Gate 2021- Instrumentation Engineering

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Question 43: Given $y(t) = e^{-3t}u(t)*u(t+3)$, where $*$ denotes convolution operation. The value of $y(t)$ as $t \to \infty$ is (GATE IN 2021)

Solution:

$$y(t) = e^{-3t}u(t) * u(t+3) \tag{1}$$

$$x(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s) \tag{2}$$

$$x(t-t_o) \overset{\mathcal{L}}{\longleftrightarrow} e^{-st_o}X(s) \tag{3}$$

$$x_1(t) * x_2(t) \overset{\mathcal{L}}{\longleftrightarrow} X_1(s)X_2(s) \tag{4}$$

$$e^{-at}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+a} \quad (ROC : Re(s) > -a) \tag{5}$$

$$u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s} \quad (ROC : Re(s) > 0) \tag{6}$$

$$u(t+3) \overset{\mathcal{L}}{\longleftrightarrow} \frac{e^{3s}}{s} \quad (ROC : Re(s) > 0) \tag{7}$$

$$\tag{8}$$

$$Y(s) = \left(\frac{1}{s+3}\right)\left(\frac{e^{3s}}{s}\right) \tag{9}$$

ROC: $(-3 < Re(s) < 0)$ By using Final Value Theorem,

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) \tag{10}$$

$$= \lim_{s\to 0} s\left(\frac{1}{s+3}\right)\left(\frac{e^{3s}}{s}\right) \tag{11}$$

$$= \frac{1}{3} \tag{12}$$

By solving the equation (9) through partial fractions,

$$Y(s) = \frac{e^{3s}}{3s} - \frac{e^{3s}}{3(s+3)} \tag{13}$$

By applying inverse fourier transform,

$$y(t) = \frac{u(t+3)}{3} - \frac{e^{-3t}u(t+3)}{3} \tag{14}$$

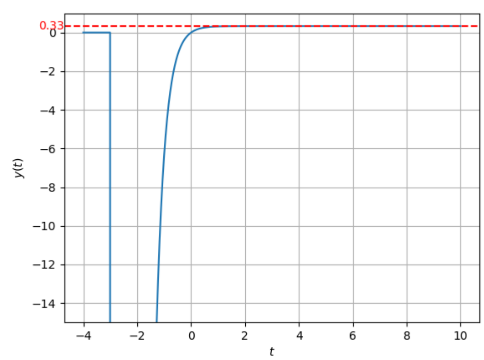

Fig. 0. plot of $y(t)$