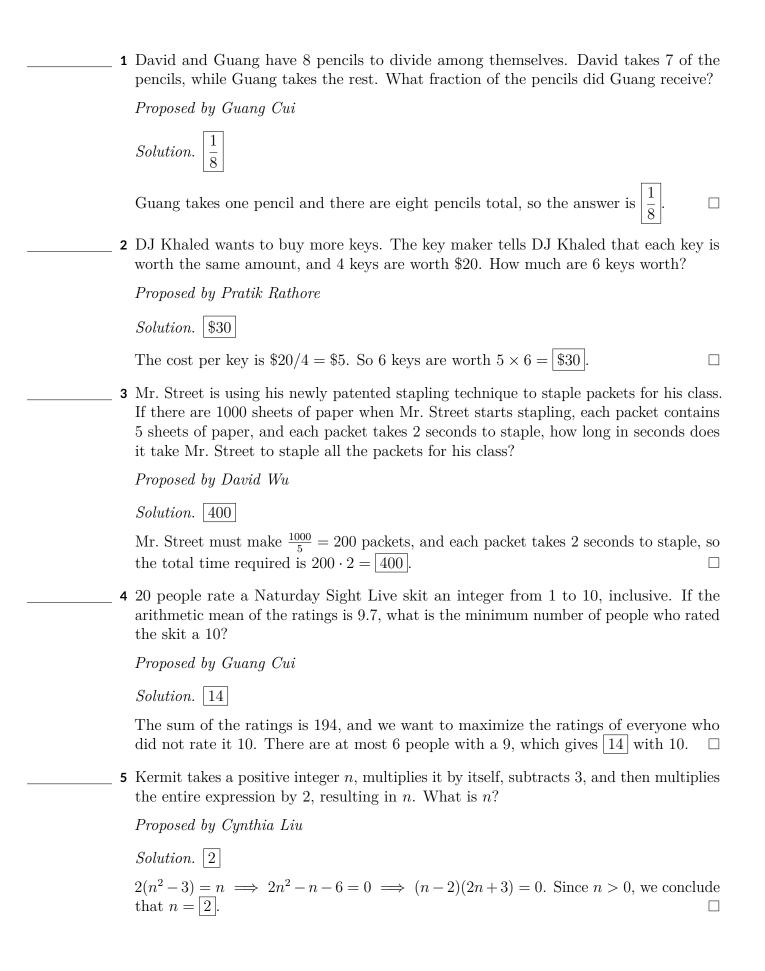# MBMT Algebra Round — Ramanujan

April 1, 2017

Full Name _____

Team Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered by fewer competitors will be weighted more heavily. Please write your answers in the simplest possible form.

1 David and Guang have 8 pencils to divide among themselves. David takes 7 of the pencils, while Guang takes the rest. What fraction of the pencils did Guang receive?

Proposed by Guang Cui

Solution. $\boxed{\dfrac{1}{8}}$

Guang takes one pencil and there are eight pencils total, so the answer is $\boxed{\dfrac{1}{8}}$. □

2 DJ Khaled wants to buy more keys. The key maker tells DJ Khaled that each key is worth the same amount, and 4 keys are worth \$20. How much are 6 keys worth?

Proposed by Pratik Rathore

Solution. $\boxed{\$30}$

The cost per key is \$20/4 = \$5. So 6 keys are worth $5 \times 6 = \boxed{\$30}$. □

3 Mr. Street is using his newly patented stapling technique to staple packets for his class. If there are 1000 sheets of paper when Mr. Street starts stapling, each packet contains 5 sheets of paper, and each packet takes 2 seconds to staple, how long in seconds does it take Mr. Street to staple all the packets for his class?

Proposed by David Wu

Solution. $\boxed{400}$

Mr. Street must make $\frac{1000}{5} = 200$ packets, and each packet takes 2 seconds to staple, so the total time required is $200 \cdot 2 = \boxed{400}$. □

4 20 people rate a Naturday Sight Live skit an integer from 1 to 10, inclusive. If the arithmetic mean of the ratings is 9.7, what is the minimum number of people who rated the skit a 10?

Proposed by Guang Cui

Solution. $\boxed{14}$

The sum of the ratings is 194, and we want to maximize the ratings of everyone who did not rate it 10. There are at most 6 people with a 9, which gives $\boxed{14}$ with 10. □

5 Kermit takes a positive integer n, multiplies it by itself, subtracts 3, and then multiplies the entire expression by 2, resulting in n. What is n?

Proposed by Cynthia Liu

Solution. $\boxed{2}$

$2(n^2 - 3) = n \implies 2n^2 - n - 6 = 0 \implies (n-2)(2n+3) = 0$. Since $n > 0$, we conclude that $n = \boxed{2}$. □

6 Find the sum of all positive integers x such that $3 < \sqrt{x} < 3.6$.

Proposed by Guang Cui

Solution. $\boxed{33}$

If $3 < \sqrt{x} < 3.6$, then $9 < x < 12.96$, so the answer is $10 + 11 + 12 = \boxed{33}$. $\qquad\square$

7 Kanye West really likes driving from his house to the studio in his Mercedes. On a day where traffic is moving smoothly, Kanye can get to the studio from his house in 30 minutes. On a day with a lot of traffic, it takes Kanye 50 minutes to reach the studio from his house, and his average speed is 20 miles per hour slower than usual. How far, in miles, is the studio from Kanye's house?

Proposed by Pratik Rathore

Solution. $\boxed{25}$

Let s be Kanye's speed in MPH. Then since $d = rt$ we have $d = \frac{s}{2} = \frac{5(s-20)}{6}$. This means that $s = 50$. Then the distance is $50/2 = \boxed{25}$. $\qquad\square$

8 Alex is playing with some goo. The goo is very magical: each morning at 7:00 AM it will make itself 60% longer. Every day, Alex wakes up at 8:00 AM, plays with it and stretches it by another 25%. If the goo is 1 meter long at 6:59 AM on Monday, on what day of the week will it surpass 2017 meters in length?

Proposed by Kevin Qian

Solution. $\boxed{\text{Thursday}}$

We claim that at noon on day i, the goo will be 2^i meters long. We can prove this claim using induction. Notice that each day, it gets longer by a factor of $1.6 \cdot 1.25 = 2$. Furthermore, it's true for the base case of $n = 1$, so the result is true for all n. Now, we take $\lceil \log_2 2017 \rceil$ to get 11 days. Now, counting 11 days with the first day being Monday, we get the answer of $\boxed{\text{Thursday}}$. $\qquad\square$