Section (4)

Physics (I)

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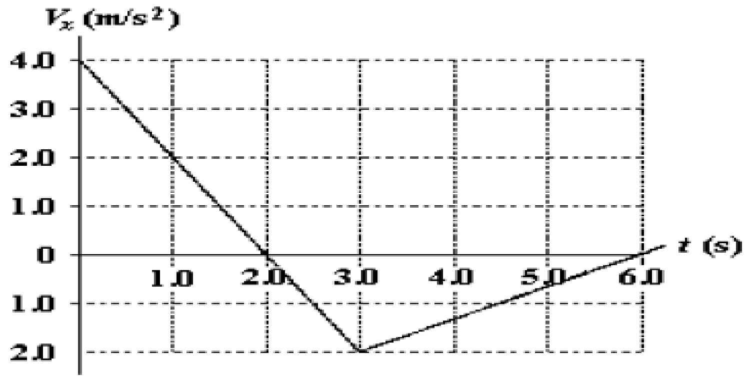
Revision questions

A baseball is thrown vertically into the air. The acceleration of the ball at its highest point is:

A. zero
B. g, down
C. g, up
D. $2g$, down
E. $2g$, up

Ans: B

V_x is the velocity of a particle moving along the x axis as shown. If $x = 2.0$ m at $t = 1.0$ s, what is the position of the particle at $t = 6.0$ s?



a. −2.0 m
b. +2.0 m
c. +1.0 m
d. −1.0 m
e. 6.0 m

Vectors

31. Consider the two vectors $\mathbf{A} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$ $\mathbf{B} = -\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$
Calculate (a) $\mathbf{A} + \mathbf{B}$, (b) $\mathbf{A} - \mathbf{B}$, (c) $|\mathbf{A} + \mathbf{B}|$, (d) $|\mathbf{A} - \mathbf{B}|$, and (e) the directions of $\mathbf{A} + \mathbf{B}$ and $\mathbf{A} - \mathbf{B}$.

P3.31

(a) $$(\mathbf{A}+\mathbf{B}) = (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) + (-\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) = \boxed{2\hat{\mathbf{i}} - 6\hat{\mathbf{j}}}$$

(b) $$(\mathbf{A}-\mathbf{B}) = (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) - (-\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) = \boxed{4\hat{\mathbf{i}} + 2\hat{\mathbf{j}}}$$

(c) $$|\mathbf{A}+\mathbf{B}| = \sqrt{2^2 + 6^2} = \boxed{6.32}$$

(d) $$|\mathbf{A}-\mathbf{B}| = \sqrt{4^2 + 2^2} = \boxed{4.47}$$

(e) $$\theta_{|\mathbf{A}+\mathbf{B}|} = \tan^{-1}\left(-\frac{6}{2}\right) = -71.6^\circ = \boxed{288^\circ}$$

$$\theta_{|\mathbf{A}-\mathbf{B}|} = \tan^{-1}\left(\frac{2}{4}\right) = \boxed{26.6^\circ}$$

Question:

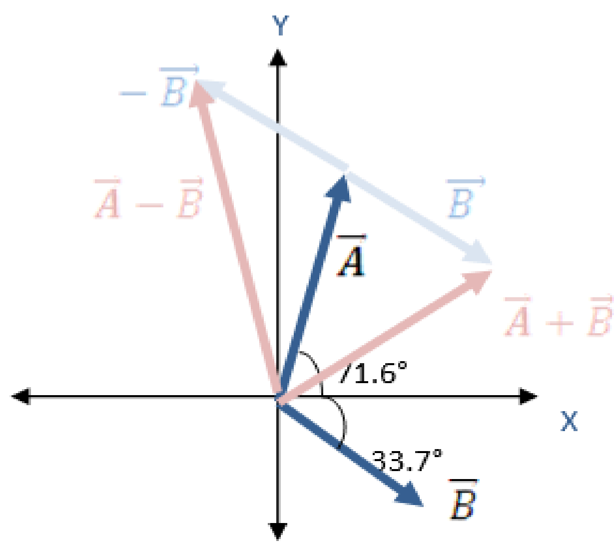
Given the vectors $\vec{A} = 2.00\ \hat{\imath} + 6.00\ \hat{\jmath}$ and $\vec{B} = 3.00\ \hat{\imath} - 2.00\ \hat{\jmath}$

a) Draw the vector sum $\vec{C} = \vec{A} + \vec{B}$ and the vector difference $\vec{D} = \vec{A} - \vec{B}$

b) Calculate C *and* D , first in terms of unit vectors and then in terms of polar coordinates, with angles measured with respect to the $+x$ *axis* .

Solution Q:

a) See **Figure 1**



b)

$$\because \vec{C} = \vec{A} + \vec{B}$$

$$\vec{C} = 2.00\,\hat{i} + 6.00\,\hat{j} + 3.00\,\hat{i} - 2.00\,\hat{j}$$

$$\vec{C} = 5.00\,\hat{i} + 4.00\,\hat{j}$$

$$|\vec{C}| = \sqrt{(5)^2 + (4)^2} = 6.4$$

at, $\tan\theta = \frac{4}{5}$ $\quad \therefore \theta = 38.7^\circ$

$$\because \vec{D} = \vec{A} - \vec{B}$$

$$\vec{D} = 2.00\,\hat{i} + 6.00\,\hat{j} - 3.00\,\hat{i} + 2.00\,\hat{j}$$

$$\vec{D} = -1.00\,\hat{i} + 8.00\,\hat{j}$$

$$|\vec{D}| = \sqrt{(-1)^2 + (8)^2} = 8.06$$

at, $\tan\theta = \frac{8}{-1}$ $\quad \therefore \theta = 97.2^\circ$

Ex.:

Find the scalar product of **A** = 8 **i** + 2**j** - 3 **k** and **B** = 3 **i** - 6**j** + 4 **k**

Solution:

$$\mathbf{A.B} = A_xB_x + A_yB_y + A_zB_z$$

$$=(8)(3) - (2)(6) - (3)(4) = 0$$

What does this result mean?

Ex.:

Find the angle between **A** = 2 **i** + **j** +2 **k** and **B** = 4 **i** - 3**j**

Solution:

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{2\times 4 - 1\times 3 + 0}{3\times 5} = \frac{1}{3} \qquad \therefore \theta = \cos^{-1}(1/3) = 70.5^0$$

Properties of Scalar Multiplication

$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$ **commutative law**

$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$ **distributive law**

$m(\vec{A} \cdot \vec{B}) = (m\vec{A}) \cdot \vec{B} = \vec{A} \cdot (m\vec{B}) = (\vec{A} \cdot \vec{B})m$ **where m is scalar**

$$\vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = 1$$

$$\vec{i} \cdot \vec{j} = \vec{i} \cdot \vec{k} = \vec{j} \cdot \vec{k} = 0$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A} \cdot \vec{A} = A_x^2 + A_y^2 + A_z^2$$

Vector Multiplication

Ex.: Find the cross product of the two vectors

$$\vec{A} = 2\vec{i} - 3\vec{j} - \vec{k} \ \ and \ \ \vec{B} = \vec{i} + 4\vec{j} - 2\vec{k} \ \ Find \ \ \vec{A} \times \vec{B}$$

Solution:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -3 & -1 \\ 1 & 4 & -2 \end{vmatrix}$$

$$= \vec{i} \begin{vmatrix} -3 & -1 \\ 4 & -2 \end{vmatrix} - \vec{j} \begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix} + \vec{k} \begin{vmatrix} 2 & -3 \\ 1 & 4 \end{vmatrix}$$

$$= 10\vec{i} + 3\vec{j} + 11\vec{k}$$

Properties of Vector Multiplication

$\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$ **commutative law fails**

$\vec{A}\times\left(\vec{B}+\vec{C}\right)=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$ **Distributive law**

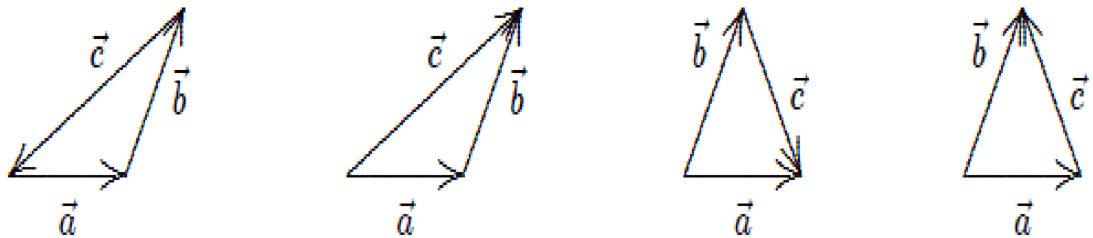
$m\left(\vec{A}\times\vec{B}\right)=\left(m\vec{A}\right)\times\vec{B}=\vec{A}\times\left(m\vec{B}\right)=\left(\vec{A}\times\vec{B}\right)m$ **where m is scalar**

$$\vec{i}\times\vec{i}=\vec{j}\times\vec{j}=\vec{k}\times\vec{k}=0$$

$$\vec{i}\times\vec{j}=\vec{k} \qquad \vec{j}\times\vec{k}=\vec{i} \qquad \vec{k}\times\vec{i}=\vec{j}$$

MCQ

The vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ are related by $\vec{c} = \vec{b} - \vec{a}$. Which diagram below illustrates this relationship?



A B C D

Ans: D

If $\vec{A} = (6\,\mathrm{m})\,\hat{\mathrm{i}} - (8\,\mathrm{m})\,\hat{\mathrm{j}}$ then $4\vec{A}$ has magnitude:

A. 10 m
B. 20 m
C. 30 m
D. 40 m
E. 50 m

Ans: D