RECITATION 4

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Problem 1. Let

$$PF = \{\langle D\rangle | D \text{ is a DFA whose language is prefix-free}\}.$$

show that PF is decidable.

Note: A prefix-free language is a set of strings such that no string is the prefix of another.

Solution 1: We build a decider for D_{PF} for language PF as follows:

$D_{PF} =$ "On input $\langle D\rangle$,

Construct a new DFA D' from D as follows:

- Add new state "Fail".
- Point all outgoing edges from accept states into "Fail".
- Decide if $L(D) = L(D')$. Accept if yes, otherwise reject."

Note that we can implement the last step because testing for equivalence of two DFAs was proven to be decidable in lecture.

Solution 2: This solution depends on the equivalence of DFAs and regular expressions.

$D_{PF} =$ "On input $\langle D\rangle$,

- Convert D into the corresponding regular expression R.
- Convert the regular expression $R\Sigma^+$ into a DFA D_1
- Build DFA D_2 such that $L(D_2) = L(D) \cap L(D_1)$
- Decide if $L(D_2) = \phi$. Accept if yes, otherwise reject."

Note that we can implement the last step because testing for emptiness of a DFA language was proven to be decidable in lecture.

Date: September 2019.

Problem 2. Let
$$PAL := \{\langle D\rangle | D \text{ is a DFA that accepts a palindrome }\}.$$
show that PAL is decidable.

First, notice that the language of palindromes if a CFG. To see that, consider the following grammar:
$$S \to aSa \ \ \forall a \in T$$
$$S \to a \ \ \forall a \in T$$
$$S \to \epsilon$$
which generates the languages of palindromes. Thus, there exists a PDA P_{PAL} that recognizes the language of palindromes. Given all this, we give the following decider D_{PAL} for PAL:

$D_{PAL} =$ " On input $\langle D\rangle$,
- Construct PDA P such that $L(P) = L(D) \cap L(P_{PAL})$
- Decide if $L(P) = \phi$. Reject if yes, otherwise accept."

We can implement the second step by following the proof that the intersection of a CFL and a regular language is a CFL. We can implement the last step because testing for emptiness of languages of CFLs/PDAs was proven to be decidable in lecture.

Problem 3. Let

$$E_{TM} := \{\langle M\rangle | M \text{ is a TM such that } L(M) = \phi\}$$

show that $\overline{E_{TM}}$ is T-recognizable.

We build a TM T recognizing the $\overline{E_{TM}}$.

T = "On input w,
- If w is not of the format $\langle M\rangle$, accept.
- Otherwise, choose an ordering of Σ^* $w_1, w_2, \ldots$
- For every $i \in \mathbb{N}$, run M for i steps on $w_1, \ldots, w_i$.
- If M accepts on any input, accept."

As we'll prove in a future lecture, E_{TM} is not decidable. As $\overline{E_{TM}}$ is T-recognizable, this implies that E_{TM} is not even T-recognizable!