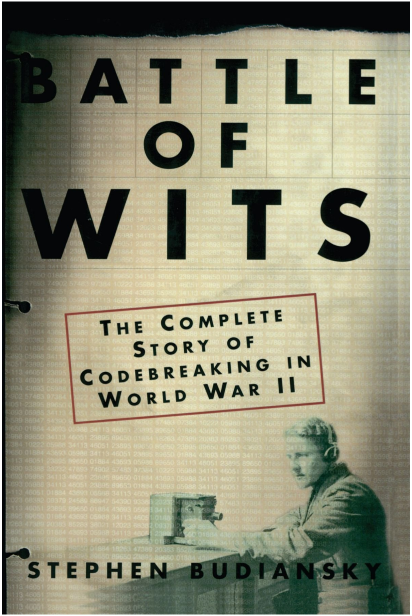



# CS 553
## CRYPTOGRAPHY

## Lecture 6
Block Cipher Cryptanalysis

Instructor
Dr. Dhiman Saha

Image Source: Google

# Part III
# Block Cipher Cryptanalysis

How to break one?

Modeling the role of Eve

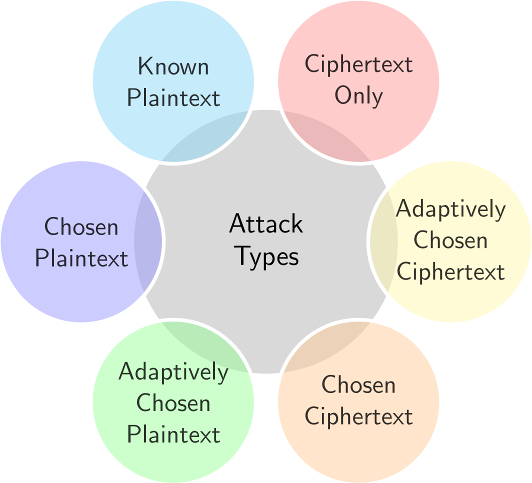
## Assumption (Oracle Access)

Assume cryptanalyst has access to black-box implementing block cipher with secret key $K$

## Aim of Cryptanalyst

- Find key $K$, or
- Find $(m, c)$ such that $\mathcal{E}_K(m) = c$ for unknown $K$, or
- Distinguish member of block cipher from randomly chosen permutation

- Modeling the power of the adversary (Eve)
- Based on the type of data required



Brute-Force → Exhaustive key-search (try all keys, one by one)

A good block cipher is one for which the **best attack** is an exhaustive search.

- Only protection is key-size

| $k$ (bits) | Search-time (operations) | Remarks on Security Level (Present Day) |
|---|---|---|
| 40 | $2^{40}$ | Easy to break |
| 64 | $2^{64}$ | Practical to break |
| 80 | $2^{80}$ | Currently infeasible |
| 128 | $2^{128}$ | Very strong |
| 256 | $2^{256}$ | Exceptionally strong |

Table: Security offered by different key lengths

**Rely on specific properties of the block-cipher**

- Differential Attacks
- Linear Attacks
- Integral Attacks
- Related Key Attacks
- Rebound Attacks
- Boomerang Attacks
- Variants

Today's Focus: **Differential Attacks**

# Differential Cryptanalysis (DC)

- Differential?
- Notion of difference of inputs

### Primary intuition

To study the propagation of differences through an SPN network focusing on the properties of the Sbox

- Trace differences of pairs of plaintexts in the decryption process.
- Deduce information about the key

# Differential Cryptanalysis (DC)

- Differential?
- Notion of difference of inputs

### Primary intuition

To study the propagation of differences through an SPN network focusing on the properties of the Sbox

- Trace differences of pairs of plaintexts in the decryption process.
- Deduce information about the key

# DC

- The discovery is generally attributed to Eli Biham and Adi Shamir in the late 1980s.
- However, in 1994, IBM claimed that DC was known to IBM as early as 1974.
- Within IBM, it was known as the ”T-attack or ”Tickle attack.
- Invented to break DES, did not succeed though
- A chosen plaintext attack.
- Applicable to many iterated block ciphers.

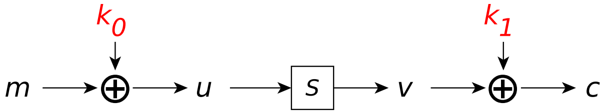
Resistance against DC a prerequisite for present-day block cipher proposals.

- Sypher001 encrypts 4 bits with two 4 bit keys

## S-box

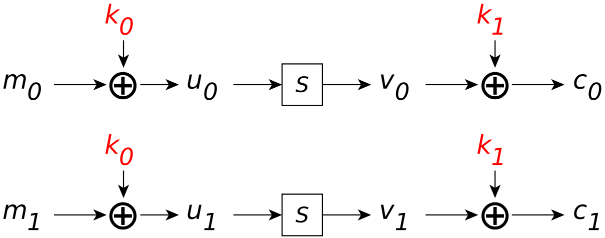
| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | a | b | c | d | e | f |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $S(x)$ | 6 | 4 | c | 5 | 0 | 7 | 2 | e | 1 | f | 3 | d | 8 | a | 9 | b |

## Encryption

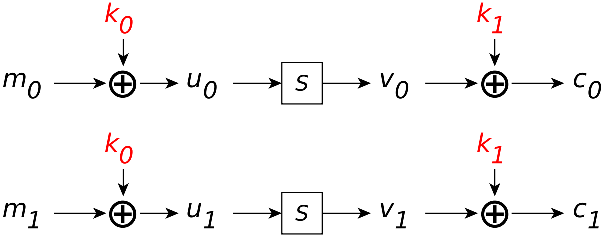


# DC of Sypher001

Assume we are given the encryptions of two messages $m_0, m_1$.



# DC of Sypher001



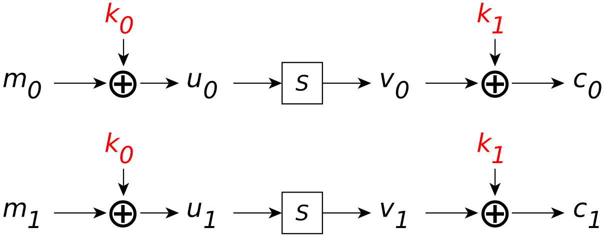

## First Observation: Key Annihilation

We Know:

$$u_0 \oplus u_1 = (m_0 \oplus k_0) \oplus (m_1 \oplus k_0) = m_0 \oplus m_1$$

even though we do not know $k_0$

# DC of Sypher001

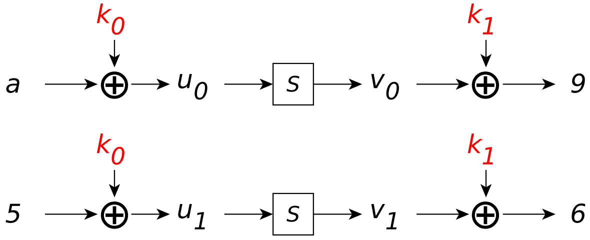



## Strategy

- Guess $k_1$
- Compute $v_0'$ and $v_1'$
- Compute $u_0' = S[v_0']^{-1}$ and $u_1' = S[v_1']^{-1}$
- Verify if $u_0 \oplus u_1 = u_0' \oplus u_1'$
- If not, then key guess was incorrect!

# Example

- Given $m_0 = a, m_1 = 5$ and $c_0 = 9, c_1 = 6$




- Computer $u_0 \oplus u_1 = a \oplus 5 = f$
- Guess $k_1$

| $k_1$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | a | b | c | d | e | f |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $u'_0 \oplus u'_1$ | e | b | e | e | d | 8 | d | **f** | **f** | d | 8 | d | e | e | b | e |

- Compare $u_0 \oplus u_1$ and $u'_0 \oplus u'_1$
- Only candidates for $k_1$ are $7, 8$

# What did we do?

## Take Away

- We know things about **differences** even though we do not know the individual values.
- We make a guess for the key and verify it by computing a bit backwards

## Is it enough if we have a good guess for the difference?

Probably not, lets see out next cipher

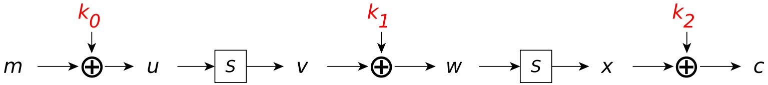
# Second Target 

- Sypher002 encrypts 4 bits with **three** 4 bit keys

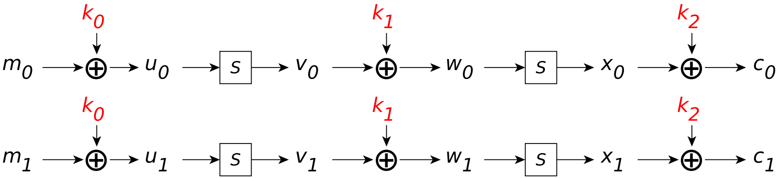
## S-box

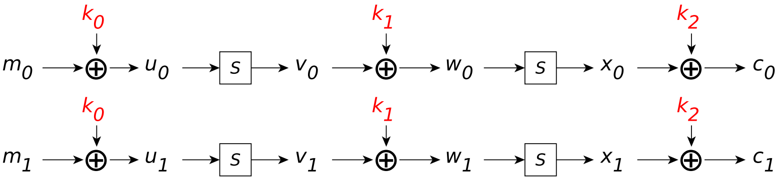
| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | a | b | c | d | e | f |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $S(x)$ | 6 | 4 | c | 5 | 0 | 7 | 2 | e | 1 | f | 3 | d | 8 | a | 9 | b |

## Encryption



Assume we are given the encryptions of two messages $m_0, m_1$.

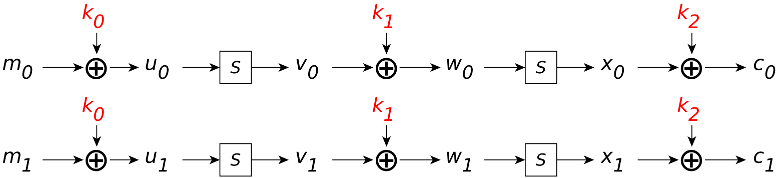





### We can compute (after guessing $k_2$)

- $u_0 \oplus u_1$
- $x_0'$ and $x_1'$
- $w_0'$ and $w_1'$
- $v_0' \oplus v_1' = w_0' \oplus w_1'$

But we still cannot check our guess for $k_2$




## More Powerful Attacker

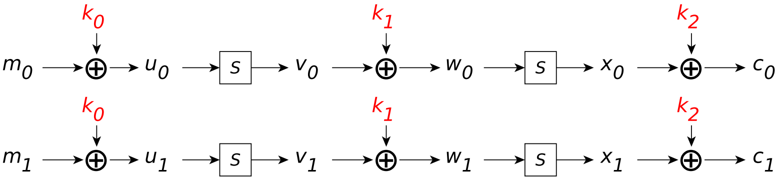
Now, we make it a **chosen plaintext attack**:

- Choose the starting difference

$$m_0 \oplus m_1 = u_0 \oplus u_1 = f$$

# The Influence of the Sbox






### We can compute (after guessing $k_2$)

- $u_0 \oplus u_1 = f$
- $v'_0 \oplus v'_1$

### Question

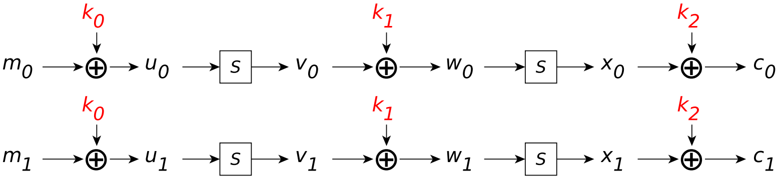
Is there anything we can say about $v_0 \oplus v_1$ given that $u_0 \oplus u_1 = f$?

| $u_0$ | $u_1 = u_0 \oplus f$ | $v_0 = S[u_0]$ | $v_1 = S[u_1]$ | $v_0 \oplus v_1$ |
|---|---|---|---|---|
| 0 | f | 6 | b | d |
| 1 | e | 4 | 9 | d |
| 2 | d | c | a | 6 |
| 3 | c | 5 | 8 | d |
| 4 | b | 0 | d | d |
| 5 | a | 7 | 3 | 4 |
| 6 | 9 | 2 | f | d |
| 7 | 8 | e | 1 | f |
| 8 | 7 | 1 | e | f |
| 9 | 6 | f | 2 | d |
| a | 5 | 3 | 7 | 4 |
| b | 4 | d | 0 | d |
| c | 3 | 8 | 5 | d |
| d | 2 | a | c | 6 |
| e | 1 | 9 | 4 | d |
| f | 0 | b | 6 | d |

### Observations

- The difference is unevenly distributed.
- Not all values occur.
- The difference $d$ occurs 10 out of 16 times.

Thus, we assume that $v_0 \oplus v_1 = d$ and this enables us to verify our guess for $k_2$ .




### We can compute (after guessing $k_2$)

- $u_0 \oplus u_1 = f$
- $v_0' \oplus v_1'$

Thus, we assume that $v_0 \oplus v_1 = d$ and this enables us to verify our guess for $k_2$ .

$$v_0 \oplus v_1 = d = v_0' \oplus v_1'$$

# What if the assumption is right/wrong?

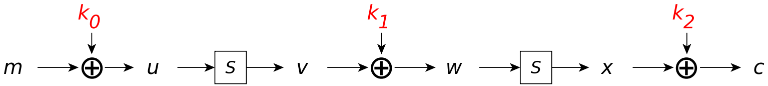
### Right

If the assumption is right, the right key is one of the possible candidates

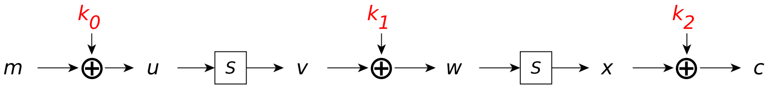
### Wrong

If the assumption is wrong, the right key might not be one of the possible candidates.

Still: If the assumption has a good probability (here : 10/16), the right key is a candidate more often than any wrong key.




- Initialize counters $T_i = 0$, one for each possible key $k_2$ .
- For each message/ciphertext pair do
  - For each guess $i$ for $k_2$ do
    - Compute $v_0' \oplus v_1'$
    - If $v_0' \oplus v_1' = d$ increase counter $T_i$
- Assume that the right key $k_2$ corresponds to the highest counter.




## Assumption

Assume that a wrong guess for $k_2$ gives a random value for $v_0 \oplus v_1$

This implies that after processing $t$ pairs we can expect

- The counter for the correct key is $\approx t \times \frac{10}{16}$
- The counter for the wrong key is $\approx t \times \frac{1}{16}$

### Observation

The attack was possible because for the input difference $f$ the output differences where highly unbalanced.

### Question

What happens for other input differences?

# The Difference Distribution Table

| in \ out | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | a | b | c | d | e | f |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 16 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 1 | - | - | 6 | - | - | - | - | 2 | - | 2 | - | - | 2 | - | 4 | - |
| 2 | - | 6 | 6 | - | - | - | - | - | - | 2 | 2 | - | - | - | - | - |
| 3 | - | - | - | 6 | - | 2 | - | - | 2 | - | - | - | 4 | - | 2 | - |
| 4 | - | - | - | 2 | - | 2 | 4 | - | - | 2 | 2 | 2 | - | - | 2 | - |
| 5 | - | 2 | 2 | - | 4 | - | - | 4 | 2 | - | - | 2 | - | - | - | - |
| 6 | - | - | 2 | - | 4 | - | - | 2 | 2 | - | 2 | 2 | 2 | - | - | - |
| 7 | - | - | - | - | - | 4 | 4 | - | 2 | 2 | 2 | 2 | - | - | - | - |
| 8 | - | - | - | - | - | 2 | - | 2 | 4 | - | - | 4 | - | 2 | - | 2 |
| 9 | - | 2 | - | - | - | 2 | 2 | 2 | - | 4 | 2 | - | - | - | - | 2 |
| a | - | - | - | - | 2 | 2 | - | - | - | 4 | 4 | - | 2 | 2 | - | - |
| b | - | - | - | 2 | 2 | - | 2 | 2 | 2 | - | - | 4 | - | - | 2 | - |
| c | - | 4 | - | 2 | - | 2 | - | - | 2 | - | - | - | - | - | 6 | - |
| d | - | - | - | - | - | - | 2 | 2 | - | - | - | - | 6 | 2 | - | 4 |
| e | - | 2 | - | 4 | 2 | - | - | - | - | - | 2 | - | - | - | - | 6 |
| f | - | - | - | - | 2 | - | 2 | - | - | - | - | - | - | 10 | - | 2 |

**How to interpret it?**

# Characteristics

## Definition

Characteristic Given an Sbox $S$, a pair $(\alpha, \beta)$ is called a *characteristic* with probability $p$, if the probability that two inputs with difference $\alpha$ provide outputs with difference $\beta$ equals $p$. This is denoted as

$$\alpha \xrightarrow{S} \beta$$

Examples for our Sbox

- $f \xrightarrow{S} d$ has probability $\frac{10}{16}$
- $d \xrightarrow{S} c$ has probability $\frac{6}{16}$
- $c \xrightarrow{S} a$ has probability 0: Impossible Characteristic