

Gate 2022- Instrumentation Engineering

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Question 38: Consider the transfer function

$$H_c(s) = \frac{1}{(s+1)(s+3)}$$

Bilinear transformation with a sampling period of $0.1s$ is employed to obtain the discrete-time transfer function $H_d(z)$. Then $H_d(z)$ is

(A) $\frac{(1+z^{-1})^2}{(19-21z^{-1})(23-17z^{-1})}$

(B) $\frac{(1-z^{-1})^2}{(21-19z^{-1})(17-23z^{-1})}$

(C) $\frac{(1+z^{-1})^2}{(21-19z^{-1})(23-17z^{-1})}$

(D) $\frac{(1+z^{-1})^2}{(21-19z^{-1})(17-23z^{-1})}$

(GATE IN 2022)

Solution:

$$H_c(s) \xrightarrow{\text{Bilinear Transform}} H_d(z) \quad (1)$$

To get $H_d(z)$, substitute s with

$$s = \frac{2}{T_s} \left(\frac{1-z^{-1}}{1+z^{-1}} \right) \quad (2)$$

where T_s is the sampling period. Then,

$$H_d(z) = \frac{1}{\left(\frac{2}{0.1} \left(\frac{1-z^{-1}}{1+z^{-1}} \right) + 1 \right) \left(\frac{2}{0.1} \left(\frac{1-z^{-1}}{1+z^{-1}} \right) + 3 \right)} \quad (3)$$

$$= \frac{(1+z^{-1})^2}{(21-19z^{-1})(23-17z^{-1})} \quad (4)$$

ROC : $|z| > \frac{19}{21}$

Using partial fractions,

$$H_d(z) = \frac{1}{323} + \frac{340}{323} \left(\frac{1}{21-19z^{-1}} \right) - \frac{380}{323} \left(\frac{1}{23-17z^{-1}} \right) \quad (5)$$

By applying inverse z-transform,

$$\delta(n) \xleftrightarrow{Z} 1 \quad (6)$$

$$x(0)r^n u(n) \xleftrightarrow{Z} \frac{x(0)}{1-rz^{-1}} \quad (7)$$

$$H_d(n) = \frac{1}{323} \delta(n) + \frac{340}{6783} \left(\frac{19}{21} \right)^n - \frac{380}{7429} \left(\frac{17}{23} \right)^n \quad (8)$$

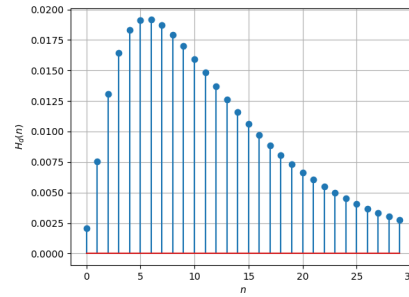


Fig. 0. stem plot of $H_d(n)$

Parameters	Value	Description
$H_c(s)$	$\frac{1}{(s+1)(s+3)}$	Transfer function in s domain
T_s	$0.1s$	Sampling period
$H_d(z)$		Transfer function of sampled signal

TABLE 0
INPUT PARAMETERS