Gate 2021- EC

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Question 4: Consider a real-valued base-band signal $x(t)$, band limited to $10kHz$. The Nyquist rate for the signal

$$y(t) = x(t)x(1 + \frac{t}{2}) \text{ is}$$

(A) $15kHz$
(B) $30kHz$
(C) $60kHz$
(D) $20kHz$

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Solution:

$$x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(j\omega) \tag{1}$$

$$x(at) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{a}X(j\omega) \tag{2}$$

$$x(t - t_o) \overset{\mathcal{F}}{\longleftrightarrow} e^{-j\omega t_o}X(j\omega) \tag{3}$$

$$x(1 + \frac{t}{2}) \overset{\mathcal{F}}{\longleftrightarrow} 2e^{j\omega}X(j2\omega) \tag{4}$$

$$y(t) = x(t)x(1 + \frac{t}{2}) \tag{5}$$

Multiplication in time-domain is convolution in frequency domain and vice-versa

$$Y(f) = X(f) * 2e^{j2\pi f}X(2f) \tag{6}$$
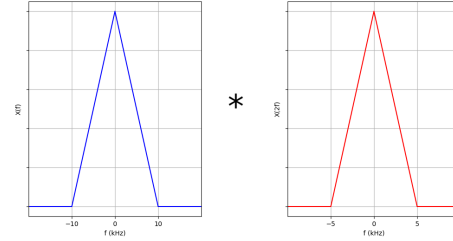
Nyquist rate is $2f_{max} = 2(15kHz)$ which is $30kHz$
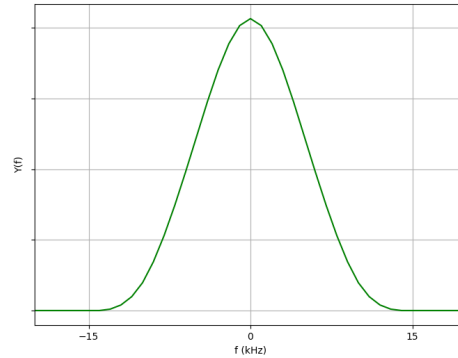

Fig. 0. Plot of $X(f)$ and $X(2f)$

Fig. 0. Plot of $Y(f)$

Parameter	Value	Description
$x(t)$		base-band signal
f	$10kHz$	Maximum frequency of $X(f)$
$y(t)$	$x(t)x(1 + \frac{t}{2})$	new signal
f_{max}		Maximum frequency of $Y(f)$

TABLE 0

INPUT PARAMETERS