MATHEMATICS (1)

Section (3)

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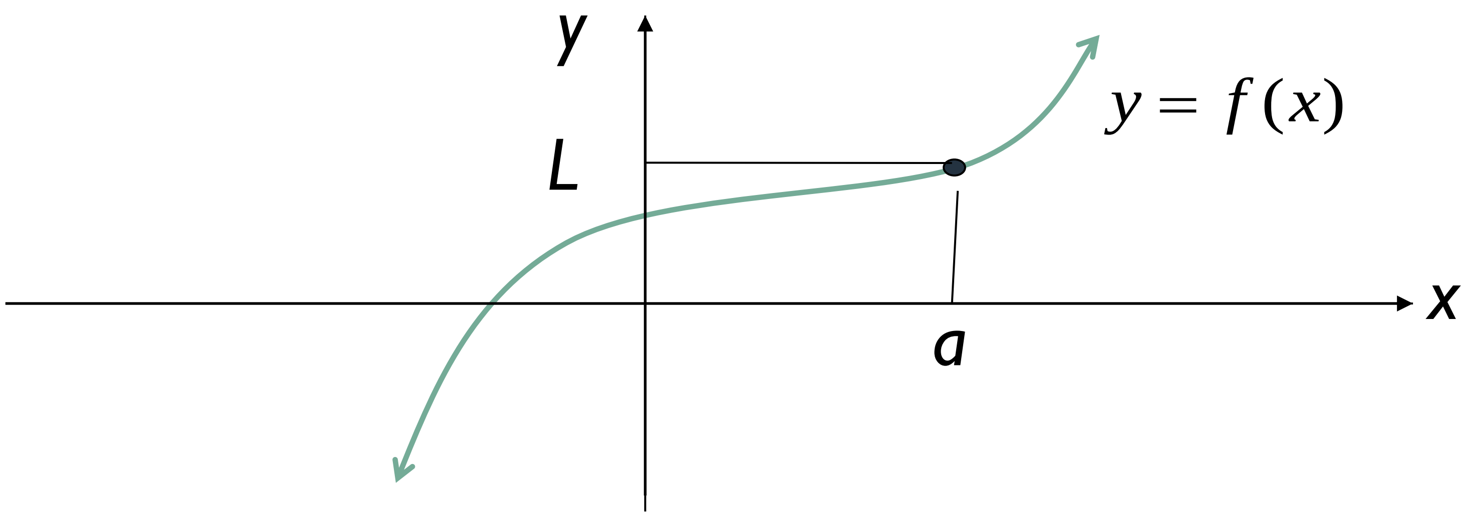
Limit of a Function

Definition

The *limit* of $f(x)$, as x tends to a, equals L

written: $$\lim_{x \to a} f(x) = L$$

if we can make the value $f(x)$ arbitrarily close to L by taking x to be sufficiently close to a.



One sided Limits

Left hand limit

The *left hand limit* of $f(x)$, as x tends to a, equals L_1

written: $$\lim_{x \to a^-} f(x) = L_1$$

if we can make the value $f(x)$ arbitrarily close to L_1 by taking x to be sufficiently close to the left of a.

Right hand limit

The *right hand limit* of $f(x)$, as x tends to a, equals L_2

written: $$\lim_{x \to a^+} f(x) = L_2$$

if we can make the value $f(x)$ arbitrarily close to L_2 by taking x to be sufficiently close to the right of a.

Remark

$$\lim_{x \to a} f(x) = L \quad \text{if and only if} \quad \lim_{x \to a^-} f(x) = L = \lim_{x \to a^+} f(x)$$

Limit properties

$$1. \lim_{x \to a} c = c$$

$$2. \lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)$$ (Scalar Multiple Law)

$$3. \lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$ (Sum Law)

$$4. \lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$$ (Difference Law)

$$5. \lim_{x \to a} [f(x)g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$$ (Product Law)

$$6.\ \lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} \quad \text{if } \lim_{x\to a} g(x)\neq 0$$ (quotient law)

$$7.\ \lim_{x\to a}\left[f(x)\right]^n=\left[\lim_{x\to a} f(x)\right]^n \quad \textit{where n is a positive integer}$$ (composite power law)

$$8.\ \lim_{x\to a}\sqrt[n]{f(x)}=\sqrt[n]{\lim_{x\to a} f(x)}$$ (composite nth root law)

9. If f is a polynomial or a rational function

and a is in the domain of f, then

$$\lim_{x\to a} f(x)=f(a)$$

Example 1

Evaluate $\lim_{x\to 1}(3x^2 + x + 1 - x^4)$.

Solution:

Note that $3x^2 + x + 1 - x^4$ is a polynomial and hence

$$\lim_{x\to 1}(3x^2 + x + 1 - x^4) = 3(1)^2 + 1 + 1 - (1)^4 = 3 + 2 - 1 = 4.$$

Example 2

Evaluate $\lim_{x\to 7}\sqrt[3]{-x^2 - 15}$

Solution:

Note that $\lim_{x\to 7}(-x^2 - 15) = -(7)^2 - 15 = -49 - 15 = -64$. This implies, by using the composite nth law, that $\lim_{x\to 7}\sqrt[3]{-x^2 - 15} = \sqrt[3]{-64} = -4$.

Example 3

Evaluate each of the following limits.

1. $\lim_{x \to 1} (3x - 2)^{10}$

2. $\lim_{x \to 2} \left(\frac{12}{x + x^2} \right)^3$

Solution

1. Note that $\lim_{x \to 1} (3x - 2) = 3 - 2 = 1$. By the composite power law, we get that $\lim_{x \to 1} (3x - 2)^{10} = (1)^{10} = 1.$

2. Note that $\lim_{x \to 2} \frac{12}{x + x^2} = \frac{12}{2 + 4} = \frac{12}{6} = 2$. By the composite power law, we get that $\lim_{x \to 2} \left(\frac{12}{x + x^2} \right)^3 = (2)^3 = 8.$

Example 4

Evaluate $\lim_{x \to -3} \sqrt{\frac{x^2 + 2x + 1}{8 + 2x}}$.

Solution:

Note that $\lim_{x \to -3} (x^2 + 2x + 1) = 4$ and $\lim_{x \to -3} (8 + 2x) = 2$. This implies, by using the composite nth root law and the quotient law, that

$$\lim_{x \to -3} \sqrt{\frac{x^2 + 2x + 1}{8 + 2x}} = \sqrt{\frac{4}{2}} = \sqrt{2}.$$

Example 5

Evaluate each of the following limits.

1. $\lim_{x\to 0}(x^3 + 2x - 3 + \sqrt{x+1}\,)$
2. $\lim_{x\to 1}(x^3 + 1)(x - x^2 + 2)(4x + 1)$

Solution:

1. Note that $\lim_{x\to 0} x^3 = 0^3 = 0$, $\lim_{x\to 0} 2x = 2(0) = 0$, $\lim_{x\to 0} 3 = 3$ and $\lim_{x\to 0}\sqrt{x+1} = \sqrt{0+1} = \sqrt{1} = 1$. Therefore,

$$\lim_{x\to 0}(x^3 + 2x - 3 + \sqrt{x+1}\,) = 0 + 0 - 3 + 1 = -2.$$

2. Note that $\lim_{x\to 1}(x^3 + 1) = 2$, $\lim_{x\to 1}(x - x^2 + 2) = 2$ and $\lim_{x\to 1}(4x + 1) = 5$. Therefore,

$$\lim_{x\to 1}(x^3 + 1)(x - x^2 + 2)(4x + 1) = (2)(2)(5) = 20\ .$$

Example 6

Evaluate $\lim\limits_{x\to 2}\dfrac{x^2-4}{x^2+x-6}$.

Solution: : Let $f(x)=x^2-4$ and $g(x)=x^2+x-6$. Since $\lim\limits_{x\to 2}f(x)=f(2)=0$ and $\lim\limits_{x\to 2}g(x)=g(2)=0$, we have to factorize both $f(x)$ and $g(x)$ and cancel out the common factors. Doing so, we get that

$$\lim_{x\to 2}\frac{x^2-4}{x^2+x-6}=\lim_{x\to 2}\frac{(x-2)(x+2)}{(x-2)(x+3)}=\lim_{x\to 2}\frac{x+2}{x+3}.$$

Now, if $f_1(x)=x+2$ and $g_1(x)=x+3$ then $\lim\limits_{x\to 2}f_1(x)=f_1(2)=4$ and $\lim\limits_{x\to 2}g_1(x)=g_1(2)=5$. This implies that $\lim\limits_{x\to 2}\dfrac{x^2-4}{x^2+x-6}=\lim\limits_{x\to 2}\dfrac{x+2}{x+3}=\dfrac{4}{5}$.

Example 7

Evaluate $\lim\limits_{x\to -2}\dfrac{x^3+8}{x^2-4}$.

Solution: Let $f(x)=x^3+8$ and $g(x)=x^2-4$. Since $\lim\limits_{x\to -2}f(x)=f(-2)=0$ and $\lim\limits_{x\to -2}g(x)=g(-2)=0$, we have to factorize both $f(x)$ and $g(x)$ and cancel out the common factors. Doing so, we get that

$$\lim_{x\to -2}\frac{x^3+8}{x^2-4}=\lim_{x\to -2}\frac{(x+2)(x^2-2x+4)}{(x+2)(x-2)}=\lim_{x\to -2}\frac{x^2-2x+4}{x-2}$$

. Now, if $f_1(x)=x^2-2x+4$ and $g_1(x)=x-2$ then $\lim\limits_{x\to -2}f_1(x)=f_1(-2)=12$ and $\lim\limits_{x\to -2}g_1(x)=g_1(-2)=-4$. This implies that

$$\lim_{x\to -2}\frac{x^3+8}{x^2-4}=\lim_{x\to -2}\frac{x^2-2x+4}{x-2}=-\frac{12}{4}=-3.$$

Example 8

Consider $f(x) = \begin{cases} x^3 + 2x + 1, & x < 1 \\ 3x - 1, & x \geq 1 \end{cases}$. Compute $\lim_{x\to 2} f(x)$ and $\lim_{x\to 1} f(x)$.

Solution: Since $f(x)$ has two different rules; one for all $x < 1$ and another one for all $x \geq 1$, we have to be careful when evaluating limits as follows:

1. Since $x = 2$ lies in $[1, \infty)$, and since we can approach 2 from both sides of it through points in $[1, \infty)$, we don't need to evaluate the left-hand and right-hand limits separately. Instead of that we evaluate the limit as x approaches 2 directly and get that

$$\lim_{x\to 2} f(x) = \lim_{x\to 2} (3x - 1) = 6 - 1 = 5.$$

2. Since the left of 1 lies in $(-\infty, 1)$ and the right of 1 lies in $[1, \infty)$, and since $f(x)$ has two different rules over these two intervals, we have to evaluate both the left-hand and the right-hand limits separately as follows:

$$\lim_{x \to 1+} f(x) = \lim_{x \to 1+} (3x - 1) = 3 - 1 = 2$$

$$\lim_{x \to 1-} f(x) = \lim_{x \to 1-} (x^3 + 2x + 1) = 1 + 2 + 1 = 4.$$

Since the left-hand and right-hand limits are different, we conclude that $\lim_{x \to 1} f(x)$ doesn't exist.

Example 9

Evaluate $\lim_{x\to 4}\frac{x+\sqrt{x}-6}{\sqrt{x}-2}$.

Solution:

$$\lim_{x\to 4}\frac{x+\sqrt{x}-6}{\sqrt{x}-2}\left(\frac{0}{0}\right)=\lim_{x\to 4}\frac{(\sqrt{x}+3)(\sqrt{x}-2)}{(\sqrt{x}-2)}$$

$$=\lim_{x\to 4}(\sqrt{x}+3)=\sqrt{4}+3=2+3=5.$$

Example 10

Evaluate $\lim_{x\to 1}\frac{x^{\frac{3}{2}}-x}{x^{\frac{1}{2}}-1}$.

Solution:

$$\lim_{x\to 1}\frac{x^{\frac{3}{2}}-x}{x^{\frac{1}{2}}-1}\left(\frac{0}{0}\right)=\lim_{x\to 1}\frac{x(x^{\frac{1}{2}}-1)}{x^{\frac{1}{2}}-1}=\lim_{x\to 1}x=1\cdot$$

Example 11

Evaluate $\lim_{x\to 1}\dfrac{2-\sqrt{x+3}}{x^2+2x-3}$.

Solution:

$$\lim_{x\to 1}\frac{2-\sqrt{x+3}}{x^2+2x-3}\left(\frac{0}{0}\right)=\lim_{x\to 1}\frac{(2-\sqrt{x+3})(2+\sqrt{x+3})}{(x^2+2x-3)(2+\sqrt{x+3})}$$

$$=\lim_{x\to 1}\frac{4-(x+3)}{(x+3)(x-1)(2+\sqrt{x+3})}$$

$$=\lim_{x\to 1}\frac{-(x-1)}{(x+3)(x-1)(2+\sqrt{x+3})}=\lim_{x\to 1}\frac{-1}{(x+3)(2+\sqrt{x+3})}$$

$$=-\frac{1}{(4)(4)}=-\frac{1}{16}.$$

Example 12

Evaluate $\lim_{x \to 4} \frac{3-\sqrt{2x+1}}{\sqrt{x}-2}$.

Solution:

$$\lim_{x \to 4} \frac{3-\sqrt{2x+1}}{\sqrt{x}-2}\left(\frac{0}{0}\right) = \lim_{x \to 4} \frac{3-\sqrt{2x+1}}{\sqrt{x}-2} \cdot \frac{3+\sqrt{2x+1}}{3+\sqrt{2x+1}} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$$

$$= \lim_{x \to 4} \frac{(9-(2x+1))(\sqrt{x}+2)}{(x-4)(3+\sqrt{2x+1})}$$

$$= \lim_{x \to 4} \frac{(9-2x-1)(\sqrt{x}+2)}{(x-4)(3+\sqrt{2x+1})} = \lim_{x \to 4} \frac{(8-2x)(\sqrt{x}+2)}{(x-4)(3+\sqrt{2x+1})}$$

$$= \lim_{x \to 4} \frac{2(4-x)(\sqrt{x}+2)}{-(4-x)(3+\sqrt{2x+1})}$$

$$= \lim_{x \to 4} \frac{-2(\sqrt{x}+2)}{(3+\sqrt{2x+1})} = \frac{-2(\sqrt{4}+2)}{3+\sqrt{8+1}} = -\frac{8}{6} = -\frac{4}{3}.$$

Example 13

Evaluate $\lim_{x\to 0} \frac{2x - |x|}{|3x| - 2x}$.

Solution: Since $|x|$ and $|3x|$ have different rules on both sides of $x = 0$, we need to evaluate the left hand and right hand limits separately as follows:

$$\lim_{x\to 0+} \frac{2x - |x|}{|3x| - 2x} = \lim_{x\to 0+} \frac{2x - x}{3x - 2x} = \lim_{x\to 0+} \frac{x}{x} = \lim_{x\to 0+} 1 = 1 \text{ and}$$

$$\lim_{x\to 0-} \frac{2x - |x|}{|3x| - 2x} = \lim_{x\to 0-} \frac{2x - (-x)}{(-3x) - 2x} = \lim_{x\to 0-} -\frac{3x}{5x} = \lim_{x\to 0-} -\frac{3}{5} = -\frac{3}{5}.$$

Thus, $\lim_{x\to 0} \frac{2x - |x|}{|3x| - 2x}$ does not exist.

Limit Properties (continued)

10. $$\lim_{x \to 0} \frac{\sin x}{x} = 1,$$

11. $$\lim_{x \to 0} \frac{\tan x}{x} = 1,$$

Example 14

Evaluate $\lim_{x \to 0} \frac{\sin 5x}{x}$

Solution

$$\lim_{x \to 0} \frac{\sin 5x}{x} = \lim_{x \to 0} \frac{\sin 5x}{x}.5 = 5\lim_{x \to 0} \frac{\sin 5x}{5x} = 5.1 = 5$$

Example 15

Evaluate $\lim_{\theta \to 0} \frac{\sin 3\theta}{\tan 5\theta}$

Solution

$$\lim_{\theta \to 0} \frac{\frac{\sin 3\theta}{\theta}}{\frac{\tan 5\theta}{\theta}} = \frac{\lim_{\theta \to 0} \frac{\sin 3\theta}{\theta}}{\lim_{\theta \to 0} \frac{\tan 5\theta}{\theta}} = \frac{3\lim_{\theta \to 0} \frac{\sin 3\theta}{3\theta}}{5\lim_{\theta \to 0} \frac{\tan 5\theta}{5\theta}} = \frac{3}{5},$$

Example 16

Evaluate $\lim_{x \to 0} \frac{1-\cos x}{x^2}$

Solution

$$\lim_{x \to 0} \frac{1-\cos x}{x^2} = \lim_{x \to 0} \frac{(1-\cos x)}{x^2} . \frac{(1+\cos x)}{(1+\cos x)}$$

$$= \lim_{x \to 0} \frac{1-\cos^2 x}{x^2} . \frac{1}{(1+\cos x)} = \lim_{x \to 0} \frac{\sin^2 x}{x^2} . \frac{1}{(1+\cos x)}$$

$$= \lim_{x \to 0} \left(\frac{\sin x}{x} \right)^2 . \frac{1}{(1+\cos x)} = \lim_{x \to 0} \left(\frac{\sin x}{x} \right)^2 . \lim_{x \to 0} \frac{1}{(1+\cos x)}$$

$$= \left(\lim_{x \to 0} \frac{\sin x}{x} \right)^2 . \lim_{x \to 0} \frac{1}{(1+\cos x)} = 1^2 . \frac{1}{2} = \frac{1}{2}$$

Example 17

Evaluate $\lim\limits_{\theta \to 0} \dfrac{\theta^2 \tan^3 \theta}{\sin^4 2\theta . \tan 3\theta}$

Solution

Dividing both of the numerator and denominator by $\boldsymbol{\theta^5}$ yields:

$$\lim_{\theta \to 0} \frac{\theta^2 \tan^3 \theta}{\sin^4 2\theta . \tan 3\theta} = \lim_{\theta \to 0} \frac{\dfrac{\tan^3 \theta}{\theta^3}}{\dfrac{\sin^4 2\theta}{(2\theta)^4} . \dfrac{\tan 3\theta}{3\theta}} . \frac{1}{48} = \lim_{\theta \to 0} \frac{\left(\dfrac{\tan \theta}{\theta}\right)^3}{\left(\dfrac{\sin 2\theta}{2\theta}\right)^4 . \dfrac{\tan 3\theta}{3\theta}} . \frac{1}{48}$$

$$= \frac{\lim\limits_{\theta \to 0} \left(\dfrac{\tan \theta}{\theta}\right)^3}{\lim\limits_{\theta \to 0} \left(\dfrac{\sin 2\theta}{2\theta}\right)^4 . \lim\limits_{\theta \to 0} \dfrac{\tan 3\theta}{3\theta}} . \frac{1}{48} = \frac{1^3}{1^4 . 1} . \frac{1}{48} = \frac{1}{48}$$

Limits at Infinity

By a limit at infinity, one means the limit of a function $f(x)$ when the variable x tends to either ∞ or $-\infty$. i.e.

$$\lim_{x \to \infty} f(x) \qquad \text{or} \qquad \lim_{x \to -\infty} f(x)$$

The next theorem is helpful in computing limits at infinity for a large class of functions.

THEOREM

1. If r is any positive real number then $\lim_{x \to \infty} \frac{1}{x^r} = 0$.
2. If n is any positive integer then $\lim_{x \to -\infty} \frac{1}{x^n} = 0$.

Example 18

Evaluate $\lim_{x\to\infty}\left(\dfrac{2x^3-3x^2+2}{x^3-x^2-100x+1}\right)$

Solution

$$\lim_{x\to\infty}\frac{2x^3-3x^2+2}{x^3-x^2-100x+1}=\frac{\infty}{\infty}$$

This is an indeterminate form. To circumvent it, we divide both the numerator and denominator by x^3 and get that

$$=\lim_{x\to\infty}\left(\frac{\dfrac{2x^3}{x^3}-\dfrac{3x^2}{x^3}+\dfrac{2}{x^3}}{\dfrac{x^3}{x^3}-\dfrac{x^2}{x^3}-\dfrac{100x}{x^3}+\dfrac{1}{x^3}}\right)$$

$$=\lim_{x\to\infty}\left(\frac{2-\dfrac{3}{x}+\dfrac{2}{x^3}}{1-\dfrac{1}{x}-\dfrac{100}{x^2}+\dfrac{1}{x^3}}\right)=\frac{2-0+0}{1-0-0+0}=2$$

Example 19

Evaluate $\lim_{x\to\infty}\left(\dfrac{x^2+2x-4}{12x+31}\right)$

Solution

$$\lim_{x\to\infty}\left(\frac{x^2+2x-4}{12x+31}\right)=\frac{\infty}{\infty}$$

This is an indeterminate form. To circumvent it, we divide both the numerator and denominator by x and get that

$$=\lim_{x\to\infty}\left(\frac{\frac{x^2}{x}+\frac{2x}{x}-\frac{4}{x}}{\frac{12x}{x}+\frac{31}{x}}\right)$$

$$=\lim_{x\to\infty}\left(\frac{x+2-\frac{4}{x}}{12+\frac{31}{x}}\right)=\frac{\infty+2-0}{12+0}=\infty$$

Example 20

Evaluate $\lim_{x\to\infty}\left(\frac{4x^2-5x+21}{7x^3+5x^2-10x+1}\right)$

Solution

$$\lim_{x\to\infty}\frac{4x^2-5x+21}{7x^3+5x^2-10x+1}=\frac{\infty}{\infty}$$

This is an indeterminate form. To circumvent it, we divide both the numerator and denominator by x^3 and get that

$$=\lim_{x\to\infty}\left(\frac{\frac{4x^2}{x^3}-\frac{5x}{x^3}+\frac{21}{x^3}}{\frac{7x^3}{x^3}+\frac{5x^2}{x^3}-\frac{10x}{x^3}+\frac{1}{x^3}}\right)$$

$$=\lim_{x\to\infty}\left(\frac{\frac{4}{x}-\frac{5}{x^2}+\frac{21}{x^3}}{7+\frac{5}{x}-\frac{10}{x^2}+\frac{1}{x^3}}\right)=\frac{0-0+0}{7+0-0+0}=0$$

Example 21

Evaluate $\lim_{x\to\infty}\left(\sqrt{x^2+1}-x\right)$

Solution

$$\lim_{x\to\infty}\left(\sqrt{x^2+1}-x\right)$$

$$= \lim_{x\to\infty}\left(\sqrt{x^2+1}-x\right).\frac{\left(\sqrt{x^2+1}+x\right)}{\left(\sqrt{x^2+1}+x\right)}$$

$$=\lim_{x\to\infty}\frac{x^2+1-x^2}{\sqrt{x^2+1}+x}$$

$$= \lim_{x\to\infty}\frac{1}{\sqrt{x^2+1}+x}=\frac{1}{\infty+\infty}=\frac{1}{\infty}=0$$

Self Test

1

Evaluate $\lim_{x \to -1} (x^3 + x^2 - 3)$.

A. -1

B. -5

C. -3

D. Does not exist

2

Evaluate $\lim_{x \to 3} \frac{x^3 - 2x^2}{x^2 + 2}$.

A. 1/2

B. 9/11

C. -1

D. Does not exist

3

Let $\lim_{x \to -4} f(x) = 10$ and $\lim_{x \to -4} g(x) = 4$. Find $\lim_{x \to -4} (f(x) + g(x))^2$.

A. 196

B. 14

C. 116

D. 6

4

Let $g(x) = x^2 - 4$ and let $L(x) = 2x - 1$. Find $\lim_{x \to 2} (2(g(x))^3 / L(x))$.

A. 0

B. 4/3

C. 1/2

D. Does not exist

5

Let $\lim_{x \to -1} f(x) = -1$ and $\lim_{x \to -1} g(x) = 6$. Find $\lim_{x \to -1}\left[\frac{-10f(x) - 4g(x)}{3 + g(x)}\right]$.

A. $34/9$

B. $-2/3$

C. -1

D. $-14/9$

6

Evaluate $\lim_{x \to -2}(-x|2x|)$.

A. 8

B. -8

C. -16

D. Does not exist

7

For $f(x) = \begin{cases} 3x^2 & \text{if } x \le -1 \\ 3 & \text{if } -1 < x \le 1 \\ 3x+1 & \text{if } x > 1 \end{cases}$, evaluate $\lim_{x \to -1^-} f(x)$.

A. 3

B. 4

C. 2

D. Does not exist

8

Given the function $f(x) = \begin{cases} \dfrac{\sqrt{x+4} - 2}{x} & \text{if } x > 0 \\ 1 & \text{if } x = 0 \\ \dfrac{1}{x+4} & \text{if } x < 0 \end{cases}$. Compute $\lim_{x \to 0} f(x)$.

A. 1 / 4

B. $-1/4$

C. 2

D. Does not exist

9

Given the function $f(x) = \begin{cases} 4 - 3x^2 & \text{if } x < 1 \\ 2 + 3x^2 & \text{if } x > 1 \end{cases}$. Find $\lim_{x \to 1} f(x)$.

A. 1

B. 5

C. 3

D. Does not exist

10

Evaluate $\lim_{x \to 2} \dfrac{x^2 - x - 2}{x^2 - 4}$.

A. 3/4

B. 1/4

C. 0

D. Does not exist

11

Evaluate $\lim_{x \to 4} \frac{6x - 1}{2x - 9}$.

A. -23

B. 23

C. $25/17$

D. Does not exist

12

Evaluate $\lim_{x \to -2} \frac{x^3 + 8}{x^4 - 16}$.

A. $-3/8$

B. $3/8$

C. 0

D. Does not exist

13

Evaluate $\lim_{x \to 2} \dfrac{(1/x) - (1/2)}{x - 2}$.

A. $-1/2$

B. $-1/4$

C. $1/2$

D. Does not exist

14

Evaluate $\lim_{x \to 0} \dfrac{\sqrt{x+5} - \sqrt{5}}{x}$.

A. $\sqrt{3}/10$

B. $1/(2\sqrt{5})$

C. $-1/10$

D. Does not exist

15

Evaluate $\lim_{x \to 4} \frac{\sqrt{x+5} - 3}{x - 4}$.

A. 2/3

B. 1

C. 1/6

D. 3/4

16

Evaluate $\lim_{x \to 1} \frac{2 - \sqrt{x+3}}{x^2 + 2x - 3}$.

A. $-1/16$

B. 1/8

C. 1/2

D. Does not exist

17

Evaluate $\lim\limits_{x \to 0} \dfrac{2}{\sqrt{3x+4}+2}$.

A. 1/2

B. 1

C. 2

D. Does not exist

18

Evaluate $\lim\limits_{x \to 3} \dfrac{x^2 - 3x}{x - \sqrt{x+1} - 1}$.

A. 1/4

B. $-1/8$

C. 4

D. 2/3

19

Evaluate $\lim_{x \to 1} \frac{x^{3/2} - x}{x^{1/2} - 1}$.

A. -1

B. 1

C. 0

D. 2

20

Evaluate $\lim_{x \to 5+} \frac{-7\sqrt{(x-5)^3}}{x-5}$.

A. $-7\sqrt{5}$

B. -7

C. 0

D. Does not exist

21

Evaluate $\lim_{x \to 1+} \frac{\sqrt{(x+25)(x-1)^2}}{11x - 11}$.

A. 1/11

B. 0

C. $\sqrt{26}/11$

D. Does not exist

22

If $\lim_{x \to 2} \frac{f(x) - 4}{x - 1} = 5$, find $\lim_{x \to 2} f(x)$.

A. 9

B. 2

C. 13

D. Does not exist

23

If $\lim\limits_{x \to 2} \dfrac{f(x)}{x^2} = 4$, find $\lim\limits_{x \to 2} \dfrac{f(x)}{x}$.

A. 4

B. 8

C. 16

D. 2

24

If $\lim\limits_{x \to 1} \dfrac{f(x) - 3}{x - 1} = 2$, find $\lim\limits_{x \to 1} f(x)$.

A. 2

B. 3

C. 1

D. Does not exist

25

For $f(x) = -3 - x^2$, evaluate $\lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$.

A. 2

B. $-3 - 2a$

C. $-2a$

D. $-a^2$

26

For $f(x) = 9/\sqrt{x}$, evaluate $\lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$.

A. $-9/(2a)$

B. $9/(2\sqrt{a})$

C. $-9/(2\sqrt{a})$

D. $-9/(2\sqrt{a^3})$

27

Evaluate $\lim_{h \to 0} \dfrac{6a^2h - 5ah + h^2}{h}$.

A. $6a - 5$

B. 0

C. $6a^2 - 5a$

D. Does not exist

ANSWER KEY

1	C	8	A	15	C	22	A
2	B	9	D	16	A	23	B
3	A	10	A	17	A	24	B
4	A	11	A	18	C	25	C
5	D	12	A	19	B	26	D
6	A	13	B	20	C	27	C
7	A	14	B	21	C		