Gate 2021- Instrumentation Engineering

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Question 43: Given $y(t) = e^{-3t}u(t) * u(t+3)$, where * denotes convolution operation. The value of $y(t)$ as $t \to \infty$ is (GATE IN 2021)

Solution:

$$y(t) = e^{-3t}u(t) * u(t+3) \tag{1}$$

$$x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(j\omega) \tag{2}$$

$$x(t-t_o) \overset{\mathcal{F}}{\longleftrightarrow} e^{-j\omega t_o}X(j\omega) \tag{3}$$

$$x_1(t) * x_2(t) \overset{\mathcal{F}}{\longleftrightarrow} X_1(j\omega)X_2(j\omega) \tag{4}$$

$$e^{-at} \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{j\omega + a} \tag{5}$$

$$u(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{j\omega} \tag{6}$$

$$Y(j\omega) = \left(\frac{1}{j\omega+3}\right)\left(\frac{e^{3j\omega}}{j\omega}\right) \tag{7}$$

By solving through partial fractions,

$$Y(j\omega) = \frac{e^{3j\omega}}{3j\omega} - \frac{e^{3j\omega}}{3(j\omega+3)} \tag{8}$$

By applying inverse fourier transform,

$$y(t) = \frac{u(t+3)}{3} - \frac{e^{-3t}u(t+3)}{3} \tag{9}$$

As $t \to \infty$, $e^{-at} \to 0 \quad (a > 0)$

$$y(t) = \frac{1}{3} \tag{10}$$

$$= 0.33 \tag{11}$$

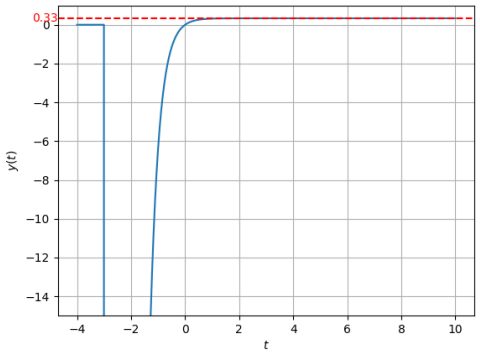

Fig. 0. plot of $y(t)$