الجامعة المصرية
للتعلم الإلكتروني الأهلية

EELU
THE EGYPTIAN E-LEARNING UNIVERSITY

GEN206
Discrete Mathematics

Section09

Faculty of Information Technology
Egyptian E-Learning University

Fall 2021-2022

Q1] Represent each of these relations on {1, 2, 3} with a matrix (with the elements of this set listed in increasing order).

a) {(1, 1), (1, 2), (1, 3)}

b) {(1, 2), (2, 1), (2, 2), (3, 3)}

c) {(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)}

d) {(1, 3), (3, 1)}

Solution:

a) $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ *b)* $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ *d)* $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

Q2] List the ordered pairs in the relations on {1, 2, 3, 4} corresponding to these matrices (where the rows and columns correspond to the integers listed in increasing order).

a) $\begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}$ c) $\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$

Solution:

a) {(1,1), (1,2), (1,4), (2,1), (2,3), (3,2), (3,3), (3,4), (4,1), (4,3), (4,4)}
b) {(1,1), (1,2), (1,3), (2,2), (3,3), (3,4), (4,1), (4,4)}
c) {(1,1), (1,3), (2,2), (2,4), (3,2), (3,3), (4,1), (4,4)}

Q3] How many nonzero entries does the matrix representing the relation R on A = {1,2,3,...,100} consisting of the first 100 positive integers have if R is {(a, b) | a > b}?

Solution:

4950 $\begin{pmatrix} 0 & 0 & \dots & 0 \\ 1 & 0 & \dots & 0 \\ 1 & 1 & \dots & 0 \\ :: & :: & :: & 0 \\ 1 & 1 & \dots 1 & 0 \end{pmatrix}$

We can see that $1 \not> n \; where \; n \; is \; 1, \dots, 100$, so the first row is zeros

For the second one it will only have one 1 as 2 is only bigger than 1 the third will have two 1s as 3 is bigger than 1 and 2 if we follow this trend till hundred, we can see that these nonzero entries will add up to the following summation $\sum_{n=1}^{100} n - 1$

Q4] Let R1 and R2 be relations on a set A represented by the matrices

$$M_{R_1} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad , \quad M_{R_2} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Find the matrices that represent

a) $M_{R_1 \cup R_2}$
b) $M_{R_1 \cap R_2}$
c) $M_{\mathrm{R2} \circ \mathrm{R1}}$
d) $M_{\mathrm{R1} \circ \mathrm{R1}}$

Solution:

a) $M_{R_1} \vee M_{R_2} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

b) $M_{R_1} \wedge M_{R_2} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

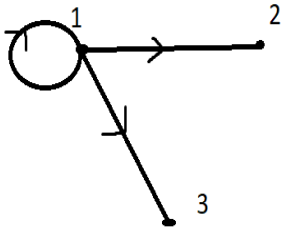
c) $M_{R_1} \circ M_{R_2} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

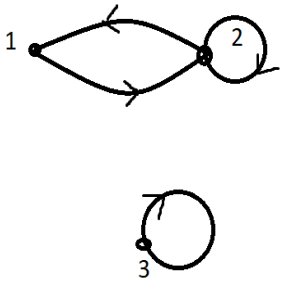
d) $M_{R_1} \circ M_{R_1} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

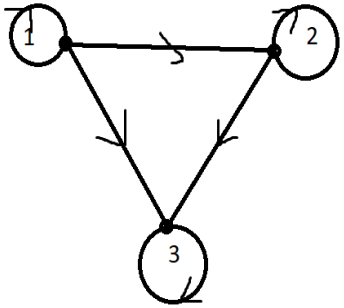
Q5]Draw the directed graphs representing each of the following relations

a) {(1, 1), (1, 2), (1, 3)}
b) {(1, 2), (2, 1), (2, 2), (3, 3)}
c) {(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)}

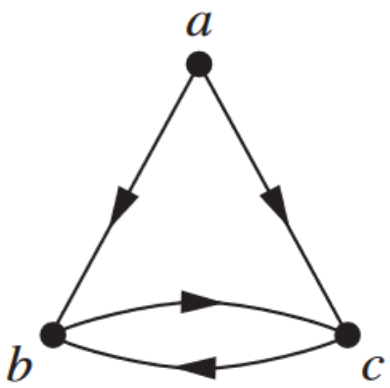
solution:



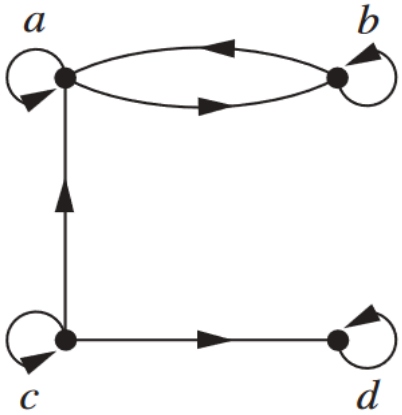




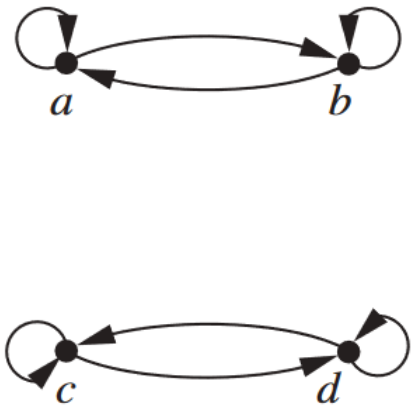
Q6] list the ordered pairs in the relations represented by the directed graphs.



{(a,b),(a,c),(b,c),(c,b)}



{(a,a),(a,b),(b,a),(b,b),(c,a),
(c,c),(c,d),(d,d)}

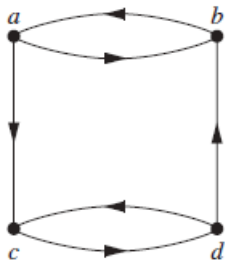


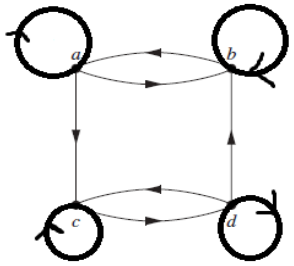
{(a,a),(a,b),(b,a),(b,b),
(c,c),(c,d),(d,c),(d,d)}

Q7] Let *R* be the relation on the set {0, 1, 2, 3} containing the ordered pairs (0, 1), (1, 1), (1, 2), (2, 0), (2, 2), and (3, 0).Find the
a) reflexive closure of *R*. b) symmetric closure of *R*.

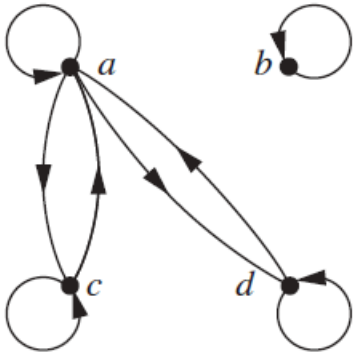
Solution:
a) {(0,0), (0, 1), (1, 1), (1, 2), (2, 0), (2, 2),(3,0),(3,3)}
b) {(0, 1), (0,2) ,(0,3), (1,0), (1, 1), (1, 2), (2, 0), (2,1), (2, 2),(3,0)}

Q9] Draw the directed graph of the reflexive closure of the relations with the directed graph shown.

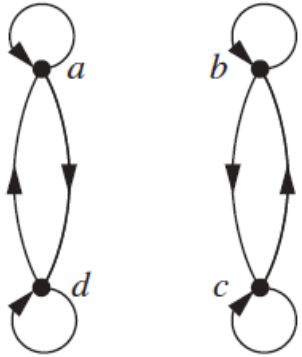




Q8] determine whether the relation with the directed graph shown is an equivalence relation.



This is not an equivalence relation as it's not transitive we have (d,a),(a,c) but we don't have (d,c)



This is an equivalence relation

Thank You