Report

q1.) a.) i) $x(t) = 2\cos(2\pi t) + \cos(6\pi t) = e^{j(2\pi t)} + e^{-j(2\pi t)} + \frac{1}{2}\left(e^{j(6\pi t)} + e^{-j6\pi t}\right)$ (1)

$T = 1$ $\quad k=1 \quad k=1 \quad k=3$

$N = 5$

since $T=1$

$x(t)$ is of the form $\sum_{k=-\infty}^{k=\infty} a_k e^{jk\left(\frac{2\pi}{T}\right)t}$ (2)

$a_k = \frac{1}{T}\int_{-\frac{1}{2}}^{\frac{1}{2}} x(t)\cdot e^{-jk\omega_0 t}\,dt$

$k\frac{2\pi}{1} = 6\pi$

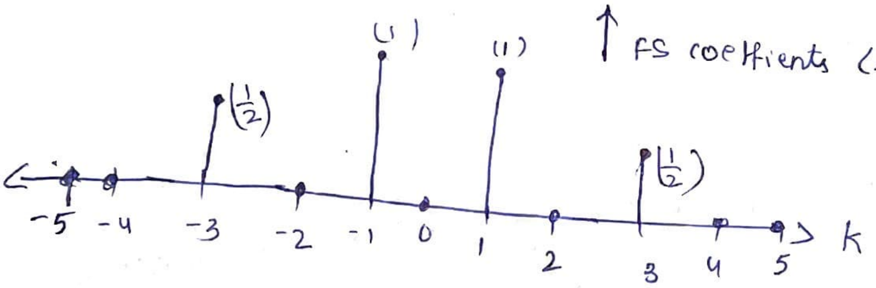
So, from comparison of (1) & (2), we get values of k:

$a_1 = 1 \qquad a_3 = \frac{1}{2}$

$a_{-1} = 1 \qquad a_{-3} = \frac{1}{2}$

$a_k = 0$ for the rest of the k's, where $k \in -\infty, \infty$.

So, graph of real part of FS coefficients is:



So, the graph matches with the one matlab plotted. Hence verified.

q1.) b.)

(ii) $x(t) = \begin{cases} 1 & -T_1 \le t \le T_1 \Rightarrow -\frac{1}{4} \le t \le \frac{1}{4} \\ 0 & T_1 < |t| < \frac{T}{2} \quad \left(\frac{1}{4} < t < \frac{1}{2} \text{ \& } -\frac{1}{2} < t < -\frac{1}{4}\right) \end{cases}$

$a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\,dt$

$a_k = 1\int_{-\frac{1}{4}}^{\frac{1}{4}} e^{-jk\frac{2\pi}{T}\times t}\,dt = \int_{-\frac{1}{4}}^{\frac{1}{4}} e^{-jk2\pi t}\,dt \Rightarrow a_k = -\frac{1}{jk2\pi}\left(e^{-jk\frac{\pi}{2}} - e^{jk\frac{\pi}{2}}\right)$

$$a_k = \left(\frac{-1}{jk2\pi}\right) \cancel{\cos k\frac{\pi}{2}} - j\sin k\frac{\pi}{2} + \cancel{\cos k\frac{\pi}{2}} - j\sin k\frac{\pi}{2}$$

$$= \left(\frac{+1}{\cancel{j}k\cancel{2}\pi}\right) + \cancel{2}\cancel{j}\sin\left(k\frac{\pi}{2}\right)$$

$$a_k = \frac{\sin k\left(\frac{\pi}{2}\right)}{k\pi}$$

$$a_{k\to 0} = \frac{1}{2}\left(\frac{\sin k\frac{\pi}{2}}{k\frac{\pi}{2}}\right) = \frac{1}{2}(1) = \frac{1}{2}$$

$a_{-5} = \dfrac{1}{5\pi}$ $\quad a_{-4} = 0$ $\quad a_2 = a_{-2} = 0$

$a_5 = \dfrac{1}{5\pi}$ $\quad a_4 = 0$ $\quad a_1 = a_{-1} = \dfrac{1}{\pi}$

$a_0 = \dfrac{1}{2}$

$a_3 = a_{-3} = \dfrac{-1}{3\pi}$

$a_6 = a_{-6} = 0$ $\quad a_8 = a_{-8} = 0$ $\quad a_{10} = a_{-10} = 0$

$a_7 = a_{-7} = \dfrac{-1}{7\pi}$

$a_9 = a_{-9} = \dfrac{1}{9\pi}$

So, when we compare it with the graph plotted by MATLAB it matches exactly.

So, thus verified.

q3.) a.) The fourier series coefficients $\{a_k\}$ for real periodic square wave that has amplitude 1 in $[-T_1, T_1]$ with period T is :

$$a_k = \frac{1}{T}\int_{-T_1}^{T_1} \underbrace{x(t)}_{\text{equal to } 1} e^{-jk\left(\frac{2\pi}{T}\right)t}\, dt \qquad \text{where } T_1 < \frac{T}{2}$$

$$= \frac{1}{\cancel{T}} \frac{-1}{jk\left(\frac{2\pi}{\cancel{T}}\right)}\left(e^{-jk\left(\frac{2\pi}{T}\right)T_1} - e^{jk\left(\frac{2\pi}{T}\right)T_1}\right)$$

$$= \frac{\cancel{-}1}{jk2\pi}\left(e^{jk\left(\frac{2\pi}{T}\right)T_1} - e^{-jk\left(\frac{2\pi}{T}\right)T_1}\right)$$

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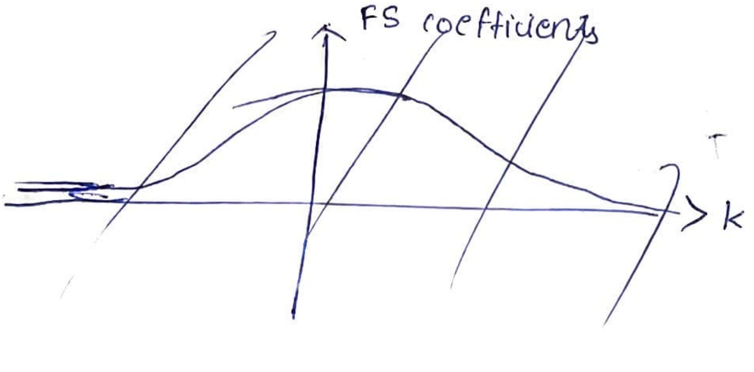
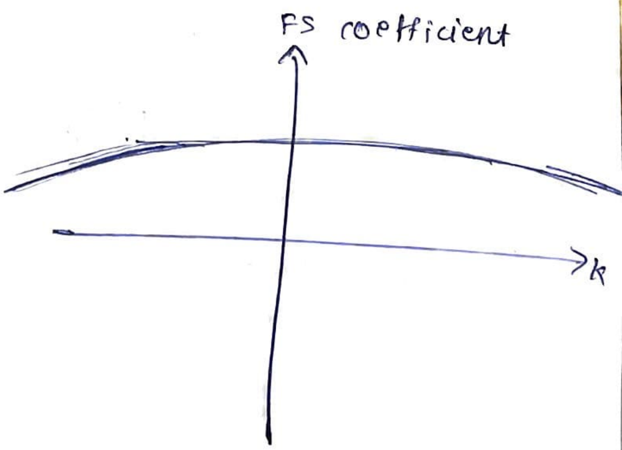
q3.) b.) as T increases, we see that N also increases, so, we get more number of coefficients. Also, we observe that FS coefficients touch the x-axis / become '0' at |k| greater than the previous curve. So, as $T \to \infty$, the k at which they become '0' also increases & the graph looks something like :





q3.) c.) As we increase N, we see that the reconstructed signal matches our square wave more accurately. This is because as N increases, we are increasing the number of frequencies (k's) we are using to reconstruct. So, more the number of frequencies more is the accuracy of our reconstruction as a square wave will have ∞ number of frequencies.

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q4.) c.) We see that the 1st signal is an even signal. So, we can say that the magnitude & phase spectrums of the FS coefficients are also even as per this derivation:

$$c_{-k} = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\, e^{-j(-k)\left(\frac{2\pi}{T}\right)t} \cdot dt \qquad c_k = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\, e^{-jk\left(\frac{2\pi}{T}\right)t} \cdot dt$$

$$= \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\, e^{jk\left(\frac{2\pi}{T}\right)t} \cdot dt$$

$$= \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(-t)\, e^{-jk\frac{2\pi}{T}t} \cdot dt \quad \text{but } x(-t) = x(t) \text{ (even)}$$

$$= \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\, e^{-jk\frac{2\pi}{T}t} \cdot dt$$

$$= c_k$$

So, $c_{-k} = c_k$ so, it is an even spectrum.

The second signal is an odd signal. So, the magnitude FS coefficients spectrum will be even but phase spectrum will be odd.