



Joan Daemen & Vincent Rijmen

CS 553

CRYPTOGRAPHY

Lecture 5

Block Ciphers

Instructor
Dr. Dhiman Saha

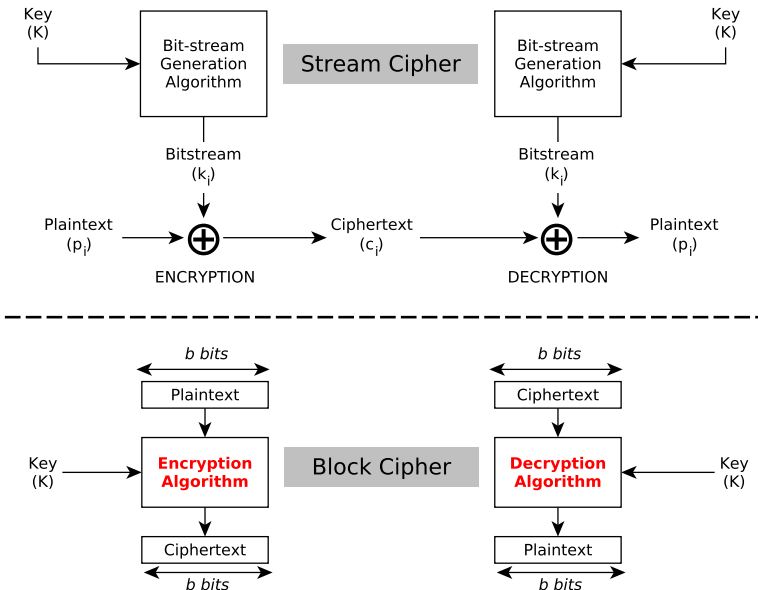
<u>Group</u>	<u>Time of Submission</u>	<u>Points Awarded</u>
BitBees	August 14, 12:20 AM	10
SPL Encrypted	August 14, 12:31 AM	7
Hex Brains	August 14, 12:47 AM	5
Bash Ciphers	August 14, 12:55 AM	-
TechHeist 3.0	August 14, 12:55 AM	-
Rook	August 14, 12:56 AM	-
Ping 999+	August 14, 01:00 AM	-
BrainFog	August 14, 01:06 AM	-
Bit by Bit	August 14, 01:07 AM	-
Kryptonian	August 14, 01:08 AM	-
..././cipher	August 14, 01:41 AM	-
Hope we "3" SDVV	August 14, 01:44 AM	-
SHA69	August 14, 03:56 AM	-
Cryptoducks	August 14, 11:35 AM	-
gugu gaga	August 17, 04:20 PM	-
Cipherbytes	August 17, 04:24 PM	-



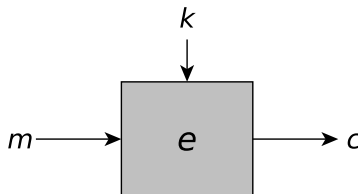
Alice and Bob use the **same** key for Encryption/Decryption

The Abstraction

Block Vs Stream




- ▶ Input block m
- ▶ Output block c
- ▶ Key k
- ▶ Block length n



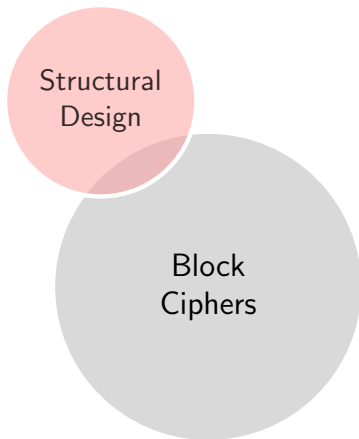
$$e : \{0, 1\}^n \times \{0, 1\}^{|k|} \rightarrow \{0, 1\}^n$$

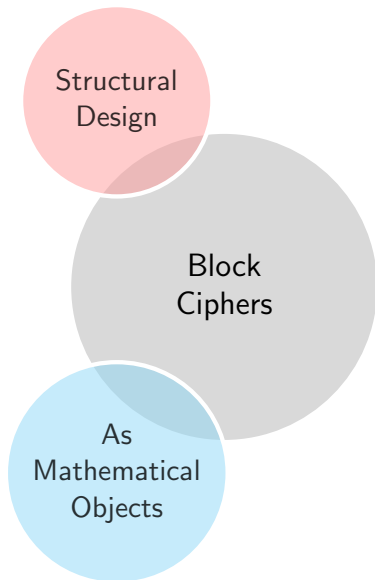
Desired

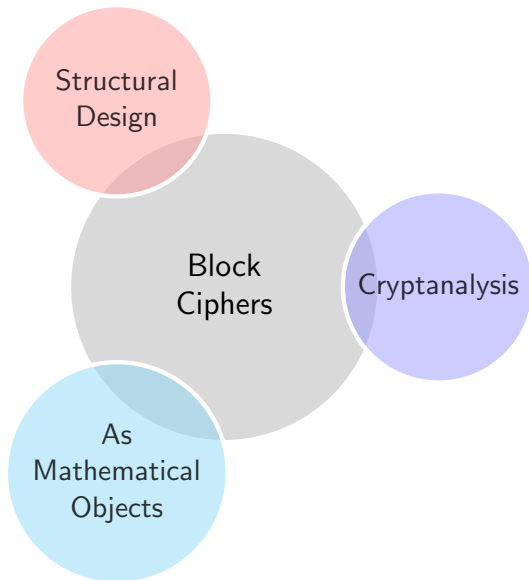
- ▶ Given k easy to encrypt and decrypt: efficiency
- ▶ Given m, c hard to compute k , such that $e_k(m) = c$
- ▶ One-way property with the key as the inversion trapdoor 
- ▶ $d(k, e(k, m_0)) = m_0$: deterministic decryption



Block
Ciphers







Part I

Inside a Block-Cipher


Is there a rule-of-thumb to design one?

The Structural Aspect

- ▶ Introduced by Shannon: “*Communication Theory of Secrecy Systems*” 1949 landmark paper
- ▶ Still most widely used principles in block cipher design
- ▶ Many interpretations: One by Massey

Confusion The ciphertext statistics should depend on the plaintext statistics in a manner too complicated to be exploited by the cryptanalyst.

Diffusion Each digit of the plaintext and each digit of the secret key should influence many digits of the ciphertext.


Block ciphers are designed to provide sufficient confusion and diffusion. 

How to get Confusion and Diffusion?

- ▶ Answer comes in the form of two very basic operations


Diffusion

Permutation (P)

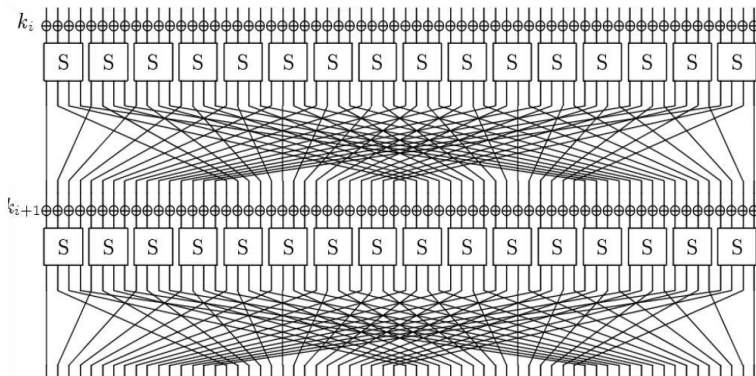
- ▶ Bit-level
- ▶ Byte-level
- ▶ Linear component 

Confusion

Substitution (S)

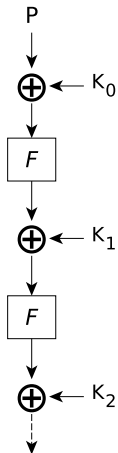
- ▶ S-box
- ▶ Look-up table
- ▶ Non-linear component 

- ▶ Block ciphers will contain some combination of S & P
- ▶ However, exact form of S & P may vary greatly




x	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
S[x]	c	5	6	b	9	0	a	d	3	e	f	8	4	7	1	2

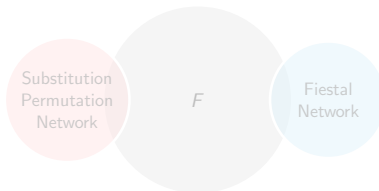
PRESENT Sbox



- ▶ What is the nature of function F ?
- ▶ Also known as the **Round Function**
- ▶ The design of F lies in the heart of block cipher design

Idea


F itself is weak, but F applied multiple times leads to a secure construction 

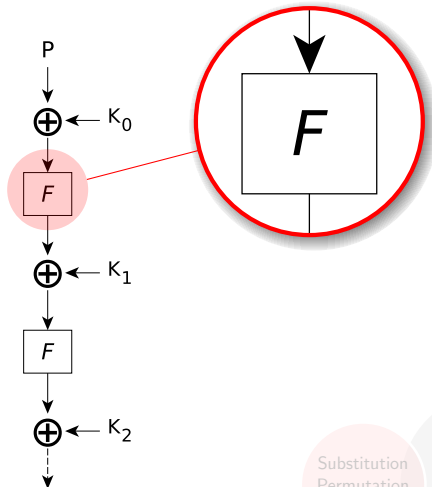


The Round Function

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Substitution
Permutation
Network


F

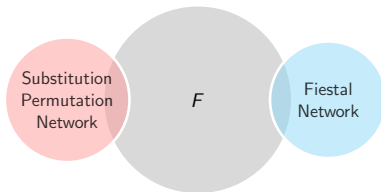
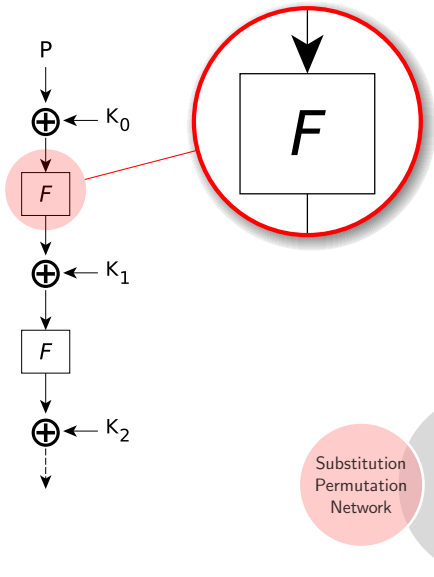
Fiestal
Network

How to design F

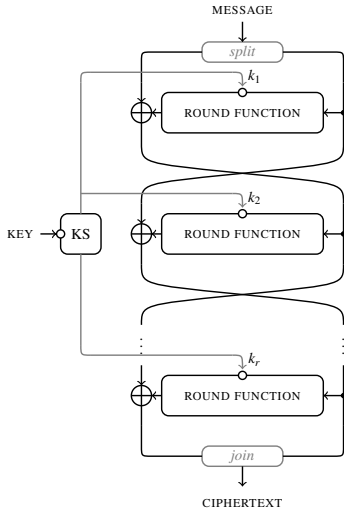
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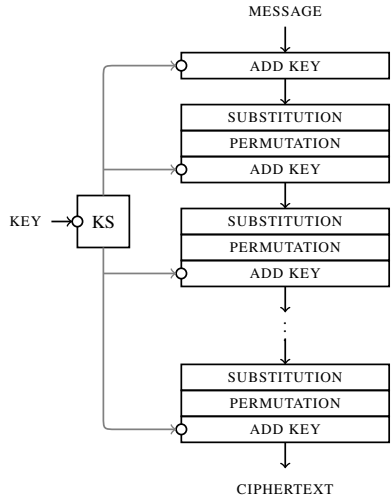
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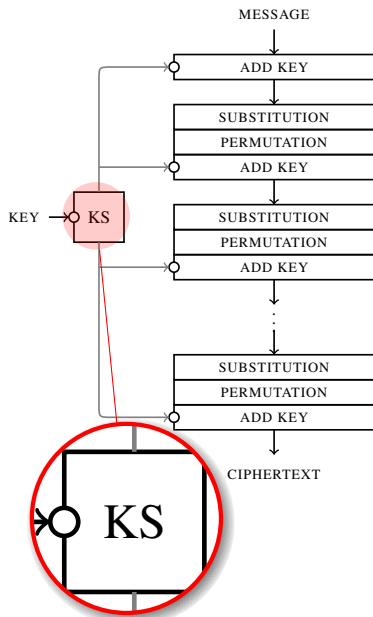
Block Cipher Design Techniques




Fiestal Structure - DES



Classical SPN - AES



Idea

Reusing the key-material
intermediately 

- ▶ The notion of Sub-keys
- ▶ Each round-key derived from the user-supplied master-key
- ▶ **Key-Scheduling/**
Key-Expansion algorithm
- ▶ Some key schedules are computationally lightweight
- ▶ Whereas others are very complex.

What if sub-keys are same?

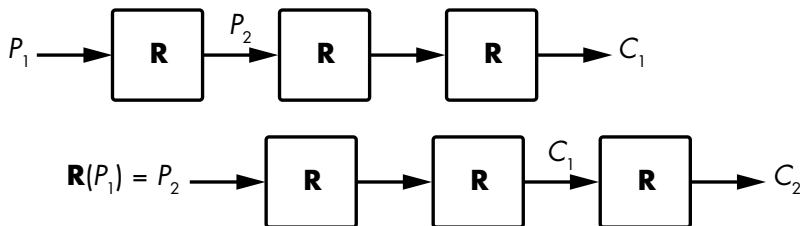
The Slide Attack




What if sub-keys are same?

The Slide Attack

When rounds are identical, the relation between the two plaintexts, $P_2 = \mathbf{R}(P_1)$, implies the relation $C_2 = \mathbf{R}(C_1)$



► Note: This is independent of the number of rounds. 

What if sub-keys are invertible?

- ▶ Invertible?
- ▶ Meaning we can derive Sub-key- n from Sub-key- $(n + 1)$

Implication

If an attacker can recover any round key K_i , he can also recover the main key K


- ▶ Typically, usefull for Side-Channel Attacks. 

Note

AES Key-schedule is invertible!!!

- ▶ A generic idea of a block cipher
- ▶ The iterated structure
- ▶ Common design techniques
- ▶ But its just processes b -bits at a time

Q: How do we deal with arbitrarily large amount of data?

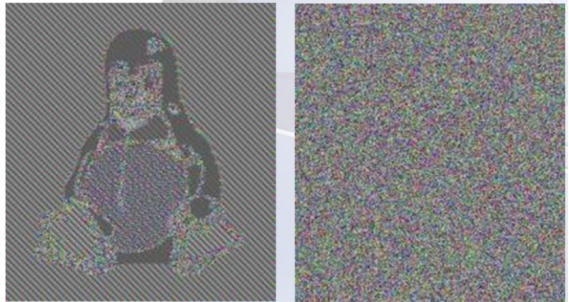
- ▶ Divide and Rule
- ▶ Repeatedly instantiate the cipher 
- ▶ Notion of **Padding**: size must be integral multiple of b

Q: Are the instantiations independent?

Determined by **Mode of Operation** 

The domain-extension algorithm

- ▶ Electronic Code Book - ECB
- ▶ Cipher Block Chaining - CBC
- ▶ Output Feedback Mode - OFB
- ▶ Cipher Feedback Mode - CFB
- ▶ Counter Mode - CTR



Stresses the need for randomization and dependency between instantiations

Part II

Block-Ciphers as Mathematical Objects

What do they represent?

Theoretical Aspect

Block ciphers as family of permutations

- ▶ A Block Cipher defines a map that takes plaintexts and keys to ciphertexts.

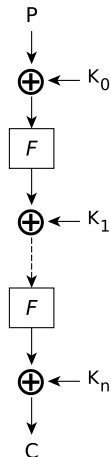
$$\mathcal{E} : \mathcal{P} \times \mathcal{K} \rightarrow \mathcal{C}$$


- ▶ fixing a key $K \in \mathcal{K}$ defines a permutation

$$\mathcal{E}_K : \mathcal{P} \rightarrow \mathcal{C}$$

- ▶ fixing all keys defines a set

$$E = \{\mathcal{E}_0, \mathcal{E}_1, \dots, \mathcal{E}_{|\mathcal{K}|-1}\}$$



Thus a block cipher is a way of generating a family of permutations and the family is indexed by a secret key K . 

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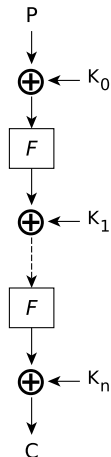
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Block cipher, n -bit blocks, k -bit key

- ▶ For a given key, a n -bit block cipher maps the set \mathcal{P} of 2^n n -bit inputs onto the same set of 2^n outputs:

$$P = \{\overbrace{0 \dots 00}^n, \overbrace{0 \dots 01}^n, \overbrace{0 \dots 10}^n, \dots, \overbrace{1 \dots 11}^n\}$$

- ▶ The block size n determines the space of all possible permutations that a block cipher might conceivably generate.
 - ▶ Number of n -bit permutations

$$(2^n)! \approx 2^{(n-1)2^n} \quad \text{Stirlings approximation}$$

- ▶ The key size k determines the number of permutations that are actually generated.
 - ▶ Number of n -bit permutations generated by block cipher

$$2^k$$

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The Problem and The Aim

- ▶ For typical values of n, k a block cipher provides only a tiny fraction of all the available permutations 
- ▶ Moreover, it will do so in a highly structured way.

For a good block cipher

A randomly chosen key is expected to “select a permutation seemingly at random from among all $2^{(n-1)2^n}$ possibilities.

- ▶ Finally, permutations from related keys should not in turn be related

Design Aim

Choose the 2^k permutations uniformly at random from the set of all $(2^n)!$ permutations 