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Report

1.) $h_{LPF}[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_{LPF}(e^{j\omega}) \cdot e^{j\omega n} \cdot d\omega$

$$= \frac{1}{2\pi} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} e^{j\omega n} \cdot d\omega$$

$$= \frac{1}{\cancel{2}\pi} \times \frac{1}{\cancel{j}n} \times \cancel{2j} \sin \frac{\pi}{6} n$$

$$\boxed{h_{LPF}[n] = \frac{\sin\left(\frac{\pi}{6}\right) n}{\pi n}}$$

$$h_d[n] = \frac{1}{2\pi} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} e^{-j\omega n_c} \cdot e^{j\omega n} \cdot d\omega$$

$$= \frac{1}{\cancel{2}\pi} \times \frac{1}{\cancel{j}(n-n_c)} \cancel{2j} \sin \frac{\pi}{6}(n-n_c)$$

$$h_d[n] = \frac{\sin\left[\frac{\pi}{6}(n-n_c)\right]}{\pi(n-n_c)}$$

$$\boxed{h_d[n] = \frac{\sin\left(\frac{\pi}{6}(n-n_c)\right)}{\pi(n-n_c)}}$$

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b.) We see that the phase graph observed is linear. The graph has no curves & just slant lines ⇒ phase is linear.

c.) We see that here too, the phase graph observed is linear.

d.) We see that the transition band of the blackman windowed filter is bigger than the rectangular windowed filter.

~~Side lobes of the~~
Side lobe levels of the rectangular window are around -50 dB but in blackman window it is around -100 dB. So, side lobe levels of blackman window are lower than the rectangular window.

f.) From the magnitude graph, we see that $h_1[n]$ acts as a highpass filter.

$$h_1[n] = (-1)^n h[n]$$

$$h_{\cancel{f}}[n] \xrightarrow{DFT} H(\omega) = H(e^{j\omega})$$

$$(-1)^n h_{\cancel{f}}[n] \xrightarrow{DFT} \cancel{H}(e^{j\omega - \pi})$$

(freq shift)

so, the freq response shifts by π & so, we get a high pass filter.

8.2

c.) **In FIR filter:**

poles → 0

So, for a filter to be stable, $|z|=1$ must be within ROC. Here, there are no poles other than 0 so ROC is the entire plane including $z=1$ & so, it is stable. ($|z|=0$ not included)

Also, for it to be causal, ROC must be outside a circle including ∞. Here ROC is $|z|>0$, so it is causal too.

In IIR Filter

poles : $r_0 e^{j\omega_0}$, $r_0 e^{-j\omega_0}$

zeroes: $e^{j\omega_0}$, $e^{-j\omega_0}$.

Now, ROC is: $|z| > |r_0 e^{j\omega_0}|$ & $|z| > |r_0 e^{-j\omega_0}|$

$|z| > |r_0|$ $\qquad$ $|z| > |r_0|$

as $r_0 < 1$, $|z|=1$ is included in the ROC & so, the filter is stable. also, $|z|=\infty$ is also included & so, filter is causal too.

d.) We see that as we $\uparrow r_0$, the effects are:

→ Magnitude response: transition band decreases & it almost allows all freq to pass except $\omega = \frac{\pi}{4}$

→ Phase response:

Phase response almost 0 for all values except $\omega_0 = \frac{\pi}{4}$. at this value, phase goes to $\pm\frac{\pi}{2}$

→ Pole Zero plot:

We see that as $r_0 \uparrow$, poles and zeroes get closer to each other on $|Z| = 1$

→ Impulse response: The impulse response is like a sinc wave for $r_0 = 0.5$ but as $r \uparrow$, the impulse response almost tends to '0' everywhere except at '0' frequency. so, it resembles $\delta[n]$ more as $r_0 \uparrow$.

f.) $F_S = 8192\ Hz$

f_0 = freq of sin = 1024

so, in notch filter $w = \frac{\pi}{4}$ is removed, so in freq domain, f removed = $f_s \times \left(\frac{1}{\frac{2\pi}{w}}\right)$

$$f_{rem} = \frac{8192 \times \cancel{\pi}}{4 \times 2\cancel{\pi}} = \frac{\cancel{8192}^{1024}}{\cancel{8}} = 1024$$

So, it must remove 1024 Hz freq which is nothing but freq of sin wave. So, it must give back our original signal before adding the sin wave. So, when we observe output, we see that the noisy sin is removed in both FIR & IIR but IIR is a more ideal filter, so output more accurately matches the input than the FIR filter.

8.3)

c.) we see that the responses of the filter designed here match those of 8.1.

d.) The comparison of both filters:

Equiripple	Least squares
→ In stop band, ripples are of equal level in the magnitude response.	→ In stop band, ripples decay a bit in mag. response

→ Position of zeroes inside & outside of $|z|=1$ are different

→ In both, stopband ripples get wider after 0.5

→ All frequencies until 0.5 are passed

→ Both have linear phase till freq = 0.5

→ Both have same number of poles and zeroes

→ position of poles is the same (50 poles at 0)

→ position of zeroes on $|z|=1$ are also same

→ Almost same impulse response