Project Quantum: Portfolio Optimization

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This is a *Quantum Inspired Optimisation(QIO*) based on the *Modern Portfolio Theory.* It performs a multi objective portfolio optimization which involves selecting the best portfolio (asset distribution) out of the given set of assets such that the expected returns from the investment is maximised and the risk is minimised. A constraint on budget is implemented to limit the number of stocks selected.
The problem definition is transformed mathematically so that it can be quantised, or made compatible with quantum computing. The quantised problem makes use of Microsoft Azure's Quantum Inspired Optimizer and is solved using the *Parallel Tempering* solver.
We compare our model with the classical solver and show that QIO converges to a better optima and therefore finds a better solution in lesser time.

Keywords: quantum inspired optimisation, azure quantum, portfolio optimisation, parallel tempering

Introduction

Quantum computing is a field that leverages the laws of quantum mechanics to solve specific problems, with enhanced performance that cannot be matched by classical computing.
Classical computers are made of bits which can hold 0 or 1, whereas quantum computers are made of qubits which can hold value 0 or 1 or a quantum superposition of these states. If there are 4 states, classical computers need 2 bits to store them as 00,01,10 or 11. Quantum computers can store any of the 4 states in a single bit as it can store 0,1 and their superpositions. This enables quantum mechanical effects such as interference, tunnelling, and entanglement, which in turn empower quantum algorithms for faster searching, better optimization, and greater security. This, when scaled, gives a significant increase of performance over classical.
Our problem is formulated as a Quantum Inspired Optimization(QIO), which is run on a classical computer but uses quantum techniques to achieve better results over traditional approaches.

Problem Definition

We have attempted to solve a Portfolio Optimization problem. This involves selection of a subset of stocks to be invested in, such that the return obtained from the investment is maximised while the risk taken is minimised. The number of shares of each stock to be invested in is also calculated.A budget constraint is also implemented so that the stocks are selected within a specified budget.

Method

Our objective function is a *Quadratic Unconstrained Binary Optimization* (QUBO). QUBO is an NP hard problem which can be solved through quantum annealing or parallel tempering. In our solution, we have submitted our model to the Parallel Tempering solver of Azure Quantum.

Parallel tempering is a computer simulation method typically used to find the lowest free energy state of a system of many interacting particles at low temperature. More specifically, parallel tempering (also known as replica exchange MCMC sampling), is a simulation method aimed at improving the dynamic properties of Monte Carlo method simulations of physical systems.

To submit the problem to Azure's Parallel Tempering solver, the objective function along with the constraints should be in the form of a list of-

Terms(c= <co-efficient>, indices = <list of indices>)

The risk term, return term and the penalty function are modelled as a list of these Terms and the linear combination of them is passed to the solver. The result obtained from the solver is in the form of a binary list, representing whether to pick that particular stock or not. Using this,the number of shares to invest in each stock is calculated by equally distributing the budget to all selected stocks. The final portfolio expected returns and portfolio risk is back-calculated using the result obtained from the solver.

Quantisation of the problem

The optimal portfolio allocation is found by maximising the expected portfolio fractional return and minimising the portfolio variance. This becomes a *mixed-integer quadratic programming problem.* The return vector **r** can be modelled as a random variable with mean **$E[r]=\mu$** and covariance **$E[(r-\mu)(r-\mu)T] = \Sigma$** .Therefore the return vector is calculated as the mean of the daily returns. The risk is modelled as the covariance matrix.
It follows that the portfolio return is also a random variable with mean **$E[R]= \mu Tx$** and variance **$Var[R] = xT\Sigma x$.**

The problem can be represented classically as -

$$\min_{x \epsilon \{0,1\}^n} \quad w^T A\, w - \mu^T w$$

where, $A = Cov\,(R_i, R_j)\,; \sum_{i=0}^{n} w_i = 1$, ***x*** - decision vector of n stocks, ***μ***- Expected fractional return vector, ***w***- weights of stocks in portfolio.

Since an investor might also have a constraint on budget which can affect the optimal portfolio available to them, the problem definition is subjected to a budget constraint. In order to quantize the problem, the weight of each stock in the portfolio is replaced by a decision vector. Consequently the budget is normalised to the stock appetite, or the number of stocks an investor can/wants to invest in.

To incorporate the constraint, a *penalty model* is used - a penalty is added to the objective function when the constraint is violated. This is done by using a soft constraint by making use of a slack variable. Finally, in an attempt to conceptualise the optimised portfolio, equal distribution of an actual given budget is done which is used to calculate the number of shares for each selected stock proportionally.

Thus the objective function used is given by the *QUBO problem's hamiltonian* as-

$$H_p = \frac{\gamma}{2}\, x^T A x - \mu^T x + \rho\,(B - J^T x)^2$$

$$x \epsilon \{0,1\}^n \ ; \ B \epsilon \{0, n\} \ ; \ A = Cov\,(R_{i\,,} R_j)$$

Here, ***Hp*** - Hamiltonian of the problem, **γ -** risk aversion index, ρ - Lagrange multiplier, ***J***-unit matrix, ***B***- normalised budget.

Results

The most optimal portfolio is determined and the expected monthly return is calculated. This monthly return percentage is compared with the actual return percentage. The actual return percentage is calculated using Bombay stock exchange data for the coming month.

The actual return percentage is 7.79% and our model predicts a return percentage of 6.43%. Therefore, the accuracy of our quantum inspired model is 82.541%.

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