AI1110 Assignment 1

Indian Institute of Technology, Hyderabad

EE22BTECH11206

Chittampalli Ananya

Question: 12.13.5.8 Suppose X has a binomial distribution . Show that X = 3 is the most likely outcome.

(Hint : $\Pr(X = 3)$ is the maximum among all $\Pr(x_i)$,x_i = 0,1,2,3,4,5,6)

Solution: Given X is any random variable whose binomial distribution is $B\left\{6, \frac{1}{2}\right\}$.

Here, 6 is the number of trials and the probability of success is $\frac{1}{2}$

Thus, $n = 6$ and $p = \frac{1}{2}$

q=$1 - p$

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$

Thus, $\Pr(X = x)$=${}^{n}C_x q^{n-x} p^x$, where $x = 0, 1, 2, 3, ...n$

$= {}^{6}C_x \left\{\frac{1}{2}\right\}^{n-x} \left\{\frac{1}{2}\right\}^x$

$= {}^{6}C_x \left\{\frac{1}{2}\right\}^6$

It can be clearly observed that $P(X = x)$ will be maximum if ${}^{6}C_x$ will be maximum.

The maximum of ${}^{6}C_k$ can be found by calculating $\frac{{}^{6}C_k}{{}^{6}C_{k+1}}$. The value of k for which the ratio $\frac{{}^{6}C_k}{{}^{6}C_{k+1}}$ first becomes greater than 1, then for that value of k, ${}^{6}C_k$ is maximum.

$\frac{{}^{6}C_0}{{}^{6}C_1} = \frac{1}{6} = 0.167$

$\frac{{}^{6}C_1}{{}^{6}C_2} = \frac{6}{15} = \frac{2}{5} = 0.4$

$\frac{{}^{6}C_2}{{}^{6}C_3} = \frac{15}{20} = \frac{3}{4} = 0.75$

$\frac{{}^{6}C_3}{{}^{6}C_4} = \frac{20}{15} = \frac{4}{3} = 1.34$

$\frac{{}^{6}C_4}{{}^{6}C_5} = \frac{15}{6} = \frac{5}{2} = 2.5$

$\frac{{}^{6}C_5}{{}^{6}C_6} = \frac{6}{1} = 6$

From the above, we can say that the value of $\frac{{}^{6}C_3}{{}^{6}C_4} = \frac{20}{15} = \frac{4}{3}$ becomes 1 for the first time. Hence we can say that ${}^{6}C_3$ is the maximum.

So, $X = 3$ is the most likely outcome.