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CSE, Section I
Sem 4
PES1201800204

LA - Assignment

(1) $y = A + Bx + Cx^2$

$(1,1), (2,-1) (3,1)$

The three eqns are:

$1 = A + B + C$

$-1 = A + 2B + 4C$

$1 = A + 3B + 9C$

$Ax = b$ form.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$[A\ b] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & -1 \\ 1 & 3 & 9 & 1 \end{bmatrix}$$

$R_2 - R_1$
$R_3 - R_1$
$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 2 & 8 & 0 \end{bmatrix}$$

$R_3 - 2R_2$
$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

We get:

$2c = 4$

$b + 3c = -2$

$a + b + c = 1$

$\therefore c = 2,\ b = -8,\ a = 7$

$\therefore$ The eqn is $y = 7 - 8x + 2x^2$

(2) $A = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 4 & 12 & 3 & -14 \\ -10 & -29 & -5 & 38 \\ 10 & 21 & 21 & -6 \end{bmatrix}$

$R_2 - 2R_1$
$R_3 + 5R_1$
$R_4 - 5R_1$
$$\begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & -4 & 5 & 13 \\ 0 & -4 & 11 & 19 \end{bmatrix}$$

$R_3 + 2R_2$
$R_4 + 2R_2$
$$\begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 9 & 11 \end{bmatrix}$$

R:

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$R_4 - 3R_3$ $\begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix}$

$U = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix}$ $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -5 & -2 & 1 & 0 \\ 5 & -2 & 3 & 1 \end{bmatrix}$

(3) $T \to R^3 \to R^3$

$T(x,y,z) = (x+2y-z,\ y+z,\ x+y-2z)$

$(1,0,0) \to (1,0,1)$

$(0,1,0) \to (2,1,1)$

$(0,0,1) \to (-1,1,-2)$

(i) $T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 2 & -1 & b_1 \\ 0 & 1 & 1 & b_2 \\ 1 & 1 & -2 & b_3 \end{bmatrix} \approx \begin{bmatrix} 1 & 2 & -1 & b_1 \\ 0 & 1 & 1 & b_2 \\ 0 & -1 & -1 & b_3-b_1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & -1 & b_1 \\ 0 & 1 & 1 & b_2 \\ 0 & 0 & 0 & b_3-b_1+b_2 \end{bmatrix}$

Basis of

$C(T) = \{(1,0,1),(2,1,1)\}$

$C(T^T) = \{(1,2,-1),(0,1,1)\}$

$N(T^T) = \{(-1,1,1)\}$

$N(T) = \{(3,-1,1)\}$

(iii) $|T - \lambda I| = 0$

$\begin{vmatrix} 1-\lambda & 2 & -1 \\ 0 & 1-\lambda & 1 \\ 1 & 1 & -2-\lambda \end{vmatrix} = 0$ $(-2-\lambda)(1-\lambda)^2 + (1-\lambda) - (-1-\lambda)$

$-\lambda^3 + 3\lambda = 0$

Eigen value of $\lambda = 0, \sqrt{3}, -\sqrt{3}$

$[T - \lambda I][x] = 0$

$\begin{bmatrix} -0.73 & 2 & -1 \\ 0 & -0.73 & 1 \\ 1 & 1 & -3.73 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

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$$\begin{bmatrix} 2.73 & 2 & -1 \\ 0 & 2.73 & 1 \\ 1 & 1 & -0.27 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$(0,0,0)$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$(3,-1,1)$

From this $y+z=0$ $\boxed{y=-z}$ $x+y-2z=0$ $\boxed{x=3z}$

(iv)

$T = QR$.

$q_1 = \frac{a}{\|a\|} = \frac{1}{\sqrt{2}}, 0, \frac{4}{\sqrt{2}}$

$q_2 = \frac{b}{\|b\|}$ $\quad B = b - (q_1^T b) q_1$

$$\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 3/2 \\ 0 \\ 3/2 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1 \\ -1/2 \end{bmatrix}$$

$q_2 = \frac{1/2}{\sqrt{3}/\sqrt{2}}, \frac{1}{\sqrt{3}/\sqrt{2}}, \frac{-1/2}{\sqrt{3/2}} = \frac{1}{\sqrt{6}}, \frac{\sqrt{2}}{\sqrt{3}}, \frac{-1}{\sqrt{6}}$

$q_3 = \frac{C}{\|c\|}$ $\quad c = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} - \begin{bmatrix} -3/2 \\ 0 \\ -3/2 \end{bmatrix} - \begin{bmatrix} 1/2 \\ 1 \\ -1/2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$q_3 = (0,0,0)$

$$Q = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 0 \\ 0 & \sqrt{2}/\sqrt{3} & 0 \\ 1/\sqrt{2} & -1/\sqrt{6} & 0 \end{bmatrix}$$

$R = Q^T T$

$T = QR$.

$$\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 0 \\ 0 & \sqrt{2}/\sqrt{3} & 0 \\ 1/\sqrt{2} & -1/\sqrt{6} & 0 \end{bmatrix} \begin{bmatrix} \sqrt{2} & 3/\sqrt{2} & -3/\sqrt{2} \\ 0 & 3/\sqrt{6} & 3/\sqrt{2} \\ 0 & 0 & 0 \end{bmatrix}$$

4. x 4 1 2 3 $\quad y = c + dx$

y 4 6 10 8

$$\overset{A}{\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}} \begin{bmatrix} c \\ d \end{bmatrix} = \overset{b}{\begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix}}$$

$\hat{x} = (A^T A)^{-1} A^T b$

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$$A^TA = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -4 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 30 \end{bmatrix}$$

$$(A^TA)^{-1} = \frac{1}{116} \begin{bmatrix} 30 & -2 \\ -2 & 4 \end{bmatrix}$$

$$A^Tb = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -4 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix} = \begin{bmatrix} 28 \\ 34 \end{bmatrix}$$

$$\hat{x} = \begin{bmatrix} 30/116 & -2/116 \\ -2/116 & 4/116 \end{bmatrix} \begin{bmatrix} 28 \\ 34 \end{bmatrix}$$

$$\hat{x} = \begin{bmatrix} 772/116 \\ 80/116 \end{bmatrix}$$

(5) $x_1 + x_2 + 3x_3 + 4x_4 = 0$

$x_1 = -x_2 - 3x_3 - 4x_4$

$x_2 = C_2 \quad x_3 = C_3 \quad x_4 = C_4$

$$V = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = C_2 \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + C_3 \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix} + C_4 \begin{bmatrix} -4 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & -3 & -4 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{Proj} = A(A^TA)^{-1}A^T$$

$$A^TA = \begin{bmatrix} -1 & 1 & 0 & 0 \\ -3 & 0 & 1 & 0 \\ -4 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & -3 & -4 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 10 & 12 \\ 4 & 12 & 17 \end{bmatrix}$$

$$(A^TA)^{-1} = \begin{bmatrix} 26/27 & -1/9 & -4/27 \\ -1/9 & 2/3 & -4/9 \\ -4/27 & -4/9 & 11/27 \end{bmatrix}$$

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$$A(A^TA)^{-1} = \begin{bmatrix} -1/27 & -1/9 & -4/27 \\ 26/27 & -1/9 & -4/27 \\ -1/9 & 2/3 & -4/9 \\ -4/27 & -4/9 & 11/27 \end{bmatrix}$$

$$A(A^TA)^{-1}A^T = \begin{bmatrix} 26/27 & -1/27 & -1/9 & -4/27 \\ -1/27 & 26/27 & -1/9 & -4/27 \\ -1/9 & -3/27 & 6/9 & -12/27 \\ -4/27 & -4/27 & 4/9 & 1\cancel{6}/27 \end{bmatrix}$$

$I = \text{Proj } v + \text{Proj } v_1$

$$\text{Proj } v_1 = \begin{bmatrix} 1/27 & 1/27 & 1/9 & 4/27 \\ 1/27 & 1/27 & 1/9 & 4/27 \\ 1/9 & 3/27 & 3/9 & 12/27 \\ 4/27 & 4/27 & 4/9 & 16/27 \end{bmatrix}$$

(6) (i) $\begin{bmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{bmatrix} \approx \begin{bmatrix} a & 2 & 2 \\ 0 & \frac{a^2-4}{a} & \frac{2a-4}{a} \\ 0 & \frac{2a-4}{a} & \frac{2a^2-4}{a} \end{bmatrix}$ $a>0 \Rightarrow \frac{a^2-4}{a} > a$

$a>2$

$a(a^2-4) - 2(2a-4) + 2(4-2a)$

$a^3 - 12a + 16 > 0$

$a > 4, 2, 2$ $\underline{2 < a < \infty}$

(ii) $[x_1 \; x_2 \; x_3] \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$\begin{bmatrix} a_{11}x_1 + a_{21}x_2 + a_{31}x_3 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \end{bmatrix}$ $\begin{aligned} a_{12} + a_{21} &= -2 \\ a_{31} + a_{13} &= 0 \\ a_{33} + a_{32} &= -2 \end{aligned}$

Symmetric

$a_{12} = a_{21} = -1$; $a_{23} = a_{32} = -1$; $a_{31} = a_{13} = 0$

Required matrix is $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

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(7) $A = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}$

$A^TA = \begin{bmatrix} -3 & 6 & 6 \\ 1 & -2 & -2 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix}$

$|A - \lambda I| = 0$

$\begin{bmatrix} 81-\lambda & -27 \\ -27 & 9-\lambda \end{bmatrix} = 0.$

$(81-\lambda)(9-\lambda) - 27^2 = 0$

$729 - 90\lambda + \lambda^2 - 729 = 0$

$\lambda = 0, 90.$

$[A - \lambda I][x] = 0.$

$\lambda = 0 \quad \begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

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$\lambda = 90 \begin{bmatrix} -9 & -27 \\ -27 & -81 \end{bmatrix} \Rightarrow (-3, 1)$

$V = \begin{bmatrix} 1/\sqrt{10} & -3/\sqrt{10} \\ 3/\sqrt{10} & 1/\sqrt{10} \end{bmatrix} \qquad \sigma = 0, \sqrt{90}.$

$u_1 = \frac{Av_1}{\sigma_1} = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix} \begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{10} \end{bmatrix} / 0$

$u_2 = \frac{Av_2}{\sigma_2} \Rightarrow \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix} \begin{bmatrix} -3/\sqrt{10} \\ 1/\sqrt{10} \end{bmatrix} / \sqrt{90}$

$u_3 = 10x - 20y - 20z = 0 \qquad x = 1; y = 1/4; z = 1/4.$

$u_3 = \begin{bmatrix} 4/\sqrt{18} \\ 1/\sqrt{18} \\ 1/\sqrt{18} \end{bmatrix}$

$A = \begin{bmatrix} 10/\sqrt{900} & 4/\sqrt{18} \\ -20/\sqrt{900} & 1/\sqrt{18} \\ -20/\sqrt{900} & 1/\sqrt{18} \end{bmatrix} \begin{bmatrix} 0 \\ \sqrt{90} \\ 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{10} & -3/\sqrt{10} \\ 3/\sqrt{10} & 1/\sqrt{10} \end{bmatrix}$