Gate 2021- Instrumentation Engineering

EE23BTECH11058 - Sindam Ananya*

Question 43: Given $y(t) = e^{-3t}u(t) * u(t+3)$, where * denotes convolution operation. The value of $y(t)$ as $t \to \infty$ is (GATE IN 2021)

$$y(t) = e^{-3t}u(t) * u(t+3) \tag{1}$$

$$x(t) \overset{\mathcal{L}}{\leftrightarrow} X(s) \tag{2}$$

$$x(t-t_o) \overset{\mathcal{L}}{\leftrightarrow} e^{-st_o}X(s) \tag{3}$$

$$x_1(t) * x_2(t) \overset{\mathcal{L}}{\leftrightarrow} X_1(s)X_2(s) \tag{4}$$

$$e^{-at} \overset{\mathcal{L}}{\leftrightarrow} \frac{1}{s+a} \tag{5}$$

$$u(t) \overset{\mathcal{L}}{\leftrightarrow} \frac{1}{s} \tag{6}$$

$$Y(s) = \left(\frac{1}{s+3}\right)\left(\frac{e^{3s}}{s}\right) \tag{7}$$

ROC : $(-3 < s < 0)$
By using Final Value Theorem,

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) \tag{8}$$

$$= \lim_{s\to 0} s\left(\frac{1}{s+3}\right)\left(\frac{e^{3s}}{s}\right) \tag{9}$$

$$= \frac{1}{3} \tag{10}$$