MBMT Number Theory Round — Euclid

Full Name _____

Team Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is **not** worth the same number of points. Questions answered by fewer competitors are weighted more heavily. Please write your answers in the simplest possible form.

1. What is the remainder when 6! is divided by 7?

2. How many integers between 0 and 100 have a remainder of 1 when they are divided by 5?

3. Alex has a dumpling cart with n dumplings. He is going to a potluck with either 4, 5, or 7 other people. Given that n is the smallest positive integer number of dumplings such that everyone at the potluck, including himself, can get an equal number of dumplings with none left over, find n.

4. Find the sum of the positive divisors of 2016 that are multiples of 6.

5. Sarah has n tomato seeds to plant. If she plants them in rows of 9 seeds each, she'll have 2 left over. If she plants them in rows of 10, she'll have 1 left over. If $50 < n < 150$, what is n?

6. What is the 200th positive integer that is not a multiple of 2, 3, or 5?

7. Let a, b be integers such that $|a|, |b| \leq 2016$. Furthermore, let b be an odd integer. Find the number of ordered pairs (a, b) such that $a^2 - 2b^2 = 1$.

8. Let a_n be a sequence such that $a_0 = 3$, $a_1 = 3^3$, $a_2 = 3^{3^3}$, $a_3 = 3^{3^{3^3}}$, ... Find the units digit of a_{2016}.