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TUTORIAL-1

What do you understand by asymptotic notations? Define different asymptotic notations with eg.

Asymptotic notations: These notations are used when input is very large. They are mathematical notations used to describe running time of algorithm.

Different asymptotic notations

Big O (O)

$f(n) = O(g(n))$

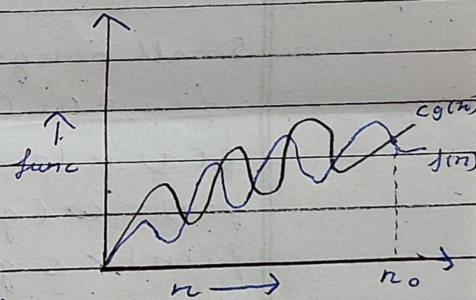
iff

$f(n) \leq c\,g(n)$

$\forall\ n > n_0$

for some constant $c > 0$

g(n) is tight upper bound of f(n)



Big Omega (Ω)

$f(n) = \Omega(g(n))$

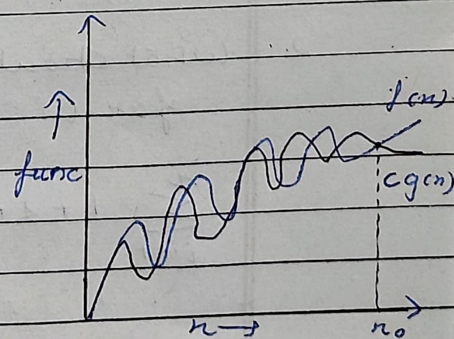
g(n) is tight lower bound of f(n)

$f(n) = \Omega\, g(n)$

iff

$f(n) > c\,g(n)$ &

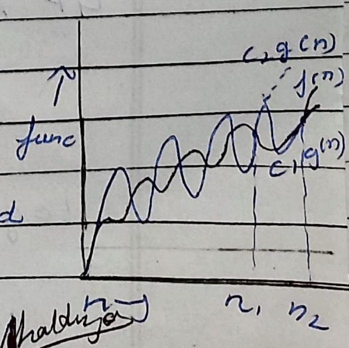
$\forall\ n \geq n_0$ for some const $c > 0$



Theta (Θ)

$f(n) = \Theta\, g(n)$

g(n) is tight upper and lower bound of f(n)



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small
~~Big~~ omega (Ω)

$f(n) = \Omega\, g(n))$

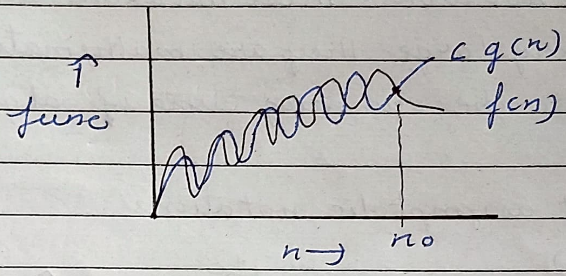
g(n) is ~~tighter~~ ~~lower~~ upper bound of f^n f(n)

$f(n) = \Omega(g(n))$

~~iff~~ when

$f(n) < c\, g(n)$

$\forall\ n \geq n_0$ for some const, $c > 0$



Small omega (ω)

$f(n) = \omega\, (g(n))$

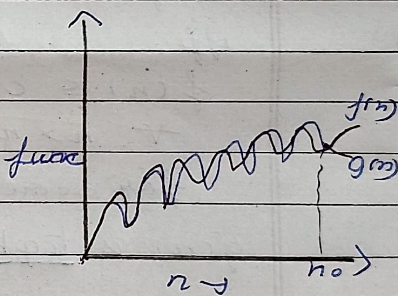
g(n) is lower bound of f(n)

$f(n) = \omega\, (g(n))$

$f(n) > c\, g(n)$

$\forall\ n > n_0$

& $\forall$ constants, $c > 0$



2. What should be time complexity of
for (i=1 to n) { i = i*2};

$$\sum_{i=1}^{n} 1 + 2 + 4 + 8 \ldots n$$

GP n^{th} value $= T_k = ar^{k-1}$

$= 1 \times 2^{k-1}$

$n = 2^{k-1}$

$2n = 2^k$

$\log 2n = k \log_2 2$

$\log 2n = k \log_2 2$

③

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$\log 2n \neq k$

$\log_2 2 + \log_2 n = k$

$k = \log_2 n + 1$

$O(1 + \log_2 n) = O(\log n)$

$= \boxed{O(\log n) \text{ answer}}$

Q-3 $T(n) = \{ 3T(n-1)$ if $n>0$, otherwise $1 \}$

$T(n) = 3T(n-1)$ —— ①

let $n = n-1$

$T(n-1) = 3T(n-2)$ —— ②

Put ② in ①

$T(n) = 3 \times 3T(n-2)$ —— ③

put $n = n-2$ in eqⁿ ①

$T(n-2) = 3T(n-3)$ —— ④

put ④ in ③

$T(n) = 3 \times 3 \times 3T(n-3)$

$T(n) = 3^k T(n-k)$

put $n-k=0$

$T(n) = 3^n T(n-1) = 3^n T(0) = 3^n$

$\boxed{= O(3^n) \text{ answer.}}$

Q-4 $T(n) = \{ 2T(n-1) - 1$ if $n>0$, otherwise $1 \}$

$T(n) = 2T(n-1) - 1$ —— ①

Let $n = n-1$

$T(n-1) = 2T(n-2) - 1$ —— ②

put ② in ①

$T(n) = 2(2T(n-2) - 1) - 1$

$4T(n-2) - 2 - 1$ —— ③

putting $n = n-2$ in eqⁿ ①

$T(n-2) = 2T(n-2) - 1$ —— ④

put ④ in ③

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$T(n) = 4[2T(n-2)-1]-2-1$

$T(n) = 8T(n-3)-4-2-1$

$T(n) = 2^k T(n-k) - 2^{k-1} - 2^{k-2} \ldots 1$

GP: $2^{k-1} + 2^{k-2} + \ldots 1$

$a = 2^{k-1}$ $r = 1/2$

$T_k = \frac{a(1-r^n)}{1-r} = 2^{k-1} \frac{(1-(1/2)^n)}{(1-1/2)}$

$2^k(1-(1/2)^k)$

$= 2^k - 1$

let $n-k=0$

$n=k$.

$T(n) = 2^n T(n-n) - (2^n - 1)$

$\cancel{2^n} - \cancel{2^n} + 1$

$\boxed{T(n) = O(1)}$ ans

5. What should be the time complexity of

```
int i=1, s=1
while (s<=n) {
    i++;
    s=s+i;
    printf("*");
}
```

let us consider

$i = 1, 2, 3, 4, 5, 6 \ldots$

$s = 1 + 3 + 6 + 10 + 15 \ldots n$

sum of s $= 1+3+6+10+15 \ldots \sqrt{n}$ — (1)

also s $= 1+3+6+10 \ldots \sqrt{n-1} + \sqrt{n}$ — (ii) (2)

(1) − (2)

$0 = 1+2+3+4+ \ldots n - \sqrt{n}$

$T(k) = 1+2+3 \ldots k$

(5)

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$T(k) = \frac{1}{2} k(n+1)$

for k iterations

$1+2+3+ \ldots k = n$

$\frac{k(k+1)}{2} <= n \Rightarrow \frac{k^2+k}{2} \leq n$

$O(k^2) <= n$

$k = O(\sqrt{n})$

$\boxed{T(n) = O(\sqrt{n}) \text{ ans.}}$

ans-6. Time complexity of

```
void function (int n) {
    int i, count = 0;
    for (i = 1; i*i <= n; i++) {
        count ++
    }
```

$i^2 <= n$, $i <= n$

$i = 1, 2, 3, 4 \ldots \sqrt{n}$

$\sum_{i=1}^{n} 1+2+3+4+ \ldots \sqrt{n}$

$T(n) = \frac{\sqrt{n}(\sqrt{n}+1)}{2} = \frac{n+\sqrt{n}}{2}$

$\boxed{T(n) = O(n)}$

Q-7) Time complexity of :

```
void fun (int n)
{ int i, j, k, count = 0;
for (i = n/2; i <= n; i++)
  { for (k = 1; k <= n, k = k*2)
    { count ++;
    }
  }
}
```

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i	j	k
1	log n	log n * log n
2	log n	log n * log n
⋮	⋮	⋮
n	log n	log n * log n

$O(n * \log n * \log n)$

$= \boxed{O(n \log^2 n).}$

Time complexity of

```
function (int n) {
    if (n==1) return;
    for( i=1 to n)
        for (j = 1 to n)
            print ("+ ");
}
}
```

function (n-3) ——— T(n-3)

for (i=1 to n)

$j = n$ times every turn

$i * j = n^2$

Now, $T(n) = n^2 + T(n-3)$

$T(n-3) = (n-3)^2 + T(n-6);$

$T(n-6) = (n-6)^2 + T(n-9);$

$T(1)$

Now substitute each value ~~of~~ in $T(n)$

$T(n) = n^2 + (n-3)^2 + (n-6)^2 \ldots + 1$

Let $n - 3k = 1$

$k = (n-1)/3$

Total ~~terms~~ terms $= k+1$

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$T(n) = n^2 + (n-3)^2 + (n-6)^2 \dots + 1$

$T(n) \simeq n^2 + n^2 + n^2 \dots (k+1)$ times

$T(n) \simeq kn^2$

$T(n) \simeq \frac{(\cancel{n-1})}{\cancel{3}} n^2 = \frac{n^3 - \cancel{n^2}}{\cancel{3}}$

$\boxed{T(n) = O(n^3)}$

Time complexity of

```
void function (int n) {
for (i=1 to n) {
 for (j=1; j<=n; j=j+i)
   printf ("*");
     }
   }
```

for	$i=1$	$j = 1+2+ \dots$	$(n \geq j+1)$
	$i=2$	$j = 1+3+5 \dots$	
	$i=3$	$j = 1+4+7 \dots$	
		1	

mth term of AP is

$T(m) = a + d \times m$

$= 1 + dm$

$(n-1)/d = m$

for	$i=1$	$(n-1)/1$
	$i=2$	$(n-1)/2$
	$i=3$	$(n-1)/3$
	$i=n-1$	1

we get,

$T(n) = i_1 j_1 + i_2 j_2 + \dots i_{n-1} j_{n-1}$

$= \frac{n-1}{1} + \frac{n-2}{2} + \frac{n-3}{3} \dots + 1$

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$n + \frac{n}{2} + \frac{n}{3} \dots + n \times 1$

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$$n\left[1+\frac{1}{2}+\frac{1}{3}+ \cdots \frac{1}{n-1}\right]-n-1$$

$$n \times \log n - n + 1$$

$$\int \frac{1}{x} = \log x$$

$$\boxed{T(n) = O(n \log n) \ \text{ans}}$$

For the f^n n^k & c^n, what is the asymtotic relationship b/w these f^ns?
Assume that $k >= 1$ & $c > 1$ are constants. Find out the value of c & n for which relation holds

$$n^k = O(c^n)$$

$$n^k \leq a(c^n)$$

$$\forall \quad n \geq n_0 \ \&$$

constant, $a > 0$

for $n = 1$

$c = 2$

$$1^k \leq a^2$$

$$\boxed{\begin{matrix} n_0 = 1 \\ c = 2 \end{matrix}}$$

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