Report

1.) a.) $p[n] = \cos\left(\frac{2\pi f_0 n}{f_s}\right)$

$$P(e^{j\omega}) = \sum_{n=-\infty}^{n=\infty} \cos\left(\frac{2\pi f_0 n}{f_s}\right) \cdot e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{n=\infty} \left(\frac{e^{\frac{2\pi f_0}{f_s} jn} + e^{-\frac{2\pi f_0}{f_s} jn}}{2}\right) e^{-j\omega n}$$

$$= \frac{1}{2}\left(\sum_{n=-\infty}^{n=\infty} e^{jn\left(\frac{2\pi f_0}{f_s} - \omega\right)} + \sum_{n=-\infty}^{n=\infty} e^{-jn\left(\frac{2\pi f_0}{f_s} + \omega\right)}\right)$$

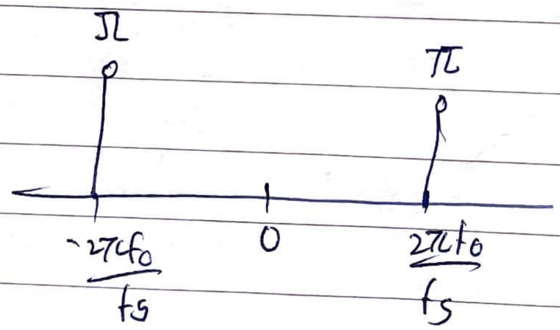
$$= \frac{\cancel{2}\pi}{\cancel{2}}\, \delta\left(\left[\frac{2\pi f_0}{f_s} - \omega\right]\right) + \frac{\cancel{2}\pi}{\cancel{2}}\, \delta\left[-\left(\frac{2\pi f_0}{f_s} + \omega\right)\right]$$

$$P(e^{j\omega}) = \pi\left(\delta\left[\frac{2\pi f_0}{f_s} - \omega\right] + \delta\left[-\frac{2\pi f_0}{f_s} - \omega\right]\right)$$

b.) location of impulses $= \pm \frac{2\pi f_0}{f_s}$

They are located at $\frac{2\pi f_0}{f_s}$ from the origin on either side of the origin.

So, they are at same distance from origin



c) $x[n] = \begin{cases} \cos \frac{2\pi f_0 n}{f_s} & 0 \le n \le L-1 \\ 0 & \text{elsewhere} \end{cases}$

So $$X(e^{j\omega}) = \sum_{0}^{L-1} \cos\left(\frac{2\pi f_0 n}{f_s}\right) \cdot e^{-j\omega n}$$

$$= \sum_{0}^{L-1} \left(\frac{e^{\frac{j2\pi f_0 n}{f_s}} + e^{\frac{-j2\pi f_0 n}{f_s}}}{2}\right) \cdot e^{-j\omega n}$$

$$= \frac{1}{2}\left[\frac{e^{jL\left(\frac{2\pi f_0}{f_s} - \omega\right)} - 1}{e^{j\frac{2\pi f_0}{f_s}} - 1} + \frac{e^{-jL\left(\frac{2\pi f_0}{f_s} + \omega\right)} - 1}{e^{-j\left(\frac{2\pi f_0}{f_s} + \omega\right)} - 1}\right]$$

Earlier without windowing, the DTFT was just combination of 2 impulses but here, we see that DTFT is totally different

Multiplication in time domain = Convolution in frequency domain

So $x[n] = p[n] \cdot w[n]$

$X(e^{j\omega}) = P(e^{j\omega}) * W(e^{j\omega})$

$$= \pi \int_{k=-\infty}^{k=\infty} \left(\delta\left[\frac{2\pi f_0}{f_s} - \omega - k\right] + \delta\left[\frac{-2\pi f_0}{f_s} - \omega - k\right]\right) \left(W e^{jk}\right) \cdot dk$$

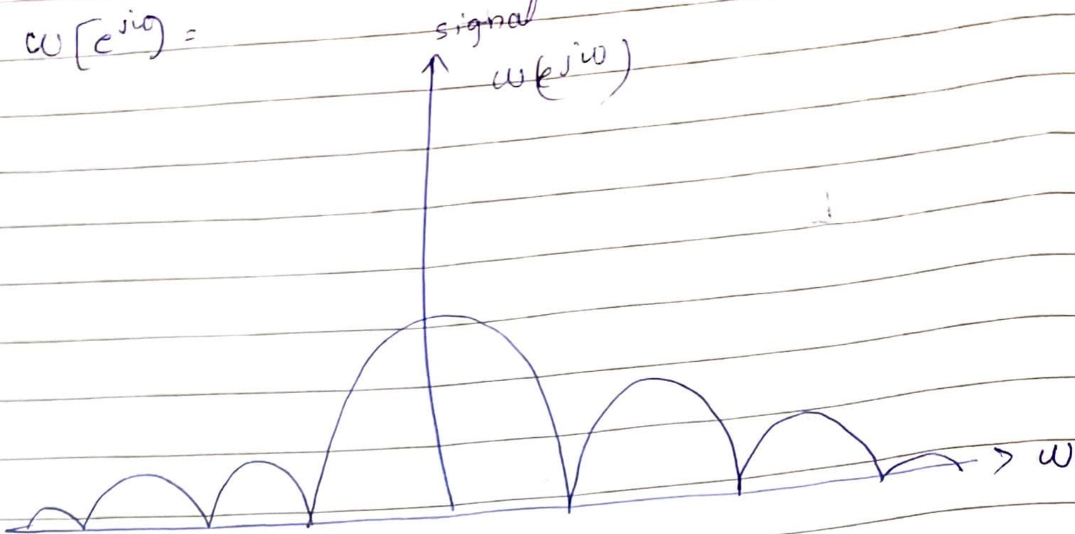
So, convolution of $\pi\left[\left(\delta\left[\frac{2\pi f_0}{f_s} - \omega\right] + \delta\left[\frac{-2\pi f_0}{f_s} - \omega\right]\right) * W(e^{j\omega})\right]$

convolution with shifted delta gives shifted signal

will give $W\left[e^{j\left(\frac{2\pi f_0}{f_s} - \omega\right)}\right] + W\left[e^{j\left(\frac{-2\pi f_0}{f_s} - \omega\right)}\right]$

Now $$W[e^{j\omega}] = \sum_{n=0}^{L-1} e^{-j\omega n} = \left(\frac{e^{-j\omega L} - 1}{e^{-j\omega} - 1}\right)$$

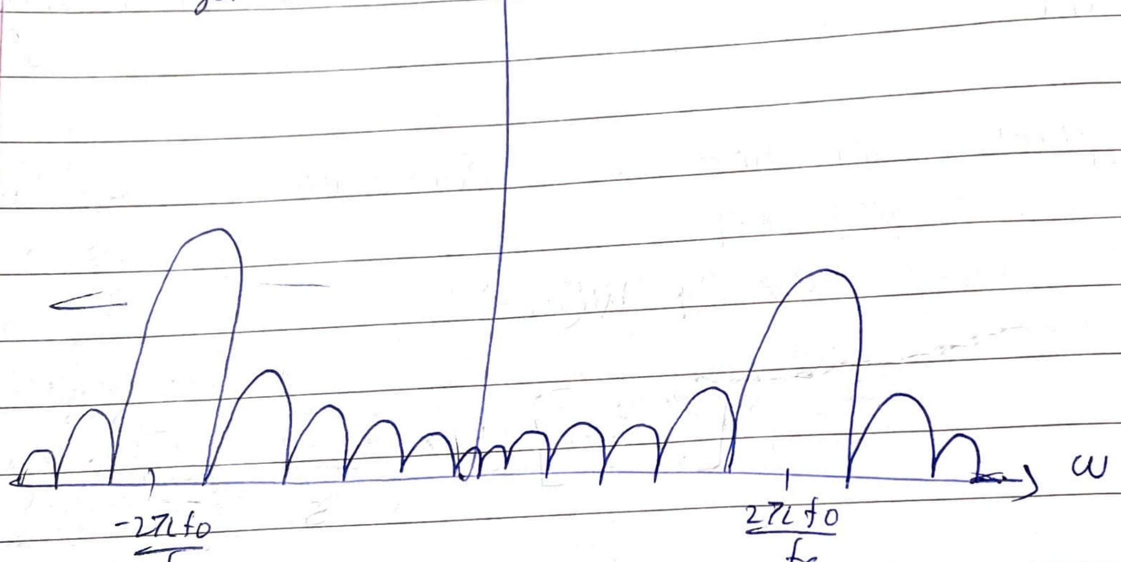
$\omega(e^{j\omega}) =$



~~ω~~ Now, convolution will give sum of shifted $\omega(e^{j\omega})$'s, as given earlier, whose graph will look like.

After windowing & then DTFT, we get →

signal
$X(e^{j\omega})$



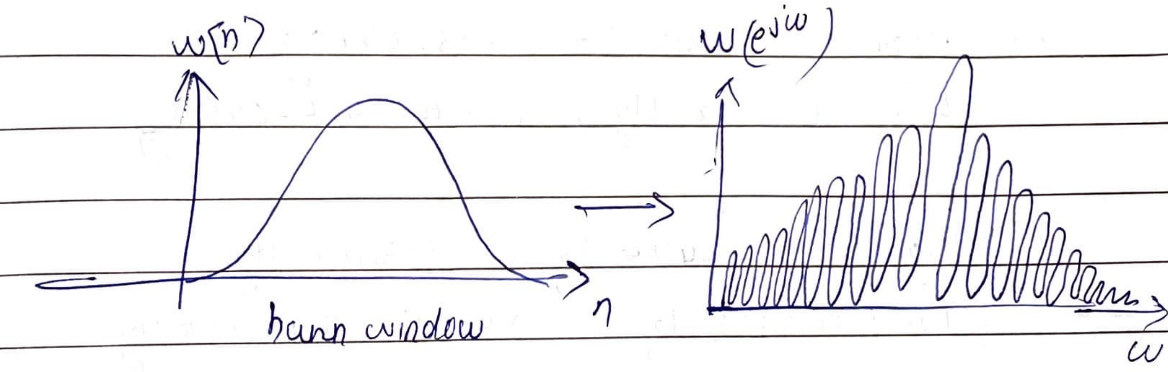
So, windowing in time domain causing smoothing in frequency domain. Earlier where there was an impulse in $P(e^{j\omega})$, now there is a main lobe at those same positions in $X(e^{j\omega})$. We also get many side lobes. This is called spectral leakage. The magnitude is distributed over the entire graph rather than just at one / two positions.

d. Yes. The observed plots match our results from part 'c'.

e.) We see that as L increases, spectrum magnitude value also increases & we see that the magnitude spectrum is getting compressed i.e frequency of the magnitude spectrum is increasing. As L value increases, we see that more frequencies can be observed & so, frequency resolution improves as L value increases.

g.) When we compare the plots of this part with the plots in part-'d'. the main lobe width increases & we see that number of side lobes also decrease => spectral leakage decreases. So hanning window is a better choice for windowing. than rectangular window

~~i) Given signal~~



i.) Given signal

top 3 frequencies:

DTFT	frequency in (Hz)
① 5124.88	0.125664, 6.15752
② 4990.93	0.125585, 6.1576
③ 4608.29	0.125507, 6.15768

Own signal

top 3 frequencies

DTFT	frequency (in Hz)
1182.08	0.0560646, 6.22712
1168.27	0.0562402, 6.22695
1155.73	0.0562292, 6.22696

~~3.) a) N=4 — N=16 — N=64~~

~~How freq: 0 Hz — 0 Hz,~~

~~High freq: 1057800 1.57 − 4.5 Hz~~

3.) a)	N=4	N=16	N=64
High	0 Hz	0 Hz	0 Hz
Low	1.57 − 4.5 (Hz)	(1.5708, 3.14, 4.712) Hz	(1.5708, 3.14, 4.71) Hz

3.b.) High: 0.942478 Hz, 5.34071 Hz
Low: 0 − 0.628 Hz, 1.2566 − 5.02655 Hz

3c.) High: 0.942478 Hz, 5.34071 Hz
Low: 0 − 0.628 Hz, 1.2566 − 5.02655 Hz

3d.) High: 0.9242478 Hz, 5.34071 Hz
Low: 0−0628 Hz, 1.2566 − 5.02655 Hz

3e.) High: 0 Hz
Low: 3 Hz

3f.) High: 3.14159
Low: 0 Hz