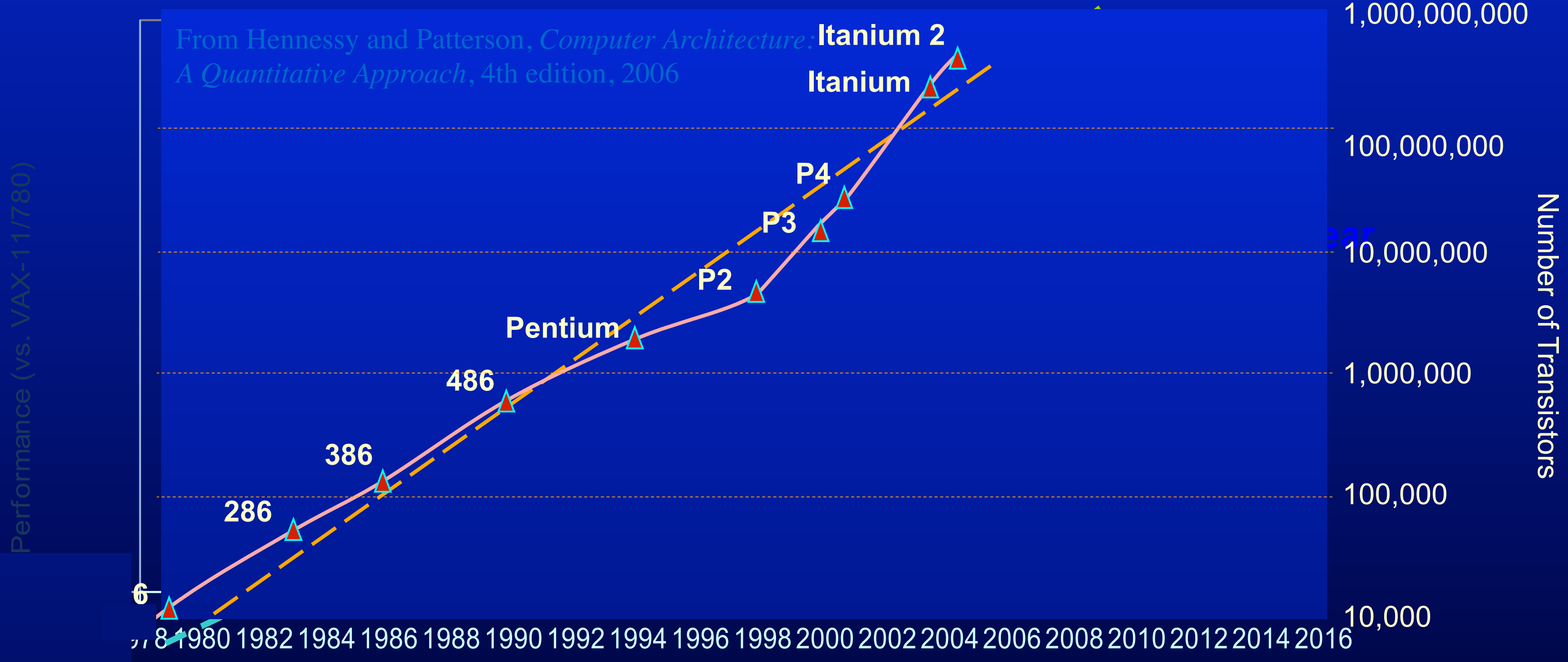
# 6.035

## Parallelization

## Outline

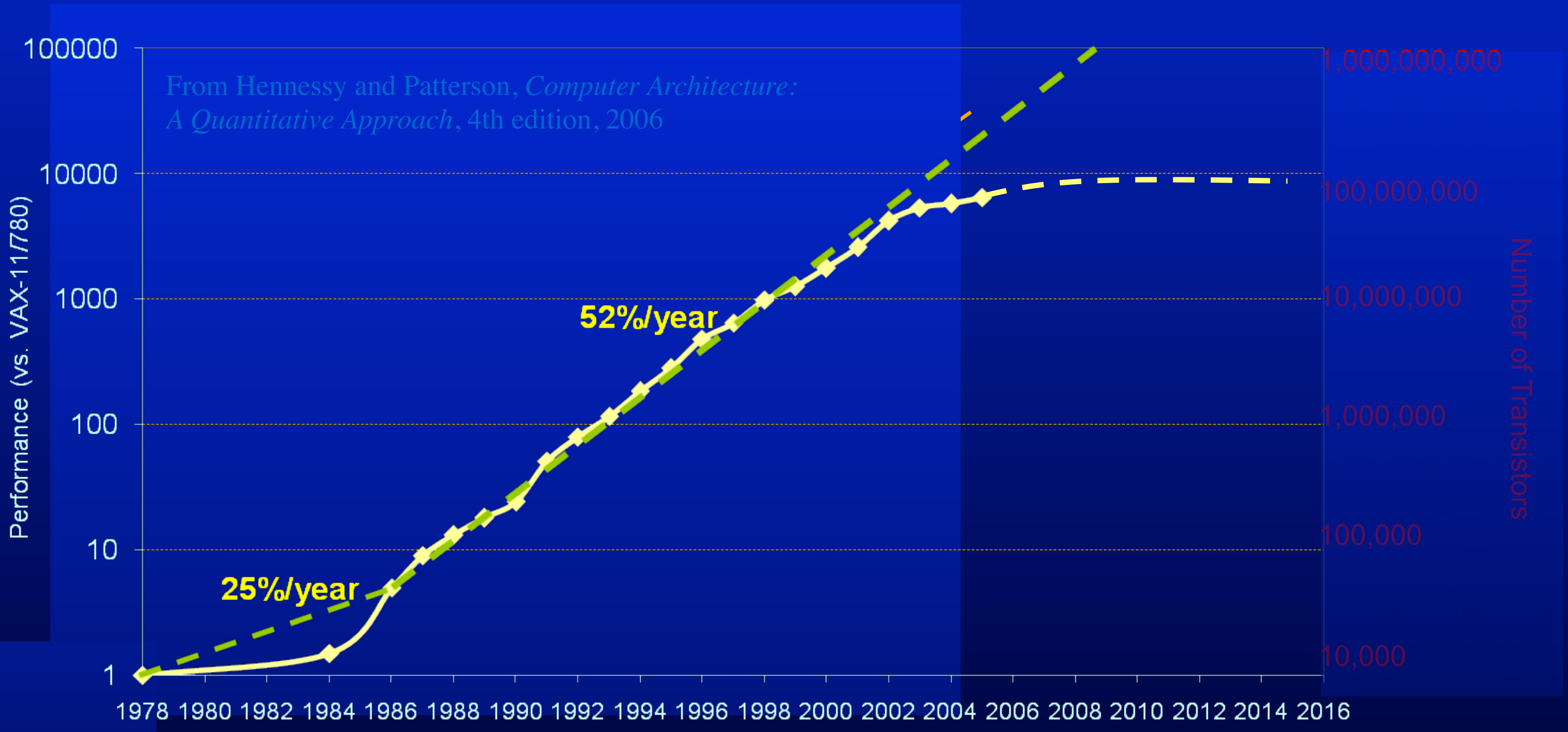
- **Why Parallelism**
- Parallel Execution
- Parallelizing Compilers
- Dependence Analysis
- Increasing Parallelization Opportunities

# Moore’s Law



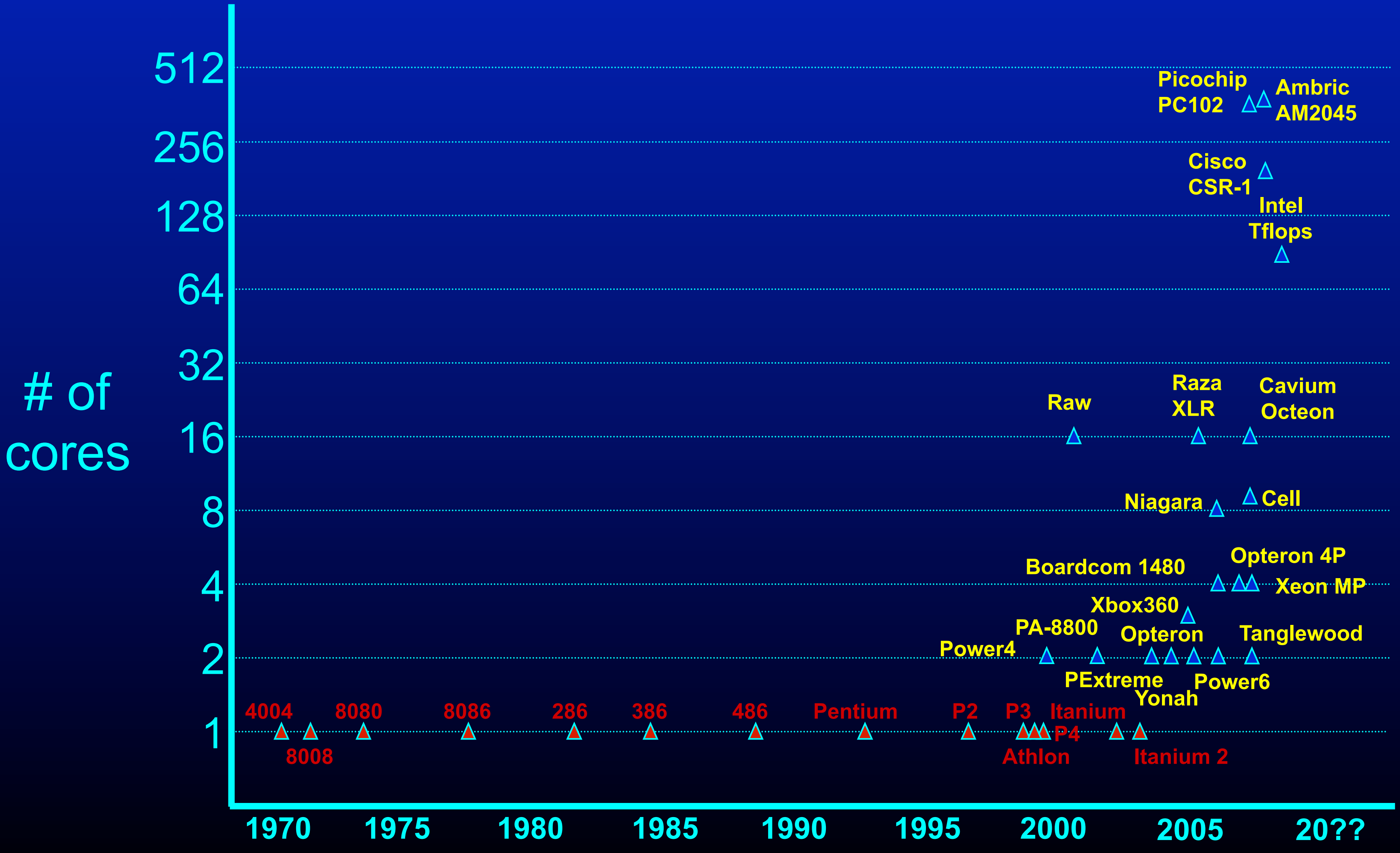
From David Patterson

# Uniprocessor Performance (SPECint)



From David Patterson

# Multicores Are Here!



## Issues with Parallelism

- Amdhal's Law
  - Any computation can be analyzed in terms of a portion that must be executed sequentially, Ts, and a portion that can be executed in parallel, Tp. Then for n processors:
  - T(n) = Ts + Tp/n
  - T(∞) = Ts, thus maximum speedup (Ts + Tp) /Ts
- Load Balancing
  - The work is distributed among processors so that ***all*** processors are kept busy when parallel task is executed.
- Granularity
  - The size of the parallel regions between synchronizations or the ratio of computation (useful work) to communication (overhead).

# Outline

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- **Parallel Execution**
- Parallelizing Compilers
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# Types of Parallelism

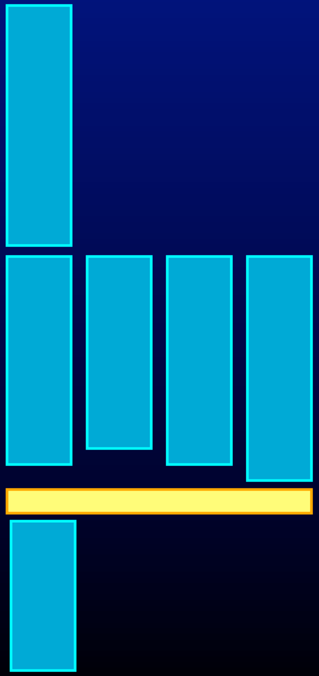
| | |
|---|---|
| • Instruction Level Parallelism (ILP) | → Scheduling and Hardware |
| • Task Level Parallelism (TLP) | → Mainly by hand |
| • Loop Level Parallelism (LLP) or Data Parallelism | → Hand or Compiler Generated |
| • Pipeline Parallelism | → Hardware or Streaming |
| • Divide and Conquer Parallelism | → Recursive functions |

## Why Loops?

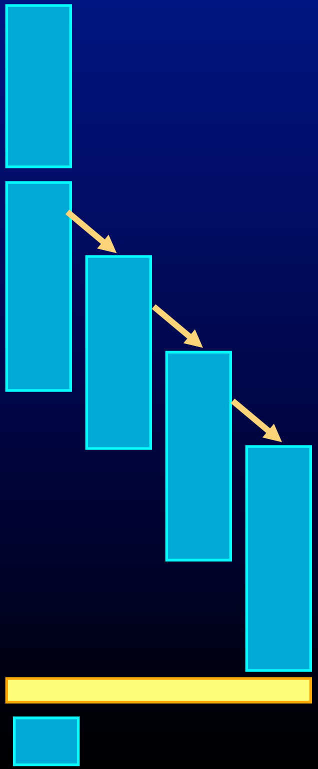
- 90% of the execution time in 10% of the code
  - Mostly in loops
- If parallel, can get good performance
  - Load balancing
- Relatively easy to analyze

# Programmer Defined Parallel Loop

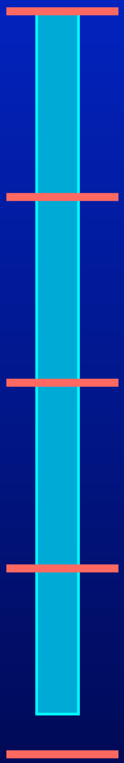
- FORALL
  - No “loop carried dependences”
  - Fully parallel



- FORACROSS
  - Some “loop carried dependences”



# Parallel Execution



- Example

```
FORPAR I = 0 to N
   A[I] = A[I] + 1
```

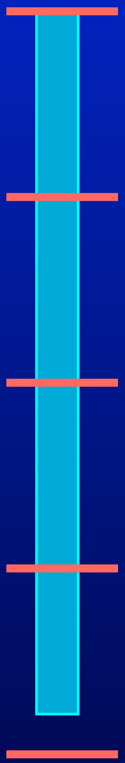
- Block Distribution: Program gets mapped into

```
Iters = ceiling(N/NUMPROC);
FOR P = 0 to NUMPROC-1
   FOR I = P*Iters to MIN((P+1)*Iters, N)
     A[I] = A[I] + 1
```

- SPMD (Single Program, Multiple Data) Code

```
If(myPid == 0) {
   …
   Iters = ceiling(N/NUMPROC);
}
Barrier();
FOR I = myPid*Iters to MIN((myPid+1)*Iters, N)
   A[I] = A[I] + 1
Barrier();
```

# Parallel Execution



- Example

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- Block Distribution: Program gets mapped into

```
Iters = ceiling(N/NUMPROC);
FOR P = 0 to NUMPROC-1
   FOR I = P*Iters to MIN((P+1)*Iters, N)
     A[I] = A[I] + 1
```

- Code fork a function

```
Iters = ceiling(N/NUMPROC);
FOR P = 0 to NUMPROC – 1 { ParallelExecute(func1, P); }
BARRIER(NUMPROC);
void func1(integer myPid)
{
   FOR I = myPid*Iters to MIN((myPid+1)*Iters, N)
     A[I] = A[I] + 1
}
```

## Parallel Execution

- SPMD
  - Need to get all the processors to execute the control flow
    - Extra synchronization overhead or redundant computation on all processors or both
  - Stack: Private or Shared?
- Fork
  - Local variables not visible within the function
    - Either make the variables used/defined in the loop body global or pass and return them as arguments
    - Function call overhead

## Parallel Thread Basics

- Create separate threads
  - Create an OS thread
    - (hopefully) it will be run on a separate core
  - pthread_create(&thr, NULL, &entry_point, NULL)
  - Overhead in thread creation
    - Create a separate stack
    - Get the OS to allocate a thread
- Thread pool
  - Create all the threads (= num cores) at the beginning
  - Keep N-1 idling on a barrier, while sequential execution
  - Get them to run parallel code by each executing a function
  - Back to the barrier when parallel region is done

## Outline

- Why Parallelism
- Parallel Execution
- **Parallelizing Compilers**
- Dependence Analysis
- Increasing Parallelization Opportunities

## Parallelizing Compilers

- Finding FORALL Loops out of FOR loops

- Examples

```
FOR I = 0 to 5
  A[I] = A[I] + 1

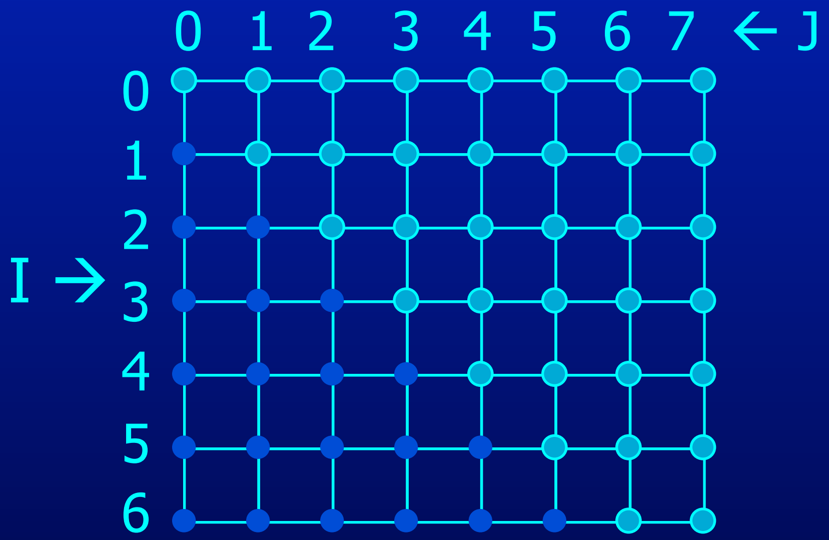
FOR I = 0 to 5
  A[I] = A[I+6] + 1

For I = 0 to 5
  A[2*I] = A[2*I + 1] + 1
```

# Iteration Space

- N deep loops → N-dimensional discrete iteration space
  - Normalized loops: assume step size = 1

```
FOR I = 0 to 6
   FOR J = I to 7
```



- Iterations are represented as coordinates in iteration space
  - $\bar{i} = [i_1, i_2, i_3, \ldots, i_n]$

## Iteration Space

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  - Normalized loops: assume step size = 1

```
FOR I = 0 to 6
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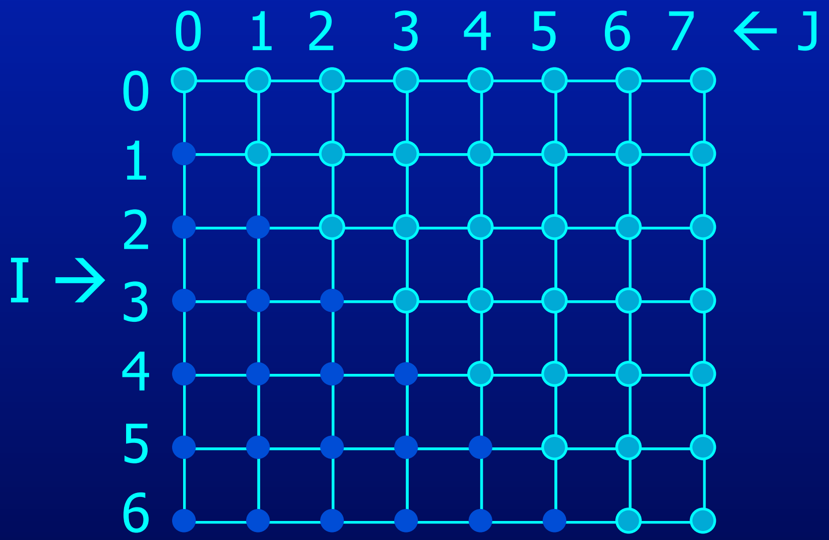

- Iterations are represented as coordinates in iteration space
- Sequential execution order of iterations → Lexicographic order

```
[0,0],  [0,1], [0,2], …, [0,6],  [0,7],
        [1,1], [1,2], …, [1,6],  [1,7],
               [2,2], …, [2,6],  [2,7],
                         ………
                         [6,6],  [6,7],
```

# Iteration Space

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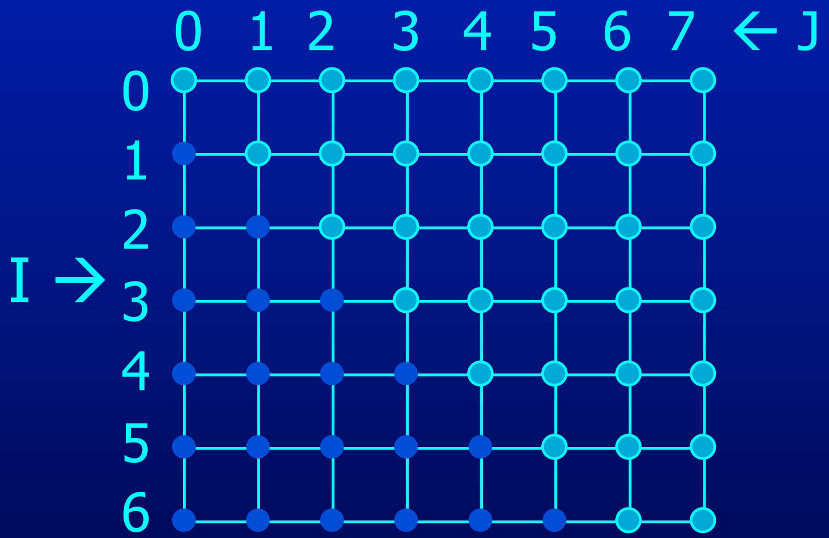
```
FOR I = 0 to 6
   FOR J = I to 7
```



- Iterations are represented as coordinates in iteration space
- Sequential execution order of iterations ➔ Lexicographic order
- Iteration $\bar{i}$ is lexicograpically less than $\bar{j}$, $\bar{i} < \bar{j}$ iff
  there exists c s.t. $i_1 = j_1$, $i_2 = j_2$,… $i_{c-1} = j_{c-1}$ and $i_c < j_c$

# Iteration Space

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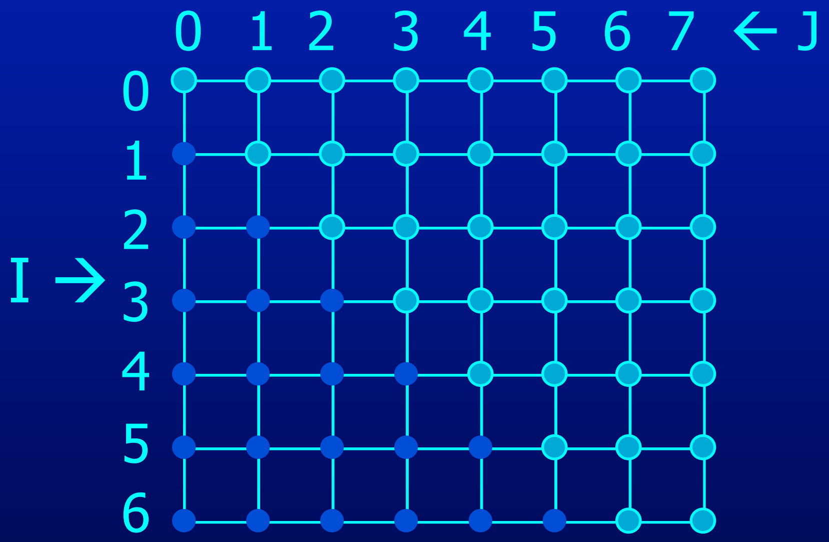
FOR I = 0 to 6
   FOR J = I to 7
```



- An affine loop nest
  - Loop bounds are integer linear functions of constants, loop constant variables and outer loop indexes
  - Array accesses are integer linear functions of constants, loop constant variables and loop indexes

# Iteration Space

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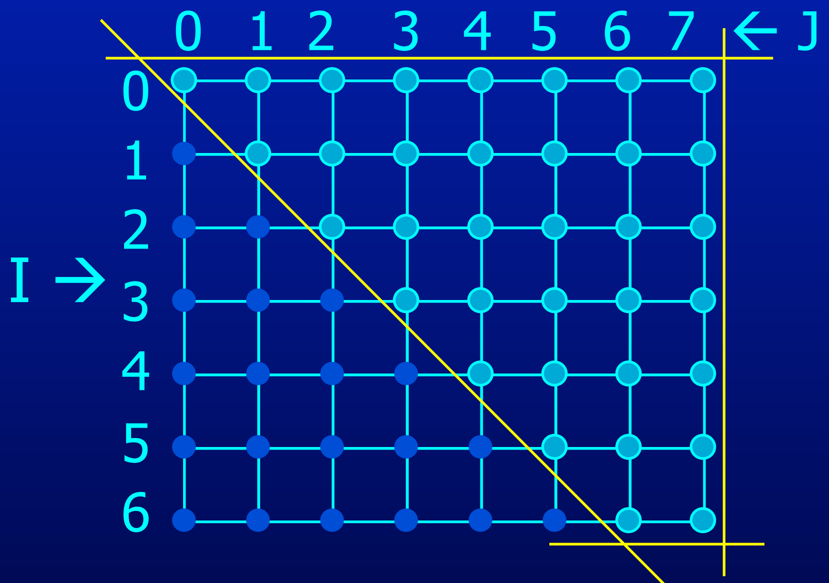
FOR I = 0 to 6
   FOR J = I to 7
```



- Affine loop nest → Iteration space as a set of linear inequalities

$0 \le I$

$I \le 6$

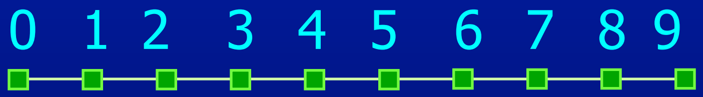
$I \le J$

$J \le 7$

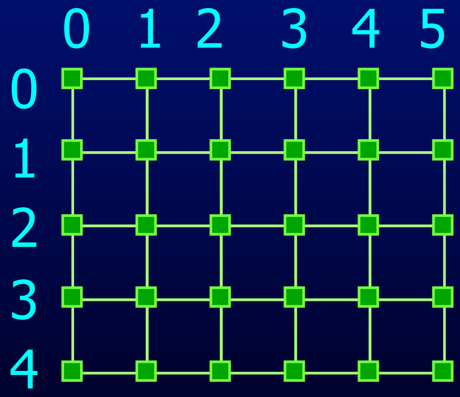
# Data Space

- M dimensional arrays → M-dimensional discrete cartesian space
  - a hypercube

```
Integer A(10)
```



```
Float B(5, 6)
```



# Dependences

- True dependence
```
a  =
   =  a
```

- Anti dependence
```
   =  a
a  =
```

- Output dependence
```
a  =
a  =
```

- Definition:
  Data dependence exists for a dynamic instance i and j iff
  - either i or j is a write operation
  - i and j refer to the same variable
  - i executes before j

- How about array accesses within loops?

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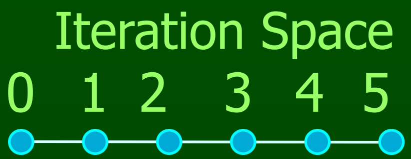
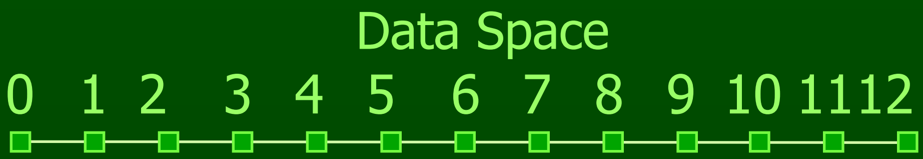
# Array Accesses in a loop

```
FOR I = 0 to 5
  A[I] = A[I] + 1
```





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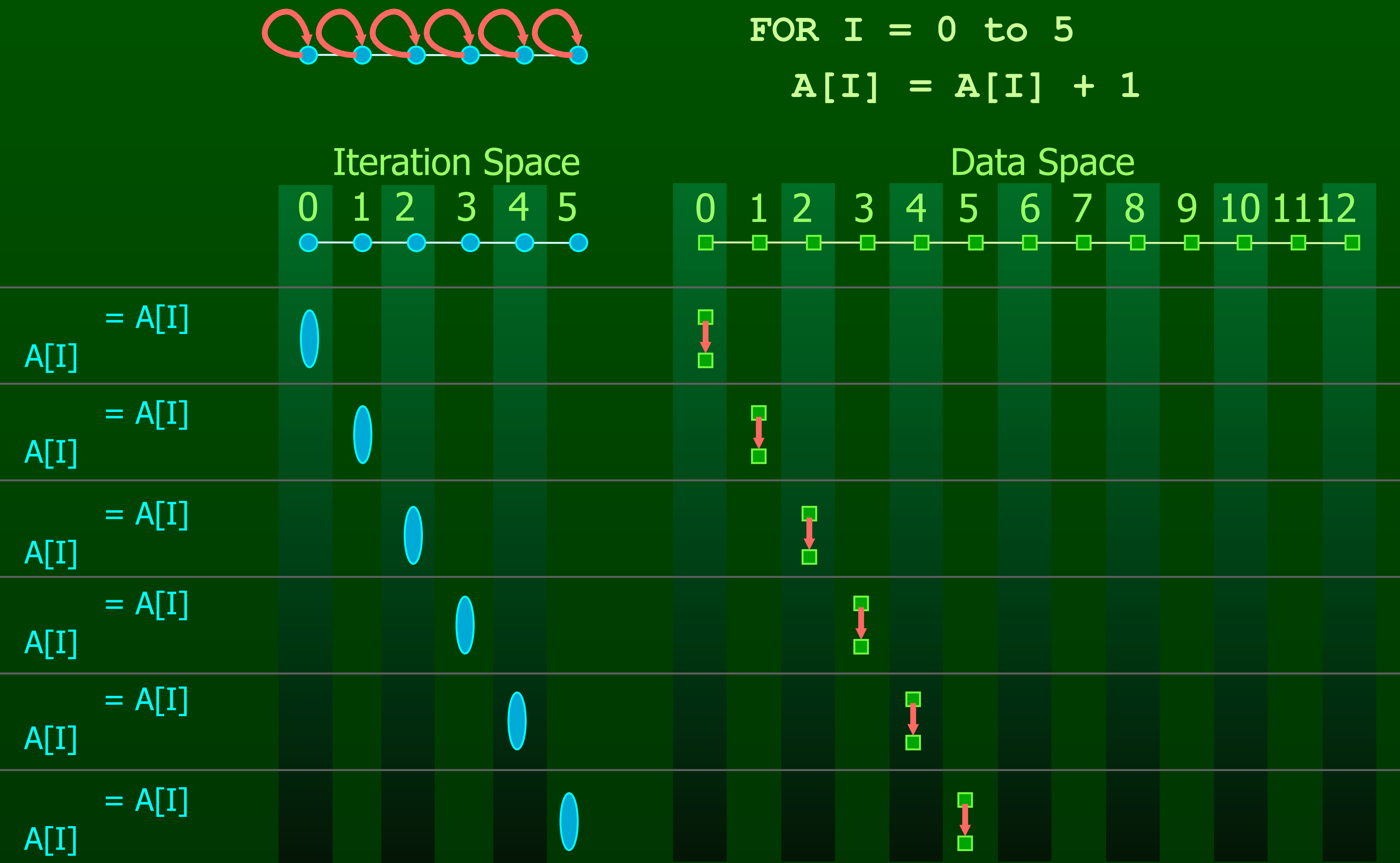
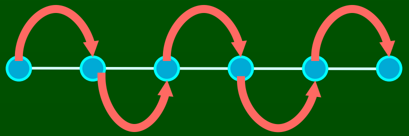
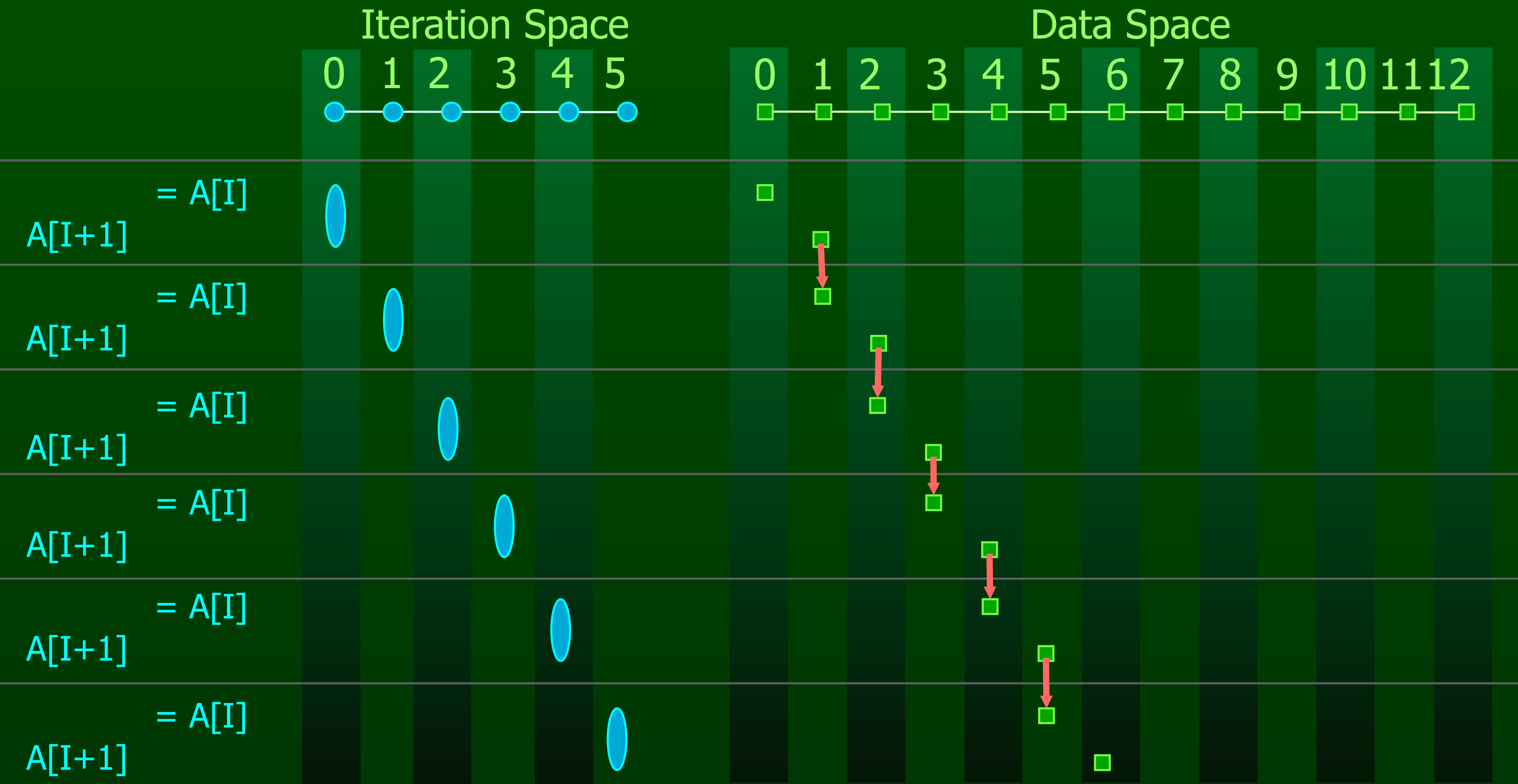


# Array Accesses in a loop

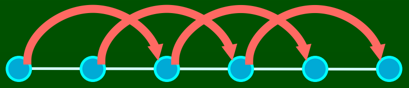


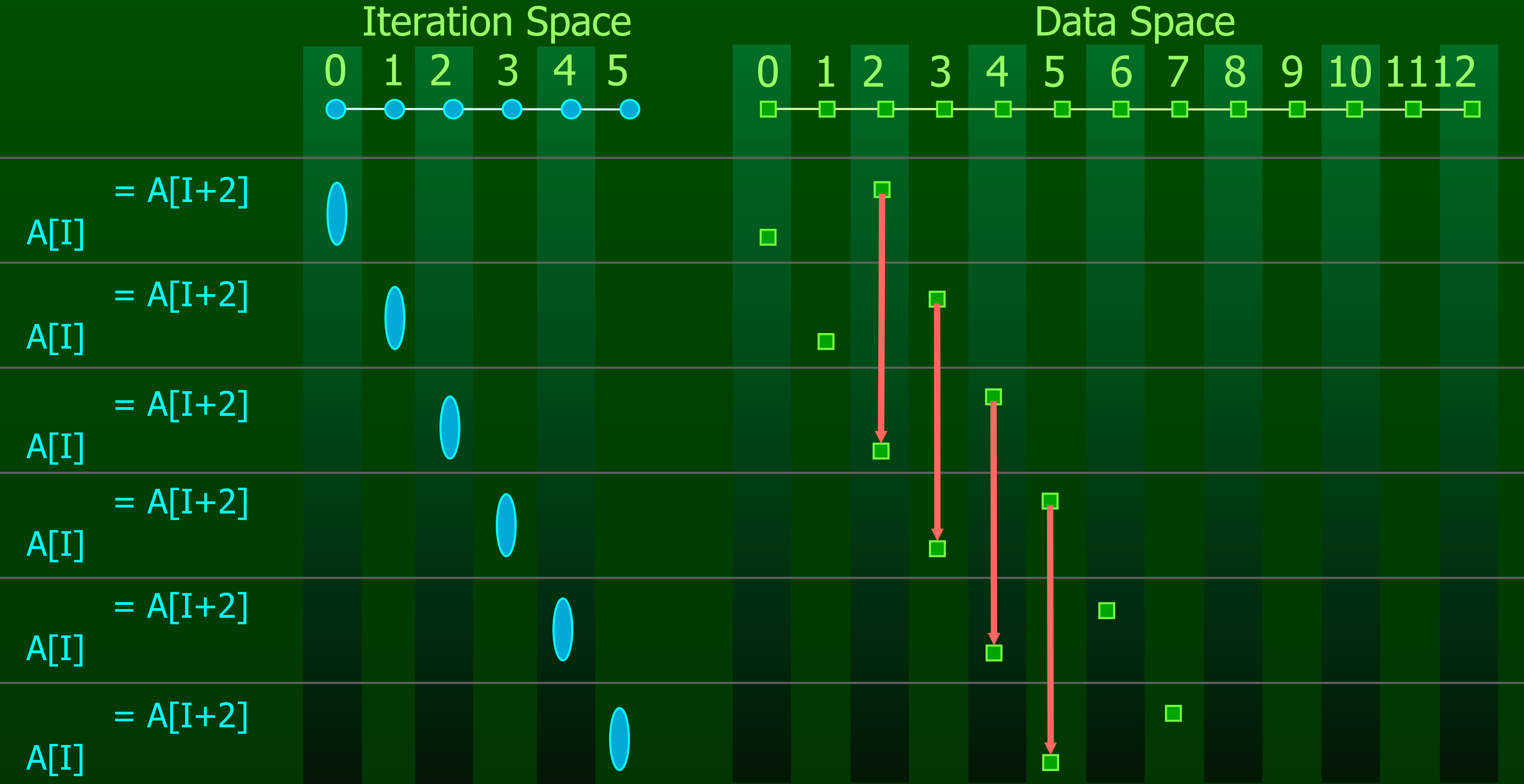
```
FOR I = 0 to 5
  A[I+1] = A[I] + 1
```



# Array Accesses in a loop

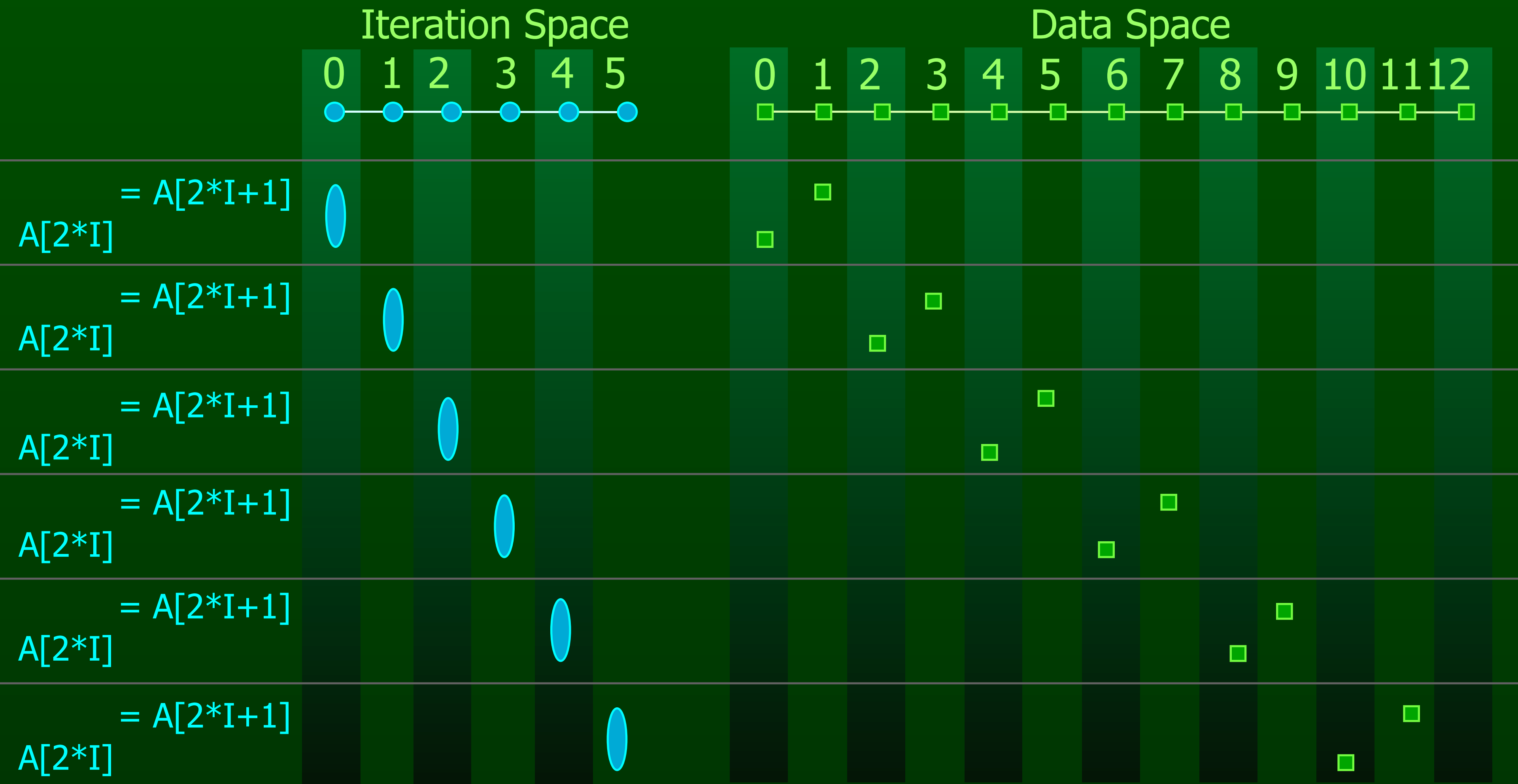
```
FOR I = 0 to 5
  A[I] = A[I+2] + 1
```





# Array Accesses in a loop

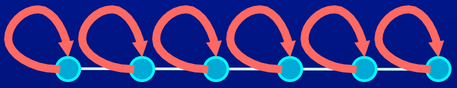
```
FOR I = 0 to 5
  A[2*I] = A[2*I+1] + 1
```



# Distance Vectors

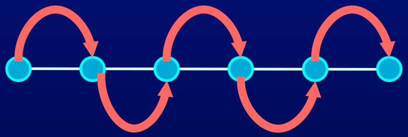
- A loop has a distance d if there exist a data dependence from iteration i to j and d = j-i

$dv = [0]$



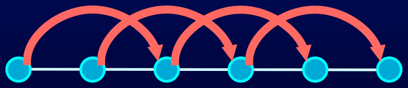
```
FOR I = 0 to 5
  A[I] = A[I] + 1
```

$dv = [1]$



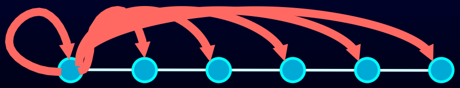
```
FOR I = 0 to 5
  A[I+1] = A[I] + 1
```

$dv = [2]$



```
FOR I = 0 to 5
  A[I] = A[I+2] + 1
```

$dv = [1], \quad [2] \quad \ldots \quad = \quad [*]$

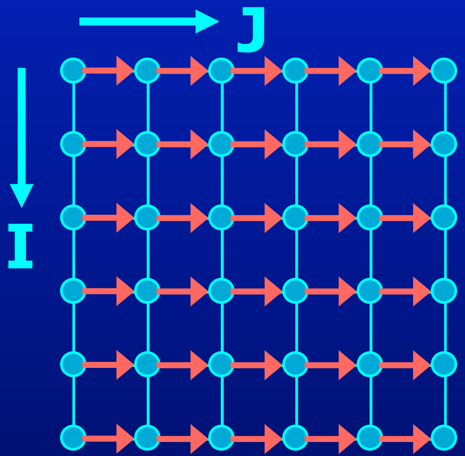


```
FOR I = 0 to 5
  A[I] = A[0] + 1
```

# Multi-Dimensional Dependence

```
FOR I = 1 to n
  FOR J = 1 to n
    A[I, J] = A[I, J-1] + 1
```

$$dv = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$



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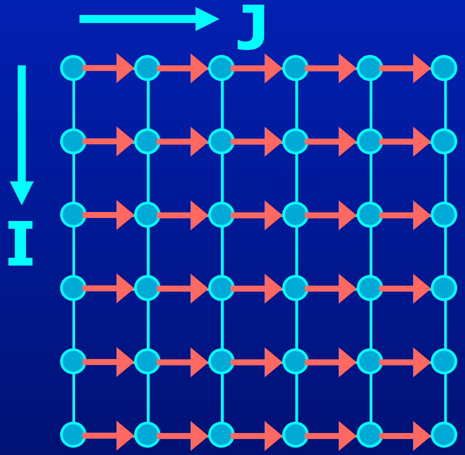
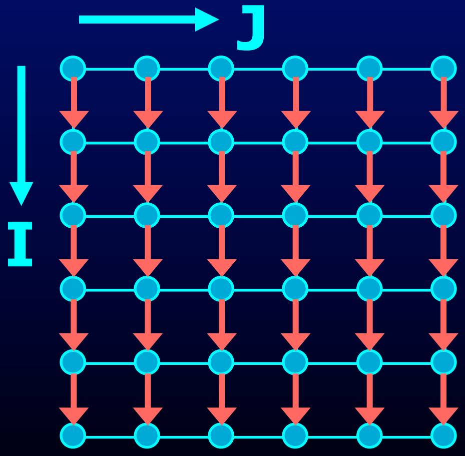


```
FOR I = 1 to n
  FOR J = 1 to n
   A[I, J] = A[I+1, J] + 1
```

$$dv = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

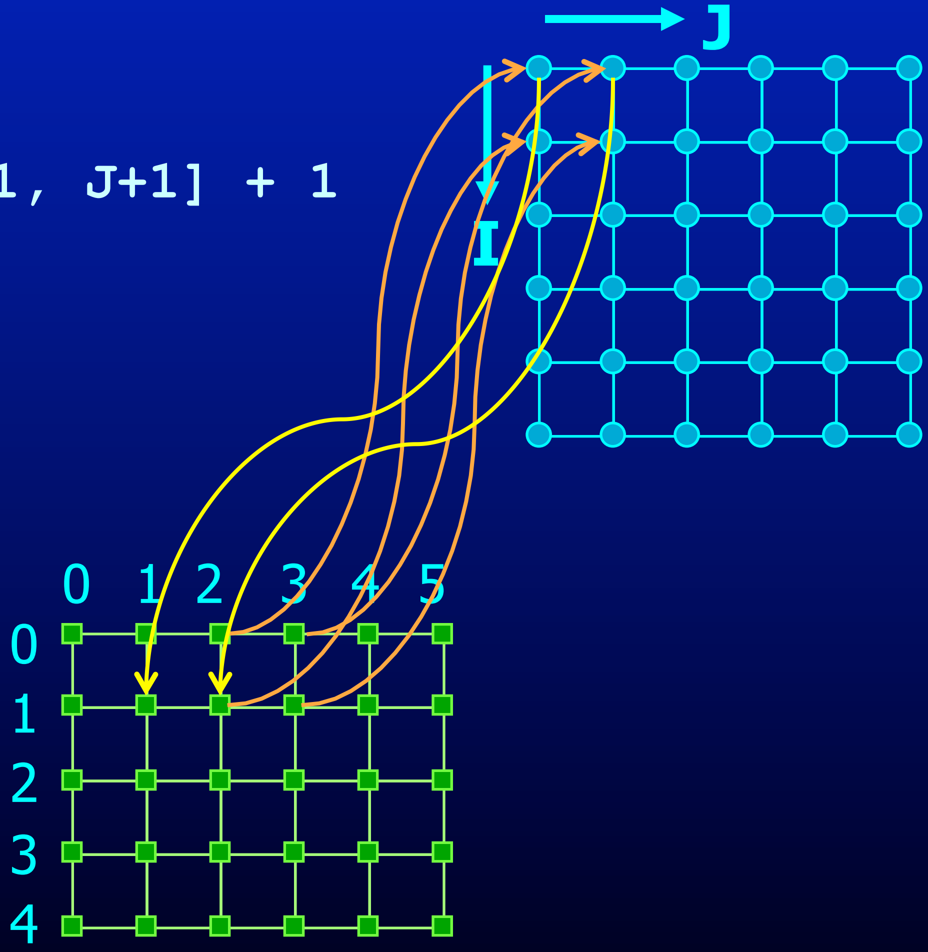


## Outline

- Dependence Analysis
- Increasing Parallelization Opportunities

# What is the Dependence?

```
FOR I = 1 to n
  FOR J = 1 to n
    A[I, J] = A[I-1, J+1] + 1
```



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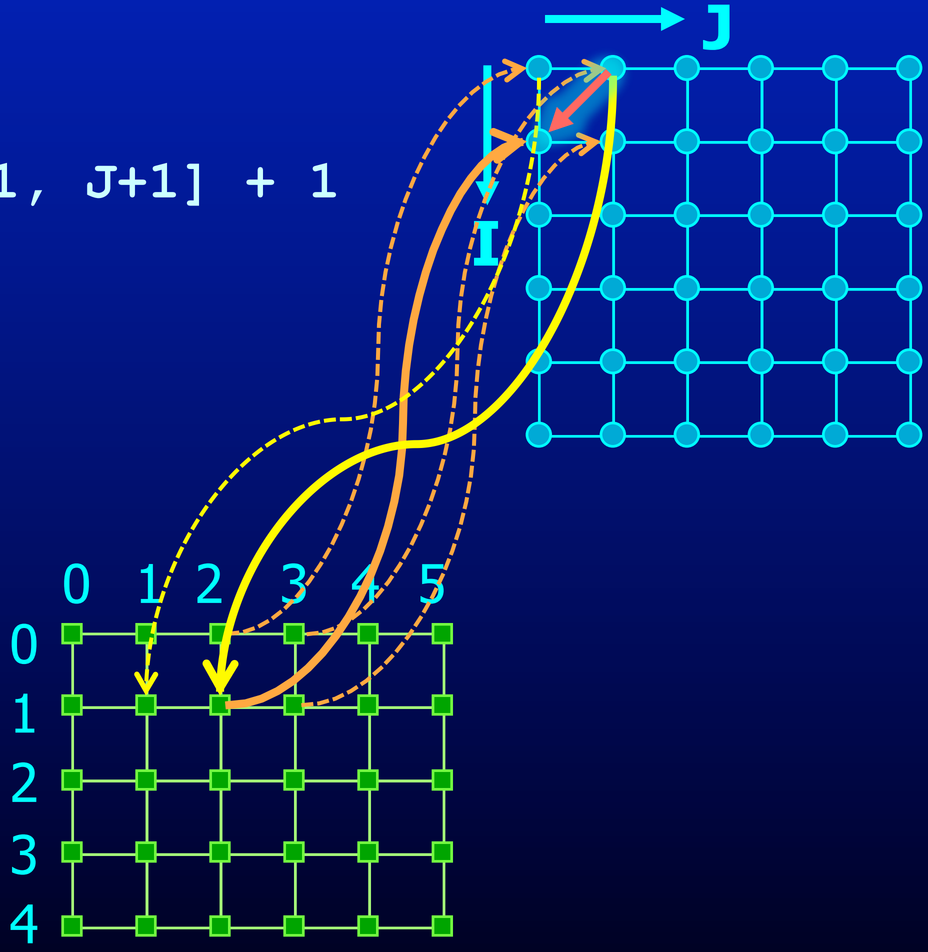
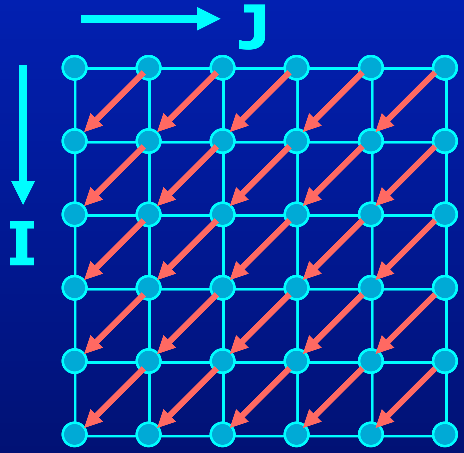


# What is the Dependence?

```
FOR I = 1 to n
  FOR J = 1 to n
    A[I, J] = A[I-1, J+1] + 1
```

$$dv = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$



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```
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  FOR J = 1 to n
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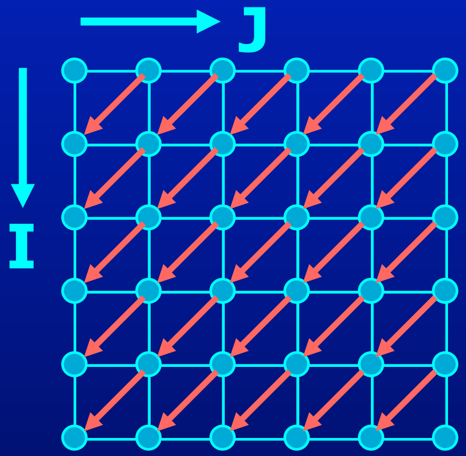
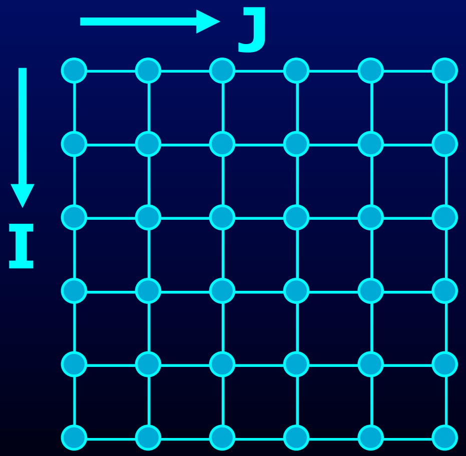
```



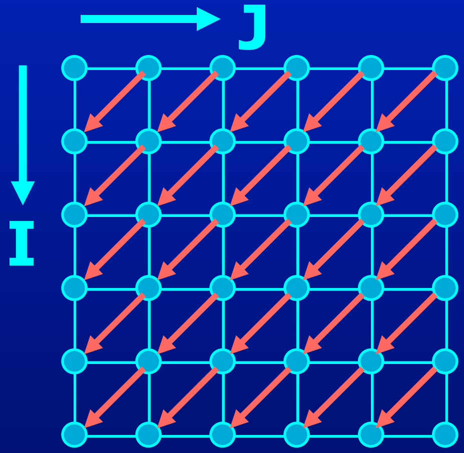
```
FOR I = 1 to n
  FOR J = 1 to n
   B[I] = B[I-1] + 1
```



# What is the Dependence?

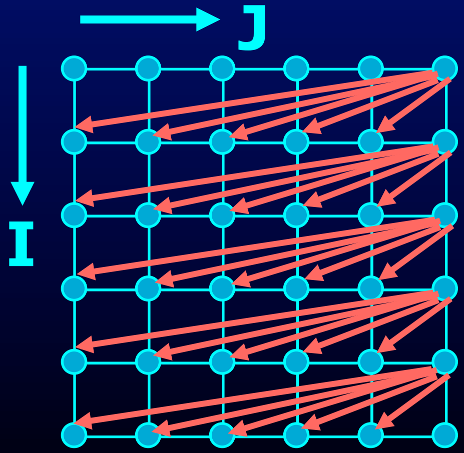
```
FOR I = 1 to n
  FOR J = 1 to n
   A[I, J] = A[I-1, J+1] + 1
```

$$dv = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$



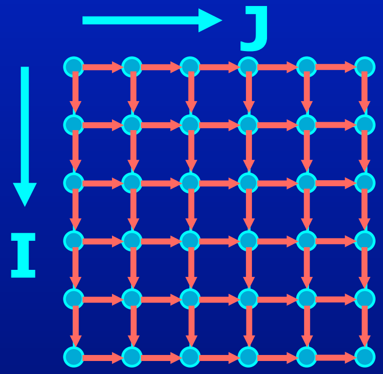
```
FOR I = 1 to n
  FOR J = 1 to n
   B[I] = B[I-1] + 1
```

$$dv = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \quad \ldots = \begin{bmatrix} 1 \\ * \end{bmatrix}$$



# What is the Dependence?

```
FOR i = 1 to N-1
  FOR j = 1 to N-1
   A[i,j] = A[i,j-1] + A[i-1,j];
```



$$dv = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

# Recognizing FORALL Loops

- Find data dependences in loop
  - For every pair of array acceses to the same array

    If the first access has at least one dynamic instance (an iteration) in which it refers to a location in the array that the second access also refers to in at least one of the later dynamic instances (iterations).

    Then there is a data dependence between the statements
  - (Note that same array can refer to itself – output dependences)
- Definition
  - Loop-carried dependence:
    dependence that crosses a loop boundary
- If there are no loop carried dependences → parallelizable

## Data Dependence Analysis

- I: Distance Vector method
- II: Integer Programming

# Distance Vector Method

- The ith loop is parallelizable for all dependence $d = [d_1,\ldots,d_i,..d_n]$
  either

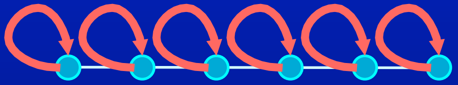
      one of $d_1,\ldots,d_{i-1}$ is $> 0$

  or

      all $d_1,\ldots,d_i = 0$

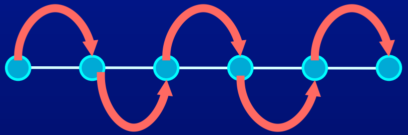
# Is the Loop Parallelizable?

$dv = \begin{bmatrix} 0 \end{bmatrix}$ Yes



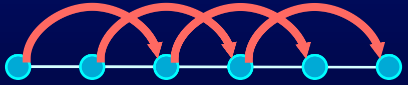
```
FOR I = 0 to 5
  A[I] = A[I] + 1
```

$dv = \begin{bmatrix} 1 \end{bmatrix}$ No



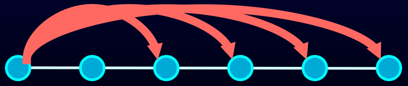
```
FOR I = 0 to 5
  A[I+1] = A[I] + 1
```

$dv = \begin{bmatrix} 2 \end{bmatrix}$ No



```
FOR I = 0 to 5
  A[I] = A[I+2] + 1
```

$dv = \begin{bmatrix} * \end{bmatrix}$ No



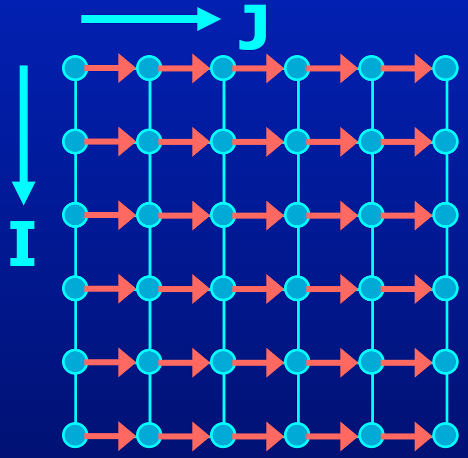
```
FOR I = 0 to 5
  A[I] = A[0] + 1
```

# Are the Loops Parallelizable?

```
FOR I = 1 to n
  FOR J = 1 to n
   A[I, J] = A[I, J-1] + 1
```

$$dv = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

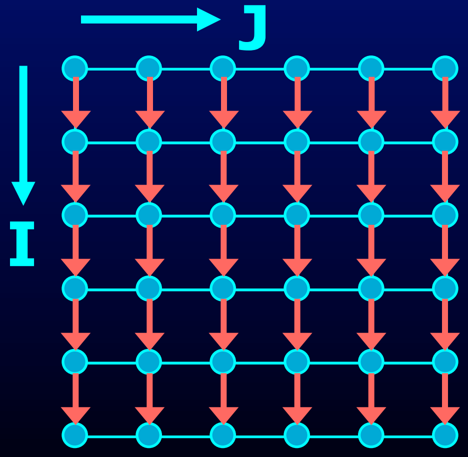
Yes
No



```
FOR I = 1 to n
  FOR J = 1 to n
   A[I, J] = A[I+1, J] + 1
```

$$dv = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

No
Yes



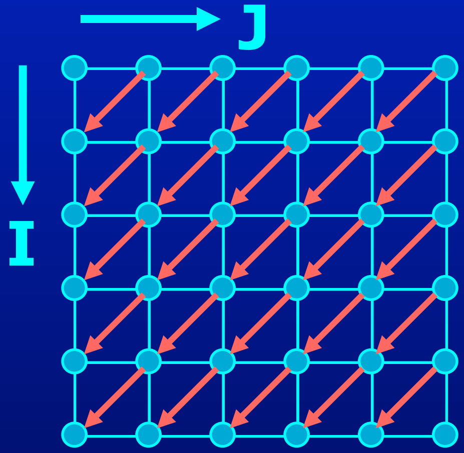
# Are the Loops Parallelizable?

```
FOR I = 1 to n
  FOR J = 1 to n
   A[I, J] = A[I-1, J+1] + 1
```

$$dv = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

No

Yes

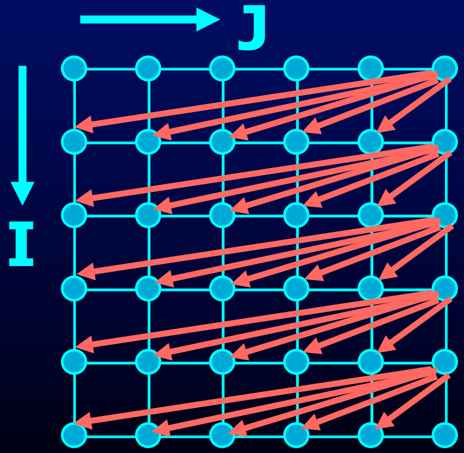


```
FOR I = 1 to n
  FOR J = 1 to n
   B[I] = B[I-1] + 1
```

$$dv = \begin{bmatrix} 1 \\ * \end{bmatrix}$$

No

Yes



# Integer Programming Method

- Example

```
FOR I = 0 to 5
  A[I+1] = A[I] + 1
```

- Is there a loop-carried dependence between A[I+1] and A[I]
  - Are there two distinct iterations $i_w$ and $i_r$ such that A[$i_w$+1] is the same location as A[$i_r$]
  - $\exists$ integers $i_w, i_r$ $\quad 0 \le i_w, i_r \le 5 \quad i_w \ne i_r \quad i_w + 1 = i_r$
- Is there a dependence between A[I+1] and A[I+1]
  - Are there two distinct iterations $i_1$ and $i_2$ such that A[$i_1$+1] is the same location as A[$i_2$+1]
  - $\exists$ integers $i_1, i_2$ $\quad 0 \le i_1, i_2 \le 5 \quad i_1 \ne i_2 \quad i_1 + 1 = i_2 + 1$

# Integer Programming Method

```
FOR I = 0 to 5
  A[I+1] = A[I] + 1
```

- Formulation
  - $\exists$ an integer vector $\bar{\imath}$ such that $\hat{A}\,\bar{\imath} \le \bar{b}$ where $\hat{A}$ is an integer matrix and $\bar{b}$ is an integer vector

# Iteration Space

```
FOR I = 0 to 5
  A[I+1] = A[I] + 1
```

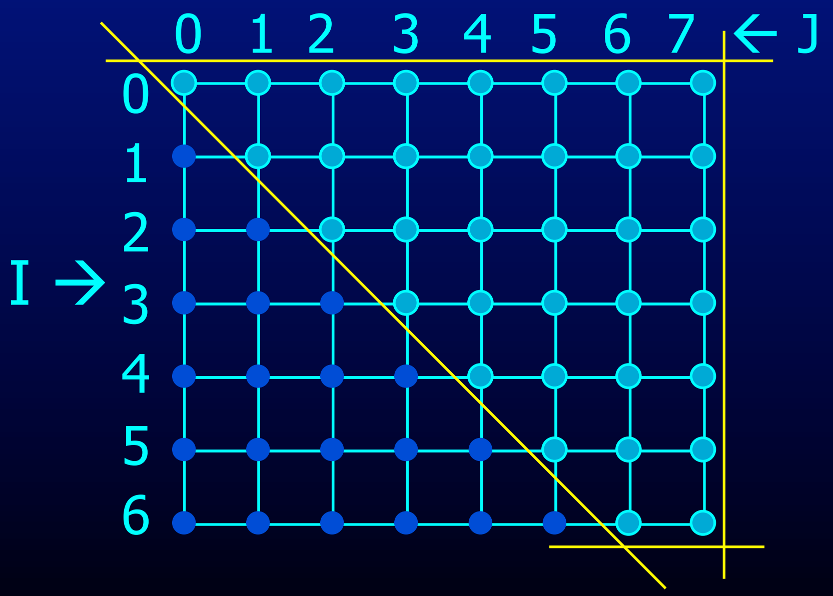
- N deep loops → n-dimensional discrete cartesian space
- Affine loop nest → Iteration space as a set of linear inequalities

$0 \le I$

$I \le 6$

$I \le J$

$J \le 7$



# Integer Programming Method

```
FOR I = 0 to 5
  A[I+1] = A[I] + 1
```

- Formulation
  - $\exists$ an integer vector $\bar{\imath}$ such that $\hat{A}\,\bar{\imath} \le \bar{b}$ where $\hat{A}$ is an integer matrix and $\bar{b}$ is an integer vector

- Our problem formulation for A[i] and A[i+1]
  - $\exists$ integers $i_w, i_r$ $0 \le i_w, i_r \le 5$ $i_w \neq i_r$ $i_w + 1 = i_r$
  - $i_w \neq i_r$ is not an affine function
    - divide into 2 problems
    - Problem 1 with $i_w < i_r$ and problem 2 with $i_r < i_w$
    - If either problem has a solution → there exists a dependence
  - How about $i_w + 1 = i_r$
    - Add two inequalities to single problem $i_w + 1 \le i_r$, and $i_r \le i_w + 1$

# Integer Programming Formulation

```
FOR I = 0 to 5
  A[I+1] = A[I] + 1
```

- Problem 1

  $0 \leq i_w$

  $i_w \leq 5$

  $0 \leq i_r$

  $i_r \leq 5$

  $i_w < i_r$

  $i_w + 1 \leq i_r$

  $i_r \leq i_w + 1$

# Integer Programming Formulation

```
FOR I = 0 to 5
  A[I+1] = A[I] + 1
```

- Problem 1

| | | |
|---|---|---|
| $0 \leq i_w$ | $\rightarrow$ | $-i_w \leq 0$ |
| $i_w \leq 5$ | $\rightarrow$ | $i_w \leq 5$ |
| $0 \leq i_r$ | $\rightarrow$ | $-i_r \leq 0$ |
| $i_r \leq 5$ | $\rightarrow$ | $i_r \leq 5$ |
| $i_w < i_r$ | $\rightarrow$ | $i_w - i_r \leq -1$ |
| $i_w + 1 \leq i_r$ | $\rightarrow$ | $i_w - i_r \leq -1$ |
| $i_r \leq i_w + 1$ | $\rightarrow$ | $-i_w + i_r \leq 1$ |

# Integer Programming Formulation

- Problem 1

$$\begin{array}{lcl} 0 \le i_w & \rightarrow & -i_w \le 0 \\ i_w \le 5 & \rightarrow & i_w \le 5 \\ 0 \le i_r & \rightarrow & -i_r \le 0 \\ i_r \le 5 & \rightarrow & i_r \le 5 \\ i_w < i_r & \rightarrow & i_w - i_r \le -1 \\ i_w + 1 \le i_r & \rightarrow & i_w - i_r \le -1 \\ i_r \le i_w + 1 & \rightarrow & -i_w + i_r \le 1 \end{array}
\qquad
\begin{array}{cc} \hat{A} & \bar{b} \\ \begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 1 & -1 \\ 1 & -1 \\ -1 & 1 \end{pmatrix} & \begin{pmatrix} 0 \\ 5 \\ 0 \\ 5 \\ -1 \\ -1 \\ 1 \end{pmatrix} \end{array}$$

- and problem 2 with $i_r < i_w$

## Generalization

- An affine loop nest

```
FOR $i_1$ = $f_{l1}(c_1…c_k)$ to $I_{u1}(c_1…c_k)$
  FOR $i_2$ = $f_{l2}(i_1,c_1…c_k)$ to $I_{u2}(i_1,c_1…c_k)$
    ……
      FOR $i_n$ = $f_{ln}(i_1…i_{n-1},c_1…c_k)$ to $I_{un}(i_1…i_{n-1},c_1…c_k)$
          A[$f_{a1}(i_1…i_n,c_1…c_k)$, $f_{a2}(i_1…i_n,c_1…c_k)$,…,$f_{am}(i_1…i_n,c_1…c_k)$]
```

- Solve 2*n problems of the form
  - $i_1 = j_1,\ i_2 = j_2, …… i_{n-1} = j_{n-1},\ i_n < j_n$
  - $i_1 = j_1,\ i_2 = j_2, …… i_{n-1} = j_{n-1},\ j_n < i_n$
  - $i_1 = j_1,\ i_2 = j_2, …… i_{n-1} < j_{n-1}$
  - $i_1 = j_1,\ i_2 = j_2, …… j_{n-1} < i_{n-1}$

  ……………………
  - $i_1 = j_1,\ i_2 < j_2$
  - $i_1 = j_1,\ j_2 < i_2$
  - $i_1 < j_1$
  - $j_1 < i_1$

## Outline

- Why Parallelism
- Parallel Execution
- Parallelizing Compilers
- Dependence Analysis
- **Increasing Parallelization Opportunities**

## Increasing Parallelization Opportunities

- Scalar Privatization
- Reduction Recognition
- Induction Variable Identification
- Array Privatization
- Loop Transformations
- Granularity of Parallelism
- Interprocedural Parallelization

## Scalar Privatization

- Example

```
FOR i = 1 to n
  X = A[i] * 3;
  B[i] = X;
```

- Is there a loop carried dependence?
- What is the type of dependence?

# Privatization

- Analysis:
  - Any anti- and output- loop-carried dependences

- Eliminate by assigning in local context

```
FOR i = 1 to n
  integer Xtmp;
  Xtmp = A[i] * 3;
  B[i] = Xtmp;
```

- Eliminate by expanding into an array

```
FOR i = 1 to n
  Xtmp[i] = A[i] * 3;
  B[i] = Xtmp[i];
```

# Privatization

- Need a final assignment to maintain the correct value after the loop nest

- Eliminate by assigning in local context

```
FOR i = 1 to n
  integer Xtmp;
  Xtmp = A[i] * 3;
  B[i] = Xtmp;
  if(i == n) X = Xtmp
```

- Eliminate by expanding into an array

```
FOR i = 1 to n
  Xtmp[i] = A[i] * 3;
  B[i] = Xtmp[i];
X = Xtmp[n];
```

## Another Example

- How about loop-carried true dependences?
- Example

```
FOR i = 1 to n
  X = X + A[i];
```

- Is this loop parallelizable?

## Reduction Recognition

- Reduction Analysis:
  - Only associative operations
  - The result is never used within the loop
- Transformation

```
Integer Xtmp[NUMPROC];
Barrier();
FOR i = myPid*Iters to MIN((myPid+1)*Iters, n)
    Xtmp[myPid] = Xtmp[myPid] + A[i];
Barrier();
If(myPid == 0) {
  FOR p = 0 to NUMPROC-1
    X = X + Xtmp[p];
  …
```

# Induction Variables

- Example

```
FOR i = 0 to N
  A[i] = 2^i;
```

- After strength reduction

```
t = 1
FOR i = 0 to N
  A[i] = t;
  t = t*2;
```

- What happened to loop carried dependences?
- Need to do opposite of this!
  - Perform induction variable analysis
  - Rewrite IVs as a function of the loop variable

# Array Privatization

- Similar to scalar privatization

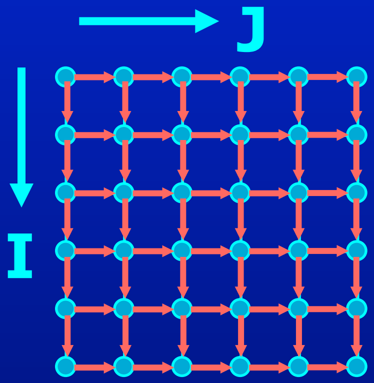
- However, analysis is more complex
  - Array Data Dependence Analysis:
    Checks if two iterations access the same location
  - Array Data Flow Analysis:
    Checks if two iterations access the same value

- Transformations
  - Similar to scalar privatization
  - Private copy for each processor or expand with an additional dimension

## Loop Transformations

- A loop may not be parallel as is
- Example

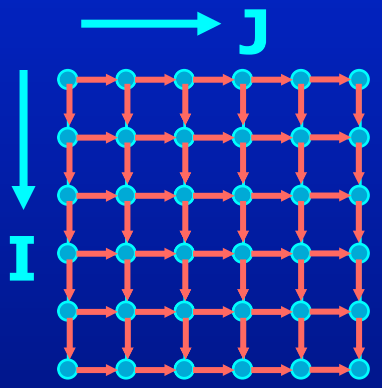
```
FOR i = 1 to N-1
  FOR j = 1 to N-1
   A[i,j] = A[i,j-1] + A[i-1,j];
```



# Loop Transformations

- A loop may not be parallel as is
- Example

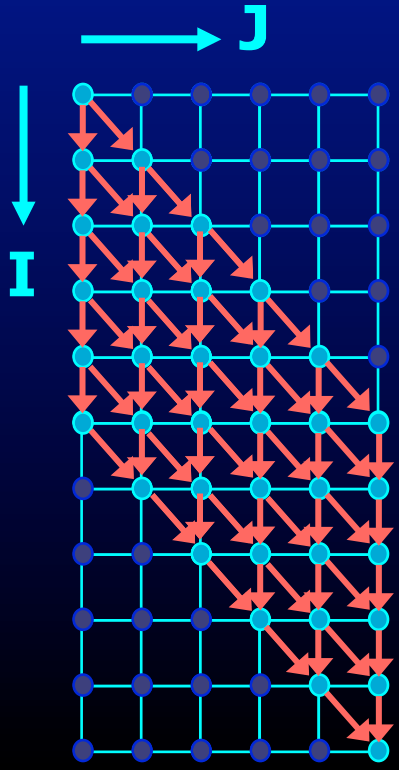
```
FOR i = 1 to N-1
  FOR j = 1 to N-1
   A[i,j] = A[i,j-1] + A[i-1,j];
```



- After loop Skewing

$$\begin{bmatrix} i_{new} \\ j_{new} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_{old} \\ j_{old} \end{bmatrix}$$

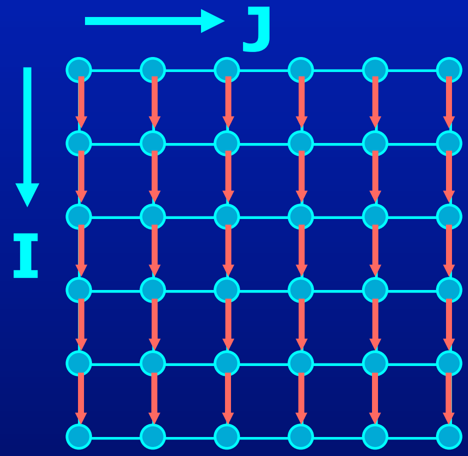
```
FOR i = 1 to 2*N-3
  FORPAR j = max(1,i-N+2) to min(i, N-1)
   A[i-j+1,j] = A[i-j+1,j-1] + A[i-j,j];
```



# Granularity of Parallelism

- Example

```
FOR i = 1 to N-1
   FOR j = 1 to N-1
     A[i,j] = A[i,j] + A[i-1,j];
```



- Gets transformed into

```
FOR i = 1 to N-1
   Barrier();
   FOR j = 1+ myPid*Iters to MIN((myPid+1)*Iters, n-1)
      A[i,j] = A[i,j] + A[i-1,j];
   Barrier();
```

- Inner loop parallelism can be expensive
  - Startup and teardown overhead of parallel regions
  - Lot of synchronization
  - Can even lead to slowdowns

## Granularity of Parallelism

- Inner loop parallelism can be expensive

- Solutions
  - Don’t parallelize if the amount of work within the loop is too small

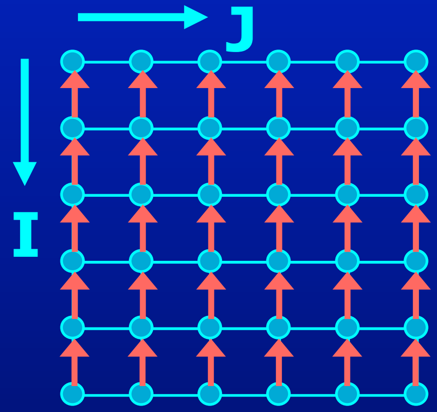
  or

  - Transform into outer-loop parallelism

# Outer Loop Parallelism

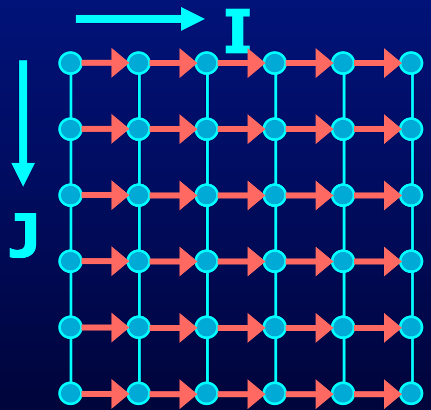
- Example

```
FOR i = 1 to N-1
   FOR j = 1 to N-1
      A[i,j] = A[i,j] + A[i-1,j];
```



- After Loop Transpose

```
FOR j = 1 to N-1
   FOR i = 1 to N-1
      A[i,j] = A[i,j] + A[i-1,j];
```



- Get mapped into

```
   Barrier();
   FOR j = 1+ myPid*Iters to MIN((myPid+1)*Iters, n-1)
      FOR i = 1 to N-1
         A[i,j] = A[i,j] + A[i-1,j];
   Barrier();
```

# Unimodular Transformations

- Interchange, reverse and skew
- Use a matrix transformation

  $I_{new} = A\ I_{old}$

- Interchange $$\begin{bmatrix} i_{new} \\ j_{new} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} i_{old} \\ j_{old} \end{bmatrix}$$
- Reverse $$\begin{bmatrix} i_{new} \\ j_{new} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_{old} \\ j_{old} \end{bmatrix}$$
- Skew $$\begin{bmatrix} i_{new} \\ j_{new} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_{old} \\ j_{old} \end{bmatrix}$$

# Legality of Transformations

- Unimodular transformation with matrix A is valid iff. For all dependence vectors v
  the first non-zero in Av is positive
- Example

```
FOR i = 1 to N-1
   FOR j = 1 to N-1
     A[i,j] = A[i,j] + A[i-1,j];
```

$$dv = \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

- Interchange $A = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ ✓
- Reverse $A = \begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix}$ $\begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix}$ ✗
- Skew $A = \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$ $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$ ✓✓

## Interprocedural Parallelization

- Function calls will make a loop unparallelizatble
  - Reduction of available parallelism
  - A lot of inner-loop parallelism
- Solutions
  - Interprocedural Analysis
  - Inlining

# Interprocedural Parallelization

- Issues
  - Same function reused many times
  - Analyze a function on each trace → Possibly exponential
  - Analyze a function once → unrealizable path problem
- Interprocedural Analysis
  - Need to update all the analysis
  - Complex analysis
  - Can be expensive
- Inlining
  - Works with existing analysis
  - Large code bloat → can be very expensive

```
HashSet h;
for i = 1 to n
    int v = compute(i);
    h.insert(i);
```

Are iterations independent?

Can you still execute the loop in parallel?

Do all parallel executions give same result?

## Summary

- Multicores are here
  - Need parallelism to keep the performance gains
  - Programmer defined or compiler extracted parallelism
- Automatic parallelization of loops with arrays
  - Requires Data Dependence Analysis
  - Iteration space & data space abstraction
  - An integer programming problem
- Many optimizations that'll increase parallelism