Notes, Solutions etc.

for George E. Martin's "Counting - The Art of Enumerative Combinatorics"

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Contents

1. Elementary Enumeration

Combination Formula

The number of ways to choose r objects from n distinct objects is given by:

$$C(n,r) = \binom{n}{r} = \frac{n!}{(n-r)!r!}$$

Also written as ${}_nC_r$ or $\binom{n}{r}$.

Quickies - I

1. Addition principle: $6 + 8 = 14$ ways.

2. Same principle, although different fruits are indistinguishable in their own class: $1 + 1 = 2$ ways.

3. 3 ways.

4. 2 B's, 2 G's, or 1 B and 1 G: 3 ways.

5. 6 students total (3 boys + 3 girls) and we choose 2: $C(6,2) = 15$ ways.

6. 1 way, since any orange we do not pick is indistinguishable from any other orange that we did not pick in a different scenario.

7. $C(6,5) = 6$ ways.

8. $C(6,1) = 6$ ways.

9. We need to pick exactly 5 fruits. Let's consider picking i oranges and $(5 - i)$ apples where $0 \leq i \leq 5$:

- 0 oranges, 5 apples
- 1 orange, 4 apples
- 2 oranges, 3 apples
- 3 oranges, 2 apples
- 4 oranges, 1 apple
- 5 oranges, 0 apples

Total: 6 ways.

10. Counting the different ways to pick each fruit:

- For oranges: 0 to 9 (10 choices)
- For apples: 0 to 6 (7 choices)

Therefore the total choice combinations are $10 \times 7 = 70$ ways. But we have to substract the one case where we pick 0 of both fruits, so we have $70 - 1 = 69$ ways.

Permutation Formula

The number of ways to arrange r objects from n distinct objects (order matters) is given by:

$$P(n,r) = \frac{n!}{(n-r)!}$$

Also written as ${}_nC_r$ or $A(n, r)$.

Relationship between Permutation and Combination:

Since permutations consider order while combinations do not, we have:

$$P(n, r) = r! \times C(n, r)$$

This is because for each combination of r objects, there are $r!$ ways to arrange them.

Quickies - II

1. Multiplication principle: We pick 1 Latin book from 5 and 1 Greek book from 7: $\mathbf{5 \times 7 = 35}$ ways.

2. Each letter can be any of the 26 letters: $\mathbf{26^2}$ ways.

3. Since we can't repeat letters, we have 26 choices for the first letter and 25 for the second: $\mathbf{26 \times 25 = 650}$ ways.

4. $\mathbf{21 \times 5 = 105}$ ways.

5. $\mathbf{3 \times 8 = 24}$ ways.

6. $\boldsymbol{P(5, 2) = 20}$ ways. (We permute here since the arrangement matters)

7. $\boldsymbol{C(5, 2) = 10}$ ways.

8. $\mathbf{26^4}$ ways.

9. Pick any row (5 choices) and any column (7 choices): $\mathbf{5 \times 7 = 35}$ ways.

10. $\boldsymbol{m \times n}$ ways.

11. Coin has 2 outcomes, die has 6 outcomes: $\mathbf{2 \times 6 = 12}$ ways.

12. $\mathbf{2 \times 6 \times 52 = 624}$ ways.

13. $\mathbf{4!}$ ways (since each ace is distinct).

14. $\mathbf{13!}$ ways.

A Discussion Question

Question: How many ways can a pair of dice fall?

Solution for this depends on how the question means, or how we interpret, "ways":

Distinguishable Dice: If we can tell the dice apart (e.g., one red die and one blue die), then each die can show any of 6 faces independently. Using the multiplication principle: $\mathbf{6 \times 6 = 36}$ ways.

This counts (1,2) and (2,1) as different outcomes since the first number represents the red die and the second represents the blue die.

Indistinguishable Dice (Unordered Pairs): If the dice are identical and we only care about which numbers appear, then we're counting unordered pairs. The possible outcomes are: (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,2), (2,3), (2,4), (2,5), (2,6), (3,3), (3,4), (3,5), (3,6), (4,4), (4,5), (4,6), (5,5), (5,6), (6,6)

This gives us $\boldsymbol{\binom{6+2-1}{2} = \binom{7}{2} = 21}$ ways (stars and bars approach).

Possible Sums: If we only care about the sum of the dice, there are 11 possible sums: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

The “correct” answer depends entirely on what the question is really asking!