Differentiation

1 General Derivative Rules

1. Constant Rule $\dfrac{d}{dx}[c] = 0$

2. Constant Multiple Rule $\dfrac{d}{dx}[cf(x)] = cf'(x)$

3. Sum Rule $\dfrac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

4. Difference Rule $\dfrac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$

5. Product Rule $\dfrac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$

6. Quotient Rule $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

7. Chain Rule $\dfrac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$

(i) $\frac{d}{dx}\log x = \frac{1}{x}$, for $x > 0$

(ii) $\frac{d}{dx}e^{x} = e^{x}$

(iii) $\frac{d}{dx}a^{x} = a^{x}\log a$, for $a > 0$

(iv) $\frac{d}{dx}\log_{a} x = \frac{1}{x\log a}$, for $x > 0$, $a > 0$, $a \neq 1$

(i) $\frac{d}{dx}\sin x = \cos x$

(ii) $\frac{d}{dx}\cos x = -\sin x$

(iii) $\frac{d}{dx}\tan x = \sec^{2} x$

(iv) $\frac{d}{dx}\sec x = \sec x \tan x$

(v) $\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x$

(vi) $\frac{d}{dx}\cot x = -\operatorname{cosec}^{2} x$

find y'.

1. $y = (4x + 1)(1 - x)^3$

(A) $-12(1 - x)^2$ (B) $(1 - x)^2(1 + 8x)$ (C) $(1 - x)^2(1 - 16x)$

(D) $3(1 - x)^2(4x + 1)$ (E) $(1 - x)^2(16x + 7)$

2. $y = \frac{2-x}{3x+1}$

(A) $-\frac{7}{(3x+1)^2}$ (B) $\frac{6x-5}{(3x+1)^2}$ (C) $-\frac{9}{(3x+1)^2}$

(D) $\frac{7}{(3x+1)^2}$ (E) $\frac{7-6x}{(3x+1)^2}$

3. $y = \sqrt{3-2x}$

(A) $\frac{1}{2\sqrt{3-2x}}$ (B) $-\frac{1}{\sqrt{3-2x}}$ (C) $-\frac{(3-2x)^{3/2}}{3}$

(D) $-\frac{1}{3-2x}$ (E) $\frac{2}{3}(3-2x)^{3/2}$

1. C
2. A
3. B

$y = \ln \dfrac{e^x}{e^x - 1}$

(A) $x - \dfrac{e^x}{e^x - 1}$ **(B)** $\dfrac{1}{e^x - 1}$ **(C)** $-\dfrac{1}{e^x - 1}$ C

(D) 0 **(E)** $\dfrac{e^x - 2}{e^x - 1}$

$y = \ln(\sec x + \tan x)$

(A) $\sec x$ **(B)** $\dfrac{1}{\sec x}$ **(C)** $\tan x + \dfrac{\sec^2 x}{\tan x}$ A

(D) $\dfrac{1}{\sec x + \tan x}$ **(E)** $-\dfrac{1}{\sec x + \tan x}$

$y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

(A) 0 **(B)** 1 **(C)** $\dfrac{2}{(e^x + e^{-x})^2}$ D

(D) $\dfrac{4}{(e^x + e^{-x})^2}$ **(E)** $\dfrac{1}{e^{2x} + e^{-2x}}$

$y = \ln(x\sqrt{x^2+1})$

(A) $1 + \dfrac{x}{x^2+1}$ (B) $\dfrac{1}{x\sqrt{x^2+1}}$ (C) $\dfrac{2x^2+1}{x\sqrt{x^2+1}}$ D

(D) $\dfrac{2x^2+1}{x(x^2+1)}$ (E) none of these

$y = e^{-x}\cos 2x$ A

(A) $-e^{-x}(\cos 2x + 2\sin 2x)$
(B) $e^{-x}(\sin 2x - \cos 2x)$
(C) $2e^{-x}\sin 2x$
(D) $-e^{-x}(\cos 2x + \sin 2x)$
(E) $-e^{-x}\sin 2x$

1. The definition of the first derivative of a function $f(x)$ is

(A) $f'(x) = \dfrac{f(x+\Delta x) + f(x)}{\Delta x}$

(B) $f'(x) = \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

(C) $f'(x) = \lim_{\Delta x \to 0} \dfrac{f(x+\Delta x) + f(x)}{\Delta x}$

(D) $f'(x) = \lim_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

Solution

The correct answer is (D).

The definition of the first derivative of the function $f(x)$ is

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

2. Given $y = 5e^{3x} + \sin x$, $\dfrac{dy}{dx}$ is

(A) $5e^{3x} + \cos x$

(B) $15e^{3x} + \cos x$

(C) $15e^{3x} - \cos x$

(D) $2.666e^{3x} - \cos x$

Use the sum rule of differentiation

$$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$$

Re-write the function as

$$y = u + v$$

where

$$u = 5e^{3x}$$

$$v = \sin x$$

Find $\dfrac{du}{dx}$ and $\dfrac{dv}{dx}$

$$\frac{d}{dx}(5e^{3x}) = 15e^{3x} \qquad \left(\frac{d}{dx}(e^{ax}) = ae^{ax}\right)$$

$$\frac{d}{dx}(\sin x) = \cos x \qquad \left(\frac{d}{dx}(\sin x) = \cos x\right)$$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$= 15e^{3x} + \cos x$$

Find $k'(s)$ if $k(s) = \dfrac{\ln s}{s^2}$.

(A) $\dfrac{1}{2s^2}$

(B) $-\dfrac{2}{s^4}$

(C) $\dfrac{1}{s^3} + \dfrac{2\ln s}{s^3}$

(D) $\dfrac{1}{s^3} - \dfrac{2\ln s}{s^3}$

Answer: D

$$k'(s) = \frac{\frac{1}{s}\cdot s^2 - (\ln s)\cdot 2s}{s^4}$$

$$= \frac{s - 2s\ln s}{s^4}$$

$$= \frac{1 - 2\ln s}{s^3} = \frac{1}{s^3} - \frac{2\ln s}{s^3}$$

If $x^2 + 2xy = y^2$, then $\frac{dy}{dx}$ is

(A) $\frac{x+y}{y-x}$
(B) $2x+2y$
(C) $\frac{x+1}{y}$
(D) $-x$

$$\frac{d}{dx}[x^2]+\frac{d}{dx}[2xy]=\frac{d}{dx}[y^2]$$

$$2x+2x\frac{dy}{dx}+2y=2y\frac{dy}{dx} \qquad \left(\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}\right)$$

$$x+y=(y-x)\frac{dy}{dx}$$

$$\frac{dy}{dx}=\frac{x+y}{y-x}$$

Given $y = x^3 \ln x$, $\dfrac{dy}{dx}$ is

(A) $3x^2 \ln x$
(B) $3x^2 \ln x + x^2$
(C) x^2
(D) $3x$

Using the product rule,

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$u = x^3$$

$$v = \ln x$$

$$\frac{du}{dx} = 3x^2$$

$$\frac{dv}{dx} = \frac{1}{x}$$

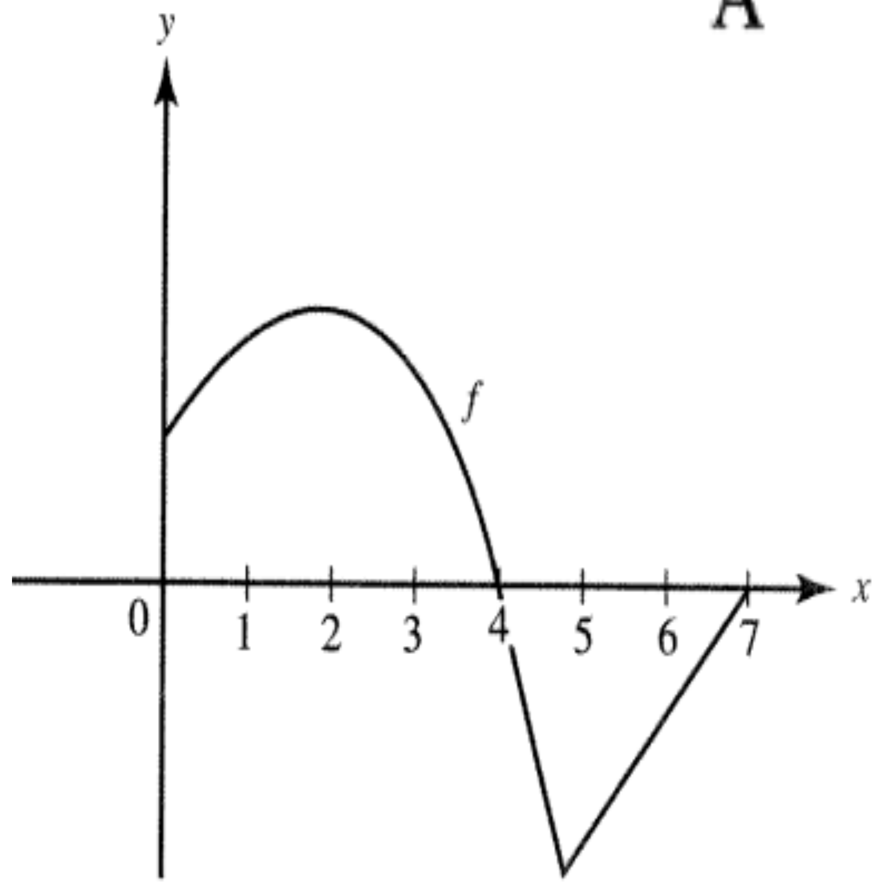
Substituting into Equation (1),

$$\frac{dy}{dx} = x^3 \times \frac{1}{x} + \ln x \times 3x^2$$

$$= x^2 + 3x^2 \ln x$$

The function f whose graph is shown has $f' = 0$ at $x =$

A



(A) 2 only
(B) 2 and 5
(C) 4 and 7
(D) 2, 4, and 7
(E) 2, 4, 5, and 7

A function f has the derivative shown. Which of the following statements is false?

(A) f is continuous at $x = a$
(B) $f(a) = 0$
(C) f has a vertical asymptote at $x = a$
(D) f has a jump discontinuity at $x = a$
(E) f has a removable discontinuity at $x = a$

C