Automata - Final - 2012-2013 - Model Answer

Please check the answer carefully; if you have a correction, please share with your friends

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Faculty of Computers and Information Technology	**Year:2012-2013 (Fall semester)**
Course Name: Automata Models	**Final Exam. (20-1-2013)**
Course code: CAS 205	**Time allowed: 3 hrs.**
Instructor: Dr. Azza Taha	**Marks: 50**

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Answer the following questions:

Question 1: Choose the correct answer: **(10 Marks)**

1. Find a correct ONTO function from f: A ⟶ A, where A = {a, b, c}
   a. f = { (a, b), (b, c), (c, c)}
   b. f = { (a, a), (b, c), (c, a)}
   $\boxed{\text{c. f = { (a, c), (b, a), (c, b)}}}$
   d. f = { (b, c), (a, c), (b, c)}
2. If |A| = 3 and |B| = 2, then |A x B| equals
   $\boxed{\text{a. 6}}$ b. 9 c. 4 d. 8
3. The function $f: R \longrightarrow R$, where $f(x) = 3x - 5$ is invertible.
   $\boxed{\text{a. True}}$ b. False
4. The function $f: R \longrightarrow R$, where $f(x) = x^2$ is bijective.
   a. True $\boxed{\text{b. False}}$
5. If L = {ab, bb}, which of the following strings is NOT in L^*:
   $\boxed{\text{a. aa}}$ b. ab c. abbb d. bb
6. Find L in {λ, a, ab}. L = {b, ab, ba, aba, abb, abba}
   $\boxed{\text{a. L = {b, ba}}}$ c. L = { ab, ba}
   b. L = {b, ab} d. L = {λ, b, ba}
7. Let G = < {D,S} , {0,1,2,…,9} , P , S > , where P is: $S \longrightarrow D \mid DS,\ D \longrightarrow 0 \mid 1 \mid \ldots \mid 9$, which of the following strings is NOT in L(G):
   a. 2123 $\boxed{\text{b. 23abb}}$ c. 10110 d. 2013

8. Let G = < {D,S} , {0,1,2,…,9} , P , S > , where P is:

$S \longrightarrow D1 \mid D3 \mid D5 \mid D7 \mid D9$

$D \longrightarrow \lambda \mid D0 \mid D1 \mid D2 \mid D3 \mid D4 \mid D5 \mid D7 \mid D8 \mid D9$

Which of the following strings is NOT in L(G):

a. 21 b. 22 c. 23 d. 27

9. Choose a *correct* grammar generating the language { λ, ab, abab, …}

a. $S \longrightarrow ab \mid ab\,S$
b. $S \longrightarrow \lambda \mid ab\,S$
c. $S \longrightarrow ab \mid b\,S\,a$
d. $S \longrightarrow ab\,S$

10. Let G = < {D,S} , {0,1,2,…,9} , P , S > , where P is: $S \longrightarrow D \mid DS,\ D \longrightarrow 0 \mid 1 \mid \ldots \mid 9$, then G is a regular grammar.

a. True b. False

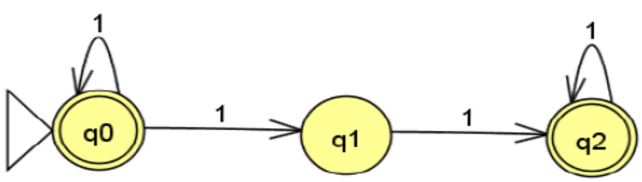
11. The complement of any finite language is a regular language.

a. True b. False

12. The language $\{a^n b^n : n \geq 0\}$ is context free language.

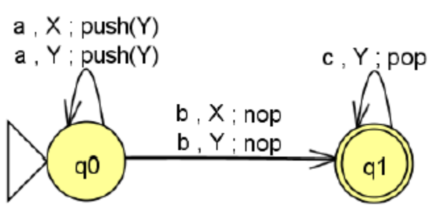
a. True b. False

13. The language accepted by the following automaton is:



a. $\{1^n : n \geq 0\}$
b. $\{1^n : n \geq 0, n \neq 1\}$
c. $\{1, 11, 111, 1111, \ldots\}$
d. $\{1^n : n \geq 0\}$

14. The language accepted by the following PDA is:



a. {abc}
b. $\{a^n b^n c^n : n \geq 0\}$
c. $\{a^n b\, c^n : n \geq 0\}$
d. $\{a^n b c^n : n \geq 1\}$

15. Regular Expressions are algebraic notations used to describe:

a. Any Language c. Context-Free Language
b. Regular Language d. Context-Sensitive Language

16. The simplest form of the regular expression **λ + ab + ab(ab)*** is:

   a. **ab(ab)***
   b. **λ + (ab)***
   c. $\boxed{\textbf{(ab)*}}$
   d. **ab + (ab)***

17. Choose a regular expression to describe the language {λ, a, aa, aaa, aaaa, aaaaa …}

   a. $\boxed{\textbf{a*}}$ b. **aa*** c. **(aa)*** d. **aaa***

18. Choose the correct language described by the regular expression **a*(a + b)**

   a. {λ, a, b, aa, ab,, aaa, aab, …}
   b. $\boxed{\{a, b, aa, ab, aaa, aab, \ldots\}}$
   c. {a, b, aa, ba, bb, ab, aaa, baa, …}
   d. { λ, a, b, aa, ba,, aaa, baa, …}

19. The Pumping lemma for regular languages is used to prove that a given infinite language is NOT regular.

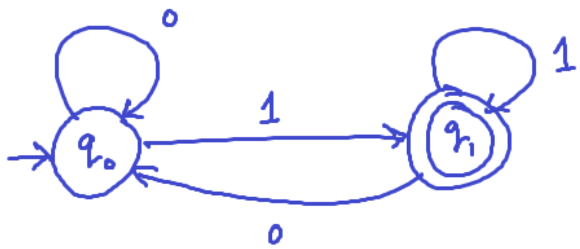
   a. $\boxed{\text{True}}$ b. False

20. Let G = < {A,S} , {a, b } , P , S > , where P consists is S ⟶ bAb, A⟶ aA|λ , then the language accepted by this grammar is L(G) = $\{ba^n b : n \geq 0\}$.

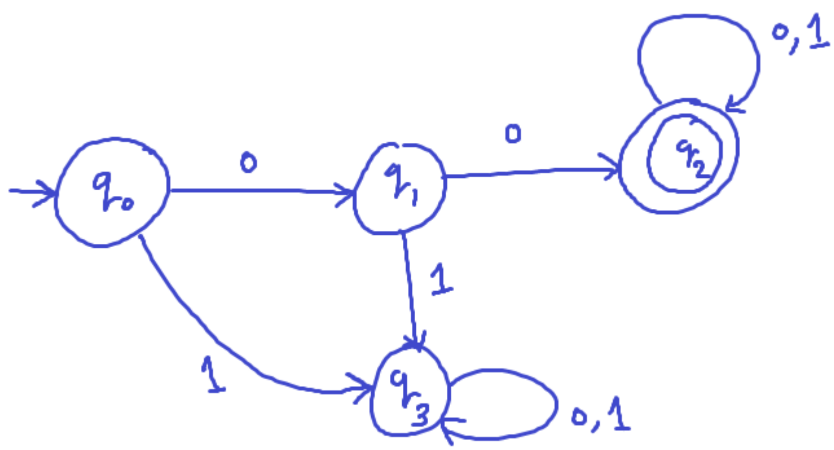
   a. $\boxed{\text{True}}$ b. False

Question 2: **(10 Marks)**

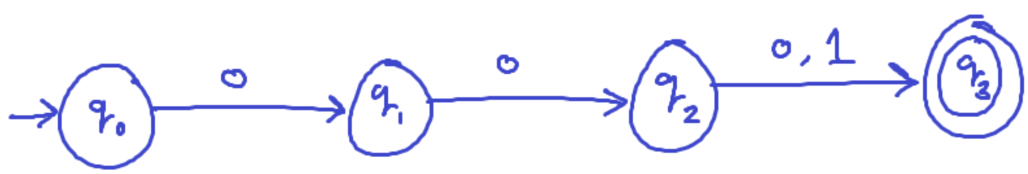
1) Construct the following machines over the alphabet {0, 1}:
   1. DFA that accepts the language of all strings ending 1.



   2. DFA that accepts the language L ={ $00w$: $w \in \{0,1\}^*$}



   3. NFA that accepts the language L = {001, 000}.

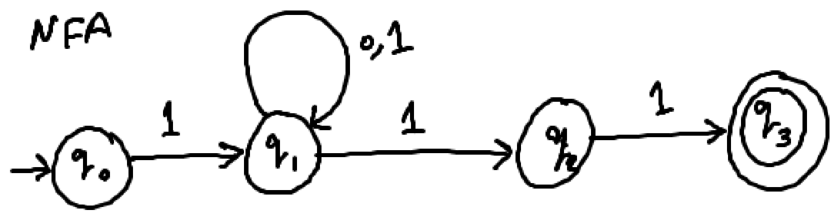


2) Show that the language $\{1w11 : w \in \{0,1\}^*\}$ is regular.

we represent it by R.E.

$$1(1+0)^*11$$

or NFA



3) Construct grammar for the set $L = \{c\,a^n\,b^n : n \geq 0\}$

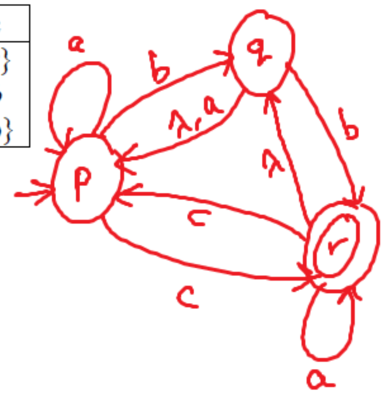
$G = \langle \{a,b,c\}, \{S, A\}, S, P \rangle$, where P:

$$S \longrightarrow cA$$
$$A \longrightarrow aAb \mid \lambda$$

Question 3:

1) Consider the NFA given by the following table: **(4 Marks)**

	λ	a	b	c
Start p	ϕ	{p}	{q}	{r}
q	{p}	{p}	{r}	ϕ
Final r	{q}	{r}	ϕ	{p}

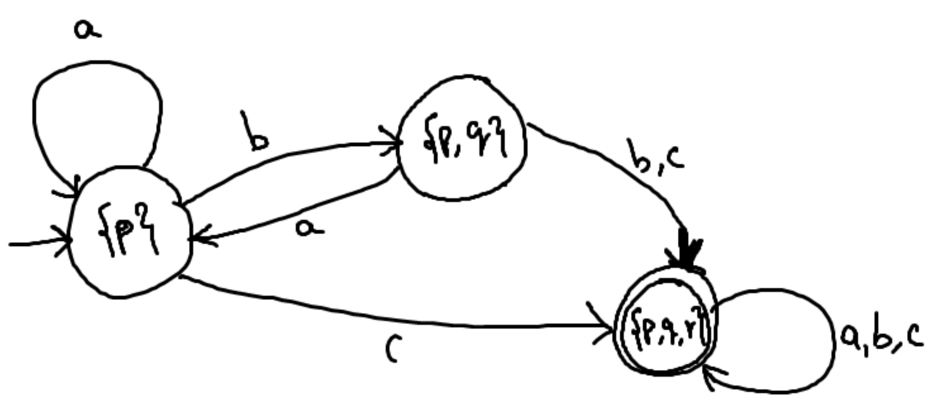


1. Find the lambda closure for all states.

state	lambda closure
p	{p}
q	{p,q}
r	{p,q,r}

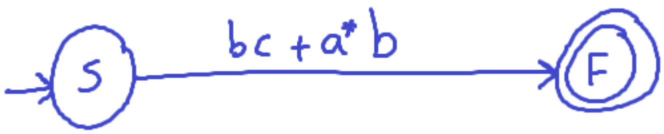
2. Convert the machine into DFA and draw the graph for the resulting machine.

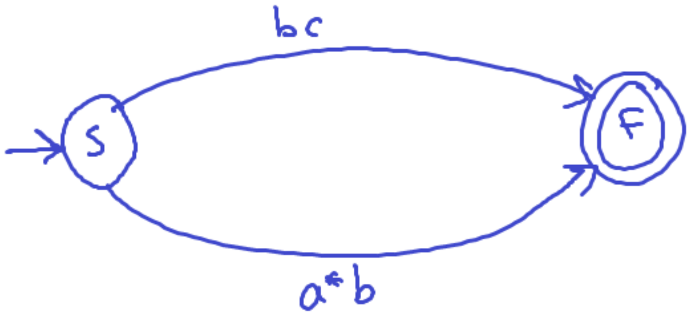
		a	b	c
Start	{p}	{p}	{p,q}	{p,q,r}
	{q}	{p}	{p,q,r}	{p,q,r}
Final	{r}	{p,q,r}	{p,q,r}	{p,q,r}
	{p,q}	{p}	{p,q,r}	{p,q,r}
Final	{p,q,r}	{p,q,r}	{p,q,r}	{p,q,r}

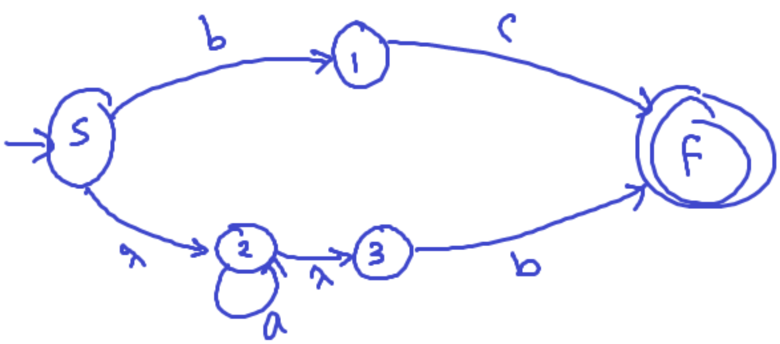


2) Construct NFA for the following regular expression using RE to FA algorithm: **(4 Marks)**

bc + a*b







3) Write regular expression to describe the language of all strings with exactly one *a* over the alphabet {a, b, c}. **(2 Marks)**

$(b+c)^* a (b+c)^*$

Question 4:

1) Show that the language $\{a^n b a^n : n \geq 0\}$ is NOT regular. **(3 Marks)**

By using Pumping lemmer :

(1) We assume that L is regular, and we can represent it by FA with # of state $= m$.

(2) we select $w \in L$, where
$w = a^n b a^n \; : \; n > m$

we write the string $w = xyz$, where $|xy| \leq m$, $|y| > 0$

$\therefore$ xy must contains a's
& y also must contains a's

$\therefore$ $xy^i z \in L$ for $i \geq 0$

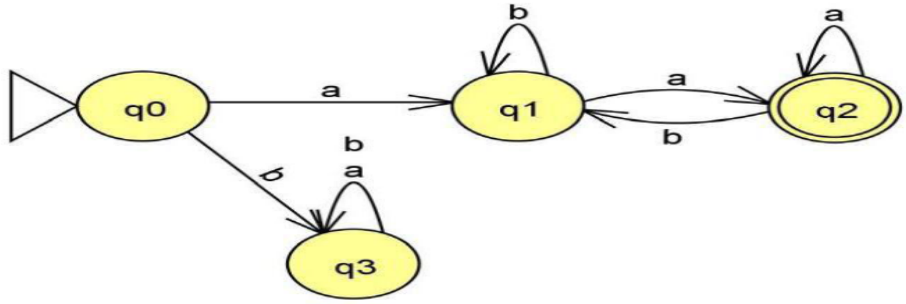
(3) If we let $i = 4$

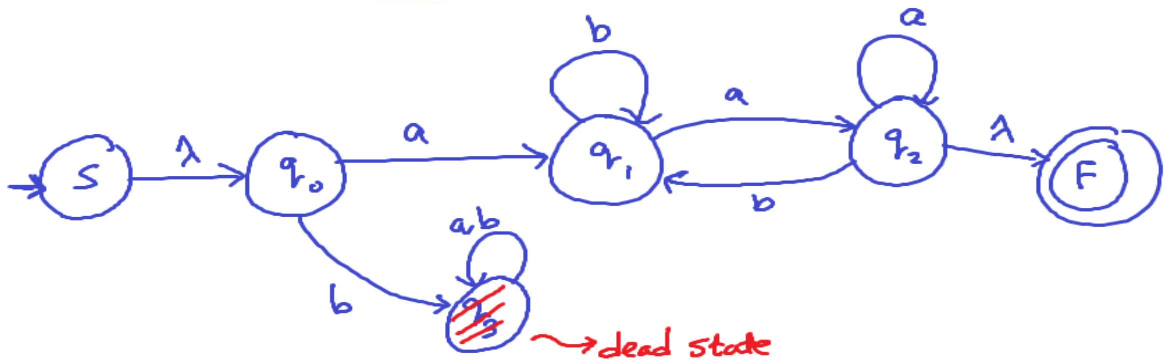
then $xy^4z \in L$, But for this Case the number of a's before the b is greater than the number of a's after the b.

$a^n b a^n$

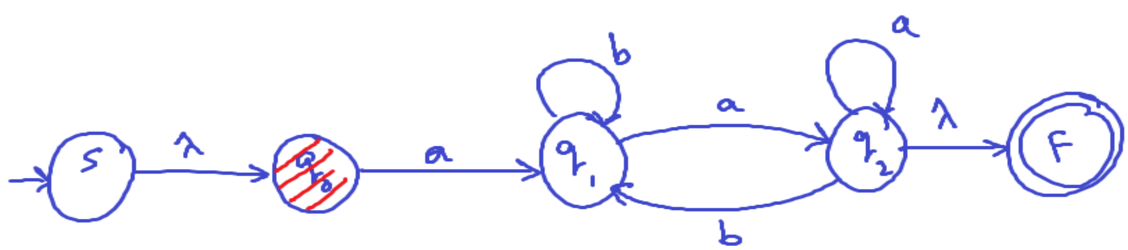
(4) by Contradiction
$\therefore$ L is Not regular langaye.

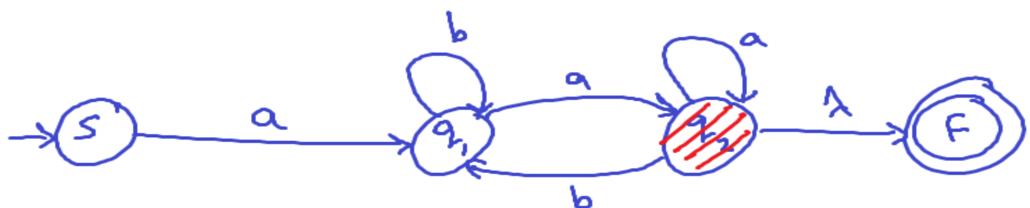
2) Step by step find a regular expression describes the language accepted by the following automaton using FA to RE algorithm: **(4 Marks)**

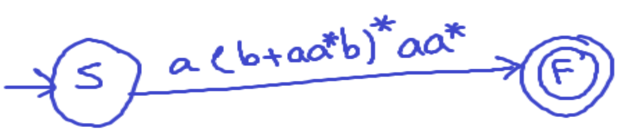




we remove the state q_3, because it's a dead state.





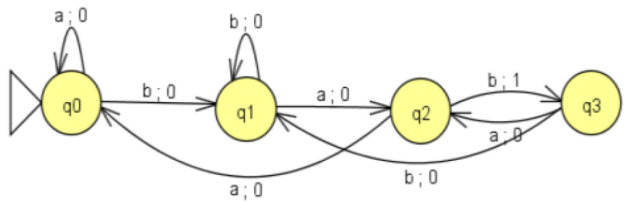


R.E. = $a(b+aa^*b)^*aa^*$

3) Consider the Mealy machine given by the following graph: **(3 Marks)**

a. Describe the parts of the machine.

b. Trace the output of this machine for the input *bababab*.



The Mealy machine Contains 6 parts :

1. $Q = \{q_0, q_1, q_2, q_3\}$ is the set of states.
2. $\Sigma = \{a, b\}$ is the set of alphabet.
3. $\Gamma = \{0, 1\}$ is the set of output symbols.
4. $q_0 \in Q$ is the start state.
5. δ_1 is the transition function:

$$\delta_1(q_0, a) = q_0 \qquad \delta_1(q_2, a) = q_0$$
$$\delta_1(q_0, b) = q_1 \qquad \delta_1(q_2, b) = q_3$$
$$\delta_1(q_1, a) = q_2 \qquad \delta_1(q_3, a) = q_2$$
$$\delta_1(q_1, b) = q_1 \qquad \delta_1(q_3, b) = q_1$$

6. δ_2 is the output function:

$$\delta_2(q_0, a) = 0 \qquad \delta_2(q_2, a) = 0$$
$$\delta_2(q_0, b) = 0 \qquad \delta_2(q_2, b) = 1$$
$$\delta_2(q_1, a) = 0 \qquad \delta_2(q_3, a) = 0$$
$$\delta_2(q_1, b) = 0 \qquad \delta_2(q_3, b) = 0$$

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b) $w =$ bababab $\Longrightarrow$ output is [0010101]

State	q_0	q_1	q_2	q_3	q_2	q_3	q_2
input	b	a	b	a	b	a	b
output	0	0	1	0	1	0	1

Question 5:

1) Show that the function f(n) = n − 2, n ≥ 2 is Turing computable where the number n is represented in unary form. **(3 Marks)**

$\lambda \longrightarrow 0$

$1 \longrightarrow 1$

$11 \longrightarrow 2$

$111 \longrightarrow 3$

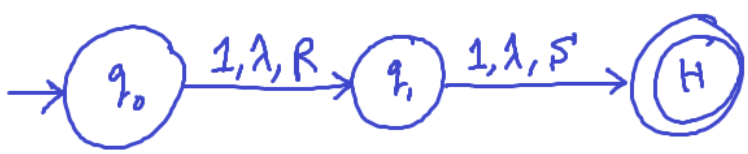
$\vdots$

we write a turing machine that take input $(n) \longrightarrow w$

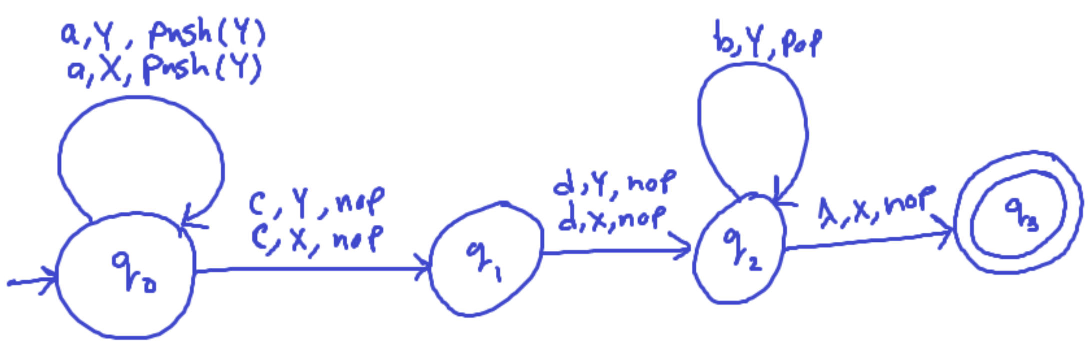
& output $(n-2) \longrightarrow w'$

i.e. $f(w) = w'$

$q_0 w \vdash^* H w'$



2) Construct PDA to describe the language $\{a^n\, c\, d\, b^n : n \geq 0\}$ **(4 Marks)**



3) Show that the language $\{ww^R : w \in \{0,1\}^*\}$ is context free. **(3 Marks)**

we represent this language by using a Context-free Grammar:

$G = \langle \{0,1\}, \{S\}, S, P \rangle$, where P:

$S \longrightarrow 0S0 \mid 1S1 \mid \lambda$