σ^2 is ~~Known~~ unknown

predict pop mean from $\bar{x}$ (sample mean)

=> use Student's t dist

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

s = sample ~~std dist~~ deviation

$$\boxed{z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}}$$

σ Not known

with n-1 degree of freedom

s → Sample it increases uncertainity

Assumption
- pop std dev → known
- pop ~ Normaly Dist
  - if Not Normal
    - large sample

Confidence interval estimation

$$\bar{x} - t_{n-1,\alpha/2}\frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{n-1,\alpha/2}\frac{s}{\sqrt{n}}$$

$$\boxed{\bar{x} - \mu = t \cdot \frac{s}{\sqrt{n}}}$$

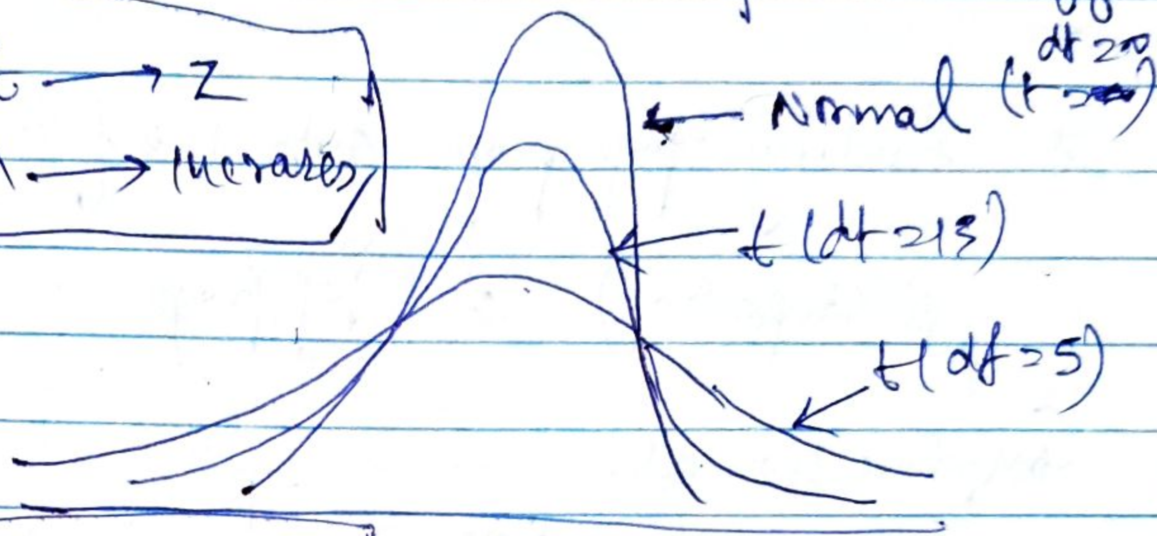
$(\bar{x} \pm ME)$

$$\boxed{ME = t_{n-1,\,\alpha/2} \frac{S}{\sqrt{n}}}$$

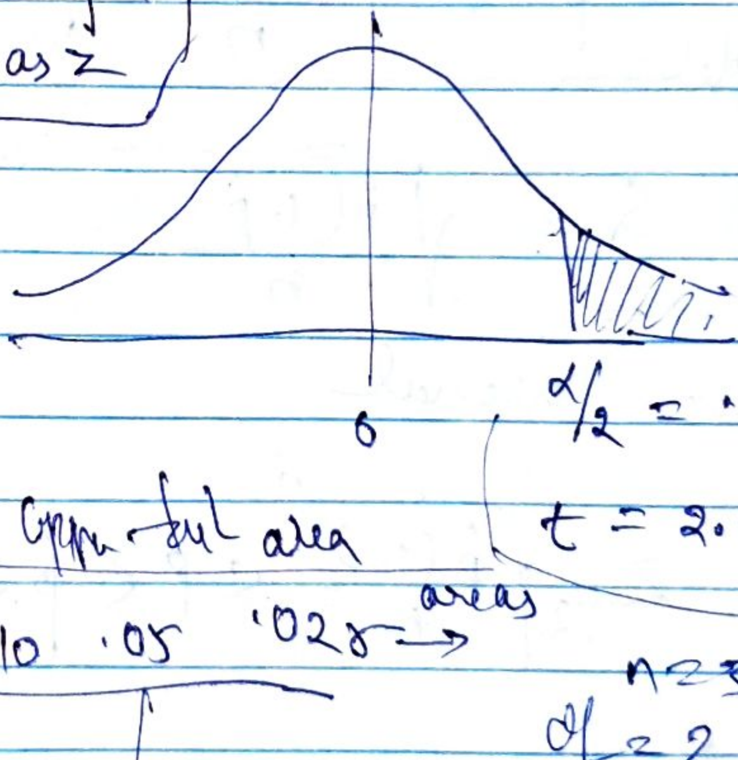
t is a family of dist

depends on deg of f

$\boxed{\begin{array}{l} t \longrightarrow z \\ n \longrightarrow \text{increases} \end{array}}$



$\boxed{\text{for large sample } t \text{ behaves as } z}$



$\alpha/2 = .05$

$t = 2.92$

df	Upper tail area			
	.10	.05	.025	→ areas
1				
2	→ 2.92			
3				

$n = 3$

$df = 2$

$\alpha = 0.10$

$\alpha/2 = 0.05$

→ t values Not probabilities

Confidence level	t df 10	t df 20	t df 30	t 40	Z
0.95	2.2	2.08	2.04	$\longrightarrow$	1.96

finding pop prop with help of sample

E(sample prop) = Pop Prop

Confidence interval

std dev sample $\boxed{\sigma_p = \sqrt{\frac{P(1-P)}{n}}}$ → Population prop P

P → Need to find out

$S_p = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Confidence Interval

$$\hat{p} - Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < P < \hat{p} + Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

lower limit

Sampling Prop

Assumption $n\hat{p} > 5$

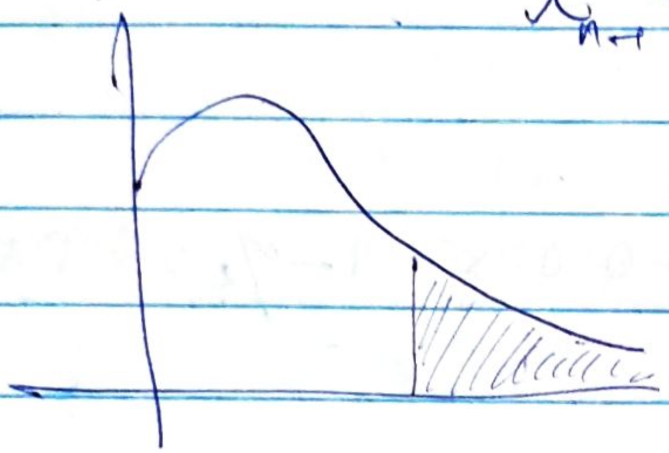
$n\hat{p}(1-\hat{p}) > 5$

Pop variance from sample var
form Confidence Interval

pop Normally for Dist

for a Random variable

$$\chi^2_{n-1} = \frac{(n-1)s^2}{\sigma^2}$$



$$\frac{(n-1)s^2}{\sigma^2}$$

$$\frac{(n-1)s^2}{\chi^2_{n-1,\alpha/2}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{n-1,\alpha/2}}$$

lower limit

(Alpha/2)
Upper limit

$(1-\alpha)$ % confidence interval

Q testing a batch of CPUs (MHz)

Sample size 17 (n)

sample mean 3004 $\bar{x}$

Sample std dev 74 s

determine σ_x^2 with 95% Conf interval,

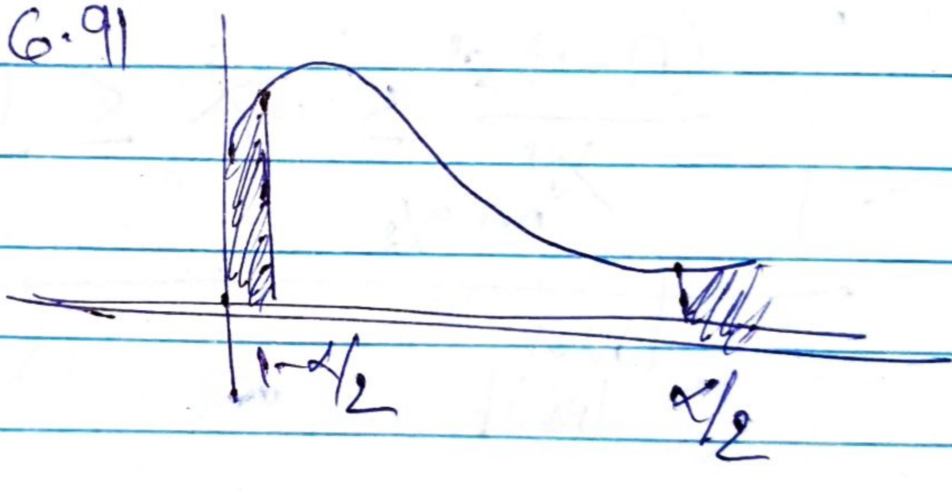
Assume population → Normal

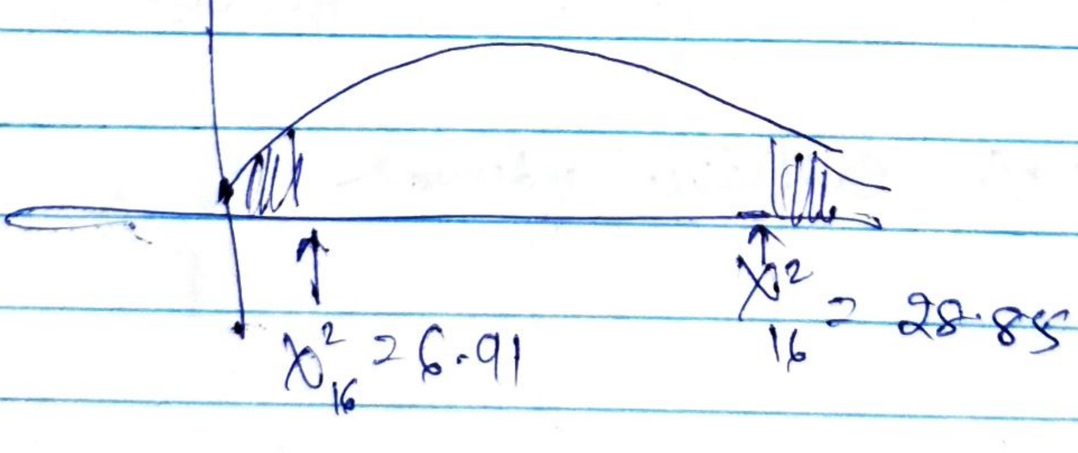
$(n-1) = df = 16$

$\alpha = 0.05$, $\alpha/2 = 0.025$ $1-\alpha/2 = 0.975$

$\chi^2_{16, .025} = 28.25$

$\chi^2_{16, .975} = 6.91$





$$\frac{(n-1)s^2}{\chi^2_{n-1,\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{n-1,1-\alpha/2}}$$

$$\frac{(17-1)(74)^2}{28.85} < \sigma^2 < \frac{(17-1)(74)^2}{6.91}$$

$$3037 < \sigma^2 < 12683$$

ans 95% confident Pop. std dev of CPU speed is between

$\sqrt{3037}$ and $\sqrt{12683}$ MHz

55.1 and 112.6

Finite population — If sample size > 5% Pop and (sampling without replacement) then a finite population correction factor be used when calculating std error (std deviation)

finite pop correction factor = $\frac{N-n}{N-1}$

$E(\mu)$

point Estimate $\Rightarrow$ $\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$

Sample variance

$\hookrightarrow$ $\hat{\sigma}^2_{\bar{x}} = \frac{s^2}{n} \left(\frac{N-n}{N-1}\right)$

population proportion : P

Sample prop $\hat{p}$

variance

Sample population $\hat{\sigma}^2_{\hat{p}} = \frac{\hat{p}(1-\hat{p})}{n} * \left(\frac{N-n}{N-1}\right)$

and $\hat{p} - Z_{\alpha/2} \hat{\sigma}_{\hat{p}} < P <$

$\hat{p} + Z_{\alpha/2} \hat{\sigma}_{\hat{p}}$

Eg Q P <20 years $=$ $P = 0.35$

$0.3 < \hat{p} < 0.4$ $\quad n = 100$

$\sigma = \sqrt{\frac{P(1-P)}{n}} \Rightarrow \sqrt{\frac{.35\,(0.65)}{100}}$

$\sigma = .048$

$P(.3 < \hat{p} < .4) \Rightarrow$

$$P\left[-\frac{.05}{.048} \le Z \le \frac{.05}{.048}\right]$$

$$\left(-1.048 \le Z \le 1.048\right)$$

$\Rightarrow$ prob $\Rightarrow$ 0.706

Revise Assignment Questions

{Info}