Report

3.1.c.) The trend of the given sequence s1[n] is increasing as per the moving average obtained. But it is not smoothly increasing & there are a few dips & ups due to the value of N taken as 5 which can be increased to get a smoother increasing trend.

3.1.d.) When we experiment with different N values starting from 5, we see that as we increase the value of N, the increase in the MA becomes smoother, i.e, the curve becomes smoother without much distortions. Finally, at $\boxed{N=100}$ we get an extremely smooth increase in the curve. So, the noise is all filtered & we get a smooth increasing curve.

3.1.j) The trend in s1[n] using convolution implementation is increasing exponentially & then decreasing exponentially & smoothly. We do find a difference in result, the result using direct moving average gives us just the increasing part of the curve whereas convolution gives us the decreasing part too & the curve obtained is much smoother compared to the one obtained using previous implementation. So, according to me, convolution is better as it gives a complete & smooth graph without any noises.

3.2.) b.) when we upsample the test sequence in a2-1.mat & interpolate, for zero order, we get the upsampled graph like a sine wave but it is a little distorted & not smooth as there are breaks in between ~~when~~ but when we do first order interpolation, we get a smooth graph resembling a sine wave.

For a_2-2.mat too, we get a very distorted graph whose trend is increasing. for zero interpolation. For linear interpolation, we get a somewhat smoother graph with an increasing trend.

3.3.) b.) $y[n] = (\cos\omega_0 n) \cdot x[n]$

$$Y[z] = \sum_{k=-\infty}^{k=\infty} (\cos\omega_0 k) \cdot x[k] \cdot z^{-k}$$

$$= \sum_{k=-\infty}^{k=\infty} \left(\frac{e^{j\omega_0 k} + e^{-j\omega_0 k}}{2}\right) x[k] \cdot z^{-k}$$

$$= \frac{1}{2}\left(\sum_{k=-\infty}^{k=\infty} e^{j\omega_0 k} \cdot x[k] \cdot z^{-k} + \sum_{k=-\infty}^{k=\infty} e^{-j\omega_0 k} x[k] z^{-k}\right)$$

$$= \frac{1}{2}\left(\sum_{k=-\infty}^{k=\infty} x[k] \left(e^{-j\omega_0} \cdot z\right)^{-k} + \sum_{k=-\infty}^{k=\infty} \left(e^{j\omega_0} \cdot z\right)^{-k} \cdot x[k]\right)$$

$$= \frac{1}{2}\left(X\left(e^{-j\omega_0} z\right) + X\left(e^{j\omega_0} \cdot z\right)\right)$$

let $x[n] = (\sin(\omega_2 n))\, u[n]$

then, we get

$$X\left(e^{-j\omega_0} z\right) = \sum_{k=-\infty}^{k=\infty} \sin\omega_2 k \left(e^{-j\omega_0} z\right)^{-k} u[n]$$

$$= \sum_{k=-\infty}^{k=\infty} (\sin\omega_2 k)\, e^{j\omega_0 k} z^{-k} \quad u[n]$$

$$= \sum_{k=-\infty}^{k=\infty} \left(\frac{e^{j\omega_2 k} - e^{-j\omega_2 k}}{2j}\right) \left(e^{j\omega_0 k}\right) \cdot z^{-k} \quad u[n]$$

$$\frac{1}{2j}\left[\sum_{k=-\infty}^{k=\infty} \left(e^{jk(\omega_2+\omega_0)} - e^{jk(\omega_0-\omega_2)}\right) \cdot z^{-k}\right] u[n]$$

$k=\infty$

$$= \frac{1}{2j} \sum_{k=0}^{k=\infty} \left(e^{jk(\omega_2+\omega_0)} - e^{jk(\omega_0-\omega_2)} \right) z^{-k}$$

$$= \frac{1}{2j} \left(\sum_{k=0}^{k=\infty} e^{jk(\omega_2+\omega_0)} \cdot z^{-k} - \sum_{k=0}^{k=\infty} e^{jk(\omega_0-\omega_2)} \cdot z^{-k} \right)$$

$$= \frac{1}{2j} \left(\sum_{k=0}^{k=\infty} \left(e^{j(\omega_2+\omega_0)} \cdot z^{-1} \right)^k - \sum_{k=0}^{k=\infty} \left(e^{j(\omega_0-\omega_2)} z^{-1} \right)^k \right)$$

$$= \frac{1}{2j} \left(\frac{1}{1 - e^{j(\omega_2+\omega_0)} \cdot z^{-1}} - \frac{1}{1 - e^{j(\omega_0-\omega_2)} z^{-1}} \right)$$

$\downarrow$ condition: $|z| > e^{j(\omega_0+\omega_2)}$

$\downarrow$ condition $|z| > e^{j(\omega_0-\omega_2)}$

also, similarly,

$$X(e^{j\omega_0} z) = \sum_{k=-\infty}^{\infty} \sin \omega_2 k \, (e^{j\omega_0} z)^{-k} \cdot u[n]$$

$$= \sum_{k=0}^{k=\infty} \sin \omega_2 k \, (e^{j\omega_0} z)^{-k}$$

$$= \sum_{k=0}^{k=\infty} \frac{1}{2j} \left(e^{j(\omega_2-\omega_0)k} - e^{jk(-(\omega_0+\omega_2))} \right) z^{-k}$$

$$= X(e^{j\omega_0} z) = \frac{1}{2j} \left(\frac{1}{1 - e^{j(\omega_2-\omega_0)} \cdot z^{-1}} - \frac{1}{1 - e^{-j(\omega_0+\omega_2)} \cdot z^{-1}} \right)$$

$\downarrow$ condition $|z| > e^{j(\omega_2-\omega_0)}$

$\downarrow$ $|z| > e^{-j(\omega_0+\omega_2)}$

So finally:

$$Y(z) = \frac{1}{2}\left(\frac{1}{2j}\right)\left(\frac{z}{z-e^{j(\omega_2+\omega_0)}} - \frac{z}{z-e^{j(\omega_0-\omega_2)}} + \frac{z}{z-e^{j(\omega_2-\omega_0)}} - \frac{z}{z-e^{-j(\omega_0+\omega_2)}}\right)$$

$$X(z) = \sum_{k=-\infty}^{\infty} (\sin \omega_2 k)\cdot u(k) z^{-k} = \sum_{k=0}^{k=\infty} (\sin \omega_2 k)\, z^{-k}$$

$$= \sum_{k=0}^{k=\infty} \left(\frac{e^{j\omega_2 k} - e^{-j\omega_2 k}}{2j}\right) z^{-k}$$

$$= \frac{1}{2j}\left(\sum_{k=0}^{\infty} \left(e^{j\omega_2}\cdot z^{-1}\right)^k - \sum_{k=0}^{k=\infty} \left(e^{-j\omega_2}\cdot z^{-1}\right)^k\right.$$

$$= \frac{1}{2j}\left(\frac{1}{1-e^{j\omega_2} z^{-1}} - \frac{1}{1-e^{-j\omega_2} z^{-1}}\right)$$

$\downarrow$ Condition: $|z| > e^{j\omega_2}$

$\downarrow$ condition $\downarrow$ $|z| > e^{-j\omega_2}$

$$\frac{Y(z)}{X(z)} = \frac{\left(\frac{1}{2}\right)\left(\frac{1}{1-e^{j(\omega_2+\omega_0)} z^{-1}} - \frac{1}{1-e^{j(\omega_0-\omega_2)} z^{-1}} + \frac{1}{1-e^{j(\omega_2-\omega_0)} z^{-1}} - \frac{1}{1-e^{-j(\omega_0+\omega_2)} z^{-1}}\right)}{\left(\frac{1}{1-e^{j\omega_2} z^{-1}} - \frac{1}{1-e^{-j\omega_2} z^{-1}}\right)}$$

1.7.)

$$H(z) = \frac{z^2 - (2\cos\theta)z + 1}{z^2 - (2r\cos\theta z) + r^2} = \frac{(z - (\cos\theta + i\sin\theta))(z - (\cos\theta - i\sin\theta))}{(z - r(\cos\theta + i\sin\theta))(z - r(\cos\theta - i\sin\theta))}$$

zeroes: $(\cos\theta + i\sin\theta)$, $(\cos\theta - i\sin\theta)$ → on a circle of magnitude 1

poles: $r(\cos\theta + i\sin\theta)$, $r(\cos\theta - i\sin\theta)$ → on a circle of magnitude r.

as θ changes the ~~poles~~ zeroes & poles are rotated about the circle of magnitude 1 & r respectively which are zero centred.

as r changes zeroes have no effect but poles will experience a change in their magnitude. ~~They~~ \\&NM