CS2201 Worksheet 09

Work out the problems on your laptop/phone (there are several nice python interpreters that work on Android - I prefer qpython - but you may find some other app more to your taste. If you do not have access to any of these, write out our answers - scan them and upload (but this should be the absolute last resort!).

You will find link to the relevant videos and lecture slides on welearn.

Q 1) Write a program that uses trapezoidal integration rule to integrate

$$\int_0^1 \sin^2 x \, dx$$

with $N = 10, 20, \ldots, 100$ intervals. Plot the absolute value of the error ϵ (the exact value of the integral can be determined analytically here) as function of the step size h. Fit the error with the formula $\epsilon = Ah^n$ to estimate the order of error for this algorithm.

Q 2) Repeat the last problem, but this time use Simpson's $\frac{1}{3}$rd rule for the integral

$$\int_0^2 xe^{-x^2} dx$$

Q 3) Use the file **gauss32.dat** from welearn to write a program that calculates the integral

$$\int_a^b f(x) \, dx$$

by 32 point Gauss quadrature. Use your program to calculate the two integrals in the two problems above. Comment on the accuracy of the 32 point Gauss quadrature compared to the two other algorithms.

Q 4) Use the file **GK7_15.txt** from welearn to write a program that calcu-

lates and estimates the error in the integral

$$\int_a^b f(x)\,dx$$

by Gauss-Kronrod quadrature. Use it to determine, along with an error estimate, the integral

$$\int_0^1 e^{-x^2} dx$$

When you use the loadtxt program from the numpy module to read the file - remember that the first seven lines give the nodes and weights for seven point Gauss quadrature G_7 and lines nine till the end contain nodes and weights for Kronrod quadrature K_{15}. So you need to use the skiprows and the max_rows options (use help(np.loadtxt to know more).