4} n_i (\bar{x}_i - \bar{x}_4)^2$

Grp (i)	$\bar{x}_i$	$\bar{x}_i - \bar{x}_4$	$(\bar{x}_i - \bar{x}_4)^2$	$n_i(\bar{x}_i - \bar{x}_4)^2$
1	5.2	-0.275	0.075	2.268
2	5.7	0.225	0.050	1.518
3	5.1	-0.375	0.140	4.218
4	5.9	0.425	0.180	5.418
				$\Sigma = 13.48$

) Sum of square within sample standar
deviations.

$$SSW = \sum_{i=1}^{4} (n_i - 1) s_i^2$$

$$= 29(0.9)^2 + 29(1.0)^2 + (29$$
$$(0.8)^2 + 2.9(\cancel{\;})$$
$$= 106.14.$$

$df_B = k-1 = 4-1 = 3$

$df_\omega = N-k = 120-4 = 116$

Mean Squares

$$MSB = \frac{SSB}{df_B} = \frac{13.425}{3} = 4.475$$

$$MSW = \frac{SSW}{df_\omega} = \frac{106.14}{116} = 0.915$$

Compute F- stastic.

$$f = \frac{MSB}{MSW} = \frac{4.475}{0.915} = 4.891$$

9) p-value (from F-distribution) $df_1 = 3$

$p = P(F_{3,116} \geq 4.891) = 0.00308$.