Physics (II)

Section 6

MCQ:

1) ……………. is characteristic of the field only, independent of a charged test particle that may be placed in the field.

A. Potential energy

B. Electric force

C. Electric Potential

Answer: C

Ex) Find the magnitude of maximum potential difference between two parallel conducting plates separated by 0.500 cm of air, given the maximum sustainable electric field strength in air to be 3.0×10^{6} V/m.

Solution

We are given the magnitude of maximum electric field E between the plates and the distance d between them. The equation $|\Delta V|=Ed$ can thus be used to calculate the magnitude of maximum voltage.

The potential difference or voltage between the plates is:

$$|\Delta V| = Ed$$

Entering the given values for E and d gives:

$$|\Delta \mathbf{V}| = (3.0\times10^{6}\ \text{V/m})\ (0.500\times10^{-2}\ \text{m}) = 15\times10^{3}\text{V} = 15\text{kV}.$$

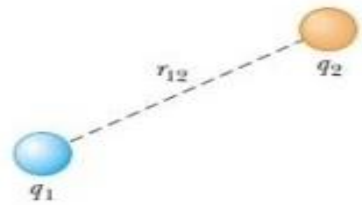
Module-3: Electric Potential

Electric Potential and Potential Energy Due to Point Charges

The total electric potential at some point *P* due to several point charges is the sum of the potentials due to the individual charges.

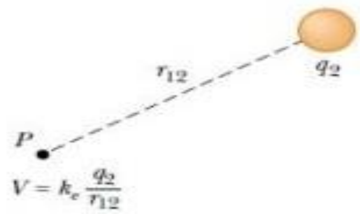
$$V = k_e \sum_i \frac{q_i}{r_i}$$

The potential energy between two point charges separated by a distance r_{12} as shown in figure can be expressed as:



$$U = k_e \frac{q_1 q_2}{r_{12}}$$

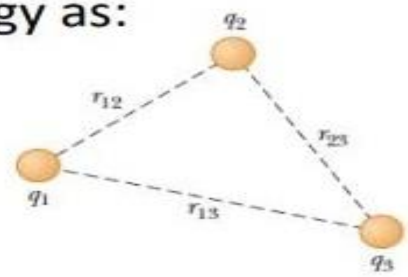
If charge q_1 have removed from point *P*, Potential due to charge q_2 as shown in figure can be expressed as:



$$V = \frac{U}{q_1} = k_e \frac{q_2}{r_{12}}$$

If the system consists of more than two charged particles as shown in figure, we can obtain the total potential energy as:

$$U = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$



Ex) What is the voltage 5.00 cm away from the center of a 1cm diameter metal sphere that has a −3.00nC static charge?

Solution

As we have discussed previously, charge on a metal sphere spreads out uniformly and produces a field like that of a point charge located at its center. Thus we can find the voltage using the equation

$$V = k\frac{q}{r}$$

Entering known values into the expression for the potential of a point charge, we obtain

$$V = k\frac{q}{r} = [(8.99\times10^{9}\ \mathrm{N\cdot m2/C2})\ (-3.00\times10^{-9}\mathrm{C})]\ /\ 5.00\times10^{-2}\mathrm{m} = -539.4\mathrm{V}$$

Ex) **(a)** What is the potential difference between two points situated 10 cm and 20 cm from a 3.0 μC point charge? **(b)** To what location should the point at 20 cm be moved to increase this potential difference by a factor of two?

Solution

Point charge: $Q = 3\ \mu\text{C} = 3 \times 10^{-6}\ \text{C}$
Distance between the charge and the first point: $r_1 = 0.1\ \text{m}$
Distance between the charge and the second point: $r_2 = 0.2\ \text{m}$
Electric force constant: $k = 9 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2$

The electric potential of a point charge:

$$V = \frac{kQ}{r}$$

So:

$$V_1 = \frac{kQ}{r_1}$$

$$V_2 = \frac{kQ}{r_2}$$

The potential difference between the two points:

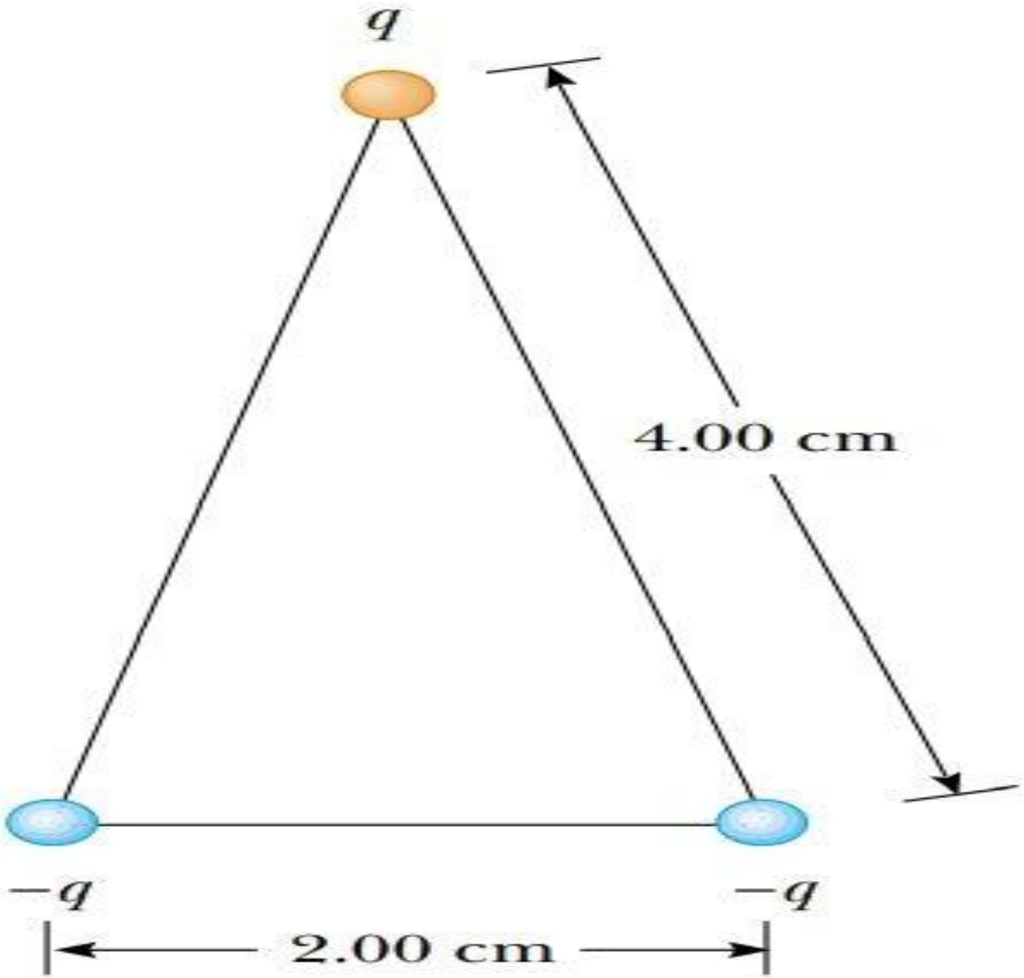
$$\Delta V = V_1 - V_2 = kQ\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\Delta V = 9 \times 10^9 \times 3 \times 10^{-6} \times \left(\frac{1}{0.1} - \frac{1}{0.2}\right) = 135 \times 10^3\ \text{V}$$

To double the potential difference, we need to make $\dfrac{1}{r_2} = 0$, so we move the charge from 20 cm to ∞

Ex)

The three charges in Figure P25.19 are at the vertices of an isosceles triangle. Calculate the electric potential at the midpoint of the base, taking $q = 7.00\ \mu C$.



Solution

The **total electric potential** at some point due to several point charges is the sum of the potentials due to the individual charges:

$$V = k_e \sum_i \frac{q_i}{r_i} \qquad (*)$$

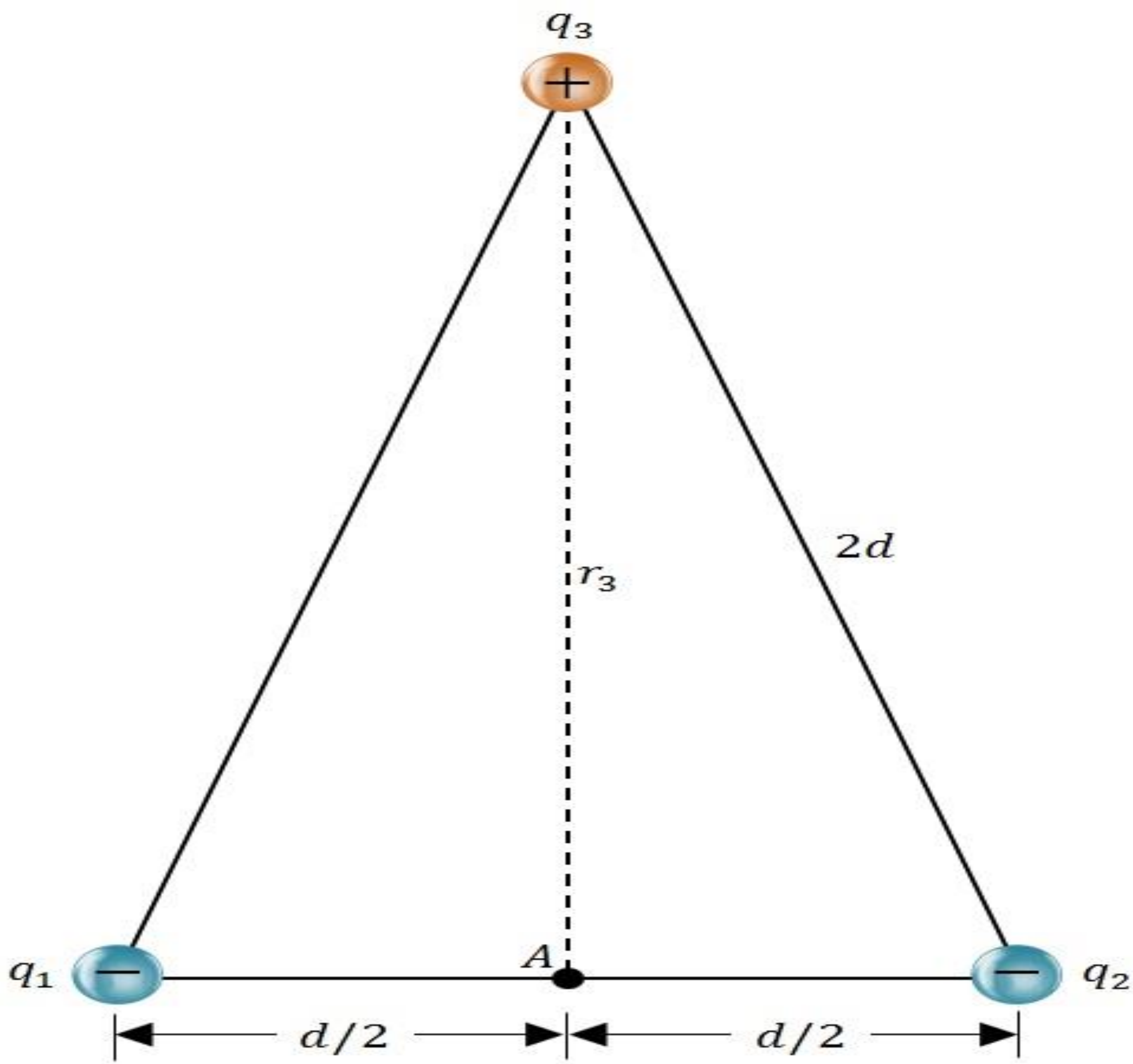
Where $k_e = 8.988 \times 10^9\ \mathrm{N \cdot m^2/C^2}$ is Coulomb constant.

$$q_1 = q_2 = -q = (-7\ \mu\mathrm{C})\left(\frac{1\ \mathrm{C}}{10^6\ \mu\mathrm{C}}\right) = -7 \times 10^{-6}\ \mathrm{C}$$

$$q_3 = q = (7\ \mu\mathrm{C})\left(\frac{1\ \mathrm{C}}{10^6\ \mu\mathrm{C}}\right) = 7 \times 10^{-6}\ \mathrm{C}$$

$$d = (2\ \mathrm{cm})\left(\frac{1\ \mathrm{m}}{100\ \mathrm{cm}}\right) = 0.02\ \mathrm{m}$$

We are asked to find the **electric potential** V_A at the midpoint of the base (point A).


q_3
r_3
$2d$
q_1
A
q_2
$d/2$
$d/2$

The distance between q_3 and point A is found by using **Pythagorean theorem**:

$$\begin{aligned} r_3 &= \sqrt{(2d)^2 - \left(\frac{d}{2}\right)^2} \\ &= \sqrt{4d^2 - \frac{d^2}{4}} \\ &= \sqrt{\frac{15}{4}d^2} \\ &= \frac{\sqrt{15}d}{2} \end{aligned}$$

The total electric potential at point A is the sum of the potentials due to the three charges as given by **Equation** (*):

$$\begin{aligned} V_A &= k_e\left(\frac{q_1}{d/2} + \frac{q_2}{d/2} + \frac{q_3}{\sqrt{15}d/2}\right) \\ &= \frac{k_e}{d/2}\left(q_1 + q_2 + \frac{q_3}{\sqrt{15}}\right) \\ &= \frac{2k_e}{d}\left(q_1 + q_2 + \frac{q_3}{\sqrt{15}}\right) \end{aligned}$$

Substitute numerical values:

$$\begin{aligned} V_A &= \frac{2(8.988 \times 10^9)}{0.02}\left[(-7 \times 10^{-6}) + (-7 \times 10^{-6}) + \frac{(7 \times 10^{-6})}{\sqrt{15}}\right] \\ &= \boxed{-1.1 \times 10^7\ \mathrm{V}} \end{aligned}$$

Ex)

Calculate the energy required to assemble the array of charges shown in Figure P25.34, where $a = 0.200$ m, $b = 0.400$ m, and $q = 6.00\ \mu C$.

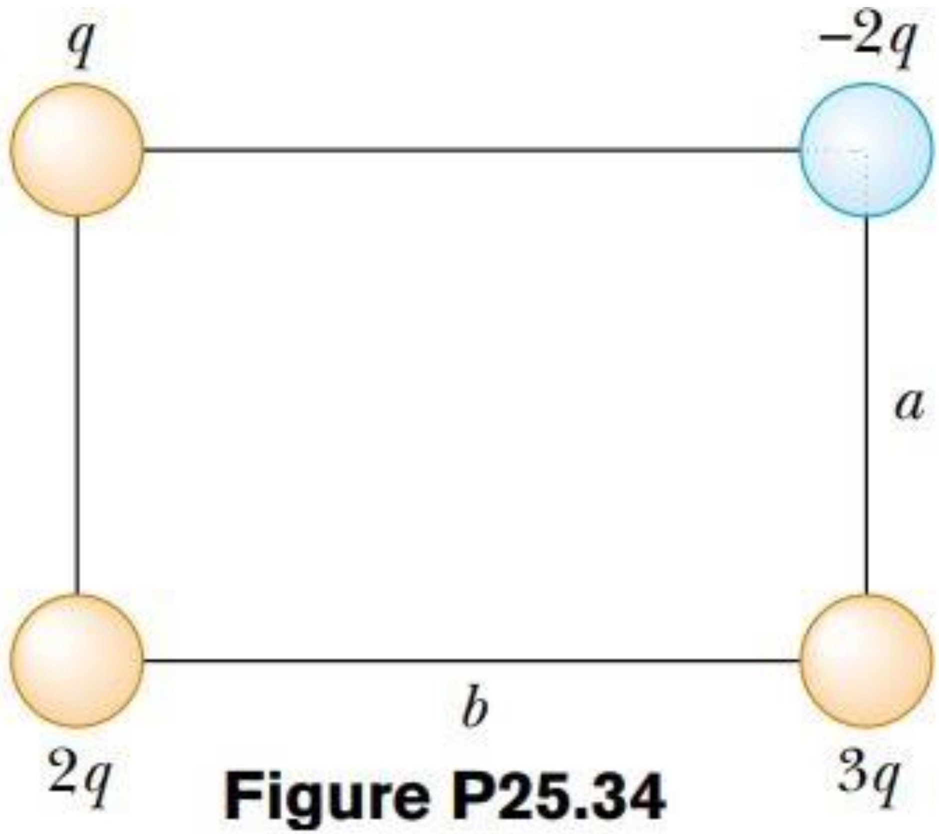


Figure P25.34

Solution

Energy needed to assemble the 4 charges as shown in the figure is actually work needed to assemble them, which is in turn equal to the total electric potential energy that of system of charges as shown in the figure. Total electric potential energy is equal to a sum of potential energies of each pair of charges. For charges q_1 and q_2 at distance r_{12}, electric potential energy U_{12} is equal to

$$U_{12} = k\frac{q_1 q_2}{r_{12}}$$

where $k = \frac{1}{4\pi\epsilon_0} = 9 \cdot 10^9 \ \frac{\text{Nm}^2}{C^2}$ is dielectric constant of vacuum. For simplicity, let's numerate the charges in the figure as follows:

$$\begin{aligned} q &= 6 \ \mu\text{C} = 6 \cdot 10^{-6} \ \text{C} \\ q_1 &= q = 6 \ \mu\text{C} \\ q_2 &= 2q = 12 \ \mu\text{C} \\ q_3 &= 3q = 18 \ \mu\text{C} \\ q_4 &= -2q = -12 \ \mu\text{C} \end{aligned}$$

Note that the total potential energy of the system is same no matter how you numerate charges. Corresponding distances are:

$$\begin{aligned} r_{12} &= r_{34} = a = 0.2 \ \text{m} \\ r_{23} &= r_{14} = b = 0.4 \ \text{m} \\ r_{24} &= r_{13} = c = \sqrt{a^2 + b^2} = 0.4472 \ \text{m} \end{aligned}$$

Total potential energy in the system will be calculated as:

$$U = k\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_1q_4}{r_{14}} + \frac{q_2q_3}{r_{23}} + \frac{q_2q_4}{r_{24}} + \frac{q_3q_4}{r_{34}}\right)$$

$$U = k\left(\frac{q\cdot 2q}{a} + \frac{q\cdot 3q}{c} + \frac{q\cdot(-2q)}{b} + \frac{2q\cdot 3q}{b} + \frac{2q\cdot(-2q)}{c} + \frac{3q\cdot(-2q)}{a}\right)$$

$$U = kq^2\left(\frac{2}{a} + \frac{3}{c} - \frac{2}{b} + \frac{6}{b} - \frac{4}{c} - \frac{6}{a}\right)$$

$$U = kq^2\left(\frac{4}{b} - \frac{4}{a} - \frac{1}{c}\right)$$

$$U = 9\cdot 10^9\,\frac{\mathrm{Nm}^2}{C^2}\cdot(6\cdot 10^{-6}\,\mathrm{C})^2\left(\frac{4}{0.4\,\mathrm{m}} - \frac{4}{0.2\,\mathrm{m}} - \frac{1}{0.4472\,\mathrm{m}}\right)$$

$$\boxed{U = -3.9649\ \mathrm{J}}$$