2016 quark
A COSMIC ODYSSEY

qSAT

(Quark Scholastic Aptitude Test)

(For Level 1: Students of Class 8th, 9th and 10th)

Time: 90 minutes | Maximum Marks: 200

Read all the Instructions carefully.

Instructions:

A. General

- This Booklet is your Question Paper and it has **8 pages**.
- Along with the Question Paper, a **rough sheet is provided** for your rough work. Do not use additional sheets for rough work.
- Use of blank papers, slide rules, calculators, cellular phones, and electronic gadgets in any form is not allowed.
- The **answer sheet is provided separately**. Make sure you fill all your details on the answer sheet provided to you.

B. Question Paper Format

- The question paper contains **23 questions**.
- The question paper is divided into **8 Sections**.
- All the 8 Sections contain different number of questions.

C. Marking Scheme

- The marking scheme for each section is different.
- Extra marks, **Section Bonus**, will be awarded if all the questions of that section are answered correctly.
- **5 marks will be deducted for an incorrectly answered question**.
- Adjacent to each section's name, the marking scheme for that section is mentioned in the following format-

Format: (No. of Questions in the section x Marks per Question; Section Bonus)

All the Best!!!

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Section 1: Appetizer (1x5; 0)

1. "Give us a puzzle, Dad", said Arthur one evening. "Very well", said Eldon, who then went into another room and returned about ten minutes later. He then put 3 cards face down on the table. On the back of each was written a statement. Eldon explained that if the card was red, the sentence written on it was true, but if the card was black, the sentence written on it was false. Here are the 3 backs.

A	B	C
Exactly one of these three cards is black	Exactly two of these three cards is black	All of three cards are black

What color is each of the three cards?

Section 2: Math Course (4x5; 10)

2. Find two integer solutions for the following equation:

$$x^y + y^x = 783$$

3. Find the value of:

$$1^2 - 2^2 + 3^2 - 4^2 + \cdots + 2009^2 - 2010^2.$$

4. The sum of n real numbers is zero. The sum of their pairwise products is also zero. What can be said regarding those n real numbers- (multiple options can be correct)

a. The sum of their cubes is zero.
b. The sum of their squares is zero.
c. The sum of their nth powers is zero.
d. The sum of, the product of three numbers taken at a time, is zero.
e. Half of the numbers are negative.

5. For any real number "a", what are the possible values of a^0?

Section 3: Diseased Chessboard (3x3; 5)

A unit square of a chessboard of size N x N gets infected if at least two of its neighbours are infected (two unit squares are called neighbours if they share a common edge). So,

6. If 2 squares are initially infected, what is the maximum number of squares that can be infected?

7. If 3 squares are initially infected, what is the maximum number of squares that can be infected?

8. Find the minimum number of unit squares that should be infected initially, so that the whole chess board gets infected.

Section 4: Mr. L and Eldon again (2x5; 5)

9. In a business meeting, each person shakes hands with each other person, with the exception of Mr. L. Since Mr. L arrives after some people have left, he shakes hands only with those present. If the total number of handshakes is exactly 100, how many people left the meeting before Mr. L arrived? (Nobody shakes hands with the same person more than once.)

10. Eldon once bought a very remarkable plant which, on the first day, increased its height by 1/2, then on the second day by 1/3, then on the third day by 1/4, and so on. How many days did it take to grow to 100 times its original height?

Section 5: Invariants (3x5; 10)

The concept of invariants is an interesting mathematical concept. We'll understand the concept with the help of chess.

Consider a chess piece which moves one square (in any direction) and then 3 squares in the perpendicular direction (similar to a knight but an extra square in the longer move). How many moves will this piece require to move from one position to its adjacent position considering and infinitely large chessboard. The answer to this problem is that this piece cannot do the required operation. When this piece makes a move, the colour on which it is standing does not change. As the colour of the adjacent position is different than its initial position, this cannot be done. Here the colour of the square on which the piece is standing is the invariant.

Invariants can also take quantitative values. Invariants may be hidden and may not be obvious at the first glance, but a bit of logic and math can uncover them like in the following problems.

11. The numbers from 1 to 50 are written on a black board. Students come and randomly erase two numbers a and b and then replace by one number a+b-1. What will be the last number remaining on the blackboard?

12. In an 8x8 table one of the boxes is colored black and all the others are white. Can we make all the boxes white by recolouring the rows and columns as many times as we want? (Note: Here 'Recolouring" is the operation of changing the colour of all boxes in a row or column.)
Give a one line explanation to support your answer.

13. What if the table has 7x7 boxes and one of the boxes is colored black? Can we make all the boxes white by recolouring the rows and columns as many times as we want?
Give a one line explanation to support your answer.

Section 6: Math Be With You (4x6; 7)

14. A positive integer N has its first, third and fifth digits equal and its second, fourth and sixth digits equal. In other words, when written in the usual decimal system it has the form XYXYXY, where X and Y are the digits. For what values of X and Y does the number form a perfect square?

15. Find all possible real x such that:

$$17^x + 14^x = \sqrt{485}^x$$

16. Find the remainder given by $3^{89} * 7^{86}$ when divided by 17.

17. Find all the possible integer solutions of x and y for the equation:

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{2}$$

Section 7: The Number Machine (2x15; 15)

A number machine has an input which accepts only specific numbers called as 'acceptable numbers' and obeys the following two rules:

- For any number X the number 2X is acceptable and returns X.
- If X returns Y, then 3X produces the associate of Y.

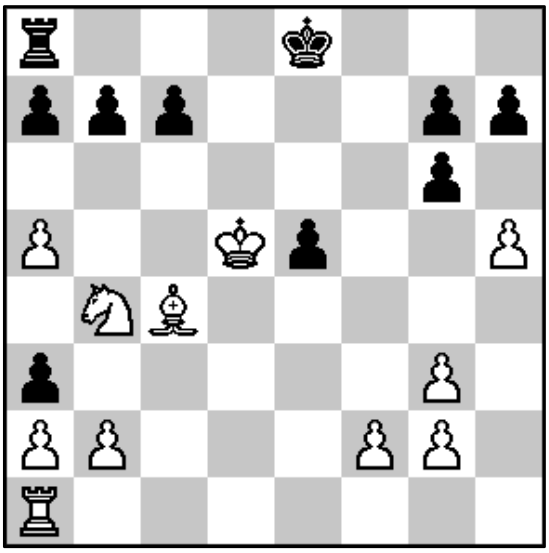
(Here NM is not N x M but is the joining of digits. E.g., if N=23, M=89 then NM is 2389 not 23 x 89; Also the associate of a number N is defined as N2N)

On the basis of the given information, find a number N such that:

18. It gives back N. Find every such number.

19. It gives back the associate of N. Find every such number.

Section 8: The Game is Afoot (4x5; 15)



It was the day just after Sherlock's birthday that Dr. Watson and Mrs. Hudson were playing chess, with Mrs. Hudson playing as white. Sherlock Holmes happened to enter the room and saw Mrs. Hudson play a knight. "Nice move Mrs. Hudson", he exclaimed. Dr. Watson was always surprised by Sherlock's reasoning and logical abilities and thus tried to put it to test. He put his left hand on the king and his right hand on the rook and was about to castle.

Sherlock raised his head and sprang up like a tiger "You can't castle! You really can't, you know!" Dr. Watson was not surprised to see this but wanted to know how Sherlock had guessed that castling was an illegal move? Sherlock left question clues for Dr. Watson to as how he got to know about this. Help Dr. Watson find the right explanation.

Castling is illegal when the king or the castling rook has moved prior to castling.
(Notation: The bottom left corner of the board where the white rook is placed is given by A1. Also the black king is at E8)

20. How many black pieces did the pawn on A5 kill?

21. How many black pieces did white pieces kill on a black square?

22. If in the black's last move, Dr. Watson moved the pawn (which is currently on A3) from A4 to A3, then what happened to the white pawn on the D5?

23. Is castling illegal? Why or Why not?

---The End---