Multivariable Calculus
Differentiability-II

Dr. S. K. Gupta

Department of Mathematics

Problem

Show that the function

$$f(x, y) = \begin{cases} \dfrac{x^3 - y^3}{x^2 + y^2}, & (x, y) \neq 0, \\ 0, & (x, y) = 0 \end{cases}$$

is continuous and possesses its partial derivatives at (0,0) but not differentiable at $(0, 0)$.

Predicting change with differentials

Let a point (x_0, y_0) be changed to $(x_0 + \triangle x, y_0 + \triangle y)$. We want to predict the approximate value of change in the value of f. Suppose f is a differentiable function and its first partial derivative at (x_0, y_0) are known.

The change in f is

$$\triangle f = f(x_0 + \triangle x, y_0 + \triangle y) - f(x_0, y_0).$$

If we move from a point (x_0, y_0) to a point $(x_0 + dx, y_0 + dy)$ nearby, the resulting differential of f is

$$df = f_x(x_0, y_0)dx + f_y(x_0, y_0)dy.$$

Here, df is also called **total derivative** of f.

Problems

1. Near the point $(1, 2)$, is $f(x, y) = x^2 - xy + y^2 - 3$ more sensitive to changes in x, or to changes in y? Explain? What can you say at point $(2, 1)$?

2. Using differentials, find an approximate value of $\sqrt{(298)^2 + (401)^2}$.

3. A certain function $z = f(x, y)$ has values $f(2, 3) = 5$, $f_x(2, 3) = 3$ and $f_y(2, 3) = 7$. Find an approximate value of $f(1.98, 3.01)$.

Absolute, relative and percentage change

If we move from (x_0, y_0) to a point nearby, then we describe the change in the values of the function $f(x, y)$ in three ways:

	True	Estimate
Absolute change	$\triangle f$	df
Relative change	$\dfrac{\triangle f}{f(x_0, y_0)}$	$\dfrac{df}{f(x_0, y_0)}$
Percentage change	$\dfrac{\triangle f}{f(x_0, y_0)} \times 100$	$\dfrac{df}{f(x_0, y_0)} \times 100.$

Problems

- Suppose the variables r and h change from the initial values of $(r_0, h_0) = (1, 5)$ by the amounts $dr = 0.03$ and $dh = -0.1$. Estimate the resulting absolute, relative and percentage changes in the values of the function $v = \pi r^2 h$.

- Find the percentage error in the computed area of an ellipse when an error of 2% is made in increasing the major and minor axes.

Continued...

- The power consumed in an electric register is given by $P = \dfrac{E^2}{R}$ (in watts). If $E = 80$ volts and $R = 5$ ohms, how much power consumption will change if E is increased by 3 volts and R is decreased by 0.1 ohms.

- Let the current I (in amperes) in an electrical circuit is related to voltage V(volts) and the resistance R(ohms) by $I = \dfrac{V}{R}$. If the voltage drops from 24 to 23 volts, and the resistance drops from 100 to 80 ohms, will I increase or decrease? Express the changes in V and R and the estimates change in I as percentage of their original values.

Continued...

- Let $y = uv$, where u and v are independent variables. If u is measured with an error of 2% and v with an error of 3%, what is the percentage error in the calculated value of y.

- If the radius r and the altitude h of a cone are measured with a percentage error of 1% in each measurement, then find the approx percentage change the curved surface area of the cone if the measured values are $r = 3$ft. and $h = 4$ft. Also, find maximum percentage error in the calculation of the volume?

Thanks!