الجامعة المصرية
للتعلم الإلكتروني الأهلية

EELU
THE EGYPTIAN E-LEARNING UNIVERSITY

Sections (7&8)

Physics (I)

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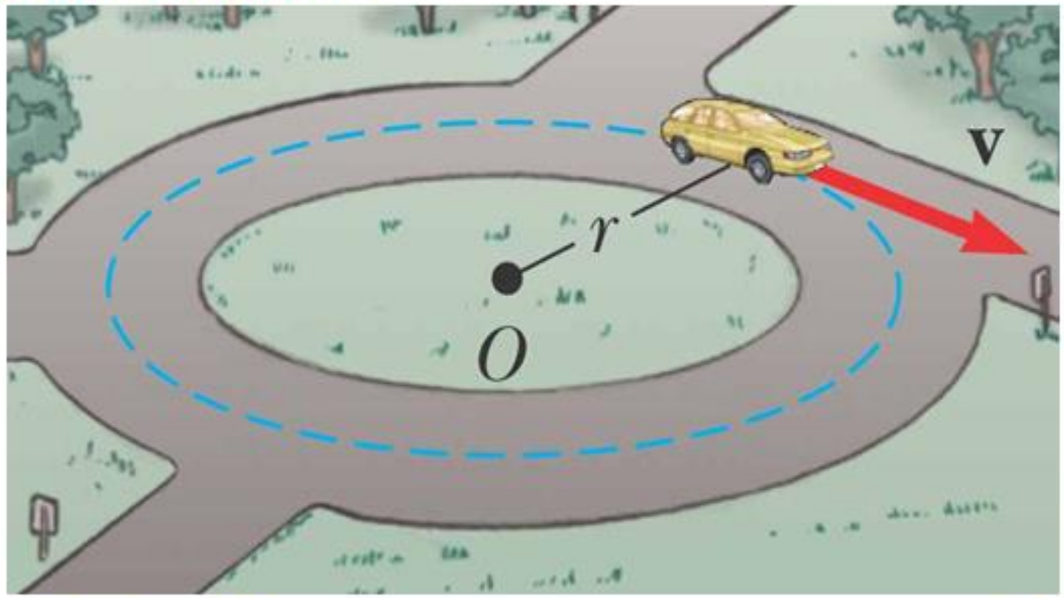
4.4 Uniform Circular Motion

• Motion of an object in a circular path with constant speed, is called **uniform circular motion**.



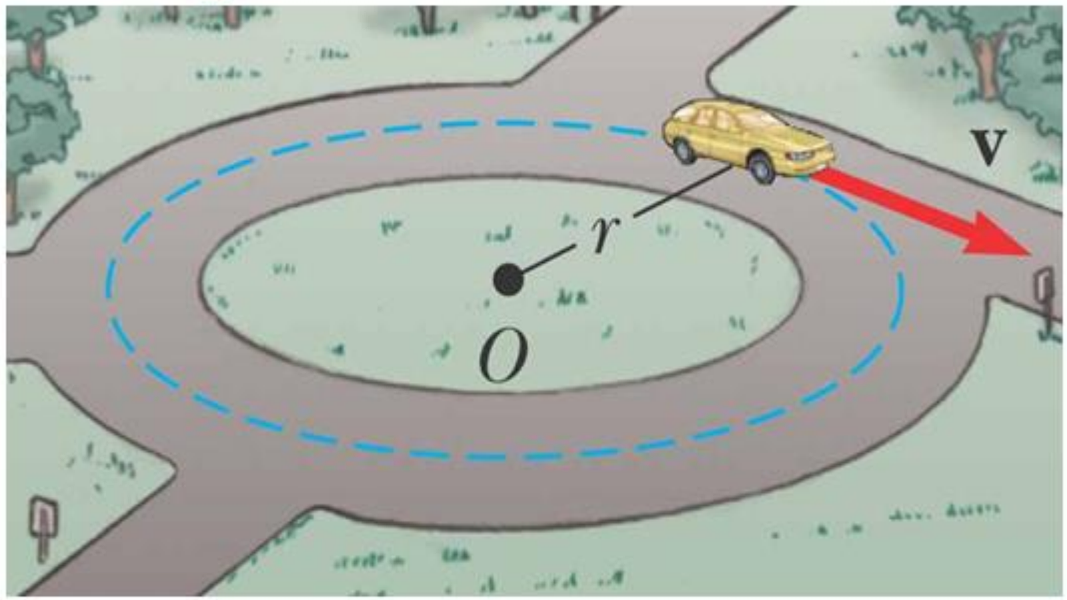
• Even though the object moves at a **constant speed** in a circular path, it still **has an acceleration** due to **change of the direction of the velocity**.

• This **acceleration** is called **centripetal acceleration** and points **toward the center**.

$$\boxed{a = \frac{v^2}{r}}$$

Period of circular motion

• In many situations, it is convenient to describe the motion of a particle moving with constant speed in a circle of radius ***r*** in terms of the period ***T***, which is defined as the time required to complete one revolution.



$$T = \frac{2\pi r}{v}$$

An object moving at a constant speed requires 6 s to go once around a circle with a diameter of 4 m. What is the magnitude of the instantaneous acceleration during this motion?

Solution:

The speed of the object can be calculated from:

$$v = \frac{x}{t} = \frac{2\pi r}{t} = \frac{4\pi}{6} = 2.1\ \mathrm{m/s}$$

The acceleration of the object moving around a circle is given by:

$$a = \frac{v^2}{r} = \frac{2.1^2}{2} = 2.2\ \mathrm{m/s^2}$$

2) A tire 0.500 m in radius rotates at a constant rate of 200 rev/min. Find the speed and acceleration of a small stone lodged in the tread of the tire (on its outer edge).

Solution:

$$r = 0.500\ \text{m};$$

$$v_t = \frac{2\pi r}{T} = \frac{2\pi(0.500\ \text{m})}{\frac{60.0\ \text{s}}{200\ \text{rev}}} = 10.47\ \text{m/s} = \boxed{10.5\ \text{m/s}}$$

$$a = \frac{v^2}{R} = \frac{(10.47)^2}{0.5} = \boxed{219\ \text{m/s}^2\ \text{inward}}$$

Newton 2nd Law: The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

Mathematical Statement

$$\sum \vec{F} = m\vec{a}$$

Components Form

$$\sum F_x = m\,a_x \qquad \sum F_y = m\,a_y \qquad \sum F_z = m\,a_z$$

Units and Dimensions of Force

$$[F] = [m].\ [a] = MLT^{-2}$$

Units of Mass, Acceleration, and Force[a]

System of Units	Mass	Acceleration	Force
SI	kg	m/s^2	$N = kg \cdot m/s^2$
U.S. customary	slug	ft/s^2	$lb = slug \cdot ft/s^2$

$1\ N = 1\ kg.\ 1\ m/s^2$

$1\ N = 1000\ gm\ 100\ cm\ /\ s^2$

$1\ N = 10^5\ gm.cm\ /\ s^2$

$1\ N = 10^5\ dyne$

15. When the engine of a 240-kg motorboat is shut off, the boat slows from 15.0 m/s to 10.0 m/s in 1.2 s, and then from 10.0 m/s to 5.0 m/s in 2.1 s. What is the average acceleration during each of these intervals? What average frictional force does the water provide during each interval?

Answer:

During the first interval the average acceleration is:

$$\mathbf{a} = \frac{\Delta v}{\Delta t} = \frac{10 - 15}{1.2} = -4.2\, m/s^2$$

During the second interval the acceleration is:

$$\mathbf{a} = \frac{\Delta v}{\Delta t} = \frac{5 - 10}{2.1} = -2.4\, m/s^2$$

During the first interval the average friction force is:

$$\mathbf{F} = \mathrm{m}\mathbf{a} = f_k$$

$$f_k = 240 \times -4.2 = -1.0 \times 10^3\, N$$

During the second interval the average friction force is:

$$f_k = 240 \times -2.4 = -5.7 \times 10^2\, N$$

47. Two heavy boxes of masses 20 kg and 30 kg sit on a smooth, frictionless surface. The boxes are in contact, and a horizontal force of 60 N pushes horizontally against the smaller box (Fig. 5.50). What is the acceleration of the two boxes? What is the force that the smaller box exerts on the larger box? What

is the force that the larger box exerts on the smaller box?

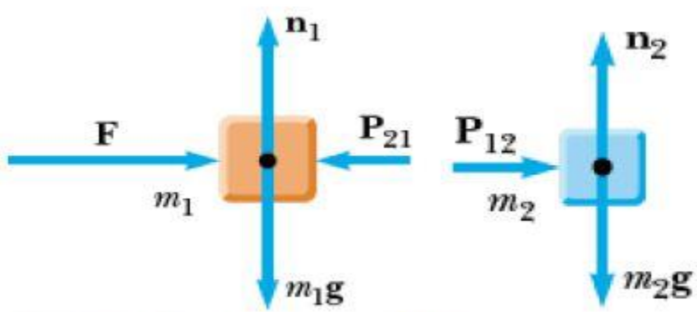
FIGURE 5.50 Two boxes in contact.

Answer:

The system of the two boxes will have an acceleration of:

$$a = \frac{F}{m_1 + m_2} = \frac{60}{20 + 30} = 1.2\, m/s^2$$

The force exerted by the small box on the large box is the force exerted by the large box on the small box but in opposite direction. This force is called contact force. To calculate this force, only the motion of one box should be considered.



Taking the first box, one can find:

$$F - P_{21} = m_1 a \Rightarrow P_{21} = F - m_1 a = F - \frac{m_1 F}{m_1 + m_2} = \frac{m_2 F}{m_1 + m_2} = \frac{30 \times 60}{20 + 30} = 36\, N$$

If the second box is considered, the result is obtained.

71. A woman pushes horizontally on a wooden box of mass 60 kg sitting on a frictionless ramp inclined at an angle of 30° (see Fig. 5.59). (a) Draw the "free-body" diagram for the box. (b) Calculate the magnitudes of all the forces acting on the box under the assumption that the box is at rest or in uniform motion along the ramp.

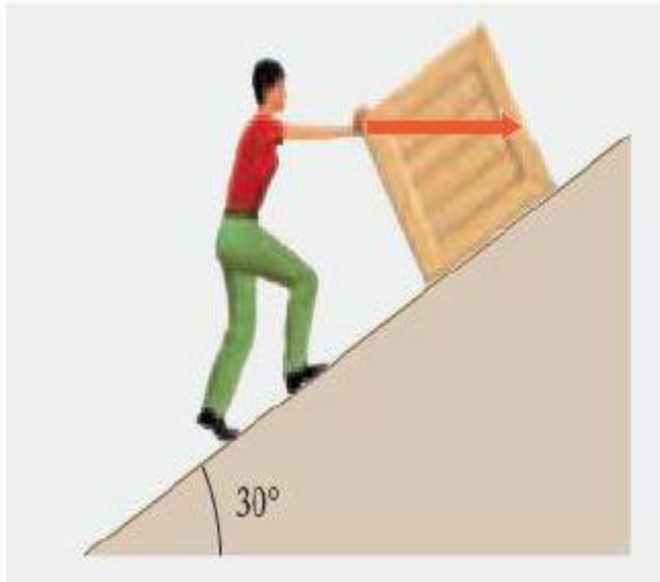
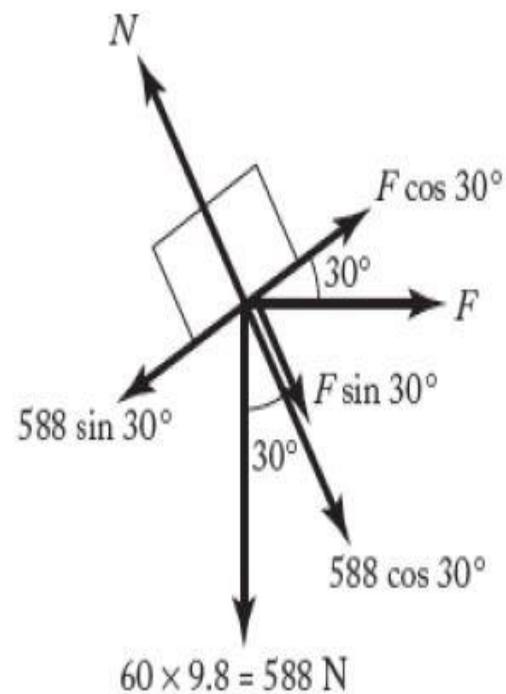


FIGURE 5.59 Woman pushing box.

Answer:

(a)



(b) To calculate the force acting on the box under the assumption that it is at rest or moving with uniform motion, N and F, equations of motion must be used.

In the x direction:

$$F\cos 30 - 588\sin 30 = 0 \Rightarrow F = 340\ N.$$

In the y direction:

$$N - F\sin 30 - 588\cos 30 = 0 \Rightarrow N = 680\ N.$$

24. A 5.00-kg object placed on a frictionless, horizontal table is connected to a cable that passes over a pulley and then is fastened to a hanging 9.00-kg object, as in Figure P5.24. Draw free-body diagrams of both objects. Find the acceleration of the two objects and the tension in the string.

*P5.24 First, consider the block moving along the horizontal. The only force in the direction of movement is T. Thus, $\sum F_x = ma$

$$T = (5\text{ kg})a \qquad (1)$$

Next consider the block that moves vertically. The forces on it are the tension T and its weight, 88.2 N.

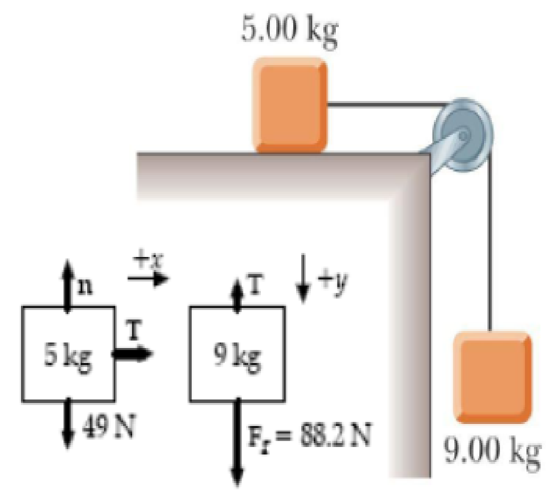


FIG. P5.24

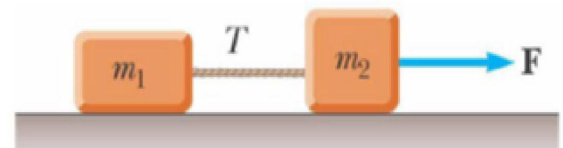
We have $\sum F_y = ma$

$$88.2\text{ N} - T = (9\text{ kg})a \qquad (2)$$

Note that both blocks must have the same magnitude of acceleration. Equations (1) and (2) can be added to give $88.2\text{ N} = (14\text{ kg})a$. Then

$$\boxed{a = 6.30\text{ m/s}^2 \text{ and } T = 31.5\text{ N}}.$$

45. Two blocks connected by a rope of negligible mass are being dragged by a horizontal force **F** (Fig. P5.45). Suppose that F = 68.0 N, m_1 = 12.0 kg, m_2 = 18.0 kg, and the coefficient of kinetic friction between each block and the surface is 0.100. (a) Draw a free-body diagram for each block. (b) Determine the tension T and the magnitude of the acceleration of the system.



P5.45 (a) See Figure to the right

(b) $68.0 - T - \mu m_2 g = m_2 a$ (Block #2)

$T - \mu m_1 g = m_1 a$ (Block #1)

Adding,

$$68.0 - \mu(m_1 + m_2)g = (m_1 + m_2)a$$

$$a = \frac{68.0}{(m_1 + m_2)} - \mu g = \boxed{1.29\ \text{m/s}^2}$$

$$T = m_1 a + \mu m_1 g = \boxed{27.2\ \text{N}}$$

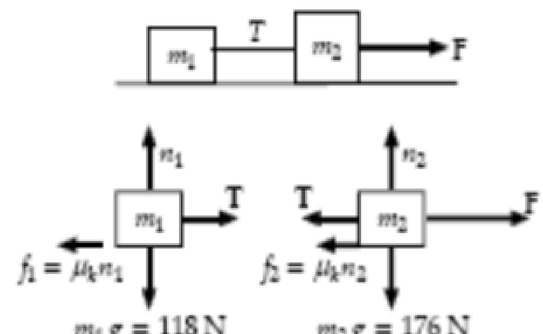


FIG. P5.45

33. Two masses $m_1 = 1.5$ kg and $m_2 = 3$ kg are connected by a thin string running over a massless pulley. One of the masses hangs from the string; the other mass slides on a 35° ramp with a coefficient of kinetic friction $\mu_k = 0.4$ (Fig. 6.33). What is the acceleration of the masses?

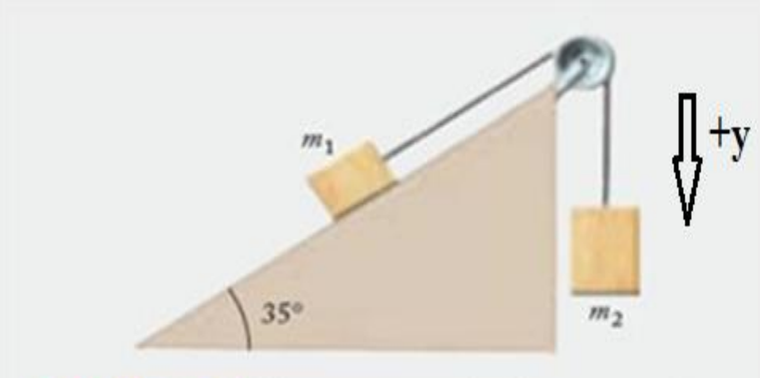


FIGURE 6.33 Two masses, an incline, and a pulley.

Answer:

Writing the equation of motion for the two masses:

m_1

$$T - m_1 g \sin\theta - f_k = m_1 a \qquad \text{...(1)}$$

Where $f_k = \mu m_1 g \cos\theta$

So:

$$T - m_1 g \sin\theta - \mu m_1 g \cos\theta = m_1 a \qquad \text{...(2)}$$

m_2

$$m_2 g - T = m_2 a \qquad \text{...(3)}$$

By adding equations 3 and 2 and solve for the acceleration:

$$a = \frac{m_2 g - m_1 g(\sin\theta - \mu\cos\theta)}{m_1 + m_2} = 3.6\ m/s^2$$

77. If the coefficient of static friction between the tires of an automobile and the road is $\mu_s = 0.80$, what is the minimum distance the automobile needs in order to stop without skidding from an initial speed of 90 km/h? How long does it take to stop?

Answer:

$$f_s = ma$$

$$-\mu_s mg = ma$$

$$a = -9.8 \times 0.8 = -7.8\, m/s^2$$

$$v^2 = v_o^2 + 2ax$$

$$x = -\frac{v_o^2}{2a} = \frac{\left(90 \times \frac{10}{36}\right)^2}{2 \times 7.8} = 40\ m$$

$$v = v_o + at$$

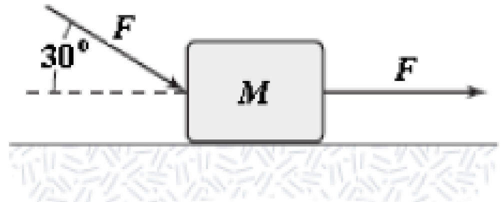
$$t = \frac{v_o}{a} = \frac{90 \times \frac{10}{36}}{7.8} = 3.2\ s$$

For a biological sample in a 1.0-m radius centrifuge to have a centripetal acceleration of $25g$ its speed must be:

A. $11\,\mathrm{m/s}$
B. $16\,\mathrm{m/s}$
C. $50\,\mathrm{m/s}$
D. $122\,\mathrm{m/s}$
E. $245\,\mathrm{m/s}$

Answer: **B**

The horizontal surface on which the block slides is frictionless. If $F = 20$ N and $M = 5.0$ kg, what is the magnitude of the resulting acceleration of the block?



5.3 m/s^2 a.
6.2 m/s^2 b.
7.5 m/s^2 c.
4.7 m/s^2 d.
3.2 m/s^2 e.

The only two forces acting on a body have magnitudes of 20 N and 35 N and directions that differ by 80°. The resulting acceleration has a magnitude of 20 m/s^2. What is the mass of the body?

2.4 kg a.
2.2 kg b.
2.7 kg c.
3.1 kg d.
1.5 kg e.

If the only forces acting on a 2.0-kg mass are $\vec{\mathbf{F}}_1 = (3\hat{\mathbf{i}} - 8\hat{\mathbf{j}})$ N and $\vec{\mathbf{F}}_2 = (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ N, what is the magnitude of the acceleration of the particle?

$1.5\ \text{m/s}^2$	a.
$6.5\ \text{m/s}^2$	b.
$4.7\ \text{m/s}^2$	c.
$9.4\ \text{m/s}^2$	d.
$7.2\ \text{m/s}^2$	e.