18.404/6.840 Michael Sipser
Theory of Computation
Due via Gradescope 2:30pm sharp, Thursday, September 10, 2020 **Problem Set 1**

Read all of Chapters 1 and 2 except Section 2.4.

0.1 Read and solve, <u>but do not turn in</u>: Book, 1.14 . [swapping NFA accept/non-accept states]

0.2 Read and solve, <u>but do not turn in</u>: Book, 1.31 . [closure under reversal]

0.3 Read and solve, <u>but do not turn in</u>: Book, 1.46c . [Pumping lemma]

1. Let
$$\Sigma_3 = \left\{ \begin{bmatrix} 0\\0\\0 \end{bmatrix}, \begin{bmatrix} 0\\0\\1 \end{bmatrix}, \begin{bmatrix} 0\\1\\0 \end{bmatrix}, \ldots, \begin{bmatrix} 1\\1\\1 \end{bmatrix} \right\}.$$
Σ_3 contains all size 3 columns of 0s and 1s. A string of symbols in Σ_3 gives three rows of 0s and 1s. Consider each row to be a binary number and let
$$B = \{w \in \Sigma_3^* |\ \text{the bottom row of } w \text{ is the sum of the top two rows}\}.$$
For example, $\begin{bmatrix} 0\\0\\1 \end{bmatrix}\begin{bmatrix} 1\\0\\0 \end{bmatrix}\begin{bmatrix} 1\\1\\0 \end{bmatrix} \in B$, but $\begin{bmatrix} 0\\0\\1 \end{bmatrix}\begin{bmatrix} 1\\0\\1 \end{bmatrix} \notin B$. Show that B is regular.
You may assume the result claimed in Problem 0.2 above.

2. Let Σ_3 be the same as in Problem 1. Let
$$M = \{w \in \Sigma_3^* |\ \text{the bottom row of } w \text{ is the product of the top two rows}\}.$$
For example, $\begin{bmatrix} 0\\0\\1 \end{bmatrix}\begin{bmatrix} 0\\0\\0 \end{bmatrix}\begin{bmatrix} 1\\1\\0 \end{bmatrix}\begin{bmatrix} 1\\1\\1 \end{bmatrix} \in M$, but $\begin{bmatrix} 0\\0\\1 \end{bmatrix}\begin{bmatrix} 1\\1\\1 \end{bmatrix}\begin{bmatrix} 1\\0\\1 \end{bmatrix} \notin M$. Show that M is not regular.

3. Let $\Sigma = \{0, 1\}$.
   (a) Let $TUT = \{tut|\ t, u \in \Sigma^*\}$. Show TUT is regular.
   (b) Let $TUTU = \{tutu|\ t, u \in \Sigma^*\}$. Show $TUTU$ is not regular.

4. Let x and y be strings over some alphabet Σ. Say x is a ***substring*** of y if $y \in \Sigma^* x\, \Sigma^*$. Define the ***avoids*** operation for languages A and B to be
$$A \textit{ avoids } B = \{w|\ w \in A \text{ and } w \text{ doesn't contain any string in } B \text{ as a substring}\}.$$
Prove that the class of regular languages is closed under the *avoids* operation.
(Hint: Theorems we've shown may be helpful. You may assume the results of 0.1 – 0.3 above.)

5. Is the class of <u>non</u>regular languages closed under (a) union? (b) concatenation? (c) star? and (d) complementation? In each part, prove your answer.

6. Let M_1 and M_2 be DFAs that have k_1 and k_2 states, respectively, where $L(M_1) \neq L(M_2)$. Using an argument similar to the proof of the pumping lemma, show that there is a string s where $|s| \leq k_1 k_2$ and where exactly one of M_1 and M_2 accepts s.

7.* ($\star$ means optional) Improve the bound in Problem 6 to show that such an s exists where $|s| \leq k_1 + k_2$.