NCERT Discrete - 11.9.3.12

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Question : 11.9.3.12 The sum of the first three terms of a G.P is 39/10 and their product is 1. Find the common ratio and the terms.

Solution:

Parameter	Value	Description
$x(0)$		First term
r		Common ratio
$x(0)^3 r^3$	1	Product of terms
$x(0) + x(0)r + x(0)r^2$	$\frac{39}{10}$	Sum of terms

TABLE 0
INPUT PARAMETERS

$$x(n) = x(0)r^n \tag{1}$$

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \tag{2}$$

$$Y(z) = X(z)U(z) \tag{3}$$

$$= \frac{x(0)}{(1 - rz^{-1})(1 - z^{-1})} \quad |z| > |r| \tag{4}$$

$$= \frac{x(0)\left(\frac{r}{1-rz^{-1}} - \frac{1}{1-z^{-1}}\right)}{r - 1} \tag{5}$$

Taking inverse z-transform using contour integration

$$y(n) = \frac{1}{2\pi j} \oint_C Y(z) z^{n-1} dz \tag{6}$$

where C is clockwise closed contour in region of convergence of $Y(z)$.

$$y(n) = x(0)\left(\frac{r^{n+1} - 1}{r - 1}\right)u(n) \tag{7}$$

From Table 0 and (7) :

$$y(2) = x(0)\left(\frac{r^3 - 1}{r - 1}\right) \tag{8}$$

$$\frac{39}{10} = x(0)\left(r^2 + r + 1\right) \tag{9}$$

$$\frac{39r}{10} = r^2 + r + 1 \quad (\because x(0)r = 1) \tag{10}$$

$$(2r - 5)(5r - 2) = 0 \tag{11}$$

$$r = \frac{2}{5} \quad or \quad \frac{5}{2} \tag{12}$$

1) If $r = \frac{2}{5}$, then terms are $\frac{5}{2}$, 1, $\frac{2}{5}$.
2) If $r = \frac{5}{2}$, then terms are $\frac{2}{5}$, 1, $\frac{5}{2}$.

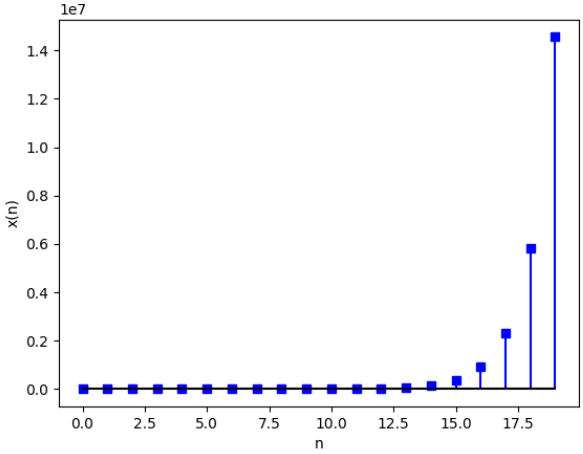

Fig. 2. stem plots of GP if $r = \frac{2}{5}$

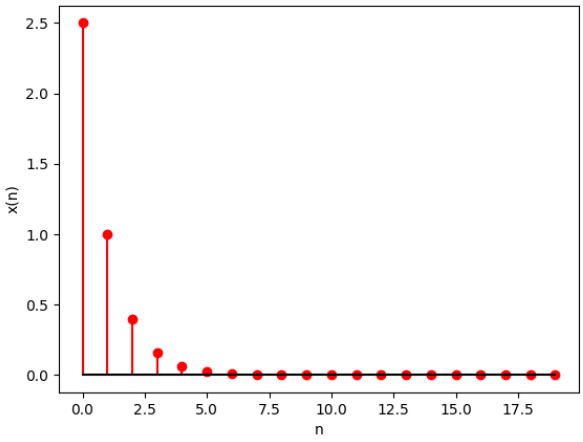

Fig. 2. stem plots of GP if $r = \frac{5}{2}$