

GAN - Theory and Applications

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"Adversarial Training (also called GAN for Generative Adversarial Networks) is the most interesting idea in the last 10 years of ML."

— Yann LeCun

Two components, the **generator** and the **discriminator**:

- The **generator** G, aim is to capture the data distribution.
- The **discriminator** D, estimates the probability that a sample came from the training data rather than from G.

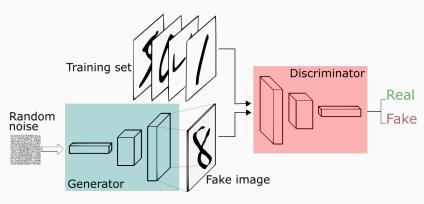


Figure 1: Credits: Reference

Generator and Discriminator compete against each other, playing the following **zero sum min-max game** with value function $V_{GAN}(D, G)$:

$$\min_{G} \max_{D} V_{GAN}(D, G) = \underset{x \sim p_{data}(x)}{\mathbb{E}} [\log D(x)] + \underset{z \sim p_{z}(z)}{\mathbb{E}} [\log (1 - D(G(z)))]$$
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4

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$$(1)$$

GANs - Discriminator

Intuitive explanation:

- **Discriminator** needs to:
 - Correctly classify real data:

$$\max_{D} \mathbb{E}_{x \sim p_{data}(x)}[\log D(x)].$$
 (2)

Correctly classify wrong data:

$$\max_{D} \mathbb{E}_{z \sim p_z(z)} [\log(1 - D(G(z)))]. \tag{3}$$

GANs - **Generator**

Intuitive explanation:

- Generator needs to fool the discriminator:
 - Generate samples similar to the real one:

$$\min_{G} \underset{z \sim p_z(z)}{\mathbb{E}} [\log(1 - D(G(z)))]. \tag{4}$$

GANs - **Generator**

Intuitive explanation:

- **Generator** needs to **fool** the discriminator:
 - Generate samples similar to the real one:

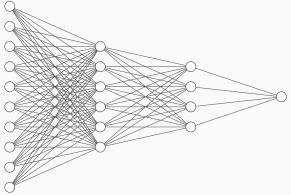
$$\min_{G} \underset{z \sim p_z(z)}{\mathbb{E}} [\log(1 - D(G(z)))]. \tag{4}$$

- Saturates easily (1).
- Change loss for generator:

$$\max_{G} \underset{z \sim p_z(z)}{\mathbb{E}} [\log(D(G(z)))]. \tag{5}$$

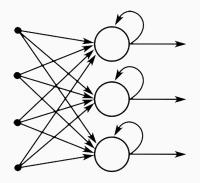
GANs - Models definition

- Both D and G can be parametrized functions (Neural Networks).
- Different architectures for different data types.
 - Tuple of numbers?



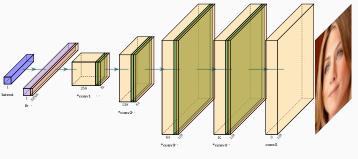
GANs - Models definition

- Both D and G can be parametrized functions (Neural Networks).
- Different architectures for different data types.
 - Text or sequences?



GANs - Models definition

- Both D and G can be parametrized functions (Neural Networks).
- Different architectures for different data types.
 - Images?





GANs - Training

- Discriminator and generator are competing against each other.
- How to train?
- Alternating execution of training steps.
- Use minibatch stochastic gradient descent/ascent.



GANs - Training - Discriminator

How to train the discriminator?

Repeat from 1 to k:

- 1. Sample minibatch of m noise samples $z^{(1)}, \ldots, z^{(m)}$ from noise prior $p_g(z)$
- 2. Sample minibatch of m examples $x^{(1)}, \ldots, x^{(m)}$ from data generating distribution $p_{data}(x)$
- 3. Train the **discriminator** by stochastic gradient **ascent**:

$$\Delta_{\theta_d} \frac{1}{m} \sum_{i=1}^m \log D(x^{(i)}) + \log(1 - D(G(z^{(i)}))$$

GANs - Training - Discriminator

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- 2. Sample minibatch of m examples $x^{(1)}, \ldots, x^{(m)}$ from data generating distribution $p_{data}(x)$
- 3. Train the discriminator by stochastic gradient ascent:

$$\Delta_{\theta_d} \frac{1}{m} \sum_{i=1}^{m} \underbrace{\log D(x^{(i)}) + \log(1 - D(G(z^{(i)}))}_{\text{Discriminator loss}}$$

GANs - Training - Discriminator

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$$\Delta_{\theta_d} \underbrace{\frac{1}{m} \sum_{i=1}^{m} \underbrace{\log D(x^{(i)}) + \log(1 - D(G(z^{(i)}))}_{\text{Discriminator loss}}}_{\text{Loss estimation using m samples}}$$

GANs - Training - Generator

How to train the generator?

The update is executed **only once** and only after the turn of the discriminator is completed:

- 1. Sample minibatch of m noise samples $z^{(1)}, \ldots, z^{(m)}$ from noise prior $p_g(z)$
- 2. Train the **generator** by stochastic gradient **ascent**:

$$\Delta_{\theta_g} \frac{1}{m} \sum_{i=1}^m \log(D(G(z^{(i)})))$$

GANs - Training - Generator

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$$\Delta_{\theta_g} \frac{1}{m} \sum_{i=1}^{m} \underbrace{\log(D(G(z^{(i)})))}_{\text{Generator loss}}$$

GANs - Training - Generator

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GANs - Training - Considerations

- Optimizers: Adam, Momentum, RMSProp
- Training phase can last for an arbitrary number of steps or epochs
- Training is completed when the discriminator is completely fooled by the generator.
- The goal of GAN training is to reach a Nash Equilibrium where the best D can do is random guessing.

Type of GANs

Types of GANs

Two big families:

- Unconditional GANs (just described)
- Conditional GANs (2)

Bibliography i

References

- [1] Ian J. Goodfellow, Jean Pouget-Abadie, Mehdi Mirza, Bing Xu, David Warde-Farley, Sherjil Ozair, Aaron Courville, and Yoshua Bengio, *Generative Adversarial Networks*.
- [2] Mehdi Mirza and Simon Osindero, *Conditional Generative Adversarial Nets*.