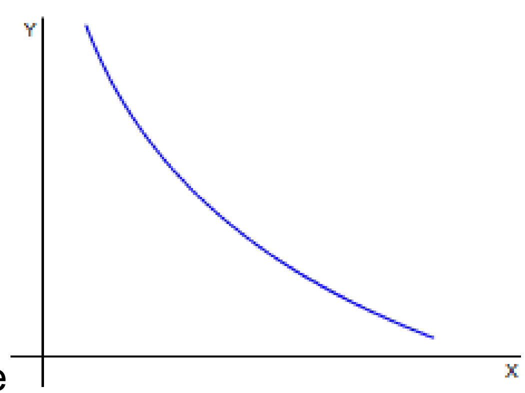
Objectives:

After completing this topic, you will be able to:

- Define functions and to give some illustrating examples.
- Represent the function symbolically.
- Demonstrate a function as an input output process.
- Introduce the notion of domain and range of a function.
- Explain the notion of piecewise functions.
- Define the graph of a function.
- Sketch the graph of a function.
- Choose points for graphing a function.
- Graph the absolute function.
- Introduce the notion of odd and even functions.
- Apply operations on functions.

Symbolic Representation of a Function:



- In each case, if it is also true that the value of y is completely determined by the value of x, we can say that y is a function of x, Euler invented a symbolic way to say "y is a function of x" by writing

$$y = \mathcal{F}(x) \longrightarrow \quad (1)$$

which we read "y equals f of x."

- This notation is shorter than the verbal statements that say the same thing. It also lets us give different functions different names by changing the letters we use.

To say that braking distance is a function of speed we can write $d = f(8)$. To say that the concentration of a medicine in the bloodstream is a function of time between doses we can write $c = g(t)$ (we use a g here because we just used f for something else). We have to know what the variables d, s, c, and t mean, of course, before these equations make sense.

Function as an Input Output Model:

In mathematics, any rule that assigns to each element in one set some element from another set is called a function.

The sets may be sets of numbers, sets of number pairs, sets of points, sets of objects of any kind.

The sets do not have to be the same.

Thus a function is like a machine that assigns an output to every allowable input. The inputs make up the domain of the function. The outputs make up the function's range

$$\xrightarrow[\text{Input}]{2} \boxed{f(x) = 5x^2 + 3x + 7} \xrightarrow[\text{Output}]{33}$$

For example:

1) $f(x) = x^2$.

2) $f(x) = x^3$.

3) $f(x) = 5x^2 + 3x + 7$

All the function has to do is assign some element from the second set to each element in the first set. You can also consider the function as an input output model which contains a processing box representing the function effect.

Now we define the function as a rule joining two sets. Also we give definitions to domain and range of a function.

Let $\mathcal{D}$ and $\mathcal{R}$ be two sets.

A function from $\mathcal{D}$ to $\mathcal{R}$ is a rule that assigns to each element of $\mathcal{D}$ a unique element of $\mathcal{R}$.

The set $\mathcal{D}$ is called the domain of the function.

Functions are usually denoted by letters as f, g, $\mathcal{F}$, $\mathcal{G}$, and so on.

If f is a function from $\mathcal{D}$ into $\mathcal{R}$, then the element $y \in \mathcal{R}$ that is assigned to the element $x \in \mathcal{D}$ by f is denoted by $f(x)$,

and is called the value of f at x, or the image of x under f.

The elements $x \in \mathcal{D}$ and $y \in \mathcal{R}$ are called the variables,

with x the independent variable and y the dependent variable.

The set of values of f, that is $\{y \in \mathcal{R} : y = f(x) \text{ for } x \in \mathcal{D}\}$, is called the range of f.

Domain and Range:

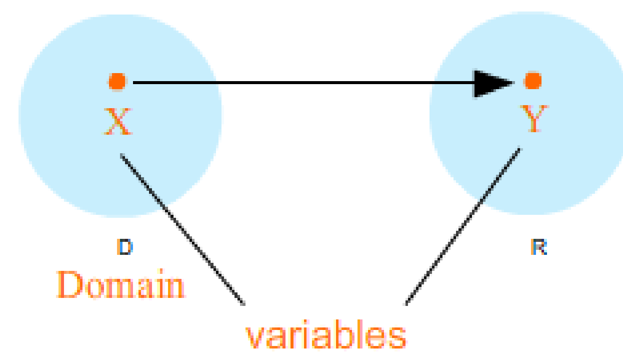
Definition 1:

Let D and R be two sets. A function from D to R is a rule that assigns to each element of D a unique element of R.

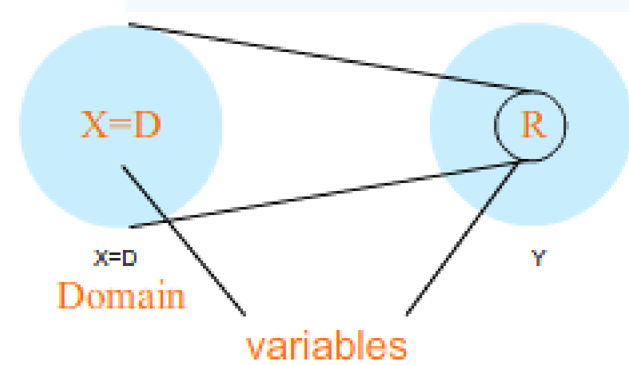
The set D is called the domain of the function. Functions are usually denoted by letters as f, g, F, G, and so on. If f is a function from D into R, then the element $y \in R$ that is assigned to the element $x \in D$ by f is denoted by $f(x)$, and is called the value of f at x, or the image of x under f.

The elements $x \in D$ and $y \in R$ are called the variables, with x the independent variable and y the dependent variable.

The set of values of f, that is $\{y \in R : y = f(x) \; for \; x \in D\}$, is called the range of f.



Throughout most of this text, $\mathcal{D}$ and $\mathcal{R}$ will be the sets of real numbers, and functions from $\mathcal{D}$ into $\mathcal{R}$ are said to be real valued functions of a real variable. In general, the rules for functions will be specified by equations, although there will be occasions when some other means, such as a set of equations, are used to define a function, as in the following examples.



You can now follow the solved examples to understand deeply the notions of domain and range of a function.

Example1:

Consider the squaring function

$$f(x) = x^2, \qquad \text{for all real numbers } x$$

the domain of f, denoted dom (f), is given explicitly as the set of all real numbers. Values of f, can be found by substituting for x in the equation.

For example,

$$f(4) = 4^2 = 16, \qquad f(-3) = -3^2 = 9, \qquad f(0) = 0^2 = 0$$

As x runs through the real numbers, x^2 runs through all the nonnegative numbers.

Therefore, the range of f is $[0, \infty)$, and the domain of f is $(-\infty, \infty)$.

Example 2:

X = [0, 6], Y = [0, 10], D = [0, 6], R = [2, 4]

Consider the function F: X $\longrightarrow$ Y where X = [0, 6], Y = [0, 10] and $g(x) = \sqrt{2x+4}$

The domain of *g* is given as the closed interval $[0,6]$.

At $x = 0$, *g* takes on the value 2:

$$g(0) = \sqrt{2(0)+4} = \sqrt{4} = 2$$

and At $x = 6$, *g* has the value 4:

$$g(6) = \sqrt{2(6)+4} = \sqrt{16} = 4$$

As *x* runs through the number from 0 to 6, $g(x)$ runs through numbers from 2 to 4.

Thus the range of *g* is closed interval $[2,4]$.

X = D = [0, 6], Y = [0, 10] and R = [2, 4]

The rule for a function might be specified by a set of equations rather than just one equation.

For example, consider the function

$$h(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ 2x+1 & \text{if } x < 0 \end{cases}$$

This function is said to be piecewise defined.

The definition of h indicates that its domain is all real numbers.

It can be verified that range (h) is all real numbers as well, although the verification here is not as straightforward as in the first two examples.

The function h maps $(-\infty, \infty)$ onto $(-\infty, \infty)$.

Definition 2:

The absolute value function defined by

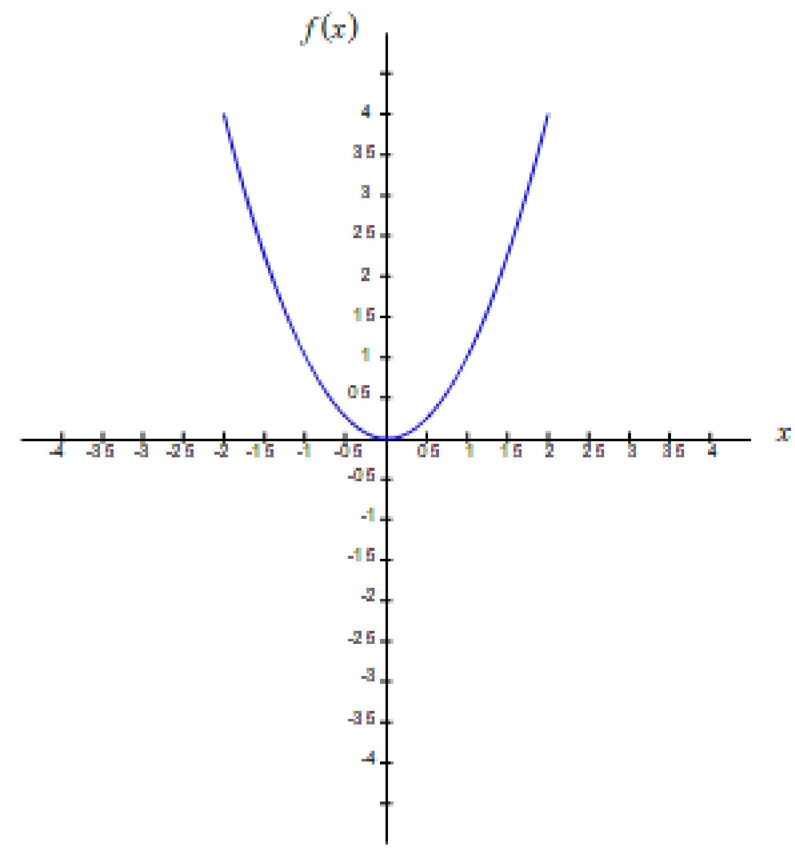
$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Is a familiar example of a piecewise defined function. This function has domain $\mathcal{D} = (-\infty, \infty)$ and range $\mathcal{R} = (0, \infty)$

Graph of Functions:

If f is a function with domain $\mathcal{D}$, then the graph of f is the set of all points $\mathcal{P}(x, f(x))$ in the plane, where $x \in \mathcal{D}$. That is, the graph of f is the graph of the equation $y = f(x)$:

Graph of $f = \{(x,y) : x \in \mathcal{D} \text{ and } y = f(x)\}$



An obvious method for sketching the graph of a function f is to plot some points $\mathcal{P}(x, f(x))$, where $x \in dom(f)$. We plot enough points so that we can "see" what the graph is and then, typically, we connect the points with a "curve"..

Of course, if we can identify the curve in advance (for example, a straight line, a parabola, and so on), then it is much easier to draw its graph.

In the following examples we graph functions whose graphs are not the straight lines.

Examples on Graph of Functions:

In the following examples we graph functions whose graphs are not the straight lines.

Example 3:

Graph the function $f(x) = x^2$

over the interval $-2 \le x \le 2$

Solution:

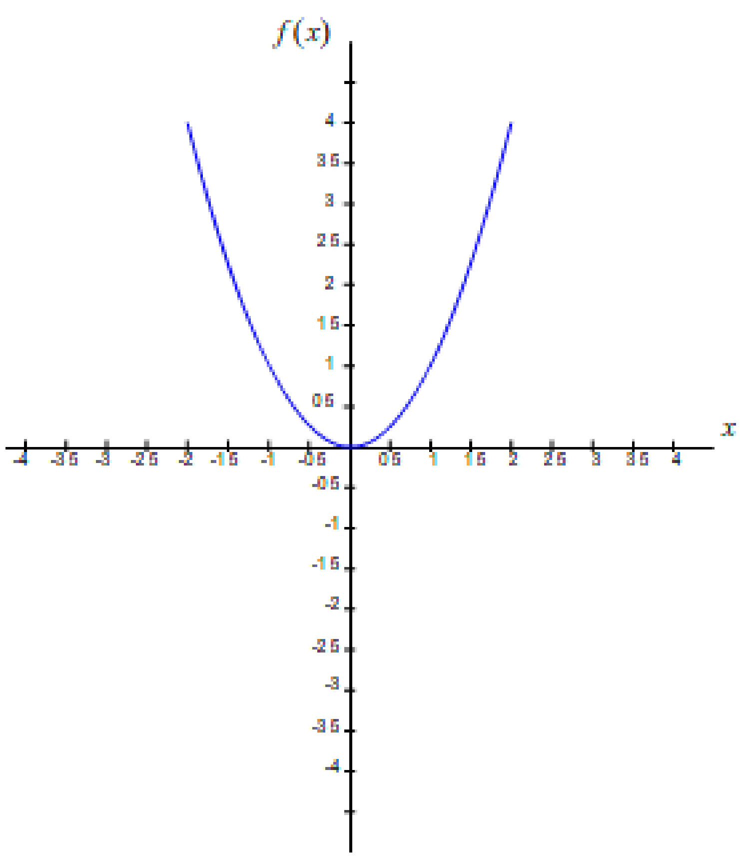
To graph the function, we carry out the following steps:

1- We make a table of input-output pairs for the function.

x	$f(x) = x^2$
-2.0	4.0
-1.75	3.0625
-1.5	2.25
-1.0	1.0
-0.5	0.25
0	0
0.5	0.25
1.0	1.0
1.5	2.25
1.75	3.0625
2	4

2- We plot the corresponding points to learn the shape of the graph.

3- We sketch the graph by connecting the point.



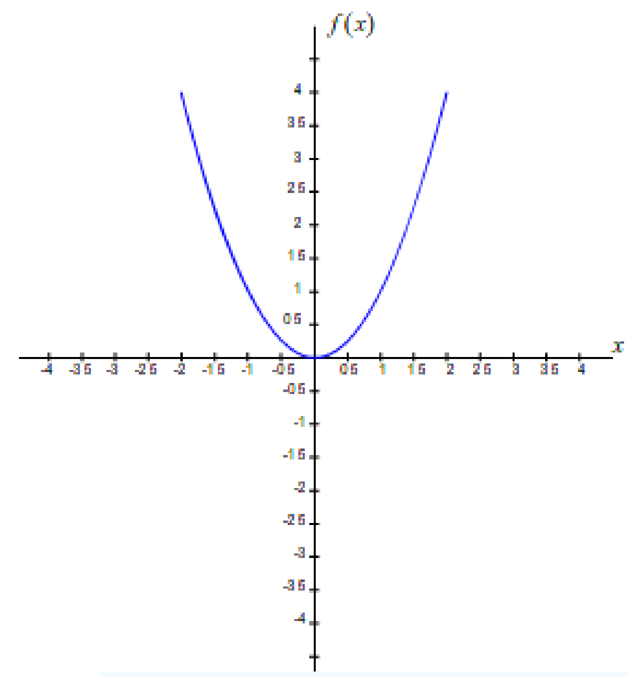
In this screen you will continue the example showing how to graph a function.

In example 3 we graphed the function $f(x) = x^2$ over the interval $-2 \le x \le 2$.

What about the rest of the graph?

The domain and the range of $f(x)$ are both infinite, so we cannot hope to draw the entire graph.

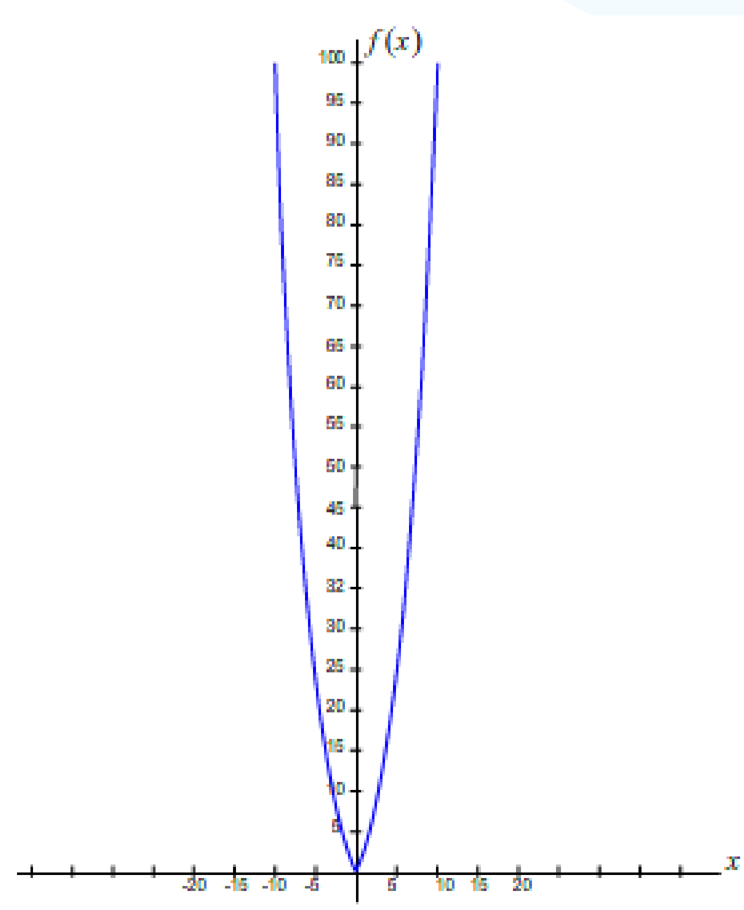
But we can imagine what the graph looks like by examining the formula $f(x) = x^2$ and by looking at the picture we already have.



As x moves away from the interval $-2 \le x \le 2$ in either direction, $f(x)$ increases rapidly.

When x is 5, $f(x)$ is 25. When x is 10, $f(x)$ is already 100.

the graph goes up as shown in Fig.1.3



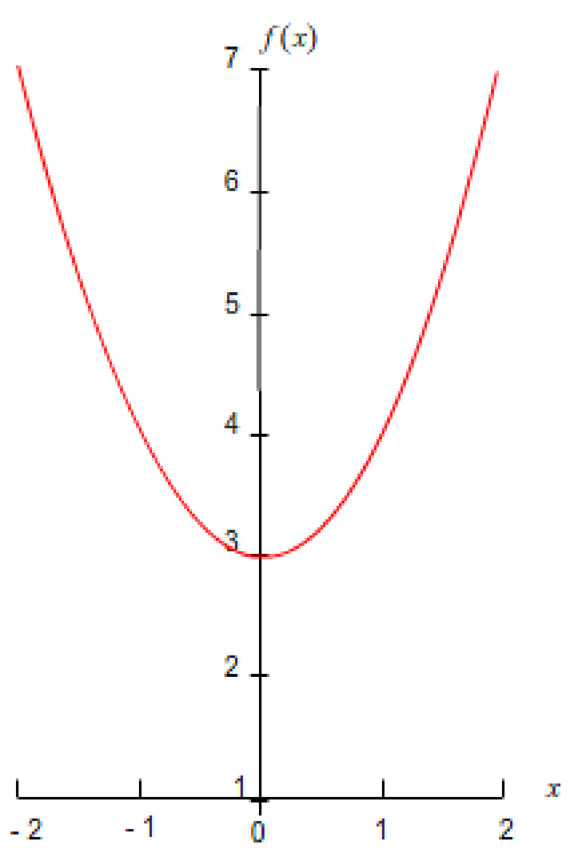
We give another example showing how to graph a function.

Example 4:

Graph the function $f(x) = x^2 + 3$ over the interval $-2 \leq x \leq 2$

Solution:

x	$f(x) = x^2 + 3$
- 2.0	7
-1.75	6.5
- 1.5	5.25
- 1.0	4
0.5	3.25
0	3
0.5	3.25
1.0	1
1.5	5.25
1.75	6.5
2.0	7



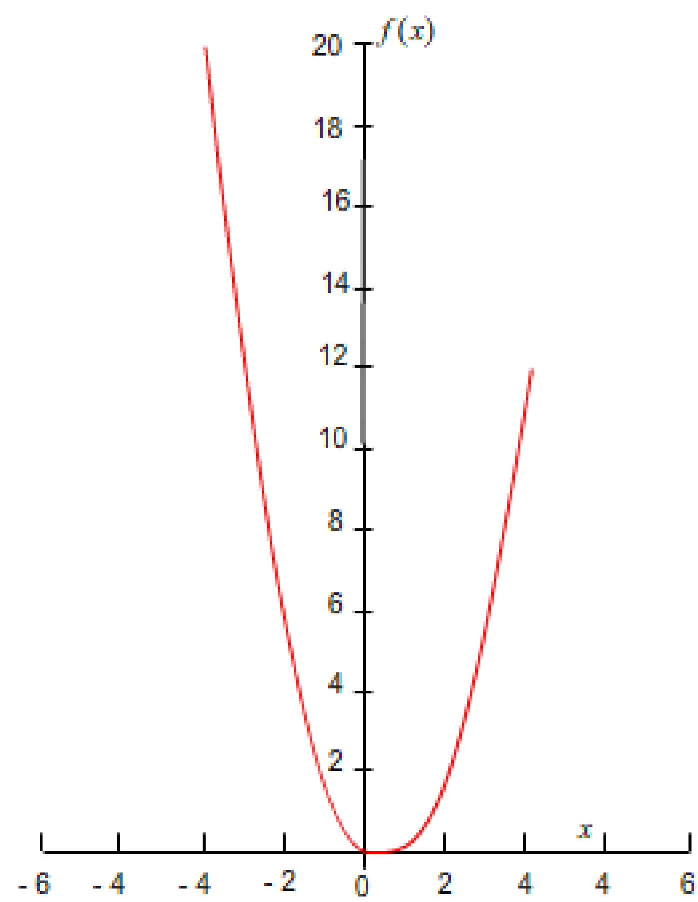
We give another example showing how to graph a function.

Example 5:

Graph the function $f(x) = x^2 - x$ over the interval $-4 \leq x \leq 4$

Solution:

x	$f(x) - x^2 - x$
- 4.0	20
- 3.0	12
- 2.5	8.75
- 2.0	6
- 1.5	3.75
0	0
1.5	0.75
2.0	2
2.5	3.75
3.0	6
4.0	12



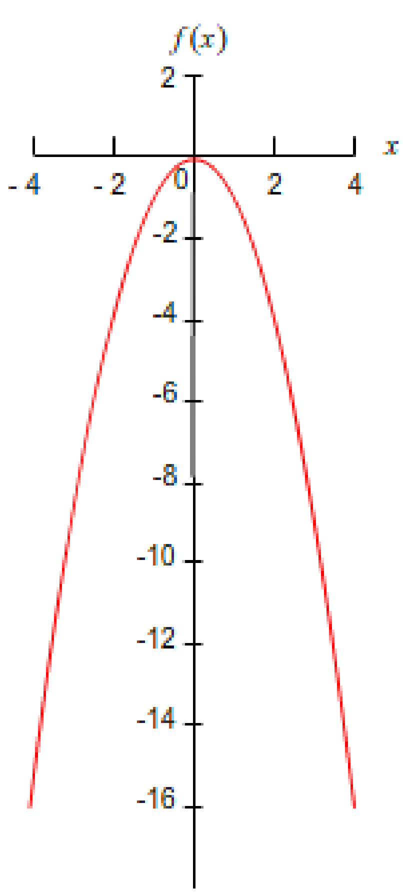
We give another example showing how to graph a function.

Example 6:

Graph the function $f(x) = -x^2$
over the interval $-4 \le x \le 4$

Solution:

x	$f(x) = -x^2$
- 4.0	-16
- 3.0	-9
- 2.5	-6.25
- 2.0	-4
- 1.5	-2.25
0	0
1.5	-2.25
2.0	-4
2.5	-6.25
3.0	-9
4.0	-16



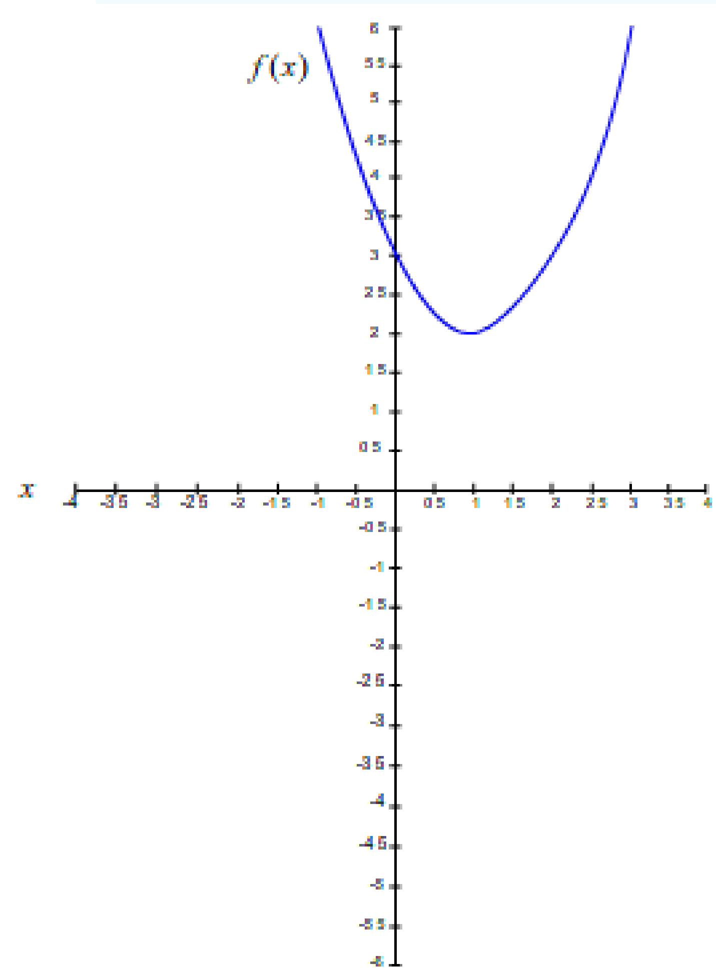
We give another example showing how to graph a function.

Example 7:

Graph the function $f(x) = x^2 - 2x + 3$
over the interval $-1 \le x \le 3$

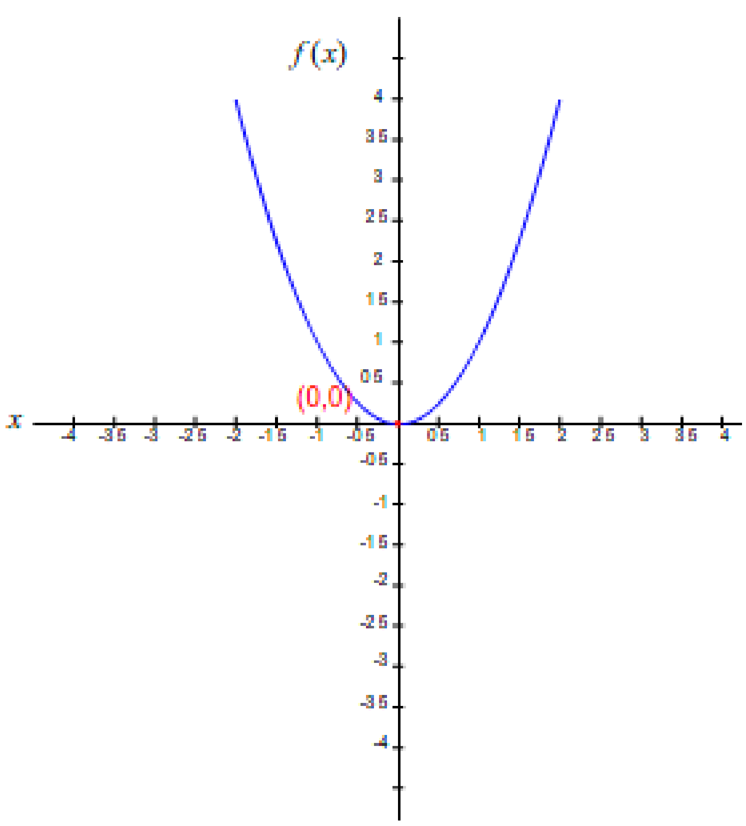
Solution:

x	$f(x)$
-1	6
0	3
0.5	2.25
1	2
2	3
3	6



In this example you will learn how to choose suitable points for graphing the function.

1- Plot any points where the graph crosses or touches the axes.
   These points are often easy to find by setting $x = 0$ and $f(x) = 0$ in the equation of $f(x)$.
2- Plot a few points near the origin.
   When the value of x are small, the values of $f(x)$ are often easy to compute or estimate.
3- Graph the function at or near any endpoints of its domain.



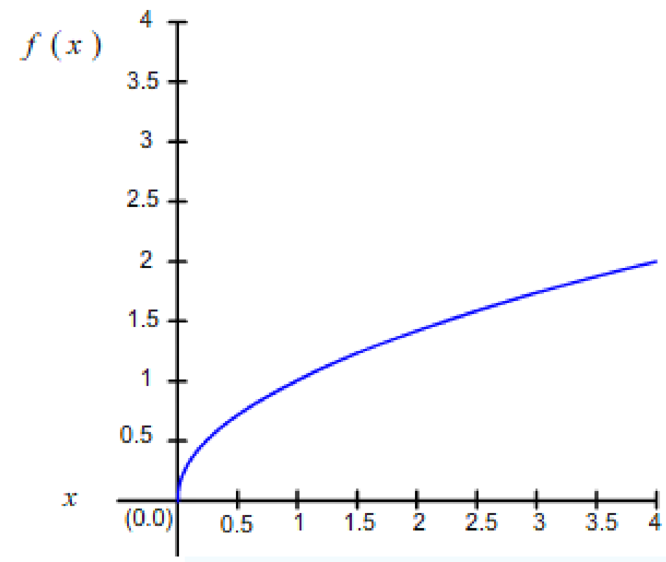
IWe give another example showing how to choose suitable points for graphing the function.

Example 8:

Graph the function $f(x) = \sqrt{x}$
over the interval $0 \leq x \leq 4$

Solution:

x	$f(x) = \sqrt{x}$
0	0
0.25	0.5
1	1
2	$\sqrt{2}$
4	2



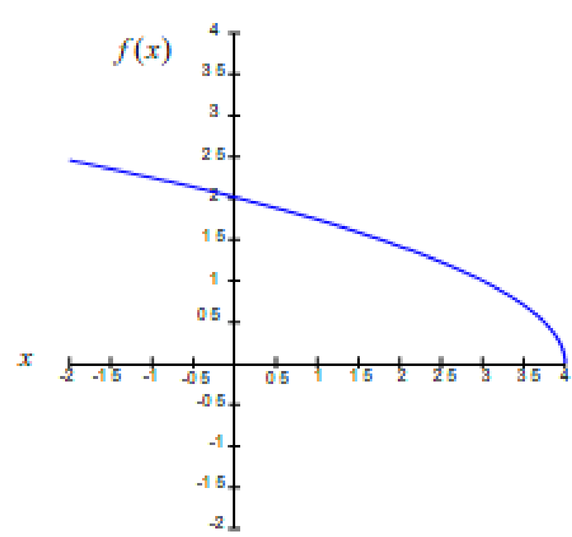
We give another example showing how to choose suitable points for graphing the function.

Example 9:

Graph the function $f(x) = \sqrt{x-4}$
over the interval $-2 \leq x \leq 4$

Solution:

x	$f(x) = \sqrt{4-x}$
-2	$\sqrt{6}$
0	2
2	$\sqrt{2}$
3.75	0.5
4	0



We give another example showing how to choose suitable points for graphing the function.

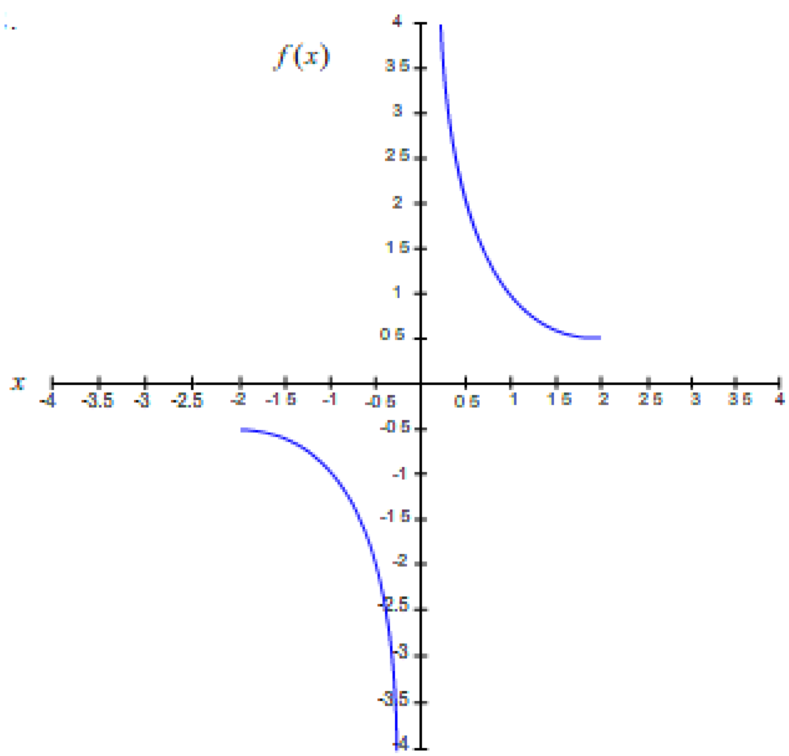
Example 10:

Graph the function $f(x) = \frac{1}{x}$

over the interval $-2 \leq x \leq 2$

Solution:

x	$f(x) = \frac{1}{x}$
-2	-0.5
-1	-1
-0.5	-2
-0.25	-4
0.25	4
0.5	2
1	1
2	0.5



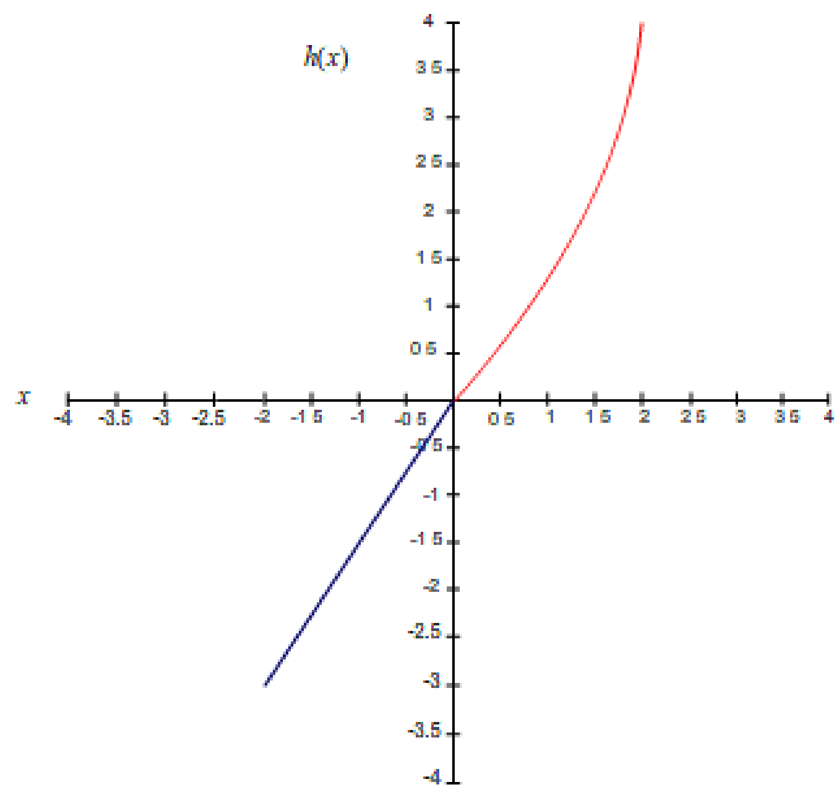
In this example you will learn how to choose suitable points for graphing a piecewise function.

Example 11:

Graph the function $h(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ 2x+1 & \text{if } x < 0 \end{cases}$

Solution:

x	$h(x)$
-2	-3
-1	-1
-.25	.5
0	0
.25	.625
1	1
2	4



Another example showing the graph of the absolute function.

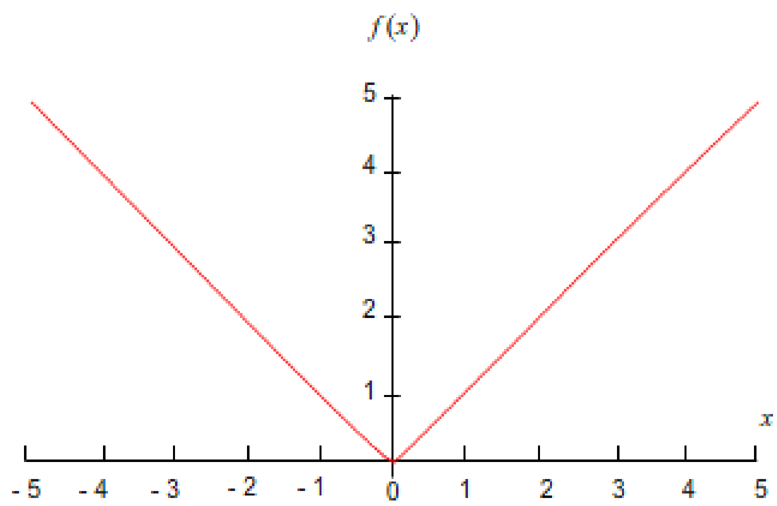
Example 12:

Graph the function $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

over the interval $-5 \leq x \leq 5$

Solution:

x	$\lvert x\rvert = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$
- 5.0	5
- 4.0	4
- 3.0	3
- 2.0	2
- 1.0	1
0	0
1.0	1
2.0	2
3.0	3
4.0	4
5.0	5



Odd and Even Functions:

In the following examples you will learn the properties of odd and even functions with illustrative examples.

A function f is said to be even if and only if $f(-x) = -f(x)$ for all $x \in dom\,(f)$.

A function f is said to be odd if and only if $f(-x) = -f(x)$ for all $x \in dom(f)$.

The graph of an even function is symmetric about y-axis and the graph of an odd function is symmetric about the origin.

The squaring function $f(x) = x^2$ is even since

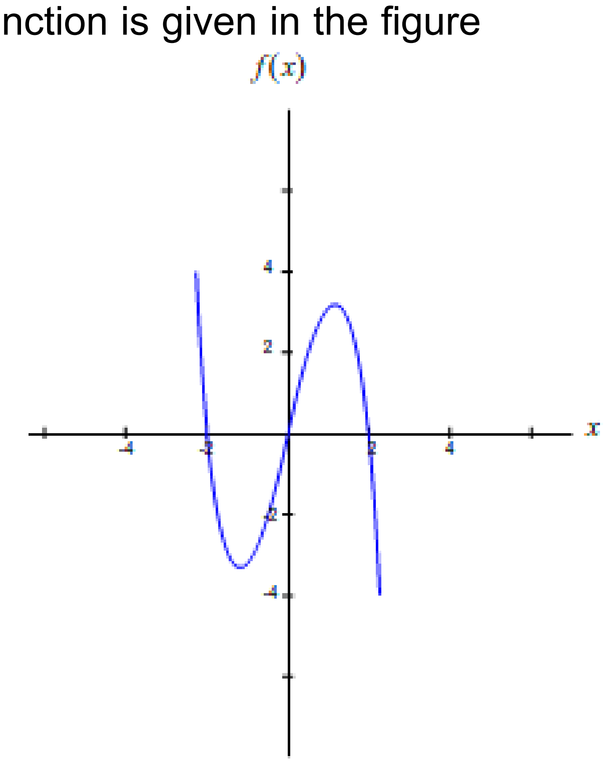
$f(-x) = (-x)^2 = x^2 = f(x)$

its graph is symmetric about the y-axis

The function $f(x) = 4x - x^3$ is odd since

$f(-x) = 4(-x) - (-x)^3 = -(4x - x^3) = -f(x)$

The graph of this function is given in the figure



Operations on Functions:

In the following examples you will learn odd and even functions and the difference in between.

In this example you will learn how to use operations with functions.

If $f(x)$ and $g(x)$ are two functions, with domain $\mathcal{D}_f$ and $\mathcal{D}_g$, then the sum $f(x) + g(x)$, the differences $f(x) - g(x)$, $g(x) - f(x)$, the product $f(x)\,g(x)$, the quotients $\frac{f(x)}{g(x)}$ and $g(x) \neq 0$, $\frac{g(x)}{f(x)}$ and $f(x) \neq 0$ are also function of x, defined for any value of x that lies in both $\mathcal{D}_f$ and $\mathcal{D}_g$

The points at which $g(x) = 0$ must be excluded, however, to obtain the domain of the quotient $\frac{f(x)}{g(x)}$.

Likewise, any points at which $f(x) = 0$ must be excluded

from the domain of the quotient $\frac{g(x)}{f(x)}$.

Now you will follow steps showing operations on functions.

Example 13:

Give the domains of $f(x) = \sqrt{x}$, $g(x) = \sqrt{1-x}$ and the corresponding domain of the sum $f(x) + g(x)$, the differences $f(x) - g(x)$, $g(x) - f(x)$, the product $f(x)\,g(x)$, the quotients $\frac{f(x)}{g(x)}$ and $g(x) \neq 0$, $\frac{g(x)}{f(x)}$ and $f(x) \neq 0$

Solution:

The domain of f and g are $D_f = [\,0\,,\infty\,)$, and $D_g = (-\infty\,,1\,]$.

The points common to these domains are the points of the closed interval $[\,0\,,1\,]$. On $[\,0\,,1\,]$, we have

Sum $\quad H_1 = f(x) + g(x) = \sqrt{x} + \sqrt{1-x}$

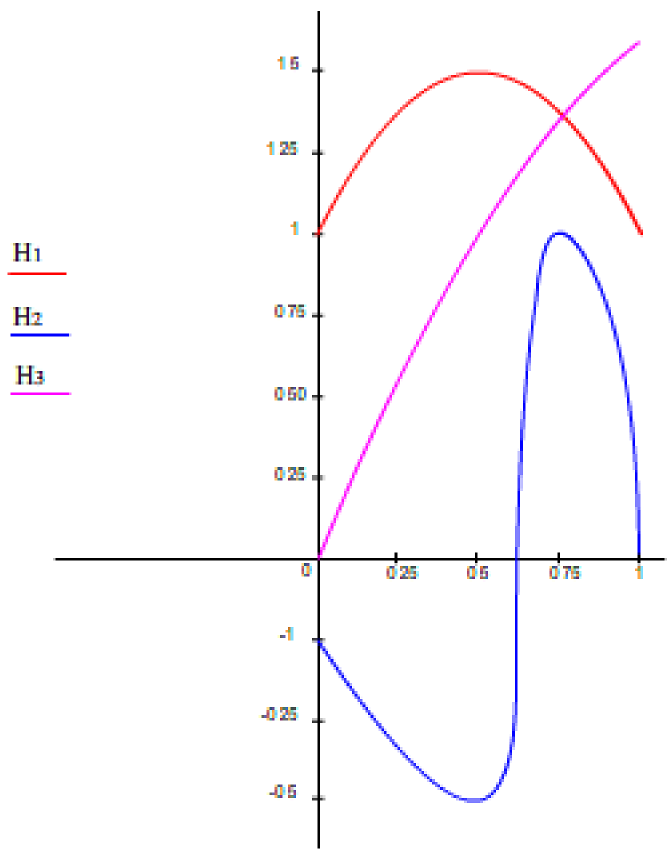
Differences $H_2 = f(x) - g(x) = \sqrt{x} - \sqrt{1-x}$

$$g(x) - f(x) = \sqrt{1-x} - \sqrt{x}$$

Product $\quad f(x)\,g(x) = \sqrt{x(1-x)}$

Quotients $H_3 = \dfrac{f(x)}{g(x)} = \sqrt{\dfrac{x}{1-x}}\ ,\quad x \neq 1$

$$\frac{g(x)}{f(x)} = \sqrt{\frac{1-x}{x}}\ ,\quad x \neq 0$$



the domain of $f+g$, $f-g$, $g-f$, fg are all the same,

namely , the closed interval $[\,0\,,1\,]$. The number $x=1$ must be excluded from the domain of f/g , however,

because $g(1)=0$.

The domain of f/g is therefore the half- open interval $[\,0\,,1\,)$.

Similarly, the number $x=0$ must be excluded from

the domain of g/f because $f(0)=0$.

The domain of g/f is therefore the half-open interval $(\,0\,,1\,]$.

Objectives:

After completing this topic, you will be able to:

- Define the distance between two points.
- Define a circle.
- Develop equation of a circle.
- Solve examples using circle equation.
- Define the general equation of the circle.
- Define a parabola.
- Understand properties of parabolas.
- Use different forms of parabolas.
- Define the general equation of a parabola.
- Solve examples on the general form of a parabola.

Distance Between Two Points:

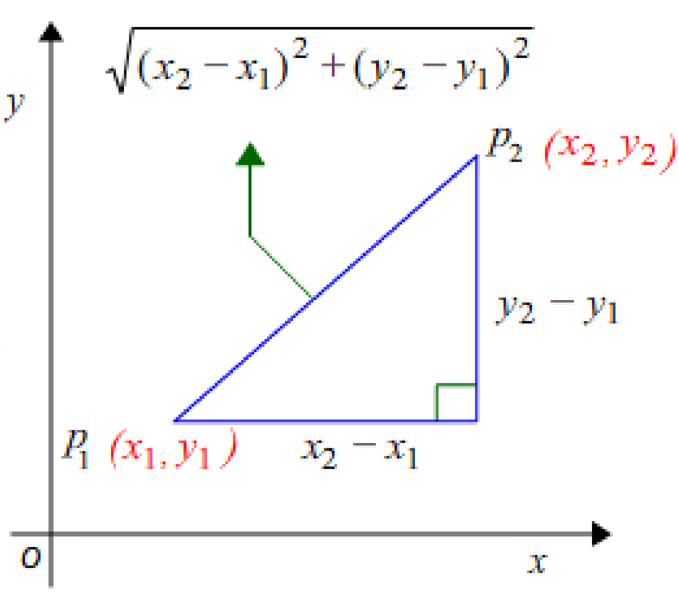
The distance formula:

To learn circles and parabola you should first know the concept of distance between two points.

The distance d between two points (x_1, y_1) and (x_2, y_2) is the plane is calculated from their coordinates by the formula $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.

This formula come form applying the Pythagorean theorem to the triangle.

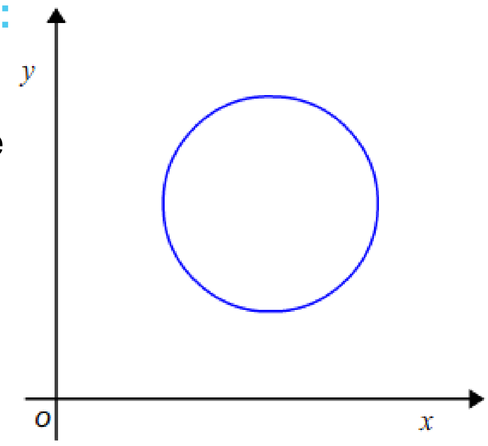
The formula is particularly useful in finding equations for curves whose geometric character depends on one or more distances, as in the following example.



You will learn in following the concept and properties of circles.

Definition of the Circle:

A circle is the set of point in a plane whose distance from a given fixed point in the plane is a constant.



Equation of a Circle:

You will learn the equation of the circle given its center and radius.

Let $C(h, k)$ be the given fixed point,(the center of the circle).

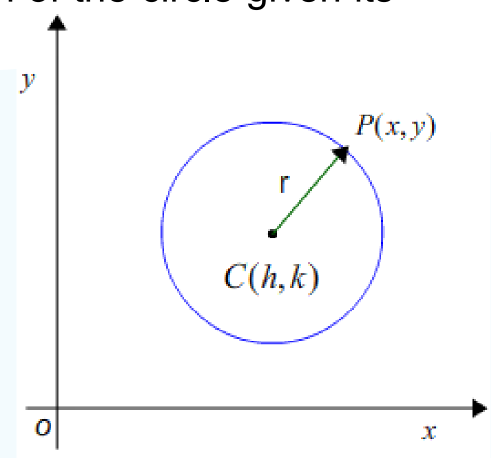
Let r be the constant distance, (the radius of the circle).

Let $P(x, y)$ be a point on the circle, Then $CP = r$

or $\sqrt{(x-h)^2+(y-k)^2}$

or $(x-h)^2+(y-k)^2 = r^2$

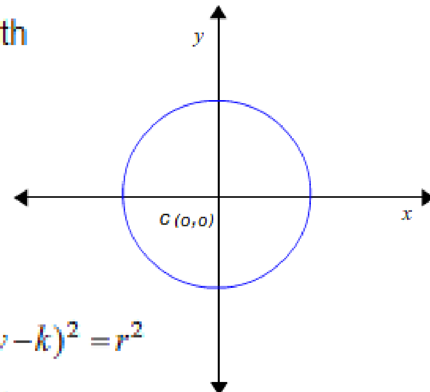
Therefore $(x-h)^2+(y-k)^2 = r^2$ is an equation for the points of the circle.



You can follow the example using the origin as the center of the circle.

Example 14:

Find an equation for the circle with center origin and with radius r.



Solution:

If $h = k = 0$,equation $(x-h)^2+(y-k)^2=r^2$

becomes $x^2+y^2=r^2$

Example 15:

Find the circle though the origin with center at $x\ (2\ ,-1)$.

Solution:

With $(h,k) = (2,-1)$, equation $(x-h)^2 + (y-k)^2 = r^2$

akes the form $(x-2)^2 + (y+1)^2 = r^2$

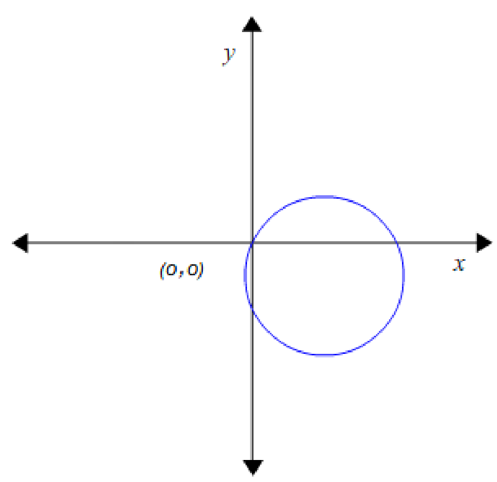
ince the circle goes through the origin, $x = y = 0$

nust satisfy the equation. Hence

$$(0-2)^2 + (0+1)^2 = r^2$$

or $r^2 = 5$.

The equation is $(x-2)^2 + (y+1)^2 = 5$

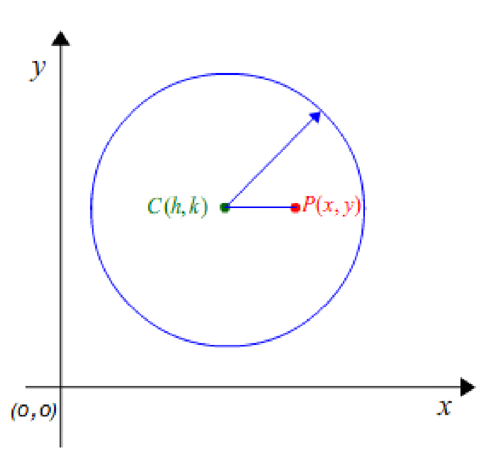


Example 16:

What points $p\ (x\ ,y)$ satisfy the inequality $(x-h)^2 + (y-k)^2 < r^2)$

Solution:

The left side of (6) is the square of the distance CP from $C(h,k)$ to $P(x,y)$. The inequality is s atisfied if and only if $CP < r$ that is, if and only if P lies inside the circle of radius r with center at $C(h,k)$. gives the interior points of the circle.



Example 17:

Find the center and radius of the circle $x^2 + y^2 + 4x - 6y = 12$

Solution:

we complete the squares in the x terms and y terms

and get $(x^2 + 4x + 4) + (y^2 - 6y + 9) = 12 + 4 + 9$.

or $(x+2)^2 + (y-3)^2 = 25$.

This is equation $(x-h)^2 + (y-k)^2 = r^2$

with $(h,k) = (2\ ,-3)$ and $r^2 = 25$.

it therefore represents a circle with

Center: $C\ (-2,\ 3)$

Radius: $r = 5$.

Parabolas:

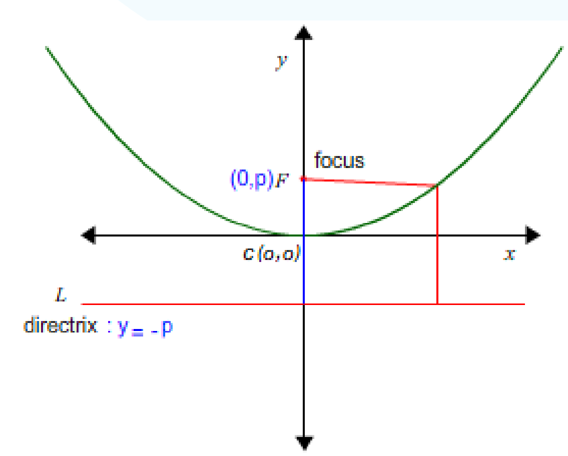
In the following you will learn what is meant by a parabola.

Definition:

A parabola is the set of points in a plane that are equidistant from a given fixed point and line in the plane.

The fixed point is called the focus of the parabola and the fixed line the directrix.

If the focus $\mathcal{F}$ line on the directrix $\mathcal{L}$, then the parabola is nothing more than the line through $\mathcal{F}$ perpendicular to $\mathcal{L}$. We consider this to be a degenerate case.



If $\mathcal{F}$ dose not lie on $\mathcal{L}$, then we may choose a coordinate system that results in a simple equation for the parabola by taking the y-$axis$ through $\mathcal{F}$ perpendicular to $\mathcal{L}$ and taking the origin halfway between $\mathcal{F}$ and $\mathcal{L}$. If the distance between $\mathcal{F}$ and $\mathcal{L}$ is $0p$, we may assign $\mathcal{F}$ coordinates $(0,\ p)$ and the equation of $\mathcal{L}$ is $y = -p$.

Example 15:

Find the circle though the origin with center at $x\ (2\ ,-1)$.

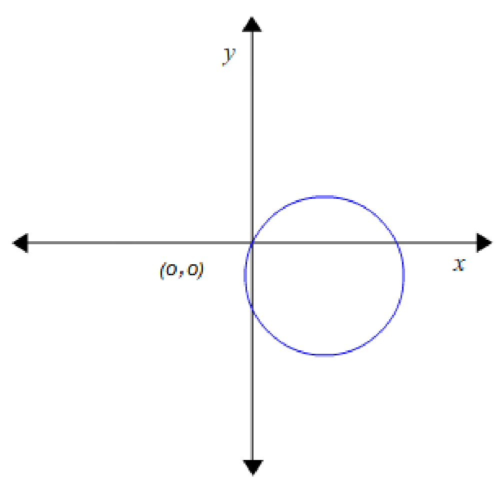
Solution:

With $(h,k)=(2,-1)$, equation $(x-h)^2+(y-k)^2=r^2$
akes the form $(x-2)^2+(y+1)^2=r^2$
ince the circle goes through the origin, $x=y=0$
nust satisfy the equation. Hence

$$(0-2)^2+(0+1)^2=r^2$$

or $\quad r^2=5$.

The equation is $\quad (x-2)^2+(y+1)^2=5$



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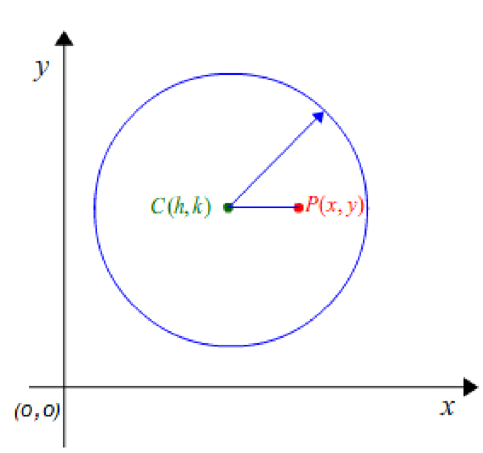

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Find the center and radius of the circle
$x^2+y^2+4x-6y=12$

Solution:

we complete the squares in the x terms and y terms

and get $\quad (x^2+4x+4)+(y^2-6y+9)=12+4+9$.

or $\quad (x+2)^2+(y-3)^2=25$.

This is equation $(x-h)^2+(y-k)^2=r^2$

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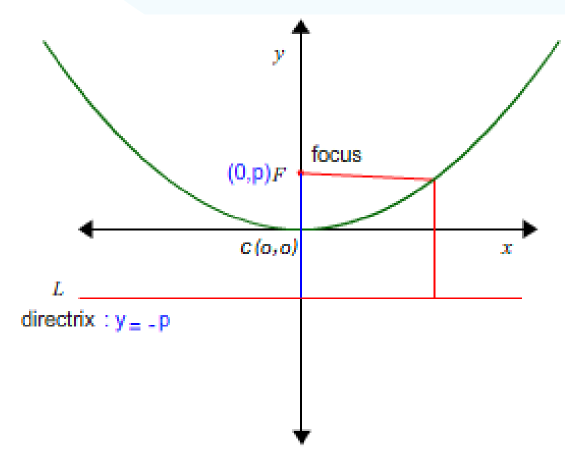


If $\mathcal{F}$ dose not lie on $\mathcal{L}$, then we may choose a coordinate system that results in a simple equation for the parabola by taking the y-$axis$ through $\mathcal{F}$ perpendicular to $\mathcal{L}$ and taking the origin halfway between $\mathcal{F}$ and $\mathcal{L}$.
If the distance between $\mathcal{F}$ and $\mathcal{L}$ is $0p$, we may assign $\mathcal{F}$ coordinates $(0,\ p)$ and the equation of $\mathcal{L}$ is $y=-p$.

In the following you can follow the properties of parabolas.

A point $P(x, y)$ lies on the parabola if and only if the distances PF and PQ are equal: $PF = PQ$

Where $Q(x,-y)$ is the foot of the perpendicular from P to L.

From the distance formula,

$$PF = \sqrt{x^2 + (y-p)^2} \qquad and \qquad PQ = \sqrt{(y+p)^2}$$

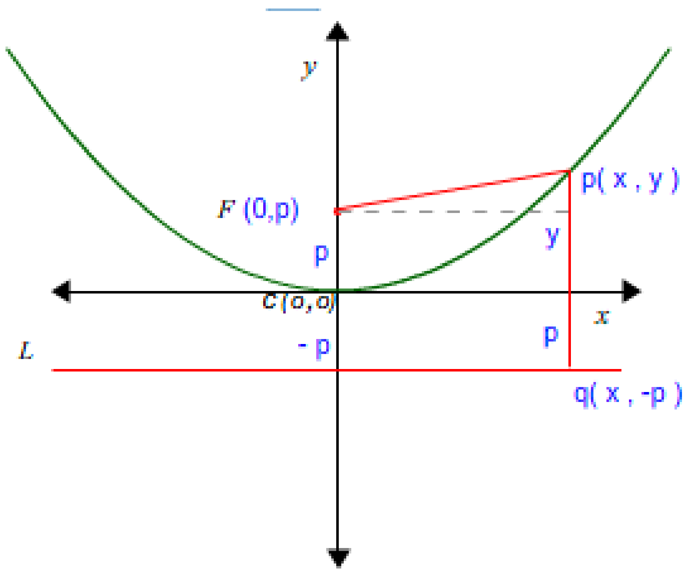
When we equate these two expressions, square, and simplify, we get

$$x^2 = 4py$$

this equation must be satisfied by any point on the parabola.

Conversely if $x^2 = 4py$ is satisfied, then we have

$$PF = \sqrt{x^2 + (y-p)^2} = \sqrt{4py + (y^2 - 2py + p^2)}$$
$$= \sqrt{(y+p)^2} = PQ$$



Example 18:

Find the focus and directrix of the parabola $x^2 = 8y$

Solution:

equation ($x^2 = 8y$) is equation ($x^2 = 4py$) with

$4p = 8, \qquad p = 2$

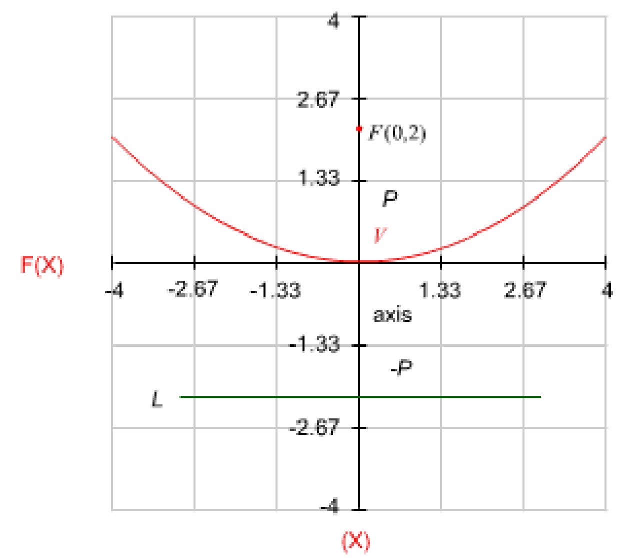
The focus is on the y-axis $p = 2$ units from the vertex, that is, at Focus: $F(0,2)$

The directrix $y = -p$ is the line $y = -2$:

Directrix: $y = -2$.

The axis of the parabola is the line through F that is perpendicular to the directrix.

The vertex of the parabola is the point V on the axis halfway from F to L.



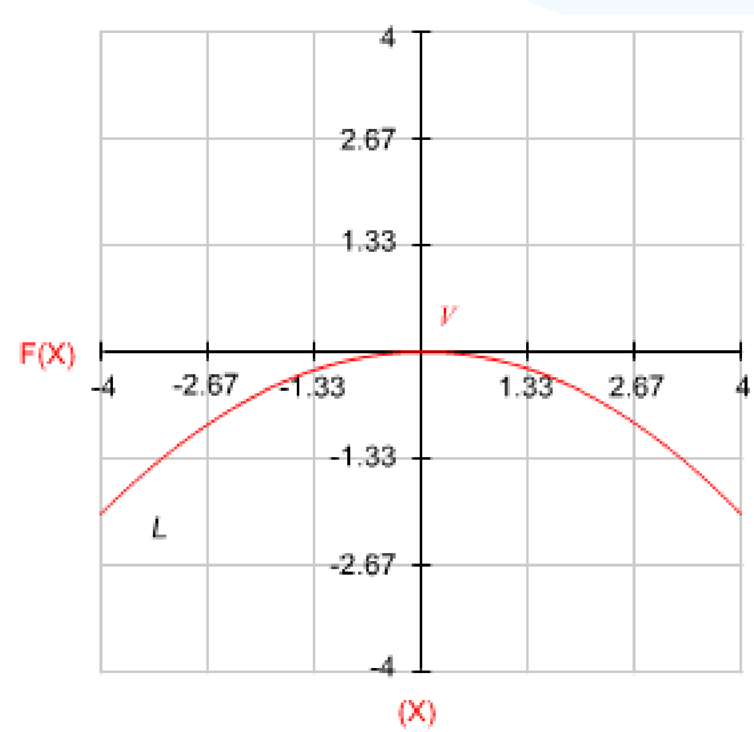
Example 19:

If the axis of the parabola is the y axis and the vertex is at the origin with focus F at $(0, p)$, then the equation of the parabola is $x^2 = 4py$

If $p > 0$, then the parabola open upward.

If $p < 0$ the parabola open downward.

Solution:



Here you find some additional examples on parabolas.

Example 20:

Interchanging the roles of x and y yields the similar equation

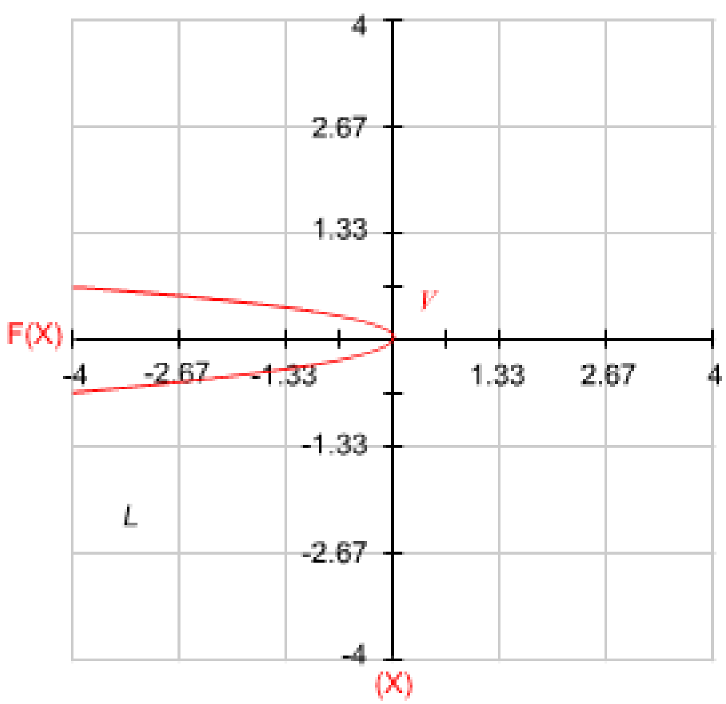
$$y^2 = 4px$$

which are the equations of the parabola with vertex at the origin and focus to F at $(p,0)$.

If $p > 0$, then the parabola open to the right.

and if $p < 0$ the parabola open to the left.

Solution:



Here you will find some examples on the general form of a parabola.

Example 21:

Interchanging the roles of x and y yields the similar equation

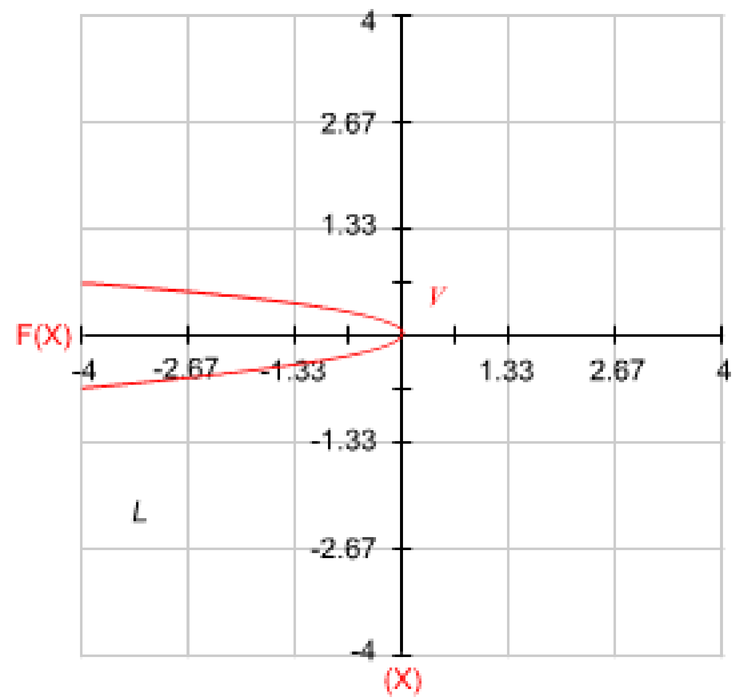
$$y^2 = 4px$$

which are the equations of the parabola with vertex at the origin and focus to F at $(p,0)$.

If $p > 0$, then the parabola open to the right.

and if $p < 0$ the parabola open to the left.

Solution:



With the parabola opening to the right if $p > 0$ and to the left if . $p < 0$

We may write equation (11) in the form

$$x = ay^2 + by + c \quad (12)$$

Where $a = 1/4p$

If the equation of the parabola is in the form $y = ax^2 + bx + c$ then we can find the vertex v algebraically by completing the square in x and changing the equation to the form $(x-h)^2 = 4p(y-k)$

Example 22:

Discuss and sketch the parabola $y = x^2 + 4x \qquad (13)$

Solution:

We complete the square in the x terms by adding 4 to both sides of equation (13):

$y + 4 = x^2 + 4x + 4$

or $(x + 2)^2 = y + 4$.

This has the form

$(x - h)2 = 4p\ (y - k)$

With $h = -2,\ k = -4,\ 4p = 1,\ p = 1/4$.

The parabola's vertex is : $V = (-2, -4)$

Vertex: $(-2, -4)$.

Its axis of symmetry is the line

$(x + 2)^2 = 0$, or $x = -2$:

Axis: $x = -2$.

Topic Summary

The distance formula

The distance d between two points (x_1, y_1) and (x_2, y_2) in the plane is calculated from their

coordinates by the formula. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Circles DEFINITION

A circle is the set of point in a plane whose distance from a given fixed point in the plane is a constant.

Equation of a circle

Let $C(h, k)$ be the given fixed point, (the center of the circle). Let r be the constant distance, (the radius of the circle).

Let $P(x, y)$ be a point on the circle.

The equation $(x - h)^2 + (y - k)^2 = r^2$

is an equation for the points of the circle

the domain of $f + g$, $f - g$, $g - f$, fg are all the same,

namely , the closed interval $[0, 1]$. The number $x = 1$ must be

excluded from the domain of f / g, however,

because $g(1) = 0$.

The domain of f / g is therefore the half- open interval $[0, 1)$.

Similarly, the number $x = 0$ must be excluded from

the domain of g / f because $f(0) = 0$.

The domain of g / f is therefore the half-open interval $(0, 1]$.

Objectives:

After completing this topic, you will be able to:

- Define trigonometric functions
- Explain and show properties of trigonometric functions
- Graph trigonometric functions.

Definition of the Trigonometric Functions:

We are concerned with real-valued function of a real variable and so we use the unit circle and the radian measure of angels as opposed to right triangles and degree measure to define the trigonometric functions.

We are concerned with real-valued function of a real variable and so we use the unit circle and the radian measure of angels as opposed to right triangles and degree measure to define the trigonometric functions.

Let θ be an angle in standard position and let $P(x, y)$ be the terminal side of θ and the unit circle.

The six trigonometric functions are defined as follows:

sine: $\sin\theta = y$

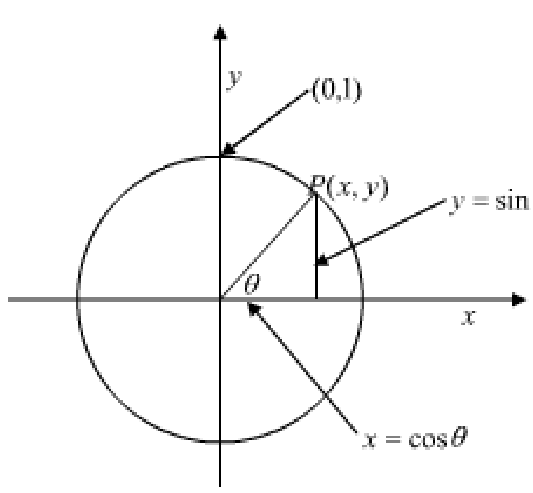
cosine: $\cos\theta = x$

tangent: $\tan\theta = \dfrac{y}{x} = \dfrac{\sin\theta}{\cos\theta}, \quad x \neq 0$

cosecant: $\csc\theta = 1/y = 1/\sin\theta, \quad y \neq 0$

secant: $\sec\theta = \dfrac{1}{x} = \dfrac{1}{\cos\theta}, \quad x \neq 0$

cotangent: $\cot\theta = \dfrac{x}{y} = \dfrac{\cos\theta}{\sin\theta}, \quad y \neq 0$



Properties of Trigonometric Functions:

You should already have those values for memorized.

	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

	$2\pi/3$	$3\pi/4$	$5\pi/6$	π	$3\pi/2$
$\sin\theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1
$\cos\theta$	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	0
$\tan\theta$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	undefined

In general, the (approximate) values of the trigonometric functions for any angle θ can be obtained with a hand calculator or form a table of values.

The sine and cosine functions are defined for all real numbers. That is, both sine and cosine have domain $(-\infty, \infty)$.

The domain of the tangent and secant functions are all real numbers except for those values of θ for which x = 0, namely $\theta = (2n+1)\pi/2$, for any integer n.

The domain of the cosecant and cotangent functions are all real numbers except for those values of θ for which y = 0, namely $\theta = n\pi$, for any integer n.

We concentrate on the sine, cosine, and tangent functions since cosecant, secant, and cotangent are simply the reciprocals of those functions. The values of the trigonometric functions for the angles $0(=0^o), \pi/6(=30^o)$ $\pi/4(=45^o)$ $\pi/3(=60^o)$, $\pi/2(=90^o)$ and selected multiples up to 2π, are given in the following tables.

In the following you will find some properties of trigonometric functions.

Periodicity:

A function f is periodic if there is a number $p, p \neq 0$, such that $f(x+p)=f(x)$ whenever x and $x+p$ are in the domain of f.

The smallest positive number with this property is called the period of f. It is easy to verify from the definitions of the sine and cosine functions that they are periodic with period 2π.

That is,

$$\sin(\theta+2\pi)=\sin\theta \text{ and } \cos(\theta+2\pi)=\cos\theta$$

It follows that the secant and cosecant function are also periodic with period 2π. It can be verified that the tangent and cotangent functions have period π.

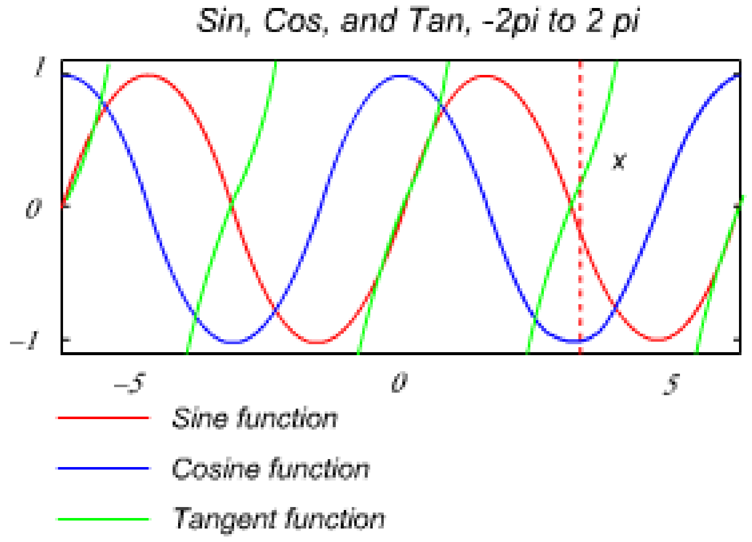
Graphs of Trigonometric Functions:

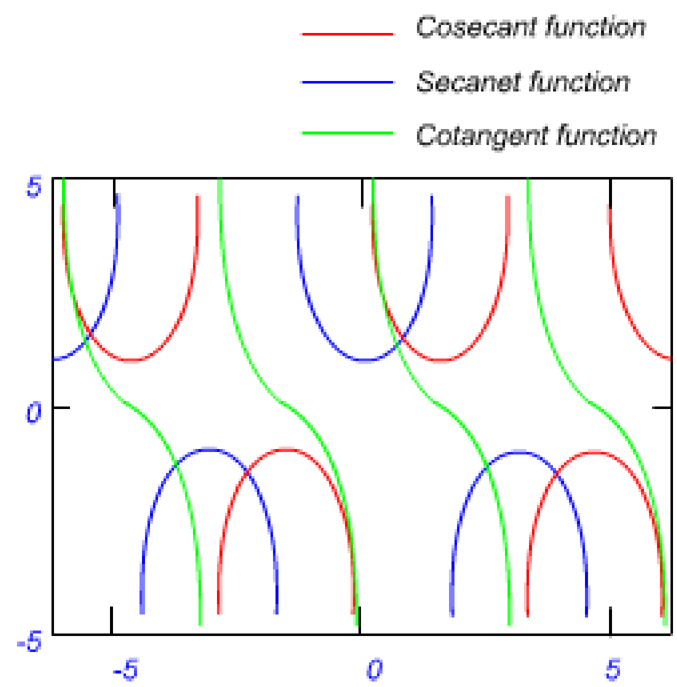
Here you find some graphs of trigonometric functions.

Graphs:

Since we normally use x and y to represent the independent and dependent variables when drawing a graph, we follow that convention here, replacing θ by x and letting y = sin x, y = cos x, and so forth.

Sin, Cos, and Tan, -2pi to 2 pi





Csc, Sec, and Cot, -2pi to 2pi

Properties of Trigonometric Functions:

Important Identities:

The graphs of the six trigonometric functions are shown in the Fig. Since we normally use x and y to represent the independent and dependent variables when drawing a graph, we follow that convention here, replacing θ by x and letting y = sin x, y = cos x, and so forth.

We assume that you are familiar with the common trigonometric Identities. The most Important of these are listed here for reference.

(i) *unit circle*

$$\sin^2\theta+\cos^2\theta = 1$$
$$\tan^2\theta+1=\sec^2\theta$$
$$1+cot^2=\csc^2$$

(ii) *addition formulas*

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$
$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$
$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

(iii) *even/odd functions*

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta.$$

Thus, the sine function is an odd function; its graph is symmetric with respect to the origin. The cosine function is even function; its graph is symmetric with respect to the y-axis.

(iv) *double-angle formulas*

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

(v) *half-angle formulas*

$$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$

$$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$$