

MIT 6.035

Top-Down Parsing

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Orientation

- Language specification
 - Lexical structure – regular expressions
 - Syntactic structure – grammar
- This Lecture - recursive descent parsers
 - Code parser as set of mutually recursive procedures
 - Structure of program matches structure of grammar

Starting Point

- Assume lexical analysis has produced a sequence of tokens
 - Each token has a type and value
 - Types correspond to terminals
 - Values to contents of token read in
- Examples
 - Int 549 – integer token with value 549 read in
 - if - if keyword, no need for a value
 - AddOp + - add operator, value +

Basic Approach

- Start with Start symbol
- Build a leftmost derivation
 - If leftmost symbol is nonterminal, choose a production and apply it
 - If leftmost symbol is terminal, match against input
 - If all terminals match, have found a parse!
 - Key: find correct productions for nonterminals

Graphical Illustration of Leftmost Derivation

Sentential Form



Grammar for Parsing Example

Start \rightarrow *Expr*

Expr \rightarrow *Expr* + *Term*

Expr \rightarrow *Expr* - *Term*

Expr \rightarrow *Term*

Term \rightarrow *Term* * Int

Term \rightarrow *Term* / Int

Term \rightarrow Int

- Set of tokens is $\{ +, -, *, /, \text{Int} \}$, where $\text{Int} = [0-9][0-9]^*$
- For convenience, may represent each Int n token by n

Parsing Example

Parse
Tree

Start



Current Position in Parse Tree

Remaining Input

2-2*2

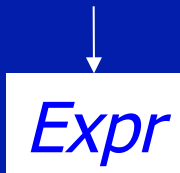
Sentential Form

Start

Parsing Example

Parse
Tree

Start



Expr

Current Position in Parse Tree

Remaining Input

2-2*2

Sentential Form

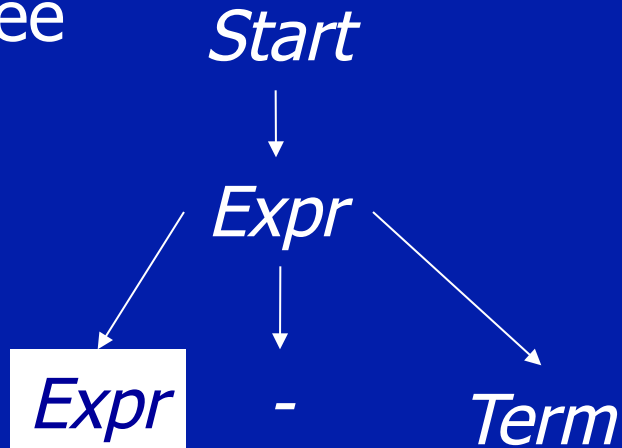
Expr

Applied Production

Start \rightarrow *Expr*

Parsing Example

Parse
Tree



Remaining Input

2-2*2

Sentential Form

$Expr - Term$

$Expr \rightarrow Expr + Term$

$Expr \rightarrow Expr - Term$

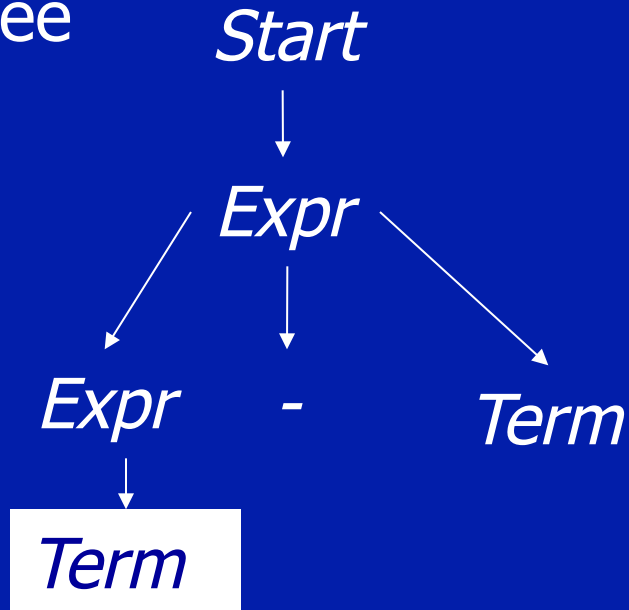
$Expr \rightarrow Term$

Applied Production

$Expr \rightarrow Expr - Term$

Parsing Example

Parse
Tree



$Expr \rightarrow Expr + Term$

$Expr \rightarrow Expr - Term$

$Expr \rightarrow Term$

Remaining Input

2-2*2

Sentential Form

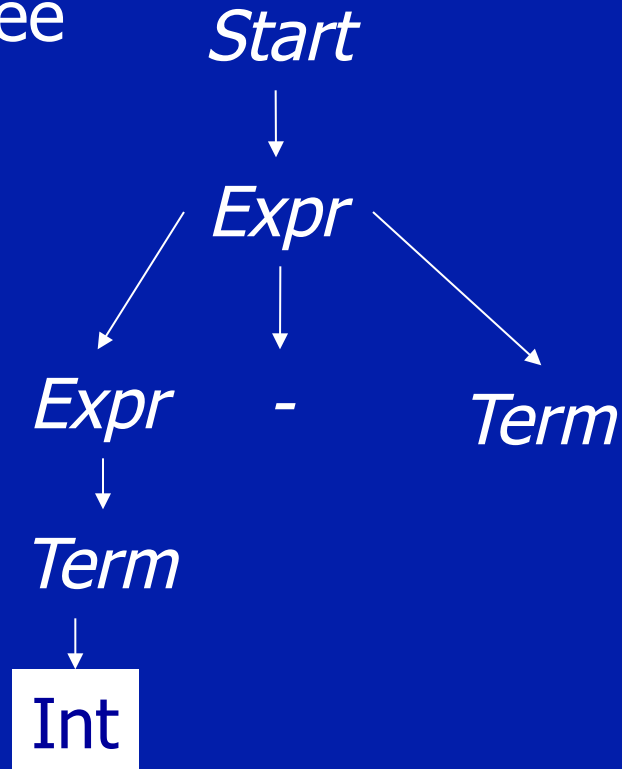
$Term - Term$

Applied Production

$Expr \rightarrow Term$

Parsing Example

Parse
Tree



Remaining Input

2-2*2

Sentential Form

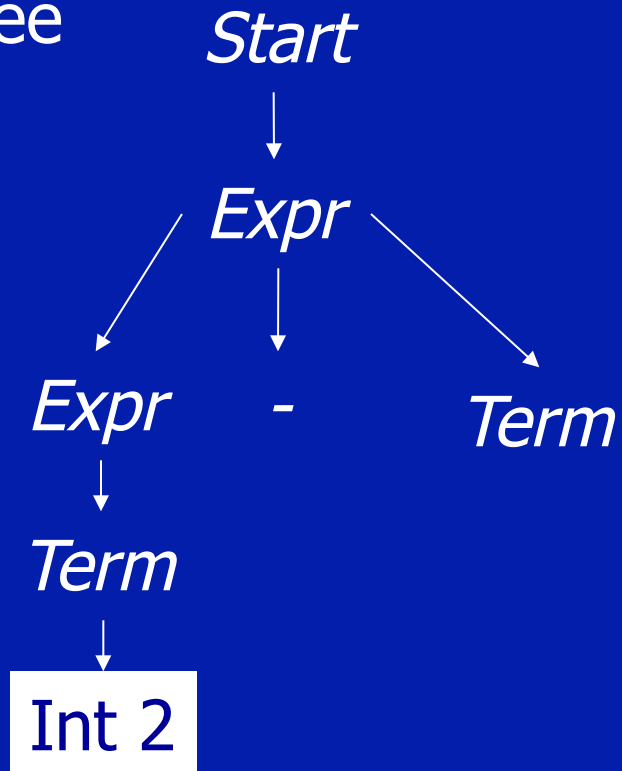
Int - *Term*

Applied Production

Term \rightarrow Int

Parsing Example

Parse
Tree



Match
Input
Token!

Remaining Input

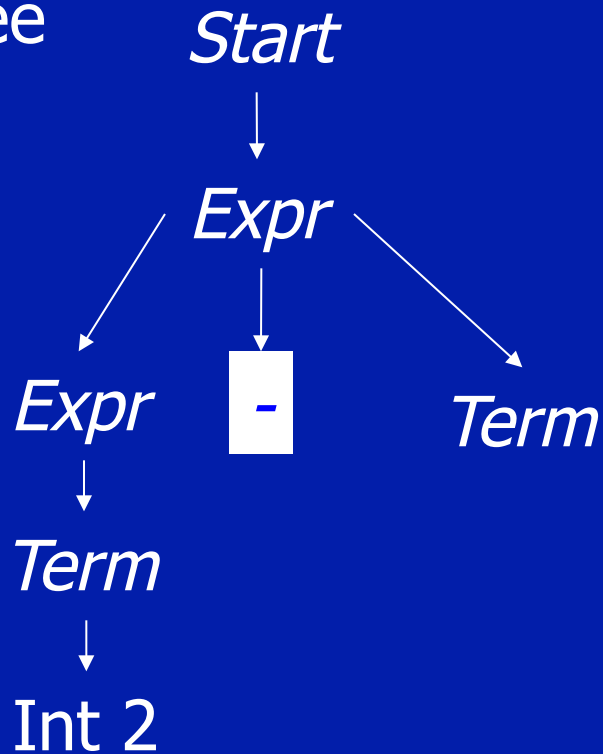
2-2*2

Sentential Form

2 - *Term*

Parsing Example

Parse
Tree



Match
Input
Token!

Remaining Input

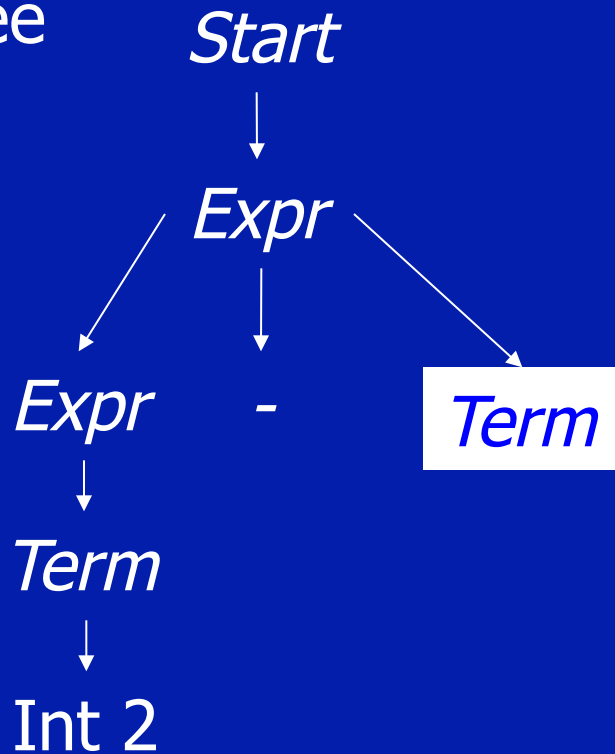
-2*2

Sentential Form

2 - Term

Parsing Example

Parse
Tree



Match
Input
Token!

Remaining Input

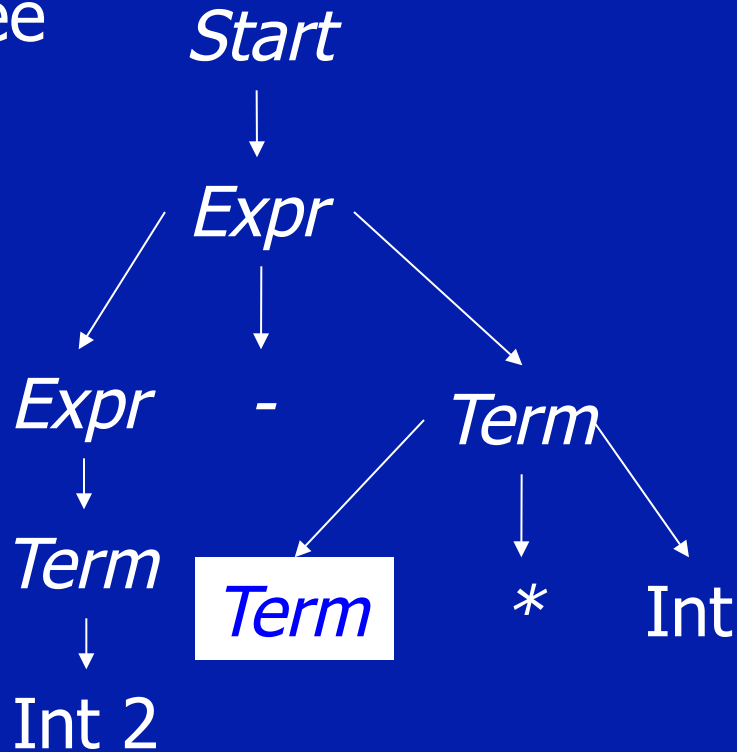
2*2

Sentential Form

2 - Term

Parsing Example

Parse
Tree



Remaining Input

2*2

Sentential Form

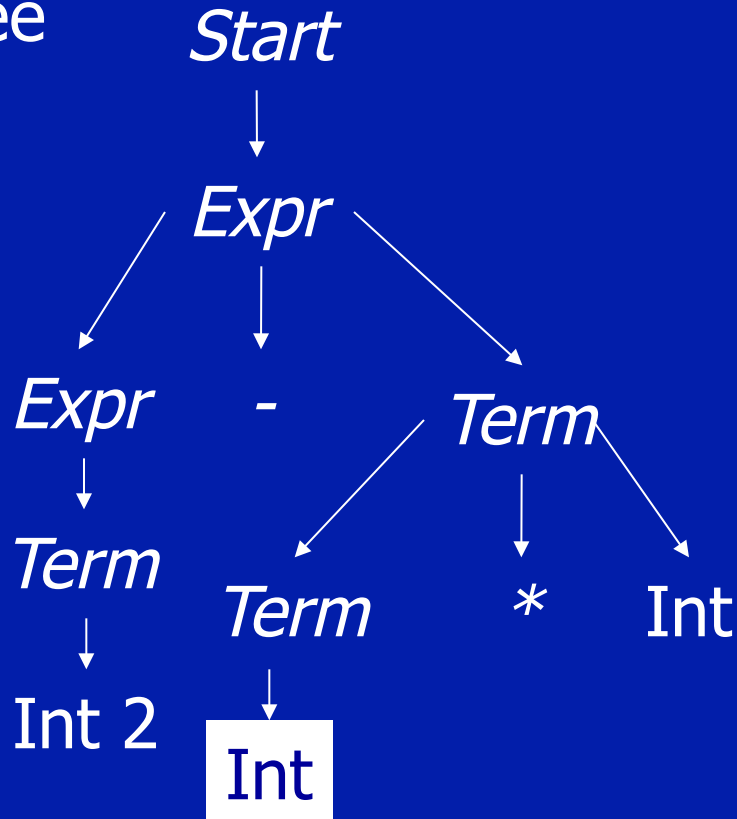
2 - *Term**Int

Applied Production

Term \rightarrow *Term* * Int

Parsing Example

Parse
Tree



Remaining Input

2*2

Sentential Form

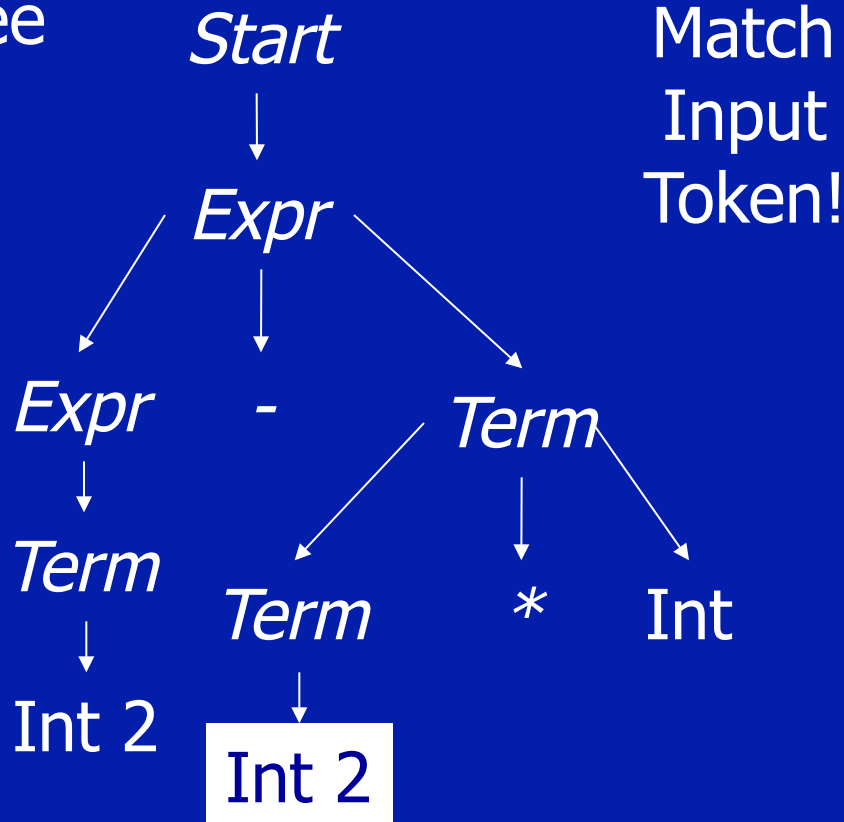
2 - Int * Int

Applied Production

$Term \rightarrow Int$

Parsing Example

Parse
Tree



Remaining Input

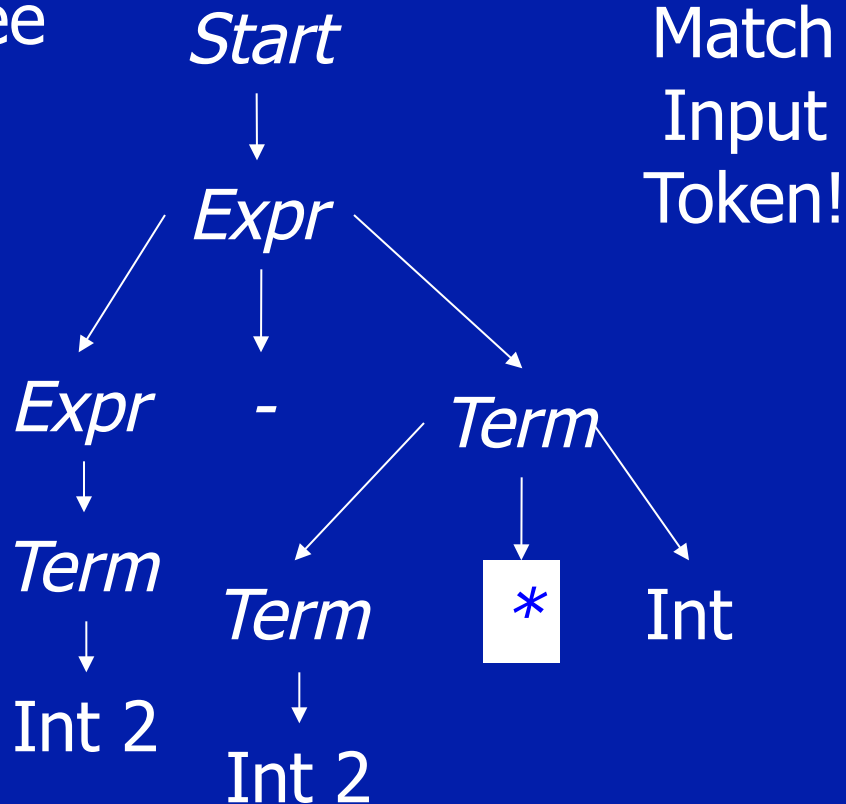
$2 * 2$

Sentential Form

$2 - 2 * \text{Int}$

Parsing Example

Parse
Tree



Match
Input
Token!

Remaining Input

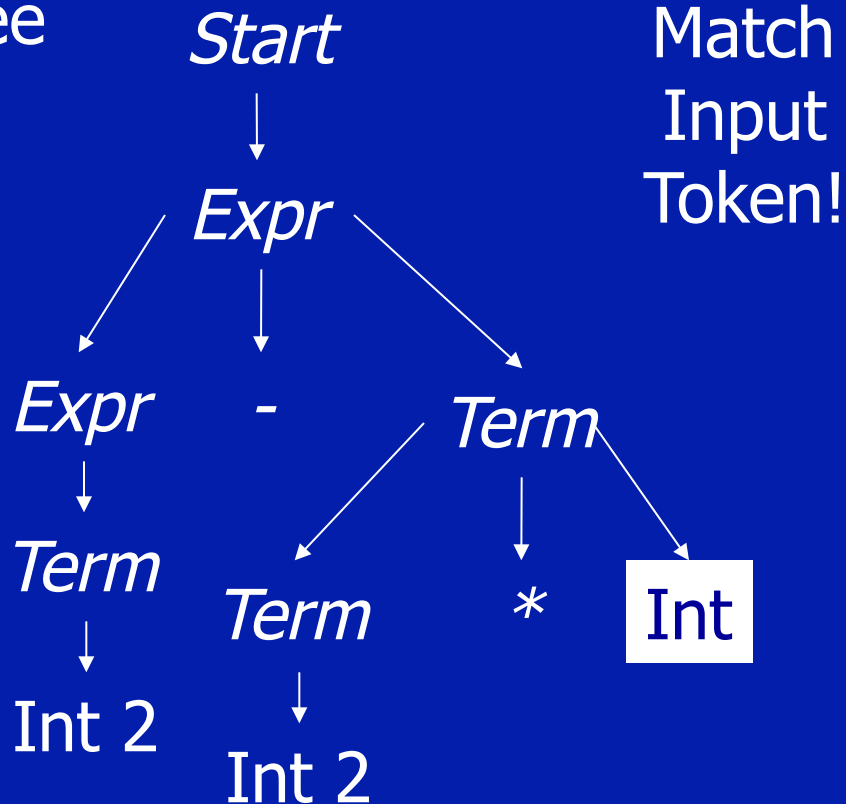
*2

Sentential Form

2 - 2 * Int

Parsing Example

Parse
Tree



Match
Input
Token!

Remaining Input

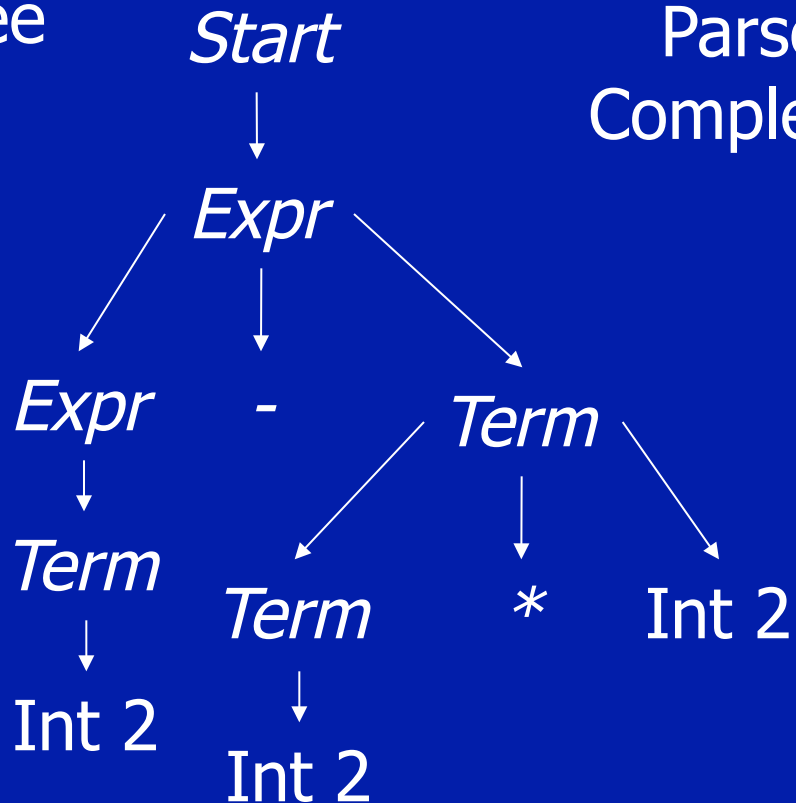
2

Sentential Form

2 - 2 * Int

Parsing Example

Parse
Tree



Parse
Complete!

Remaining Input

2

Sentential Form

2 - 2*2

Summary

- Three Actions (Mechanisms)
 - Apply production to expand current nonterminal in parse tree
 - Match current terminal (consuming input)
 - Accept the parse as correct
- Parser generates preorder traversal of parse tree
 - visit parents before children
 - visit siblings from left to right

Policy Problem

- Which production to use for each nonterminal?
- Classical Separation of Policy and Mechanism
- One Approach: Backtracking
 - Treat it as a search problem
 - At each choice point, try next alternative
 - If it is clear that current try fails, go back to previous choice and try something different
- General technique for searching
- Used a lot in classical AI and natural language processing (parsing, speech recognition)

Backtracking Example

Parse
Tree

Start

Remaining Input

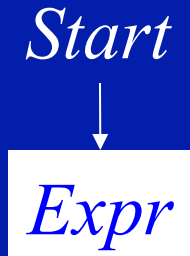
$2-2*2$

Sentential Form

Start

Backtracking Example

Parse
Tree



Remaining Input

2-2*2

Sentential Form

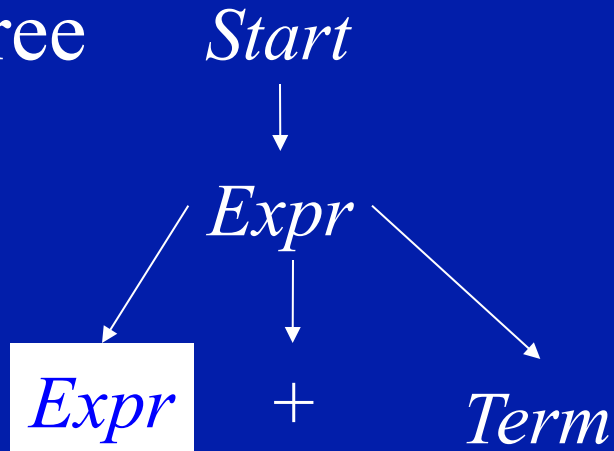
Expr

Applied Production

Start \rightarrow *Expr*

Backtracking Example

Parse
Tree



Remaining Input

2-2*2

Sentential Form

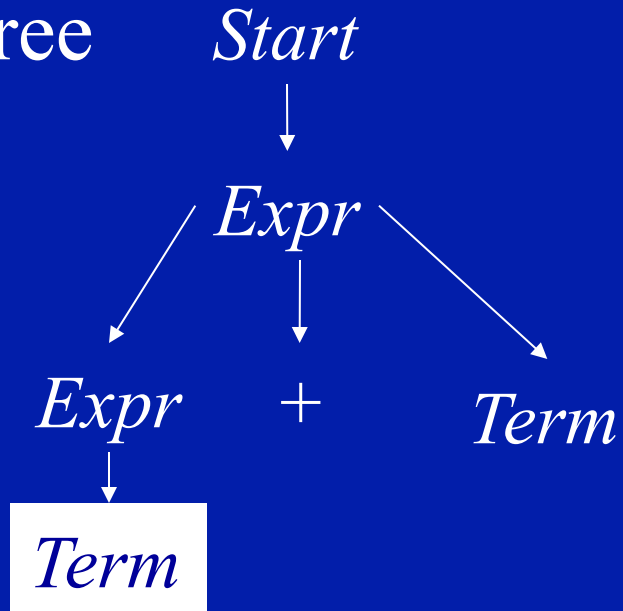
$Expr + Term$

Applied Production

$Expr \rightarrow Expr + Term$

Backtracking Example

Parse
Tree



Remaining Input

2-2*2

Sentential Form

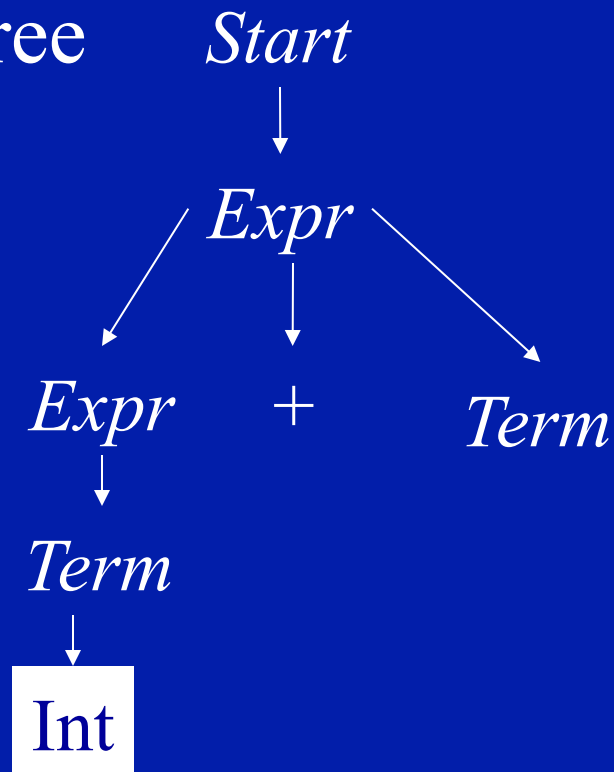
$Term + Term$

Applied Production

$Expr \rightarrow Term$

Backtracking Example

Parse
Tree



Match
Input
Token!

Remaining Input

2-2*2

Sentential Form

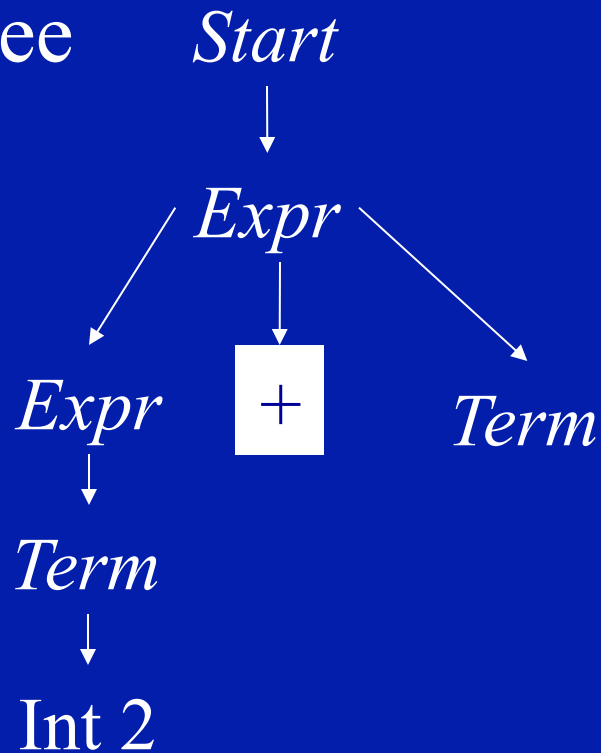
$\text{Int} + \text{Term}$

Applied Production

$\text{Term} \rightarrow \text{Int}$

Backtracking Example

Parse
Tree



Can't
Match
Input
Token!

Remaining Input

-2*2

Sentential Form

2 - *Term*

Applied Production

Term → Int

Backtracking Example

Parse
Tree

Start



Expr

So
Backtrack!

Remaining Input

2-2*2

Sentential Form

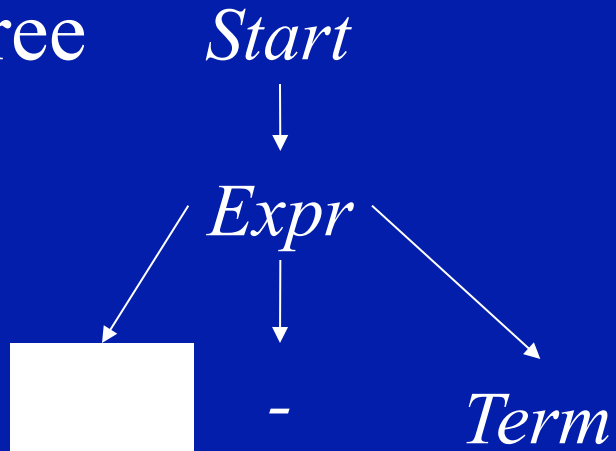
Expr

Applied Production

Start \rightarrow *Expr*

Backtracking Example

Parse
Tree



Remaining Input

2-2*2

Sentential Form

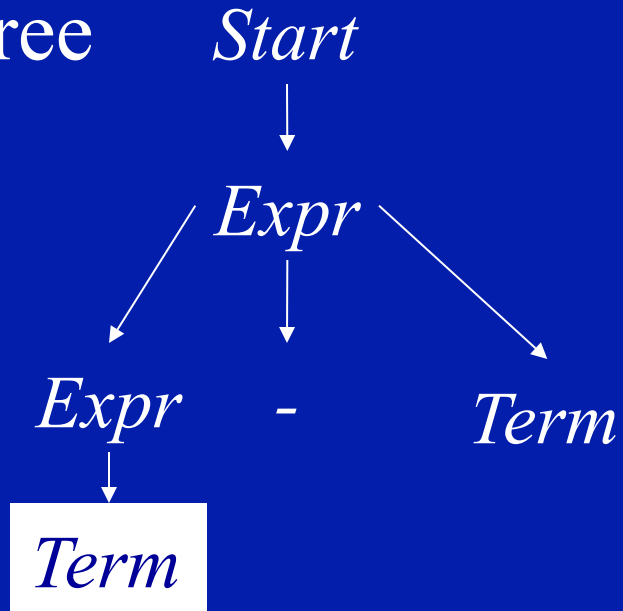
$Expr - Term$

Applied Production

$Expr \rightarrow Expr - Term$

Backtracking Example

Parse
Tree



Remaining Input

2-2*2

Sentential Form

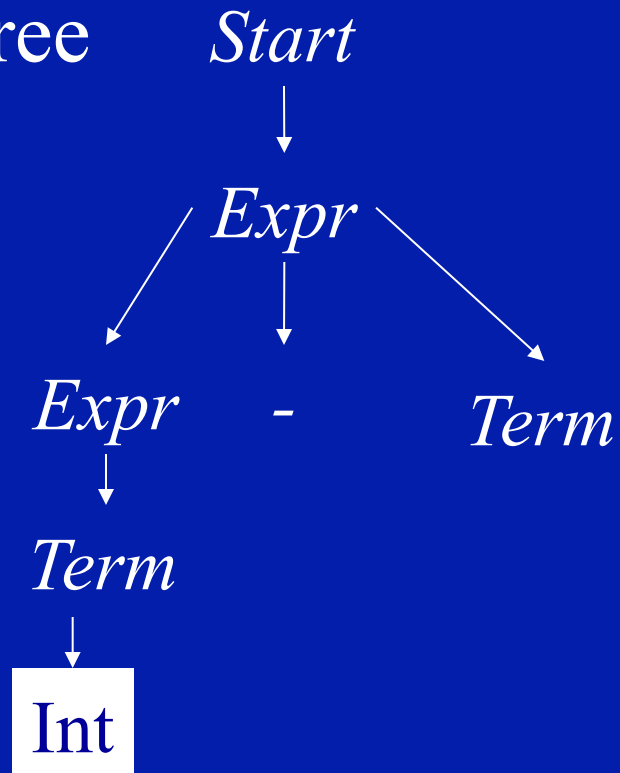
$Term - Term$

Applied Production

$Expr \rightarrow Term$

Backtracking Example

Parse
Tree



Remaining Input

2-2*2

Sentential Form

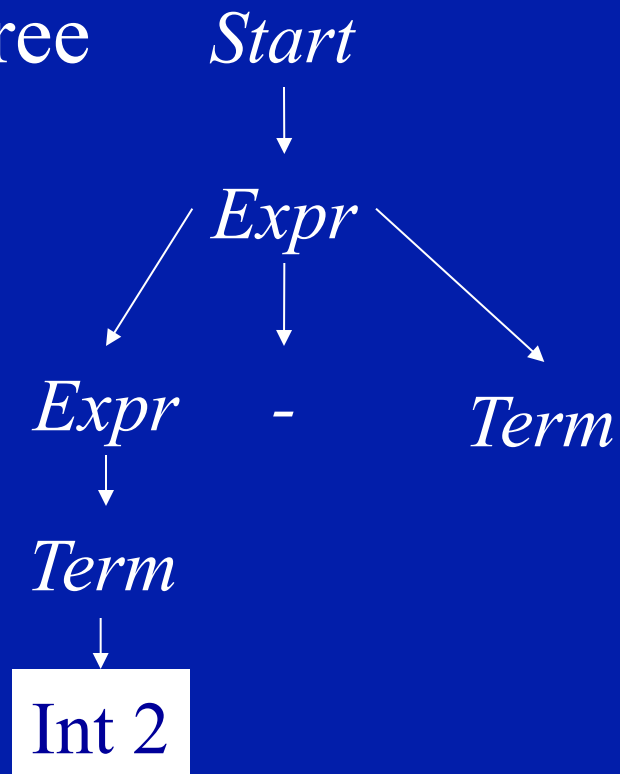
Int - *Term*

Applied Production

Term \rightarrow Int

Backtracking Example

Parse
Tree



Match
Input
Token!

Remaining Input

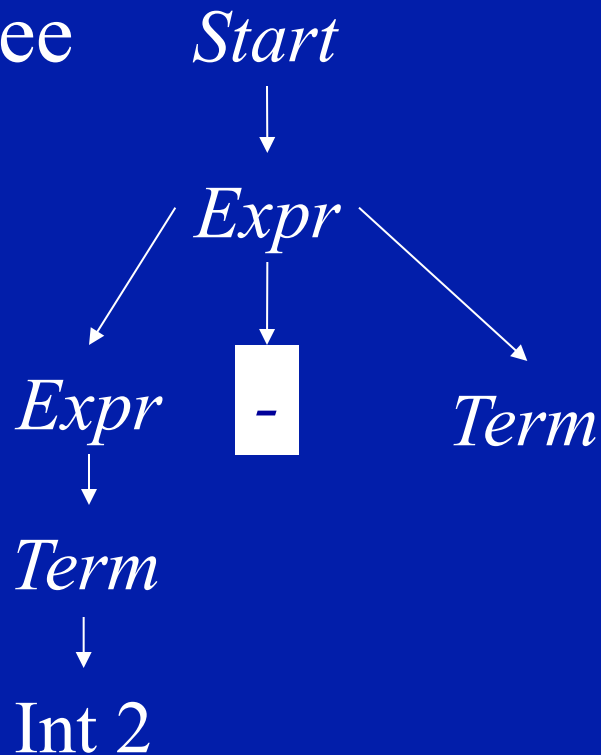
$-2*2$

Sentential Form

$2 - \textit{Term}$

Backtracking Example

Parse
Tree



Match
Input
Token!

Remaining Input

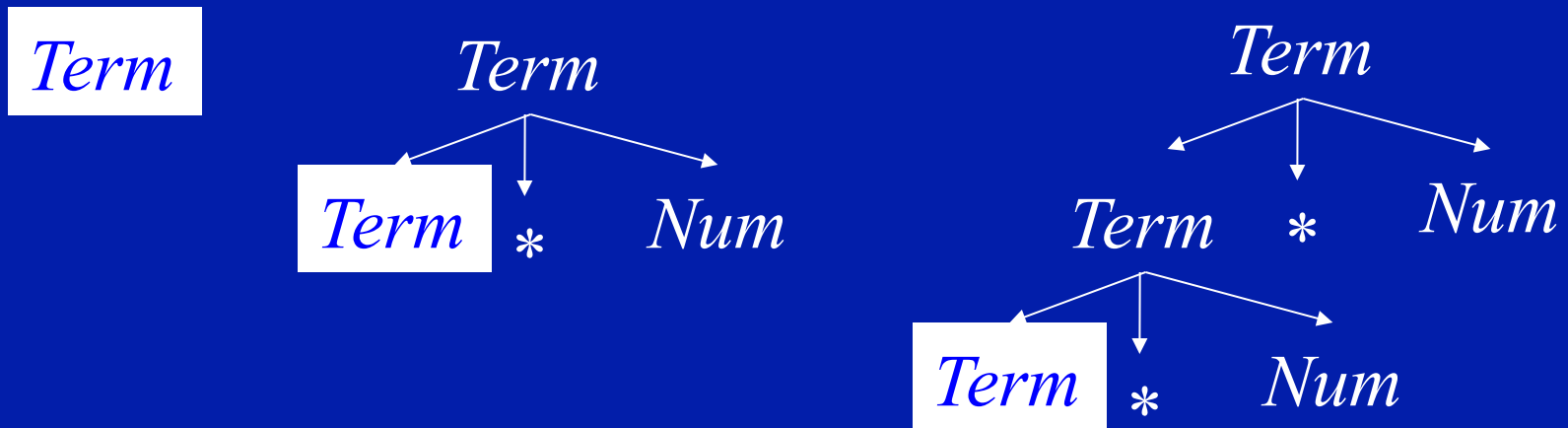
$2 * 2$

Sentential Form

$2 - \textit{Term}$

Left Recursion + Top-Down Parsing = Infinite Loop

- Example Production: $Term \rightarrow Term * Num$
- Potential parsing steps:

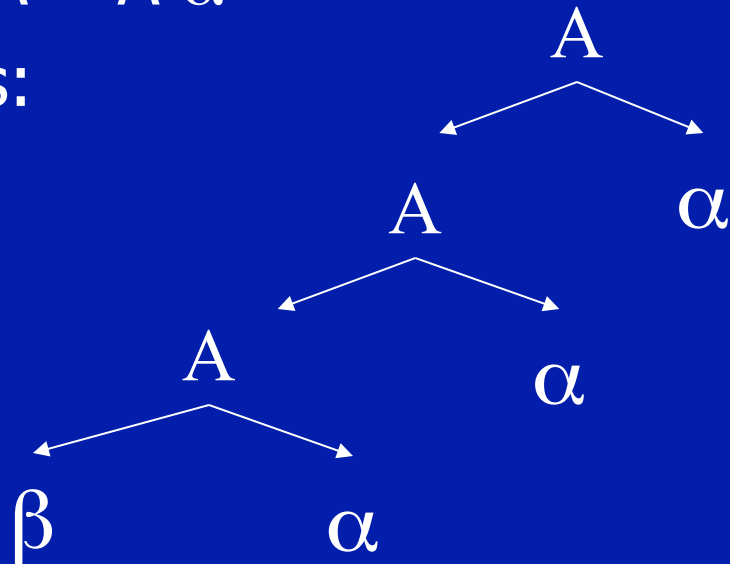


General Search Issues

- Three components
 - Search space (parse trees)
 - Search algorithm (parsing algorithm)
 - Goal to find (parse tree for input program)
- Would like to (but can't always) ensure that
 - Find goal (hopefully quickly) if it exists
 - Search terminates if it does not
- Handled in various ways in various contexts
 - Finite search space makes it easy
 - Exploration strategies for infinite search space
 - Sometimes one goal more important (model checking)
- For parsing, hack grammar to remove left recursion

Eliminating Left Recursion

- Start with productions of form
 - $A \rightarrow A \alpha$
 - $A \rightarrow \beta$
 - α, β sequences of terminals and nonterminals that do not start with A
- Repeated application of $A \rightarrow A \alpha$ builds parse tree like this:



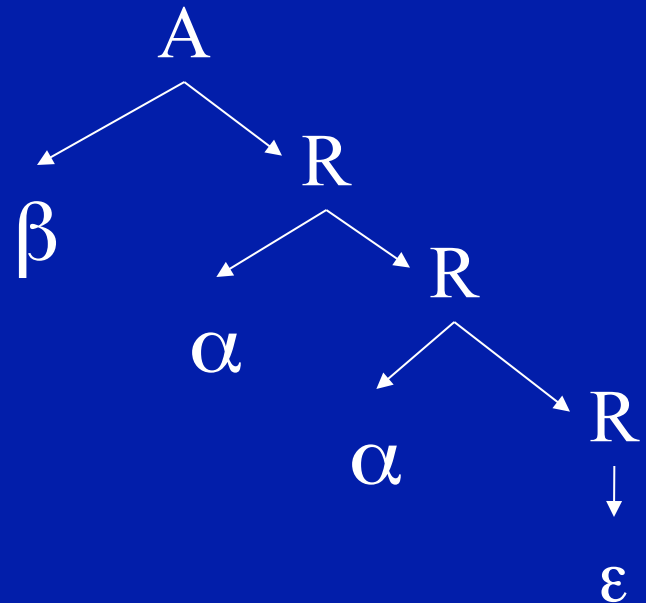
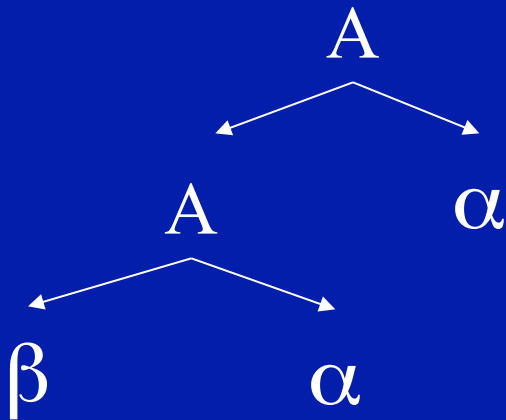
Eliminating Left Recursion

- Replacement productions

- $A \rightarrow A \alpha$ $A \rightarrow \beta R$ R is a new nonterminal
- $A \rightarrow \beta$ $R \rightarrow \alpha R$
- $\quad\quad\quad R \rightarrow \varepsilon$

New Parse Tree

Old Parse Tree



Hacked Grammar

Original Grammar Fragment

$Term \rightarrow Term * Int$

$Term \rightarrow Term / Int$

$Term \rightarrow Int$

New Grammar Fragment

$Term \rightarrow Int Term'$

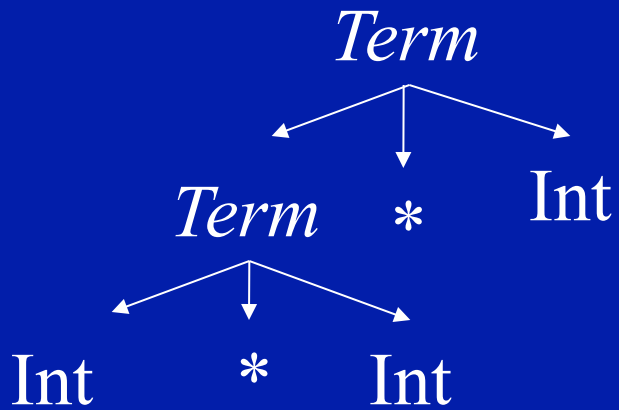
$Term' \rightarrow * Int Term'$

$Term' \rightarrow / Int Term'$

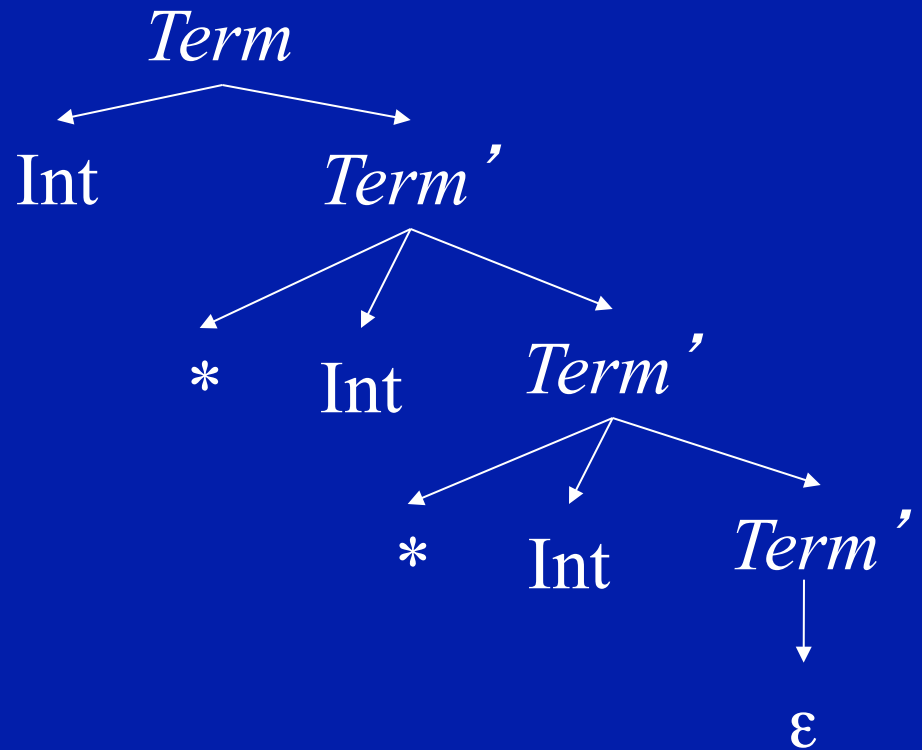
$Term' \rightarrow \epsilon$

Parse Tree Comparisons

Original Grammar



New Grammar



Eliminating Left Recursion

- Changes search space exploration algorithm
 - Eliminates direct infinite recursion
 - But grammar less intuitive
- Sets things up for predictive parsing

Predictive Parsing

- Alternative to backtracking
- Useful for programming languages, which can be designed to make parsing easier
- Basic idea
 - Look ahead in input stream
 - Decide which production to apply based on next tokens in input stream
 - We will use one token of lookahead

Predictive Parsing Example Grammar

Start \rightarrow *Expr*

Expr \rightarrow *Term Expr'*

Expr' \rightarrow + *Term Expr'*

Expr' \rightarrow - *Term Expr'*

Expr' \rightarrow ϵ

Term \rightarrow Int *Term'*

Term' \rightarrow * Int *Term'*

Term' \rightarrow / Int *Term'*

Term' \rightarrow ϵ

Choice Points

- Assume $Term'$ is current position in parse tree
- Have three possible productions to apply
$$Term' \rightarrow * Int Term'$$
$$Term' \rightarrow / Int Term'$$
$$Term' \rightarrow \varepsilon$$
- Use next token to decide
 - If next token is $*$, apply $Term' \rightarrow * Int Term'$
 - If next token is $/$, apply $Term' \rightarrow / Int Term'$
 - Otherwise, apply $Term' \rightarrow \varepsilon$

Predictive Parsing + Hand Coding = Recursive Descent Parser

- One procedure per nonterminal NT
 - Productions $NT \rightarrow \beta_1, \dots, NT \rightarrow \beta_n$
 - Procedure examines the current input symbol T to determine which production to apply
 - If $T \in \text{First}(\beta_k)$
 - Apply production k
 - Consume terminals in β_k (check for correct terminal)
 - Recursively call procedures for nonterminals in β_k
 - Current input symbol stored in global variable `token`
- Procedures return
 - true if parse succeeds
 - false if parse fails

Example

Boolean Term()

```
if (token = Int n) token = NextToken(); return(TermPrime())  
else return(false)
```

Boolean TermPrime()

```
if (token = *)  
    token = NextToken();  
    if (token = Int n) token = NextToken(); return(TermPrime())  
    else return(false)  
else if (token = /)  
    token = NextToken();  
    if (token = Int n) token = NextToken(); return(TermPrime())  
    else return(false)  
else return(true)
```

$Term \rightarrow \text{Int } Term'$

$Term' \rightarrow * \text{Int } Term'$

$Term' \rightarrow / \text{Int } Term'$

$Term' \rightarrow \varepsilon$

Multiple Productions With Same Prefix in RHS

- Example Grammar

$NT \rightarrow \text{if then}$

$NT \rightarrow \text{if then else}$

- Assume NT is current position in parse tree, and if is the next token
- Unclear which production to apply
 - Multiple k such that $T \in \text{First}(\beta_k)$
 - $\text{if} \in \text{First}(\text{if then})$
 - $\text{if} \in \text{First}(\text{if then else})$

Solution: Left Factor the Grammar

- New Grammar Factors Common Prefix Into Single Production

$NT \rightarrow \text{if then } NT'$

$NT' \rightarrow \text{else}$

$NT' \rightarrow \varepsilon$

- No choice when next token is if!
- All choices have been unified in one production.

Nonterminals

- What about productions with nonterminals?

$$NT \rightarrow NT_1 \alpha_1$$

$$NT \rightarrow NT_2 \alpha_2$$

- Must choose based on possible first terminals that NT_1 and NT_2 can generate
- What if NT_1 or NT_2 can generate ε ?
 - Must choose based on α_1 and α_2

NT derives ε

- Two rules
 - $NT \rightarrow \varepsilon$ implies NT derives ε
 - $NT \rightarrow NT_1 \dots NT_n$ and for all $1 \leq i \leq n$ NT_i derives ε implies NT derives ε

Fixed Point Algorithm for Derives ε

for all nonterminals NT

 set NT derives ε to be false

for all productions of the form $NT \rightarrow \varepsilon$

 set NT derives ε to be true

while (some NT derives ε changed in last iteration)

 for all productions of the form $NT \rightarrow NT_1 \dots NT_n$

 if (for all $1 \leq i \leq n$ NT_i derives ε)

 set NT derives ε to be true

First(β)

- $T \in \text{First}(\beta)$ if T can appear as the first symbol in a derivation starting from β
 - 1) $T \in \text{First}(T)$
 - 2) $\text{First}(S) \subseteq \text{First}(S\beta)$
 - 3) NT derives ε implies $\text{First}(\beta) \subseteq \text{First}(NT\beta)$
 - 4) $NT \rightarrow S\beta$ implies $\text{First}(S\beta) \subseteq \text{First}(NT)$
- Notation
 - T is a terminal, NT is a nonterminal, S is a terminal or nonterminal, and β is a sequence of terminals or nonterminals

Rules + Request Generate System of Subset Inclusion Constraints

Grammar

$Term' \rightarrow * Int Term'$

$Term' \rightarrow / Int Term'$

$Term' \rightarrow \varepsilon$

Rules

- 1) $T \in First(T)$
- 2) $First(S) \subseteq First(S \beta)$
- 3) NT derives ε implies
 $First(\beta) \subseteq First(NT \beta)$
- 4) $NT \rightarrow S \beta$ implies
 $First(S \beta) \subseteq First(NT)$

Request: What is $First(Term')$?

Constraints

$First(* Num Term') \subseteq$
 $First(Term')$

$First(/ Num Term') \subseteq$
 $First(Term')$

$First(*) \subseteq First(* Num Term')$

$First(/) \subseteq First(/ Num Term')$

$* \in First(*)$

$/ \in First(/)$

Constraint Propagation Algorithm

Constraints

$$\text{First}(* \text{ Num } Term') \subseteq \text{First}(Term')$$

$$\text{First}(/ \text{ Num } Term') \subseteq \text{First}(Term')$$

$$\text{First}(*) \subseteq \text{First}(* \text{ Num } Term')$$

$$\text{First}(/) \subseteq \text{First}(/ \text{ Num } Term')$$

$$* \in \text{First}(*)$$

$$/ \in \text{First}(/)$$

Solution

$$\text{First}(Term') = \{ \}$$

$$\text{First}(* \text{ Num } Term') = \{ \}$$

$$\text{First}(/ \text{ Num } Term') = \{ \}$$

$$\text{First}(*) = \{ * \}$$

$$\text{First}(/) = \{ / \}$$

Initialize Sets to $\{ \}$

Propagate Constraints Until
Fixed Point

Constraint Propagation Algorithm

Constraints

$$\text{First}(* \text{ Num } \text{Term}') \subseteq \text{First}(\text{Term}')$$

$$\text{First}(/ \text{ Num } \text{Term}') \subseteq \text{First}(\text{Term}')$$

$$\text{First}(*) \subseteq \text{First}(* \text{ Num } \text{Term}')$$

$$\text{First}(/) \subseteq \text{First}(/ \text{ Num } \text{Term}')$$

$$* \in \text{First}(*)$$

$$/ \in \text{First}(/)$$

Solution

$$\text{First}(\text{Term}') = \{ \}$$

$$\text{First}(* \text{ Num } \text{Term}') = \{ \}$$

$$\text{First}(/ \text{ Num } \text{Term}') = \{ \}$$

$$\text{First}(*) = \{ * \}$$

$$\text{First}(/) = \{ / \}$$

Grammar

$$\text{Term}' \rightarrow * \text{Int } \text{Term}'$$

$$\text{Term}' \rightarrow / \text{Int } \text{Term}'$$

$$\text{Term}' \rightarrow \varepsilon$$

Constraint Propagation Algorithm

Constraints

$$\text{First}(* \text{ Num } Term') \subseteq \text{First}(Term')$$

$$\text{First}(/ \text{ Num } Term') \subseteq \text{First}(Term')$$

$$\text{First}(*) \subseteq \text{First}(* \text{ Num } Term')$$

$$\text{First}(/) \subseteq \text{First}(/ \text{ Num } Term')$$

$$* \in \text{First}(*)$$

$$/ \in \text{First}(/)$$

Solution

$$\text{First}(Term') = \{ \}$$

$$\text{First}(* \text{ Num } Term') = \{ * \}$$

$$\text{First}(/ \text{ Num } Term') = \{ / \}$$

$$\text{First}(*) = \{ * \}$$

$$\text{First}(/) = \{ / \}$$

Grammar

$$Term' \rightarrow * \text{ Int } Term'$$

$$Term' \rightarrow / \text{ Int } Term'$$

$$Term' \rightarrow \varepsilon$$

Constraint Propagation Algorithm

Constraints

$$\text{First}(* \text{ Num } \text{Term}') \subseteq \text{First}(\text{Term}')$$

$$\text{First}(/ \text{ Num } \text{Term}') \subseteq \text{First}(\text{Term}')$$

$$\text{First}(*) \subseteq \text{First}(* \text{ Num } \text{Term}')$$

$$\text{First}(/) \subseteq \text{First}(/ \text{ Num } \text{Term}')$$

$$* \in \text{First}(*)$$

$$/ \in \text{First}(/)$$

Solution

$$\text{First}(\text{Term}') = \{ *, / \}$$

$$\text{First}(* \text{ Num } \text{Term}') = \{ * \}$$

$$\text{First}(/ \text{ Num } \text{Term}') = \{ / \}$$

$$\text{First}(*) = \{ * \}$$

$$\text{First}(/) = \{ / \}$$

Grammar

$$\text{Term}' \rightarrow * \text{Int } \text{Term}'$$

$$\text{Term}' \rightarrow / \text{Int } \text{Term}'$$

$$\text{Term}' \rightarrow \varepsilon$$

Constraint Propagation Algorithm

Constraints

$$\text{First}(* \text{ Num } Term') \subseteq \text{First}(Term')$$

$$\text{First}(/ \text{ Num } Term') \subseteq \text{First}(Term')$$

$$\text{First}(*) \subseteq \text{First}(* \text{ Num } Term')$$

$$\text{First}(/) \subseteq \text{First}(/ \text{ Num } Term')$$

$$* \in \text{First}(*)$$

$$/ \in \text{First}(/)$$

Solution

$$\text{First}(Term') = \{ *, / \}$$

$$\text{First}(* \text{ Num } Term') = \{ * \}$$

$$\text{First}(/ \text{ Num } Term') = \{ / \}$$

$$\text{First}(*) = \{ * \}$$

$$\text{First}(/) = \{ / \}$$

Grammar

$$Term' \rightarrow * \text{ Int } Term'$$

$$Term' \rightarrow / \text{ Int } Term'$$

$$Term' \rightarrow \varepsilon$$

Building A Parse Tree

- Have each procedure return the section of the parse tree for the part of the string it parsed
- Use exceptions to make code structure clean

Building Parse Tree In Example

Term()

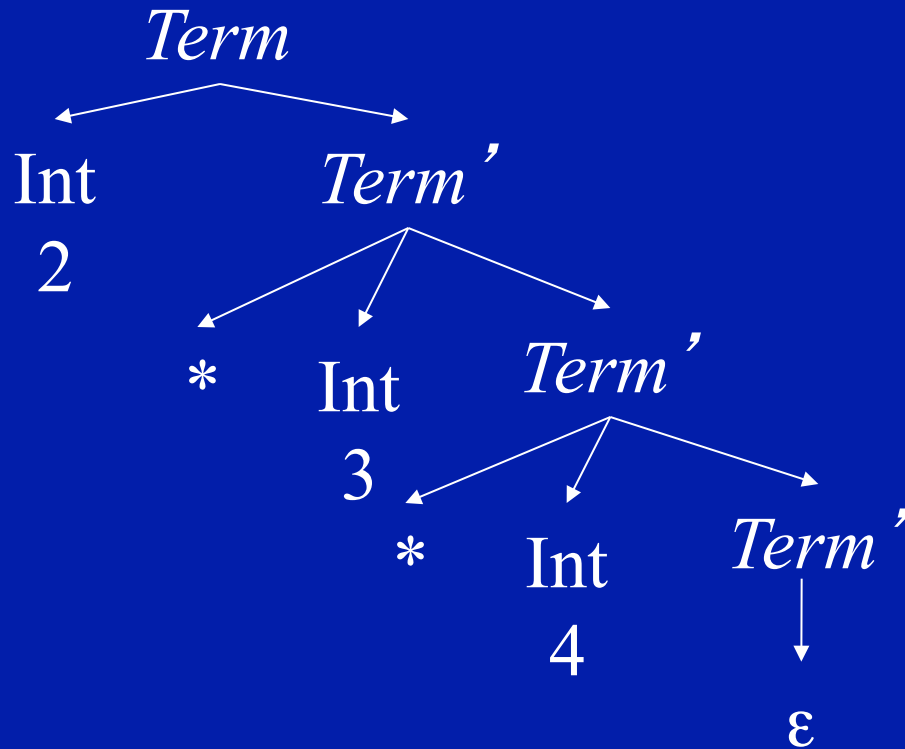
```
if (token = Int n)
    oldToken = token; token = NextToken();
    node = TermPrime();
    if (node == NULL) return oldToken;
    else return(new TermNode(oldToken, node);
else throw SyntaxError
```

TermPrime()

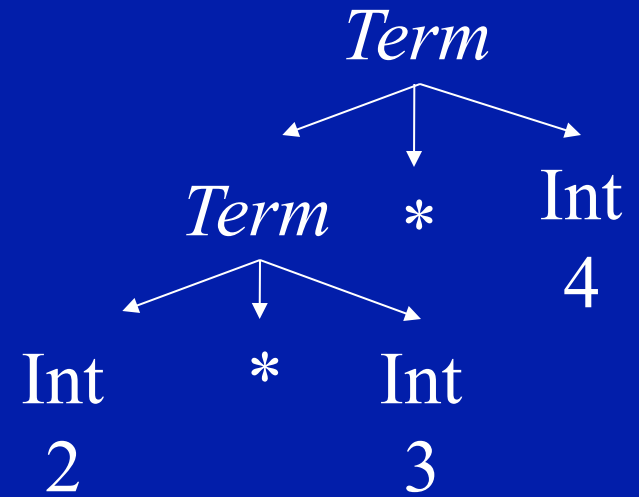
```
if (token = *) || (token = /)
    first = token; next = NextToken();
    if (next = Int n)
        token = NextToken();
        return(new TermPrimeNode(first, next, TermPrime()))
    else throw SyntaxError
else return(NULL)
```

Parse Tree for $2*3*4$

Concrete
Parse Tree



Desired
Abstract
Parse Tree



Why Use Hand-Coded Parser?

- Why not use parser generator?
- What do you do if your parser doesn't work?
 - Recursive descent parser – write more code
 - Parser generator
 - Hack grammar
 - But if parser generator doesn't work, nothing you can do
- If you have complicated grammar
 - Increase chance of going outside comfort zone of parser generator
 - Your parser may NEVER work

Bottom Line

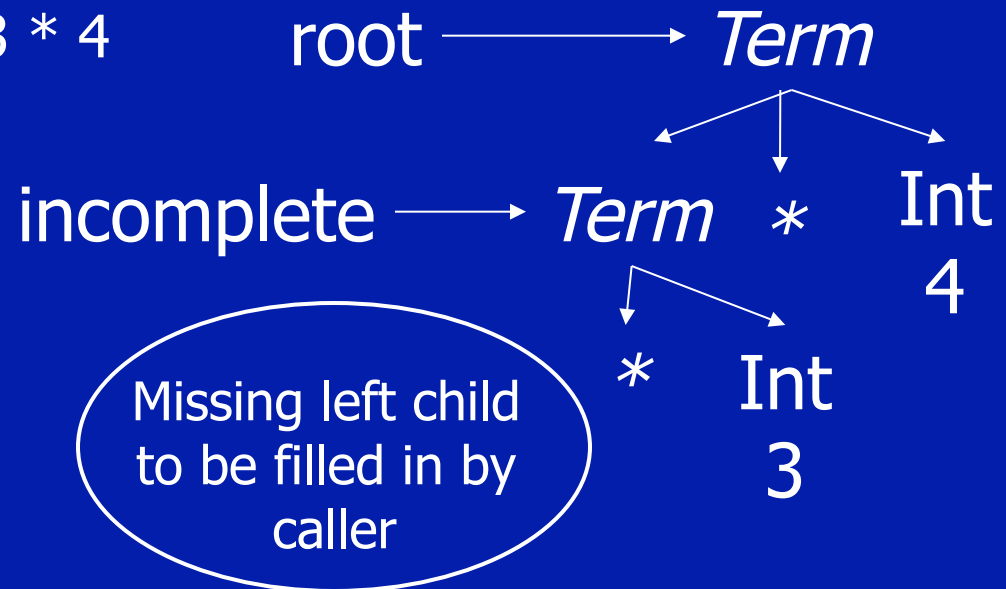
- Recursive descent parser properties
 - Probably more work
 - But less risk of a disaster - you can almost always make a recursive descent parser work
 - May have easier time dealing with resulting code
 - Single language system
 - No need to deal with potentially flaky parser generator
 - No integration issues with automatically generated code
- If your parser development time is small compared to rest of project, or you have a really complicated language, use hand-coded recursive descent parser

Summary

- Top-Down Parsing
- Use Lookahead to Avoid Backtracking
- Parser is
 - Hand-Coded
 - Set of Mutually Recursive Procedures


Direct Generation of Abstract Tree

- TermPrime builds an incomplete tree
 - Missing leftmost child
 - Returns root and incomplete node
- $(\text{root}, \text{incomplete}) = \text{TermPrime}()$
 - Called with token = *
 - Remaining tokens = 3 * 4




Code for Term

Term()

```
if (token = Int n)   
    leftmostInt = token; token = NextToken();  
    (root, incomplete) = TermPrime();  
    if (root == NULL) return leftmostInt;  
    incomplete.leftChild = leftmostInt;  
    return root;  
else throw SyntaxError
```

Input to
parse

2*3*4


token  Int
2

Code for Term

Term()

```
if (token = Int n)
    leftmostInt = token; token = NextToken(); ←
    (root, incomplete) = TermPrime();
    if (root == NULL) return leftmostInt;
    incomplete.leftChild = leftmostInt;
    return root;
else throw SyntaxError
```

Input to
parse

2*3*4
↑

token → Int
2

Code for Term

Term()

```
if (token = Int n)
    leftmostInt = token; token = NextToken();
    (root, incomplete) = TermPrime(); ←
    if (root == NULL) return leftmostInt;
    incomplete.leftChild = leftmostInt;
    return root;
else throw SyntaxError
```

Input to
parse

2*3*4
↑

token → Int
2

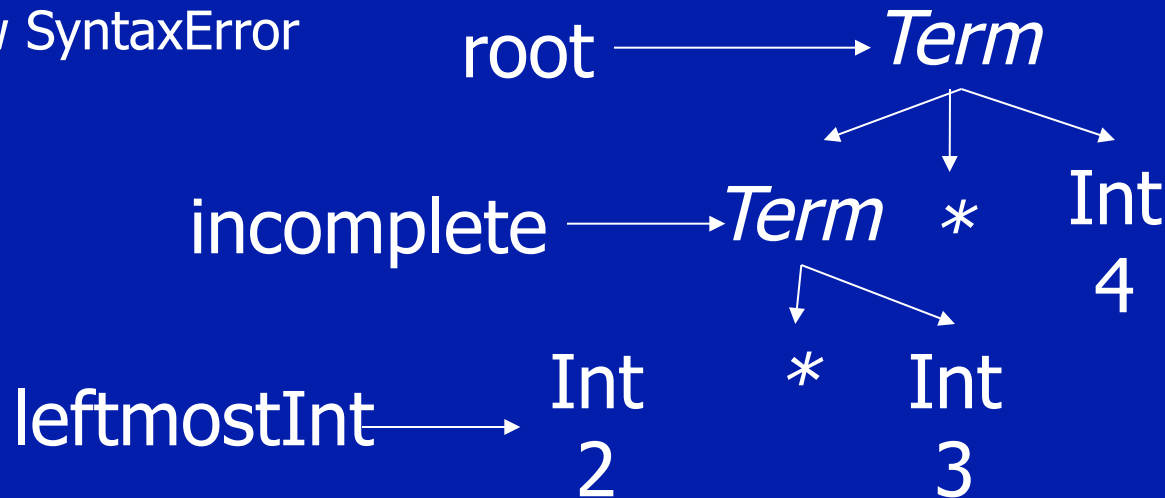
Code for Term

Term()

```
if (token = Int n)
    leftmostInt = token; token = NextToken();
    (root, incomplete) = TermPrime();
    if (root == NULL) return leftmostInt;
    incomplete.leftChild = leftmostInt;
    return root;
else throw SyntaxError
```

Input to
parse

2*3*4
↑



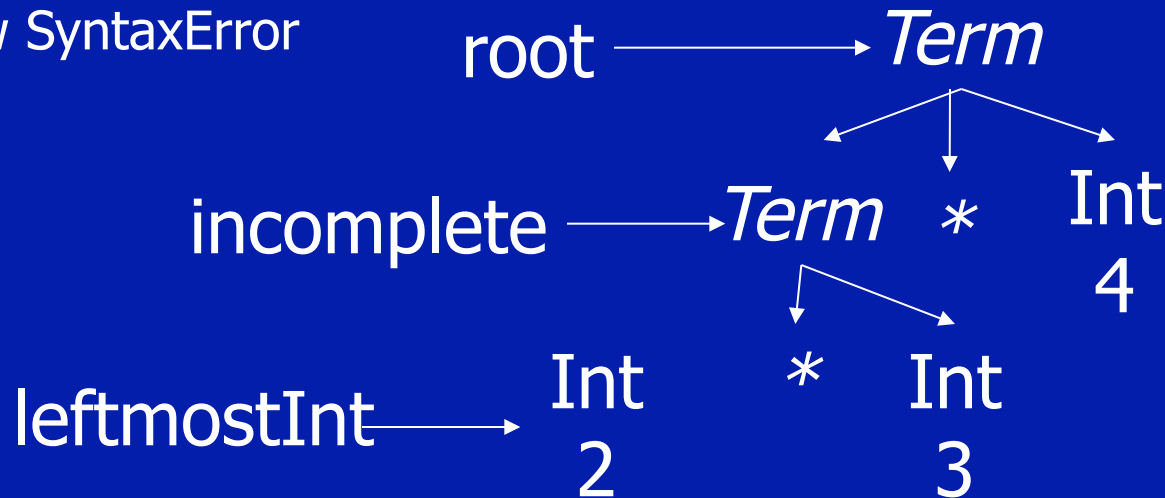
Code for Term

Term()

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if (token = Int n)
    leftmostInt = token; token = NextToken();
    (root, incomplete) = TermPrime();
    if (root == NULL) return leftmostInt;
    incomplete.leftChild = leftmostInt;
    return root;
else throw SyntaxError
```

Input to
parse

2*3*4
↑



Code for Term

Term()

if (token = Int n)

leftmostInt = token; token = NextToken();

(root, incomplete) = TermPrime();

if (root == NULL) return leftmostInt;

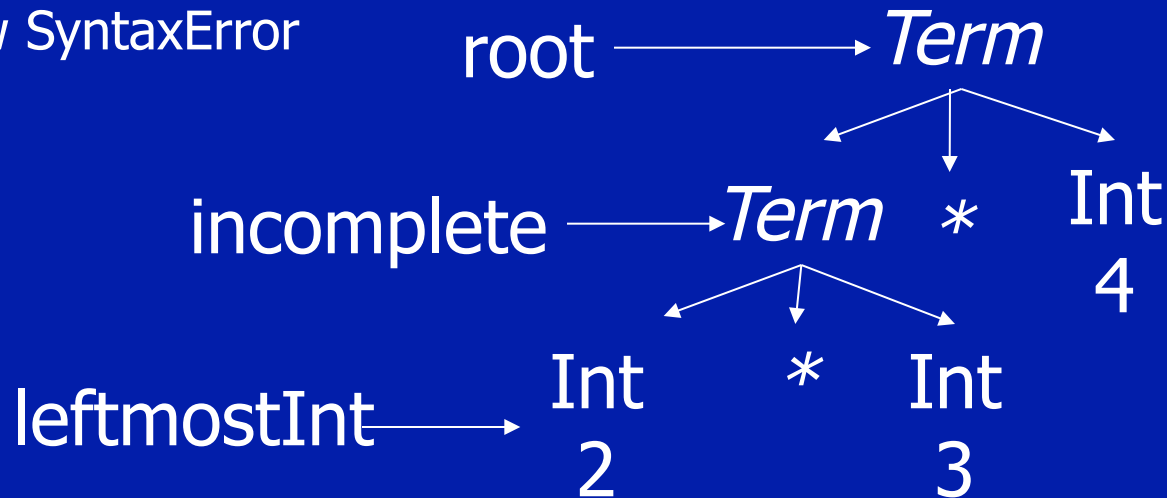
incomplete.leftChild = leftmostInt;

return root; ←

else throw SyntaxError

Input to
parse

2*3*4
↑



Code for TermPrime

TermPrime()

```
if (token = *) || (token = /)
    op = token; next = NextToken();
    if (next = Int n)
        token = NextToken();
        (root, incomplete) = TermPrime();
        if (root == NULL)
            root = new ExprNode(NULL, op, next);
            return (root, root);
        else
            newChild = new ExprNode(NULL, op, next);
            incomplete.leftChild = newChild;
            return(root, newChild);
    else throw SyntaxError
else return(NULL, NULL)
```

Missing left child
to be filled in by
caller

