Assignment 1

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- Course Name: Numerical methods
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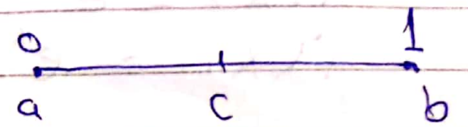
"Assignment of Numerical Methods"

1- Using Bisection method find the root of $\cos(x) - x e^{x} = 0$ with $a = 0$ and $b = 1$, $\epsilon = 0.01$

$f(a) = \cos(0) - (0) \times e^{(0)} = -1$

$f(b) = \cos(1) - (1) \times e^{1} = 2.17.$

$f(a) f(b) < \epsilon$



Number	a	b	c	f(c)
1	0	1	0.5	0.053
2	0.5	1	0.75	-0.856
3	0.5	0.75	0.625	-0.356690
4	0.5	0.625	0.5625	-0.14129
5	0.5	0.5625	0.53125	0.77913.
6	0.5	0.53125	0.515625	6.475×10^{-3}
7	0.515625	0.53125	0.5234375	-0.017362
8	0.515625	0.5234375	0.1953125	-5.404×10^{-3}
9	0.515625	0.51953	~~0.51578~~ $\boxed{0.517578}$	5.9518×10^{-4}

Solution = 0.517578.

Solution

(2) find the Solution of $f(x) = e^{x} - 1.5 - \tan^{-1}(x) = 0$ by Nawton Method. $x = -10$ and $\epsilon = .10^{-5}$ z.

$$f(x) = e^{x} - 1.5 - \tan^{-1}(x)$$

$$f'(x) = e^{x} - \frac{1}{1+x^{2}}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = \cancel{x_0}\ -10 - \frac{e^{-10} - 1.5 - \tan^{-1}(-10)}{e^{-10} - \frac{1}{1+(-10)^2}} = -12.9243146.$$

$$x_2 = -12.924 - \frac{e^{-12.924} - 1.5 - \tan^{-1}(-12.924)}{e^{-12.924} - \frac{1}{1+(-12.924)^2}} = -14.00381$$

$$x_3 = -14.00381 - \frac{e^{-14.00381} - 1.5 - \tan^{-1}(-14.00381)}{e^{-14.00381} - \frac{1}{1+(-14.00381)^2}} = -14.100600$$

$$x_4 = -14.100 - \frac{e^{-14.100} - 1.5 - \tan^{-1}(-14.100)}{e^{-14.100} - \frac{1}{1+(-14.100)^2}} = -14.10126974$$

Solution $= -14.10126974.$ ~~-14.10126~~

(3) ~~Find~~ using the fixed point iterative method to find a root of $f(x) = 4x^2 - 2x - 1$ $\epsilon = 0.005$ with interval $[0, 1]$.

$f(a) = 4(0) - 2(0) - 1 = -1$ (−)

$f(b) = 4(1) - 2(1) - 1 = 5.$ (+)

1st method:-

$4x^2 - 2x + 1$

$x^2 = \frac{-x}{2} + \frac{1}{4}$

$= -\frac{2x+1}{4}$

$x = \frac{\pm\sqrt{-1-2x}}{2}$

not relevant

2nd method:-

$4x^2 + 2x - 1$

$4x^2 + 2x = 1$

$x(4x + 2) = 1$

$x = \frac{1}{4x+2}$ ⇒

$x_1 = 0.166667$

$x_2 = 0.37500375$

$x_3 = 0.28571$

$x_4 = 0.31818$

$x_5 = 0.305555$

$x_6 = 0.3103448$

$\boxed{x_7 = 0.3085108}$ ← Solution

(4) Find a solution of $x^3 + 1 = 0$ by Newton method for $x_0 = 3$ and $\epsilon = 5\%$.

$f(x) = x^3 + 1$ $\quad f'(x) = 3x^2$ $\quad \epsilon = 0.05.$

$f x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = -3 - \frac{(-3)^3 + 1}{3(-3)^2} = -2.037037.$

$x_2 = -2.037 - \frac{(-2.037)^3 + 1}{3(-2.037)^2} = -1.43835$

$x_3 = -1.4383552\cancel{7}$

$x_4 = -1.0124026$

$x_5 = -1.00015\cancel{9}311$

~~A~~ Solution:-

-1.00015311

(5) Find the equation $f(x) = e^{-x} - x = 0$ Find the root of the function by using Fixed Point iteration method with $x_0 = 0.5$ and $\epsilon = 5\%$.

$f(x) = e^{-x} - x$

equation : $x = e^{-x}$

$x_1 = 0.606530$ $\quad$ $x_2 = 0.545239$

$x_3 = 0.579703$ $\quad$ $\boxed{x_4 = 0.5600076462}$ ← Solution

(6) Consider finding the root of $f(x) = x^2 - 3$ and start with the interval $[1, 2]$ with $N = 3$ iterations.

$f(a) = (1)^2 - 3 = -2 \ (-)$

$f(b) = (2)^2 - 3 = 1 \ (+)$

$f(a) f(b) < \epsilon$. $\quad$ $c = \frac{a+b}{2} = \frac{1+2}{2} = 1.5$

Number	a	b	c	f(c)
0	1	2	1.5	-0.75
1	1.5	2	1.75	0.0626
2	1.5	1.75	1.625	-0.3593
3	1.625	1.75	1.6875	-0.15234

(7) Use Newton's method to find the root of $x^4 - 5x^3 + 9x + 3 = 0$ accurate to 6 decimal places in the interval [4, 6].

$F(x) = x^4 - 5x^3 + 9x + 3$

$F'(x) = 4x^3 - 15x^2 + 9$

$x_0 = \frac{a+b}{2} = \frac{4+6}{2} = 5$

$x_1 = 5 - \frac{(5)^4 - 5(5)^3 + 9(5) + 3}{4(5)^3 - 15(5)^2 + 9} = 4.64179$

$x_2 = 4.64179 - \frac{(4.64)^4 - 5(4.64) + 9(4.64) + 3}{4(4.64)^3 - 15(4.64)^2 + 9}$

$= 4.5375$

$x_3 = 4.53 - \frac{(4.53)^4 - 5(4.53)^3 + 9(4.53) + 3}{4(4.53)^3 - 15(4.53)^2 + 9} = \boxed{4.52891796}$

Solution

(8) (a) Put $(11100101.1101)_2$ in Single Precision IEEE 754 standard.

$(11100101.1101)_2$

$1.00101101 \times 2^7 \leftarrow$ exponent.

Exponent = 7 + 127 = 134.

134	2	0
67	2	1
33	2	1
16	2	0
8	2	0
4	2	0
2	2	0
1	2	1

10000110.

Solution:

0 10000110 1100 1011 1010 0000 0000 000

(8) [b) Find IEEE 754 single precision of 85.125

85	2	1
42	2	0
21	2	1
10	2	0
5	2	1
2	2	0
1	2	1

$0.125 \times 2 = 0.25 \quad 0$

$0.25 \times 2 = 0.5 \quad 0$

$0.5 \times 2 = 1 \quad 1$

1010101.001

1.010101001×2^6

Exponent $= 6 + 127 = 133$

133	2	1
66	2	0
33	2	1
16	2	0
8	2	0
4	2	0
2	2	0
1	2	1

10000101

Solution: 0 1000 0101 0101 0100 1000 0000 0000 000

(9) Suppose we would like to determine the minimum no. of iterations needed in Bisection algorithm given to $a_0 = 3$, $b_0 = 4.5$ and $\epsilon = 10^{-5}$.

$$N \geq \frac{[\log_{10}(b_0 - a_0) - \log_{10}(\epsilon)]}{\log_{10}(2)}$$

$$N \geq \frac{[\log_{10}(4.5 - 3) - \log_{10}(10^{-5})]}{\log_{10}(2)} = 17.1946$$

$\simeq 17$ iterations.

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(10) (a) Determine the absolute error and relative errors when approximating p by when $p = 1.32 \times 10^2$ a, $p^* = 1.35 \times 10^2$

Absolute error $= |p - p^*| = |1.32 \times 10^2 - 1.35 \times 10^2|$
$= 3.$

Relative error $= \dfrac{|p - p^*|}{p} = \dfrac{|1.32 \times 10^2 - 1.35 \times 10^2|}{1.32 \times 10^2}$
$= 0.0227 \simeq 2.2727\%$

(10)(b) What is the decimal number represented by word:

$\boxed{1}$ $\underbrace{1000\ 0001}_{\text{Exponent.}}$ 0100 0000 0000 0000 0000 000

S=1

$Exp = 2^0 + 2^7 = 129.$

$Mantissa = 2^{-2} = \frac{1}{4} = 0.25.$

Form:- $(-1)^s \ 2^{e-127}$ 1. Mantissa.

$= (-1)^1 (2^2)(1.25) = -5$