DOMAIN AND RANGE

Revision Math 1

Find the domain of these functions:

- $f(t) = 2^t$

Domain is: $(-\infty, \infty)$

- $f(x) = \sqrt{4 + 3x - x^2}$

$4 + 3x - x^2 \geq 0$

$-(x^2 - 3x - 4) \geq 0$

$-(x - 4)(x + 1) \geq 0$

$(x - 4)(x + 1) \leq 0$ -> $x \leq 4$ & $x \geq -1$

Domain is: [-1 , 4]

- $f(x) = \frac{1}{x^3-3x^2+2x}$

$$x^3 - 3x^2 + 2x = 0$$
$$x(x-2)(x-1) = 0$$

Domain: R – { 0 , 2 , 1}

- $f(v) = \sqrt{\frac{1}{(1-v)}}$

$$1 - v > 0$$
$$1 > v$$

Domain is : $(-\infty, 1)$

Find the domain and the range of these functions:

- $f(x) = x^2 + 2x + 3$

Domain is $(-\infty, \infty)$, Range is (k , ∞) (quadratic eq)

k = $f(h)$ & $h = -\frac{b}{2a}$

$h = -1,$

$k = f(-1) = 2$

Range: (2 , ∞)

- $f(x) = \frac{1}{x-1}$

Domain is R – { 1 }

Range is R – { 0 }

- $h(t) = \sqrt{t^2 + 1}$

Range:

$h^2 = t^2 + 1$

$t = \sqrt{h^2 - 1}$

- Range: [1 , ∞)
- Domain is: R

Limits and continuity

$$f(x) = \begin{cases} \frac{x^2-9}{x-3} &, \; x < 3 \\ cx^2 + 10, & x \geq 3 \end{cases}$$

Find the value of *c* so that f(x) is continuous at x = 3?

Solution:

at x = 3, $f(3) = c(3)^2 + 10 = 9c + 10$

$$\lim_{x \to 3-} \frac{x^2-9}{x-3} = \lim_{x \to 3-} \frac{(x-3)(x+3)}{x-3} = 6$$

$$\lim_{x \to 3+} f(x) = \lim_{x \to 3-} f(x)$$

9c + 10 = 6 -> c = -4/9

Estimate the value of:

- $\lim_{t \to -1} \frac{2 - \sqrt{t^2+3}}{t+1}$

Solution:

- $\lim_{t \to -1} \frac{2 - \sqrt{t^2+3}}{t+1} * \frac{2 + \sqrt{t^2+3}}{2 + \sqrt{t^2+3}}$
- $\lim_{t \to -1} \frac{4-(t^2+3)}{(t+1)(2+\sqrt{t^2+3})}$
- $\lim_{t \to -1} \frac{-(t^2+1)}{(t+1)(2+\sqrt{t^2+3})}$
- $\lim_{t \to -1} \frac{-1}{(2+\sqrt{t^2+3})} = 1/4$

- $\lim_{y\to 7} \frac{y^2-4y-21}{3y^2-17y-28}$

Solution:

$$\lim_{y\to 7} \frac{(y-7)(y+3)}{(y-7)(3y+4)}$$

$$\lim_{y\to 7} \frac{(y+3)}{(3y+4)} = \frac{2}{5}$$

- $\lim_{h\to 0} \frac{(6+h)^2-36}{h}$

$$\lim_{h\to 0} \frac{36+12h+h^2-36}{h}$$

$$\lim_{h\to 0} \frac{12h+h^2}{h}$$

$$\lim_{h\to 0} \frac{h(12+h)}{h} = 12$$

- $$\lim_{t\to -1} \frac{t+1}{|t+1|}$$

Solution:

- $|t+1| = \ (t+1) \quad at\ (t+1) \geq 0$

$$\lim_{t\to -1} \frac{t+1}{t+1} = 1$$

- $|t+1| = -(t+1) \quad at\ (t+1) < 0$

$$\lim_{t\to -1} \frac{t+1}{-(t+1)} = -1$$

$$\lim_{x\to\infty} \frac{8-4x^2}{9x^2+5x}$$

Solution:

- $$\lim_{x\to\infty} \frac{\frac{8}{x^2} - \frac{4x^2}{x^2}}{\frac{9x^2}{x^2} + 5\frac{x}{x^2}}$$
- $$\lim_{x\to\infty} \frac{0-4}{9+0} = -\frac{4}{9}$$
- $$\lim_{x\to\infty} \frac{\sqrt{7+9x^2}}{1-2x}$$

$$\lim_{x\to\infty} \frac{\sqrt{\frac{7}{x^2} \quad + \quad 9x^2/x^2}}{\frac{1}{X} \quad - \quad \frac{2x}{x}}$$

$$\lim_{x\to\infty} \frac{\sqrt{0 \ + 9}}{0 \ - \ 2} = -3/2$$

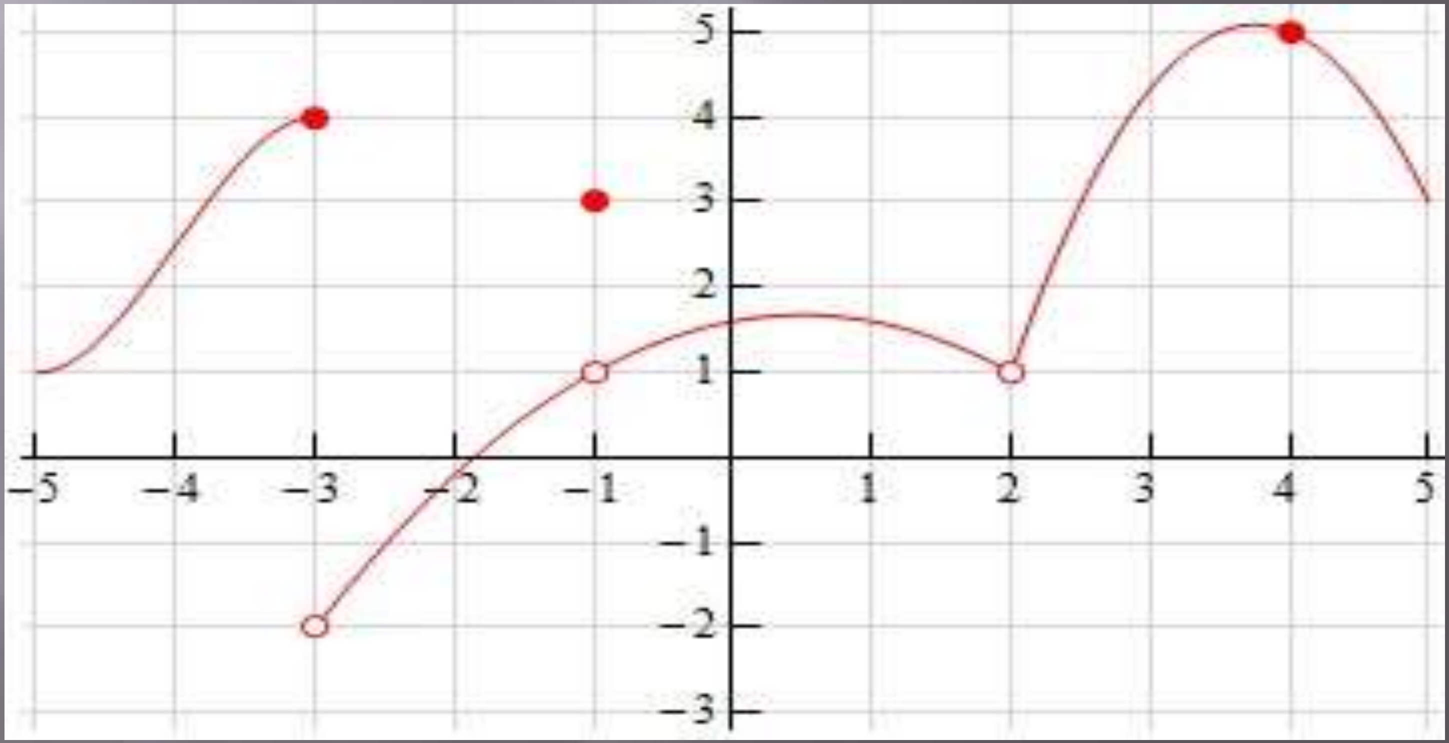
- $\lim_{z\to 0} \frac{\sin(10z)}{z}$
- $\lim_{z\to 0} \frac{\sin(10z)}{z} * \frac{10}{10}$
- $\lim_{z\to 0} \ 10 * \frac{\sin(10z)}{10\ z} = 10$

- $$\lim_{\alpha \to 0} \frac{\sin(12\alpha)}{\sin(5\alpha)}$$

- $$\lim_{\alpha \to 0} \frac{\sin(12\alpha)}{\sin(5\alpha)} * \frac{12}{12} * \frac{5}{5}$$
- $$\lim_{\alpha \to 0} \frac{\sin(12\alpha)}{12} * \frac{5}{\sin(5\alpha)} * \frac{12}{5} = \frac{12}{5}$$

- Below is the graph of f(x). for each of the given points determine the value of f(a) and $\lim_{x \to a} f(x)$. if any of the quantities do not exist, explain why

(a) a = -8 (b) a = -2 (c) a = 6 (d) a = 10



Derivative

- Find the derivative of the given functions:

1. $f(x) = 10\sqrt[5]{x^3} - \sqrt{x^7} + 6\sqrt[3]{x^8} - 3$
2. $g(y) = (y - 4)(2y - y^2)$
3. $f(y) = \frac{y^5 - 5y^3 + 2y}{y^3}$
4. $g(z) = 10\tan(z) - 2\cot(z)$
5. $g(t) = 4\log_3(t) - \ln(t)$
6. $f(t) = 3^t \log(t)$
7. $f(x) = 2\sin(3x + \tan(x))$
8. $f(x) = e^{1-\cos x}$
9. $f(x) = \ln(1 - 5y^2 + y^3)$

- Determine where the function is not changing (critical points):
- $f(x) = \frac{x+4}{2x^2+x+8}$ ans: $x = -4 \pm 3\sqrt{2}$
- $f(x) = \sin\left(\frac{x}{3}\right) + \frac{2x}{9}$ ans: $\frac{y}{3} = 2.3005 + 2\pi n$
- $f(x) = e^{x^3-2x^2-7x}$ ans: x = -1, x = 7/3

- Find the tangent line to :

1. $f(x) = 7^x + 4e^x \ \ at\ x = 0$ ans: 5.9459
2. $f(x) = x^3 - 5x^2 + x$ is parallel to the line $y = 4x + 23$
3. Ans: $x = \frac{5 \pm \sqrt{34}}{3}$

- Sketch the graph of $f(x) = x^2 - 4x$ and identify minimum and maximum of the function on the following intervals:

a)[-1 , 4] b) (-1 , 5]

Integral applications

Arc Length

- If we want to determine the length of continuous function y = f(x) on the interval [a , b], we will assume the derivative is continuous on [a , b].
- The function curve will be divided into a series of small straight lines connectiong the points.

When $L \approx \sum_1^n l1 + l2 + l3 + \cdots \ldots\ldots + ln$

and where : $l1 = |p2 - p1|$

$= \sqrt{(x2 - x1)^2 + (y2 - y1)^2}$

and so on……

But we can write length in this form:

$$l = \sqrt{\Delta x^2 + \Delta y^2}$$

$$l = \sqrt{\left(1 + \left(\frac{\Delta y}{\Delta x}\right)^2\right)\Delta x^2}$$

$$l = \sqrt{\left(1 + \left(\frac{dy}{dx}\right)^2\right)dx^2}$$

- Then the length is :

$$L = \lim_{x \to \infty} \sum_{1}^{n} \sqrt{1 + (\grave{f}(x))^2}\ dx$$

and using the definition of definite integral, we can write the length as:

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

Volumes of Solid

- Simply, we can get the volume of solid object by getting the cross sectional area of the object on the interval [a , b], then the area is integrated to give the volume.

$$V = \int_a^b A\,(x)\; dx$$

Ex.1

- Determine the volume of the solid which bounded by $y = x^2 - 4x + 5$, x = 1 and x = 4

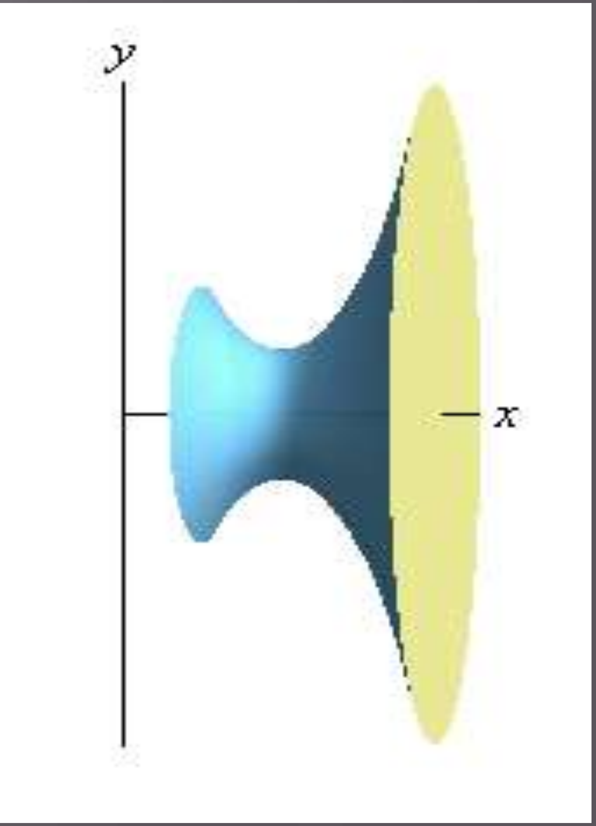
- **Solution:**

at this case the surface is a circle , then:

$A = \pi r^2$

But we have outer circle and inner circle, then:

$$A = \pi\left(\binom{outer}{radius}^2 - \binom{inner}{radius}^2\right)$$



- This example the radius is a function $f(x) = x^2 - 4x + 5$

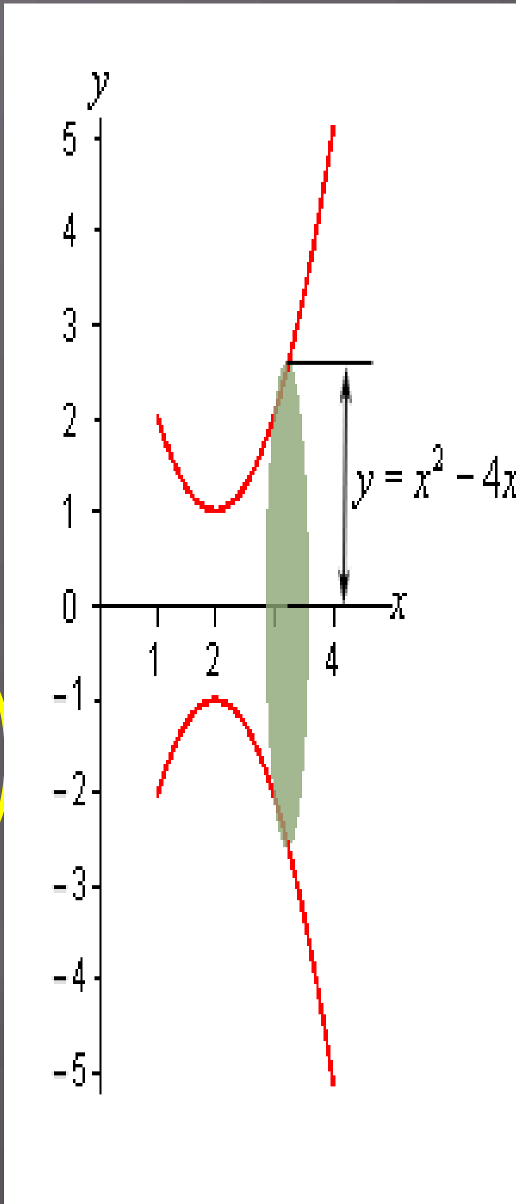
$$A = \pi(x^2 - 4x + 5)^2$$

$$V = \int_a^b A(x)\, dx$$

$$V = \pi \int_1^4 x^4 - 8x^3 + 26x^2 - 40x + 25\, dx$$

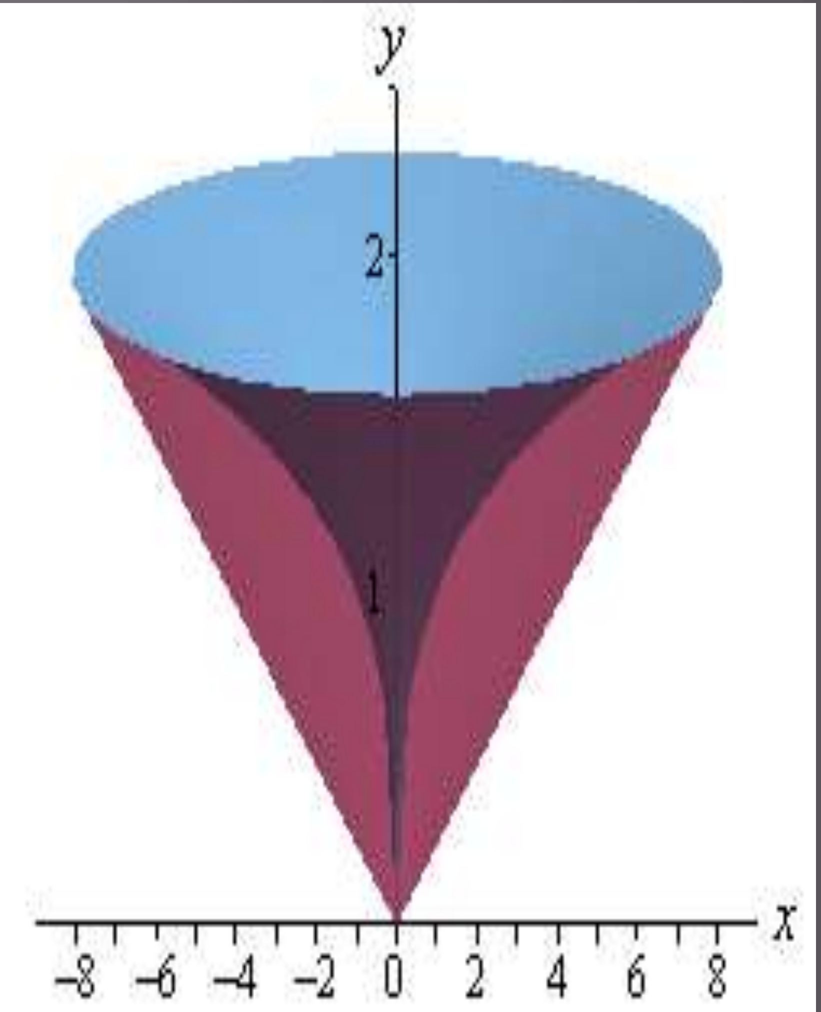
$$V = \pi \left(\frac{1}{5}x^5 - 2x^4 + \frac{26}{3}x^3 - 20x^2 + 25x\right)$$

$$V = 78\frac{\pi}{5}$$



Ex.2

- Determine the volume of the solid which bounded by $y = \sqrt[3]{x}$ and $y = \frac{x}{4}$,that lies in the first quadrant of y-axis.



- **Solution:**
- $y = \sqrt[3]{x}$ -> $x = y^3$
- $y = \frac{x}{4}$ -> $x = 4y$

$$A = \pi\left(\begin{pmatrix} outer \\ radius \end{pmatrix}^2 - \begin{pmatrix} inner \\ radius \end{pmatrix}^2\right)$$

$$A = \pi((4y)^2 - (y^3)^2)$$

$$A = \pi(16y^2 - y^6)$$

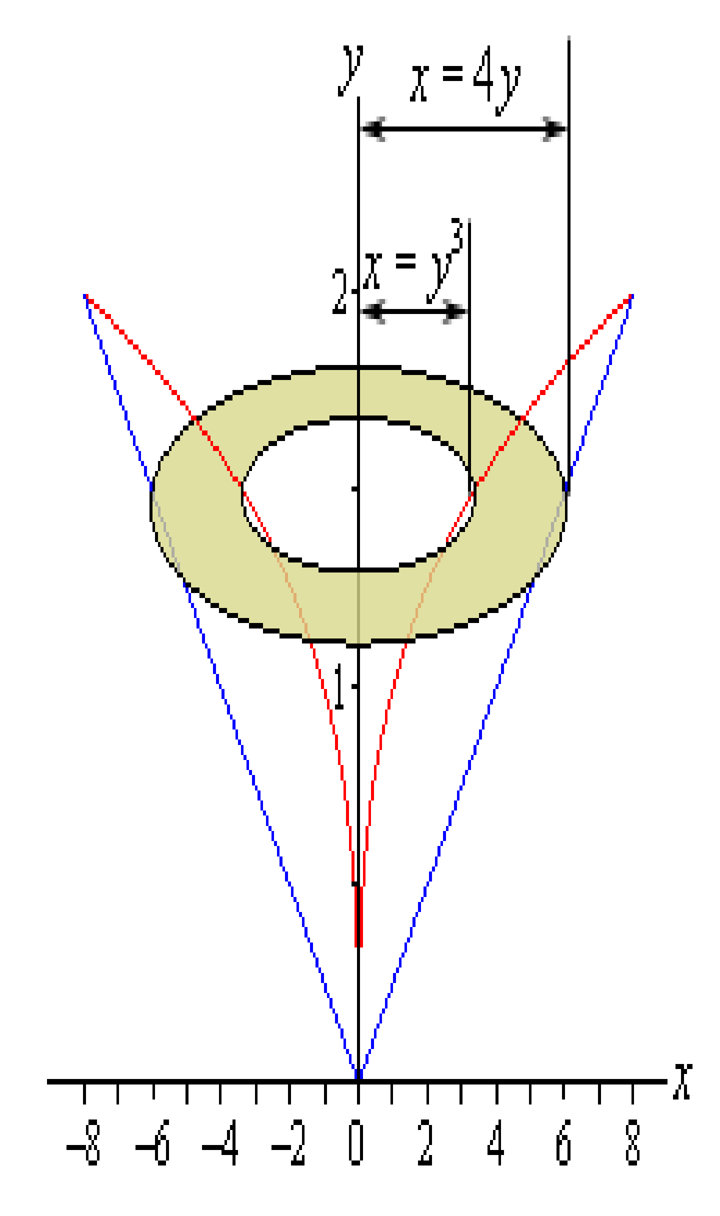
from the image, the first cross section is at y = 0 and end at y = 2, then:

$$V = \int_a^b A\,(y)\,dy$$

$$V = \int_0^2 \pi(16y^2 - y^6)\ dy$$

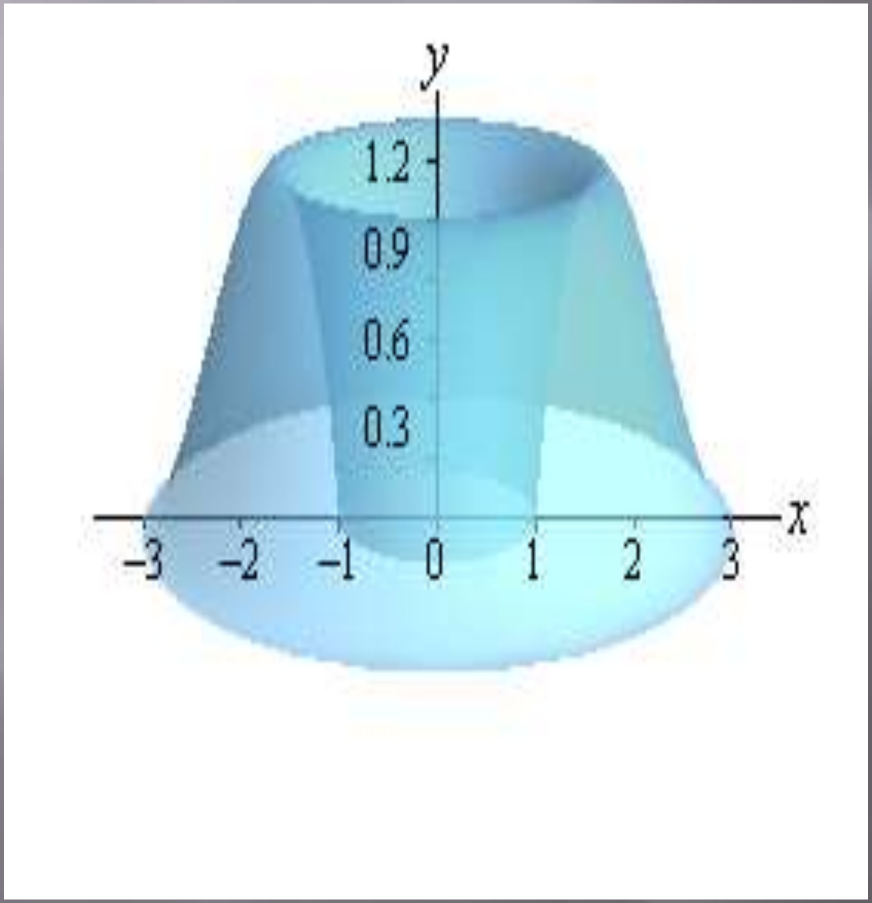
$$V = \pi(\frac{16}{3}y^3 - \frac{y^7}{7})$$

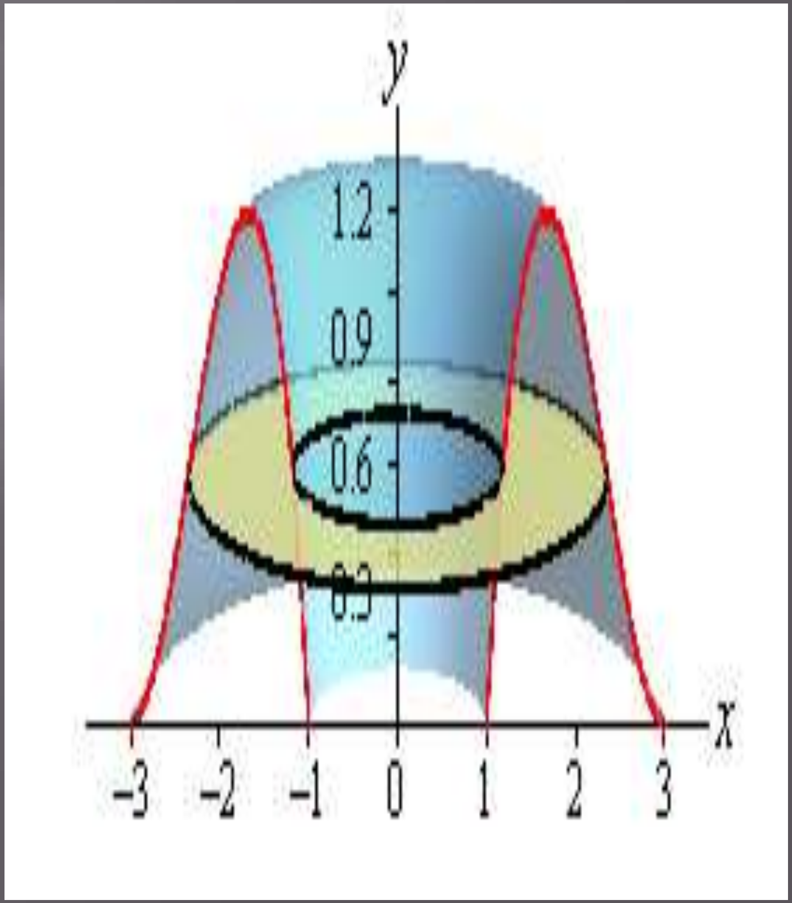
$$V = \frac{512\pi}{21}$$



Ex.3

- Determine the volume of the solid which bounded by $y = (x-1)(x-3)^2$





- The inner and outer radius is defined by same function.
- The shape is converted to Cylinder

From the graph, x changed from x = 1 to x = 3 and the area of cylinder is:

$A(x) = 2\pi\ (radius)\ (height)$

and where radius is (X), then:

$A(x) = 2\pi(x)(x-1)(x-3)^2$

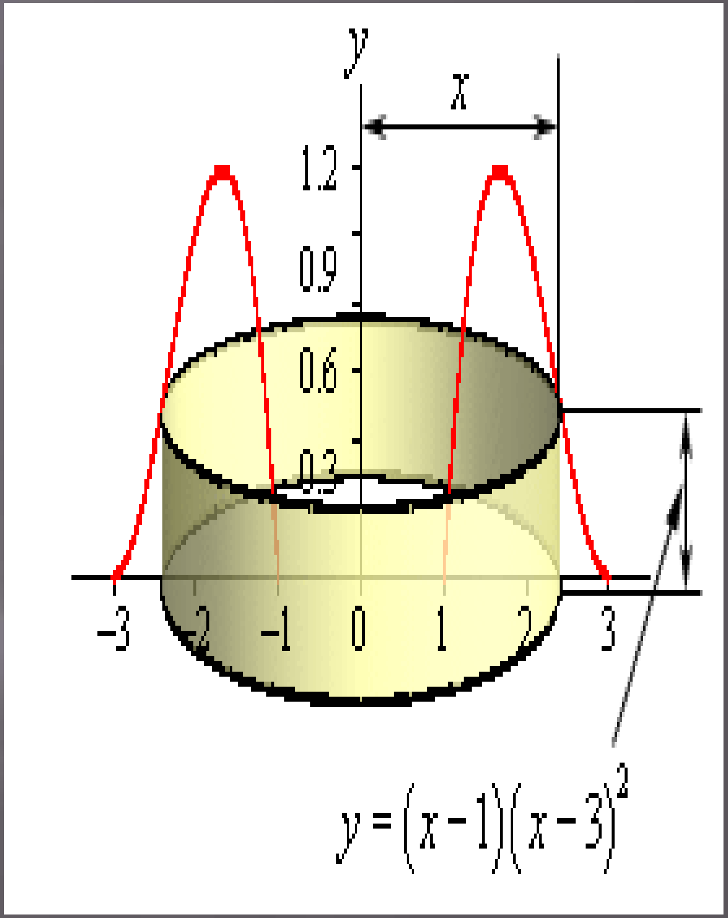
$A(x) = 2\pi(x^4 - 7x^3 + 15x^2 - 9x)$

$V = \int_1^3 A\,(x)\,dx$

$$V = 2\pi \int_1^3 (x^4 - 7x^3 + 15x^2 - 9x)\ dx$$

$$V = 2\pi\left(\frac{1}{5}x^5 - \frac{7}{4}x^4 + 5x^3 - \frac{9}{2}x^2\right)$$

$$V = \frac{24\pi}{5}$$



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