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EELU

THE EGYPTIAN E-LEARNING UNIVERSITY

GEN206
Discrete Mathematics

Section 10

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The Basics of Counting

THE PRODUCT RULE Suppose that a procedure can be broken down into a sequence of two tasks. If there are n_1 ways to do the first task and for each of these ways of doing the first task, there are n_2 ways to do the second task, then there are n_1n_2 ways to do the procedure.

THE SUM RULE If a task can be done either in one of n_1 ways or in one of n_2 ways, where none of the set of n_1 ways is the same as any of the set of n_2 ways, then there are $n_1 + n_2$ ways to do the task

THE SUBTRACTION RULE If a task can be done in either n_1 ways or n_2 ways, then the number of ways to do the task is $n_1 + n_2$ minus the number of ways to do the task that are common to the two different ways.

1) **A multiple-choice test contains 10 questions. There are four possible answers for each question.**

   A. **In how many ways can a student answer the questions on the test if the student answers every question?**

   B. **In how many ways can a student answer the questions on the test if the student can leave answers blank?**

Solution

A. The product rule applies, since the student will perform each of 10 tasks, one after the other. There are 4 ways to do each task. Therefore there are $4 \cdot 4 \cdot \cdots 4 = 4^{10} = 1{,}048{,}576$ ways to answer the questions on the test.

B. This is identical to part (a), except that now there are 5 ways to answer each question-give any of the 4 answers or give no answer at all. Therefore there are $5^{10} = 9{,}765{,}625$ ways to answer the questions on the test.

2) **Six different airlines fly from New York to Denver and seven fly from Denver to San Francisco. How many different pairs of airlines can you choose on which to book a trip from New York to San Francisco via Denver, when you pick an airline for the flight to Denver and an airline for the continuation flight to San Francisco?**

Solution

The product rule applies here, since a flight is determined by choosing an airline for the flight from New York to Denver (which can be done in 6 ways) and then choosing an airline for the flight from Denver to San Francisco (which can be done in 7 ways). Therefore there are $6 \cdot 7 = 42$ different possibilities for the entire flight.

3) How many different three-letter initials can people have?

Solution

Three-letter initials are determined by specifying the first initial (26 ways), then the second initial (26 ways),

and then the third initial (26 ways). Therefore by the product rule there are $26 \cdot 26 \cdot 26 = 26^3 = 17{,}576$

possible three-letter initials

4) How many bit strings are there of length six or less ?

Solution

We use the sum rule, adding the number of bit strings of each length up to 6. If we include the empty string, then we

get $2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 = 2^7 - 1 = 127$

5)   How many strings are there of four lowercase letters that have the letter x in them?

Solution

We can subtract from the number of strings of length 4 of lower case letters the number of strings of length 4

of lower case letters other than x. Thus the answer is $26^4 - 25^4 = 66,351$.

First count the number of strings of any four letters. That's 26^4. Then count the number of strings that don't have any x. That'll be 25^4 since each of the letters can be any of 25 possibilities. A string will have at least one x if it's one of the 26^4 strings of length 4, but not one of the 25^4 strings that don't have an x in them. Thus, the answer is $26^4 - 25^4$.

Thank You