18.404 Recitation 3

Sept 18, 2020

Today's Topics

- CFL Pumping Lemma
  - What it is?
  - Why it works?
- Example: Proving Non-CFL Languages
  - { ww }
  - $\{ a^i b^j c^k \mid i > j > k \}$
- Definition: Turing Machines
  - Multitape equivalence, Nondeterministic equivalence
  - (reset, R) TM equivalent to regular TM
- Church-Turing Thesis
- Turing-Recognizable, Turing-Decidable Languages
  - A is T-decidable iff $\neg$A is T-decidable
  - **Not True:** A is T-recognizable iff $\neg$A is T-recognizable
- Recap

CFL Pumping Lemma (What it is)

- A tool to prove languages are non-context free
- CFLs are *always true* under the CFL Pumping Lemma
  - To prove non- CFL , need to find only **1** counter example
- Many more possible pumpings of a given string
  - Need to make sure all cases are not pump-able

Formal Statement

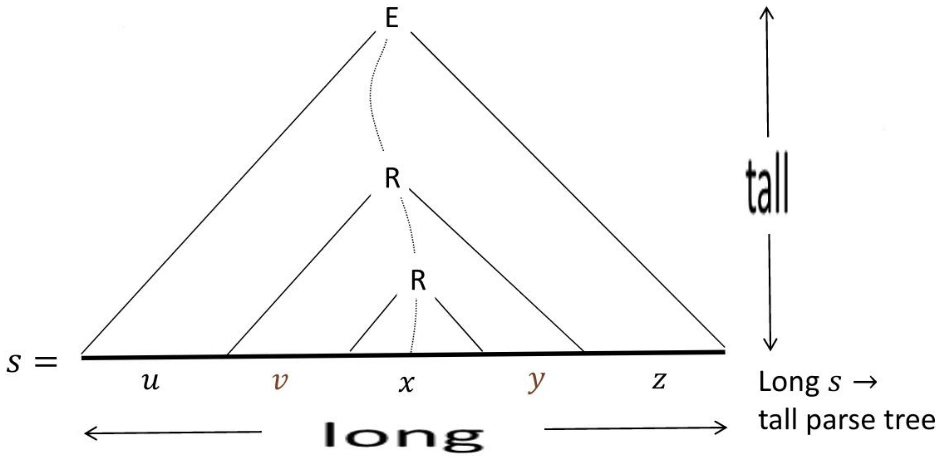
For every CFL, there exists a pumping number $p \geq 1$ such that every string of length at least p can be written as $s=uvxyz$ and satisfies:

- $(\forall n \geq 0)\ (uv^n xy^n z \in L)$
- $|vy| \geq 1$
- $|vxy| \leq p$

CFL Pumping Lemma (Why it works)

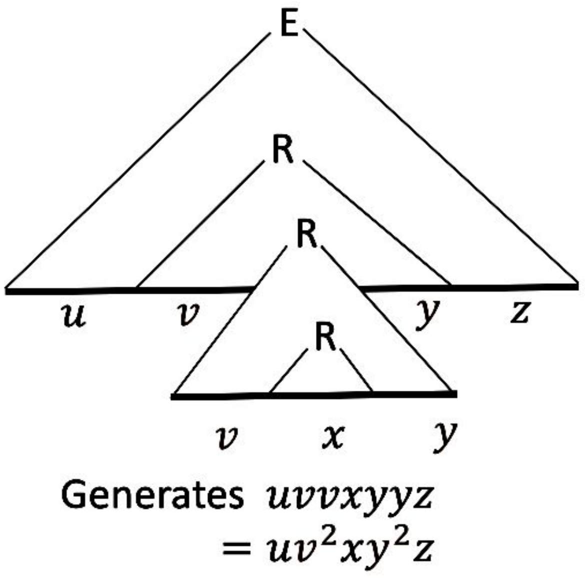
A long enough string *s* generated by a CFG will have repeated variables

- Loop Form: $R \rightarrow vRy$ where $|vy| \geq 1$
- Assume minimal CFG so never R $\rightarrow \ldots \rightarrow R$



CFL Pumping Lemma (Why it works)

Pumping Up

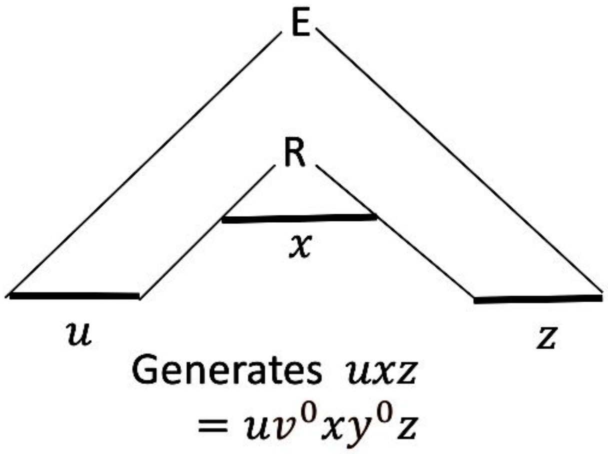


- $(\forall n \geq 0)\ (uv^n xy^n z \in L)$
- $|vy| \geq 1$
- $|vxy| <= p$

CFL Pumping Lemma (Why it works)

Pumping Down

- $(\forall n \geq 0)\ (uv^nxy^nz \in L)$
- $|vy| \geq 1$
- $|vxy| <= p$



Example: Proving Non-CFL Languages

Prove that { ww } is not a CFL

- $(\forall n \geq 0)\ (uv^nxy^nz \in L)$
- $|vy| \geq 1$
- $|vxy| <= p$

$$0^p 1^p 0^p 1^p$$

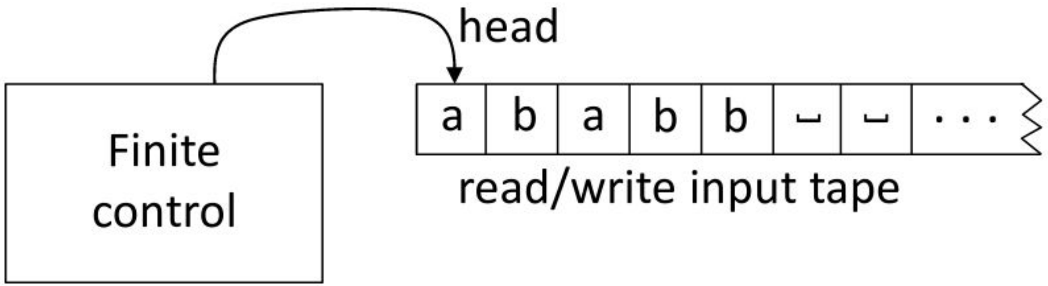
Example: Proving Non-CFL Languages

Prove that { $a^ib^jc^k$ | $i > j > k$ } is not a CFL

- $(\forall n \geq 0)\ (uv^nxy^nz \in L)$
- $|vy| \geq 1$
- $|vxy| <= p$

$$a^{p+2}b^{p+1}c^{p}$$

Definition: Turing Machines

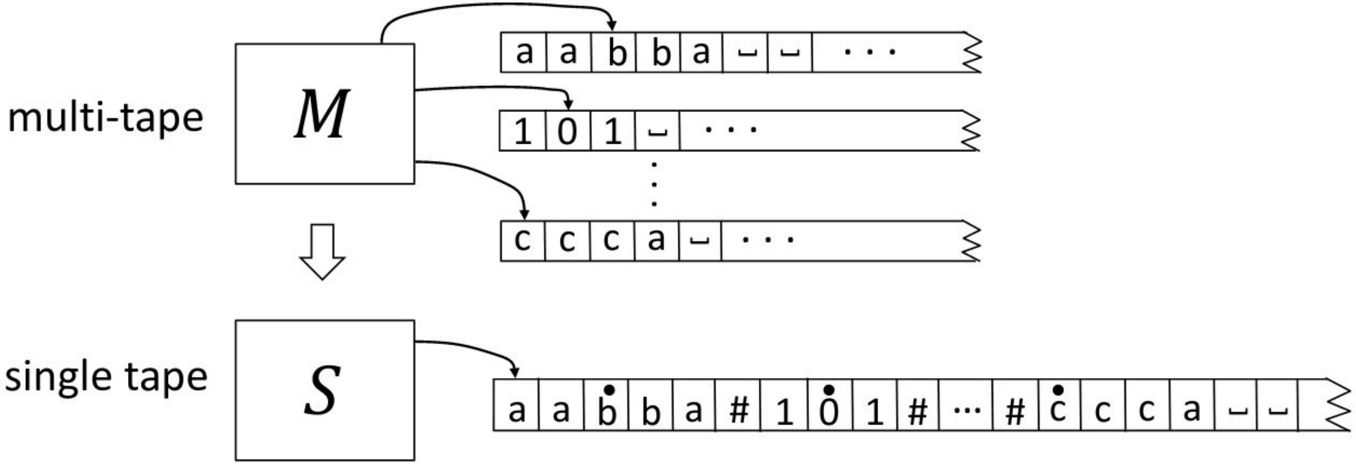


Big Upgrades

- Head can read and <u>write</u>
- Head is two-way (can move left and right)
- Tape is infinite to the right with blank spaces as scratch area
- Can accept/reject at any time, not limited to end of the input

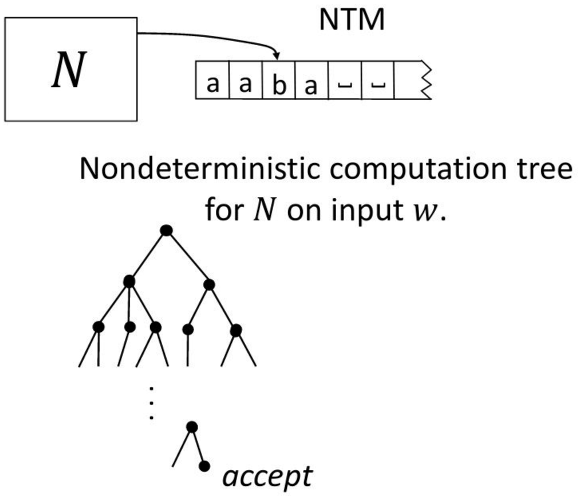
TM Equivalence

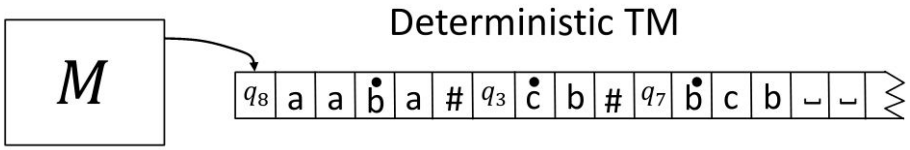
Multi-Tape vs Single Tape TM equivalence



TM Equivalence

Non-Deterministic vs Deterministic TM equivalence





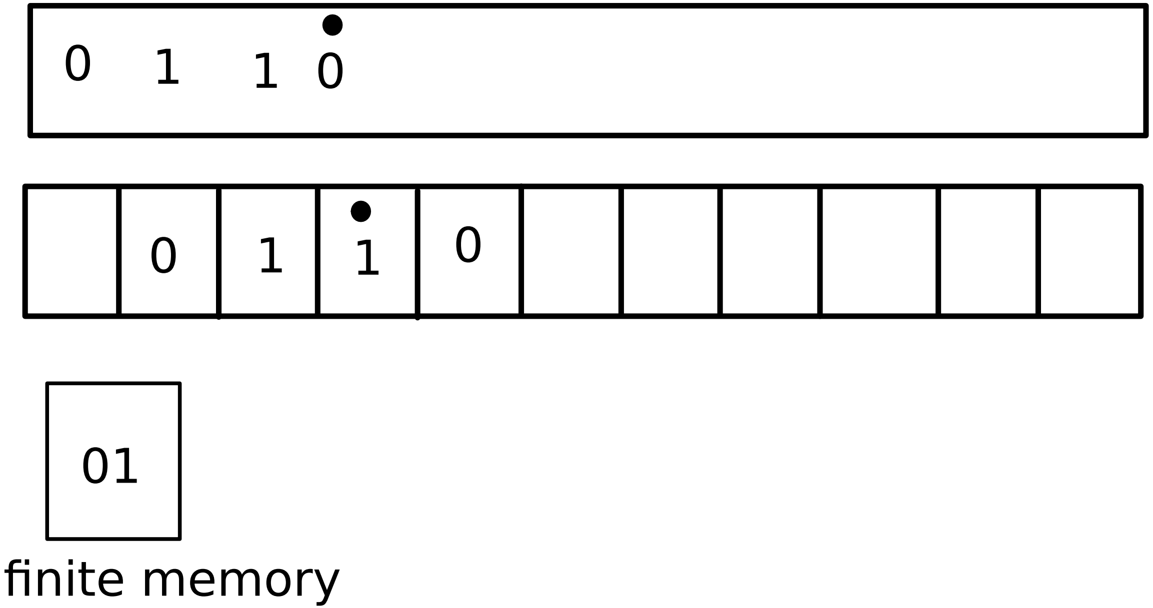
TM Equivalence

(reset, R) TM equivalent to regular TM

- (reset, R) TM ≡ a TM that can only move right or RESET to the beginning of the tape. It cannot move left space-by-space

Show how to get back left-move using only (reset, R)

TM Equivalence



Church-Turing Thesis

Algorithm	=	Turing machine
Intuitive		Formal

Instead of Turing machines,
can use any other "reasonable" model
of unrestricted computation:
λ-calculus, random access machine,
your favorite programming language, ...

Definition: TM-Recog, TM-Decidable

Language *A* is T-Recognizable language if exists TM M s.t. L(M)=*A*

- Corollary: M must accept only and all strings in *A*
  - Can loop or reject on all others

Language *B* is T-Decidable language if exists TM <u>decider</u> M s.t. L(M)=*B*

- Corollary: M must accept only and all strings in B
  - <u>Must</u> reject on all others

Example: T-Decidable

Show A is T-decidable iff ¬A is T-decidable

M' = "on input s
- Run TM M on s
- Return opposite of result"

Example: T-Recognizable

M

M'

Not True: Show A is T-recognizable iff ¬A is T-recognizable

D = "on input s
- Non det. run s on both M and M'
- return result of whichever terminates"

Recap