Gate 2022- Instrumentation Engineering

EE23BTECH11058 - Sindam Ananya*

Question 11: The input $x(t)$ to a system is related to its output $y(t)$ as

$$\frac{dy(t)}{dt} + y(t) = 3x(t-3)u(t-3)$$

Here $u(t)$ represents a unit-step function.
The transfer function of this system is

(A) $\frac{e^{-3s}}{s+3}$

(B) $\frac{3e^{-3s}}{s+1}$

(C) $\frac{3e^{-(s/3)}}{s+1}$

(D) $\frac{e^{-(s/3)}}{s+3}$

(GATE IN 2022)

Solution:

$$\frac{dy(t)}{dt} + y(t) = 3x(t-3)u(t-3) \tag{1}$$

By applying Laplace Transform on both sides

$$x(t) \overset{\mathcal{L}}{\leftrightarrow} X(s) \tag{2}$$

$$x(t-t_o) \overset{\mathcal{L}}{\leftrightarrow} X(s)e^{-st_o} \tag{3}$$

$$sY(s) + Y(s) = 3X(s)e^{-3s} \tag{4}$$

$$Y(s)\,(s+1) = 3X(s)e^{-3s} \tag{5}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{3e^{-3s}}{s+1} \quad (Re(s) > 0) \tag{6}$$

$$H(j\omega) = \frac{3e^{-3j\omega}}{1+j\omega} \tag{7}$$

$$= \frac{3\,(cos3\omega - jsin3\omega)}{1+j\omega} \tag{8}$$

$$|H(j\omega)| = \frac{3}{\sqrt{1+\omega^2}} \tag{9}$$

$$phase = \tan^{-1}\left(\frac{\omega\cos(3\omega) + \sin(3\omega)}{\omega\sin(3\omega) - \cos(3\omega)}\right) \tag{10}$$

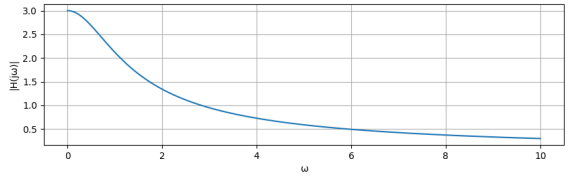

Fig. 0. Plot for magnitude of transfer function

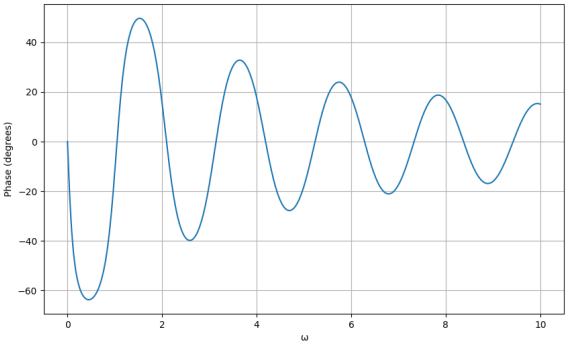

Fig. 0. Plot for phase of transfer function