الجامعة المصرية
للتعلم الإلكتروني الأهلية

EELU
THE EGYPTIAN E-LEARNING UNIVERSITY

GEN206
Discrete Mathematics

Section 4

Faculty of Information Technology
Egyptian E-Learning University

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29. Let $A = \{a, b, c, d\}$ and $B = \{y, z\}$. Find

a) $A \times B$. **b)** $B \times A$.

Solution :

PART A: $\{(a, y), (a, z), (b, y), (b, z), (c, y), (c, z), (d, y), (d, z)\}$

PART B: $\{(y, a), (y, b), (y, c), (y, d), (z, a), (z, b), (z, c), (z, d)\}$

34. Let $A = \{a, b, c\}$, $B = \{x, y\}$, and $C = \{0, 1\}$. Find

a) $A \times B \times C$.

Solution :

(a) $A \times B \times C = \{(a,x,0),(a,x,1),(a,y,0),(a,y,1),(b,x,0),(b,x,1),$
$(b,y,0),(b,y,1),(c,x,0),(c,x,1),(c,y,0),(c,y,1)\}$

3. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{0, 3, 6\}$. Find

a) $A \cup B$.

b) $A \cap B$.

c) $A - B$.

d) $B - A$.

Solution :

PART A: $\{0, 1, 2, 3, 4, 5, 6\}$
PART B: $\{3\}$
PART C: $\{1, 2, 4, 5\}$
PART D: $\{0, 6\}$

Prove :

$a) A \cap \emptyset \equiv \emptyset$

Solution:

$$A \cap \emptyset = \{x | (x \in A) \wedge (x \in \emptyset)\}$$

$$= \{x | (x \in A) \wedge F\}$$

$$= \{x | \mathrm{F}\} = \emptyset$$

b) $A \cap$ U=A

$$A \cap u = \{x | (x \in A) \wedge (x \in u)\}$$

$$= \{x | (x \in A) \wedge T\}$$

$$= \{x | (x \in A)\}$$

$$= A$$

Prove :

a) $A \cup \overline{A} = U.$ **b)** $A \cap \overline{A} = \emptyset.$

Solution:

$$A \cap \bar{A} = \{x | (x \in A) \vee (x \notin A)\}$$

$$= \{x | (x \in A) \vee \neg(x \in A)\}$$

$$= \{x | P \vee \neg P\} = \{x | T\}$$

$$= U$$

$$A \cap \bar{A} = \{x | (x \in A) \cap (x \notin A)\}$$

$$= \{x | (x \in A) \wedge \neg(x \in A)\}$$

$$= \{x | P \wedge \neg P\}$$

$$= \{x | f\} = \emptyset$$

If A,B, C are sets , show that : $A \cap (A \cup B) = A$

Let $x \in (A \cap (A \cup B))$

$(x \in A) \wedge \left(x \in (A \cup B)\right)$=True

$so\ (x \in A)\ must\ be\ True$

i.e $(x \in A)$

$\therefore\ A \cap (A \cup B) \subseteq A$ 1

Let $(x \in A) = true$

$so\ (x \in A) \vee (x \in B)\ is\ also\ true$

$and\ (x \in A) \wedge (x \in (A \cup B)\ )is\ also\ true$

$x \in (A \cap (A \cup B))$

$\therefore A \subseteq (A \cap (A \cup B))$ 2

From 1 & 2 A=$A \cap A \cup B$

Use set builder notation and logical equivalences to establish the first De Morgan law $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

$$\begin{aligned}
\overline{A \cap B} &= \{x \mid x \notin A \cap B\} \\
&= \{x \mid \neg(x \in (A \cap B))\} \\
&= \{x \mid \neg(x \in A \wedge x \in B)\} \\
&= \{x \mid \neg(x \in A) \vee \neg(x \in B)\} \\
&= \{x \mid x \notin A \vee x \notin B\} \\
&= \{x \mid x \in \overline{A} \vee x \in \overline{B}\} \\
&= \{x \mid x \in \overline{A} \cup \overline{B}\} \\
&= \overline{A} \cup \overline{B}
\end{aligned}$$

Determine whether the function $f(x) = x^2$ from the set of integers to the set of integers is one-to-one.

Solution: The function $f(x) = x^2$ is not one-to-one because, for instance, $f(1) = f(-1) = 1$, but $1 \neq -1$.

23. Determine whether each of these functions is a bijection from **R** to **R**.

a) $f(x) = 2x + 1$
b) $f(x) = x^2 + 1$
c) $f(x) = x^3$
d) $f(x) = (x^2 + 1)/(x^2 + 2)$

We can prove not bijective by proving either not onto or not one to one

a) bijctive

b) Not bijctive let x_1= a , x_2= -a so $(x_1 \neq x_2)$ but $\mathrm{f(a)} = \mathrm{f(-a)}$so it is not one to one

c) bijctive

d) Not bijctive let x_1= a , x_2= -a so $(x_1 \neq x_2)$ but $\mathrm{f(a)} = \mathrm{f(-a)}$so it is not one to one