18.100B Problem Set 5

Due in class Monday, March 18. You may discuss the problems with other students, but you should write solutions entirely on your own.

1. Text, page 78, number 7.

2. Text, page 81, number 16.

3. Suppose that X and Y are metric spaces, $f: X \to Y$ is any function, and $p \in X$. Show (using Definition 4.5) that f is continuous at p if and only if the following condition is satisfied: for every sequence $\{p_n\}$ in X that converges to p, we have

$$\lim_{n \to \infty} f(p_n) = f(p)$$

in Y.

4. (If you've seen one convergent sequence, you've seen them all.) Let X be the metric space consisting of the points $\{1/n | n = 1, 2, 3, \dots\}$ and 0 in $\mathbb{R}$, with the distance function coming from $\mathbb{R}$. Let Y be any metric space, $\{p_n\}$ a sequence in Y, and p_0 any point of Y. Define a function $f: X \to Y$ by

$$f(1/n) = p_n, \qquad f(0) = p_0.$$

Show that f is continuous if and only if the sequence $\{p_n\}$ converges to p_0.