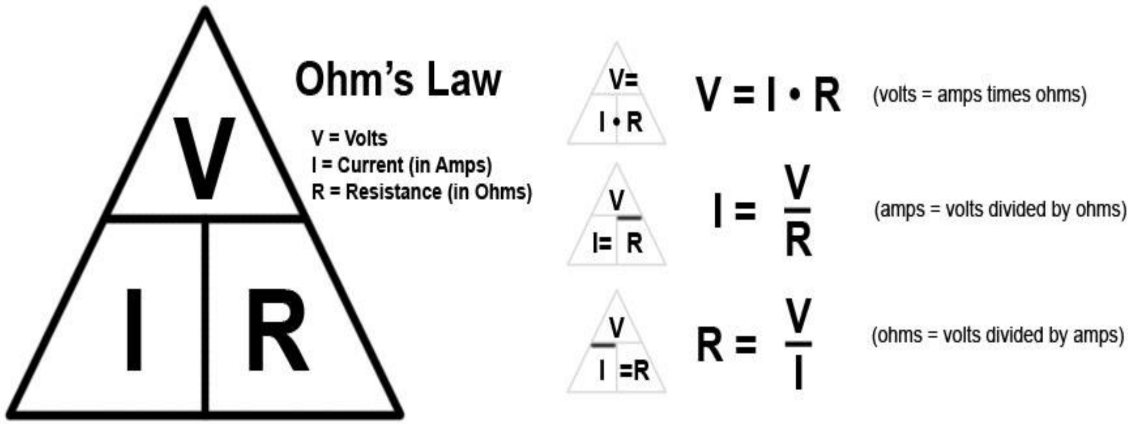
Section (9)
Physics II

“Resistances and their connections”

- In Ohm’s Law, the current is directly proportional to the voltage and the proportionality constant is called “Resistance”



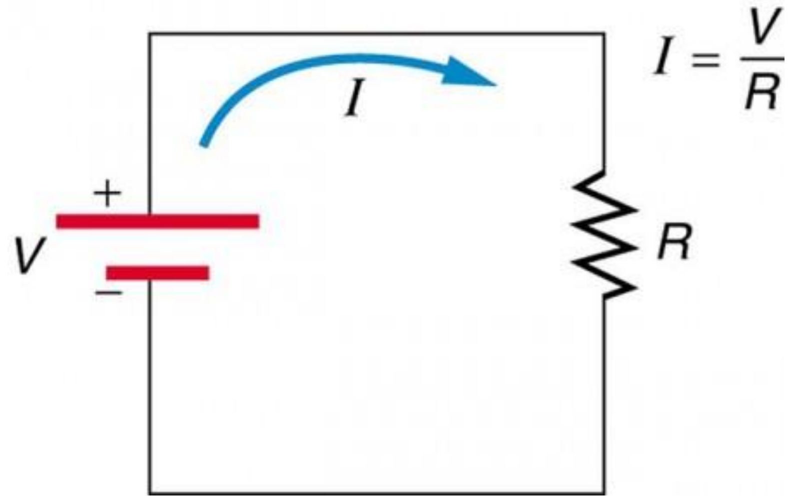
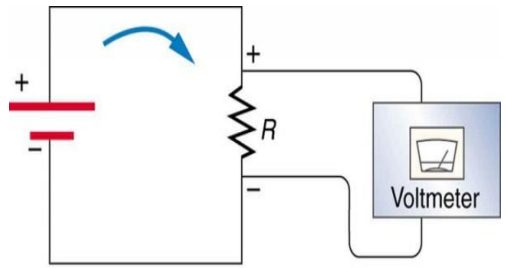


Fig (1)

Example 1 :

What is the resistance of an automobile headlight through which 2.50 A flows when 12.0 V is applied to it?



(Solution)

$$R = \frac{V}{I} = \frac{12.0\,(V)}{2.50\,(A)} = 4.80\Omega$$

But when it comes to more than a single resistance, our calculations will be a bit different and will depend on the type of the connections between the resistances such as:

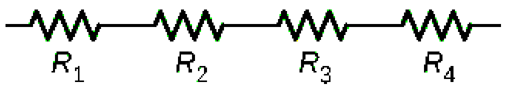
- Series connection
- Parallel connection

1- Series connection of resistors:

Current: $I_{Total} = I_{R_1} = I_{R_2} = \cdots$

Equivalent resistance: $R_{total} = R_1 + R_2 + R_3 + \ldots\ldots R_n$ etc.

Voltage: $V_{Total} = V_{R1} + V_{R2} + V_{R3} + \ldots\ldots + V_N$



(a) Resistors connected in series

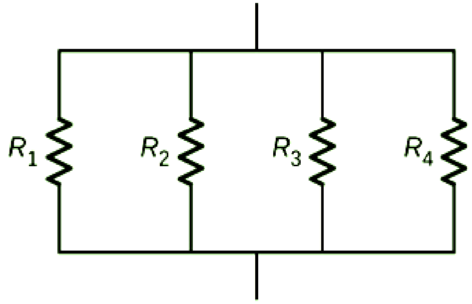
Fig(2)

2- Parallel connection of resistors:

Current: $I_{Total} = I_1 + I_2 + I_3 + \cdots + I_n$

Equivalent resistance: $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \ldots\ldots + \frac{1}{R_n}$ etc

Voltage: $V_{Total} = V_{R_1} = V_{R_2} = V_{R_3} = V_{R_n}$



(b) Resistors connected in parallel

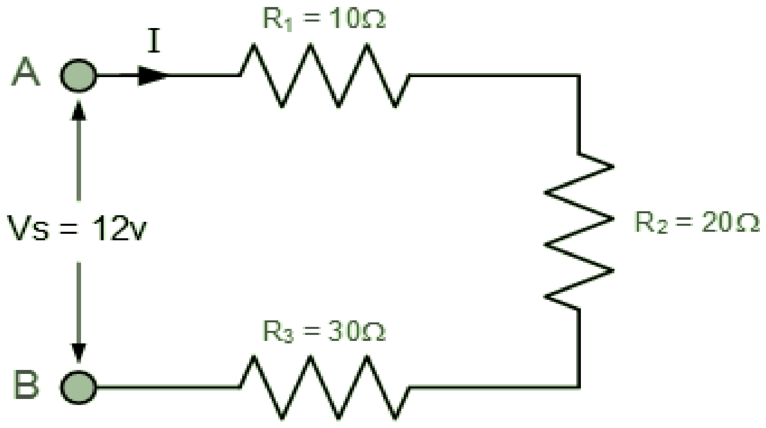
Fig(3)

- For the case of two resistances in parallel connection

$$R_T = \frac{R_1 \times R_2}{R_1 + R_2}$$

Example2:

Using Ohms Law, calculate the equivalent series resistance, the series current, voltage drop and power for each resistor in the following resistors in series circuit.



(Solution)

$$R_{eq} = R_1 + R_2 + R_3 \quad R_{eq} = 10 + 20 + 30 = 60\Omega$$

$$I = I_{R_1} = I_{R_2} = I_{R_3} = \frac{V_{total}}{R_{eq}} \quad I\,(A) = \frac{12\,(V)}{60\,(\Omega)} = 200\ mA = 0.2\ A$$

$$V_1 = R_1 I = 10 \times 0.2 = 2\ V\ , V_2 = R_2 I = 20 \times 0.2 = 4\ V\ , V_3 = R_3 I = 30 \times 0.2 = 6V$$

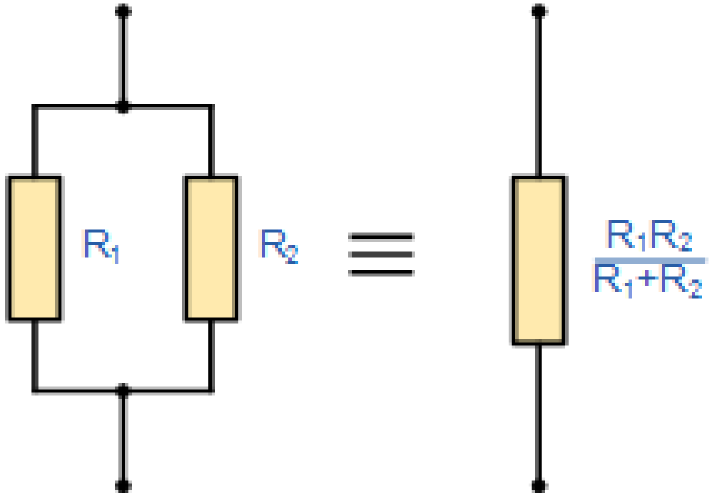
$$V_{total} = V_1 + V_2 + V_3 = 2 + 4 + 6 = 12V$$

$$P_1 = V_1 I = 2 \times 0.2 = 0.4W\ \ , P_2 = V_2 I = 4 \times 0.2 = 0.8W\ , P_3 = V_3 I = 6 \times 0.2 = 1.2W$$

$$P_{total} = P_1 + P_2 + P_3 = 0.4 + 0.8 + 1.2 = 2.4\ W$$

Example 3:

Consider the following circuit which has only two resistors in a parallel combination, knowing that $R_1 = 22\ k\Omega$, and $R_2 = 47\ k\Omega$, Calculate the total or the equivalent resistance of the following circuit .



Solution

Using our formula above for two resistors connected together in parallel we can calculate the total circuit resistance, R_T as:

$$R_T = \frac{22k\Omega \times 47k\Omega}{22k\Omega + 47k\Omega} = 14{,}985\Omega \text{ or } 15k\Omega$$

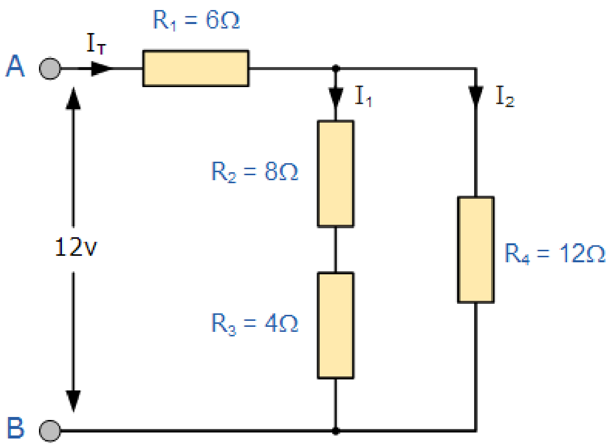
Another solution:

$$\frac{1}{R_T} = \frac{1}{R_a} + \frac{1}{R_b} = \frac{1}{22k\Omega} + \frac{1}{47k\Omega}$$

$$then\ R_T = 15k\Omega$$

Example 4:

in the following circuit calculate the total current (I_T) taken from the 12v supply.



Solution

$R_2 + R_3 = 8\Omega + 4\Omega = 12\Omega$

$$\frac{1}{R_{eq}} = \frac{1}{R_A} + \frac{1}{R_4} = \frac{1}{12} + \frac{1}{12} = 0.1667\Omega^{-1}$$

$$R_{eq} = \frac{1}{0.1667} = 6\Omega$$

$$R_{(ab)} = R_{eq} + R_1 = 6\Omega + 6\Omega = 12\Omega$$

$$\text{Circuit Current (I)} = \frac{V}{R} = \frac{12}{12} = 1\,\text{Ampere}$$

$$V_{R_1} = I \times R_1 = 1 \times 6 = 6\,V$$

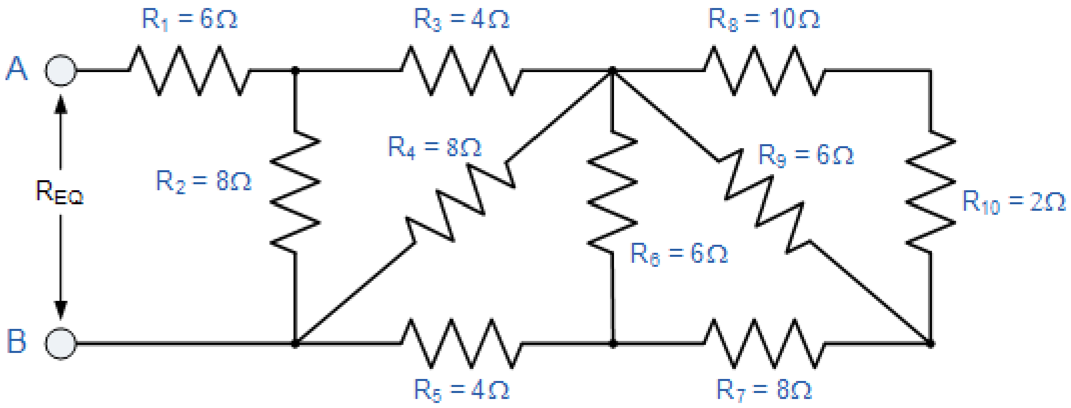
$$V_{R_a} = I \times R_4 = (12 - V_{R_1})\text{=12-6=6}\,V$$

$I_1 = 6V \div R_A = 6 \div 12 = 0.5A$ or $500mA$

$I_2 = 6V \div R_4 = 6 \div 12 = 0.5A$ or $500mA$

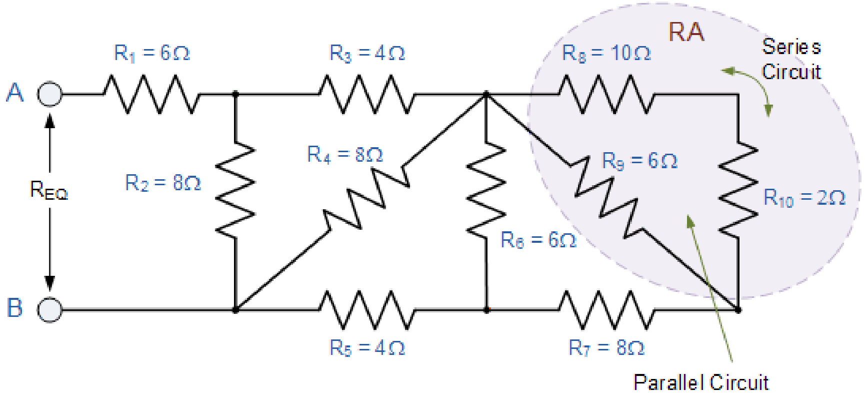
Example 5:

Find the equivalent resistance, R_{EQ} for the following resistor combination circuit.



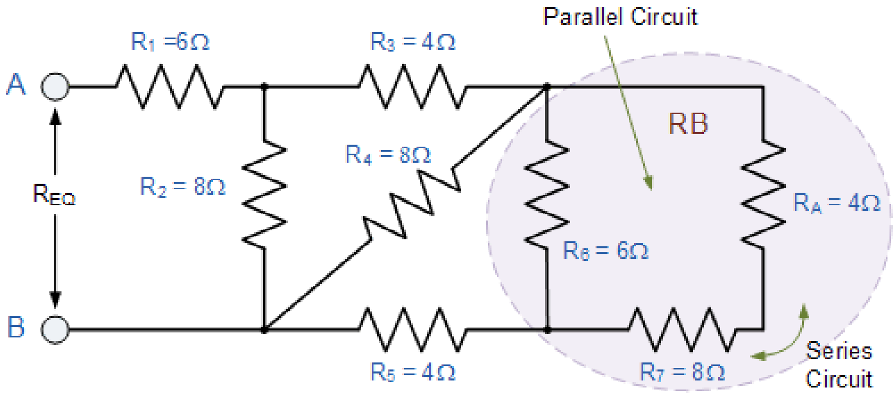
Solution

1)



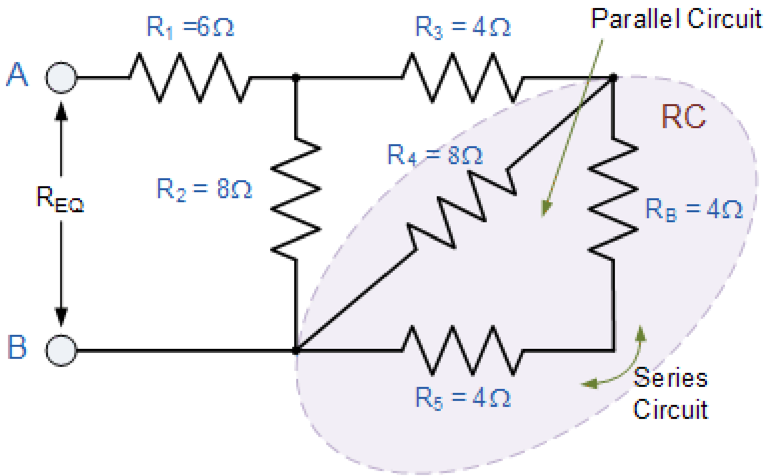
$$R_A = \frac{R_9 \times (R_8 + R_{10})}{R_9 + R_8 + R_{10}} = \frac{6 \times (10 + 2)}{6 + 10 + 2} = 4\Omega$$

2)



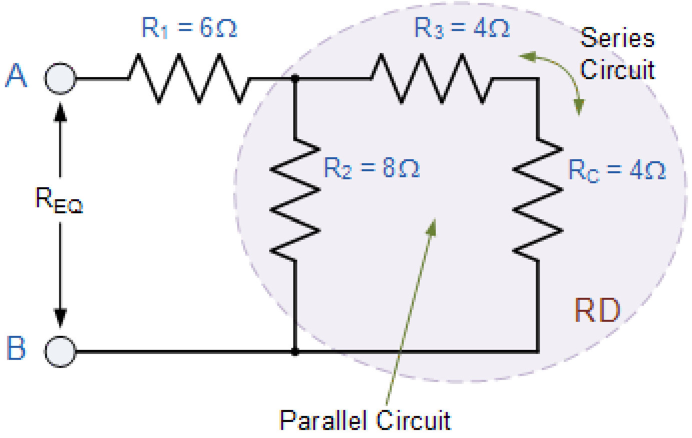
$$R_B = \frac{R_6 \times (R_A + R_7)}{R_6 + R_A + R_7} = \frac{6 \times (4 + 8)}{6 + 4 + 8} = 4\Omega$$

3)



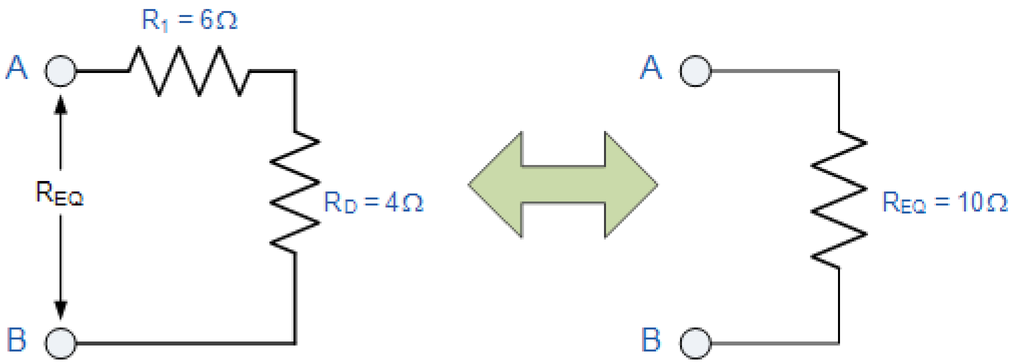
$$R_C = \frac{R_4 \times (R_B + R_5)}{R_4 + R_B + R_5} = \frac{8 \times (4 + 4)}{8 + 4 + 4} = 4\Omega$$

4)



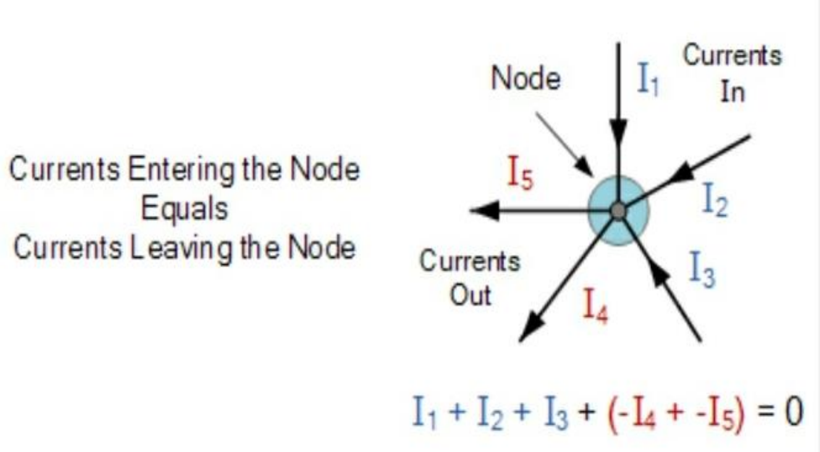
$$R_D = \frac{R_2 \times (R_C + R_3)}{R_2 + R_C + R_3} = \frac{8 \times (4 + 4)}{8 + 4 + 4} = 4\Omega$$

5)

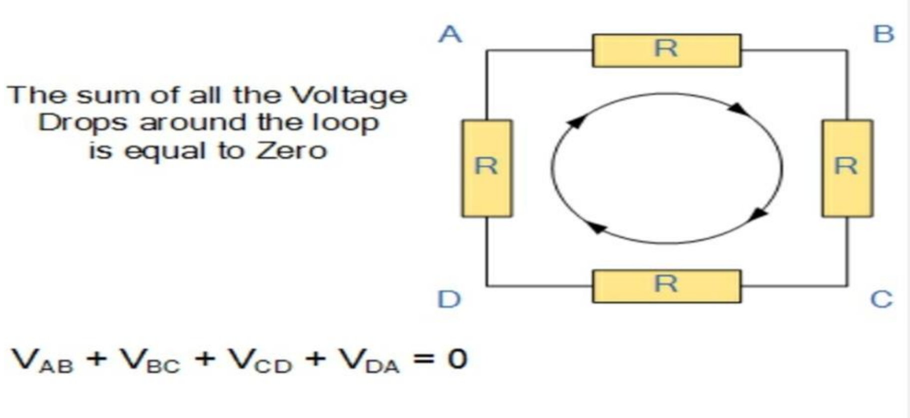


Kirchhoffs Circuit Law

1^{st} KVL

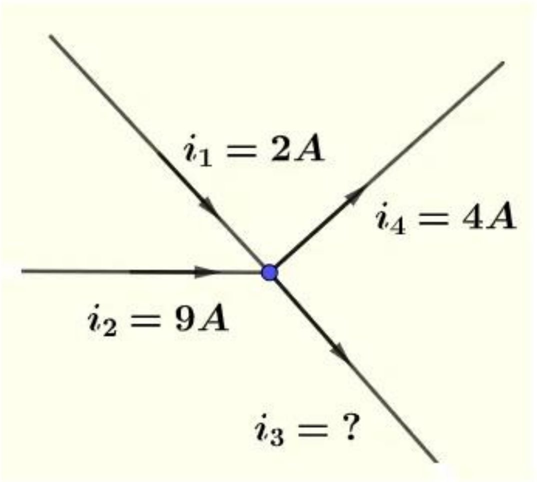


2^{nd} KVL



Example6:

Find current *i*3 at the node shown below.



Solution

Currents i_1 and i_2 are flowing into the node and currents i_3 and i_4 are flowing out of the node. Apply Kirchhoff's law of current at the given node.

$i_1 + i_2 = i_3 + i_4$

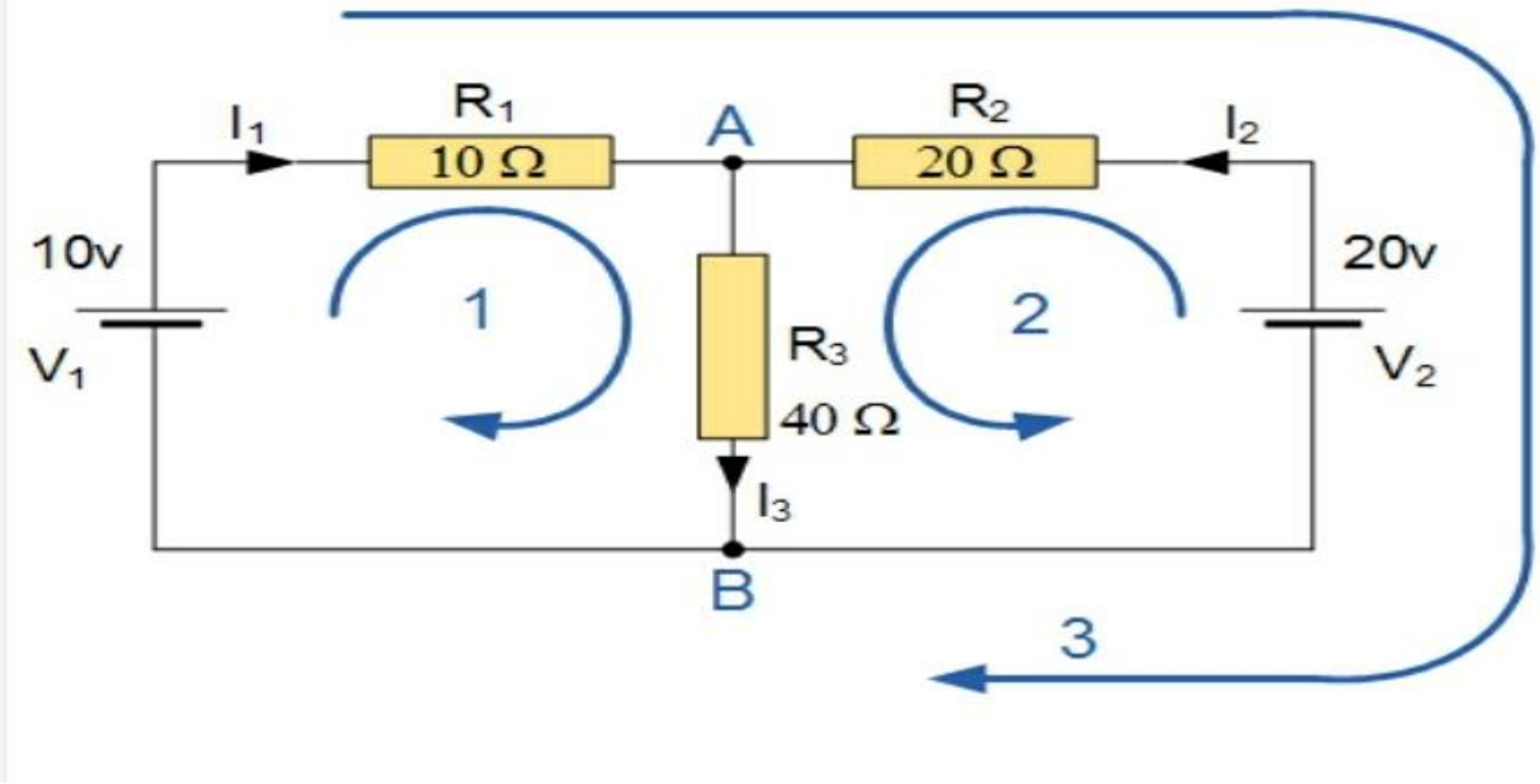
Substitute the known quantities

$2 + 9 = i_3 + 4$

Solve for i_3

$i_3 = 7$ A

Example7:Determine the voltage across the resistance R_3 and the currents across the following circuit.



Solution

The circuit has 3 branches, 2 nodes (A and B) and 2 independent loops.

Using **Kirchhoffs Current Law**, **KCL** the equations are given as:

At node A : $I_1 + I_2 = I_3$

At node B : $I_3 = I_1 + I_2$

Using **Kirchhoffs Voltage Law**, **KVL** the equations are given as:

Loop 1 is given as : $10 = R_1 I_1 + R_3 I_3 = 10I_1 + 40I_3$

Loop 2 is given as : $20 = R_2 I_2 + R_3 I_3 = 20I_2 + 40I_3$

Loop 3 is given as : $10 - 20 = 10I_1 - 20I_2$

As I_3 is the sum of $I_1 + I_2$ we can rewrite the equations as;

Eq. No 1 : $10 = 10I_1 + 40(I_1 + I_2) = 50I_1 + 40I_2$

Eq. No 2 : $20 = 20I_2 + 40(I_1 + I_2) = 40I_1 + 60I_2$

We now have two "**Simultaneous Equations**" that can be reduced to give us the values of I_1 and I_2

Substitution of I_1 in terms of I_2 gives us the value of I_1 as -0.143 Amps

Substitution of I_2 in terms of I_1 gives us the value of I_2 as +0.429 Amps

As : $I_3 = I_1 + I_2$

The current flowing in resistor R_3 is given as : $-0.143 + 0.429 = 0.286$ Amps

and the voltage across the resistor R_3 is given as : $0.286 \times 40 = 11.44$ volts

The negative sign for I_1 means that the direction of current flow initially chosen was wrong, but never the less still valid. In fact, the 20v battery is charging the 10v battery.

MCQ:

1)What is the maximum power can be generated from an 18-V emf using any combination of a 6.0Ω resistor and a 9.0Ω resistor?

a)54W

b)71W

c)90W

d)80W

2)Consider a circuit with two unequal resistances in parallel, then…

a) large current flows in large resistor

b) current is same in both

c) potential difference across each is same

d) smaller resistance has smaller conductance

3)Correct form of ohm's law

a) I = VR

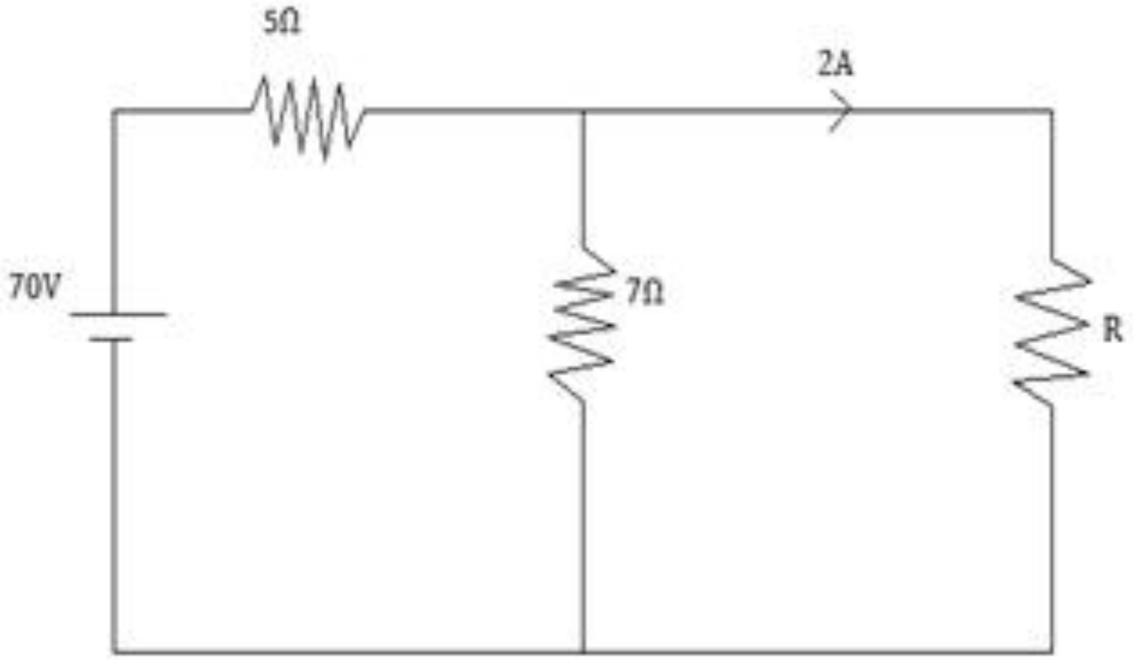
b)V ∝ I

c)V = IR

d)V= $\frac{I}{R}$

4)Find R in the following circuit

a) 17.5 Ω
b) 15 Ω
c) 24 Ω
d) 10.3Ω



Answers:

1) c
2)c
3) b&c
4) a

Thanks!

Any questions?