18.100B Problem Set 7

Due in class Wednesday, April 17. You may discuss the problems with other students, but you should write solutions entirely on your own.

1. Let f be the real-valued function on $[0, 1]$ defined by

$$f(x) = \begin{cases} 1/q & \text{if } x \text{ is rational and } x = p/q \text{ in lowest terms} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$$

Prove that f is Riemann-integrable, and that

$$\int_0^1 f(x)\,dx = 0.$$

2. Text, page 138, number 8.

3. Text, page 139, number 10, parts (a)–(c).

4. Text, page 141, number 16.