# CS2201 Worksheet 07

*Create a directory called Worksheet07 under your home directory. Change to that directory and do all your work there. Open a file called Worksheet07.txt (using gedit) inside this directory where you will write down the answers to all questions asked in this worksheet. Once you are done, archive the directory and upload it to welearn.*

*You will find descriptions of the integration algorithms in the file uploaded to welearn.*

**Q 1)** By modifying the program **LR.py** that you will find on welearn write a program called **Euler.py** to use the Euler algorithm to solve the first order differential equation

$$\frac{dy}{dx} = \frac{2y - x}{y + 2x} \tag{1}$$

subject to $y(0) = 1$. The program should write the values of $x$ and $y$, for $x$ ranging from 0 to 20 in steps of 0.01 to a file. Use gnuplot to plot $y$ versus $x$.

The differential equation has a solution (you should be able to work this out) given by

$$\log\left(x^2 + y^2\right) + 4\tan^{-1}\left(\frac{y}{x}\right) = 2\pi \tag{2}$$

To see how well your code is doing plot the left hand side of (2) against $x$ using gnuplot.

Reduce $h$ to 0.001 and repeat. Comment on how, if at all the accuracy increases with a reduction of the step size.

**Q 2)** Repeat the previous problem, but this time with the modified Euler method.

**Q 3)** Solve the differential equation

$$\frac{dy}{dx} - \frac{y}{x} = x^3, \quad y(1) = 1$$

using the midpoint method from $x = 1$ to $x = 5$ with step size 0.01.

Plot the error in the solution (you should be able to solve the differential equation analytically) versus $x$.

Repeat for a step size of $h = 0.001$. Comment on the change in the error with step size.

**Q 4)** Solve the equation in problem 3 using the third order Runge Kutta method outlined below:

- To solve

  $$\frac{dy}{dx} = f(x, y)$$

  use

  $$y_{n+1} = y_n + \frac{h}{6}(p_n + 4q_n + r_n)$$

  where

  - $p_n = f(x_n, y_n)$
  - $q_n = f\left(x_n + \frac{h}{2},\, y_n + \frac{h}{2}p_n\right)$
  - $r_n = f(x_n + h,\, y_n + h(2q_n - p_n))$

Use $h = 0.1$ and 0.05. What can you say about the order of global error in this method from your results?