18.404 Recitation 2

Sept 11, 2020

Today's Topics

- Pumping Lemma
  - What it is?
  - Why it works?
- Example: Proving Non-regular Languages
  - Pumping up: $\{ 0^n1^m0^n \}$
  - Pumping down: $\{ 0^i1^j \mid i \geq j \}$
  - Pumping Lemma with Closure Properties: { w | w ≠ number of 0s and 1s }
- Context Free Languages
  - Designing a PDA for: $\{ 0^n1^m0^n \}$
  - Designing a CFG for: $\{ 0^n1^m0^n \}$
  - Converting CFG to PDA
  - CFL Closure Properties
- Recap

Pumping Lemma (What it is)

- A tool to prove languages are non-regular
- Regular languages are *always true* under the Pumping Lemma
  - To prove non-regular, need to find only **1** counter example

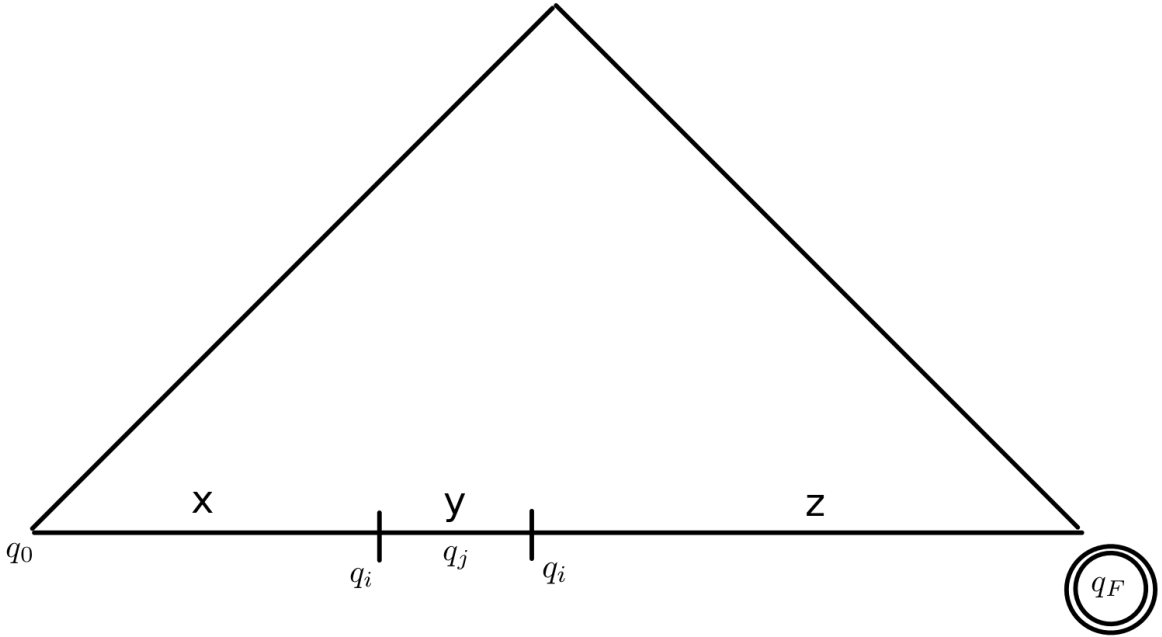
Formal Statement

For every regular language, there exists a pumping number $p \geq 1$ such that every string of length at least p can be written as $w=xyz$ and satisfies:

- $|y| \geq 1$
- $|xy| \leq p$
- $(\forall n \geq 0)\ (xy^n z \in L)$

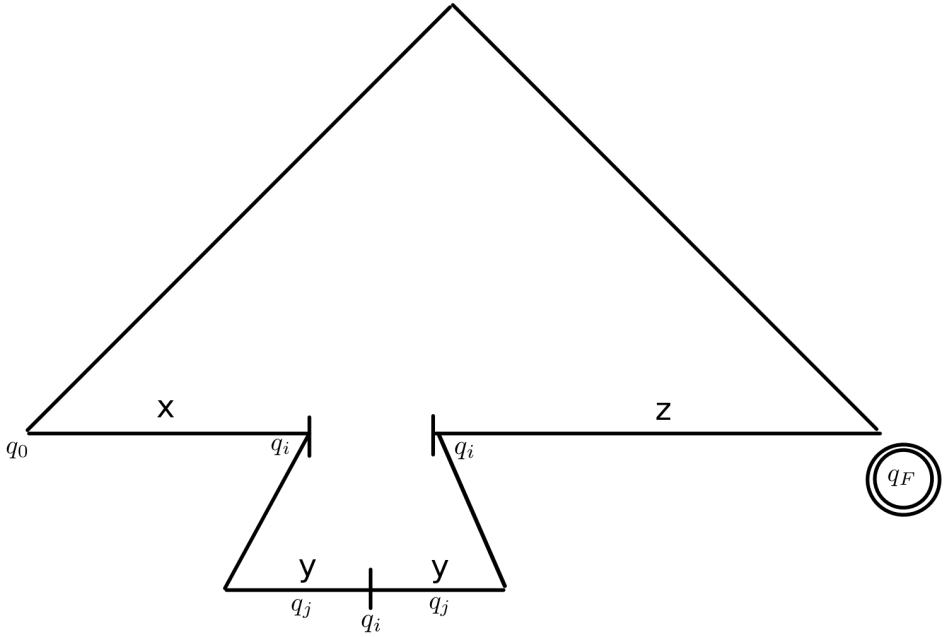
Pumping Lemma (Why it works)

Un-pumped String



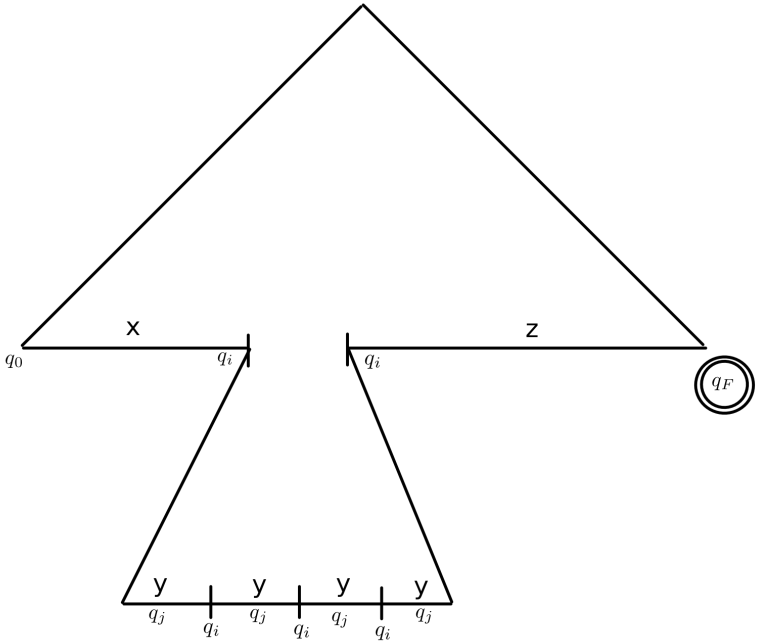
Pumping Lemma (Why it works)

Once Pumped Up String



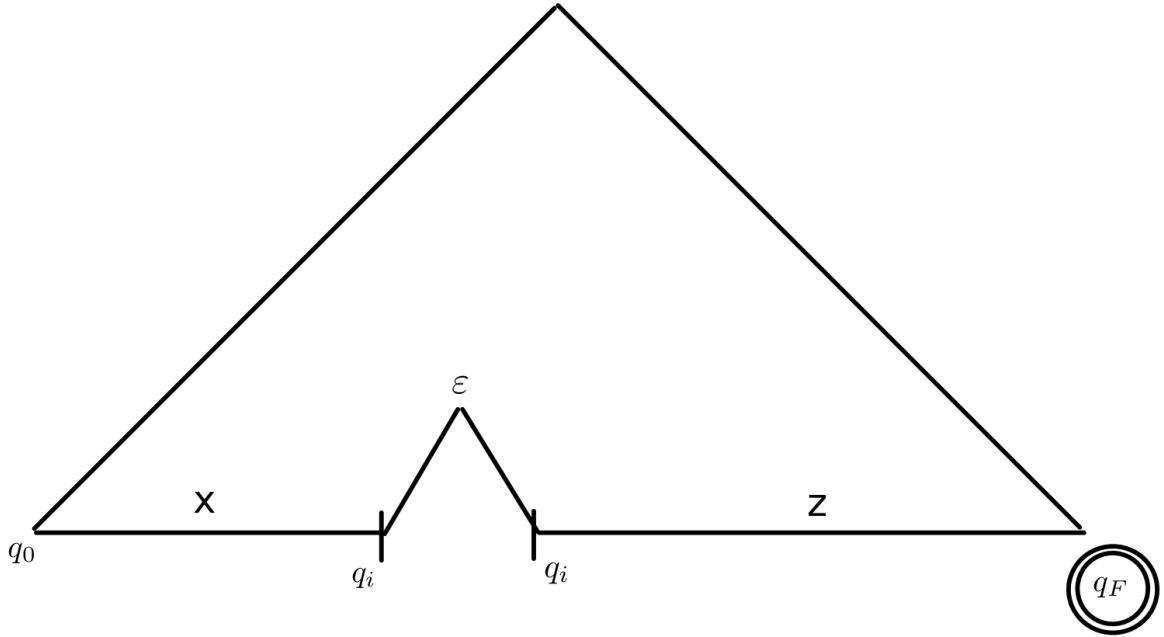
Pumping Lemma (Why it works)

Pumped Up String



Pumping Lemma (Why it works)

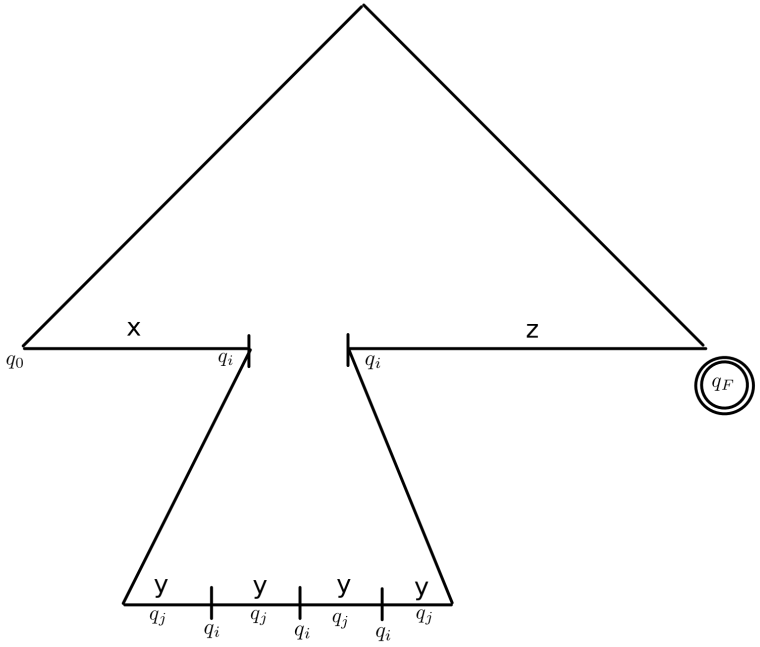
Pumped Down String



Pumping Lemma (Why it works) 5/5

Once Pumped Up String

- $|y| \geq 1$
  - Enforces that loop exists
- $|xy| \leq p$
  - Useful bound that enforces that there is a loop within first *p* input characters
- $(\forall n \geq 0)\ (xy^n z \in L)$
  - Loops can be repeated and still stay in language



Example: Proving Non-Regular Languages

ex) $\{ 0^n 1^m 0^n \}$

$\{0^p 1 0^p\}$

$\{0^{2p} 1 0^p\}$

- $|y| \geq 1$
- $|xy| \leq p$
- $(\forall n \geq 0)\ (xy^n z \in L)$

Example: Proving Non-Regular Languages

ex) $\{ 0^i 1^j \mid i \geq j \}$

- $|y| \geq 1$
- $|xy| \leq p$
- $(\forall n \geq 0)(xy^n z \in L)$

$$\{0^p 1^p\}$$

$$\{0^{p-1} 1^p\}$$

Example: Proving Non-Regular Languages

Hint: Use closure properties

ex) { w | w ≠ number of 0s and 1s }

$$\{w | w = \text{number of 0s and 1s}\}$$ Non-regular

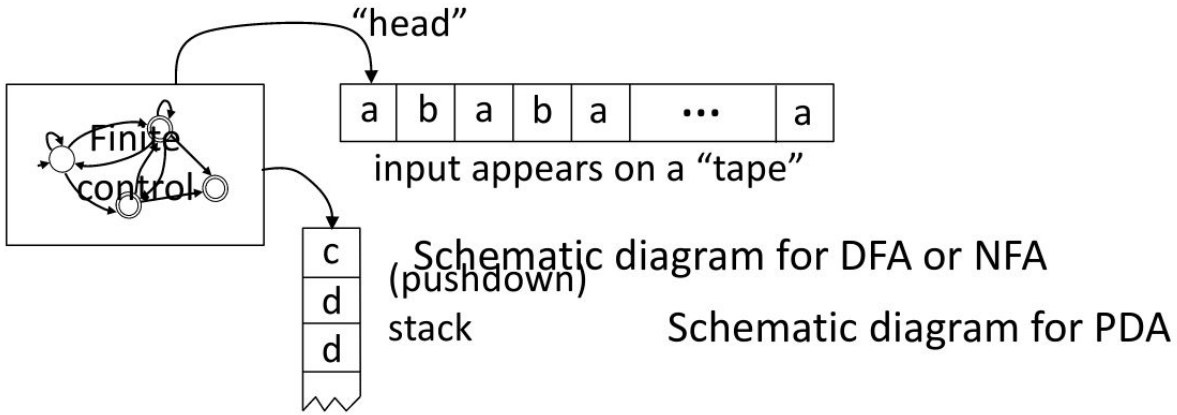
$$\{0^p 1^p\}$$

- $|y| \geq 1$
- $|xy| \leq p$
- $(\forall n \geq 0)(xy^n z \in L)$

Context Free Languages (PDA)

Designing a PDA for: $\{ 0^n 1^m 0^n \}$

Definition of Pushdown Automata (PDA)



Context Free Languages (CFG)

Designing a CFG for: { $0^n1^m0^n$ }

$S \rightarrow 0S0|R$ S is the starting variable

$R \rightarrow 1R|\varepsilon$

0110

0S0

0R0

01R0

011R0

0110

Defn: A Context Free Grammar (CFG) G is a 4-tuple (V, Σ, R, S)

- V finite set of variables
- Σ finite set of terminal symbols
- R finite set of rules
- S start variable

ex) $E \rightarrow E+T \mid T$

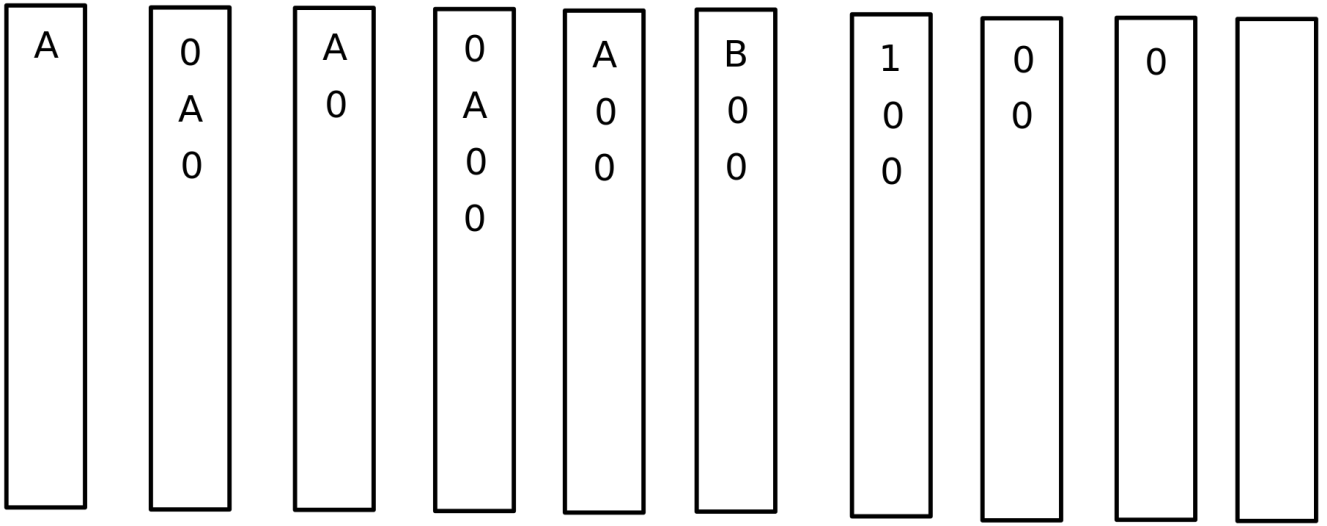
$T \rightarrow T\times F \mid F$

$F \rightarrow (\ E\ ) \mid a$

Converting CFG to PDA

CFG for: { $0^n1^m0^n$ } ex) 00100

$$A \rightarrow 0A0$$
$$A \rightarrow B$$
$$B \rightarrow 1B \mid \varepsilon$$



CFL Closure Properties

- Union
- Concatenation
- Kleene Star Operation

Recap

	Recognizer	Generator
Regular language	DFA or NFA	Regular expression
Context Free language	PDA	Context Free Grammar