18.404/6.840 Lecture 16

Last time:

- NP-completeness
-
-

Today:

- Cook-Levin Theorem: is NP-complete
- is NP-complete

Quick Review

Defn: is NP-complete if

1) NP
2) For all NP,

If is NP-complete and P then P NP.

Importance of NP-completeness

1) Evidence of computational intractability.
2) Gives a good candidate for proving P NP.

To show some language is NP-complete, show .

or some other previously shown NP-complete language

Check-in 16.1

The big sigma notation means summing over some set.

The big AND (or OR) notation has a similar meaning.

For example, if and are two strings of length , when does the following hold?

(a) Whenever and agree on some symbol.
(b) Whenever .

Or: P = NP

Check-in 16.1

Cook-Levin Theorem (idea)

Theorem: is NP-complete

Proof: 1) (done)

2) Show that for each we have :

Let be decided by NTM in time .

Give a polynomial-time reduction mapping to .

formulas

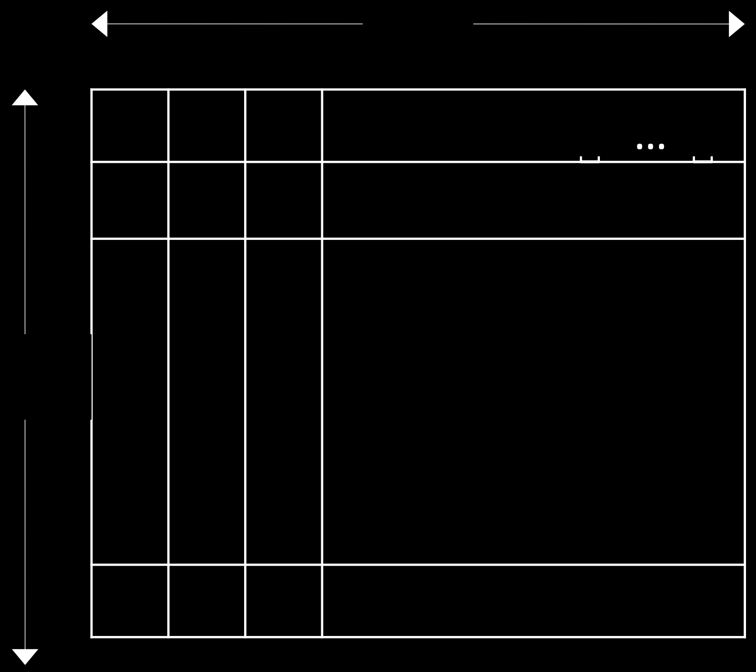
iff is satisfiable

Idea: simulates on . Design to "say" accepts .

Satisfying assignment to is a computation history for on .

Tableau for on

Defn: An (accepting) tableau for NTM on is an table representing an computation history for on on an accepting branch of the nondeterministic computation.



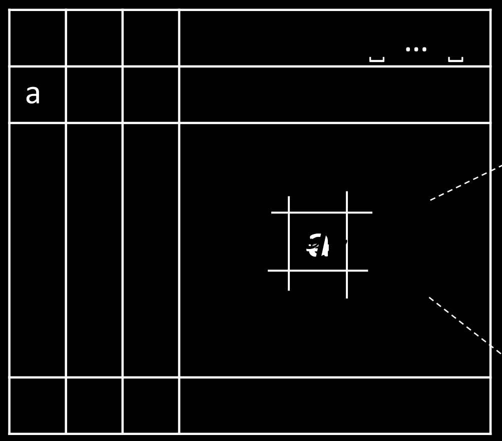
Start configuration for on

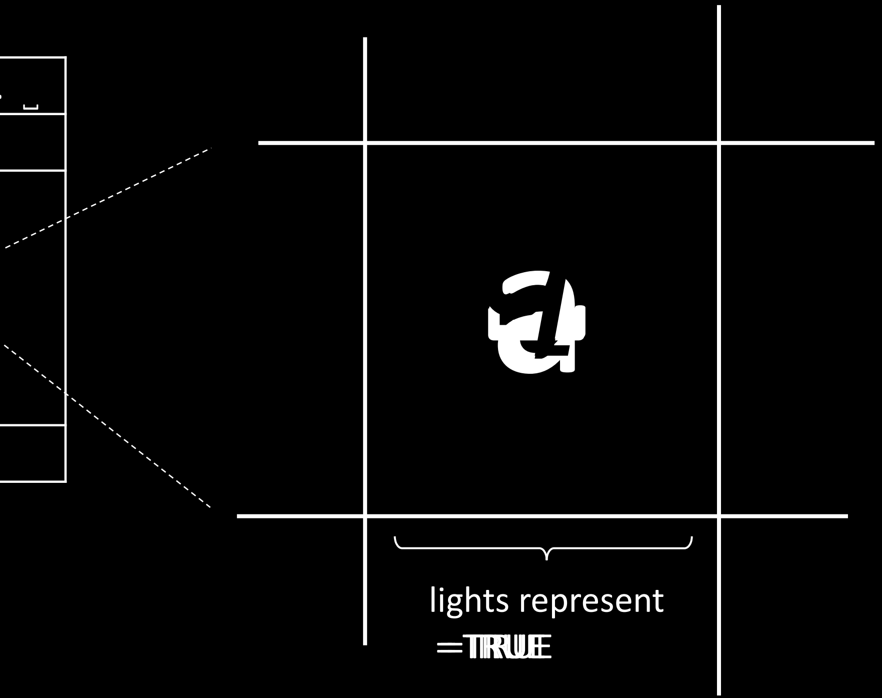
Accepting configuration

Construct to “say” accepts .

“says” a tableau for on exists.

Constructing :





Cell can contain
any symbol in

The variables of are
for and .

TRUE means cell contains .

Check-in 16.2

How many variables does have?
Recall that .

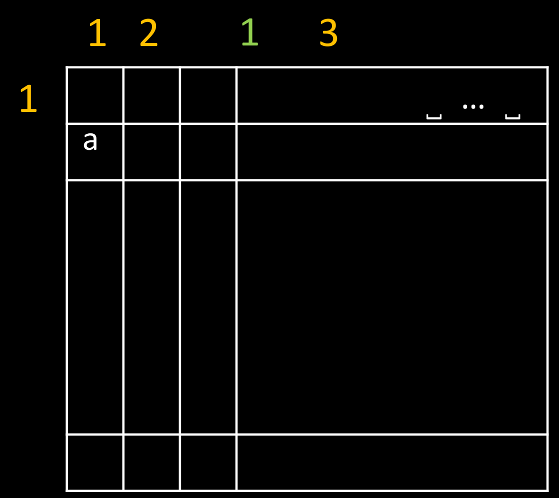
(a)

(b)

(c)

(d)

Constructing : and



Start configuration

Accepting configuration

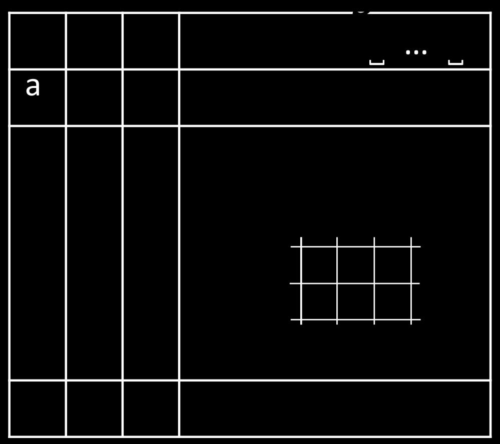
“says” a tableau for on exists.

done ✓

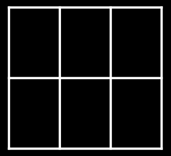
i


Coffee
Break

Constructing :



neighborhood



“says” a tableau for on exists.

✓ ✓ ✓

Legal neighborhoods: consistent with ’s transition function

potential examples:

a		b
	a	c

a	b	c
a	b	c

a	b	c
a	b	

a	b	c
d	b	c

Illegal neighborhoods: not consistent with ’s transition function

examples:

a	b	c
a	d	c

a	b	c
a		c

a		c
a	b	c

a		c
	d	

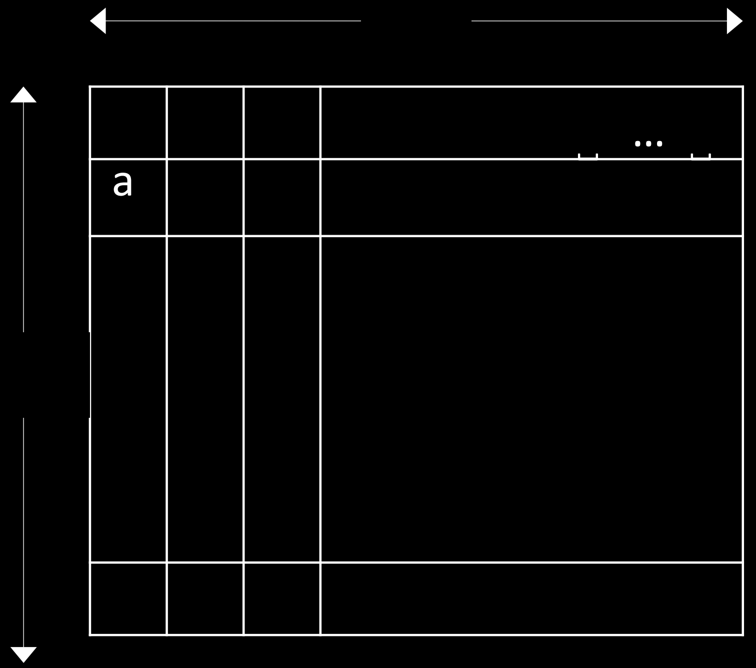
Claim: If every neighborhood is legal then tableau corresponds to a computation history.

Legal

r	s	t
v	y	z

Says that the neighborhood at is legal

Conclusion: is NP-complete



Summary:
For decided by NTM ,
we gave a reduction from to :
formulas

iff is satisfiable.

The size of is roughly the size of the tableau for on , so size is .

Therefore is computable in polynomial time.

is NP-complete

a	b	a
1	1	1
0	1	1
1	0	1
0	0	0

Theorem: is NP-complete

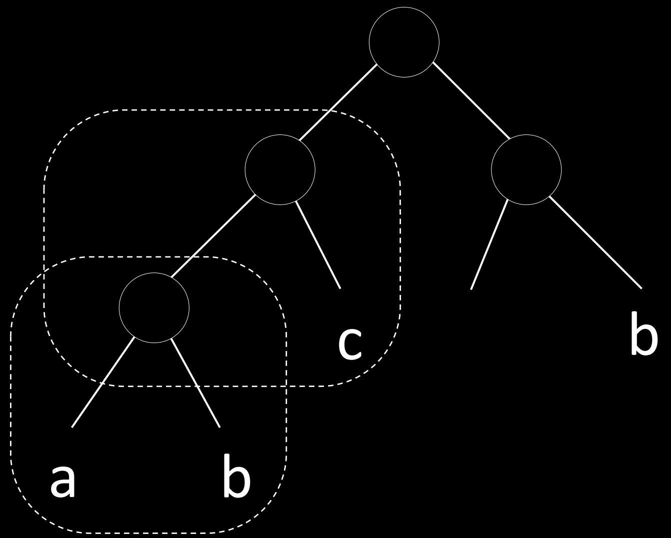
Proof: Show

Give reduction converting formula to 3CNF formula , <u>preserving satisfiability</u>.

(Note: and are not logically equivalent)

Example: Say

Tree structure for :



Logical equivalence: and and

repeat for each

Check-in 16.3

If has operations (and), how many clauses has ?

(a) (c)

(b) (d)

Quick review of today

1. is NP-complete
2. is NP-complete