

NCERT Discrete - 11.9.3.12

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Question : 11.9.3.12 The sum of the first three terms of a G.P is $39/10$ and their product is 1. Find the common ratio and the terms.

Solution:

Parameter	Value	Description
$x(0)$		First term
r		Common ratio
$x(0)^3 r^3$	1	Product of terms
$x(0) + x(0)r + x(0)r^2$	$\frac{39}{10}$	Sum of terms

TABLE 0
INPUT PARAMETERS

$$x(n) = x(0)r^n \quad (1)$$

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \quad (2)$$

$$Y(z) = X(z)U(z) \quad (3)$$

$$= \frac{x(0)}{(1 - rz^{-1})(1 - z^{-1})} \quad |z| > |r| \quad (4)$$

$$= \frac{x(0) \left(\frac{r}{1 - rz^{-1}} - \frac{1}{1 - z^{-1}} \right)}{r - 1} \quad (5)$$

Taking inverse z-transform using contour integration

$$y(n) = \frac{1}{2\pi j} \oint_C Y(z) z^{n-1} dz \quad (6)$$

where C is clockwise closed contour in region of convergence of $Y(z)$.

$$y(n) = x(0) \left(\frac{r^{n+1} - 1}{r - 1} \right) u(n) \quad (7)$$

From Table 0 and (7) :

$$y(2) = x(0) \left(\frac{r^3 - 1}{r - 1} \right) \quad (8)$$

$$\frac{39}{10} = x(0) (r^2 + r + 1) \quad (9)$$

$$\frac{39r}{10} = r^2 + r + 1 \quad (\because x(0)r = 1) \quad (10)$$

$$(2r - 5)(5r - 2) = 0 \quad (11)$$

$$r = \frac{2}{5} \quad \text{or} \quad \frac{5}{2} \quad (12)$$

- 1) If $r = \frac{2}{5}$, then terms are $\frac{5}{2}, 1, \frac{2}{5}$.
- 2) If $r = \frac{5}{2}$, then terms are $\frac{2}{5}, 1, \frac{5}{2}$.

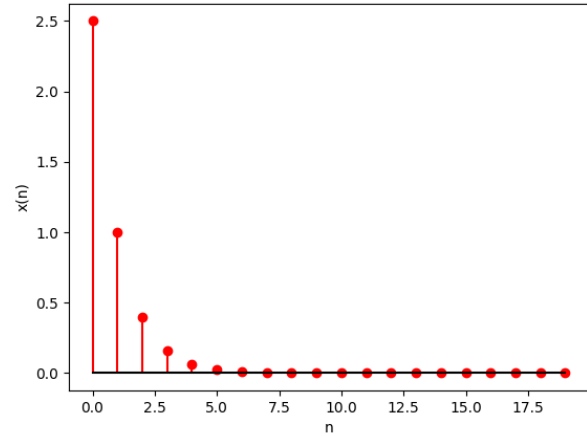


Fig. 2. stem plots of GP if $r = \frac{2}{5}$

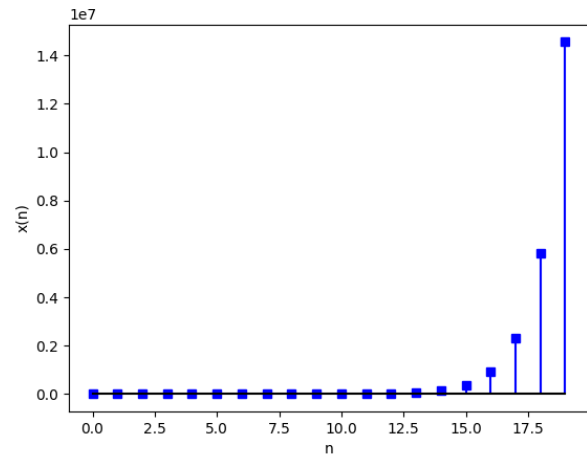


Fig. 2. stem plots of GP if $r = \frac{5}{2}$