

# Gate 2022- Instrumentation Engineering

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**Question 11:** The input  $x(t)$  to a system is related to its output  $y(t)$  as

$$\frac{dy(t)}{dt} + y(t) = 3x(t-3)u(t-3)$$

Here  $u(t)$  represents a unit-step function. The transfer function of this system is

- (A)  $\frac{e^{-3s}}{s+3}$   
 (B)  $\frac{3e^{-3s}}{s+1}$   
 (C)  $\frac{3e^{-(s/3)}}{s+1}$   
 (D)  $\frac{e^{-(s/3)}}{s+3}$

(GATE IN 2022)

**Solution:**

$$\frac{dy(t)}{dt} + y(t) = 3x(t-3)u(t-3) \quad (1)$$

By applying Laplace Transform on both sides

$$x(t) \xleftrightarrow{\mathcal{L}} X(s) \quad (2)$$

$$x(t-t_0) \xleftrightarrow{\mathcal{L}} X(s)e^{-st_0} \quad (3)$$

$$sY(s) + Y(s) = 3X(s)e^{-3s} \quad (4)$$

$$Y(s)(s+1) = 3X(s)e^{-3s} \quad (5)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{3e^{-3s}}{s+1} \quad (Re(s) > 0) \quad (6)$$

$$H(j\omega) = \frac{3e^{-3j\omega}}{1+j\omega} \quad (7)$$

$$= \frac{3(\cos 3\omega - j\sin 3\omega)}{1+j\omega} \quad (8)$$

$$|H(j\omega)| = \frac{3}{\sqrt{1+\omega^2}} \quad (9)$$

$$phase = \tan^{-1} \left( \frac{\omega \cos(3\omega) + \sin(3\omega)}{\omega \sin(3\omega) - \cos(3\omega)} \right) \quad (10)$$

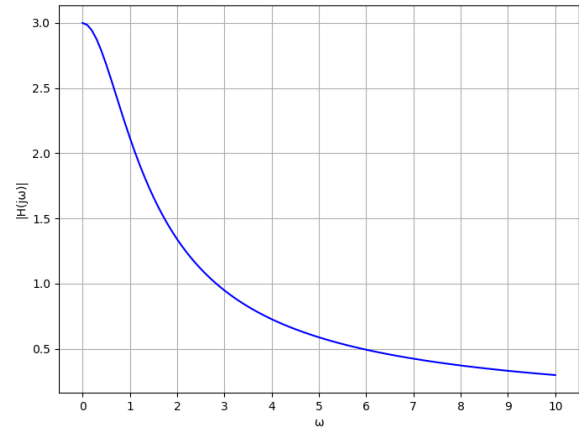


Fig. 0. Plot for magnitude of transfer function

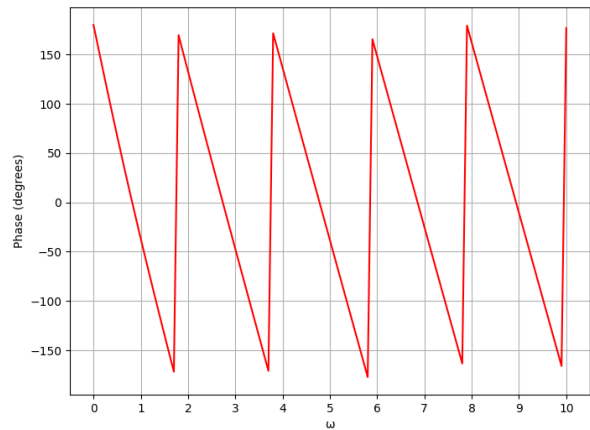


Fig. 0. Plot for phase of transfer function