

# NCERT Discrete - 11.9.3.12

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**Question : 11.9.3.12** The sum of the first three terms of a G.P is  $39/10$  and their product is 1. Find the common ratio and the terms.

**Solution:** Let the G.P be  $x(0)$ ,  $x(0)r$ ,  $x(0)r^2$ ,  $x(0)r^3$ ,  
.....

Parameter	Value	Description
$x(0)$		Second term
$r$		Common ratio
$x(0)^3 r^3$	1	Product of terms
$x(0) + x(0)r + x(0)r^2$	$\frac{39}{10}$	Sum of terms

TABLE 0  
INPUT PARAMETERS

$$x(n) = x(0)r^n \quad (1)$$

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \quad (2)$$

$$Y(z) = X(z)U(z) \quad (3)$$

$$= \frac{x(0)}{(1 - rz^{-1})(1 - z^{-1})} \quad |z| > |r| \quad (4)$$

$$= \frac{x(0)(\frac{r}{1-rz^{-1}} - \frac{1}{1-z^{-1}})}{(r-1)} \quad (5)$$

The inverse of  $Y(z)$  is  $y(n)$  which is:

$$y(n) = x(0)(\frac{r^{n+1} - 1}{r - 1})u(n) \quad (6)$$

From Table 0 and (6) :

$$y(2) = x(0)\frac{r^3 - 1}{r - 1} \quad (7)$$

$$\frac{39}{10} = x(0)(r^2 + r + 1) \quad (8)$$

$$\frac{39r}{10} = (r^2 + r + 1) \quad x(0)r = 1 \quad (9)$$

$$(2r - 5)(5r - 2) = 0 \quad (10)$$

$$r = \frac{2}{5} \text{ or } \frac{5}{2} \quad (11)$$

- 1) If  $r = \frac{2}{5}$ , then terms are  $\frac{5}{2}$ , 1,  $\frac{2}{5}$ .
- 2) If  $r = \frac{5}{2}$ , then terms are  $\frac{2}{5}$ , 1,  $\frac{5}{2}$ .