

# CS553 Cryptography

BitBees

## Question 7

### Number Theory with SageMath

We have implemented the Euclidean GCD and Extended Euclidean GCD on Sage. Please refer to `q7_part1.sage` for the implementation of the same.

The first output simply is the modular inverse of 11 in  $\mathbb{Z}_4$

The second output, separated by `*` is the number of invertible elements in  $\mathbb{Z}_4$  along with checking for each element in  $\mathbb{Z}_4$ .

These inputs can be customized according to your choice.

The implementation using inbuilt functions can be found in `q7_part2.sage`. A more efficient way to calculate the number of invertible elements in is by using Eulers Totient function.

$$\Phi(n) = |\{1 \leq a < n \mid (a, n) = 1\}|$$

And  $\Phi(p) = p - 1$  for prime numbers, we can factorize our given  $m$  to its prime factors.

$$\Phi(mn) = \Phi(m)\Phi(n)$$

$$\Phi(35) = \Phi(7)\Phi(5) = 6 \cdot 4 = 24$$

Implementation of this can be found in `q7_part3.sage`