الجامعة المصرية للتعلم الإلكتروني الأهلية

EELU

THE EGYPTIAN E-LEARNING UNIVERSITY

GEN206
Discrete Mathematics

Section 3

Faculty of Information Technology
Egyptian E-Learning University

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1. Use a direct proof to show that the sum of two odd integers is even.

Let
p: n, m are odd integers
q : (n+m) is even

$$p \rightarrow q$$

1. We assume that p is true
2. We try to prove that q is also true
3. Then $p \rightarrow q$ is true.

1- we assume that p is true :

n= 2k+1 , m=2j+1 where k,j are integers

2- we try to prove that q is also true:

$(n+m) = (2k+1)+(2j+1) = (2k+2j+2) = 2(k+j+1)$ = even integer so $q \equiv$ true

3- Hence $(p \rightarrow q) \equiv$ True

10. Use a direct proof to show that the product of two rational numbers is rational.

p: n ,m are rational numbers

q: nxm is rational

We assume p is true and try to prove that q is also true

p is true so n= x/y , m= k/z

n*m = xk/yz

xk is integer and yz is also integer

and by axiom xk has no common factor with yz

So n*m Is rational i.e q is true

16. Prove that if x, y, and z are integers and $x + y + z$ is odd, then at least one of x, y, and z is odd.

By contraposition

p: x+y+z is odd where x,y,z are integers
q: x is odd or y is odd or z is odd

$p \rightarrow q$

$\neg q \rightarrow \neg p$

1.$we\ assumet\ hat\ \neg q\ is\ true$

2.$We\ try\ to\ provet\ hat \neg p\ is\ also\ true$

3.$Then\ \neg q \rightarrow \neg p\ is\ true$.

4.$The\ p \rightarrow q\ is\ also\ true$

¬q = (x is even and y is even and z is even)

So x= 2a , y =2b, z= 2c

Now let us see p:

x+y+z = 2(a+b+c) = even so $\neg q$ is $true$

Therefore $\neg q \rightarrow \neg p\ is\ true$ then $The\ p \rightarrow q\ is\ also\ true$

8. Prove that if n is a perfect square, then $n + 2$ is not a perfect square.

By contradiction

P: n is a perfect square i.e. n= a^2 where a is an integer
q: n+2 is not perfect square
To use contradiciton method
we try to prove that (p∧¬q) → False
i.e ¬(p ∧ ¬q) ≡ True i.e (p ∧ ¬q) ≡ False
so we assume that p is true
and prove that ¬q is false

p: n is a perfect square i.e. n= a^2 where a is an integer

q: n+2 is not perfect square ($\neg$q) = n+2 is a perfect square

n+2 = y^2 where y is integer

So $a^2 + 2 = y^2$

$y^2 - a^2 = 2$

(y+a) (y-a) =2

since y and a are integers then (y+a) and (y-a) are integers

So either (1) (y+a) =2 and (y-a)=1 or (2) (y+a)= 1 and (y-a) =2

If we solve the set of equations (1) or (2) we get y= 3/2 and a= 3/2

Which proves that ($\neg$q) is always false i.e (n+2) is not a perfect square

Therefore $\neg$(p$\wedge$ $\neg$q) = true then $The\ p \rightarrow q\ is\ also\ true$

1. List the members of these sets.

a) $\{x \mid x \text{ is a real number such that } x^2 = 1\}$

b) $\{x \mid x \text{ is a positive integer less than } 12\}$

c) $\{x \mid x \text{ is the square of an integer and } x < 100\}$

d) $\{x \mid x \text{ is an integer such that } x^2 = 2\}$

a) $\{-1, 1\}$

b) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

c) $\{0, 1, 4, 9, 16, 25, 36, 49, 64, 81\}$

d) $\emptyset$

2. Use set builder notation to give a description of each of these sets.

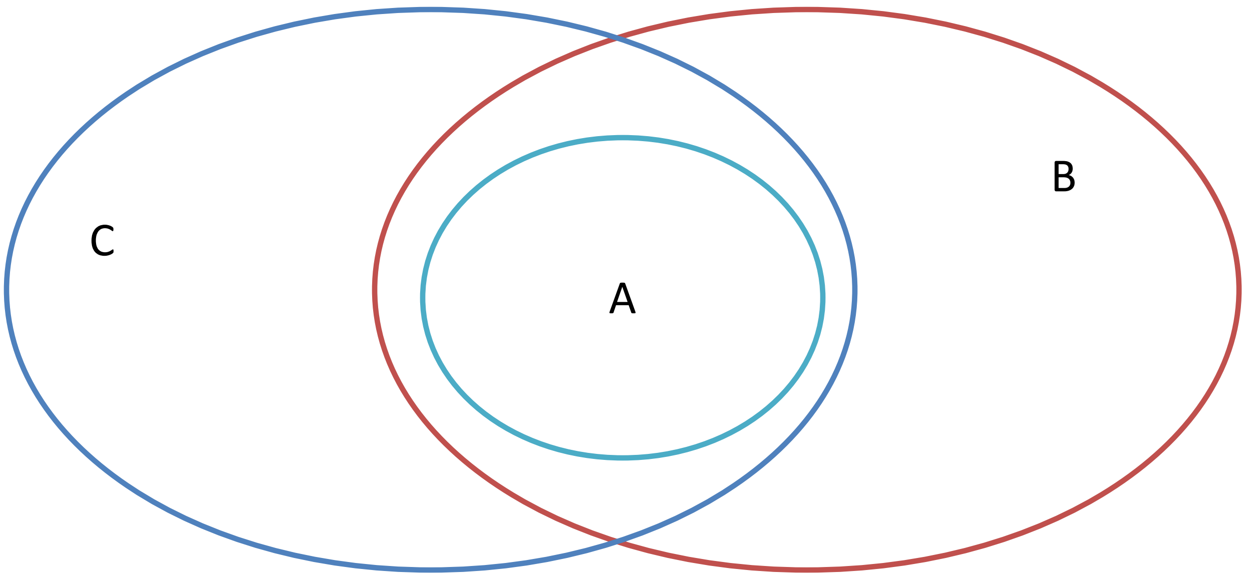
a) $\{0, 3, 6, 9, 12\}$

b) $\{-3, -2, -1, 0, 1, 2, 3\}$

c) $\{m, n, o, p\}$

(a) $\{x \in \mathbb{N} | x \text{ is a multiple of } 3 \text{ and } x \leq 12\}$
(b) $\{x \in \mathbb{Z} | -3 \leq x \leq 3\}$
(c) $\{x | x \text{ is a letter in the alphabet from } m \text{ to } p\}$

18. Use a Venn diagram to illustrate the relationships $A \subset B$ and $A \subset C$.



22. What is the cardinality of each of these sets?

a) $\emptyset$

b) $\{\emptyset\}$

c) $\{\emptyset, \{\emptyset\}\}$

d) $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

a) 0, b) 1, c) 2, d) 3.

23. Find the power set of each of these sets, where a and b are distinct elements.

a) $\{a\}$ **b)** $\{a, b\}$ **c)** $\{\emptyset, \{\emptyset\}\}$

(a) $P(\{a\}) = \{\emptyset, \{a\}\}$
(b) $P(\{a, b\}) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
(c) $P(\{\emptyset, \{\emptyset\}\}) = \{\emptyset, \{\emptyset, \}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$

25. How many elements does each of these sets have where a and b are distinct elements?

a) $\mathcal{P}(\{a, b, \{a, b\}\})$

b) $\mathcal{P}(\{\emptyset, a, \{a\}, \{\{a\}\}\})$

c) $\mathcal{P}(\mathcal{P}(\emptyset))$

(a) 8
(b) 16
(c) 2

Thank You