

NCERT Discrete - 10.5.2.2

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Question 10.5.2.2:

- 1) 30th term of the AP: 10, 7, 4, ... is
- 2) 11th term of the AP: $-3, -\frac{1}{2}, 2, \dots$ is

Solution: 1) Let the AP be a function $x_1(n)$ where

Parameter	i value	value	Description
$x_i(0)$	1	10	First term
	2	-3	
d_i	1	-3	Common difference
	2	$\frac{5}{2}$	
$x_i(29)$	1	?	30th term
$x_i(10)$	2	?	11th term

TABLE 2
INPUT PARAMETERS

$x_1(n)$ is the $(n + 1)th$ term of AP(1).

Let the common difference be d_1 .

So, the first term is $x_1(1 - 1)$ which is $x_1(0)$; given $x_1(0) = 10$

For the 30th term of the series we need to find $x_1(30 - 1)$ which is $x_1(29)$.

Let Z-transform of $x_1(n)$ be $X_1(z)$. Let $U(z)$ be the Z-transform of $u(n)$.

where $u(n)$ is the step function.

$$x_1(n) = [x_1(0) + (n) \times d_1] \times u(n) \quad (1)$$

$$X_1(z) = x_1(0).U(z) + d_1(Z\{nu(n)\}) \quad (2)$$

$$= \frac{x_1(0)}{1 - z^{-1}} + \frac{d_1 \times z^{-1}}{(1 - z^{-1})^2} \quad (3)$$

$$= \frac{10}{1 - z^{-1}} + \frac{(-3)z^{-1}}{(1 - z^{-1})^2} \quad (4)$$

$$= \frac{10}{1 - z^{-1}} - \frac{3z^{-1}}{(1 - z^{-1})^2} \quad (5)$$

$$= \frac{10 - 13z^{-1}}{(1 - z^{-1})^2} \quad \forall \quad |z| > 1 \quad (6)$$

From the values given in table:1 :

$$x_1(29) = (10 + (29)(-3))(u(n)) \quad (7)$$

$$= (10 + 29(-3))(u(n)) \quad (8)$$

$$= (10 + (-87))(u(n)) \quad (9)$$

$$= -77 \quad (10)$$

(where $u(n) = 1$ if $n \geq 0$)

So, the 30th term of the AP is -77 .

2) Let the AP be a function $x_2(n)$ where $x_2(n)$ is the $(n + 1)th$ term of AP(2).

Let the common difference be d_2 .

So, the first term is $x_2(1 - 1)$ which is $x_2(0)$; given $x_2(0) = -3$

For the 11th term of the series we need to find $x_2(11 - 1)$ which is $x_2(10)$.

Let Z-transform of $x_1(n)$ be $X_1(z)$. Let $U(z)$ be the Z-transform of $u(n)$.

where $u(n)$ is the step function.

$$x_2(n) = [x_2(0) + (n) \times d_2] \times u(n) \quad (11)$$

$$X_2(z) = x_2(0).U(z) + d_2(Z\{nu(n)\}) \quad (12)$$

$$= \frac{x_2(0)}{1 - z^{-1}} + \frac{d_2 \times z^{-1}}{(1 - z^{-1})^2} \quad (13)$$

$$= \frac{-3}{1 - z^{-1}} + \frac{(2.5)z^{-1}}{(1 - z^{-1})^2} \quad (14)$$

$$= \frac{0.5z^{-1} - 3}{(1 - z^{-1})^2} \quad \forall \quad |z| > 1 \quad (15)$$

From the values given in table:1 :

$$x_2(10) = (-3 + (10)\left(\frac{5}{2}\right))(u(n)) \quad (16)$$

$$= (-3 + 10(2.5))(u(n)) \quad (17)$$

$$= (-3 + 25)(u(n)) \quad (18)$$

$$= 22 \quad (19)$$

(where $u(n) = 1$ if $n \geq 0$)

so, the 11th term of the AP is 22.