NCERT Discrete - 10.5.2.2

EE23BTECH11058 - Sindam Ananya*

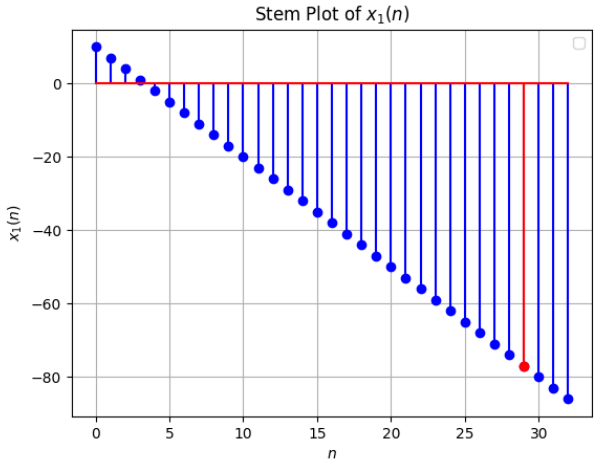
Question 10.5.2.2:

1) 30th term of the AP: 10, 7, 4, ... is
2) 11th term of the AP: $-3, -\frac{1}{2}, 2, \ldots$ is

Solution:

Parameter	value	Description
$x_i(0)$	10	First term
	-3	
d_i	-3	Common difference
	$\frac{5}{2}$	
$x_1(29)$	?	30th term
$x_2(10)$	?	11th term

TABLE 2
INPUT PARAMETERS



The $(n+1)th$ term of the AP is given by:

$$x_i(n) = [x_i(0) + (n) \times d_i] \times u(n) \tag{1}$$

1)Let the AP be a function $x_1(n)$ where $x_1(n)$ is the $(n+1)th$ term.
For the 30th term of the series we need to find $x_1(30-1)$ which is $x_1(29)$.
From the equation (1) and the values from the table Table 2 :

$$\begin{aligned} x_1(29) &= (10 + (29)(-3))(u(n)) && (2)\\ &= (10 + 29(-3))(1) && (3)\\ &= 10 + (-87) && (4)\\ &= -77 && (5) \end{aligned}$$

$$\begin{aligned} X_1(z) &= \sum_{n=-\infty}^{\infty} x_1(n) \times u(n) \cdot z^{-n} && (6)\\ &= \sum_{n=0}^{\infty} x_1(n) \times u(n) \cdot z^{-n} && (7)\\ &= x_1(0) \times U(z) + d_1 \times \frac{d \times U(z)}{dz} && (8)\\ &= \frac{10 - 13z^{-1}}{(1 - z^{-1})^2} \quad \forall \quad |z| > 1 && (9) \end{aligned}$$

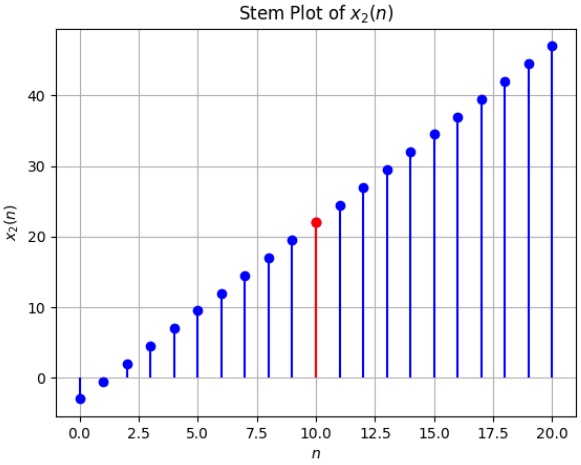
So, the 30th term of the AP is -77.

2)Let the AP be a function $x_2(n)$ where $x_2(n)$ is the $(n+1)th$ term.
For the 11th term of the series we need to find $x_2(11-1)$ which is $x_2(10)$.
From the equation (1) and the values from the table Table 2 :

$$\begin{aligned} x_2(10) &= (-3 + (10)\left(\frac{5}{2}\right))(u(n)) && (10)\\ &= (-3 + 10(2.5))(1) && (11)\\ &= -3 + 25 && (12)\\ &= 22 && (13) \end{aligned}$$

$$\begin{aligned} X_2(z) &= \sum_{n=-\infty}^{\infty} x_2(n) \times u(n) \cdot z^{-n} && (14)\\ &= \sum_{n=0}^{\infty} x_2(n) \times u(n) \cdot z^{-n} && (15)\\ &= x_2(0) \times U(z) + d_2 \times \frac{d \times U(z)}{dz} && (16)\\ &= \frac{0.5z^{-1} - 3}{(1 - z^{-1})^2} \quad \forall \quad |z| > 1 && (17) \end{aligned}$$

so, the 11th term of the AP is 22.


Stem Plot of $x_2(n)$
$x_2(n)$
n
0
10
20
30
40
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0