

Assignment 1

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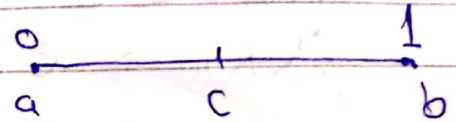
"Assignment of Numerical Methods"

1- Using Bisection method find the root of $\cos(x) - xe^x = 0$
with $a = 0$ and $b = 1$, $\epsilon = 0.01$

$$f(a) = \cos(0) - (0) \times e^0 = -1$$

$$f(b) = \cos(1) - (1) \times e^1 = 2.17$$

$$f(a)f(b) < \epsilon$$



Number	a	b	c	f(c)
1	0	1	0.5	0.053
2	0.5	1	0.75	-0.856
3	0.5	0.75	0.625	-0.356690
4	0.5	0.625	0.5625	-0.14129
5	0.5	0.5625	0.53125	0.77913
6	0.5	0.53125	0.515625	6.475×10^{-3}
7	0.515625	0.53125	0.5234375	-0.017362
8	0.515625	0.5234375	0.51953125	-5.404×10^{-3}
9	0.515625	0.51953	0.51578 0.517578	5.4518×10^{-4}

Solution = 0.517578.

Solution

(2) Find the solution of $f(x) = e^x - 1.5 - \tan^{-1}(x) = 0$ by Newton's Method. $x = -10$ and $\epsilon = 10^{-5}$.

$$f(x) = e^x - 1.5 - \tan^{-1}(x)$$

$$f'(x) = e^x - \frac{1}{1+x^2}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = -10 - \frac{e^{-10} - 1.5 - \tan^{-1}(-10)}{e^{-10} - \frac{1}{1+(-10)^2}} = -12.9243146$$

$$x_2 = -12.924 - \frac{e^{-12.924} - 1.5 - \tan^{-1}(-12.924)}{e^{-12.924} - \frac{1}{1+(-12.924)^2}} = -14.00381$$

$$x_3 = -14.00381 - \frac{e^{-14.00381} - 1.5 - \tan^{-1}(-14.00381)}{e^{-14.00381} - \frac{1}{1+(-14.00381)^2}} = -14.10060$$

$$x_4 = -14.100 - \frac{e^{-14.100} - 1.5 - \tan^{-1}(-14.100)}{e^{-14.100} - \frac{1}{1+(-14.100)^2}} = -14.10126974$$

$$\text{Solution} = -14.10126974$$

~~-14.10126~~

(3) Find using the fixed point iterative method to find a root of $f(x) = 4x^2 - 2x - 1$ $\epsilon = 0.005$ with interval $[0, 1]$.

$$f(a) = 4(0) - 2(0) - 1 = -1 \quad \ominus$$

$$f(b) = 4(1) - 2(1) - 1 = 1 \quad \oplus$$

1st method:-

$$4x^2 - 2x + 1$$

$$x^2 = \frac{-x}{2} + \frac{1}{4}$$

$$= -\frac{2x+1}{4}$$

$$x = \frac{\pm \sqrt{-1-2x}}{2}$$

not relevant

2nd method:-

$$4x^2 + 2x - 1$$

$$4x^2 + 2x = 1$$

$$x(4x+2) = 1$$

$$x = \frac{1}{4x+2} \quad \checkmark$$

$$x_1 = 0.16667$$

$$x_2 = 0.37500375$$

$$x_3 = 0.28571$$

$$x_4 = 0.31818$$

$$x_5 = 0.305555$$

$$x_6 = 0.3103448$$

$$x_7 = 0.3085108 \quad \leftarrow$$

Solution

(4) Find a solution of $x^3 + 1 = 0$ by Newton method for $x_0 = 3$ and $\epsilon = 5\%$.

$$f(x) = x^3 + 1$$

$$f'(x) = 3x^2$$

$$\epsilon = 0.05$$

$$f_{x_{n+1}} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = -3 - \frac{(-3)^3 + 1}{3(-3)^2} = -2.037037$$

$$x_2 = -2.037 - \frac{(-2.037)^3 + 1}{3(-2.037)^2} = -1.43835$$

$$x_3 = -1.43835527$$

$$x_4 = -1.01240226$$

$$x_5 = -1.000151311$$

A solution:-

$$-1.000151311$$

(5) Find the equation $f(x) = e^{-x} - x = 0$ Find the root of the function by using Fixed Point iteration method with $x_0 = 0.5$ and $\epsilon = 5\%$.

$$f(x) = e^{-x} - x$$

$$\text{Equation: } x = e^{-x} \quad \checkmark$$

$$x_1 = 0.606530$$

$$x_2 = 0.545239$$

$$x_3 = 0.579703$$

$$x_4 = 0.560007462 \quad \leftarrow$$

Solution

(6) Consider finding the root of $f(x) = x^2 - 3$ and start with the interval $[1, 2]$ with $N = 3$ iterations.

$$f(a) = (1)^2 - 3 = -2 \quad (-)$$

$$f(b) = (2)^2 - 3 = 1 \quad (+)$$

$$f(a) f(b) < 0$$

$$c = \frac{a+b}{2} = \frac{1+2}{2} = 1.5$$

Number	a	b	c	f(c)
0	1	2	1.5	-0.75
1	1.5	2	1.75	0.0625
2	1.5	1.75	1.625	-0.3593
3	1.625	1.75	1.6875	-0.15234

(7) Use Newton's method to find the root of $x^4 - 5x^3 + 9x + 3 = 0$ accurate to 6 decimal places in the interval $[4, 6]$.

$$f(x) = x^4 - 5x^3 + 9x + 3$$

$$f'(x) = 4x^3 - 15x^2 + 9$$

$$x_0 = \frac{a+b}{2} = \frac{4+6}{2} = 5$$

$$x_1 = 5 - \frac{(5)^4 - 5(5)^3 + 9(5) + 3}{4(5)^3 - 15(5)^2 + 9} = 4.64179$$

$$x_2 = 4.64179 - \frac{(4.64179)^4 - 5(4.64179)^3 + 9(4.64179) + 3}{4(4.64179)^3 - 15(4.64179)^2 + 9} = 4.5375$$

$$x_3 = 4.53 - \frac{(4.53)^4 - 5(4.53)^3 + 9(4.53) + 3}{4(4.53)^3 - 15(4.53)^2 + 9} = \boxed{4.52891796}$$

Solution

(8) (a) Put $(1100101.1101)_2$ in Single Precision IEEE 754 standard.

$$(1100101.1101)_2$$

$$1.00101101 \times 2^{7+} \text{ exponent.}$$

$$\text{Exponent} = 7 + 127 = 134.$$

134	2	0
67	2	1
33	2	1
16	2	0
8	2	0
4	2	0
2	2	0
1	2	1

Solution?

$$0 \ 1000110 \ 11001011010 \ 0000 \ 0000 \ 000$$

(8) (b) Find IEEE 754 single Precision B of 85.125

85	2	1
42	2	0
21	2	1
10	2	0
5	2	1
2	2	0
1	2	1

$$\begin{aligned} 0.125 \times 2 &= 0.25 \quad 0 \\ 0.25 \times 2 &= 0.5 \quad 0 \\ 0.5 \times 2 &= 1 \quad 1 \end{aligned}$$

1010101.001

$$1.010101001 \times 2^6$$

$$\text{Exponent} = 6 + 127 = 133$$

133	2	1
66	2	0
33	2	1
16	2	0
8	2	0
4	2	0
2	2	0
1	2	1

10000 101

Solution: 0 1000 101 0101 0100 1000 0000 0000 000

(9) Suppose we would like to determine the minimum no. of iterations needed in Bisection algorithm given to $a_0 = 3$, $b_0 = 4.5$ and $\epsilon = 10^{-5}$.

$$N \geq \frac{\log_{10}(b_0 - a_0) - \log_{10}(\epsilon)}{\log_{10}(2)}$$

$$N \geq \frac{\log_{10}(4.5 - 3) - \log_{10}(10^{-5})}{\log_{10}(2)} = 17.1946$$

≈ 17 iterations.

(10) (a) Determine the absolute error and relative errors when approximating p by when $p = 1.32 \times 10^2$ and $p^* = 1.35 \times 10^2$

$$\text{Absolute error} = |p - p^*| = |1.32 \times 10^2 - 1.35 \times 10^2| = 3$$

$$\text{Relative error} = \frac{|p - p^*|}{p} = \frac{|1.32 \times 10^2 - 1.35 \times 10^2|}{1.32 \times 10^2} = 0.0227 \approx 2.2727\%$$

(10) (b) what is the decimal number represented by word.

$S = 1$ $\boxed{1}$ 1000 0001 0100 0000 0000 0000 0000 0000
Exponent.

$$\text{Exp} = 2^0 + 2^7 = 129$$

$$\text{Mantissa} = 2^{-2} = \frac{1}{4} = 0.25$$

$$\text{Form: } (-1)^S 2^{e-127} \text{ 1. Mantissa}$$

$$= (-1)^1 (2^2) (0.25) = -5$$