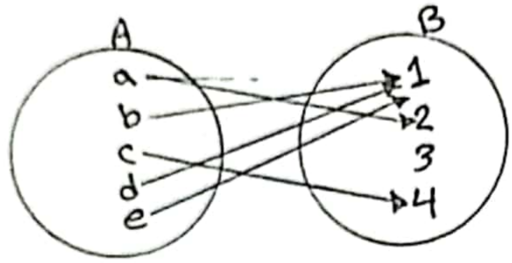
nada.

* **EX:** Let $A = \{a, b, c, d, e\}$ and $B = \{1, 2, 3, 4\}$ with $f(a) = 2$, $f(b) = 1$, $f(c) = 4$, $f(d) = 1$, and $f(e) = 1$. Find $f(S)$ if $S = \{b, c, d\} \subseteq A$.

ans:



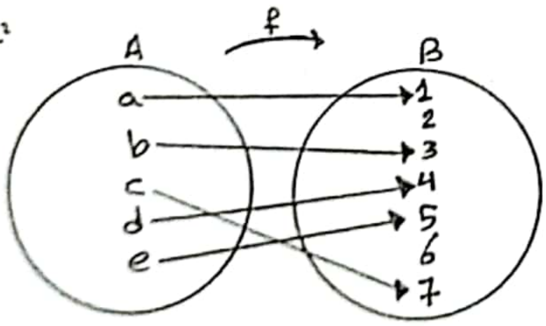
$f(S) = \{1, 4\}$

---

1 one-to-one function (injective):-

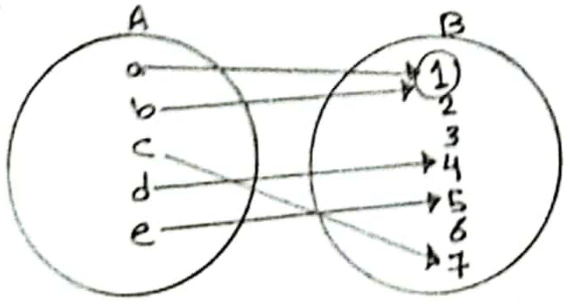
* A function f is said to be one-to-one, or injective, if and only if $f(a) = f(b)$ implies that $a = b$ for all a and b in the domain of f.

* **EX:**



one-to-one

* **EX:**

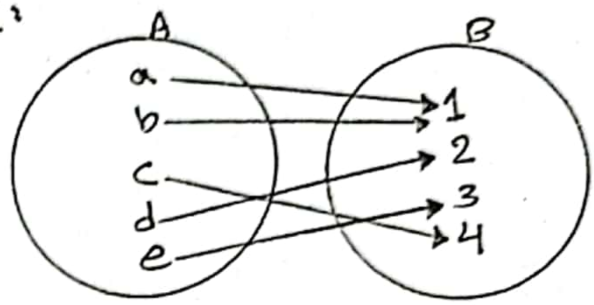


Not one-to-one (Not injective).

2 Onto function (surjective):-

* If and only if for every element $b \in B$ there is an element $a \in A$ with $f(a) = b$.

* **EX:**



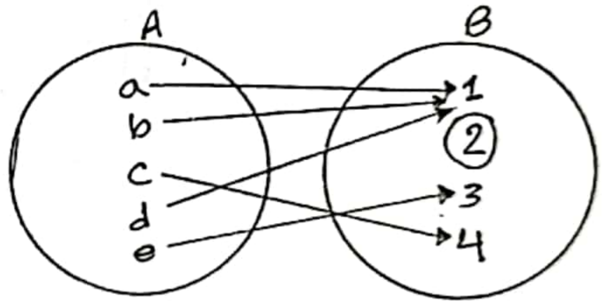
onto.

Co-domain $= \{1, 2, 3, 4\}$

Range $= \{1, 2, 3, 4\}$

*يعني عشان تبقى onto لازم ال Co-domain يساوي ال Range "يعني ال2 واحد".

* **Ex:**



Not onto (Not surjective).

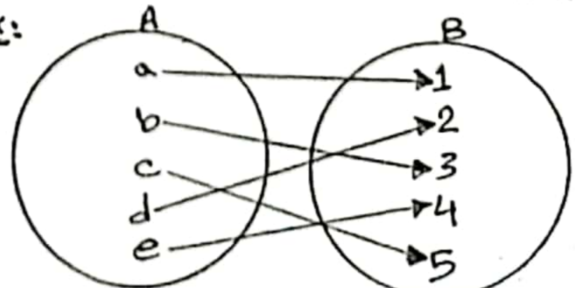
Co-domain $= \{1, 2, 3, 4\}$

Range $= \{1, 3, 4\}$.

3 One-to-one correspondence (bijective)

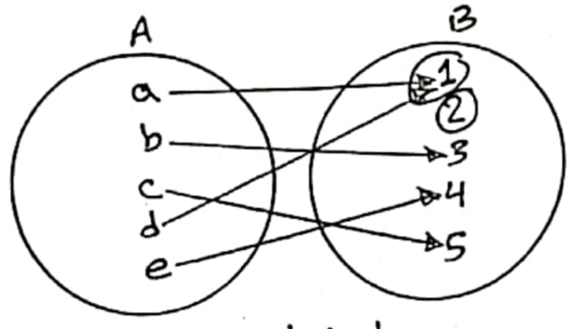
* if it is both one-to-one and onto.

* **EX:**



bijective.

* Ex.



" Not bijective "

* **Ex (1):** Determine whether the function $f(x) = x+1$ from the set of integers to the set of integers is one-to-one.

ans: $\boxed{\begin{matrix} f(a) = f(b) \\ a = b \end{matrix}}$

$f(a) = a+1$

$f(b) = b+1$

$f(x)$ is one-to-one (if $f(a) = f(b)$, and a equal b then).

$f(a) = f(b)$

$a+1 = b+1$

$a = b$

$\therefore f(x) \Rightarrow$ is one-to-one.

* **Ex (2):** Determine whether the function $f(x) = x^2$ from the set of integers to the set of integers is one-to-one?

ans.

$f(a) = a^2$, $f(b) = b^2$

$f(x)$ is one-to-one (if $f(a) = f(b)$, and a equal b then).

$f(a) = f(b)$

$a^2 = b^2$

$\pm a = \pm b$

a may be not equal b

$\therefore f(x)$ is NOT one-to-one. ###

4 Inverse Functions :-

* عشان اقدر أجيب معكوس الدالة لازم الدالة تكون bijective يعني لازم تكون onto و one-to-one.

$f(a) = b$

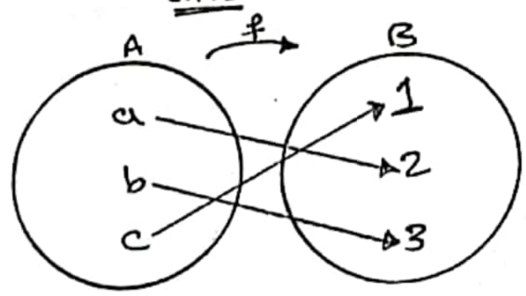
$f^{-1}(b) = a.$

* **Invertable :-**

  - A one-to-one correspondence is called invertible.

* **Ex:** Let f be the function from $\{a, b, c\}$ to $\{1, 2, 3\}$ such that $f(a) = 2$, $f(b) = 3$, and $f(c) = 1$. Is Invertable, and if it is, what is its inverse?

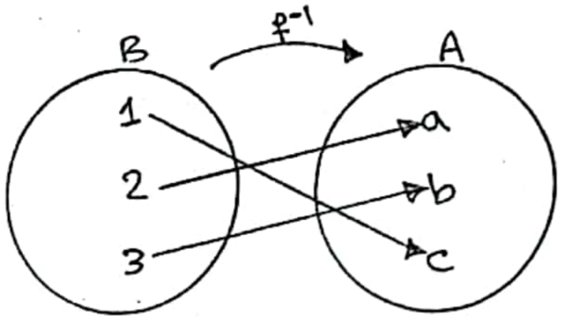
ans.



one-to-one and onto
" bijective "

$\therefore$ This function f is invertable

$\therefore f^{-1}(1) = c$, $f^{-1}(2) = a$, $f^{-1}(3) = b$



$\boxed{3}$ **Sequences :-**

* A sequence is a set of things (usually numbers) that are in order.

بترتيب

* we use the notation $\{a_n\}$ to describe the sequence.

$$\{a_n\} = \{a_1, a_2, a_3, \dots\}.$$

*EX: Consider the sequence $\{a_n\}$ where $a_n = \frac{1}{n}$

The list of terms of this sequence beginning with a_1, namely

$a_1, a_2, a_3, \dots$

Start with

$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$

*Type of the sequences :-

(1) Geometric.

(2) Arithmetic.

(1) Geometric :- $\boxed{\{ar^n\}}$ الحد الأول أساس المتتالية

* A geometric progression is a sequence of the form

$$\boxed{a, ar, ar^2, \dots, ar^n, \dots}$$

→ a : initial term.

→ r : Common ratio.

يعني لو قسمت أي عددين ورا بعض لازم يطلع r

*EX: $2, 10, 50, 250, \dots$

توضيح الحد الأول على إيه الدال

$\boxed{a=2}$, $r = \frac{10}{2} = \frac{50}{10} = \frac{250}{50} = 5.$

$\boxed{9}$

*Ex: $1, -1, 1, -1, 1, \dots$

$a = 1$

$r = \frac{-1}{1} = -1$

$\because$ notation $\{ar^n\}$, $n = 0, 1, 2, \dots$

$\therefore$ Notation: $\{1(-1)^n\}$, $n = 0, 1, 2, \dots$

$= \{(-1)^n\}$, $n = 0, 1, 2, \dots$

*Ex(3): $6, 2, \frac{2}{3}, \frac{2}{9}, \frac{2}{27}, \dots$

$a = 6$

$r = \frac{2}{6} = \frac{1}{3}$

$\because \{ar^n\}$, $n = 0, 1, 2, \dots$

$\therefore \{6 * (\frac{1}{3})^n\}$, $n = 0, 1, 2, 3, \dots$

*Ex(4): Find a, r?

$\{3 * 4^n\}$, $n = 0, 1, 2, \dots$

ans. $3 \times 4^0 = 3$

$\because \{ar^n\}$, $n = 0, 1, 2, 3, \dots$

$\therefore \{3 * 4^n\}$

$\therefore \boxed{a=3}$, $\boxed{r=4}$.

*EX(5): Find a, r?

$\{3 * 4^n\}$, $n = 1, 2, 3, \dots$

ans:

$\therefore a = 3*(4)^1 = 3 * 4^{(1)} = 12 \Rightarrow \boxed{a=12}$

initial term يعني أول حد

عشان تحسب ال a تعوض بـ أول قيمة لل n.

$\therefore \boxed{r=4}$

(2) Arithmetic :- $\boxed{\{a+nd\}}$

An Arithmetic progression is a sequence of the form

$\boxed{a, a+d, a+2d, \ldots, a+nd + \ldots}$ تخفظ

→ a : initial term.

→ d : Common difference.

~*~

* Ex(1): $-1, 3, 7, 11, \ldots$

$a = -1$

$d = 3-(-1) = 7-3 = 11-7 = 4$

$\because$ Notation $\{a+nd\}$, $n = 0, 1, 2, \ldots$

$\therefore$ Notation $\{-1+4n\}$, $n = 0, 1, 2, \ldots$

* Ex(2): $7, 4, 1, -2, \ldots$ (−3, −3, −3)

$a = 7$

$r = 4-7 = 1-4 = -2-1 = -3$

$\because \{a+nd\}$, $n = 0, 1, 2, \ldots$

$\therefore \{7-3n\}$, $n = 0, 1, 2, \ldots$

* Fibonacci Sequence :-

* $f_0, f_1, f_2, \ldots$

* $f_0 = 0$. $f_1 = 1$

* $f_n = f_{n-1} + f_{n-2}$, $n = 2, 3, 4, \ldots$

$0, 1, 1, 2, 3, 5, 8, 13, \ldots$

Geme → ar^2 عدد

A.P → جمع $a+nd$

[4] Summations :- $\sum$

* $\boxed{\sum_{i=m}^{n} a_i}$ $\boxed{\sum_{i=m}^{n} a_i}$ $\boxed{\sum_{m \le i \le n} a_i}$

* (i) is called the index of summation

$a_m + a_{m+1} + a_{m+2} + \cdots + a_n$.

* (m) is called lower limit.

* (n) is called upper limit.

* Ex(1): Express the sum of the first 100 terms of the sequence $\{a_n\}$, where $a_n = \frac{1}{n}$ for $n = 1, 2, 3, \ldots$

Ans.

$\because a_n = \frac{1}{n}$ for $n = 1, 2, 3, \ldots$

$\therefore \{a_n\} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{100}$

$\therefore \sum_{n=1}^{100} \frac{1}{n}$.

* Ex(2): what is the value of $\sum_{j=1}^{5} j^2$?

ans.

$\because \sum_{j=1}^{5} j^2 = (1)^2 + (2)^2 + (3)^2 + (4)^2 + (5)^2$

$= 1 + 4 + 9 + 16 + 25$

$= 55$.

* Theorem :-

$$\sum_{j=0}^{n} ar^j = \begin{cases} \dfrac{ar^{n+1} - a}{r-1} & \text{if } r \neq 1 \\ (n+1)a & \text{if } r = 1 \end{cases}$$

* if a and r are real numbers and $r \neq 0$.

---

* Ex(1): Find $\sum_{i=0}^{2} 3 * 4^i$

ans

$\because a = 3, \ r = 4, \ n = 2$

$\because r = 4 \neq 1$

$$\therefore \sum = \text{Sum} = \frac{ar^{n+1} - a}{r-1}$$

$$= \frac{3 * (4)^{2+1} - 3}{4-1}$$

$$= \boxed{63}.$$

---

* Ex(2): Find $\sum_{k=1}^{4} 2 * 3^{k-1}$

ans.

$\because \boxed{k-1 = j}$

$$\therefore \sum_{k=1}^{4} 2 * 3^{k-1} = \sum_{j=0}^{3} 2 * 3^j$$

$\therefore a = 2, \ r = 3, \ n = 3$

$$\therefore \text{Sum} = \frac{ar^{n+1} - a}{r-1} = \frac{(2)(3)^{3+1} - 2}{3-1}$$

$$= \boxed{80}$$

---

* Double Summation :-

- Find $\sum_{i=1}^{4} \sum_{j=1}^{3} ij = ?$

ans.

$$\sum_{i=1}^{4} \sum_{j=1}^{3} ij = \sum_{i=1}^{4} (i + 2i + 3i)$$

$$= \sum_{i=1}^{4} 6i$$

$$= 6 + 6(2) + 6(3) + 6(4)$$

$$= 60.$$

---

* Some useful Summation :-

$\sum_{k=0}^{n} ar^k \ (r \neq 0)$	$\dfrac{ar^{n+1} - a}{r-1}, \ r \neq 1$
$\sum_{k=1}^{n} k$	$\dfrac{n(n+1)}{2}$
$\sum_{k=1}^{n} k^2$	$\dfrac{n(n+1)(2n+1)}{6}$
$\sum_{k=1}^{n} k^3$	$\dfrac{n^2(n+1)^2}{4}$

* Ex(1): Find $\sum_{k=1}^{100} k^2 = ?$

ans.

$$\sum_{k=1}^{100} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{100(100+1)(200+1)}{6} = 33835\ldots$$

12

* Ex(2): Find $\sum_{K=50}^{100} K^2 = ??$

ans.

$$\because \sum_{K=50}^{100} K^2 = \sum_{K=1}^{100} K^2 - \sum_{K=1}^{49} K^2$$

$$\because \sum_{K=1}^{100} K^2 = \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{100(101)(201)}{6} = 338350$$

$$\because \sum_{K=1}^{49} K^2 = \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{49(50)(99)}{6} = 40425$$

$$\therefore \sum_{K=50}^{100} K^2 = 338350 - 40425$$

$$= 297925.$$

* Ex(3): Find $\sum_{K=1}^{5}(K+1)$

ans.

$$\sum_{K=1}^{5}(K+1) = \sum_{K=1}^{5} K + \sum_{K=1}^{5} 1$$

$$= \frac{n(n+1)}{2} + (1*5)$$

$$= \frac{5(5+1)}{2} + 5$$

$$= 15 + 5$$

$$= 20.$$

* Ex(4): $\sum_{K=1}^{10} 3 = ?$

ans.

$$\sum_{K=1}^{10} 3 = 3*10$$

$$= 30.$$