

## Logic and Bit operations

question 10

Truth Value	Bit
T	1
F	0

### \* Computer Bit operations:-

- OR =  $\vee$
- AND =  $\wedge$
- XOR =  $\oplus$

x	y	$x \vee y$	$x \wedge y$	$x \oplus y$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

### \* Bit strings:-

- سلسلة من 0 و 1
- bit string Length = عدد البتات
- String

\* Ex: Find the bitwise OR, bitwise AND, and bitwise XOR of the bit strings

01 1011 0110 and 11 0001 1101

ans.

01 1011 0110

11 0001 1101

11 1011 1111 bitwise OR

01 0001 0100 bitwise AND

10 1010 1011 bitwise XOR

A(1), Q(15).

### \* Applications of Propositional Logic:-

- (1) Translating English sentences
- (2) System specifications.
- (3) Boolean Searches.
- (4) Logic Puzzles.
- (5) Logic circuits.

### II Translating English sentences:

EX(1):

"You can access the Internet from Campus only if You are a computer science major or You are not a student"

Ans.

Let P, q, r be the Propositions:

P: You can access the Internet from Campus.

q: You are a computer science.

r: You are a student.

$$P \rightarrow (q \vee \neg r)$$

- only if =  $\rightarrow$
- or =  $\vee$

Logic

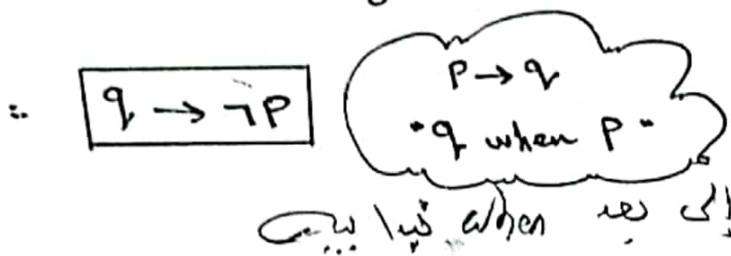
"The automated replay cannot be sent ~~by~~ when the file system is full".

ans.

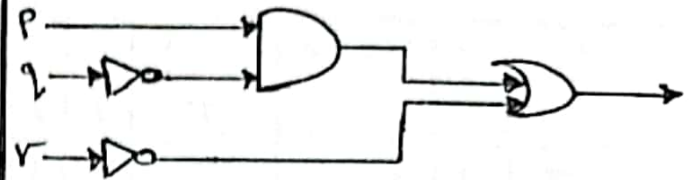
Let  $P$ , and  $q$  be the Propositions.

$P$ : The automated replay can be sent.

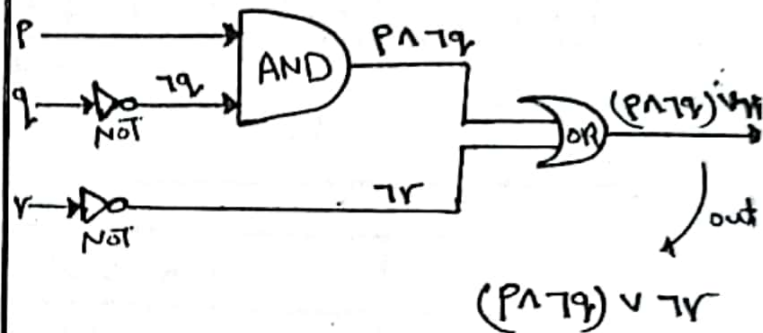
$q$ : The File system is full.



\*EX(1): Find the output for the Combinatorial circuit in the following graph/figure:-



ans.

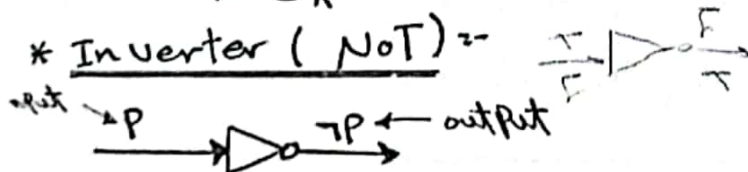


3) Logic Circuits:-

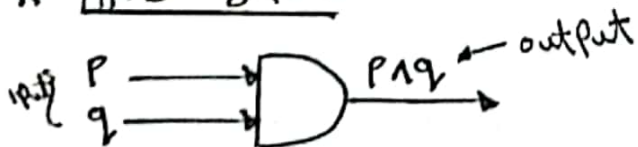
• "F" → "0" → "off"

• "T" → "1" → "on"

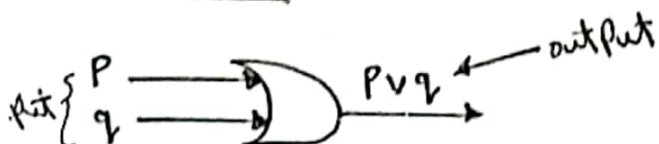
• 3 Gates:-



\* AND gate:-



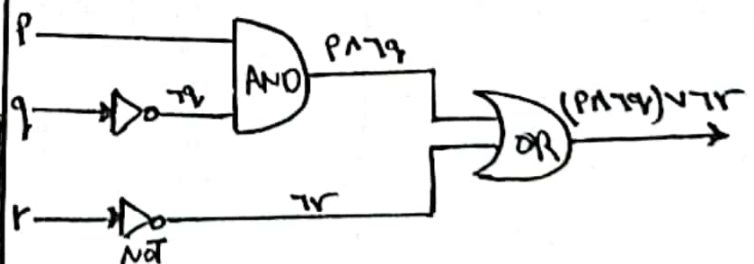
\* OR gate:-



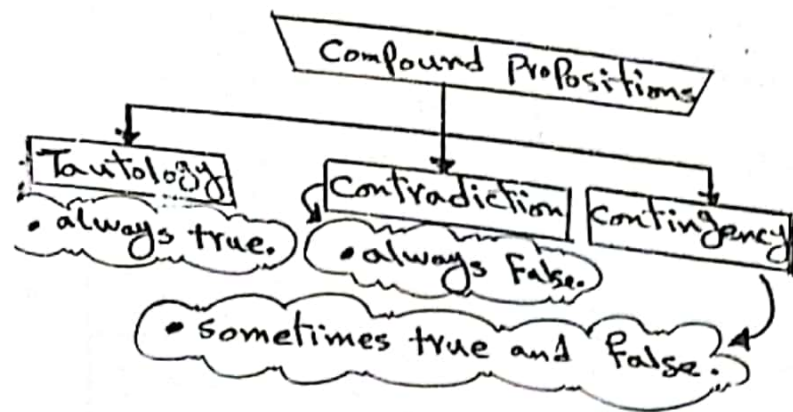
A(1), Q(6).

\*EX(2): Build a digital circuit that produces the output  $(P \wedge \neg q) \vee \neg r$ .

ans.



A(1), Q(7).



\* Ex(1): Show that following conditional statement is a tautology by using truth table.

$$(P \wedge q) \rightarrow P$$

ans.

P	q	$P \wedge q$	$(P \wedge q) \rightarrow P$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

كل قيم المقود = T

∴ This statement is a tautology.

Logically equivalent:

- Compound Propositions that have the same truth values in all possible cases.

لوا المقودين لهم نفس القيمة.

الرمز  $\Leftrightarrow$  أو  $\equiv$  #

FUT = T  
TUT = T  
...  
TTF = T

\* Ex(1): Show that  $\neg(P \vee q)$  and  $\neg P \wedge \neg q$  are logically equivalent.

ans.

P	q	$P \vee q$	$\neg(P \vee q)$	$\neg P$	$\neg q$	$\neg P \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

$$\therefore \neg(P \vee q) \equiv \neg P \wedge \neg q$$

\*

Equivalence	Name
$P \wedge T \equiv P$ $P \vee F \equiv P$	Identity Laws
$P \vee T \equiv T$ $P \wedge F \equiv F$	Domination Laws
$P \vee P \equiv P$ $P \wedge P \equiv P$	Idempotent Laws
$\neg(\neg P) \equiv P$	Double negation law
$P \vee q \equiv q \vee P$ $P \wedge q \equiv q \wedge P$	Commutative Laws
$(P \vee q) \vee r \equiv P \vee (q \vee r)$ $(P \wedge q) \wedge r \equiv P \wedge (q \wedge r)$	Associative Laws
$\neg(P \wedge q) \equiv \neg P \vee \neg q$ $\neg(P \vee q) \equiv \neg P \wedge \neg q$	De Morgan's Laws
$P \vee (P \wedge q) \equiv P$ $P \wedge (P \vee q) \equiv P$	Absorption Laws
$P \vee \neg P \equiv T$ $P \wedge \neg P \equiv F$	Negation Laws



Equivalence	Name.
$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$ $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$	Distributive Laws

\* 2nd

Logical Equivalences Involving Conditional statement " $\rightarrow$ "
$P \rightarrow Q \equiv \neg P \vee Q$ $P \rightarrow Q \equiv \neg Q \rightarrow \neg P$ $P \vee Q \equiv \neg P \rightarrow Q$ $P \wedge Q \equiv \neg(P \rightarrow \neg Q)$

\* 2nd

Logical Equivalence Involving Biconditional statement " $\leftrightarrow$ "
$P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$ $P \leftrightarrow Q \equiv \neg P \leftrightarrow \neg Q$ $P \leftrightarrow Q \equiv (P \wedge Q) \vee (\neg P \wedge \neg Q)$ $\neg(P \leftrightarrow Q) \equiv P \leftrightarrow \neg Q$

Ex: Show that  $\neg(P \vee (\neg P \wedge Q))$  and  $\neg P \wedge \neg Q$  are logically Equivalent.

$$\begin{aligned}
 \therefore \neg(P \vee (\neg P \wedge Q)) &\equiv \neg P \wedge \neg(\neg P \wedge Q) \quad \text{by the 2nd De Morgan Law.} \\
 &\equiv \neg P \wedge \neg(\neg P) \vee \neg Q \quad \text{// // 1st // // //} \\
 &\equiv \neg P \wedge (P \vee \neg Q) \quad \text{// // double negation Law.} \\
 &\equiv (\neg P \wedge P) \vee (\neg P \wedge \neg Q) \quad \text{// // second distributive Law.} \\
 &\equiv F \vee (\neg P \wedge \neg Q) \quad \text{// // negation Law.} \\
 &\equiv (\neg P \wedge \neg Q) \vee F \quad \text{// // commutative Law.} \\
 &\equiv \neg P \wedge \neg Q \quad \text{// // Identity Law.}
 \end{aligned}$$

$$\therefore \neg(P \vee (\neg P \wedge Q)) \equiv \neg P \wedge \neg Q.$$



Assignment	Announcement	Due
Two	Saturday, November 14, 2020 at 11:00 am	Thursday, November 26, 2020 at 11:59 pm

### Submission Instructions

Each student will write a combined report for their work that has the following:

1. You must submit one file with all answers in one report with a cover page that includes: course code, course name, academic year, semester, instructor, assignment #, student name, IDs and emails, etc.
2. The file must be named **GEN206-A2-Assiut-StudentID.docx** Where Assiut is an example of an EELU center name. Write the name of your center.

StudentID should be your EELU ID.

3. The file must be in either MS Word format or in PDF format.
4. Upload the file in the Assignment Solution Library on <http://moodlelms.eelu.edu.eg/>.
5. Make sure you test this process of uploading in advance before the deadline.

Answer the following questions:

- 1) Use truth tables to verify the absorption laws.  
a)  $p \vee (p \wedge q) \equiv p$       b)  $p \wedge (p \vee q) \equiv p$
- 2) Show that each of these conditional statements is a tautology by using truth tables.  
a)  $(p \wedge q) \rightarrow p$       b)  $p \rightarrow (p \vee q)$   
c)  $\neg p \rightarrow (p \rightarrow q)$       d)  $(p \wedge q) \rightarrow (p \rightarrow q)$
- 3) Let  $P(x)$  be the statement " $x$  spends more than five hours every weekday in class," where the domain for  $x$  consists of all students. Express each of these quantifications in English.  
a)  $\exists x P(x)$       b)  $\forall x P(x)$   
c)  $\exists x \neg P(x)$       d)  $\forall x \neg P(x)$
- 4) Translate these statements into English, where  $C(x)$  is " $x$  is a comedian" and  $F(x)$  is " $x$  is funny" and the domain consists of all people.  
a)  $\forall x (C(x) \rightarrow F(x))$       b)  $\forall x (C(x) \wedge F(x))$
- 5) Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.  
a) Someone in your class can speak Hindi.  
b) Everyone in your class is friendly.
- 6) Use a direct proof to show that the sum of two odd integers is even.
- 7) Show that if  $n$  is an integer and  $n^3 + 5$  is odd, then  $n$  is even using  
a) a proof by contraposition.  
b) a proof by contradiction.