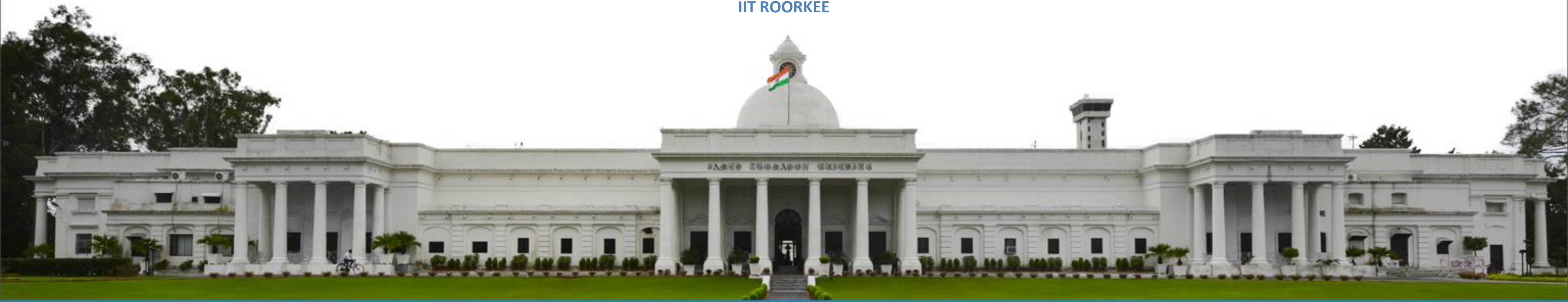
IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

mean $\longrightarrow$ Proportion $\longrightarrow$ Variance

Hypothesis Testing: Two sample test

Dr. A. Ramesh
DEPARTMENT OF MANAGEMENT
IIT ROORKEE



Agenda

- Comparing two population variances
- Choosing z or t test
- Sample size

Compare 2 pop variances

Hypothesis Tests for Two Variances

F distribution

Right skewed distribution

Goal: Test hypotheses about two population variances

Tests for Two Population Variances

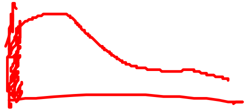
F test statistic

F test

2 popu = Normally dist

$H_0: \sigma_1^2 \geq \sigma_2^2$
$H_1: \sigma_1^2 < \sigma_2^2$

Lower-tail test



$H_0: \sigma_1^2 \leq \sigma_2^2$
$H_1: \sigma_1^2 > \sigma_2^2$

Upper-tail test

$H_0: \sigma_1^2 = \sigma_2^2$
$H_1: \sigma_1^2 \neq \sigma_2^2$

Two-tail test

The two populations are assumed to be independent and normally distributed

Hypothesis Tests for Two Variances

Both pop = Equal variance

$F = s_1^2 / s_2^2$; $\alpha_1 = \frac{\alpha}{2}$

Tests for Two Population Variances

F test statistic

The random variable

$$\boxed{F = \frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2}}$$

Has an F distribution with $(n_1 - 1)$ numerator degrees of freedom and $(n_1 - 1)$ denominator degrees of freedom

Denote an F value with ν_1 numerator and ν_2 denominator degrees of freedom by

Test Statistic

Tests for Two Population Variances

F test statistic

The critical value for a hypothesis test about two population variances is

$$\boxed{F = \frac{s_1^2}{s_2^2}}$$

where F has $(n_x - 1)$ numerator degrees of freedom and $(n_y - 1)$ denominator degrees of freedom

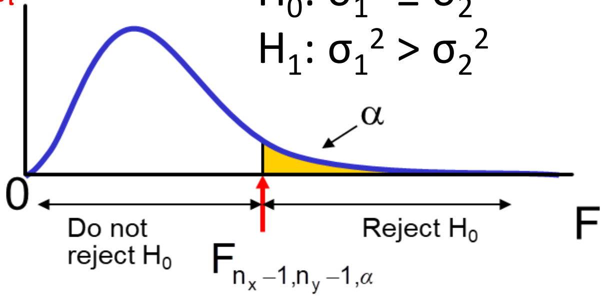
Decision Rules: Two Variances

we are here bothered about upper limit of variance so mostly it will be right tailed test

Bcoz upper limit shd not cross

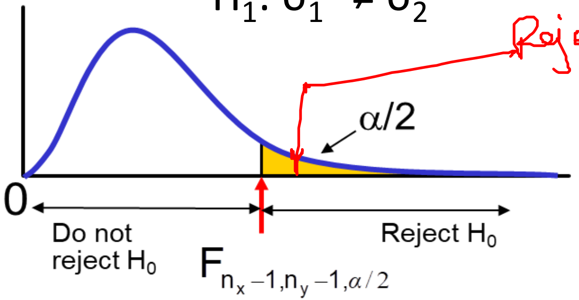
eg catching bus before time = OK
after time = NOT ok

$H_0: \sigma_1^2 \le \sigma_2^2$
$H_1: \sigma_1^2 > \sigma_2^2$



$\boxed{\text{Reject } H_0 \text{ if } F > F_{n_x-1, n_y-1, \alpha}}$

$H_0: \sigma_1^2 = \sigma_2^2$
$H_1: \sigma_1^2 \ne \sigma_2^2$



$s_1^2 > s_2^2$

- rejection region for a two-tail test is:

$\boxed{\text{Reject } H_0 \text{ if } F > F_{n_x-1, n_y-1, \alpha/2}}$

where s_x^2 is the larger of the two sample variances

Problem

- A company manufactures impellers for use in jet-turbine engines.
- One of the operations involves grinding a particular surface finish on a titanium alloy component.
- Two different grinding processes can be used, and both processes can produce parts at identical mean surface roughness.
- The manufacturing engineer would like to select the process having the least variability in surface roughness. (small variation)
- A random sample of $n1 = 11$ parts from the first process results in a sample standard deviation $s1 = 5.1$ micro inches, and a random sample of $n2 = 16$ parts from the second process results in a sample standard deviation of $s2 = 4.7$ micro inches.
- We will find a 90% confidence interval on the ratio of the two standard deviations.

Compare variances.

Problem

- Form the hypothesis test:

  $H_0: \sigma_1^2 = \sigma_2^2$ (there is no difference between variances) → Equal variances

  $H_1: \sigma_1^2 \neq \sigma_2^2$ (there is a difference between variances)

- Find the F critical values for $\alpha = .10/2$: $F_\alpha = ?$ $\alpha = 0.1/2$

Degrees of Freedom:

- Numerator
  - $n_1 - 1 = 11 - 1 = 10$ d.f.
- Denominator:
  - $n_2 - 1 = 16 - 1 = 15$ d.f.

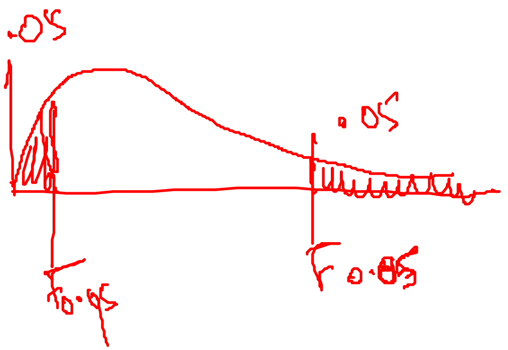
Problem

- Assuming that the two processes are independent and that surface roughness is normally distributed

$\frac{s_1^2}{s_2^2} f_{0.95,15,10}$ nume deno

$$\frac{s_1^2}{s_2^2} f_{0.95,15,10} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{s_1^2}{s_2^2} f_{0.05,15,10}$$

$$\frac{(5.1)^2}{(4.7)^2} 0.39 \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{(5.1)^2}{(4.7)^2} 2.85$$



F table read right to left

or upon completing the implied calculations and taking square roots,

Reverse degree of freedom

$$0.678 \le \frac{\sigma_1}{\sigma_2} \le 1.887$$

$f_{0.95,15,10} = 1/f_{0.05,10,15}$

Table of F-statistics P=0.05

t-statistics
F-statistics with other P-values: P=0.01 | P=0.001
Chi-square statistics

df2\df1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74	8.73	8.71	8.70
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91	5.89	5.87	5.86
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.70	4.68	4.66	4.64	4.62
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00	3.98	3.96	3.94
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57	3.55	3.53	3.51
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28	3.26	3.24	3.22
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07	3.05	3.03	3.01
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91	2.89	2.86	2.85
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79	2.76	2.74	2.72
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69	2.66	2.64	2.62
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60	2.58	2.55	2.53
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53	2.51	2.48	2.46
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48	2.45	2.42	2.40

Problem

- $f_{0.95,15,10} = 1 / f_{0.05,10,15} = 1/2.54 = 0.39$
- Since this confidence interval includes unity, we cannot claim that the standard deviations of surface roughness for the two processes are different at the 90% level of confidence.

$\boxed{\frac{\sigma_1}{\sigma_2} = 1}$ Possibility

$\therefore \sigma_1 = \sigma_2$

```
In [1]: import pandas as pd
        import numpy as np
        import math
        from scipy import stats
        import scipy
```

$f_{0.95, 15, 10}$

0.95

```
In [45]: scipy.stats.f.ppf(q=1-0.05, dfn= 15, dfd=10)
Out[45]: 2.845016526958436
```

```
In [44]: scipy.stats.f.ppf(q=0.05, dfn=15, dfd=10)
Out[44]: 0.3931252536255495
```

F Test example:

In [9]:
```python
X = [3,7,25,10,15,6,12,25,15,7]
Y = [48,44,40,38,33,21,20,12,1,18]
import numpy as np
```

$H_0: \sigma_x^2 = \sigma_y^2$

$H_1: \sigma_x^2 \neq \sigma_y^2$ $\quad \alpha = 0.5$

In [11]:
```python
F = np.var(X) /np.var(Y)
dfn = len(X) -1
dfd = len(Y) -1
```

$F = \frac{\sigma_1}{\sigma_2}$

In [12]:
```python
p_value = scipy.stats.f.cdf(F, dfn, dfd)
```

In [13]:
```python
p_value
```

$\sigma_1^2 \neq \sigma_y^2$

Out[13]: 0.024680183438910465

$0.025 < .05 \therefore$ Reject H_0

Z Vs t

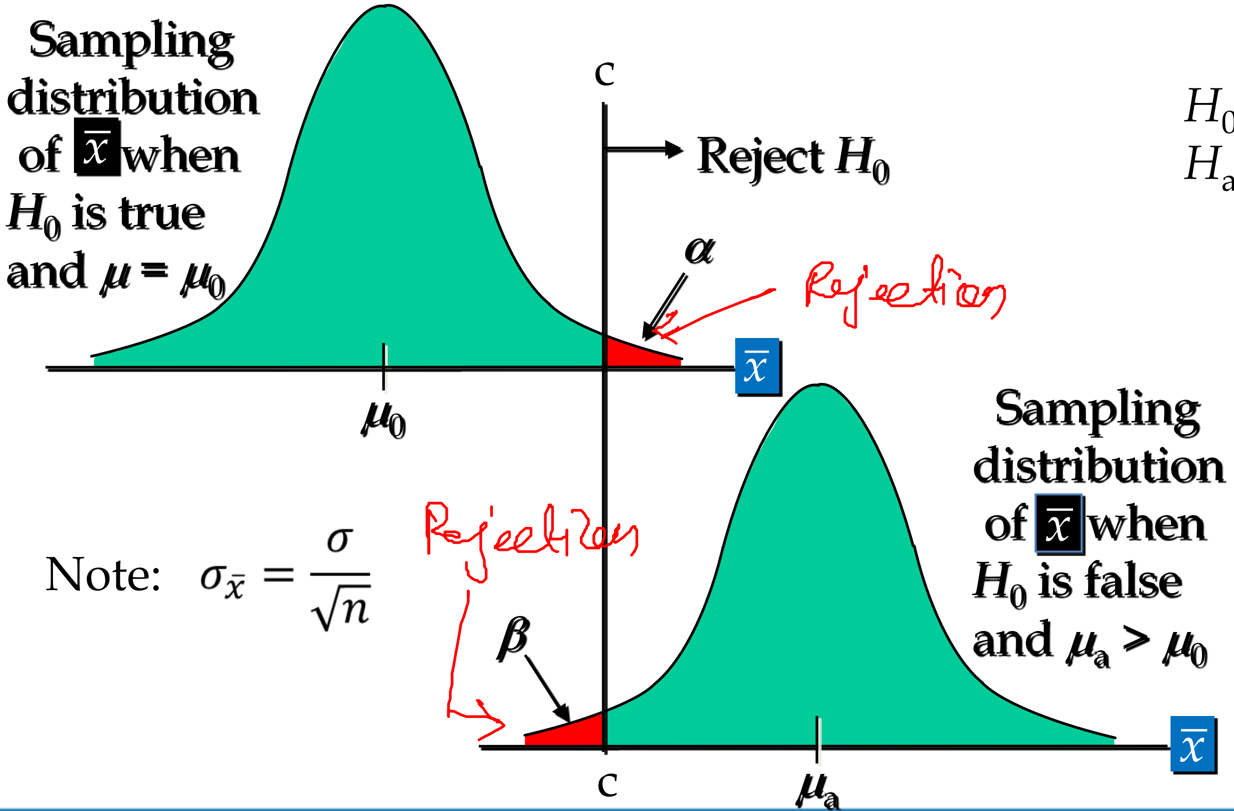
Pop std deviation

	σ –known	σ –unknown
n ≤ 30	Z-test	t-test
n > 30	Z-test	Z-test Use Sample standard deviation

as n → High ⇒ t → Z

Determining the Sample Size for a Hypothesis Test About a Population Mean

$$H_0: \mu \le \mu_0$$
$$H_a: \mu > \mu_0$$



Note: $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$

Determining the Sample Size for a Hypothesis Test About a Population Mean

$$\boxed{n = \frac{(z_\alpha + z_\beta)^2 \sigma^2}{(\mu_0 - \mu_a)^2}}$$

$$\frac{(Z_\alpha + Z_\beta)^2 \sigma^2}{(\mu_0 - \mu_a)^2}$$

where

z_α = *z* value providing an area of α in the tail

z_β = *z* value providing an area of β in the tail

σ = population standard deviation

μ_0 = value of the population mean in H_0

μ_a = value of the population mean used for the Type II error

$$Z_\alpha = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$Z_\beta = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

Note: In a two-tailed hypothesis test, use $z_{\alpha/2}$ not z_α

Determining the Sample Size for a Hypothesis Test About a Population Mean

- Let's assume that the manufacturing company makes the following statements about the allowable probabilities for the Type I and Type II errors:

- If the mean diameter is $\mu = 12$ mm, I am willing to risk an $\alpha = .05$ probability of rejecting H_0.

α = type1

- If the mean diameter is 0.75 mm over the specification ($\mu = 12.75$), I am willing to risk a $\beta = .10$ probability of not rejecting H_0.

β = type2

Calculate n (sample)

Determining the Sample Size for a Hypothesis Test About a Population Mean

$\alpha = .05, \;\; \beta = .10$

$z_\alpha = 1.645, \;\; z_\beta = 1.28$

$\mu_0 = 12, \;\; \mu_a = 12.75$

$\sigma = 3.2$

$$\boxed{\frac{\bar{x} - \mu_a}{\sigma/\sqrt{n}} \to z_\beta}$$

$$n = \frac{(z_\alpha + z_\beta)^2 \sigma^2}{(\mu_0 - \mu_a)^2} = \frac{(1.645 + 1.28)^2 (3.2)^2}{(12 - 12.75)^2}$$

$$= 155.75 \approx 156$$

In [2]:
```python
import pandas as pd
import numpy as np
from scipy import stats
```

In [5]:
```python
import math
```

In [22]:
```python
def samplesize(alfa,beta,mu1,mu2,sigma):
    z1 = -1*stats.norm.ppf(alfa)
    z2 = -1*stats.norm.ppf(beta)
    n= (((z1+z2)**2)*(sigma**2))/((mu1-mu2)**2)
    print (n)
```

In [23]:
```python
samplesize(0.05,0.1,12,12.75,3.2)
```

```
155.900083325938
```

close;

Thank You