الجامعة المصرية
للتعلم الإلكتروني الأهلية

EELU
THE EGYPTIAN E-LEARNING UNIVERSITY

GEN206
Discrete Mathematics

Section 8

Faculty of Information Technology
Egyptian E-Learning University

Fall 2021-2022

1)

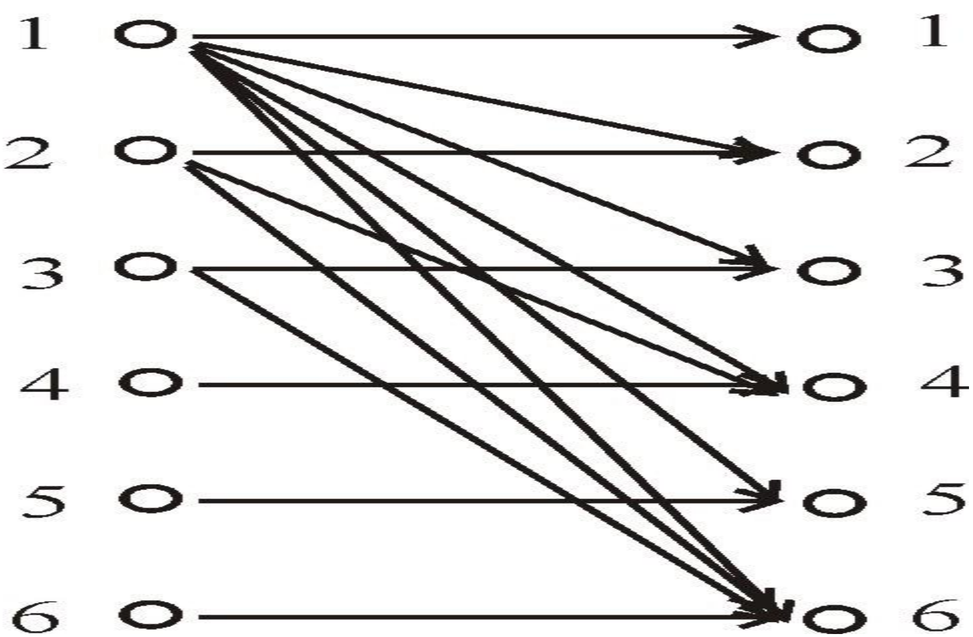
a) List all the ordered pairs in the relation $R = \{(a, b) \mid a \text{ divides } b\}$ on the set $\{1, 2, 3, 4, 5, 6\}$.

b) Display this relation graphically,

c) Display this relation in tabular form.

Answer

R={(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,2),(2,4),(2,6),(3,3),(3,6),(4,4),(5,5),(6,6)}



R	1	2	3	4	5	6
1	x	x	x	x	x	x
2		x		x		x
3			x			x
4				x		
5					x	
6						x

3. For each of these relations on the set $\{1, 2, 3, 4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

a) $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$

b) $\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$

c) $\{(2, 4), (4, 2)\}$

d) $\{(1, 2), (2, 3), (3, 4)\}$

e) $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$

f) $\{(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)\}$

(a) Transitive
(b) Reflexive, Symmetric and Transitive
(c) Symmetric
(d) Antisymmetric
(e) Reflexive, Symmetric, Antisymmetric and Transitive
(f) None

6) How many relations are there on a set {a,b,c} ?

a) 432
b) 512
c) 234
d) 290

Solution: b) 512

A relation on a set A is a subset of $A \times A$. Because $A \times A$ has n^2 elements when A has n elements, and a set with m elements has 2^n subsets, there are 2^{n2} subsets of $A \times A$. Thus,

There are 2^{n2} relations on a set with n elements. For example, there are $2^{32}= 2^9 =$ 512 relations on
the set $\{a, b, c\}$

7) Is the "divides" relation on the set of positive integers reflexive?

a) True

b) False

Solution: b) true

Because $a \mid a$ whenever a is a positive integer, the "divides" relation is reflexive. (Note that if we replace the set of positive integers with the set of all integers the relation is not reflexive because by definition 0 does not divide 0.)

8) Is the “divides” relation on the set of positive integers symmetric? Is it antisymmetric?

a) True

b) False

Solution: b) False

This relation is not symmetric because $1|2$, but $2 \nmid 1$.However, it is antisymmetric. To see this, note that if *a* and *b* are positive integers with $a|b$ and $b|a$, then $a = b$

1. List the ordered pairs in the relation R from $A = \{0, 1, 2, 3, 4\}$ to $B = \{0, 1, 2, 3\}$, where $(a, b) \in R$ if and only if

a) $a = b$.
b) $a + b = 4$.
c) $a > b$.
d) $a \mid b$.
e) $\gcd(a, b) = 1$.
f) $\operatorname{lcm}(a, b) = 2$.

(a) $R = \{(0,0), (1,1), (2,2), (3,3)\}$
(b) $R = \{(1,3), (2,2), (3,1), (4,0)\}$
(c) $R = \{(1,0), (2,0), (3,0), (4,0), (2,1), (3,1), (4,1), (3,2), (4,2), (4,3)\}$
(d) $R = \{(1,0), (2,0), (3,0), (4,0), (1,1), (1,2), (2,2), (1,3), (3,3)\}$
(e) $R = \{(1,0), (0,1), (1,1), (1,2), (1,3), (2,1), (3,1), (4,1), (2,3), (3,2), (4,3)\}$
(f) $R = \{(1,2), (2,1), (2,2)\}$

9) Let *R*
1 = {(1, 2), (2, 3), (3, 4)} and *R*2 = {(1, 1), (1, 2),
(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4)} be relations
from {1, 2, 3} to {1, 2, 3, 4}. Find

a) $R1 \cup R2$.
b) $R1 \cap R2$.
c) $R1 - R2$.
d) $R2 - R1$
e) $R1 \oplus R2$

a) R1 U R2={(1,2), (2, 3), (3, 4),(1, 1),(2, 1), (2, 2), (3, 1), (3, 2), (3, 3)}

b) R1 ∩ R2={(1,2), (2, 3), (3, 4)}

c) R1 - R2= $\emptyset$,

d) R2 –R1={(1, 1),(2, 1), (2, 2), (3, 1), (3, 2), (3, 3)}

e) $R1 \oplus R2$ = R1 U R2 - R1 ∩ R2={(1, 1),(2, 1), (2, 2), (3, 1), (3, 2), (3, 3)}

11 Let $R = \{(1,1),(2,1),(3,2),(4,3)\}$.

Find the powers R^n, $n = 2,3,4,\ldots$

$$R^2 = R \circ R = \{(1,1),(2,1),(3,1),(4,2)\}$$
$$R^3 = R^2 \circ R = \{(1,1),(2,1),(3,1),(4,1)\}$$
$$R^4 = R^3 \circ R = \{(1,1),(2,1),(3,1),(4,1)\}$$

R^4 is the same R^3, it is also follow that $R^n = R^3$, $n = 5,6,7,\ldots$

Thank You