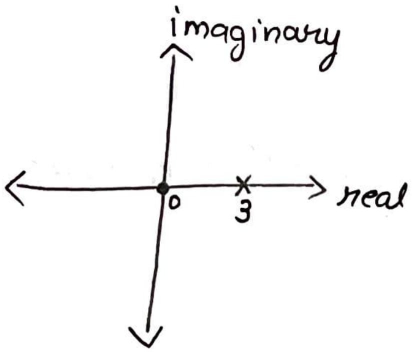
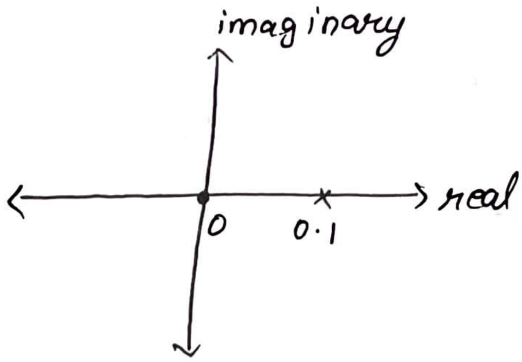
q1_b: No zeros as numerator = 1

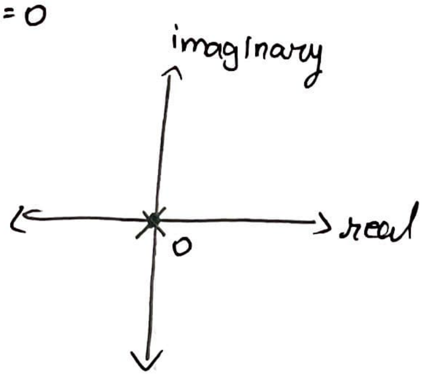
p = 3



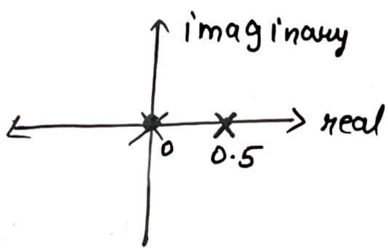
p = 0.1



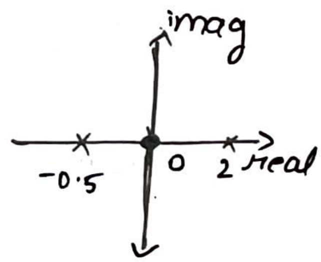
p = 0



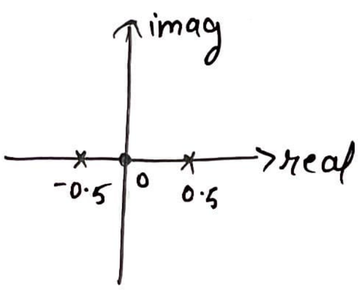
p = [0, 0.5]



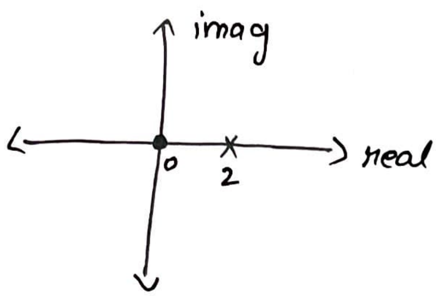
p = [2, -0.5]



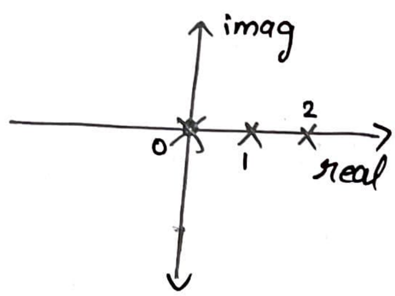
p = [0.5, -0.5]



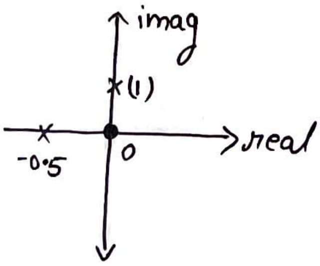
p = [2, 2, 2]



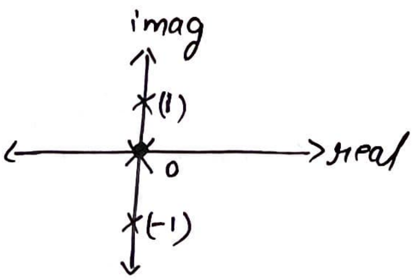
p = [0, 1, 2]



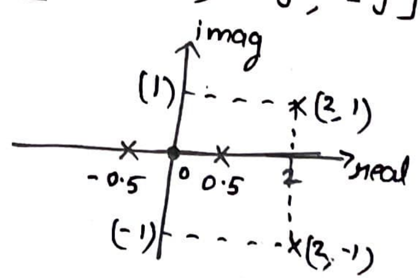
p [-0.5, j]



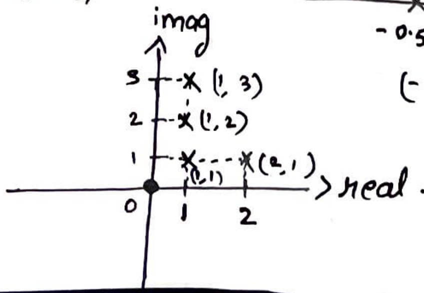
p [0, j, -j]



p [0.5, -0.5, 2+j, 2-j]



p = [1+j, 1+2j, 1+3j, 2+j] →



$q_2 - c:$

For the given system function, since $p \in (-1, 1)$, only 2 kinds of impulse responses are possible: $h[n] = (-p)^n u[n]$

I → alternately decaying impulse response when $\underline{0 < p < 1}$

→ it is a decaying response.
→ its sign keeps changing alternately.
→ when $n = odd$, $h[n] = -ve$
when $n = even$ $h[n] = +ve$

II → decaying when $-1 < p < 0$

→ it is a decaying & +ve response.

By impz() for $p = 0.8$, we get type I impulse response.

Q2 - d:

$H(z) = \dfrac{1}{1+P\left(\frac{1}{z}\right)}$

Now, converting this impulse response to operator R form:

$H(R) = \dfrac{1}{1+PR}$ (as $R = \frac{1}{z}$)

~~H(R)/(+)~~ Now, converting this in terms of difference equation, we get

$h[n] + p\,h[n-1] = \delta[n]$

$h[n] = \delta[n] - p\,h[n-1]$

When $H(R) = \dfrac{1}{1-\alpha R}$, the output $h[n] = \alpha^n u[n]$
[accumulator with a scaling factor α]

so, here,

$\alpha = -P.$

So $h[n] = (-P)^n u[n]$

Case 1: when $P = 0.8$

The impulse response is $(-0.8)^n u[n]$

So, for $n>0$ & $n =$ even, the $h[n]$ is positive

for $n>0$ & $n =$ odd, $h[n] =$ negative

So, it keeps alternating signs & the impulse response is decaying.

Case 2: when $P = -0.8$

Impulse response $= (0.8)^n u[n]$

This is a decaying response.

Case - 3 : $p = 0.1$

Here, Impulse response $h[n] = (-0.1)^n u[n]$

So, for odd & +ve n, $h[n] = -ve$

for even & -ve n, $h[n] = +ve$

& it is a decaying signal with its sign alternating for even & odd values of n.

So, Impulse response: $(-p)^n u[n]$

growing	$\alpha > 1 \Rightarrow$	$p < -1$
decaying	$0 < \alpha < 1 \Rightarrow$	$-1 < p < 0$
alternating & decaying	$-1 < \alpha < 0$	$0 < p < 1$
alternating & growing	$\alpha < -1$	$p > 1$

Frequency response:

for $p = 0.1$, it is increasing for $0 < freq < \pi$

for $p = -0.8$, it is decreasing for $0 < freq < \pi$

for $p = 0.8$, it is increasing for $0 < freq < \pi$

→ for $p > 0$, freq response in increasing with increasing freq from 0 to π.

→ for $p < 0$, frequency response is decreasing with increasing freq from 0 to π.

Q3-a

for $r = 0.5$

$\theta = 0°$

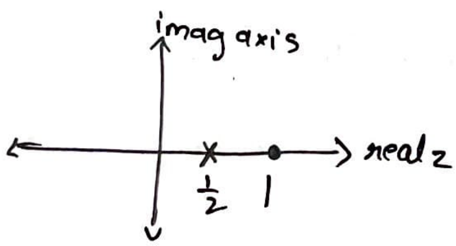
$$H(z) = \frac{z^2 - 2z + 1}{z^2 - z + \left(\frac{1}{4}\right)}$$

$$H(z) = \frac{4\,(z-1)^2}{4z^2 - 4z + 1}$$

$$H(z) = \frac{4\,(z-1)^2}{(2z-1)^2}$$

so zeroes $= 1, 1$

poles $= \frac{1}{2}$



pole-zero plot

for $r = 0.2$

$\theta = \frac{\pi}{4}$

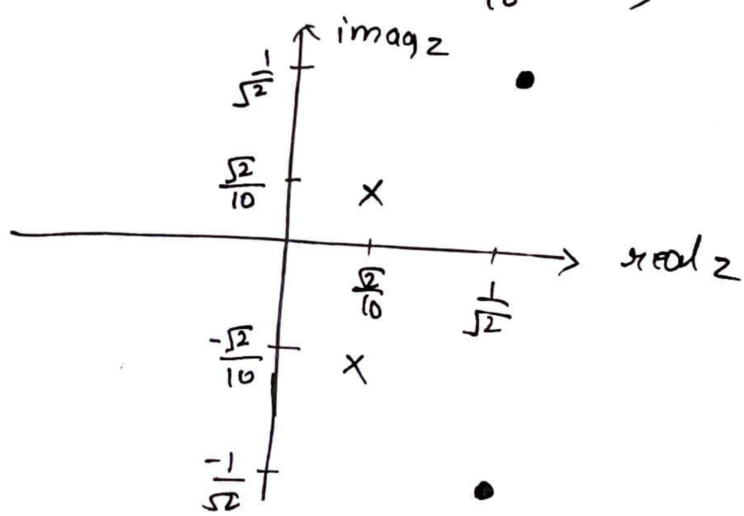
$$H(z) = \frac{z^2 - \sqrt{2}z + 1}{z^2 - \frac{\sqrt{2}}{5}z + \frac{1}{25}}$$

$$H(z) = \frac{(25)\left(z^2 - \sqrt{2}z + 1\right)}{\left(25z^2 - 5\sqrt{2}z + 1\right)}$$

$$H(z) = \frac{(25)\left(z - \frac{1}{\sqrt{2}}(1+i)\right)\left(z - \frac{1}{\sqrt{2}}(1-i)\right)}{\left(z - \frac{\sqrt{2}}{10}(1+i)\right)\left(z - \frac{\sqrt{2}}{10}(1-i)\right)}$$

zeroes $= \frac{1}{\sqrt{2}}(1+i), \ \frac{1}{\sqrt{2}}(1-i)$

poles $= \frac{\sqrt{2}}{10}(1+i), \ \frac{\sqrt{2}}{10}(1-i)$



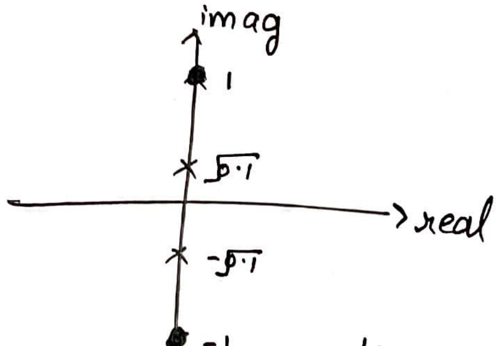
for $r = 0.1$

$\theta = \frac{\pi}{2}$

$$H(z) = \frac{z^2 + 1}{z^2 + (0.1)^2}$$

Zero: $\pm j$

Poles: $\pm \sqrt{0.1}\, j$



pole-zero plot

Thus all verified

for $r = 0.8$

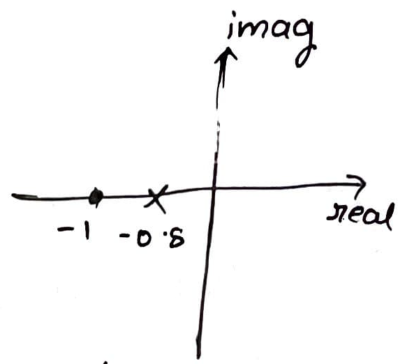
$\theta = \pi$

$$H(z) = \frac{z^2 + 2z + 1}{z^2 + 1.6z + (0.8)^2}$$

$$= \frac{(z+1)^2}{(z+0.8)^2}$$

zeroes: -1

poles = -0.8



pole-zero plot

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q3 - b:

for the system to be causal, ROC = right sided for $h[n]$

so, $|z| > a$

& for stability, $|z| = 1$ must be included in the ROC.

so $a < 1$

So, pole must be of the form $(z - a)$ where $a > 0$ & $a < 1$

ROC: $\boxed{|z| > a}$ a = poles or $-1 < a < 1$

$$H(z) = \frac{\left(z - (\cos\theta + i\sin\theta)\right)\left(z - (\cos\theta - i\sin\theta)\right)}{\left(z - \left(r(\cos\theta + i\sin\theta)\right)\right)\left(z - r(\cos\theta - i\sin\theta)\right)}$$

for this ~~sys~~ to be causal & stable,

$$-1 < \left|\frac{r}{2}(\cos\theta \pm i\sin\theta)\right| < 1$$

$$-2 < r\left|(\cos\theta \pm i\sin\theta)\right| < 2$$

$-2 < r < 2$, then the system is causal & stable

which is happening $\forall r$ as $r \in (0\ 1)$.

So, the system is always both causal & stable.

Q3 – c.) as θ goes from 0 to π, we see that the frequency response in increasing first & as θ increasing freq response decreased & then increases, shows a downward peak & as θ increases this peak is achieved at a higher frequency & finally, there is only decrease / dip & no ~~decrease~~ increase at $f_{req} = \pi$

Q3 – d.)

We see that as r increases the upward parabola increases in height, i.e max freq. response value increases & then it also starts flattening at the top & after a point, it starts flattening more & decreasing its height i.e max freq response value decreases.