Gate 2022- Instrumentation Engineering

EE23BTECH11058 - Sindam Ananya*

Question 38: Consider the transfer function

$$H_c(s) = \frac{1}{(s+1)(s+3)}$$

Bilinear transformation with a sampling period of $0.1s$ is employed to obtain the discrete-time transfer function $H_d(z)$. Then $H_d(z)$ is

(A) $\frac{(1+z^{-1})^2}{(19-21z^{-1})(23-17z^{-1})}$

(B) $\frac{(1-z^{-1})^2}{(21-19z^{-1})(17-23z^{-1})}$

(C) $\frac{(1+z^{-1})^2}{(21-19z^{-1})(23-17z^{-1})}$

(D) $\frac{(1+z^{-1})^2}{(21-19z^{-1})(17-23z^{-1})}$

(GATE IN 2022)

Solution:

Parameters	Value	Description
$H_c(s)$	$\frac{1}{(s+1)(s+3)}$	Transfer function in s domain
T_s	$0.1s$	Sampling period
$H_d(z)$		Transfer function of sampled signal

TABLE 0
Input Parameters

$$H_c(s) \xleftrightarrow{BilinearTransform} H_d(z) \tag{1}$$

To get $H_d(z)$, substitute s with

$$s = \frac{2}{T_s}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) \tag{2}$$

where T_s is the sampling period. Then,

$$H_d(z) = \frac{1}{\left(\frac{2}{0.1}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+1\right)\left(\frac{2}{0.1}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+3\right)} \tag{3}$$

$$= \frac{(1+z^{-1})^2}{(21-19z^{-1})(23-17z^{-1})} \tag{4}$$

ROC : $|z| > \frac{19}{21}$

Using partial fractions,

$$H_d(z) = \frac{1}{323} + \frac{340}{323}\left(\frac{1}{21-19z^{-1}}\right) - \frac{380}{323}\left(\frac{1}{23-17z^{-1}}\right) \tag{5}$$

By applying inverse z-transform,

$$\delta(n) \overset{\mathcal{Z}}{\leftrightarrow} 1 \tag{6}$$

$$x(0)r^n u(n) \overset{\mathcal{Z}}{\leftrightarrow} \frac{x(0)}{1-rz^{-1}} \tag{7}$$

$$H_d(n) = \frac{1}{323}\delta(n) + \frac{340}{6783}\left(\frac{19}{21}\right)^n - \frac{380}{7429}\left(\frac{17}{23}\right)^n \tag{8}$$

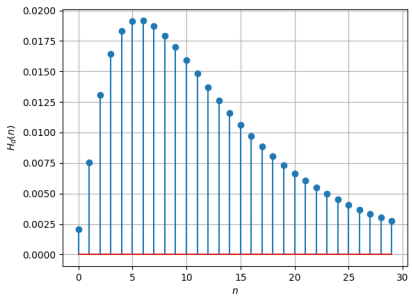

Fig. 0. stem plot of $H_d(n)$