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Hypothesis Testing: Two sample test

this is a comment

this is [illegible]

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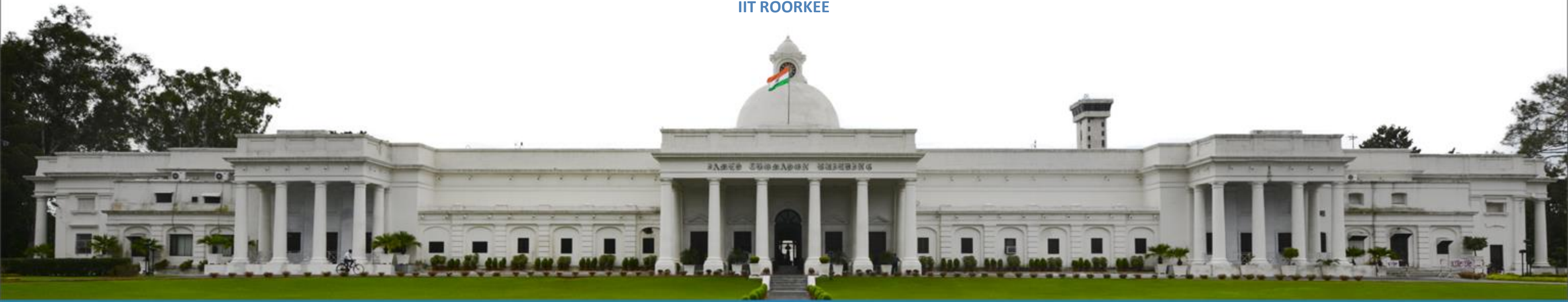


σ_1^2 and σ_2^2 Unknown, Assumed Unequal

- Population means, independent samples
  - σ_1^2 and σ_2^2 known
  - σ_1^2 and σ_2^2 unknown
    - σ_1^2 and σ_2^2 assumed equal
    - σ_1^2 and σ_2^2 assumed unequal

Assumptions:

- Samples are randomly and independently drawn
- Populations are normally distributed
- Population variances are unknown and assumed unequal

σ_1^2 and σ_2^2 Unknown: Assumed Unequal

Pooled var = doesn't suit here.

- Population means, independent samples
  - σ_1^2 and σ_2^2 known
  - σ_1^2 and σ_2^2 unknown
    - σ_1^2 and σ_2^2 assumed equal
    - σ_1^2 and σ_2^2 assumed unequal

df =

Forming interval estimates:

- The population variances are assumed unequal, so a pooled variance is not appropriate
- Use a t value with ν degrees of freedom, where

$$v = \frac{\left[\left(\frac{s_1^2}{n_1}\right) + \left(\frac{s_2^2}{n_2}\right)\right]^2}{\left(\frac{s_1^2}{n_1}\right)^2 / (n_1 - 1) + \left(\frac{s_2^2}{n_2}\right)^2 / (n_2 - 1)}$$

Test Statistic: σ_1^2 and σ_2^2 Unknown, Unequal

σ_1^2 and σ_2^2 unknown

- σ_1^2 and σ_2^2 assumed equal
- σ_1^2 and σ_2^2 assumed unequal

The test statistic for $\mu_1 - \mu_2$ is:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

Where t has ν degrees of freedom:

$$\nu = \frac{\left[\left(\dfrac{s_1^2}{n_1}\right) + \left(\dfrac{s_2^2}{n_2}\right)\right]^2}{\left(\dfrac{s_1^2}{n_1}\right)^2 / (n_1 - 1) + \left(\dfrac{s_2^2}{n_2}\right)^2 / (n_2 - 1)}$$

Problem:Test Statistic: σ_1^2 and σ_2^2 Unknown, Unequal

Example

2 → independent Samples

As in H_2O

- Arsenic concentration in public drinking water supplies is a potential health risk.
- An article in the Arizona Republic (Sunday, May 27, 2001) reported drinking water arsenic concentrations in parts per billion (ppb) for 10 metropolitan Phoenix communities and 10 communities in rural Arizona.
- The data as shown:

ppb

2 arrays
2 Samples

Metro Phoenix		Rural Arizona	
Phoenix,	3	Rimrock,	48
Chandler,	7	Goodyear,	44
Gilbert,	25	New River,	40
Glendale,	10	Apachie Junction,	38
Mesa,	15	Buckeye,	33
Paradise Valley,	6	Nogales,	21
Peoria,	12	Black Canyon City,	20
Scottsdale,	25	Sedona,	12
Tempe,	15	Payson,	1
Sun City,	7	Casa Grande,	18

$\bar{x}_1 = 12.5$
$s_1 = 7.63$

Calculated
Std deviation

$\bar{x}_1 = 27.5$
$s_2 = 15.3$

Problem:Test Statistic: σ_1^2 and σ_2^2 Unknown, Unequal

- We wish to determine it there is any difference in mean arsenic concentrations between metropolitan Phoenix communities and communities in rural Arizona.

μ_1 = phoenix μ_2 = Arizona

(mean) difference between the two independent samples

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

$$t = \frac{\bar{x}_1 - \bar{x}_2 - (0)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Problem:Test Statistic: σ_1^2 and σ_2^2 Unknown, Unequal

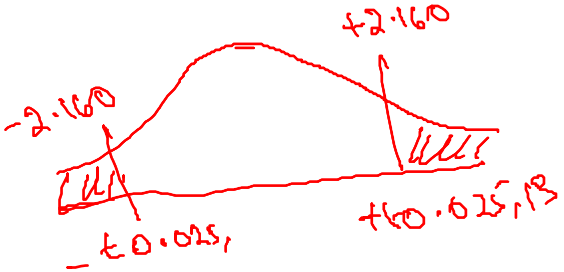
1. The parameters of interest are the mean arsenic concentrations for the two geographic regions, say, μ_1 and μ_2, and we are interested in determining whether $\mu_1 - \mu_2 = 0$.
2. H_0: $\mu_1 - \mu_2 = 0$, or H_0: $\mu_1 = \mu_2$
3. H_1: $\mu_1 \neq \mu_2$
4. $\alpha = 0.05$ (say)
5. The test statistic is

$\mu_1 = \mu_2$ as per H_0

$\therefore$ zero

$$\boxed{t_0^* = \frac{\bar{x}_1 - \bar{x}_2 - 0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}}$$

Problem:Test Statistic: σ_1^2 and σ_2^2 Unknown, Unequal

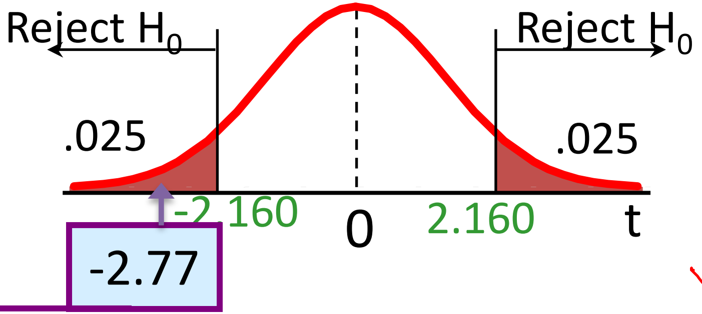


6. The degrees of freedom

$$v = \frac{\left[\left(\frac{s_1^2}{n_1}\right) + \left(\frac{s_2^2}{n_2}\right)\right]^2}{\left(\frac{s_1^2}{n_1}\right)^2 / (n_1 - 1) + \left(\frac{s_2^2}{n_2}\right)^2 / (n_2 - 1)} = \frac{\left[\left(\frac{7.63^2}{10}\right) + \left(\frac{15.3^2}{10}\right)\right]^2}{\left(\frac{7.63^2}{10}\right)^2 / (10 - 1) + \left(\frac{15.3^2}{10}\right)^2 / (10 - 1)} = 13.2 = 13$$

Therefore, using $\alpha = 0.05$, we would reject H_0: $\mu_1 = \mu_2$ if $t_0^* > t_{0.025,13} = 2.160$ or if $t_0^* < -t_{0.025,13} = -2.160$

Problem: Test Statistic: σ_1^2 and σ_2^2 Unknown, Unequal



7. Computations:

$$t = \frac{(12.5 - 27.5) - 0}{\sqrt{\left(\frac{7.63^2}{10} + \frac{15.3^2}{10}\right)}} = \boxed{-2.77}$$

Decision:

Reject H_0 at $\alpha = 0.05$

Some cities have more economic not all equal

Conclusion:

There is evidence of a difference in means.

Problem:Test Statistic: σ_1^2 and σ_2^2 Unknown, Unequal

8. Conclusions: Because $t_0^* = -2.77 < t_{0.025,13} = -2.160$,

- Reject the null hypothesis.
- There is evidence to conclude that mean arsenic concentration in the drinking water in rural Arizona is different from the mean arsenic concentration in metropolitan Phoenix drinking water.

Problem:Test Statistic: σ_1^2 and σ_2^2 Unknown, Unequal

```
In [17]: stats.t.ppf(0.025,13) #critical  t value

Out[17]: -2.160368656461013
```

$t_{\alpha/2}$ → $t_{0.025,13}$ (2 tailed test by default) → t_α

```
In [18]: metro = [3,7,25,10,15,6,12,25,15,7]
         rural = [48,44,40,38,33,21,20,12,1,18]
```

equal variance = FALSE

(t, p)

```
In [20]: stats.ttest_ind(metro,rural, equal_var = False)

Out[20]: Ttest_indResult(statistic=-2.7669395785560558, pvalue=0.015827284816100885)
```

$p < 0.05$ ∴ Reject H_0

Dependent Samples

2 Sample test

Tests Means of 2 Related Populations

- Paired or matched samples
- Repeated measures (before/after)
- Use difference between paired values:

$$\boxed{d_i = x_i - y_i}$$

Before treatment After

- Assumptions:
  - Both Populations Are Normally Distributed

$d = x_1 - y_1$

$\bar{d} = \bar{x} - \bar{y}$

$\left[\frac{x_1 + x_2 + x_3 + \ldots - (y_1 + y_2 + \ldots)}{n}\right]$

Test Statistic: Dependent Samples

The test statistic for the mean difference is a t value, with n – 1 degrees of freedom:

Sd = Std deviation of differenced data

Assumed difference in the population

$$t = \frac{\bar{d} - D_0}{\frac{s_d}{\sqrt{n}}}$$

$$\bar{d} = \frac{\sum d_i}{n} = \bar{x} - \bar{y}$$

D_0 = hypothesized mean difference

s_d = sample standard dev. of differences

n = the sample size (number of pairs)

Decision Rules: Dependent Samples

$\mu_1 \neq \mu_2$ Based on type of Study

Lower-tail test:

$H_0: \mu_1 - \mu_2 \geq 0$

$H_1: \mu_1 - \mu_2 < 0$

Upper-tail test:

$H_0: \mu_1 - \mu_2 \leq 0$

$H_1: \mu_1 - \mu_2 > 0$

$\mu_1 \leq \mu_2$

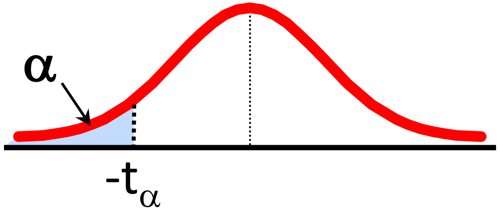
Two-tail test:

$H_0: \mu_1 - \mu_2 = 0$

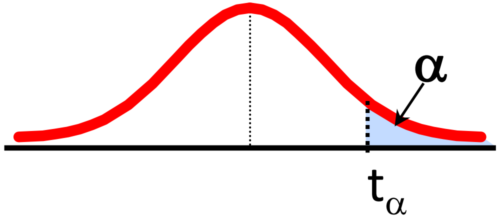
$H_1: \mu_1 - \mu_2 \neq 0$

$\mu_1 = \mu_2$

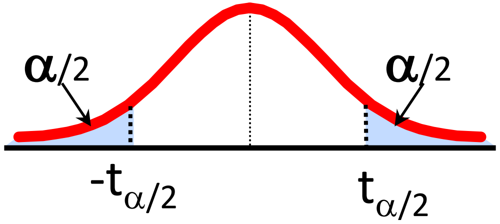
Decision Rules: Dependent Samples



Reject H_0 if $t < -t_{n-1,\,\alpha}$



Reject H_0 if $t > t_{n-1,\,\alpha}$



Reject H_0 if $t < -t_{n-1\,,\alpha/2}$
or $t > t_{n-1\,,\alpha/2}$

Where $$t = \frac{\bar{d} - D_0}{\frac{s_d}{\sqrt{n}}}$$ has n - 1 d.f.

Dependent Samples: Example

- An article in the Journal of Strain Analysis (1983, Vol. 18, No. 2) compares several methods for predicting the shear strength for steel plate girders.
- Data for two of these methods, the Karlsruhe and Lehigh procedures, when applied to nine specific girders, are shown in Table .
- We wish to determine whether there is any difference (on the average) between the two methods.

Table : Strength Predictions for Nine Steel Plate Girders

(Predicted Load/Observed Load)

Girder	Karlsruhe Method X	Lehigh Method Y	$\boxed{\text{Difference dj}}$ X − Y
S11	1.186	1.061	0.119
S21	1.151	0.992	0.159
S31	1.322	1.063	0.259
S41	1.339	1.062	0.277
S51	1.200	1.065	0.138
S21	1.402	1.178	0.224
S22	1.365	1.037	0.328
S23	1.537	1.086	0.451
S24	1.559	1.052	0.507

Inferences About the Difference Between Two Population Means: Matched Samples

1. The parameter of interest is the difference in mean shear strength between the two methods, say, $\mu_D = \mu_1 - \mu_2 = 0$.
2. H_0: $\mu_D = 0$
3. H_1: $\mu_D \neq 0$
4. $\alpha = 0.05$
5. The test statistic is

$$t_0 = \frac{\bar{d}}{s_D/\sqrt{n}}$$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$\frac{\bar{d} - 0}{s_d/\sqrt{n}}$$

Inferences About the Difference Between Two Population Means: Matched Samples

6. Reject H_0 if $t_0 > t_{0.025,8} = 2.306$ or if $t_0 < -t_{0.025,8} = -2.306$.
7. Computations: The sample average and standard deviation of the differences d_j are $\bar{d} = 0.2736$ and $s_D = 0.1356$, so the test statistic is

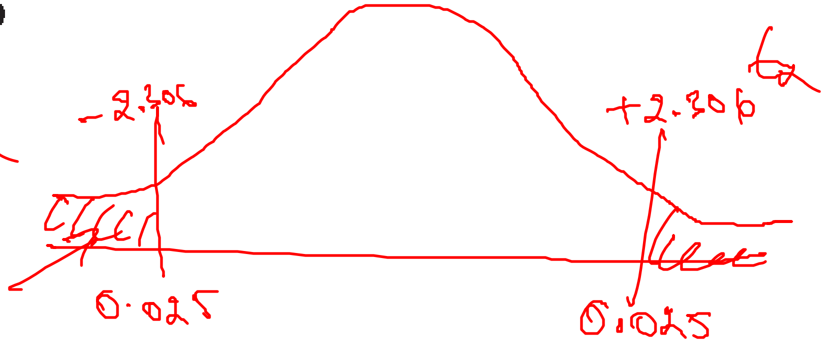
$$t_0 = \frac{\bar{d}}{s_D/\sqrt{n}} = \frac{0.2736}{0.1356/\sqrt{9}} = 6.05$$

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$ $\therefore$ effect significant.

t_α

$t = 6.05$



8. Conclusions: Since $t_0 = 6.05 > 2.306$,

we conclude that the strength prediction methods yield different results.

The P-value for $t_0 = 6.05$ is $P = 0.0002$,

```
In [37]: KARL= [1.186,1.151,1.322,1.339,1.200,1.402,1.365,1.537,1.559]
         LEH= [1.061,0.992,1.063,1.062,1.065,1.178,1.037,1.086,1.052]
```

arrays

relative (dependent)

```
In [38]: stats.ttest_rel(KARL,LEH)

Out[38]: Ttest_relResult(statistic=6.0819394375848255, pvalue=0.00029529546278604066)
```

t

p

Inferences About the Difference Between Two Population Proportions

- Inferences About the Difference Between Two Population Proportion

Categorical variable.

Inferences About the Difference Between Two Population Proportions

- Interval Estimation of $p_1 - p_2$

- Hypothesis Tests About $p_1 - p_2$

Sampling Distribution of $\bar{p}_1 - \bar{p}_2$

- Expected Value

mean → $$E(\bar{p}_1 - \bar{p}_2) = p_1 - p_2$$

var $\frac{p_1 q_1}{n_1}$

$\frac{p_2 q_2}{n_2}$

- Standard Deviation (Standard Error)

$$\sigma_{\bar{p}_1 - \bar{p}_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

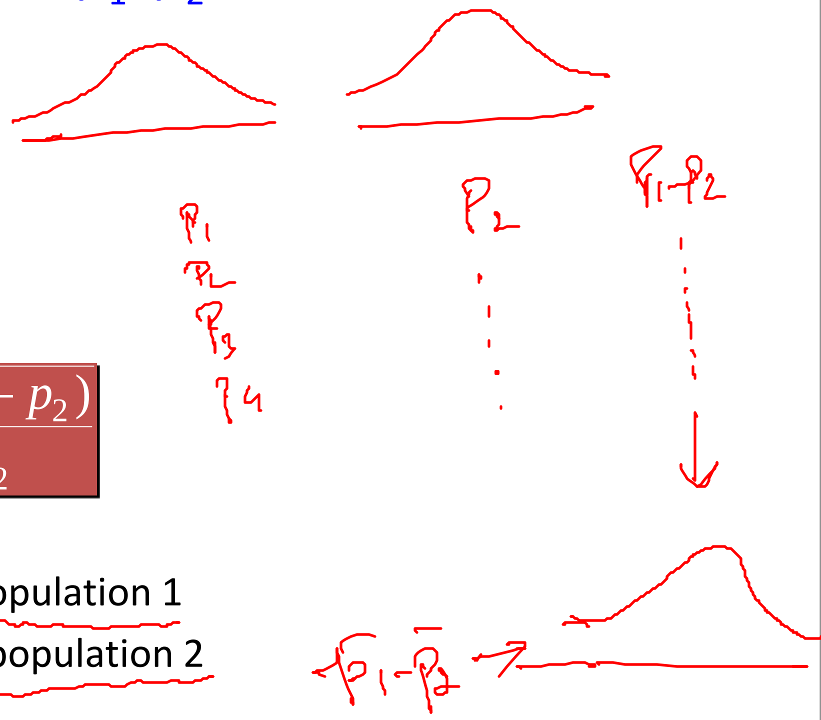
variance $= \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$

where: n1 = size of sample taken from population 1

n2 = size of sample taken from population 2

$E = \frac{pq}{n}$

$\sigma^2 = \frac{pq}{n}$



$$\sqrt{\frac{P_1(1-P_1)}{n_1}+\frac{P_2(1-P_2)}{n_2}}$$

Sampling Distribution of $\bar{p}_1 - \bar{p}_2$

- If the sample sizes are large, the sampling distribution of $\bar{p}_1 - \bar{p}_2$ can be approximated by a normal probability distribution.

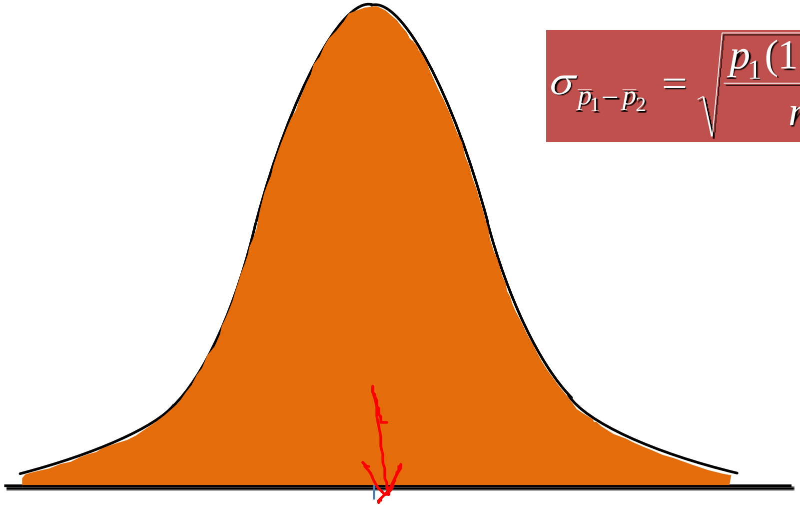
- The sample sizes are sufficiently large if all of these conditions are met:

$np > 5$ or
$nq > 5$

$n_1p_1 \geq 5$	$n_1(1 - p_1) \geq 5$
$n_2p_2 \geq 5$	$n_2(1 - p_2) \geq 5$

Sampling Distribution of $\bar{p}_1 - \bar{p}_2$

$$\sigma_{\bar{p}_1-\bar{p}_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$



$\bar{p}_1 - \bar{p}_2$

$p_1 - p_2$

Interval Estimation of p_1 - p_2

- Interval Estimation

$$\bar{p}_1 - \bar{p}_2 \pm z_{\alpha/2}\sqrt{\frac{\bar{p}_1(1-\bar{p}_1)}{n_1} + \frac{\bar{p}_2(1-\bar{p}_2)}{n_2}}$$

(Proportion)

$$\bar{p} \pm Z_{\alpha/2}\sqrt{\frac{pq}{n}}$$

$$\bar{x} \pm Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$(\bar{p}_1 - \bar{p}_2) \pm Z_{\alpha/2}\sqrt{\frac{\bar{p}_1(1-\bar{p}_1)}{n_1} + \frac{\bar{p}_2(1-\bar{p}_2)}{n_2}}$$

Point Estimator of the Difference Between Two Population Proportions

Example

- p1 = proportion of the population of households "aware" of the product after the new campaign
- p2 = proportion of the population of households "aware" of the product before the new campaign
- $\bar{p}_1$ = sample proportion of households "aware" of the product after the new campaign
- $\bar{p}_2$ = sample proportion of households "aware" of the product before the new campaign

Aware? Before

(Promotion)

After Aware?

After 120 out of 250 Aware.

Before 60 out of 150 Aware

$$\bar{p}_1 - \bar{p}_2 = \frac{120}{250} - \frac{60}{150} = .48 - .40 = .08$$

ie 8%

Hypothesis Tests about p_1 - p_2

- Hypothesis

We focus on tests involving no difference between the two population proportions (i.e. $p_1 = p_2$)

Left-tailed	Right-tailed	Two-tailed
$H_0\colon\ p_1 - p_2 \geq 0$	$H_0\colon\ p_1 - p_2 \leq 0$	$H_0\colon\ p_1 - p_2 = 0$
$H_a\colon\ p_1 - p_2 < 0$	$H_a\colon\ p_1 - p_2 > 0$	$H_a\colon\ p_1 - p_2 \neq 0$

Hypothesis Tests about p_1 - p_2

- Standard Error of $\bar{p}_1 - \bar{p}_2$ when $p_1 = p_2 = p$

$$\sigma_{\bar{p}_1 - \bar{p}_2} = \sqrt{p(1-p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

- Pooled Estimator of p when $p_1 = p_2 = p$

Why Pooled?

If two population proportions are same

$$\bar{p} = \frac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1 + n_2}$$

Hypothesis Tests about p_1 - p_2

- Test Statistic

$$z = \frac{(\bar{p}_1 - \bar{p}_2)}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$\longrightarrow S$

$$\boxed{Z = \frac{\bar{P}_1 - \bar{P}_2}{S}}$$

$$S = \sqrt{\frac{\bar{p}(1-\bar{p})}{n_1 + n_2}}$$

$$\sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}}$$

$P_1 q_1 = P_2 q_2$

Pooled variance

Problem: Hypothesis Tests about $p_1 - p_2$

- Extracts of St. John's Wort are widely used to treat depression.
- An article in the April 18, 2001 issue of the *Journal of the American Medical Association* ("Effectiveness of St. John's Worton Major Depression: A Randomized Controlled Trial") compared the efficacy of a standard extract of St. John's Wort with a placebo in 200 outpatients diagnosed with major depression.
- Patients were randomly assigned to two groups; one group received the St. John's Wort, and the other received the placebo.
- After eight weeks, 19 of the placebo-treated patients showed improvement, whereas 27 of those treated with St. John's Wort improved.
- Is there any reason to believe that St. John's Wort is effective in treating major depression? Use 0.05.

Problem: Hypothesis Tests about p_1 - p_2

1. The parameters of interest are p_1 and p_2, the proportion of patients who improve following treatment with St. John's Wort (p_1) or the placebo (p_2).

   Proportion : ratio
   a out of b

2. H_0: $p_1 = p_2$ → No effect
3. H_1: $p_1 \neq p_2$ → Some effect

   } we decide 1 tail or 2 tail

4. $\alpha = 0.05$
5. The test statistic is

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where $\hat{p}_1 = 27/100 = 0.27, \hat{p}_2 = 19/100 = 0.19, n_1 = n_2 = 100,$ and

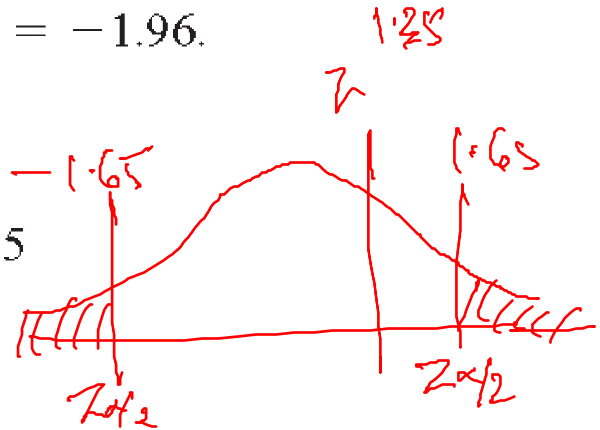
$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{19 + 27}{100 + 100} = 0.23$$

Problem: Hypothesis Tests about p_1 - p_2

6. Reject H_0: $p_1 = p_2$ if $z_0 > z_{0.025} = 1.96$ or if $z_0 < -z_{0.025} = -1.96$.

7. Computations: The value of the test statistic is

$$z_0 = \frac{0.27 - 0.19}{\sqrt{0.23(0.77)\left(\frac{1}{100} + \frac{1}{100}\right)}} = 1.35$$



8. Conclusions: Since z_0 1.35 does not exceed z 0.025, we cannot reject the null hypothesis.

The P-value is $P \cong 0.177$. There is insufficient evidence to support the claim that St. John's Wort is effective in treating major depression.

∴ Can't say significant effect of treatment

In [29]:
```python
import math
def two_samp_proportion(p1,p2,n1,n2):
    p_pool = ((p1*n2)+(p2*n1))/(n1+n2)
    x = (p_pool*(1- p_pool)*((1/n1)+(1/n2)))
    s = math.sqrt(x)
    z = (p1- p2)/s
    if (z < 0):
        p_val = stats.norm.cdf(z)
    else:
        p_val = 1 - stats.norm.cdf(z)
    return z, p_val*2
```

LT

RT

$$\left[\frac{p_1 n_1 + p_2 n_2}{n_1 + n_2}\right] = \bar{p}$$

$$s = \sqrt{\bar{p}(1-\bar{p})\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}$$

In [30]:
```python
two_samp_proportion(0.27,0.19,100,100)
```

Out[30]: (1.3442056254198995, 0.17888190308175567)

z p

$$Z = \frac{(\bar{p}_1 - \bar{p}_2) - (P_1 - P_2)}{s}$$

$$\boxed{P_1 - P_2 = 0}$$

In [27]:
```python
stats.norm.cdf(1.3442056254198995)
```

Out[27]: 0.9105590484591222

Accept Null hypothesis.

Thank You