Electronics
Section 06

Faculty of Information Technology Egyptian E-Learning University

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Capacitors

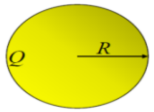
A **capacitor** consists of two conducting plates separated by an insulator (or dielectric).

Capacitance is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads (F).

$$\boxed{q = Cv}$$

Calculation of capacitance

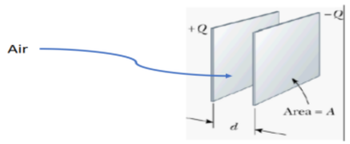
Isolated charged sphere



$$C_0 = \frac{R}{k_e} = 4\pi\varepsilon_0 R$$

$$k_e = \frac{1}{4\pi\varepsilon_0}$$

Parallel Plates



$$C_0 = \frac{\varepsilon_0 A}{d}$$

$\varepsilon_0 = 8.854187817.. \times 10^{-12}$ F/m

Capacitors with any dielectric

$$C = kC_o$$

K is a dimensionless factor , called the **dielectric constant** where C_0 is the capacitance in the absence of the dielectric

A capacitor is an open circuit to dc.

The voltage on a capacitor cannot change abruptly.

$$\boxed{i = C\frac{dv}{dt}}$$

The equivalent capacitance of N parallel-connected capacitors is the sum of the individual capacitances.

$$\boxed{C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N}$$

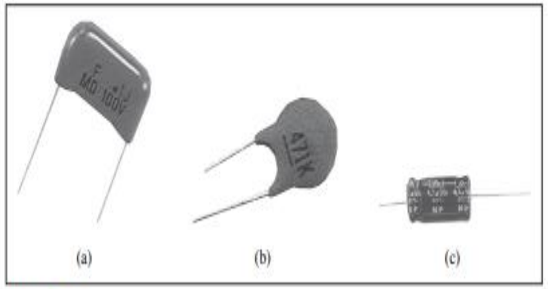


Figure
Fixed capacitors: (a) polyester capacitor, (b) ceramic capacitor, (c) electrolytic capacitor. Courtesy of Tech America.

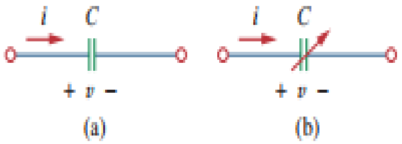


Figure
Circuit symbols for capacitors: (a) fixed capacitor, (b) variable capacitor.

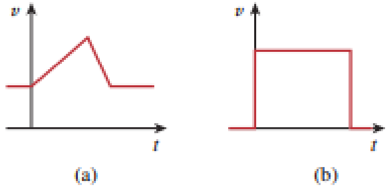


Figure
Voltage across a capacitor: (a) allowed, (b) not allowable; an abrupt change is not possible.

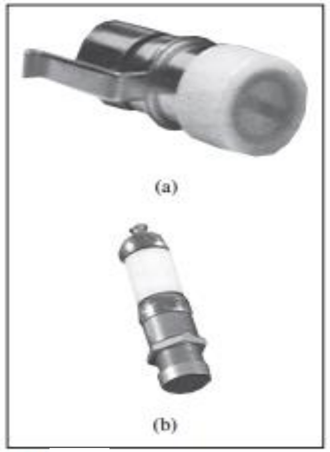


Figure
Variable capacitors: (a) trimmer capacitor, (b) filmtrim capacitor. Courtesy of Johanson.

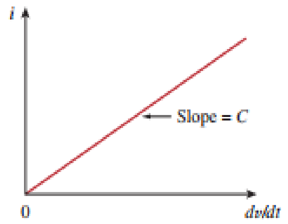


Figure
Current-voltage relationship of a capacitor.

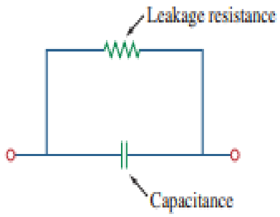


Figure
Circuit model of a nonideal capacitor.

The equivalent capacitance of series-connected capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\boxed{C_{eq} = \frac{C_1 C_2}{C_1 + C_2}}$$

Energy stored in charged capacitor

$$U_E = \frac{Q^2}{2C} = \tfrac{1}{2}Q\Delta V = \tfrac{1}{2}C(\Delta V)^2$$

Inductors

An **inductor** consists of a coil of conducting wire.

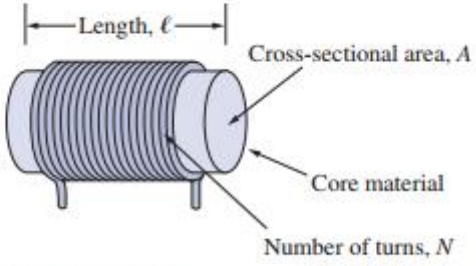


Figure 6.21
Typical form of an inductor.

L = Inductance of coil in Henrys

N = Number of turns in wire coil (straight wire = 1)

μ = Permeability of core material (absolute, not relative)

μ_r = Relative permeability, dimensionless (μ_0 = 1 for air)

μ_0 = 1.26×10^{-6} T-m/A permeability of free space

A = Area of coil in square meters = πr^2

l = Average length of coil in meters

$$L = \frac{N^2 \mu A}{\ell} \qquad \mu = \mu_r \mu_0$$

An inductor acts like a short circuit to dc.

The current through an inductor cannot change instantaneously.

$$L = \frac{\Phi}{i} = \frac{vt}{i}$$

$$\boxed{v = L\frac{di}{dt}}$$

$$\boxed{w = \frac{1}{2}Li^2}$$

The **equivalent inductance** of parallel inductors is the reciprocal of the sum of the reciprocals of the individual inductances.

$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}}$$

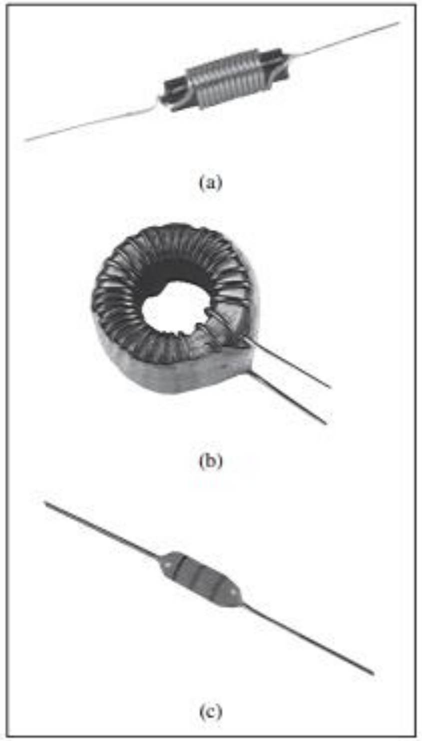


Figure
Various types of inductors: (a) solenoidal wound inductor, (b) toroidal inductor, (c) chip inductor.
Courtesy of Tech America.

Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).

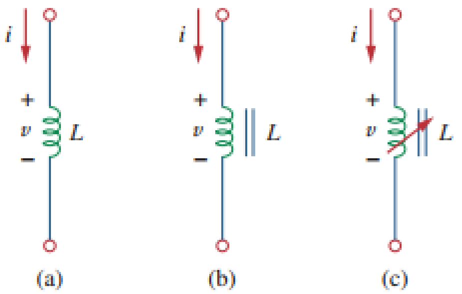


Figure
Circuit symbols for inductors: (a) air-core, (b) iron-core, (c) variable iron-core.

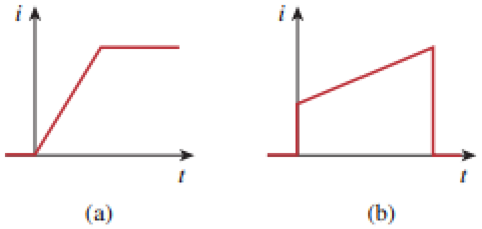


Figure
Current through an inductor: (a) allowed, (b) not allowable; an abrupt change is not possible.

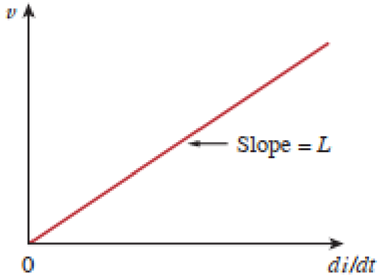


Figure
Voltage-current relationship of an inductor.

Since an inductor is often made of a highly conducting wire, it has a very small resistance.

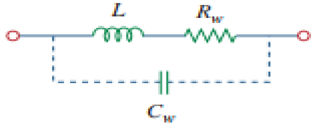


Figure
Circuit model for a practical inductor.

The **equivalent inductance** of series-connected inductors is the sum of the individual inductances.

$$\boxed{L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N}$$

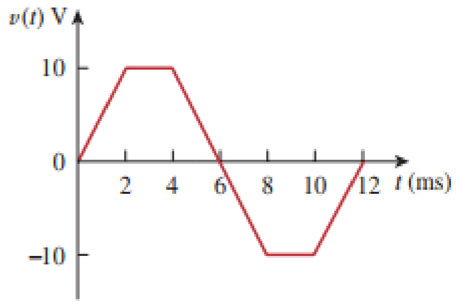
Q1. **If the voltage across a 7.5-F capacitor is $2t\,e^{-3t}$ V, find the current and the power.**

Solution

$$i = C\frac{dv}{dt} = 7.5\left(2e^{-3t} - 6te^{-3t}\right) = \mathbf{15(1 - 3t)e^{-3t}\ A}$$

$$p = vi = 15(1-3t)e^{-3t} \cdot 2t\,e^{-3t} = \mathbf{30t(1 - 3t)e^{-6t}\ W}.$$

Q2. The voltage waveform in the Figure is applied across a **55-µF** capacitor. Draw the current waveform through it.



Solution

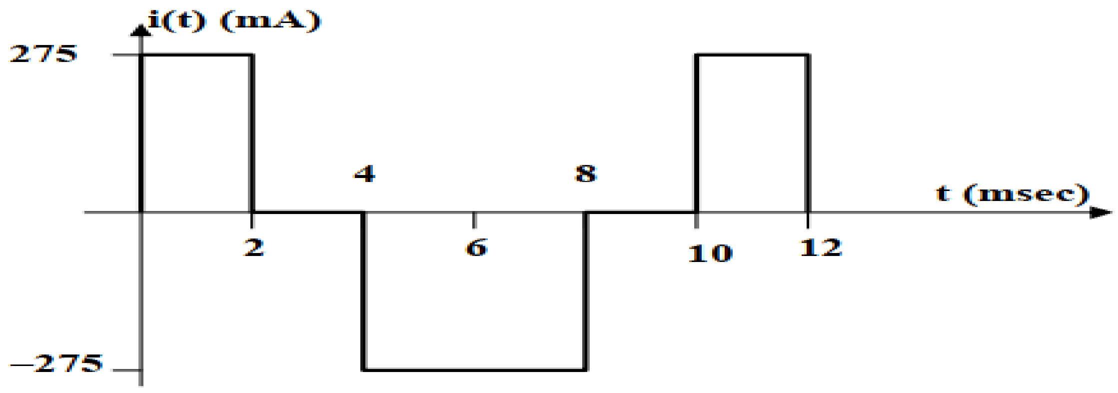
$i = C\frac{dv}{dt} = 55x10^{-6}$ times the slope of the waveform.

For example, for $0 < t < 2$,

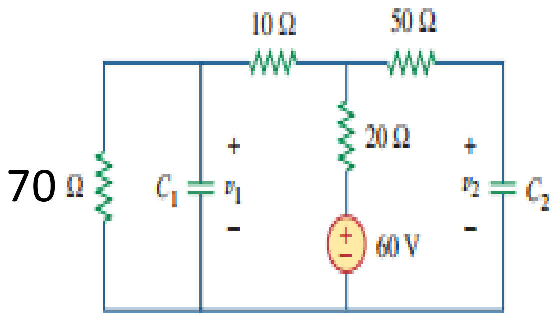
$$\frac{dv}{dt} = \frac{10}{2x10^{-3}}$$

$$i = C\frac{dv}{dt} = (55x10^{-6})\frac{10}{2x10^{-3}} = 275mA$$

Thus the current i(t) is sketched below.

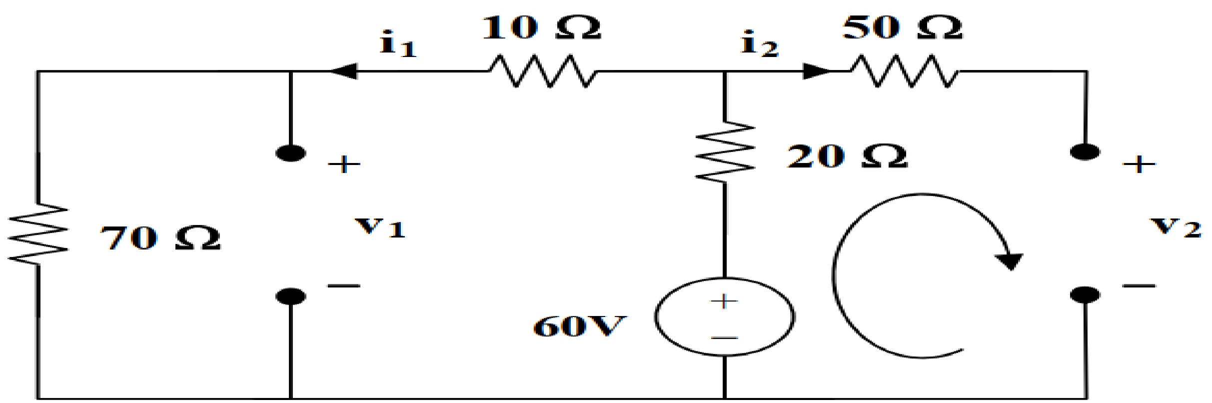


Q3. **Find the voltage across the capacitors in the circuit of the Figure under dc conditions.**



Solution

Under dc conditions, the circuit becomes that shown below:



$i_2 = 0$, $i_1 = 60/(70+10+20) = 0.6$ A

$v_1 = 70i_1 = 42$ V, $v_2 = 60–20i_1 = 48$ V

Thus, $v_1 =$ **42 V**, $v_2 =$ **48 V**.

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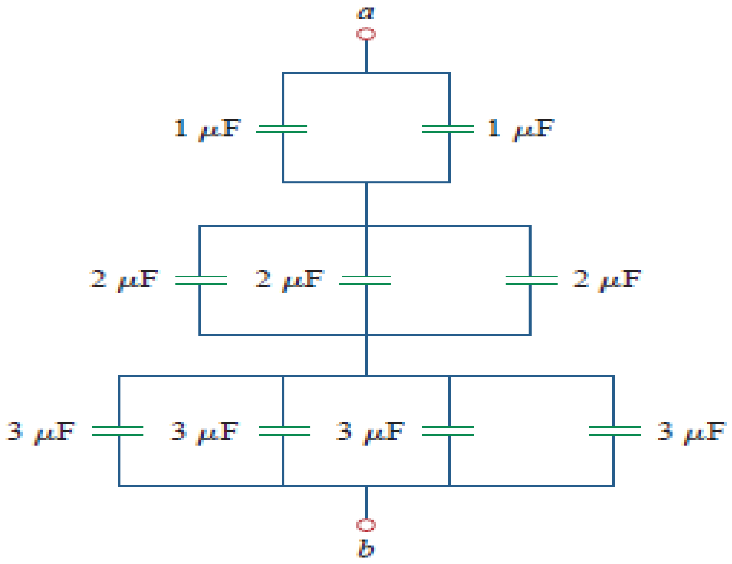
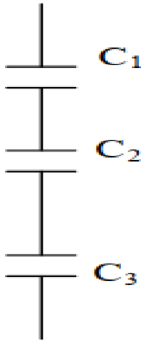
Q4. Find the equivalent capacitance at terminals a-b of the circuit in the figure.



Solution

Consider the circuit shown below.



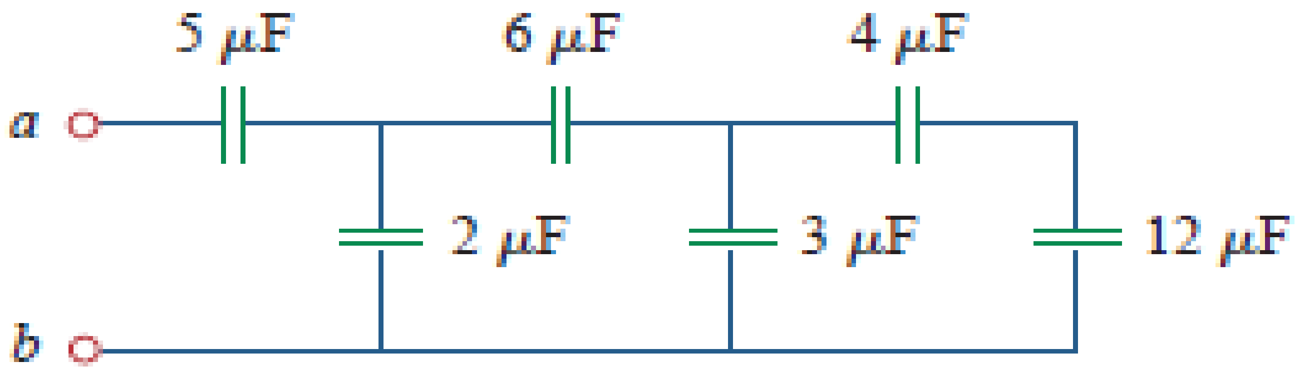
$$C_1 = 1 + 1 = 2\mu F$$
$$C_2 = 2 + 2 + 2 = 6\mu F$$

$$C_3 = 4x3 = 12\mu F$$

$$1/C_{eq} = (1/C_1) + (1/C_2) + (1/C_3) = 0.5 + 0.16667 + 0.08333 = 0.75x10^6$$

$$C_{eq} = \mathbf{1.3333\ \mu F}.$$

Q5. Determine the equivalent capacitance at terminals a – b of the circuit in the Figure.



Solution

4μF in series with 12μF = (4x12)/16 = 3μF
3μF in parallel with 3μF = 6μF
6μF in series with 6μF = 3μF
3μF in parallel with 2μF = 5μF
5μF in series with 5μF = 2.5μF

Hence C_{eq} = **2.5μF**

Q6. **The current through a 10-mH inductor is 10 e $^{-t/2}$ A. Find the voltage and the power at t = 3 s**

Solution

$$i = 10e^{-t/2}$$

$$v = L\frac{di}{dt} = 10x10^{-3}(10)\left(\frac{1}{2}\right)e^{-t/2}$$

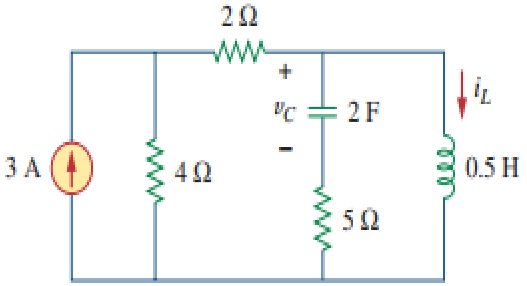
$$= -50e^{-t/2}\ \text{mV}$$

$$v(3) = -50e^{-3/2}\ \text{mV} = \mathbf{-11.157\ mV}$$

$$p = vi = -500e^{-t}\ \text{mW}$$

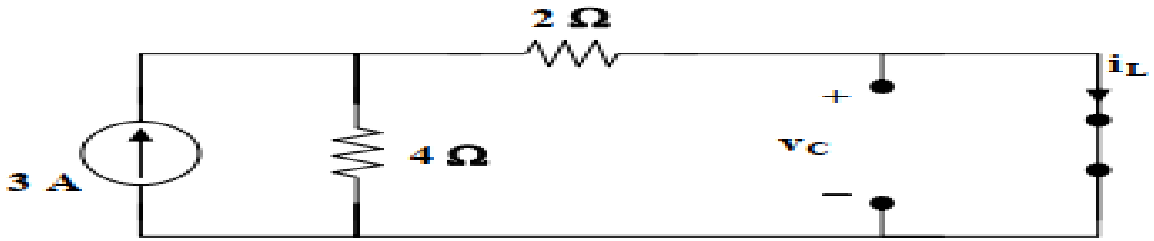
$$p(3) = -500e^{-3}\ \text{mW} = \mathbf{-24.89\ mW}.$$

Q7. **Find v_C, I_L and the energy stored in the capacitor and inductor in the circuit of the Figure under dc conditions.**



Solution

Under dc conditions, the circuit is as shown below:



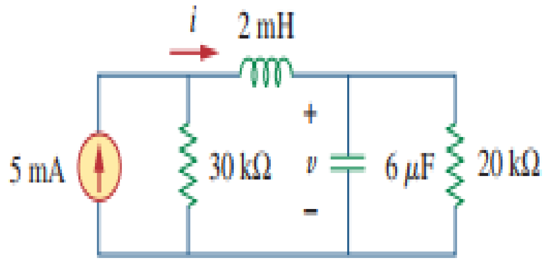
By current division,

$$i_L = \frac{4}{4+2}(3) = 2A, \quad v_c = 0V$$

$$w_L = \frac{1}{2} L\, i_L^2 = \frac{1}{2}\left(\frac{1}{2}\right)(2)^2 = 1J$$

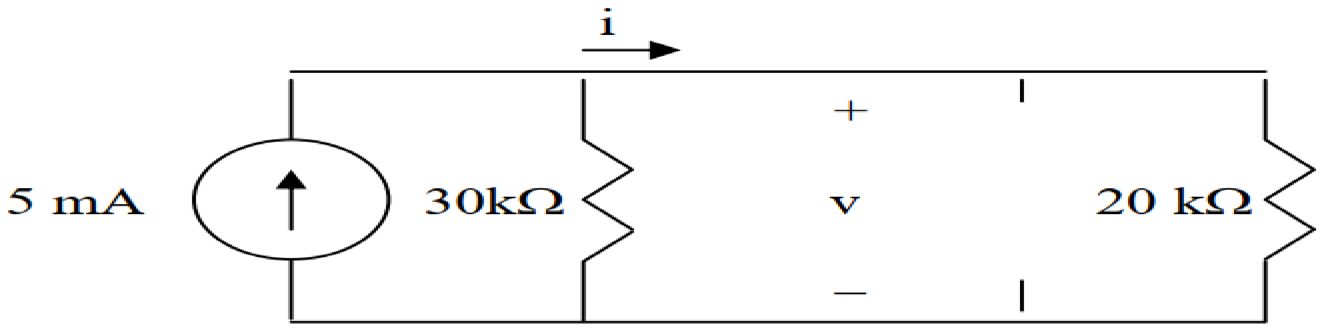
$$w_c = \frac{1}{2} C\, v_c^2 = \frac{1}{2}(2)(v) = 0J$$

Q8. Under steady-state dc conditions, find *i* and v in the circuit in Figure.



Solution

Under steady-state, the inductor acts like a short-circuit, while the capacitor acts like an open circuit as shown below.



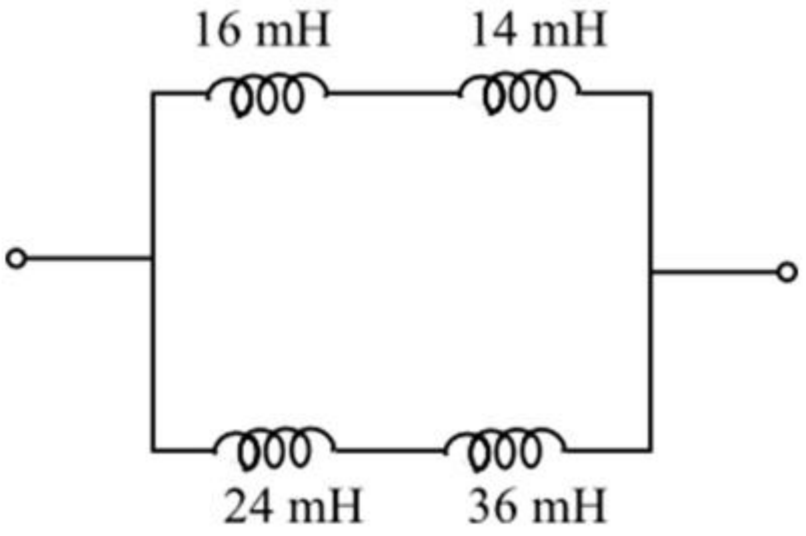
Using current division,

$$i = (30k/(30k+20k))(5mA) = \mathbf{3\ mA}$$

$$v = 20ki \quad = \mathbf{60\ V}$$

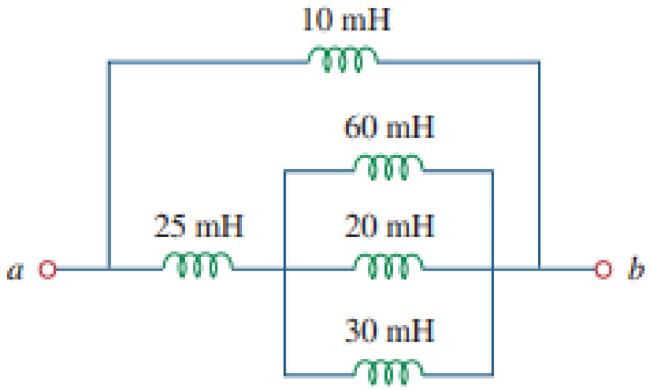
Q9. An energy-storage network consists of series- connected 16-mH and 14mH inductors; in parallel with series-connected 24-mH and 36-mH inductors. Calculate the equivalent inductance.

Solution



16mH in series with 14 mH = 16+14=30 mH
24 mH in series with 36 mH = 24+36=60 mH
30mH in parallel with 60 mH = 30x60/90 = **20 mH**

Q10. **Determine L_{eq} *equivalent* at terminals a-b of the circuit in the figure.**



Solution

$$\frac{1}{L} = \frac{1}{60} + \frac{1}{20} + \frac{1}{30} = \frac{1}{10}$$

$$L = 10 \text{ mH}$$

$$L_{eq} = 10 \| (25 + 10) = \frac{10x35}{45}$$

$$= \mathbf{7.778\ mH}$$

MCQ

1) By what factor is the capacitance of a metal sphere multiplied if its volume is tripled?

(a) 3

(b) $3^{1/3}$

(c) 1

(d) $3^{-\frac{1}{3}}$

(e) $\frac{1}{3}$

Answer: B