MBMT Counting and Probability Round — Lobachevsky Answers

1. William chooses a random integer between 1 and 5, inclusive. Wendy chooses a random integer between 6 and 10, inclusive. What is the probability that the sum of their chosen integers is odd?
Answer: $\frac{13}{25}$

2. A coin is flipped 8 times and its sequence of heads and tails is recorded. What is the probability that the sequence doesn't contain "HT"?
Answer: $\frac{9}{256}$

3. A ladybug starts on vertex A of hexagon $ABCDEF$. Every minute it randomly chooses one of the vertices that it is not on and walks to it. On average, how many minutes will it take for the ladybug to reach vertex B?
Answer: 5

4. How many ways can a set of 4 not necessarily distinct numbers be chosen from the set of integers from 0 to 9? Two such sets are 5,2,4,2 and 0,0,0,7, but 2,2,4,5 is considered the same as 5,2,4,2.
Answer: 715

5. How many 3-digit numbers have the property that the product of the digits is a positive multiple of 3?
Answer: 513

6. Mr. Smith wants to bake 6 pies. For each pie, Mr. Smith chooses whether the pie will be apple, pumpkin, or cherry, each with probabilities of $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{4}$, respectively. What is the probability that there will be at least 3 pies of the same type?
Answer: $\frac{467}{512}$

7. A standard deck of 52 cards is shuffled and dealt. What is the probability that the 4 Aces are dealt before any of the face cards (4 each of J, Q, K)?
Answer: $\frac{1}{1820}$

8. Alice picks a random number between 0 and 0.75. Charlie picks a random number between 0.25 and 1. Bob picks a random number between 0 and 1. What is the probability that Bob's number is between Charlie and Alice's numbers?
Answer: $\frac{35}{108}$