Gate 2022- Instrumentation Engineering

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Question 38: Consider the transfer function

$$H_c(s) = \frac{1}{(s+1)(s+3)}$$

Bilinear transformation with a sampling period of $0.1s$ is employed to obtain the discrete-time transfer function $H_d(z)$. Then $H_d(z)$ is

(A) $\frac{(1+z^{-1})^2}{(19-21z^{-1})(23-17z^{-1})}$

(B) $\frac{(1-z^{-1})^2}{(21-19z^{-1})(17-23z^{-1})}$

(C) $\frac{(1+z^{-1})^2}{(21-19z^{-1})(23-17z^{-1})}$

(D) $\frac{(1+z^{-1})^2}{(21-19z^{-1})(17-23z^{-1})}$

(GATE IN 2022)

Solution:

Parameters	Value	Description
$H_c(s)$	$\frac{1}{(s+1)(s+3)}$	Transfer function in s domain
T_s	$0.1s$	Sampling period
$H_d(z)$		Transfer function of sampled signal

TABLE 0
INPUT PARAMETERS

$$H_c(s) \xleftrightarrow{BilinearTransform} H_d(z) \tag{1}$$

To get $H_d(z)$, substitute s with

$$s = \frac{2}{T_s}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) \tag{2}$$

where T_s is the sampling period. Then,

$$H_d(z) = \frac{1}{\left(\frac{2}{0.1}\left(\frac{1-z^{-1}}{1+z^{-1}}+1\right)\right)\left(\frac{2}{0.1}\left(\frac{1-z^{-1}}{1+z^{-1}}+3\right)\right)} \tag{3}$$

$$= \frac{(1+z^{-1})^2}{(21-19z^{-1})(23-17z^{-1})} \tag{4}$$

ROC : $|z| > \frac{19}{21}$