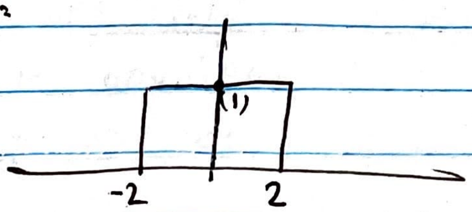
Report

Lab-6

1.) b.) $x(t) =$



$$\text{CTFT}(x(t)) = X(j\Omega) = \int_{t=-\infty}^{t=\infty} x(t)\cdot e^{-j\Omega t}\cdot dt$$

$$= \int_{t=-2}^{2} x(t)\cdot e^{-j\Omega t}\, dt$$

$$= \int_{-2}^{2} e^{-j\Omega t}\cdot dt$$

$$= \frac{\left(e^{-j\Omega 2} - e^{+j\Omega 2}\right)}{-j\Omega}$$

$$= \frac{\cancel{\cos 2\Omega} - j\sin 2\Omega - \cancel{\cos 2\Omega} - j\sin 2\Omega}{-j\Omega}$$

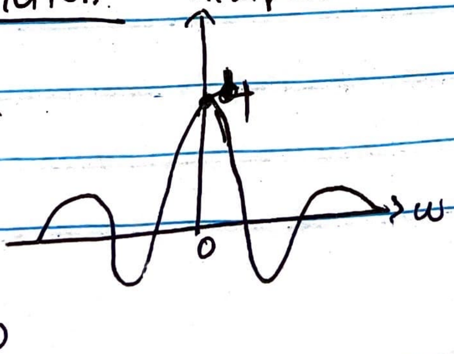
$$= \frac{\cancel{-}2\cancel{j}\sin 2\Omega}{\cancel{-j}\Omega} = \frac{2\sin 2\Omega}{\Omega}$$

$$= 4\left(\frac{\sin 2\Omega}{2\Omega}\right)$$

This is sinc function.

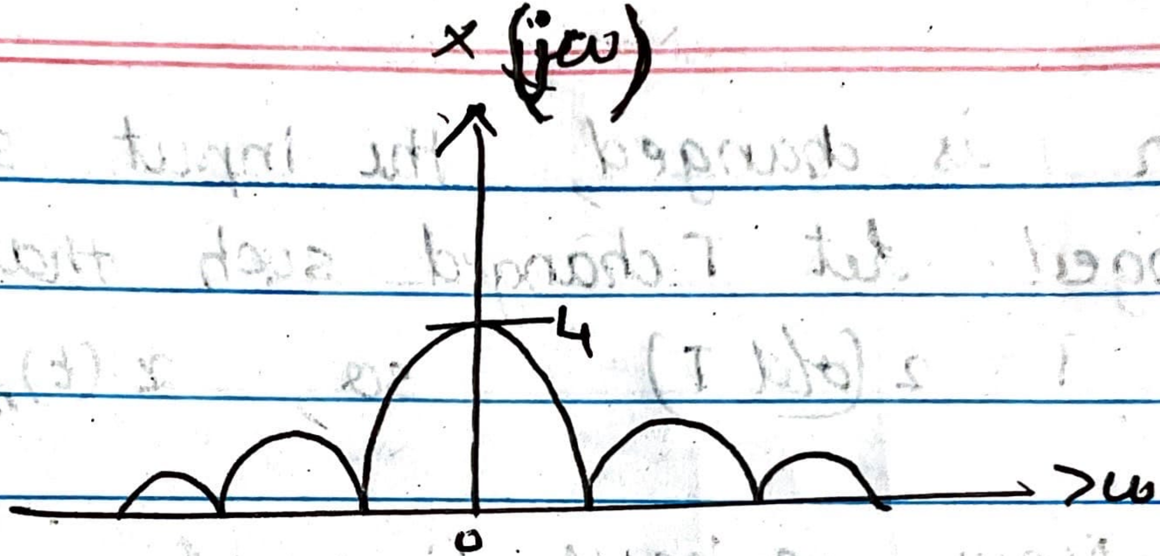
realpart

So, real part =

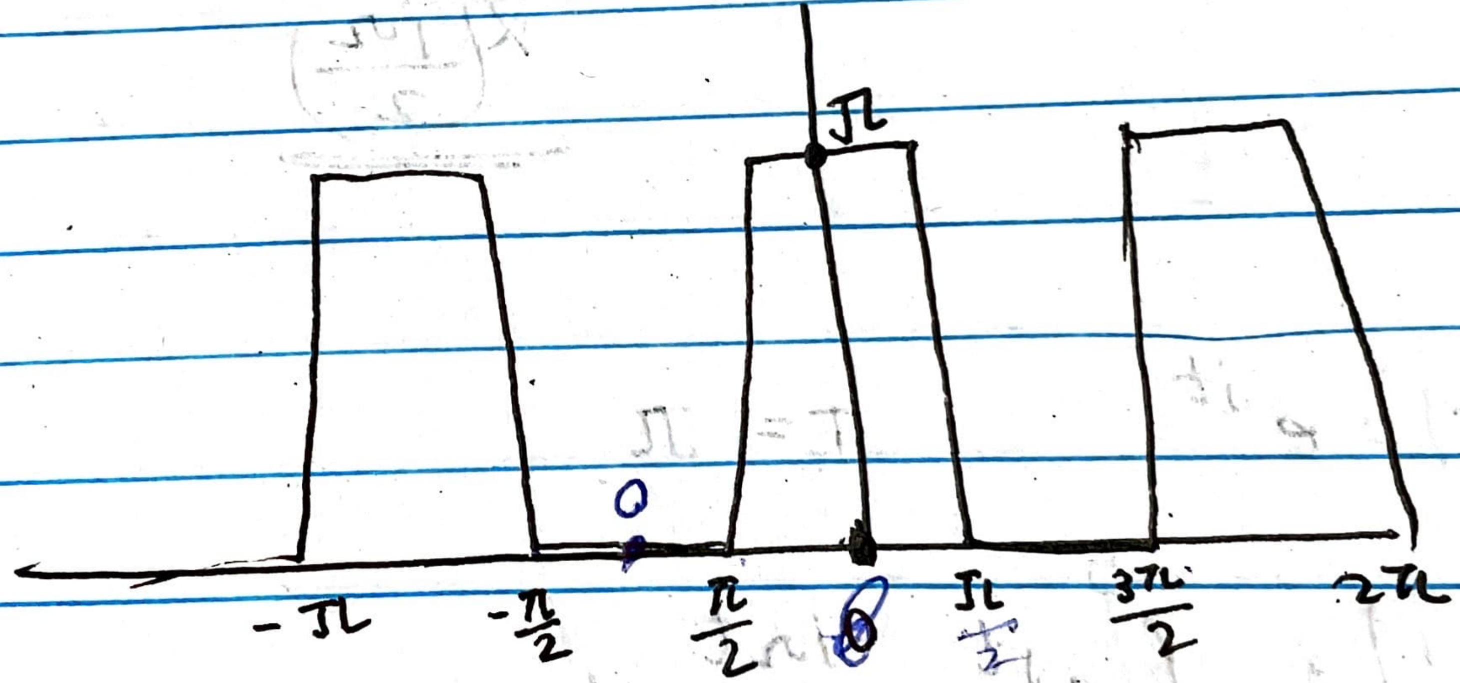


imaginary part $= 0$

Absolute value:



Phase



c.) When T is changed, the input signal is also changed. Let T changed such that it is 4.

so $T = 2(\text{old } T)$ so, $x(t)_{new} = x_{old}\left(\frac{t}{2}\right)$

Now, we have time scaling property

$$x(t) \xrightarrow{CTFT} X(j\Omega)$$

$$x(at) \xrightarrow{CTFT} \frac{1}{|a|} X\left(\frac{j\Omega}{a}\right)$$

so, for T = 4, the magnitude of the graphs will be double of T=2 & also, the graph compresses as $X\left(\frac{j\Omega}{a}\right) = X(2j\Omega)$

= compressed version of $X(j\Omega)$.

When T=1, a = 2

so the CTFT graphs magnitude is halved than T=2 & it expands as we get

$$\underline{X\left(\frac{j\Omega}{2}\right)}$$

d.) when $x(t) = e^{jt}$ $T = \pi$

$$CTFT[x(t)] = \int_{-\infty}^{\infty} e^{jt} \cdot e^{-j\Omega t} \, dt$$

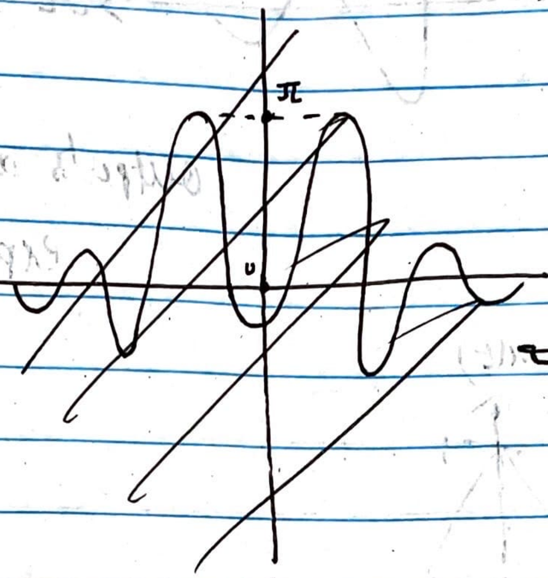
$$= \int_{-\pi}^{\pi} e^{jt(1-\Omega)} \, dt = \frac{1}{j(1-\Omega)}\left(e^{j\pi(1-\Omega)} - e^{-j\pi(1-\Omega)}\right.$$

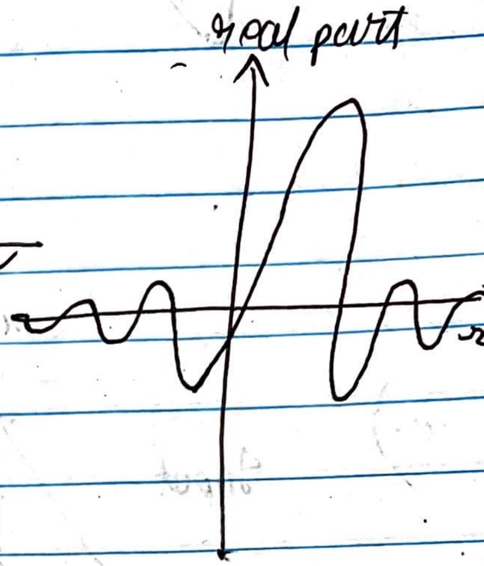
$$= \frac{1}{\cancel{j}(1-\Omega)} \cancel{2j} \sin \pi(1-\Omega)$$

Imaginary part = 0

$$= \frac{2\sin(\pi(1-\Omega))}{(1-\Omega)}$$

Real part:





When $x(t) = \cos t$

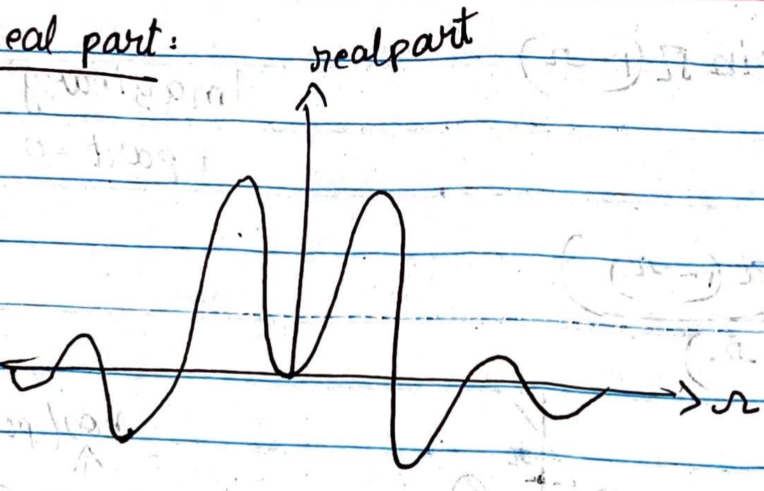
$$CTFT\ x(t) = \int_{-\infty}^{\infty} \left(\frac{e^{jt} + e^{-jt}}{2}\right) e^{-j\Omega t} \cdot dt$$

$$= \frac{1}{2} \int_{-\pi}^{\pi} \left(e^{jt(1-\Omega)} + e^{-jt(1+\Omega)}\right) \cdot dt$$

$$= \frac{1}{\cancel{2}}\left(\frac{\cancel{2}\sin \pi(1-\Omega)}{1-\Omega}\right) + \frac{1}{\cancel{2}}\left(\frac{\cancel{2}\sin \pi(1+\Omega)}{1+\Omega}\right)$$

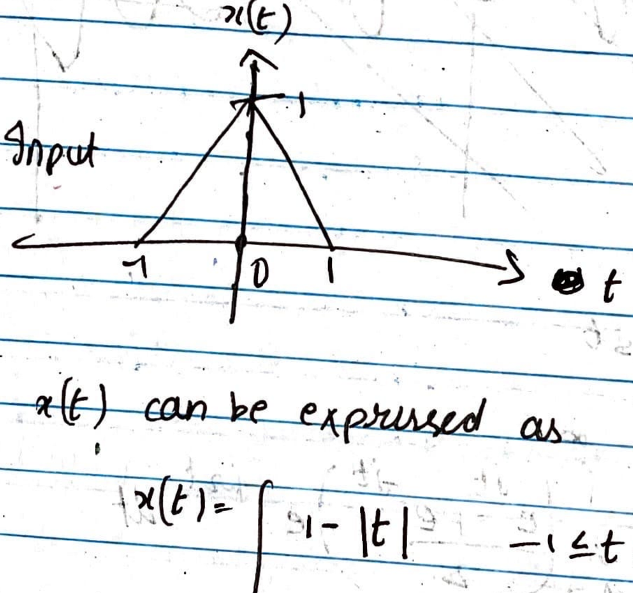
$$= \pi\left(\frac{\sin \pi(1-\Omega)}{\pi(1-\Omega)}\right) + \frac{\pi \sin \pi(1+\Omega)}{\pi(1+\Omega)}$$

Real part:



outputs are as expected.

e.) Input



$x(t)$ can be expressed as

$$x(t) = \begin{cases} 1 - |t| & -1 \le t \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

expected FT

$$X(j\Omega) = \int_{-1}^{1} (1 - |t|) \cdot e^{-j\Omega t} \, dt$$

$$= \int_{-1}^{0} (1+t) e^{-j\Omega t} \, dt + \int_{0}^{1} (1-t) e^{-j\Omega t} \, dt$$

$$= \frac{-1}{j\Omega}\left[\left(e^{0} - e^{j\Omega}\right)\right] + \left[\left(t\left(\frac{e^{-j\Omega t}}{-j\Omega}\right) - \frac{e^{-j\Omega t}}{(-j\Omega)^2}\right)_{-1}^{0}\right]$$

$$+ \left(\frac{-1}{j\Omega}\right)\left[e^{-j\Omega} - e^{0}\right] - \left[\left(t\,\frac{e^{-j\Omega t}}{j\Omega} - \frac{e^{-j\Omega t}}{(-j\Omega)^2}\right)_{0}^{1}\right]$$

$$= \frac{-1}{j\Omega}\left(1 - e^{j\Omega}\right) + \left(\frac{-1}{-1\Omega^2} - \left(\frac{(-1)e^{j\Omega}}{+j\Omega} - \frac{e^{j\Omega}}{(-1)\Omega^2}\right)\right)$$

$$-\frac{1}{j\Omega}\left(e^{-j\Omega} - 1\right) - \left(\frac{e^{-j\Omega}}{j\Omega} - \frac{e^{-j\Omega}}{(-1)\Omega^2} + \frac{1}{(-1)\Omega^2}\right)$$

$$= \cancel{\frac{-1}{j\Omega}} + \cancel{\frac{e^{j\Omega}}{j\Omega}} + \frac{1}{\Omega^2} - \cancel{\frac{e^{j\Omega}}{j\Omega}} + \frac{e^{j\Omega}}{\Omega^2} - \frac{e^{-j\Omega}}{j\Omega}$$

$$+ \cancel{\frac{1}{j\Omega}} - \frac{e^{-j\Omega}}{j\Omega} + \frac{e^{-j\Omega}}{\Omega^2} + \frac{1}{\Omega^2}$$

$$= \frac{2}{\Omega^2} - \frac{2e^{-j\Omega}}{j\Omega} = \left(\frac{e^{j\Omega} + e^{-j\Omega}}{\Omega^2}\right)$$

$$= \frac{2}{\Omega^2} - \frac{2e^{-j\Omega}}{j\Omega} - \frac{2\cos\Omega}{\Omega^2} = \frac{2}{\Omega^2} + \frac{2je^{-j\Omega}}{\Omega} - \frac{2\cos\Omega}{\Omega^2}$$

6.3 When we plot FFT for input cost using inbuilt FFT function & radix2fft method, we see that they match.

6.4. When we multiply the (FS) of input to $H(\omega)$, we will get ~~convolution of~~ FS of the output (As convolution in time domain is multiplication in frequency domain).

So, FS coefficients of output will be the coefficients of $e^{jk\omega}$ of $X(\omega) \cdot H(\omega)$.

The periodicity of the output signal will be LCM of the period of $H(\omega)$ & $X(\omega)$.

u.) b.) Low-pass filter:

We see that the FS coefficients of cost are present only for $k = -1, 1$

$\Rightarrow \omega = -1\omega_0, \omega_0$

as $\omega_0 = 1$

$\omega = \{-1, 1\}$

so, when $\omega_c = 2$, as cutoff is high, it will allow all $|\omega| \leq 2$, so, full cos wave is passed as it is.

But when $\omega_c = 0.5$ as cutoff is low, it doesn't allow $|\omega| > 0.5$ so, as we have $\omega = \{1, -1\}$, it doesn't pass both & so, it doesn't pass the $\cos(t)$ & we get zero.

4.c.) **High pass filter:**

Like earlier case, we can analyse it this way:

when $\omega_c = 2$, it will allow only $|\omega| \geq 2$, so it doesn't allow $\omega = \{-1, 1\}$ & thus it doesn't pass our $\cos(t)$ signal

when $\omega_c = 0.5$, it allows $|\omega| \geq \frac{1}{2}$ so $\omega = \{1, 1\}$ is allowed to pass & thus, our entire $\cos t$ signal is passed as it is

4.d.) **Non-Ideal Filter**

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{G}{a+\omega}$$

when ω = low,

$$Y(\omega) = \left(\frac{G}{a}\right) X(\omega)$$

$$= (k)\, X(\omega)$$

it passes all $X(\omega)$

if ω = high $Y(\omega) \approx 0$ it doesn't pass high ω's so it acts like a low pass filter.

d.) We see that the output signal also has an imaginary part due to the complex valued nature of the LTI system frequency response.

6.5 When we compare quality of the interpolated signals, we see that sinc has the highest quality quality wise, it is like

zero < Linear < sinc

MAE is also least for sinc, so it has more accuracy.

6.6.) We see that as sampling interval i.e T_s decreases. the reconstructed signal matches with the original signal more & more i.e it has more accuracy. It is because as we are $\downarrow T_s$, no. of samples $\uparrow$, so, we have more accuracy & less loss of information.

6.8-) Nyquist rate = $\frac{1}{2\times}$ freq of signal

$$= \frac{1}{2} \times 5\pi = \frac{5\pi}{2}$$

→ As we see, if ~~~~ we have the frequency sampling, if it goes ~~greater~~ lesser than nyquist frequency, aliasing takes place. So, we get a distorted signal for higher T_s i.e lower f_s. when f_s goes less than nyquist freq.