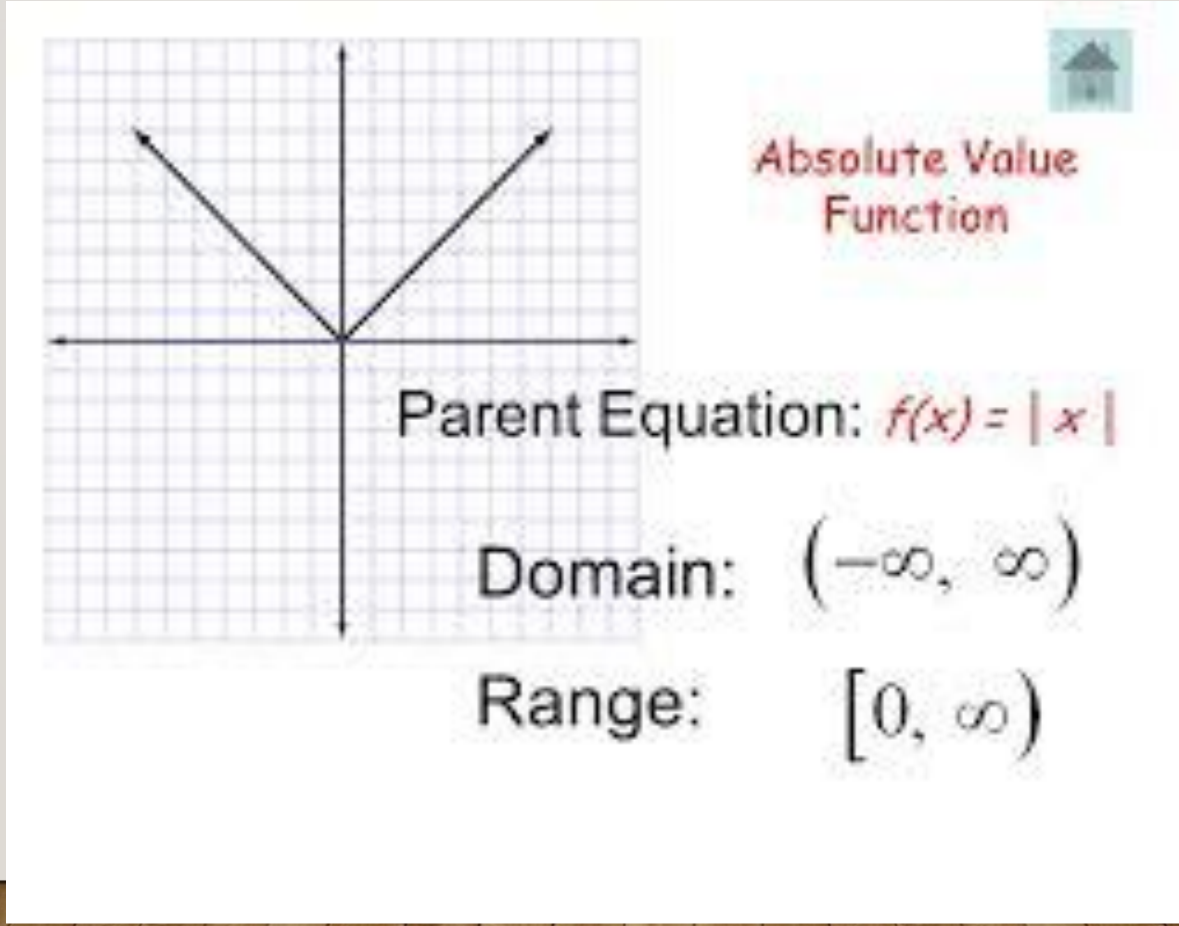
الجامعة المصرية للتعلم الإلكتروني الأهلية
THE EGYPTIAN E-LEARNING UNIVERSITY

Mathematics 1

Section 2

Classification of Functions



Classification of Functions

odd and even Functions

Even Functions	Odd Functions
$f(-x) = f(x)$	$f(-x) = -f(x)$

(i) Even Functions is symmetry about y-axis
(ii) odd Functions is symmetry about origin

Determine whether each of the following functions is even, odd, or neither even nor odd.

(a) $f(x) = x^5 + x$ (b) $g(x) = 1 - x^4$ (c) $h(x) = 2x - x^2$

(a) $f(-x) = (-x)^5 + (-x) = (-1)^5 x^5 + (-x)$

$= - x^5 + (-x) = -(x^5 + x)$

$= -f(x)$

Therefore f(x) is an odd function

(b) $g(-x) = 1 - (-x)^4$

$= 1 - x^4$

$= g(x)$

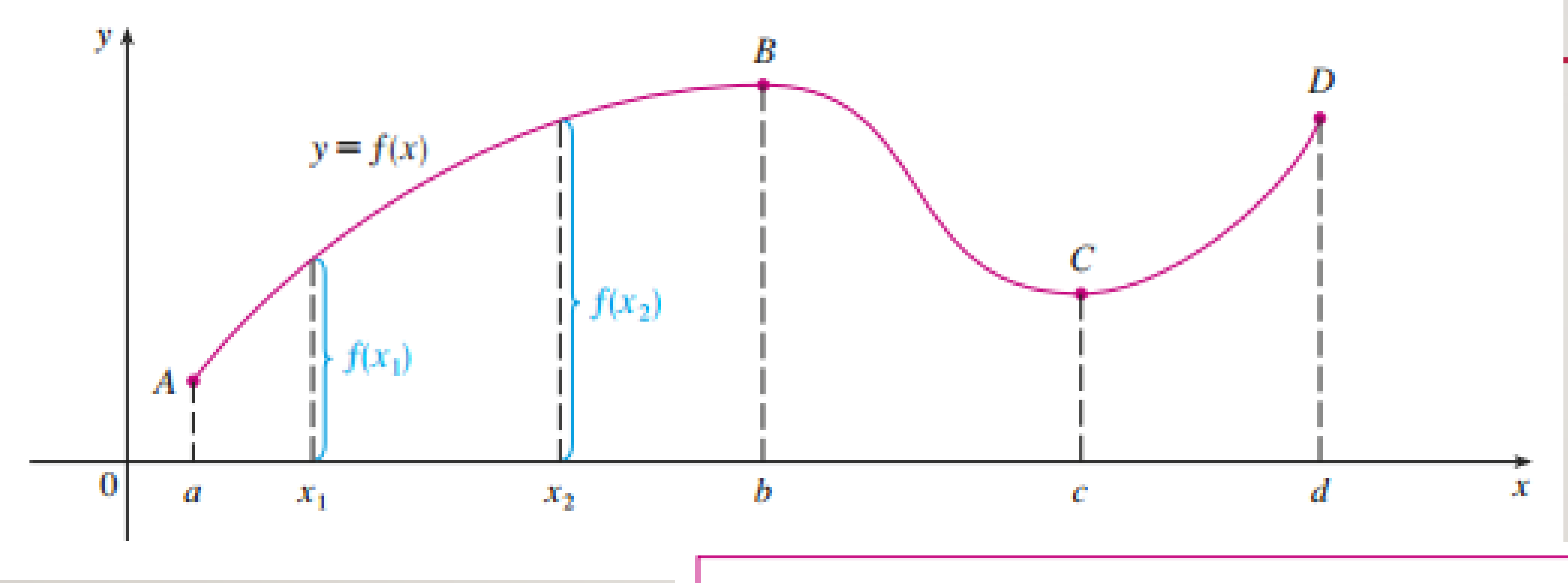
Therefore g(x) is an even function

(c) $h(-x) = 2(-x) - (-x)^2 = -2x + (-1)^2 x^2$

$= -2x + x^2$

Therefore f(x) is neither even nor odd function

CLASSIFICATION OF FUNCTIONS



A function f is called **increasing** on an interval I if

$$f(x_1) < f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ in } I$$

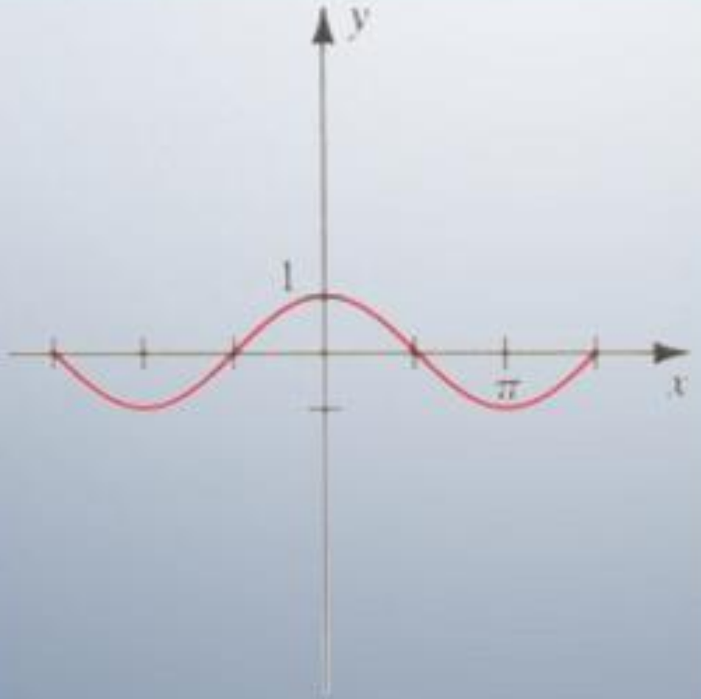
It is called **decreasing** on I if

$$f(x_1) > f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ in } I$$

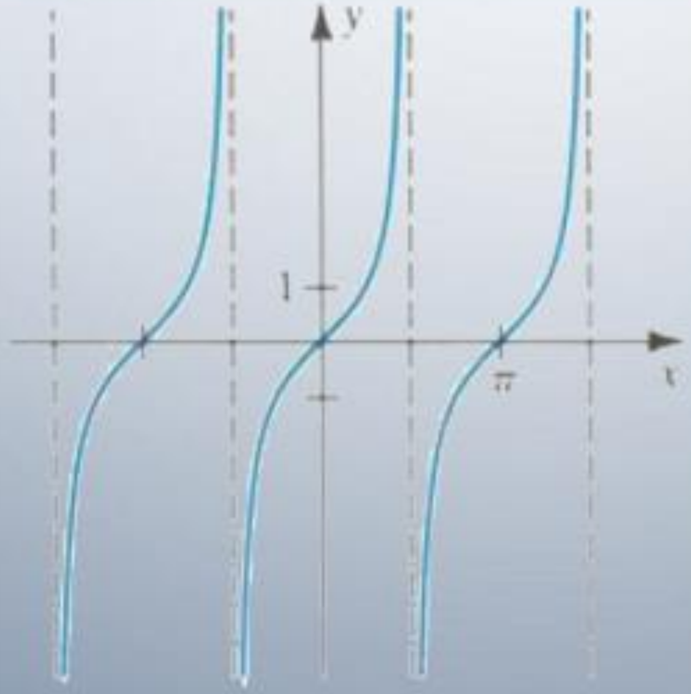
TRIGONOMETRIC FUNCTIONS

Function f	Graph	Domain and Range
$f(x) = \sin x$		Domain $= \mathbb{R}$ or $(-\infty, \infty)$ Range $= [-1, 1]$ Period $= 2\pi$ Odd function ; $\sin(-x) = -\sin x$

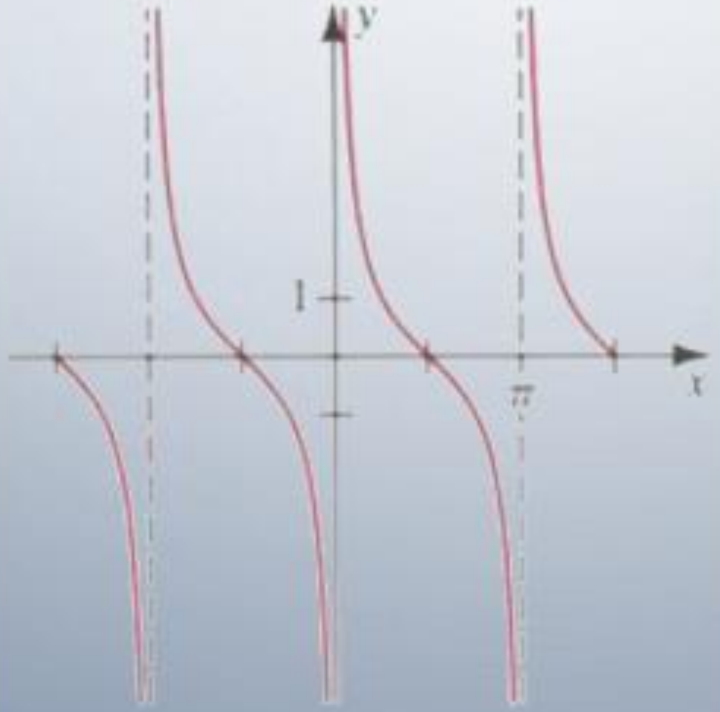
TRIGONOMETRIC FUNCTIONS

Function f	*Graph*	*Domain and Range*
$f(x) = \cos x$		$Domain = \mathbb{R}$ *or* $(-\infty, \infty)$ $Range = [-1, 1]$ $Period = 2\pi$ *Even function ;* $\cos(-x) = \cos x$

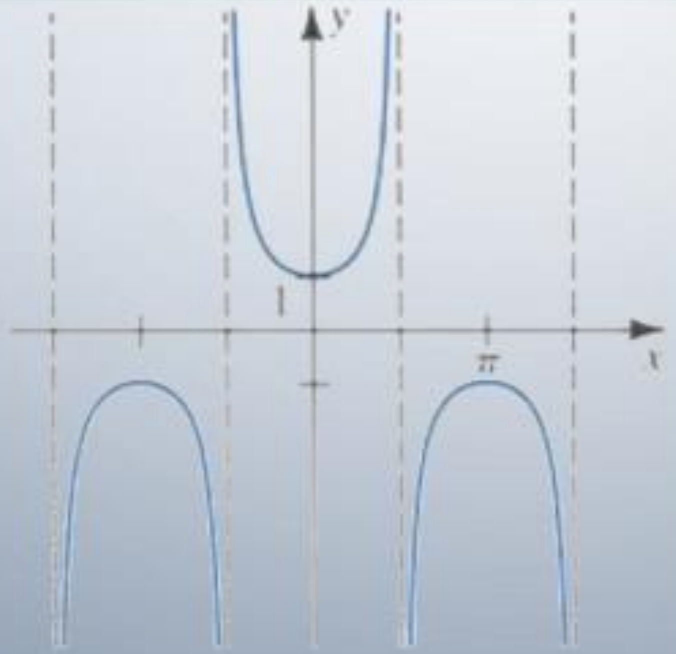
TRIGONOMETRIC FUNCTIONS

Function f	Graph	Domain and Range
$f(x) = \tan x = \dfrac{\sin x}{\cos x}$		Domain $= \mathbb{R} - \left\{ (2n+1)\dfrac{\pi}{2}, n \in \mathbb{Z} \right\}$ Range $= \mathbb{R}$ or $(-\infty, \infty)$ Period $= \pi$ Odd function ; $\tan(-x) = -\tan x$

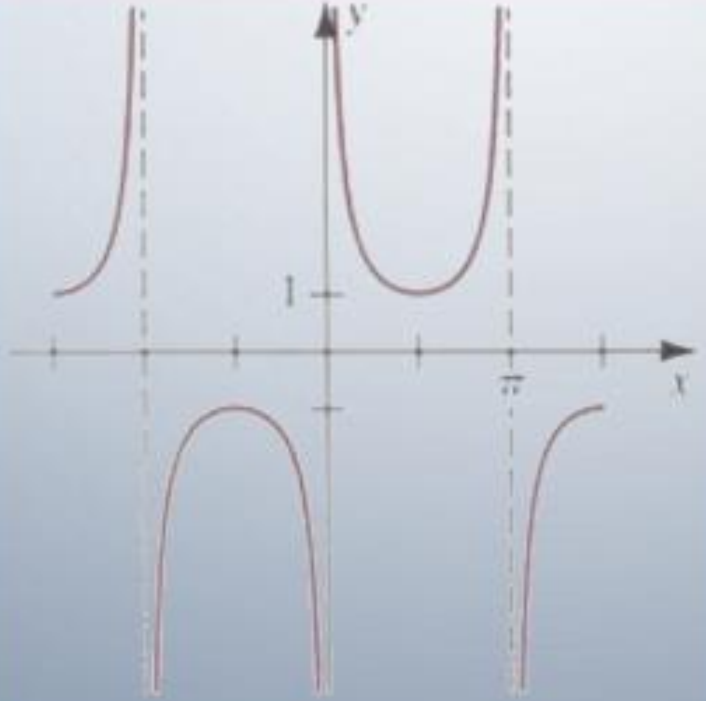
TRIGONOMETRIC FUNCTIONS

Function f	*Graph*	*Domain and Range*
$f(x) = \cot x$ $= \dfrac{\cos x}{\sin x}$		*Domain* = $\mathbb{R} - \{n\pi , n \in \mathbb{Z}\}$ *Range* $= \mathbb{R}$ *or* $(-\infty, \infty)$ *Period* $= \pi$ *Odd function ;* $\cot(-x) = -\cot x$

TRIGONOMETRIC FUNCTIONS

Function f	Graph	Domain and Range
$f(x) = \sec x$ $= \dfrac{1}{\cos x}$		Domain = $\mathbb{R} - \left\{(2n+1)\dfrac{\pi}{2}, n \in \mathbb{Z}\right\}$ Range $= (-\infty, -1] \cup [1, \infty)$ or $\mathbb{R} - (-1, 1)$ Period $= 2\pi$ Even function ; $\sec(-x) = \sec x$

TRIGONOMETRIC FUNCTIONS

Function f	Graph	Domain and Range
$f(x) = \csc x$ $= \dfrac{1}{\sin x}$		Domain = $\mathbb{R} - \{n\pi, n \in \mathbb{Z}\}$ Range $= (-\infty, -1] \cup [1, \infty)$ or $\mathbb{R} - (-1, 1)$ Period $= 2\pi$ Odd function ; $\csc(-x) = -\csc x$

TRIGONOMETRIC IDENTITIES

Trigonometric Identities

(1) $\cos^2\theta + \sin^2\theta = 1$

(2) $1 + \tan^2\theta = \sec^2\theta$

(3) $1 + \cot^2\theta = \text{cosec}^2\theta$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\cot(A \pm B) = \frac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$$

$$\sin(2a) = 2\sin(a)\cos(a)$$

$$\cos(2a) = \cos^2(a) - \sin^2(a)$$

$$\cos(2a) = 2\cos^2(a) - 1$$

$$\cos(2a) = 1 - 2\sin^2(a)$$

$$\tan(2a) = \frac{2\tan(a)}{1 - \tan^2(a)}$$

EXAMPLE:

- **Solve the trig equations:** $\boldsymbol{2\sin 3\theta = \sqrt{3}}$

$\sin 3\theta = \frac{\sqrt{3}}{2}$

$3\theta = \frac{\pi}{3} + 2n\pi$

Or $3\theta = \frac{2\pi}{3} + 2n\pi$

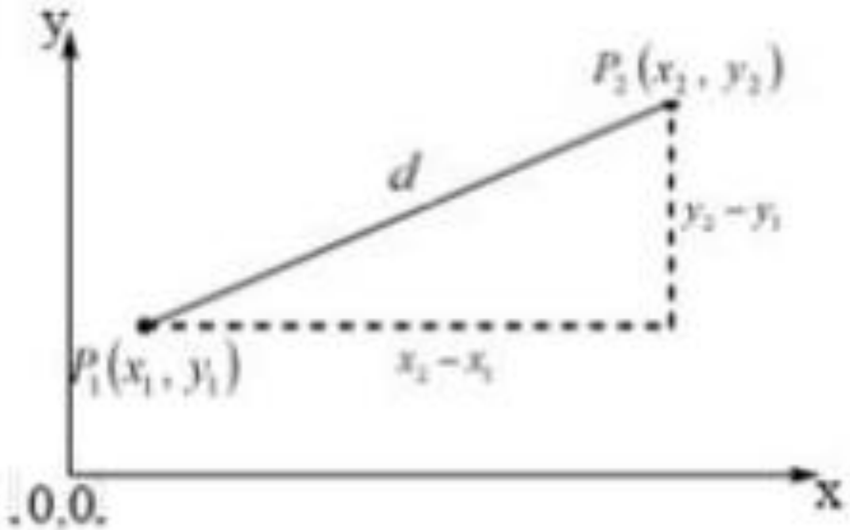
$\theta = \frac{\pi}{9} + \frac{2}{3}n\pi$

Or $\theta = \frac{2\pi}{9} + \frac{2}{3}n\pi$ $\quad$ $n = 0, \pm1, \pm2, \pm3$………etc.

Graphs

Distance between two points:

Applying Pythagorean theorem, the distance d between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is



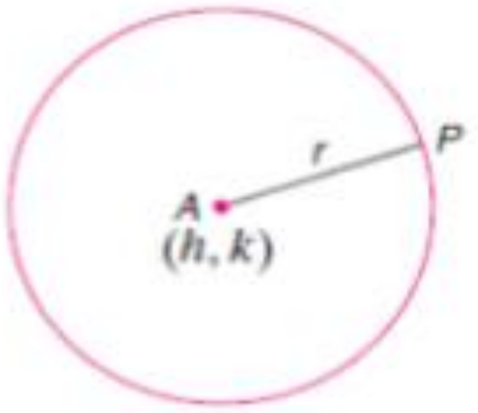
$$d^2 = \overline{P_2P_2}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2,$$

or $$d = \overline{P_1P_2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

CIRCLE

The Circle

Definition: Circle is the locus of a point $P(x,y)$ moving such that its distance from a fixed point $A(h,k)$ is a constant r. This fixed point is called *center* of the circle and the constant distance is known as *radius of the circle*.



$$\left(x-h\right)^2+\left(y-k\right)^2=r^2$$

EXAMPLE:

Find the equation of the circle which passes through the point (4, 5) and has its center at (2, 2)

Solution

As the circle is passing through the point (4,5) and its center is (2,2) so its radius is

$$r=\sqrt{(4-2)^2+(5-2)^2}=\sqrt{13}$$

Therefore

$$(x-2)^2+(y-2)^2=13$$

EXAMPLE:

Find the center and radius of the the circle: $x^2+y^2+6x-8y-11=0$

Solution

$$\Rightarrow\left(x^2+6x\right)+\left(y^2-8y\right)-11=0$$

$$\Rightarrow\left[(x+3)^2-(3)^2\right]+\left[(y-4)^2-(4)^2\right]-11=0$$

$$\Rightarrow(x+3)^2-9+(y-4)^2-16-11=0$$

$$\Rightarrow(x+3)^2+(y-4)^2-36=0$$

$$\Rightarrow(x+3)^2+(y-4)^2=36$$

$\therefore$ The center is $(-3,4)$ and the radius: $r=\sqrt{36}=6$

EXAMPLE:

Find the elements of the circle $(2x+7)^2+4(y-3)^2=100$

Solution

$$\Rightarrow 4\left(x+\frac{7}{2}\right)^2+4(y-3)^2=100$$

Dividing both sides by 4:

$$\Rightarrow \left(x+\frac{7}{2}\right)^2+(y-3)^2=25$$

$\therefore$ The center is $\left(-\frac{7}{2},3\right)$ and the radius: $r=\sqrt{25}=5$

EXAMPLE:

Find the equation of the circle that has a diameter with endpoints (11,8) and (5,10)

Solution

The center of the circle is in the middle of the diameter:

$$\left(\frac{11+5}{2},\frac{8+10}{2}\right)=(8,9)$$

The diameter: $d=\sqrt{(11-5)^2+(10-2)^2}=\sqrt{40}=2\sqrt{10}$

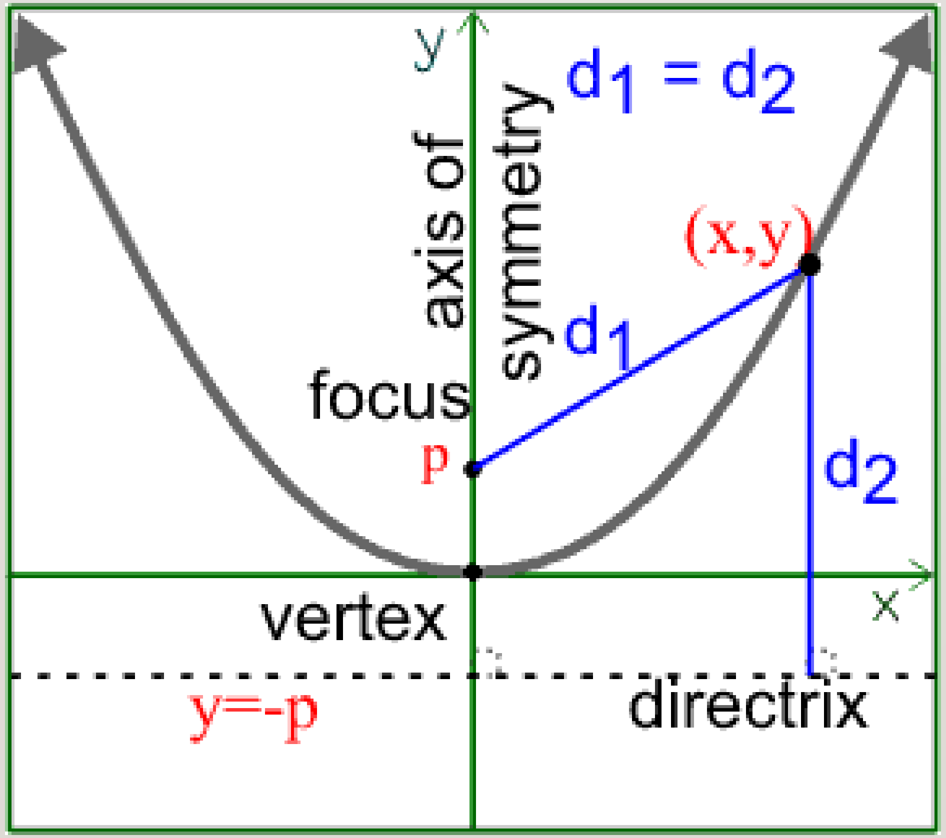
$$\therefore r=\sqrt{10}$$

The circle equation is

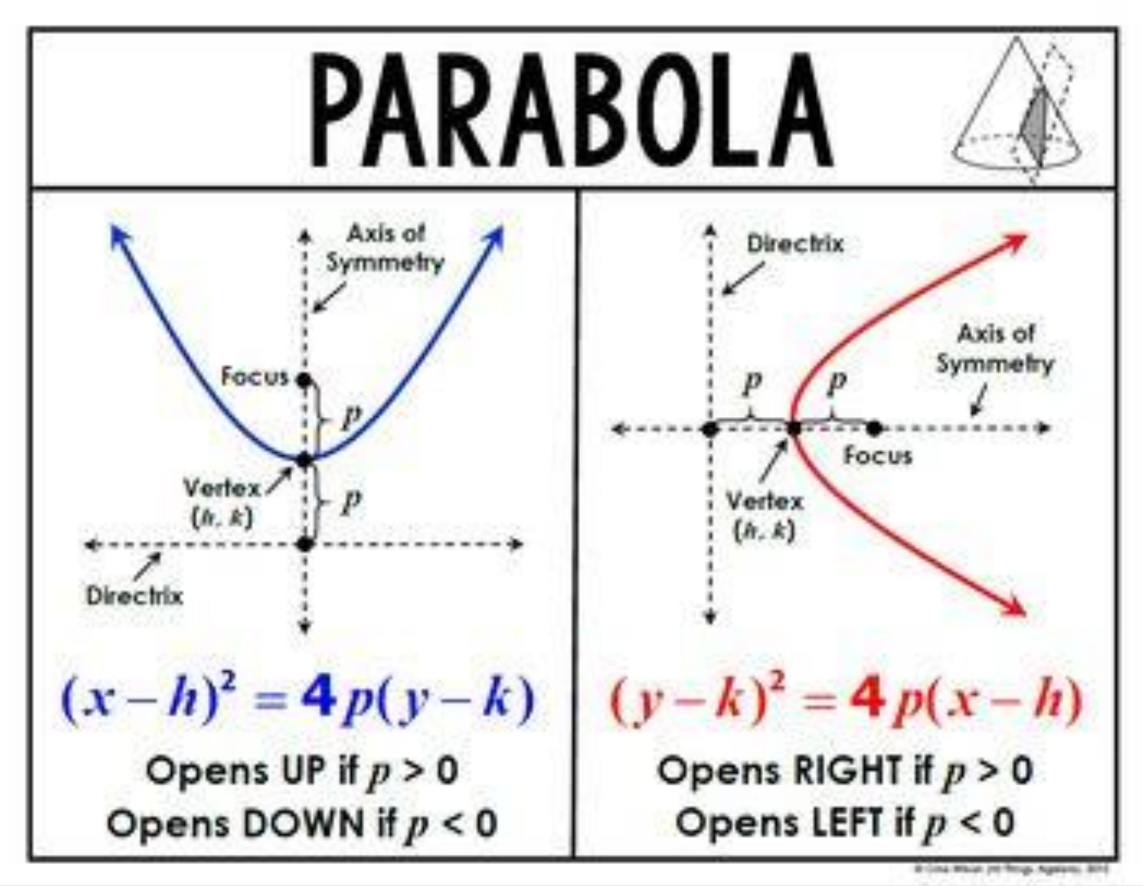
$$(x-8)^2+(y-9)^2=10$$

PARABOLA

PARABOLA



PARABOLA



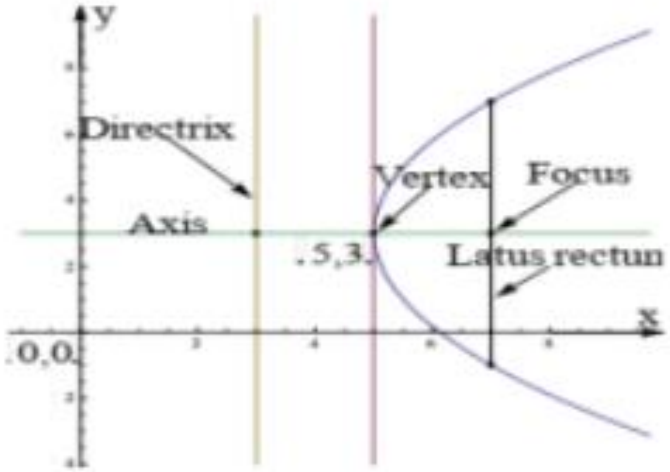
	Horizontal axis of symmetry $(y-k)^2$=4p(x-h)	**Vertical axis of symmetry** $(x-h)^2$=4p(y-k)
Vertex	(h, k)	(h, k)
Focus	(h+p, k)	(h, k+p)
Directrix	x = h-p	y = k-p
Axis of symmetry	y = k	x = h

EXAMPLE:

Find the elements of the parabola $(y-3)^2 = 8(x-5)$ and sketch the curve.

Solution

The vertex is $(5,3)$. Since $4a = 8$ then $a = 2$. The symmetry axis is parallel to x-axis and its equation is $y = 3$. This parabola opens to the right.



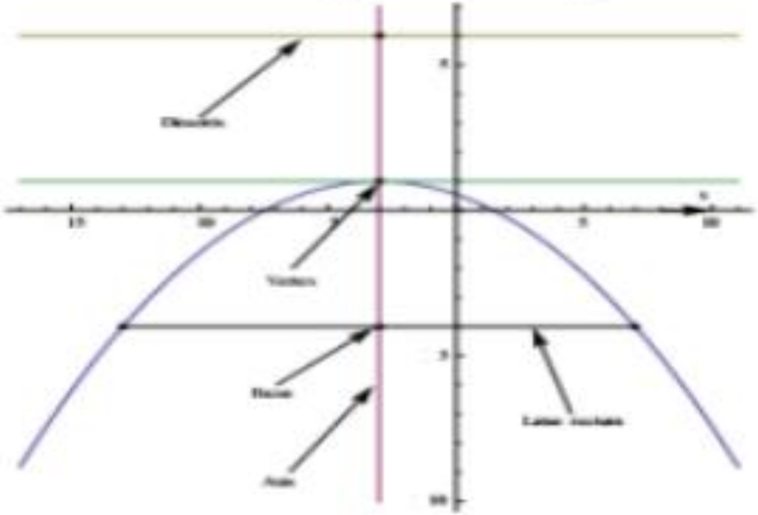
The focus is $(5+2,3) = (7,3)$, the directrix is $x = 3$ and the latus rectum length $= 4a = 8$.

EXAMPLE

Find the elements of the parabola $(x+3)^2 = -20(y-1)$ and sketch the curve.

Solution

The vertex is $(-3,1)$. Since $4a = 20$ then $a = 5$. The symmetry axis is parallel to y-axis and its equation is $x = -3$. This parabola opens to the down.



The focus is $(-3, 1-5) = (-3, -4)$, the directrix is $y = 6$ and the latus rectum length $= 4a = 20$.

EXAMPLE:

State the vertex, the focus, and the directrix of the parabola having the equation

$$x^2 - 4x + 4y - 4 = 0.$$

Solution

We shall rewrite the given equation in the standard form by completing square of the L.H.S,

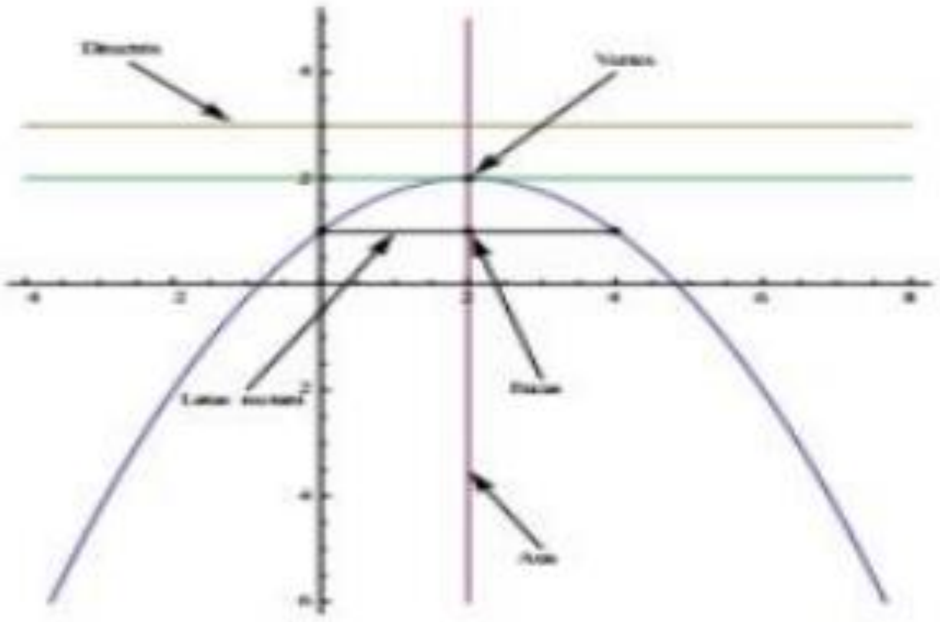
$$\Rightarrow (x^2 - 4x) + 4y - 4 = 0 \Rightarrow \left[(x-2)^2 - 4\right] + 4y - 4 = 0$$

$$\Rightarrow (x-2)^2 + 4y - 8 = 0$$

$$\Rightarrow (x-2)^2 = -4y + 8$$

$$\Rightarrow (x-2)^2 = -4(y-2)$$

The vertex is $(2,2)$. Since $4a = 4$ then $a = 1$. The symmetry axis is parallel to y-axis and its equation is $x = 2$. This parabola opens to the down.



The focus is $(2, 2-1) = (2,1)$,

The directrix is $y = 3$,

The latus rectum length $= 4a = 4$.

THANK YOU