2016

quark

A COSMIC ODYSSEY

qSAT

(Quark Scholastic Aptitude Test)

(For Level 2: Students of Class 11th and 12th)

Time: 90 minutes | Maximum Marks: 200

Read all the Instructions carefully.

Instructions:

A. General

- This Booklet is your Question Paper and it has **8 pages.**
- Along with the Question Paper, a **rough sheet is provided** for your rough work. Do not use additional sheets for rough work.
- Use of blank papers, slide rules, calculators, cellular phones, and electronic gadgets in any form is not allowed.
- The **answer sheet is provided separately.** Make sure you fill all your details on the answer sheet provided to you.

B. Question Paper Format

- The question paper contains **35 questions.**
- The question paper is divided into **10 Sections.**
- All the 10 sections contain different number of questions.

C. Marking Scheme

- The marking scheme for each section is different.
- Extra marks, **Section Bonus**, will be awarded if all the questions of that section are answered correctly.
- **5 marks will be deducted for an incorrectly answered question.**
- Adjacent to each section's name, the marking scheme for that section is mentioned in the following format-

Format: (No. of Questions in the section x Marks per Question; Section Bonus)

All the Best!!!

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Section 1: Warm Up (1x2; 0)

1. "Give us a puzzle, Dad", said Arthur one evening. "Very well", said Eldon, who then went into another room and returned about ten minutes later. He then put 3 cards face down on the table. On the back of each was written a statement. Eldon explained that if the card was red, the sentence written on it was true, but if the card was black, the sentence written on it was false. Here are the 3 backs.

A	B	C
Exactly one of these three cards is black	Exactly two of these three cards is black	All of three cards are black

What color is each of the three cards?

Section 2: Mr. L (1x3; 0)

2. In a business meeting, each person shakes hands with each other person, with the exception of Mr. L. Since Mr. L arrives after some people have left, he shakes hands only with those present. If the total number of handshakes is exactly 100, how many people left the meeting before Mr. L arrived? (Nobody shakes hands with the same person more than once.)

Section 3: Diseased Chessboard (3x2; 2)

A unit square of a chessboard of size NxN gets infected if at least two of its neighbours are infected (two unit squares are called neighbours if they share a common edge). So,

3. If 2 squares are initially infected, what is the maximum number of squares that can be infected?

4. If 3 squares are initially infected, what is the maximum number of squares that can be infected?

5. Find the minimum number of unit squares that should be infected initially, so that the whole chess board gets infected.

Section 4: Ah, Math (3x3; 3)

6. A 3-digit number 'abc' in base 6 is equal to the 3-digit number 'cba' in base 9. Find a, b, c.

7. If a, b, c > 1, then,

$$\left(\frac{1}{1+\log_{a^2c}\frac{c}{a}}\right)+\left(\frac{1}{1+\log_{b^2c}\frac{a}{b}}\right)+\left(\frac{1}{1+\log_{c^2a}\frac{b}{c}}\right)=?$$

8. Find the remainder given by $3^{89} * 7^{86}$ when divided by 17.

Section 5: Pompeiu (3x4; 8)

Given two points A and B, if one rotates B around A through 60◦ to a point B', then the triangle ABB' is equilateral. A consequence of this is the Pompeiu's theorem, discovered by the Romanian mathematician D. Pompeiu in 1936.
Pompeiu's theorem is a simple fact that was (surprisingly) never discovered by Euclid.

Get into Pompeiu's shoes and state whether the following properties are true, or false:
(Give a one line Explanation to support your answer)

9. Given an equilateral triangle ABC and a point P that lies on the circumcircle of ABC, one can construct a triangle of side lengths equal to PA, PB, and PC.

10. If P lies on the circumcircle, then one of these three lengths is equal to the sum of the other two.

11. Given that, in a triangle ABC, AB is the longest side. For any point P in the interior of the triangle, PA+PB > PC.

Section 6: No Square is Negative (2x4; 3)

Given a function such that

$$f(x^2) - (f(x))^2 \geq 0.25$$

12. Find the value of f(0) and f(1).

13. Give a one-to-one function with the above property.

Section 7: Zero (1x4; 4)

14. The sum of n real numbers is zero. The sum of their pairwise products is also zero. What can be said regarding those n real numbers?
a) The sum of their cubes is zero.
b) The sum of their squares is zero.
c) The sum of their nth powers is zero.
d) The sum of, the product of three numbers taken at a time, is zero.
e) Half of the numbers are negative.

Section 8: Count till 10 (10x4; 35)

15. Integers 1; 2; _ _ _ ; n are placed in such a way that each value is either bigger than all preceding values or smaller than all preceding values. In how many ways this can be done? (For example in case of n = 5, 3 2 4 1 5 is valid and 3 2 5 1 4 is not valid.)

16. Find all pairs (x; y) of integers such that y^2 = x(x + 1)(x + 2).

17. What is the number of subsets of {1,2,....,10} which contains at least one odd number?

18. What is the smallest positive integer n for which 50!/24^n is not an integer?

19. A positive integer N has its first, third and fifth digits equal and its second, fourth and sixth digits equal. In other words, when written in the usual decimal system it has the form XYXYXY, where X and Y are the digits. For what values of X and Y does the number form a perfect square?

20. $If\ f(x) = \sum_{i=0}^{n} \left\{\frac{x^i}{i!}\right\}$; Then does f(x) have no repeated roots? Explain in one line.

21. Find the GCD of $2^{13} - 2, 3^{13} - 3, 4^{13} - 4, \dots \dots, 12^{13} - 12$

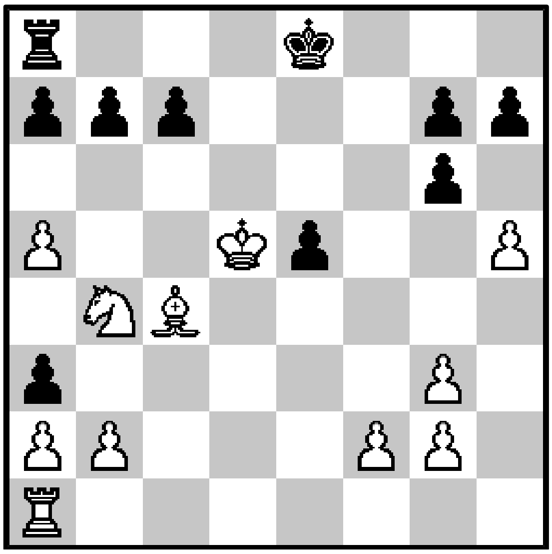
22. Suppose a; b; c are all real numbers such that a+b+c > 0; abc > 0 and ab+bc+ac > 0:
Can either a, b or c be negative?

23. How many pairs of integers (x, y) are there such that 11x + 8y + 17 = xy.

24. Find all the possible integer solutions of x and y for the equation:

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{2}$$

Section 9: Chess, Sherlock? (6x4; 6)



It was the day just after Sherlock’s birthday that Dr. Watson and Mrs. Hudson were playing chess, with Mrs. Hudson playing as white. Sherlock Holmes happened to enter the room and saw Mrs. Hudson play a knight. “Nice move Mrs. Hudson”, he exclaimed. Dr. Watson was always surprised by Sherlock’s reasoning and logical abilities and thus tried to put it to test. He put his left hand on the king and his right hand on the rook and was about to castle.
Sherlock raised his head and sprang up like a tiger “You can't castle! You really can't, you know!” Dr. Watson was not surprised to see this but wanted to know how Sherlock had guessed that castling was an illegal move? Sherlock left question clues for Dr. Watson to as how he got to know about this. Help Dr. Watson find the right explanation.

Castling is illegal when the king or the castling rook has moved prior to castling.

Notation: the bottom left corner of the board where the white rook is placed is given by A1. Also the black king is at E8.

25. How many black pieces did the pawn on h5 kill?

a) 1
b) 2
c) 3
d) None of the above.

26. How many black pieces did the pawns on a5 kill?

a) 1
b) 2
c) 3
d) 4

27. How many black pieces did white kill on a black square?

a) 1
b) 2
c) 3
d) 4

28. Which of the following paths are impossible for the pawn on e5 to follow in the game?

a) e7-d6-e5
b) e7-f6-e5
c) e7-e6-e5
d) e7-e5
e) Both a, b

29. If in the black's last move, Dr. Watson moved the pawn (which is currently on a3) from a4 to a3, then what happened to the white pawn on the d file?

a) It was killed by a black pawn
b) It was killed by the knight on b4
c) It was killed by the black rook
d) It got promoted

30. If Dr. Watson states that his last move was with a pawn, is he lying?

a) Yes
b) No
c) Cannot Be Determined

Section 10: The Number Machine (5x7; 10)

A number machine has an input which accepts only specific numbers called as 'acceptable numbers' and obeys the following two rules:

- For any number X the number 2X is acceptable and returns X.
- If X returns Y, then 3X produces the associate of Y.

(Here NM is not N x M but is the joining of digits. E.g., if N=23, M=89 then NM is 2389 not 23 x 89; Also the associate of a number N is defined as N2N)

On the basis of the given information, find a number N such that:

31. It gives back N. Find every such number.

32. It gives back the associate of N. Find every such number.

33. Gives back 7N

After the following attempts the scientist who made this machine decided to complicate it and added the following extra rules:

- If X returns Y, then 4X returns the reverse of Y
- If X returns Y, then 5X produces YY (repeat of Y)

(The reverse of a number is defined as the number formed after reversing all the digits. E.g. reverse of 2345 is 5432)

On the basis of the information given before and the new rules just mentioned, find a number N such that:

34. It produces the reverse of its own repeat.

35. It produces the associate of its own repeat.

---The End---