Report

1.b.) 1.) $x[n] = \delta[n]$

DTFT is: $\sum_{n=-\infty}^{n=\infty} \delta[n] \cdot e^{-j\omega n} = X(e^{j\omega})$

$= (1)\, e^{0} = \underline{1}$

$\boxed{X(e^{j\omega}) = 1}$

Magnitude $= 1 \quad \forall\, \omega$

Phase $= 0 \quad \forall\, \omega$

Real part $= 1 \quad \forall\, \omega$

Imaginary part $= 0 \quad \forall\, \omega$

2.) $\delta[n+3] = x[n]$

DTFT is: $\sum_{n=-\infty}^{n=\infty} \delta[n+3]\, e^{-j\omega n} = X(e^{j\omega})$

$X(e^{j\omega}) = e^{3j\omega} = \cos 3\omega + j\sin 3\omega$

Magnitude $= 1 \quad \forall\, \omega$

Phase $= 3\omega$

Real part $= \cos 3\omega$

Imaginary $= \sin 3\omega$

3.) rectangular pulse from -3 to 3

$$x[n] = \begin{cases} 1 & -3 \le n \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

so $X(e^{j\omega}) = \sum_{n=-\infty}^{n=\infty} x(n)\, e^{-j\omega n}$

$= \sum_{n=-3}^{n=3} (1) \cdot e^{-j\omega n}$

$= \sum_{n=-3}^{n=3} \left(e^{-j\omega}\right)^{n} = \dfrac{\left(e^{-j\omega}\right)^{-3}\left(\left(e^{-j\omega}\right)^{7} - 1\right)}{\left(e^{-j\omega} - 1\right)} = X(e^{j\omega})$

$$\boxed{= \frac{e^{3j\omega}\left(e^{-7j\omega} - 1\right)}{\left(e^{-j\omega} - 1\right)}}$$

4.) $x[n] = \sin(w_0 n)$ $\quad w_0 = \frac{\pi}{4}$ $\quad$ -200 to 200

$$x[n] = \sin\left(\frac{\pi}{4} n\right)$$

so $$X(e^{j\omega}) = \sum_{n=-200}^{n=200} \sin\left(\frac{\pi}{4}\right)n \cdot e^{-j\omega n}$$

$$= \sum_{n=-200}^{n=200} \left(\frac{e^{\frac{jn\pi}{4}} - e^{\frac{-jn\pi}{4}}}{2j}\right) e^{-j\omega n}$$

$$\frac{1}{2j} \sum_{n=-200}^{n=200} \left(e^{jn\left(\frac{\pi}{4}-\omega\right)} - e^{-jn\left(\omega+\frac{\pi}{4}\right)}\right)$$

$$\boxed{X(e^{j\omega}) = \frac{1}{2j}\left(\frac{e^{-j200\left(\frac{\pi}{4}-\omega\right)}\left(e^{jn\left(\frac{\pi}{4}-\omega\right)401} - 1\right)}{e^{j\left(\frac{\pi}{4}-\omega\right)}} - \frac{e^{200j\left(\frac{\pi}{4}+\omega\right)}\left(e^{-401j\left(\frac{\pi}{4}+\omega\right)} - 1\right)}{e^{-j\left(\frac{\pi}{4}+\omega\right)}}\right)}$$

c.) $x[n] = a^n u[n]$

$$\text{DTFT} = X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} a^n u[n] \cdot e^{-j\omega n}$$

$$= \sum_{n=0}^{n=\infty} a^n \cdot e^{-j\omega n}$$

$$= \sum_{n=0}^{n=\infty} (ae^{-j\omega})^n$$

$$\Rightarrow \frac{1}{1-ae^{-j\omega}} = \boxed{\frac{1}{1-ae^{-j\omega}}}$$

$$X(e^{j\omega}) = \frac{1}{1-ae^{-j\omega}}$$

Magnitude $X(e^{j\omega}) = \frac{1}{1-a(\cos\omega - j\sin\omega)}$

$$= \frac{1}{1-a\cos\omega + aj\sin\omega}$$

$$= \frac{1}{\sqrt{(1-a\cos\omega)^2 + (a\sin\omega)^2}}$$

$$= \frac{1}{\sqrt{1+a^2-2a\cos\omega}}$$

We observe from the plots that as b is increasing, the graph strechus vertically, i.e the maxima value increases & minima value decreases. Also, the curve flatten at the bottom & becomes more sharper at the top. So, the maxima becomes sharper & minima part flattens. Also, when we take a=b & then a=-b, the magnitude spectrum shifts by π.

when $a=0.01$, when $\omega=0 \rightarrow$ maxima

$$|X(e^{j\omega})| = \frac{1}{1-0.01} = \frac{1}{0.99}$$

When $a=0.5$ $\omega=0$ maxima

$$|X(e^{j\omega})| = \frac{1}{1-0.5} = \frac{1}{0.5}$$

maxima is increasing as b increasing.

when $a=0.01$ & $\omega=\pi \rightarrow$ minima

$$|X(e^{j\omega})| = \frac{1}{1+0.01} = \frac{1}{1.01}$$

when $a=0.5$ & $\omega=\pi \rightarrow$ minima

$$|X(e^{j\omega})| = \frac{1}{1+0.5} = \frac{1}{1.5}$$

minima decreases as b is increasing.

also, when $a = 0.01$ when $w = \pi$

$|X(e^{jw})|$ when $w = 0$ is $= \frac{1}{0.99}$ → maxima $|X(e^{jw})| = \frac{1}{1.01}$

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when $a = -0.01$ & $w = 0$,

when $w = \pi$

$$|X(e^{jw})| = \frac{1}{1+0.01} = \frac{1}{1.01} \rightarrow \text{minima} \quad |X(e^{jw})| = \frac{1}{0.99}$$

maxima

So at $w = 0$, when $a = 0.01$ ~~where~~ maxima is observed, maxima ~~minima~~ is observed at $w = \pi$ ~~there~~ when $a = -0.01$. So, the graph shifts by π.

4.2.)

a.) $h[n] = \frac{1}{M} \sum_{m=0}^{M-1} \delta[n-m]$ $h[n]$ = output when input = $\delta[n]$.

$$= \begin{cases} 1 & m = 0 \text{ to } m = M-1 \\ 0 & \text{elsewhere} \end{cases}$$

e.) We know that in a moving average filter, the noise is filtered out. So, as we increase M, we are taking the average of more number of samples & so, the curve smoothens out more & the noise is filtered out more. But the tradeoff we see is that even though we are getting curve ridden of noise, the curve is shifting slightly & so, we get a shifted curve.

f.) We observe that as we increase M & filter the signal & plot DTFT, the filtered signal matches the original signal more closely for a higher value of M. For lesser M values, the DTFT is little distorted. ⇒ we get similar values of DTFT for more number of frequencies but as M increases, the distortion reduces as the noise is filtered out.

g.) We see that when we filter the noisy signal & then plot DTFT of original signal & filtered signal, we see that DTFT of the noisy filtered signal is more distorted & ~~the~~ noise is highlighted / enhanced / sharpened there. Also, we see that the plot of the filtered signal is even more noisy & jaggered than the noisy signal. So, this filter basically enhances / sharpens the curve & makes it more jaggered.

h.) In terms of frequency selection, we observe that the moving average filter accents ~~very less~~ small range of frequencies compared to the difference filter, as the first filter removes noise whereas second filter enhances the noise i.e it takes greater range of frequencies while plotting.

h[n] of digital differentiator

$$h[n] = \delta[n] - \delta(n-1)$$

$$h[n<0] = \delta(n<0) - \delta\left[n(<-1)\right]$$

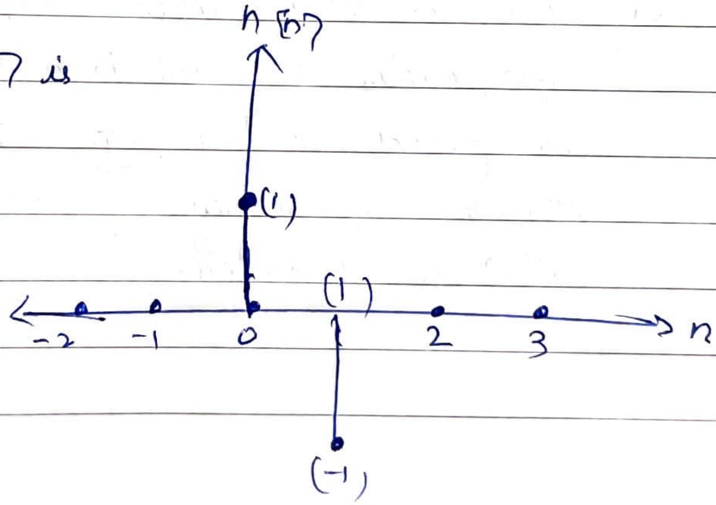
$$= 0$$

$h[0] = 1$ so, $h[n]$ is

$h[1] = -1$

$h[2] = 0$

$h[n>2] = 0$



3.) a.) $x(e^{j\omega}) = \begin{cases} 1 & -\omega_c < \omega < \omega_c \\ 0 & \text{elsewhere} \end{cases}$

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n} . d\omega$$

$$= \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} (1) \cdot e^{j\omega n} . d\omega$$

$$x[n] = \frac{1}{2\pi j n} \left(e^{j\omega_c n} - e^{-j\omega_c n}\right)$$

$$= \frac{1}{\cancel{2}\pi \cancel{j} n} \left(\cancel{\cos\omega_c n} + \cancel{j}\sin\omega_c n - \cancel{\cos\omega_c n} + \cancel{j}\sin\omega_c n\right)$$

$$x[n] = \frac{\sin\omega_c n}{\pi n}$$ (so, $x[n]$ is real & not complex)

b.) we see that as we are increasing ω_c, we are increasing the range of frequencies over which we are integrating $X(e^{j\omega})$, so as we increasing the value of maximum frequency taken into consideration, the frequency of $x[n]$ also increases & so, we see a compression in the graph of $x[n]$ as ω_c increases.

c.) We see that when $\omega_1 = \frac{\pi}{10}$ & $\omega_2 = \frac{\pi}{2}$, the range of frequencies over which we are integrating increases when compared to $\omega_1 = \frac{\pi}{8}$ & $\omega_2 = \frac{\pi}{4}$ & so, as mentioned earlier, frequency of $x[n]$ increases & so, the compression of $x[n]$ is observed.