18.404/6.840 Lecture 19

Last time:

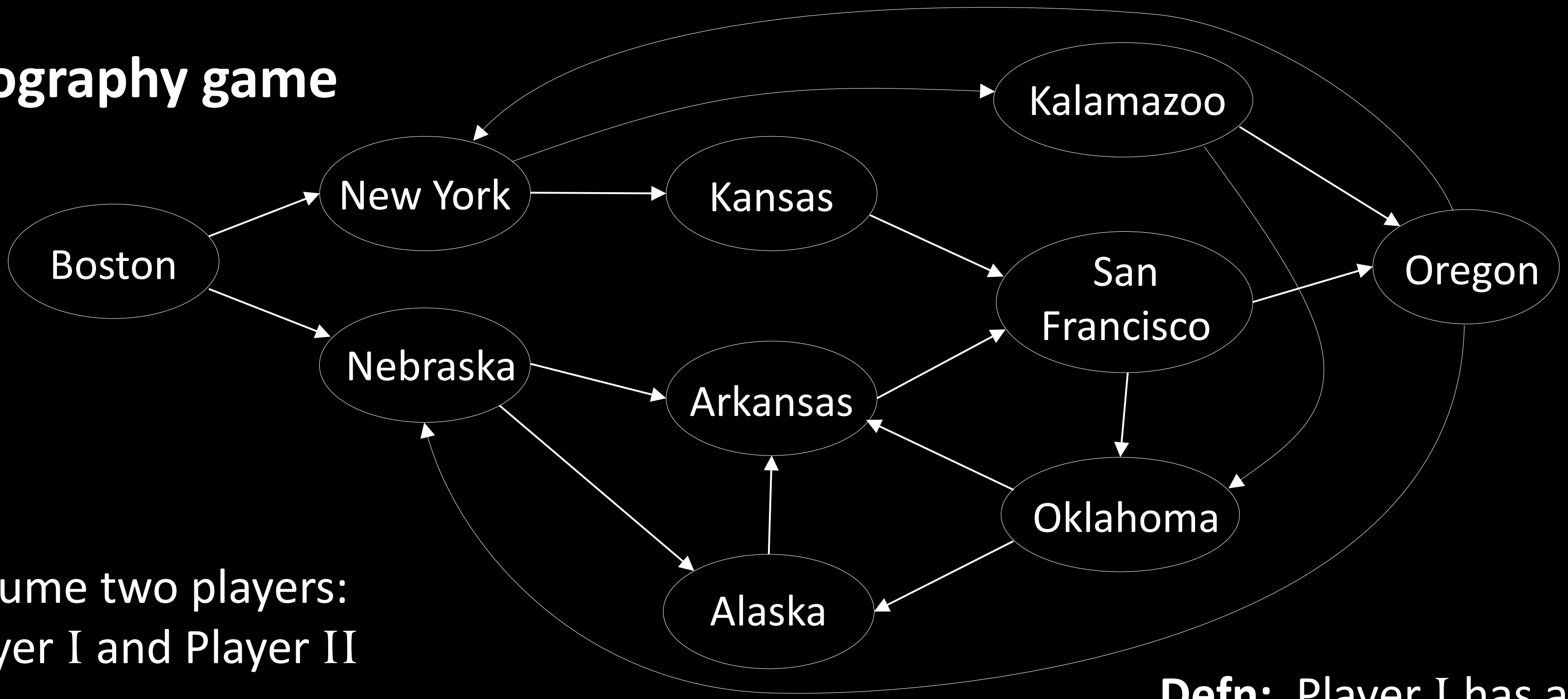
- Review PSPACE
- Savitch's Theorem: NSPACESPACE
- is PSPACE-complete

Today:

- Games and Quantifiers
- The Formula Game
- Generalized Geography is PSPACE-complete
- Logspace: L and NL

Games and Complexity

Geography game



Assume two players:
Player I and Player II

Players take turns picking places that start with the letter which ended the previous place. No repeats allowed. The first player stuck (= cannot move) loses.

Generalized Geography Game
Played on any directed graph.
Players take turns picking nodes that form a simple path.
The first player stuck loses.

Defn: Player I has a <u>forced win</u> in Generalized Geography on graph starting at node .

"forced win" also called a "winning strategy" means that the player will win if both players play optimally.

Theorem: is PSPACE-complete

Games and Quantifiers

The Formula Game

Given QBF

There are two Players “” and “”.

Player assigns values to the -quantified variables.

Player assigns values to the -quantified variables.

The players choose the values according to the order of the quantifiers in .

After all variables have been assigned values, we determine the winner:

Player wins if the assignment satisfies.

Player wins if not.

Claim: Player has a forced win in the formula game on iff is TRUE.

Therefore Player has a forced win on .

Next: show.

is PSPACE-complete

Theorem: is PSPACE-complete

Proof: 1) PSPACE (recursive algorithm, exercise)

2)

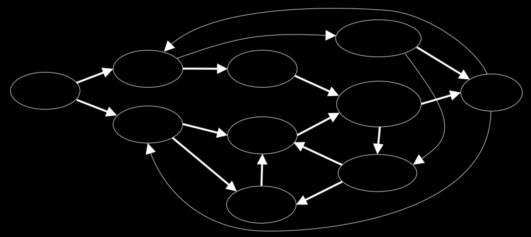
Give reduction from to .

Construct to mimic the formula game on .

has Players I and II

Player I plays role of -Player in . Ditto for Player II and the -Player.

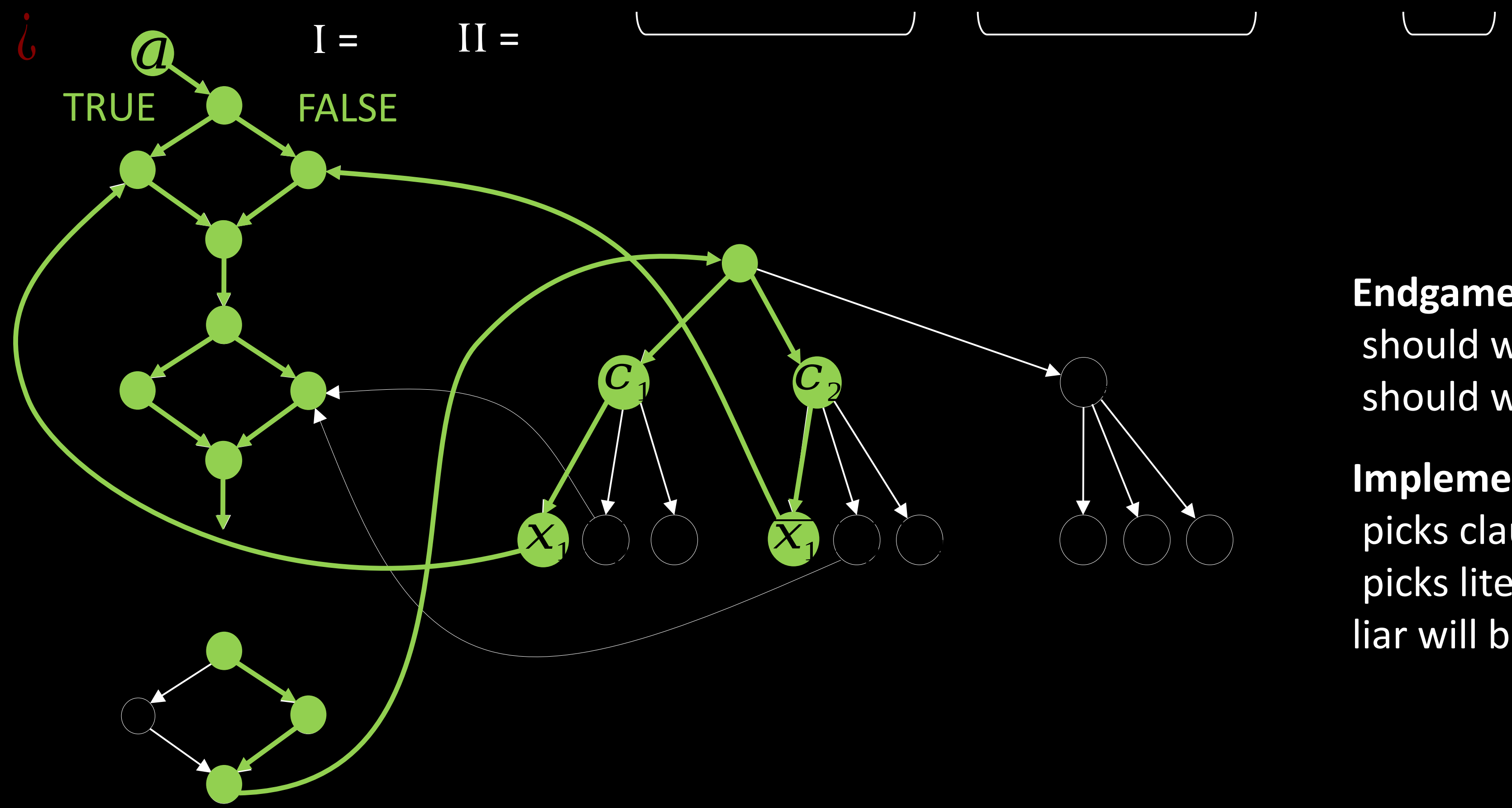
assume in cnf



Constructing the graph

Illustrate construction by example

Say



Endgame

should win if assignment satisfied all clauses

should win if some unsatisfied clause

Implementation

picks clause node claimed unsatisfied

picks literal node claimed to satisfy the clause

liar will be stuck

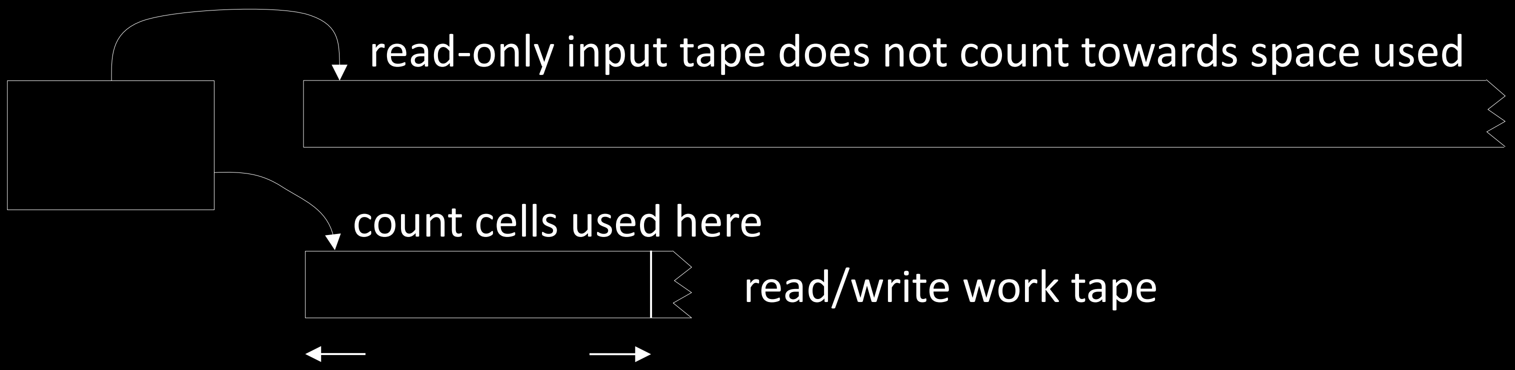

Coffee
Break

Log space

To define sublinear space computation, do not count input as part of space used.
Use 2-tape TM model with read-only input tape.

Defn: L = SPACE
NL = NSPACE

Log space can represent a constant number of pointers into the input.



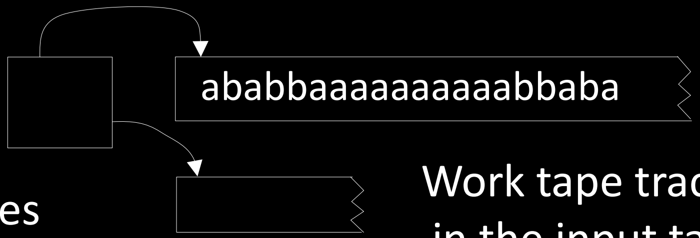
Examples

1. L

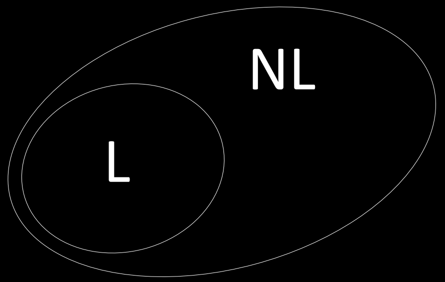
2. NL

Nondeterministically select the nodes

of a path connecting to .



Work tape tracks corresponding locations in the input tape.



L = NL? Unsolved

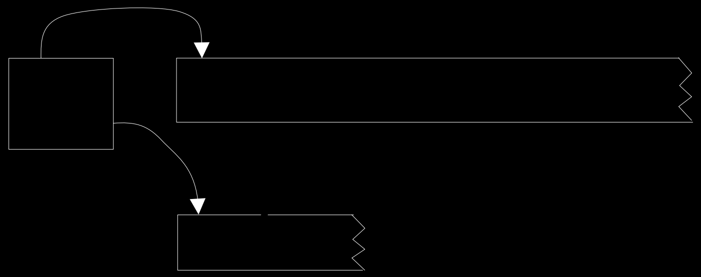
Log space properties

Theorem: L P

Proof: Say decides in space .

Defn: A configuration for on is where is a state,
and are the tape head positions, and is the tape contents.
The number of such configurations is for some .

Therefore runs in polynomial time.
Conclusion: P



Theorem: NL
Proof: Savitch's theorem works for log space

NL properties

Theorem: NL P

Proof: Say NTM decides in space .

Defn: The <u>configuration graph</u> for on has

nodes: all configurations for on

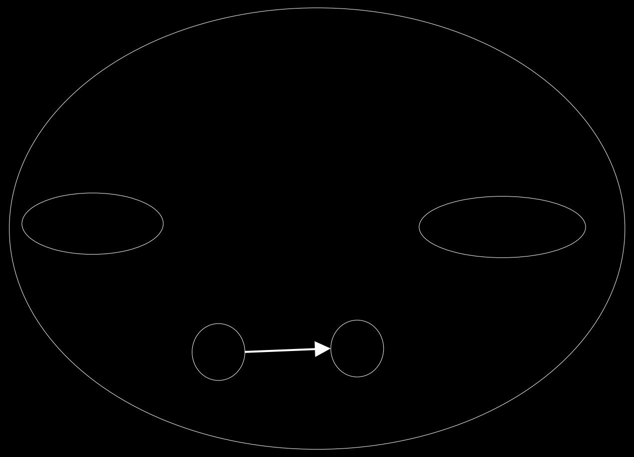
edges: edge from if can yield in 1 step.

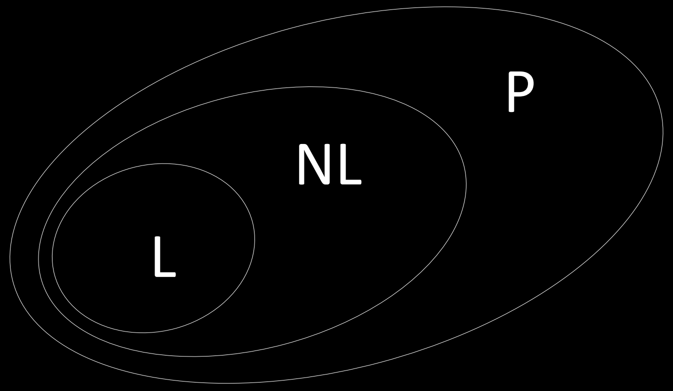
Claim: accepts iff the configuration graph has a path from to

Polynomial time algorithm for :

"On input

1. Construct the .
2. *Accept* if there is a path from to .
   Reject if not."





L = P? Unsolved

Quick review of today

1. The Formula Game
2. Generalized Geography is PSPACE-complete
3. Log space: L and NL
4. Configuration graph
5. NL P