
بسم الله الرحمن الرحيم

EELU

الجامعة المصرية للتعلم الإلكترونى

Egyptian E-Learning University

MATH – 1

BY

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CHAPTER 3

Derivatives and their applications

LECTURE 5.

Slopes, tangent Lines and derivatives

Aims and Objectives:

(1) Understand the notion of slopes, tangent lines and derivatives.
(2) Use the relation between limits and derivatives.
(3) Understand the concepts of higher-order derivatives.
(4) Have a strong intuitive feeling for these important concepts.

- **APPLICATIONS OF CALCULUS IN COMPUTER SCIENCE**

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In Computer Science, Calculus is used for:

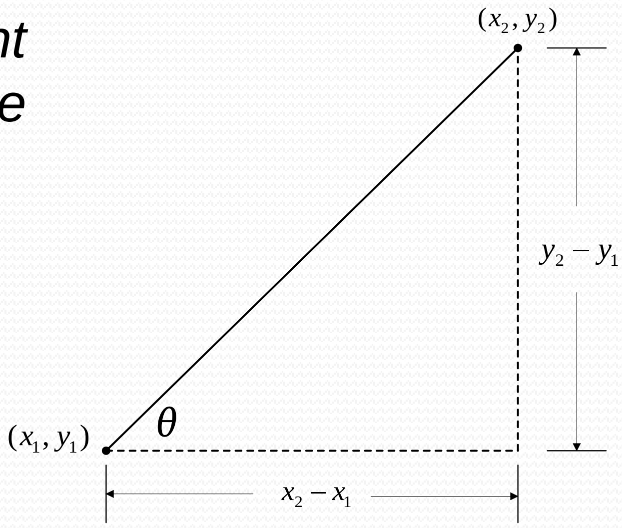
- ✓ Machine learning
- ✓ Data mining
- ✓ Scientific computing
- ✓ Image processing
- ✓ Creating the graphics and physics engines for video games
- ✓ 3D visuals for simulations
- ✓ In a wide array software programs that require it
- ✓ 3D models using multiple variable equations to utilize these models is through game development

- **APPLICATIONS OF CALCULUS IN COMPUTER SCIENCE**

- They use calculus for general problem solving applications, simulations, and physics engines.

- Physics engines create realistic situations in video games and probability simulations.

Slope *of a line is the change in the dependent variable* *y* *between two points divided by the relative change in the independent variable* *x*



The slope is the $\dfrac{\text{increase in } y}{\text{increase in } x} = \dfrac{\text{height moved}}{\text{length moved}}$

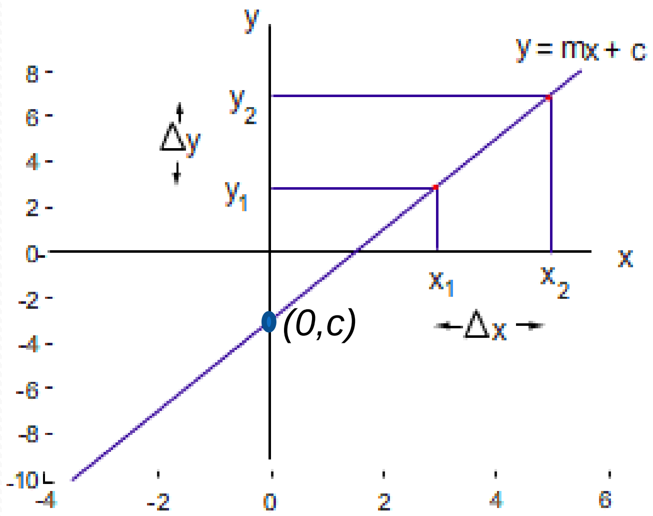
$$Slope = m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

Differentiation *is all about calculating the slope of a curve* *y(x)*, *at a given point,* *x*.

For a ***straight-line*** *graph of equation*

y(x) = mx + c,

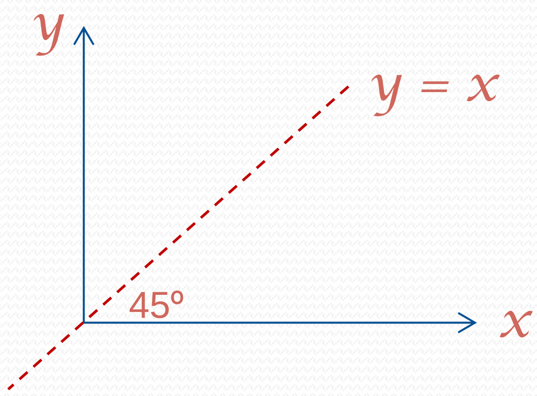
the slope is given simply by the value of *m* *and* *c* *is the* *y**-intercept.*



$$m = \frac{y - y_1}{x - x_1} = \frac{y - c}{x - 0} \Rightarrow y = mx + c$$

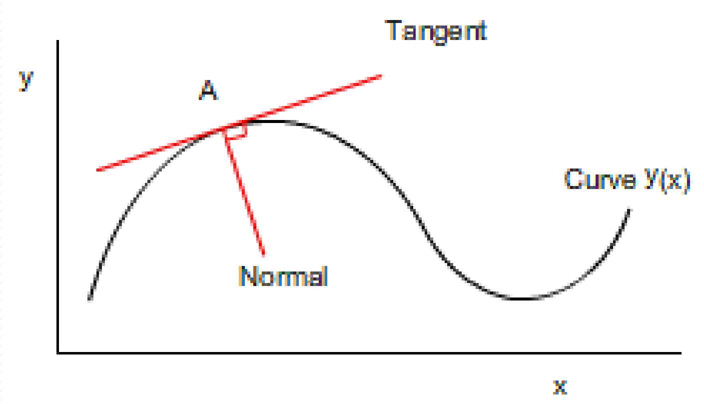
Example 1:

- $y = x$, slope $= 1$, $\theta = 45^\circ$, y-intercept $= 0$
- $y = 3x + 6$, slope $= 3$, y-intercept $= 6$
- $y = 5x - 3$, slope $= 5$, y-intercept $= -3$
- $y = -2x + 1$, slope $= -2$, y-intercept $= 1$
- $y = 6 - 3x$, slope $= -3$, y-intercept $= 6$



Notice:

The slope of any curve at point $\mathcal{A}$ is the same as that of the tangent at point $\mathcal{A}$.



Example 2:

Values of y and x are given below, what is the slope?

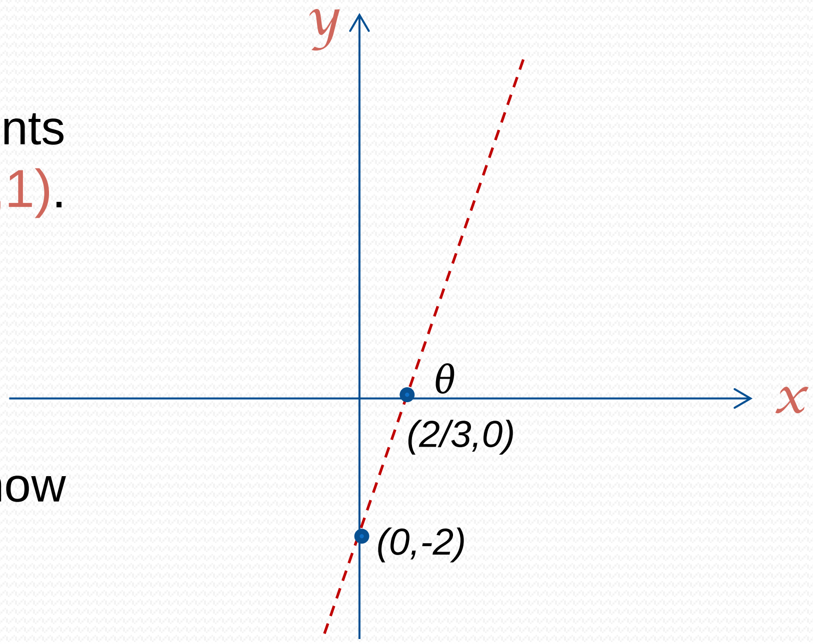
x	−3	−2	−1	0	1	2	3
y	−11	−8	−5	−2	1	4	7

i) Plotting the graph

ii) Choose any 2 points along the line (x_1,y_1) and (x_2,y_2)

iii) Draw the triangles (as in the diagram), or just calculate x and y.

iv) Calculate slope from: slope = $\Delta y/\Delta x$



Numerical Method: Choose any two points we have values for, say, (-2, -8) and (1,1). We now have:

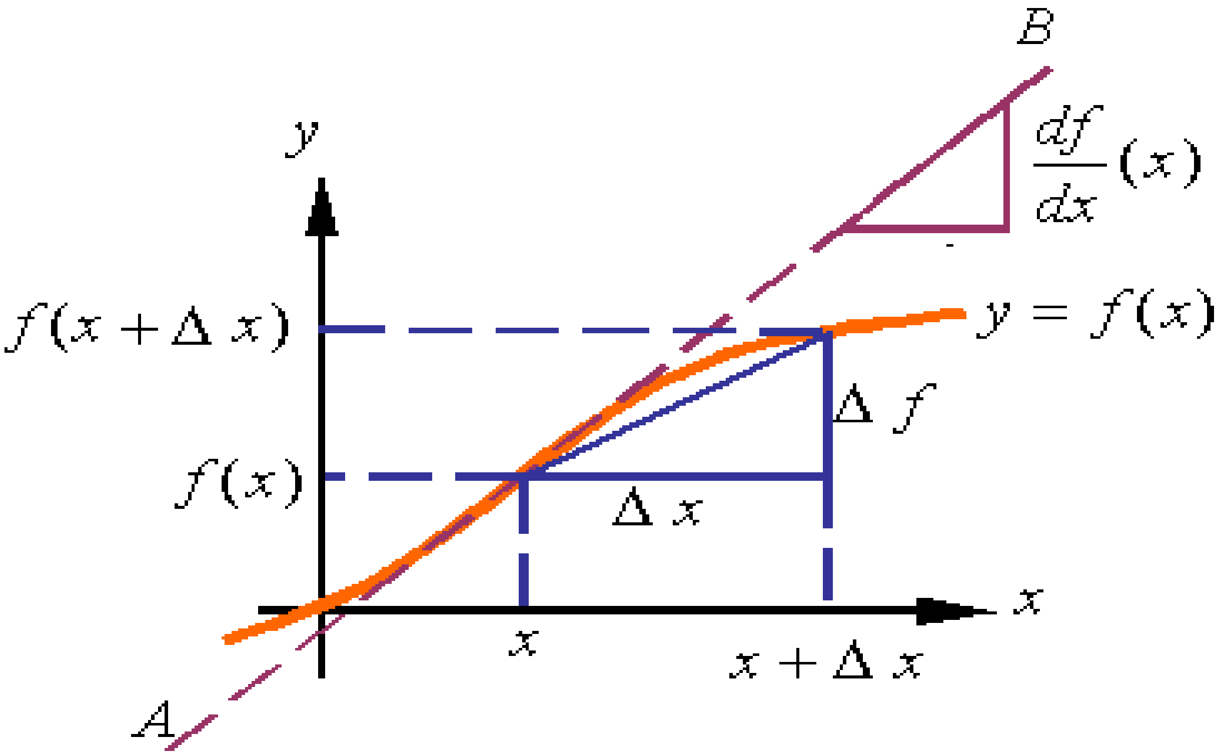
$$Slope = \frac{\Delta y}{\Delta x} = \frac{1-(-8)}{1-(-2)} = 3$$

$$\tan\theta = 3, \qquad \text{then} \qquad \theta \cong 71°$$

Since the intercept is at $y = -2$, we know that the equation of this line must be

$$y(x) = 3x - 2.$$

The derivative of a function is the slope at a given point



$$\lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = f'(x) = m_t(\text{at } x)$$

Definition: $$\frac{df}{dx} = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\delta y}{\delta x} = f'(x) = y'$$

For s(t), we have $\frac{ds}{dt}$, for E(ν), we have $\frac{dE}{d\nu}$, and for φ(λ), we have $\frac{d\varphi}{d\lambda}$.

Differentiation 'magic formula' (for standard polynomials)

$$\frac{d}{dx}(ax^n) = a * n * x^{n-1}$$

To differentiate a polynomial function, multiply together the leading factor, a, and the exponent (power), n, then subtract one from the exponent.

Examples:

1- $y = x^2$, $dy/dx = 2x$

2- $y = 2x^3$, $dy/dx = 6x^2$

3- $y = 9x^{27}$, $dy/dx = 243x^{26}$

4- $u = 3m^6$, $du/dm = 18m^5$

5- $\varphi = 7\lambda$, $d\varphi/d\lambda = 7$

6- $\Psi = x^3/12$, $d\Psi/dx = x^2/4$

7- $p = -5q^2$, $dp/dq = -10q$

8- $y = 5$, $dy/dx = 0$

The differential of a constant is always zero, i.e. its slope is zero, as we expect.

Differentiation Rules :

If f and g are differentiable functions , c is a constant , and n is any real number , then

1. Derivative of a constant function

$$\frac{d}{dx}c=0$$

2. The constant multiple rule

$$\frac{d}{dx}\left[cf(x)\right]=c\frac{d}{dx}f(x)$$

3. The sum rule

$$\frac{d}{dx}\left[f(x)+g(x)\right]=\frac{d}{dx}f(x)+\frac{d}{dx}g(x)$$

4. The difference rule

$$\frac{d}{dx}\left[f(x)-g(x)\right]=\frac{d}{dx}f(x)-\frac{d}{dx}g(x)$$

Differentiation Rules :

5. The product rule

$$\frac{d}{dx}\left[f(x).g(x)\right]=f(x)\cdot\frac{d}{dx}g(x)+g(x)\cdot\frac{d}{dx}f(x)$$

6. The quotient rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)\cdot\frac{d}{dx}f(x)-f(x)\cdot\frac{d}{dx}g(x)}{\left[g(x)\right]^{2}}$$

7. The power rule

$$\frac{d}{dx}\left[x^{n}\right]=n\,x^{n-1}$$

8. Derivative of natural exponential function

$$\frac{d}{dx}e^{x}=e^{x}$$

Derivatives of roots:

Roots - Fractional powers of x

$$\text{e.g.}\ \sqrt{x} = x^{1/2}, \sqrt[3]{x} = x^{1/3}$$

$$\sqrt[3]{x^2} = x^{2/3}, 1/\sqrt{x} = x^{-1/2}$$

$$\sqrt{x+1} = (x+1)^{1/2}, 1/\sqrt[3]{x^2+3} = (x^2+3)^{-1/3}$$

In the following you find some examples illustrating how to convert roots to fractional powers and then using the given rules to find the derivative.

Example 4:

1- $y = \sqrt{x} = x^{1/2}, \quad \frac{dy}{dx} = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$

2- $y = \sqrt[3]{x^2} = x^{2/3}, \quad \frac{dy}{dx} = \frac{2}{3} x^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x}}$

3- $y = 1/\sqrt{x} = x^{-1/2}, \quad \frac{dy}{dx} = -\frac{1}{2} x^{-3/2} = -\frac{1}{2\sqrt{x^3}}$

4- $\varphi(\lambda) = \sqrt{\lambda} - \frac{3}{\sqrt{\lambda^3}} = \lambda^{1/2} - 3\lambda^{-3/2}, \quad \frac{d\varphi}{d\lambda} = \frac{1}{2\sqrt{\lambda}} + \frac{9}{2\sqrt{\lambda^5}}$

Using Differentiation to calculate slopes:

Now we have a method to calculate the value of a slope at any point along a curve, without having to draw the graph and construct the tangent.

Example 5:

1- What is the slope of $y(x) = x^2 - 4x - 1$ at the point $x = 4$

Note: this is the same function we solved graphically, earlier.

Slope $= \dfrac{dy}{dx} = 2x - 4$

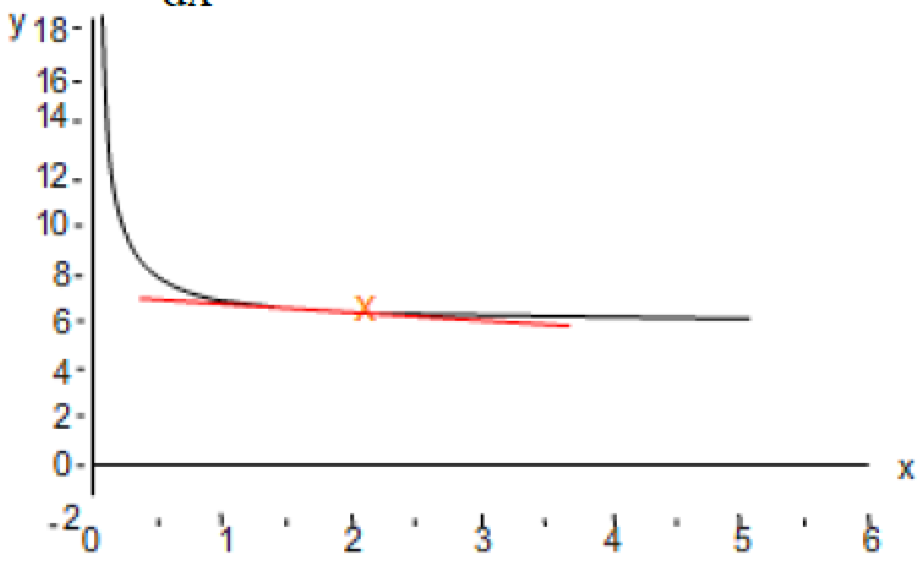
so, when $x = 4$

$\dfrac{dy}{dx} = (2 \times 4) - 4 = 4$ (as we found before)

2- What is the slope of $y(x) = \frac{1}{x} + 6$

at the point $x = 2$ $\frac{dy}{dx}$ = -$\frac{1}{x^2}$

so, when $x = 2$, $\frac{dy}{dx} = -\frac{1}{4}$



3- What is the slope of $p(q) = \frac{q^3}{3} - 2q^2$ at $q = 3$

$\frac{dp(q)}{dq} = q^2 - 4q$, so $\frac{dp(q)}{dq} = -3$

Higher Derivatives :

For the function $y = f(x)$

(i) The first derivative of f *is*

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx} f(x) = D_x f(x)$$

(ii) The second derivative of f *is*

$$f''(x) = y'' = \frac{d^2 y}{dx^2} = \frac{d^2 f}{dx^2} = \frac{d^2}{dx^2} f(x) = D_x^2 f(x)$$

(iii) The third derivative of f *is*

$$f'''(x) = y''' = \frac{d^3 y}{dx^3} = \frac{d^3 f}{dx^3} = \frac{d^3}{dx^3} f(x) = D_x^3 f(x)$$

$$\vdots$$

(iv) The nth derivative of f *is*

$$f^{(n)}(x) = y^{(n)} = \frac{d^n y}{dx^n} = \frac{d^n f}{dx^n} = \frac{d^n}{dx^n} f(x) = D_x^n f(x)$$

Illustrating Example :

Find the first four derivatives of $y = x^3 - 3x^2 + 2$.

Solution

First derivative :	$y' = 3x^2 - 6x$
Second derivative :	$y'' = 6x - 6$
Third derivative :	$y''' = 6$
Fourth derivative :	$y^{(4)} = 0$

Applications: Maxima and Minima

- ***1. Determine the first derivative.***
- ***2. Set the derivative to 0 and solve for values that satisfy the equation.***
- ***3. Determine the second derivative.***
  - ***(a) If second derivative > 0, point is a minimum.***
  - ***(b) If second derivative < 0, point is a maximum.***
  - ***(c) If second derivative = 0, point of inflection.***

THANK YOU