SECTION 1.

Math 2

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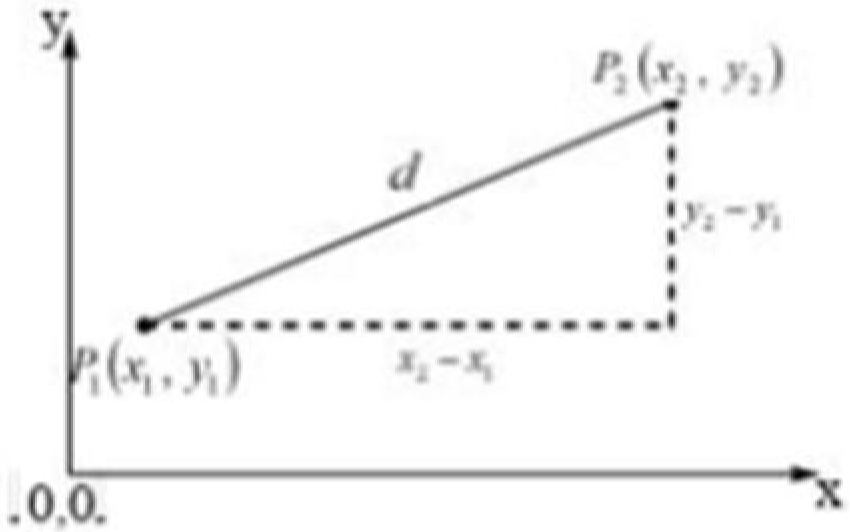
Section 1

Distance between two points:

Applying Pythagorean theorem, the distance d between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is

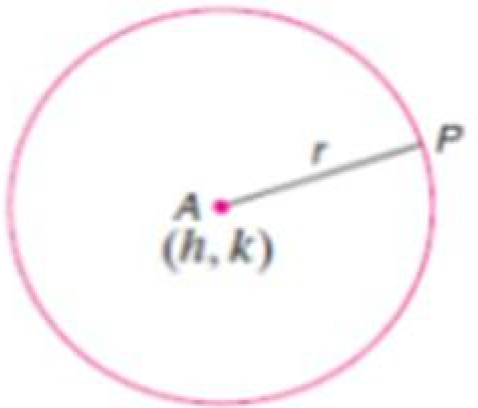


$$d = \overline{P_1P_2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Circle

The Circle

Definition: Circle is the locus of a point $P(x,y)$ moving such that its distance from a fixed point $A(h,k)$ is a constant r. This fixed point is called *center* of the circle and the constant distance is known as *radius of the circle*.



$$(x-h)^2+(y-k)^2=r^2$$

Example:

Find the equation of the circle which passes through the point (4, 5) and has its center at (2, 2)

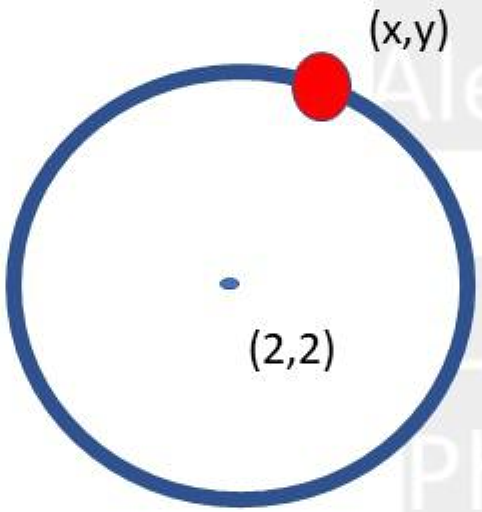
Solution

As the circle is passing through the point (4,5) and its center is (2,2) so its radius is

$$r=\sqrt{(4-2)^2+(5-2)^2}=\sqrt{13}$$

Therefore

$$(x-2)^2+(y-2)^2=13$$



Example:

Perfect Square Quadratic Expression
(with leading coefficient of 1)

$$x^2 + Bx = \left(x + \tfrac{B}{2}\right)^2 - \left(\tfrac{B}{2}\right)^2$$

Find the center and radius of the the circle: $x^2 + y^2 + 6x - 8y - 11 = 0$

Solution

$$\Rightarrow \left(x^2 + 6x\right) + \left(y^2 - 8y\right) - 11 = 0$$

$$\Rightarrow \left[(x+3)^2 - (3)^2\right] + \left[(y-4)^2 - (4)^2\right] - 11 = 0$$

$$\Rightarrow (x+3)^2 - 9 + (y-4)^2 - 16 - 11 = 0$$

$$\Rightarrow (x+3)^2 + (y-4)^2 - 36 = 0$$

$$\Rightarrow (x+3)^2 + (y-4)^2 = 36 \qquad \boxed{(x-h)^2 + (y-k)^2 = r^2}$$

$\therefore$ The center is $(-3, 4)$ and the radius: $r = \sqrt{36} = 6$

Perfect Square Quadratic Expression

(with leading coefficient of 1)

$$x^2 + Bx = \left(x + \frac{B}{2}\right)^2 - \left(\frac{B}{2}\right)^2$$

Example:

$$\boxed{(x-h)^2+(y-k)^2=r^2}$$

Find the elements of the circle $(2x+7)^2+4(y-3)^2=100$

Solution

$$\Rightarrow 4\left(x+\frac{7}{2}\right)^2+4(y-3)^2=100$$

Dividing both sides by 4:

$$\Rightarrow \left(x+\frac{7}{2}\right)^2+(y-3)^2=25 \qquad \boxed{(x-h)^2+(y-k)^2=r^2}$$

$\therefore$ The center is $\left(-\frac{7}{2},3\right)$ and the radius: $r=\sqrt{25}=5$

Example:

Find the equation of the circle that has a diameter with endpoints (11,8) and (5,10)

Solution

The center of the circle is in the middle of the diameter:

$$\left(\frac{11+5}{2},\frac{8+10}{2}\right)=(8,9)$$

The diameter: $d=\sqrt{(11-5)^2+(10-2)^2}=\sqrt{40}=2\sqrt{10}$

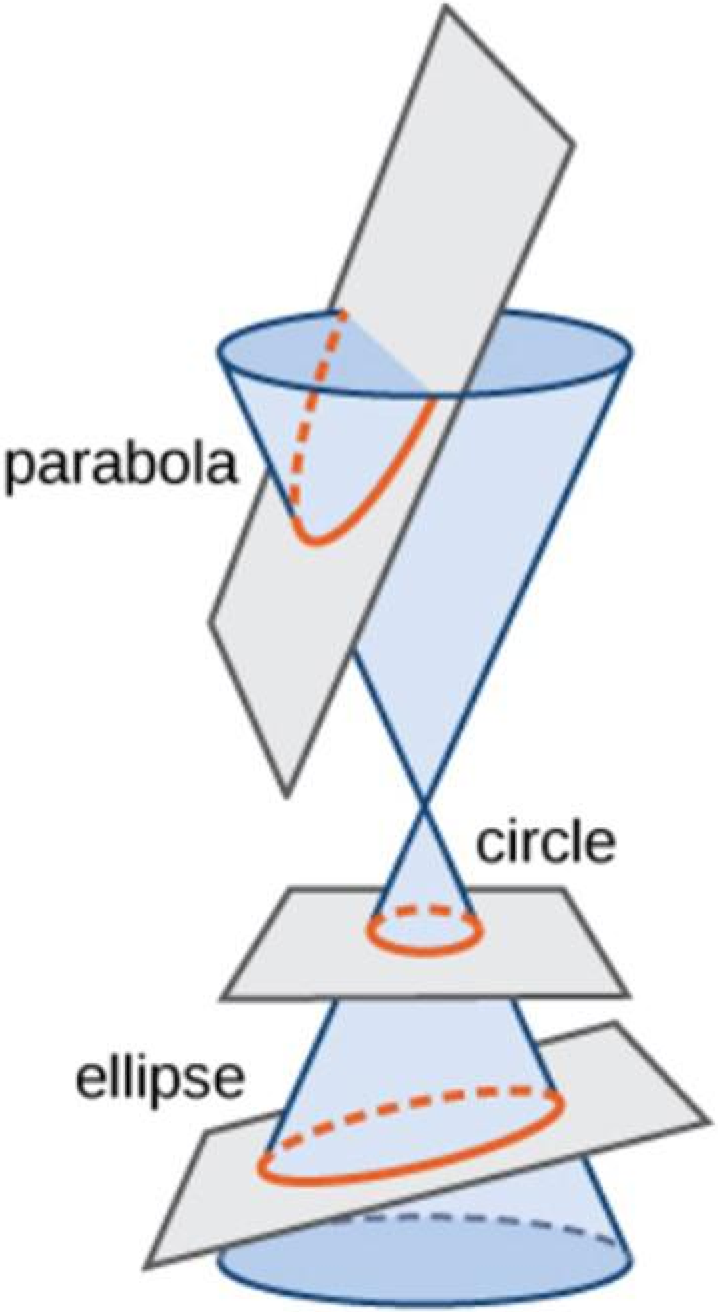
$$\therefore r=\sqrt{10}$$

The circle equation is $\boxed{(x-h)^2+(y-k)^2=r^2}$

$$(x-8)^2+(y-9)^2=10$$

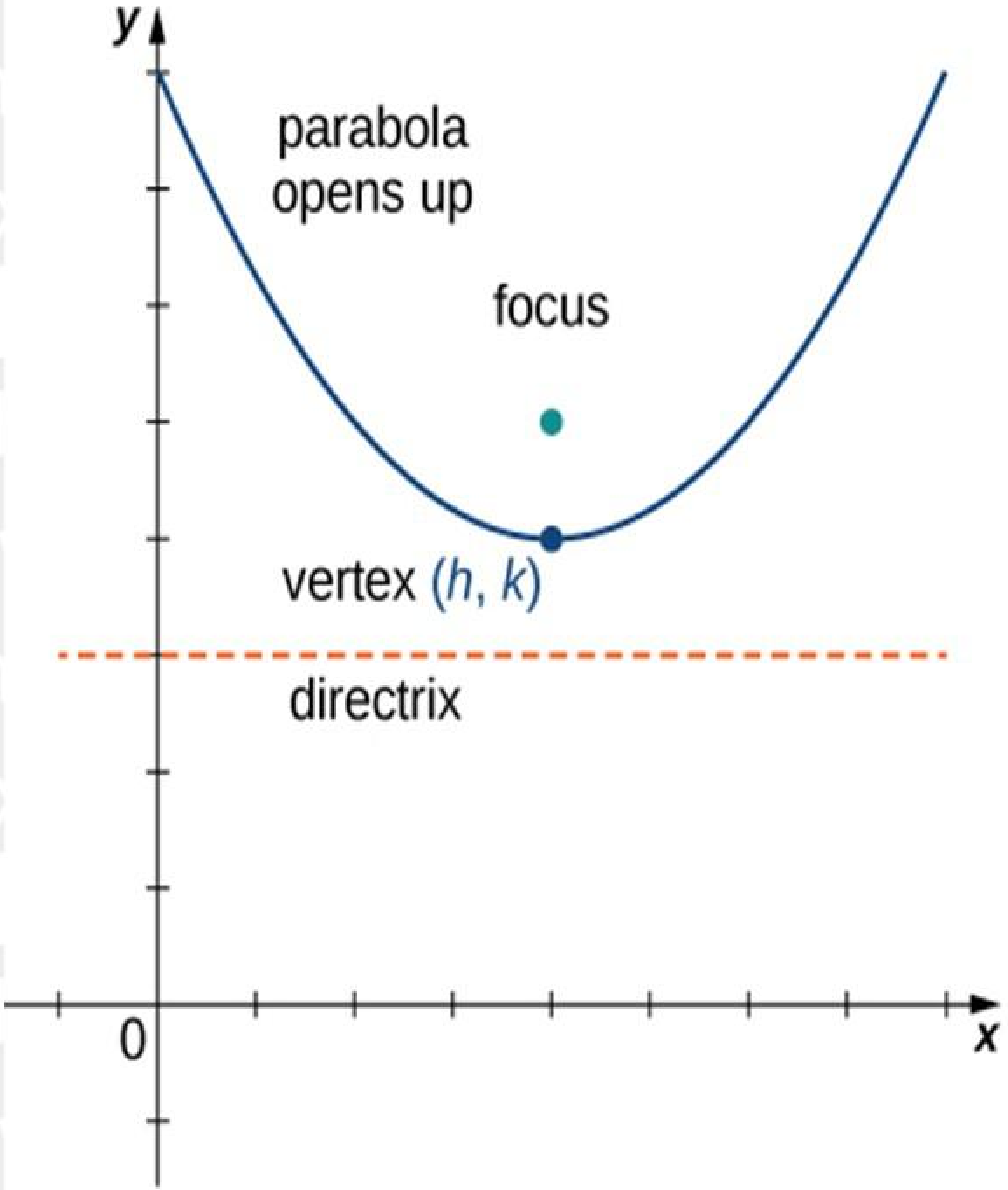
parabola

Parabola



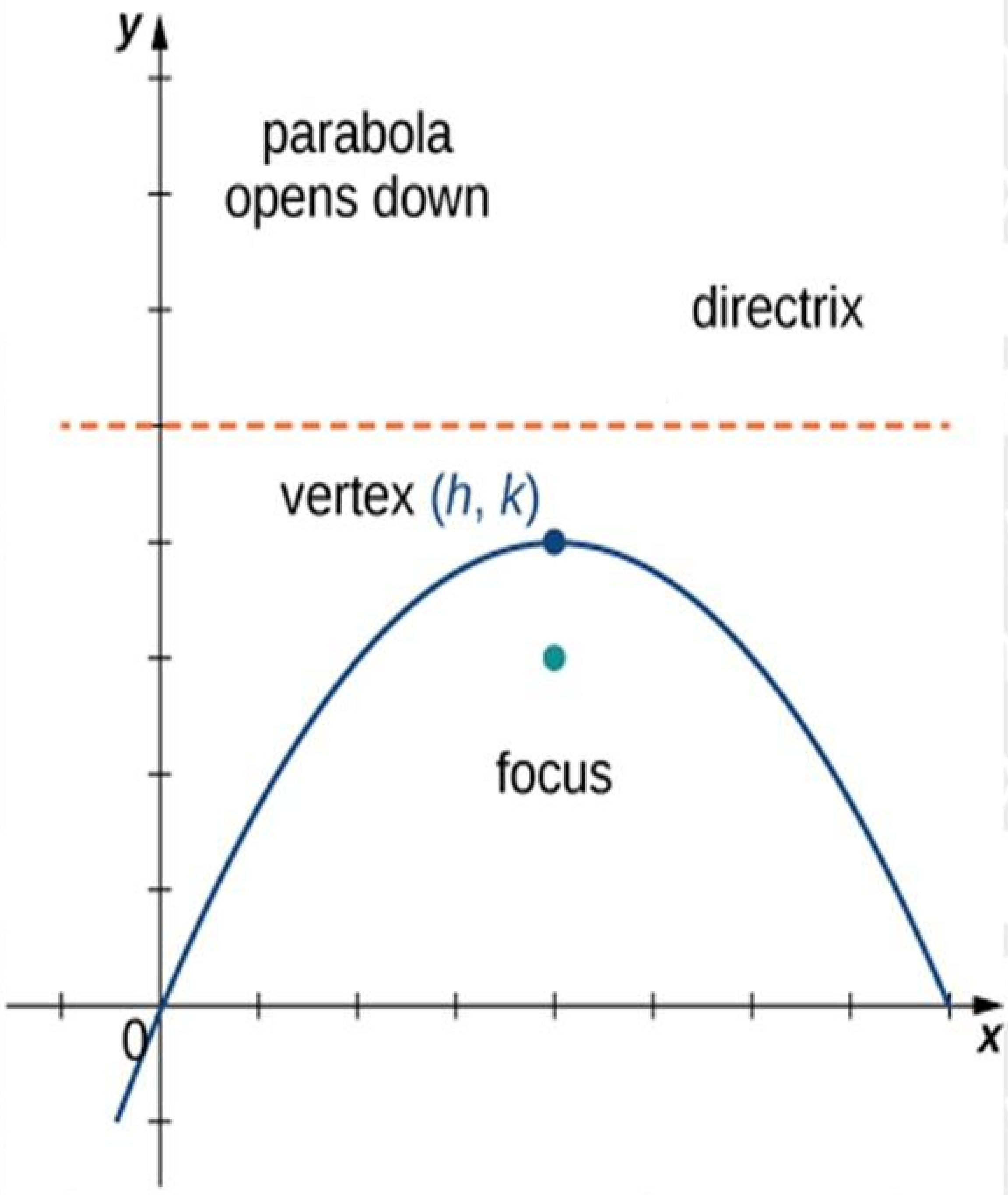
Case 1

Horizontal axis of symmetry $(\mathbf{x}-\mathbf{h})^2$=+4a(y-k)	
Vertex	(h, k)
Focus	(h, k+a)
Directrix	y = k-a
Axis of symmetry	x=h



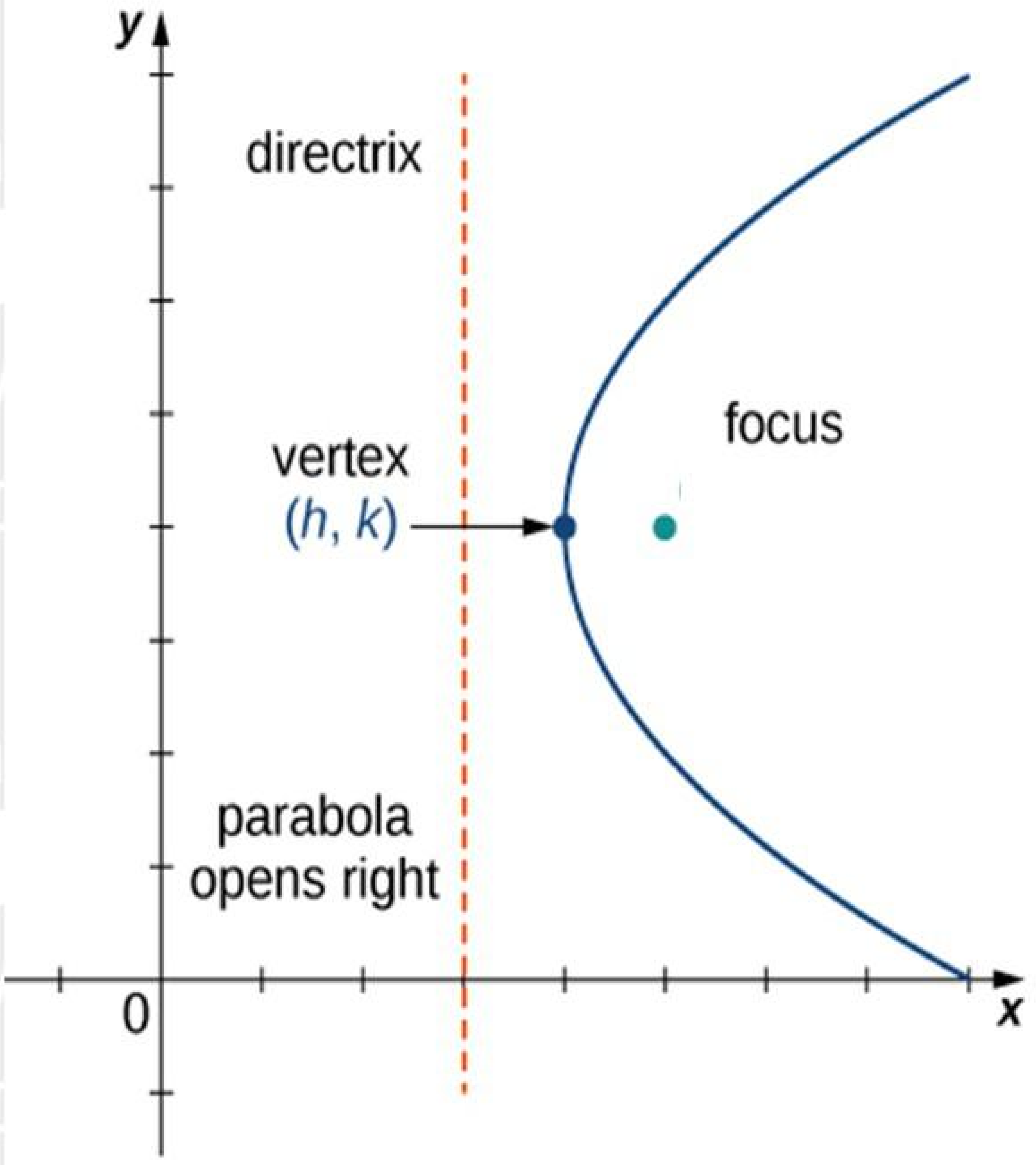
Case 2

Horizontal axis of symmetry $(x-h)^2=-4a(y-k)$	
Vertex	(h, k)
Focus	(h, k-a)
Directrix	y = k+a
Axis of symmetry	x=h



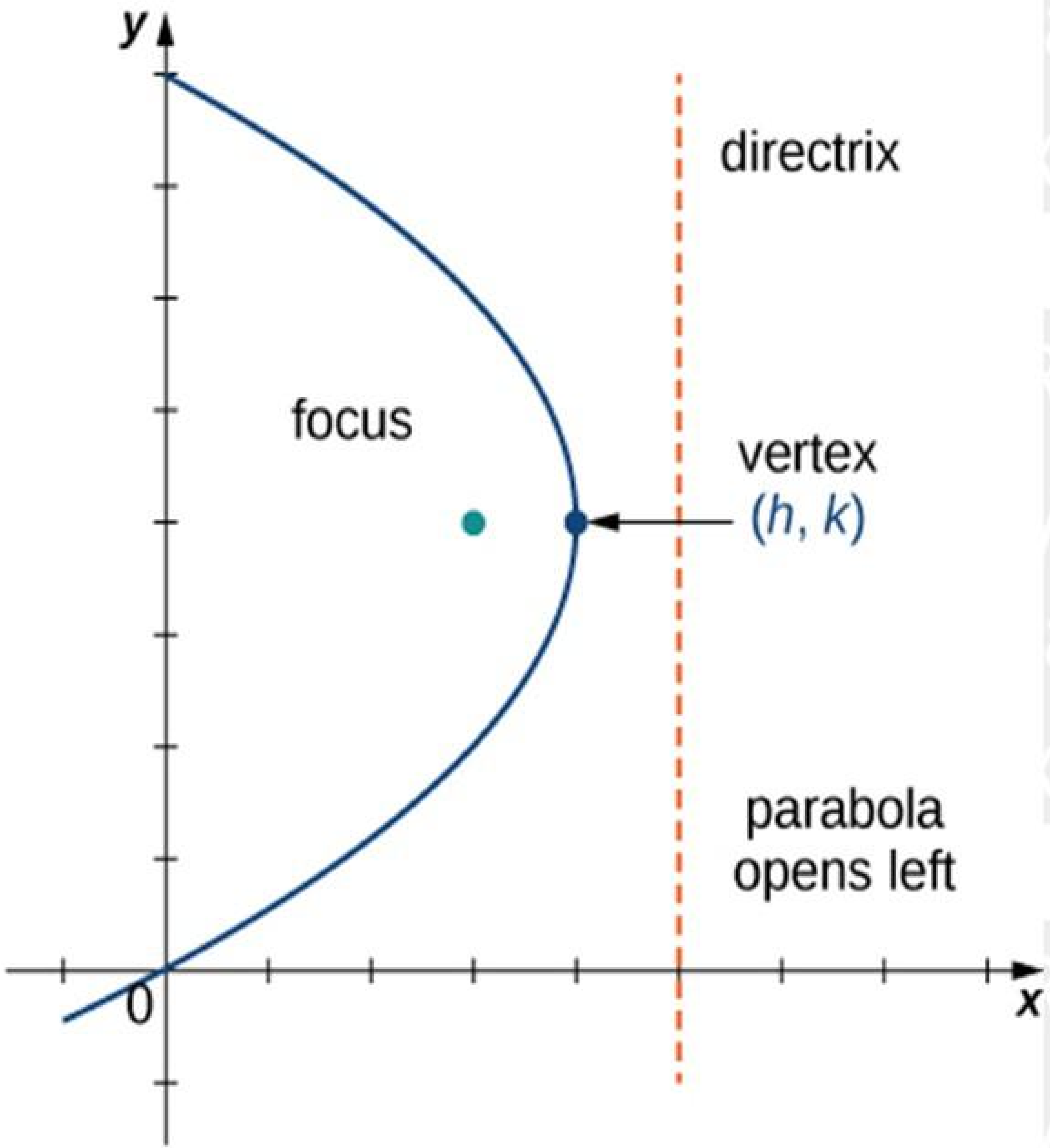
Case 3

Horizontal axis of symmetry $(y-k)^2 = +4a(x-h)$	
Vertex	(h, k)
Focus	(h+a, k)
Directrix	x = h-a
Axis of symmetry	y=k



Case 4

Horizontal axis of symmetry $(y-k)^2=-4a(x-h)$	
Vertex	(h, k)
Focus	(h-a, k)
Directrix	x = h+a
Axis of symmetry	y=k



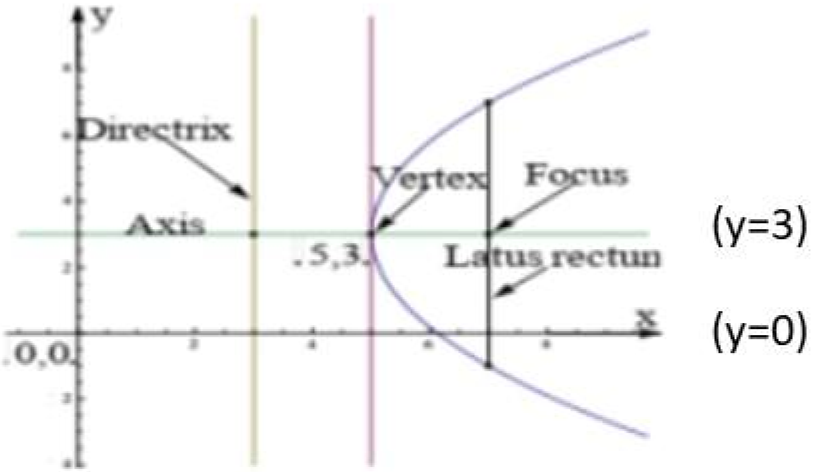
Example:

Find the elements of the parabola $(y-3)^2 = 8(x-5)$ and sketch the curve.

4a = 8 ----- a = 2 , k = 3 , h = 5

Solution Case 3

The vertex is (5,3). Since $4a = 8$ then $a = 2$. The symmetry axis is parallel to x-axis and its equation is $y = 3$. This parabola opens to the right.



The focus is $(5+2,3) = (7,3)$, the directrix is $x = 3$ and the latus rectum length $= 4a = 8$.

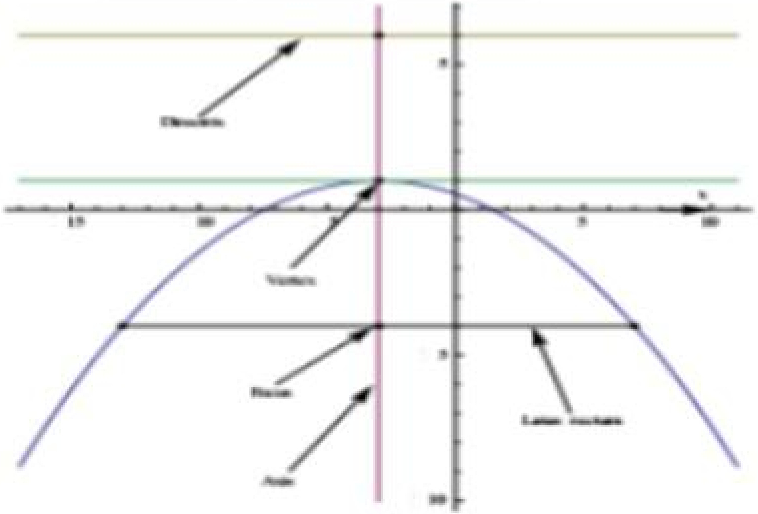
example

Find the elements of the parabola $(x+3)^2 = -20(y-1)$ and sketch the curve.

Solution

Case 2

$$(x-(-3))^2 = -20(y-1)$$
$$h = -3\,, k = 1$$
$$4a = 20 \qquad \therefore a = 5$$
$$Axis\ of\ symmetry\ equation -- \quad x = -3$$



The focus is $(-3, 1-5) = (-3, -4)$, the directrix is $y = 6$ and the latus rectum length $= 4a = 20$.

Example:

State the vertex, the focus, and the directrix of the parabola having the equation

$$x^2 - 4x + 4y - 4 = 0.$$

Solution

We shall rewrite the given equation in the standard form by completing square of the L.H.S,

$$\because (x^2 - 4x) = (x^2 + (-4)x) = \left(x + \frac{-4}{2}\right)^2 - \left(\frac{-4}{2}\right)^2 = (x-2)^2 - 4$$

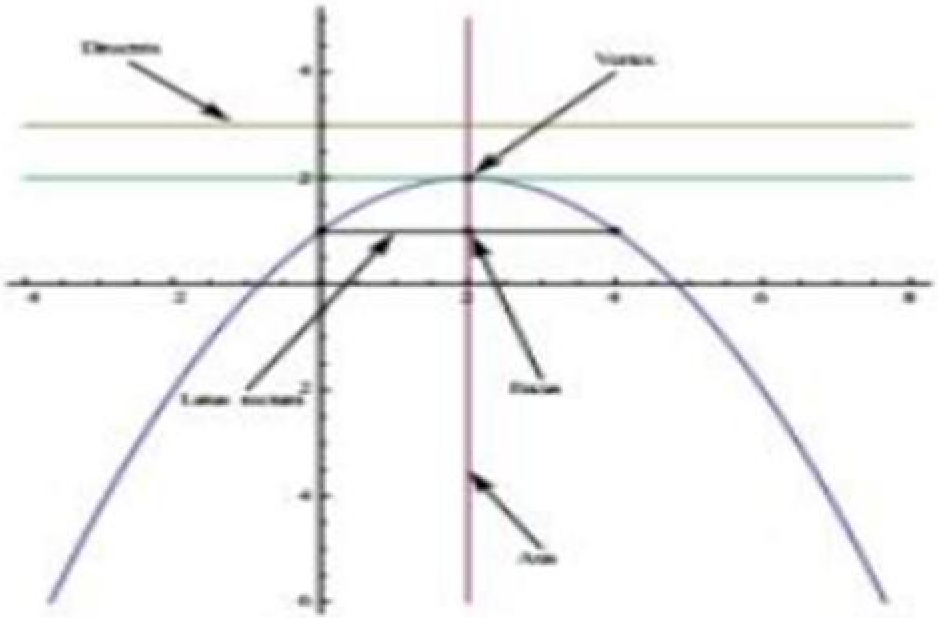
$$\because (x^2 - 4x) + 4y - 4 = 0$$

$$\therefore [(x-2)^2 - 4] + 4y - 4 = 0$$

$$\therefore (x-2)^2 = -4y + 8$$

$$\therefore (x-2)^2 = -4(y-2) \quad \text{Case 2}$$

The vertex is $(2,2)$. Since $4a = 4$ then $a = 1$. The symmetry axis is parallel to y-axis and its equation is $x = 2$. This parabola opens to the down.



The focus is $(2,2-1)=(2,1)$,

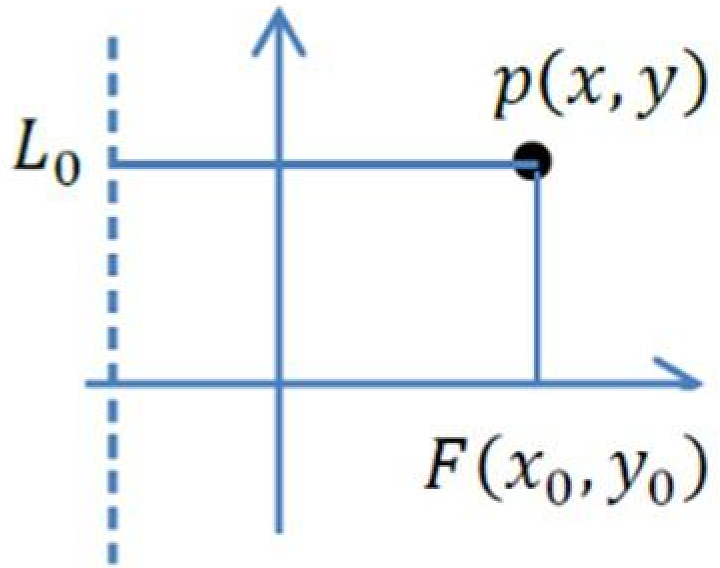
The directrix is $y=3$,

The latus rectum length $= \boxed{4a = 4}$.

Conic sections

Definition: The path of a point which moves in the plane so that its distance from a fixed point to its distance from a fixed line is in constant ratio is called *a conic section*. The fixed point is *the focus of the conic*, the fixed line is *the directrix*, and the constant ratio is *the eccentricity* " e "

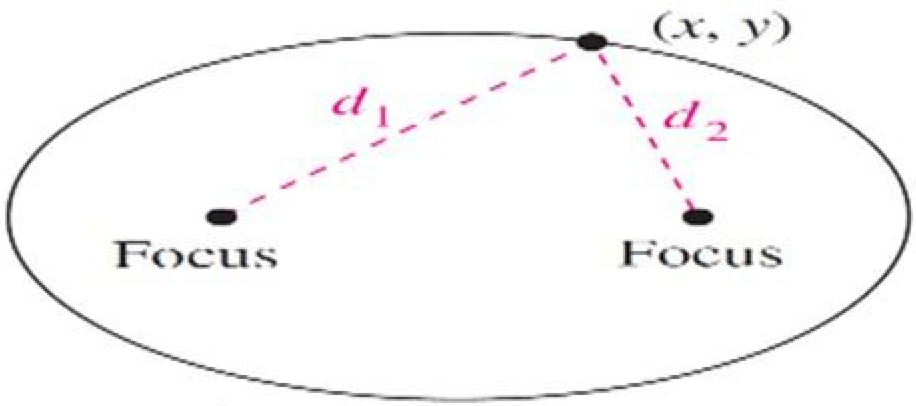
$$\frac{\overline{PF}}{\overline{PL_0}} = \text{constant} = e$$



- If $e = 1$, then the conic is a parabola.
- If $e < 1$, then the conic is an ellipse.
- If $e > 1$, then the conic is a hyperbola.

Ellipse

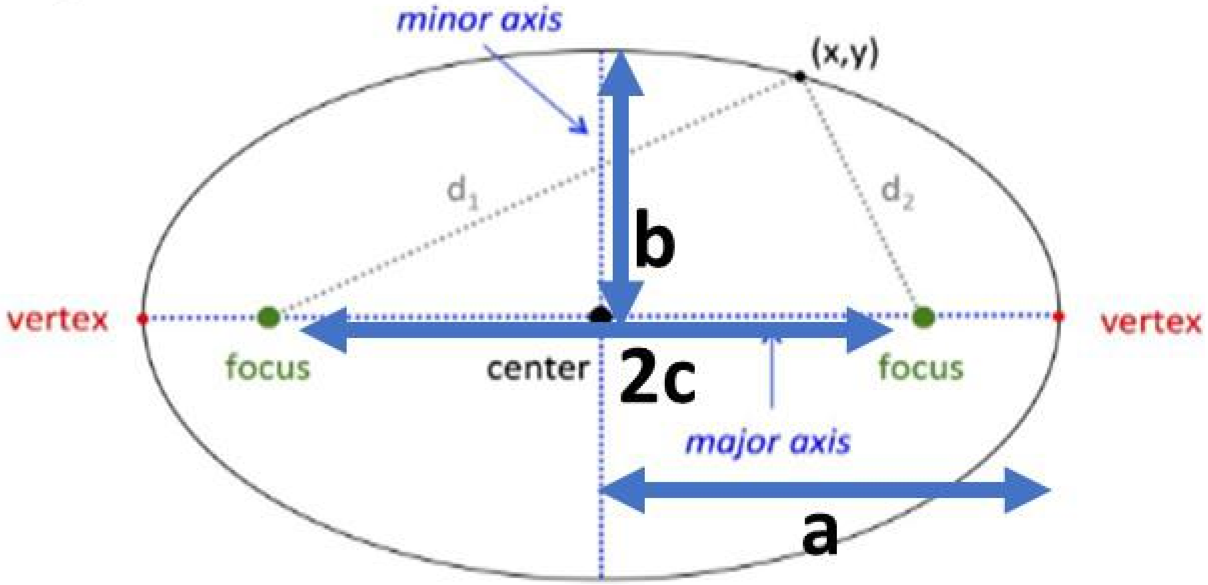
Definition: An ellipse is the set of all points (x, y) in the plane, such that the *sum* of their distances from two distinct fixed points, called the *foci* (plural of focus) of the ellipse , is a constant=$2a$.



$d_1 + d_2$ is constant.

- The distance between the two foci equals **2c**.

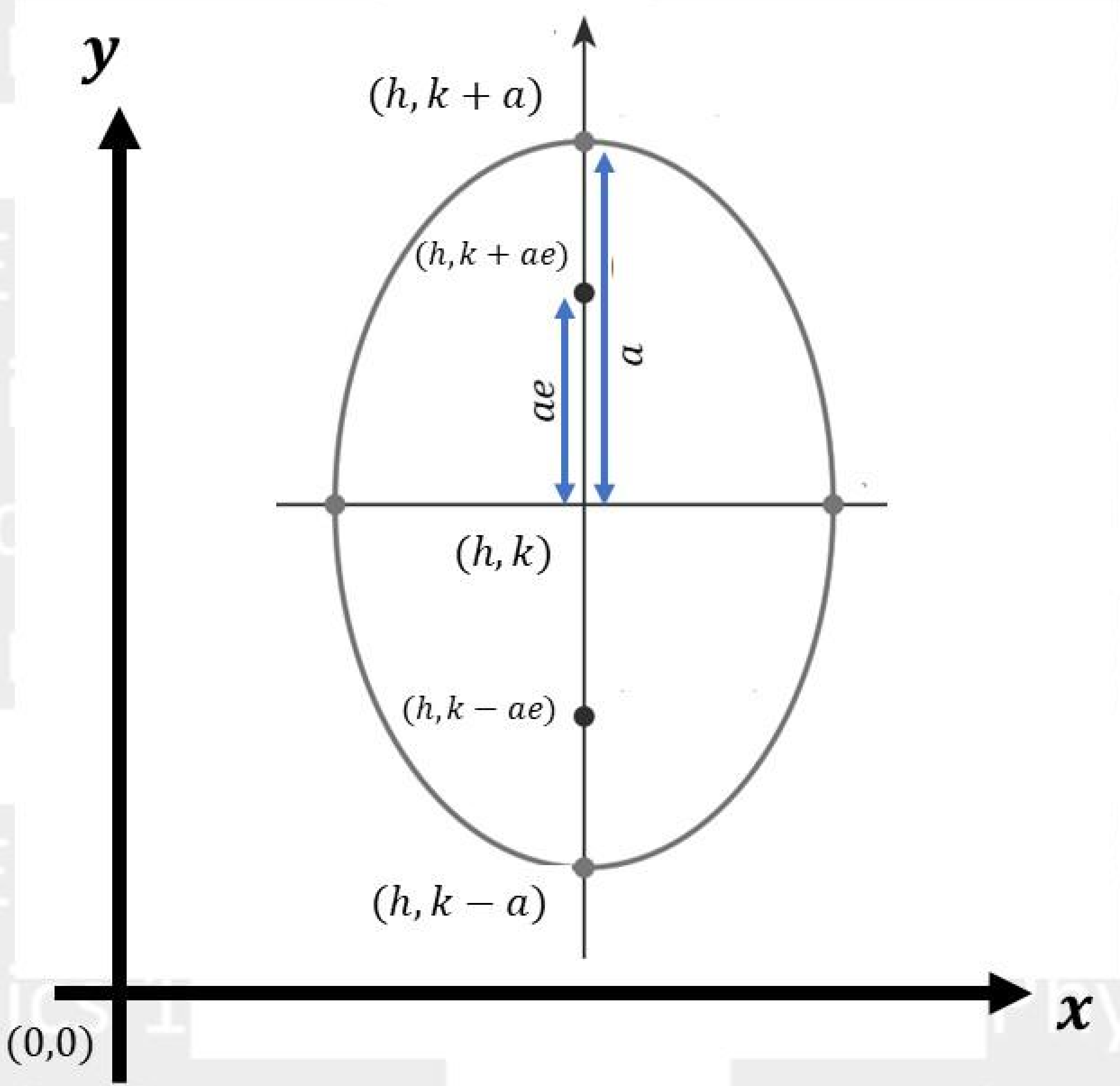
- The chord through the foci is called ***major axis*** and its length $=2a$.
- The ***major axis*** intersects the ellipse at two points called ***vertices***.
- The ***center*** is the midpoint between the vertices (or the midpoint between the foci) .
- The chord perpendicular to the major axis at the center is ***the minor axis*** and its length $=2b$.
- The endpoints of the minor axis are called ***co-vertices***.
- For the ellipse, we always have $a > b, a > c, c^2 = a^2 - b^2$ and the eccentricity $e = \dfrac{c}{a}$.



c = ae

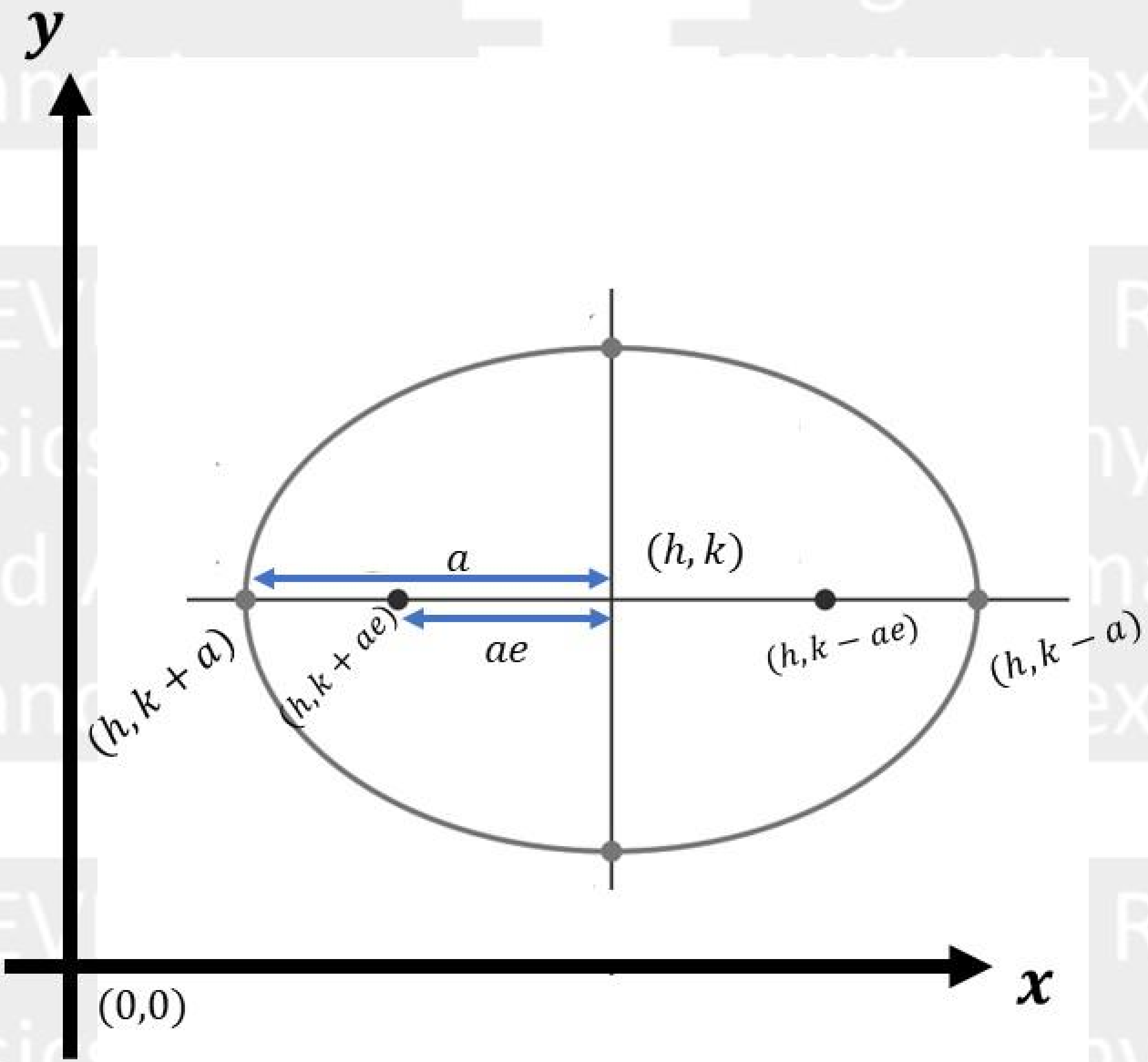
Case 1 – Eclipse having the major axes parallel to the y-axis

Standard Equation form $\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$	
Center	(h, k)
Focus points	(h, k+ae) , (h, k-ae)
Vertices	(h, k+a) , (h, k-a)
Directrix equations	$y=\frac{a}{e}+k$, $y=-\frac{a}{e}+k$
Equation of the major axis	$x=h$
The length of the major axis	Length = 2a
Equation of the minor axis	$y=k$
The length of the minor axis	Length = 2b



Case 2 – Eclipse having the major axes parallel to the x-axis

Standard Equation form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} += 1$	
Center	(h, k)
Focus points	(h+ae , k) , (h-ae , k)
Vertices	(h+a, k) , (h-a, k)
Directrix equations	$x = \frac{a}{e} + h$, $x = -\frac{a}{e} + h$
Equation of the major axis	$y = k$
The length of the major axis	Length = 2a
Equation of the minor axis	$x = h$
The length of the minor axis	Length = 2b



Examples

Example 3

Find the elements and sketch the ellipse $3x^2+4y^2+12x-8y+4=0$

Solution

$$\left(3x^2+12x\right)+\left(4y^2-8y\right)+4=0,$$

$$3\left(x^2+4x\right)+4\left(y^2-2y\right)+4=0,$$

By completing square of the L.H.S, we get

Perfect Square Quadratic Expression
(with leading coefficient of 1)

$$x^2+Bx=\left(x+\tfrac{B}{2}\right)^2-\left(\tfrac{B}{2}\right)^2$$

$$3\left[(x+2)^2-4\right]+4\left[(y-1)^2-1\right]+4=0,$$

$$3(x+2)^2-12+4(y-1)^2-4+4=0,$$

$$3(x+2)^2+4(y-1)^2=12, \qquad \div 12$$

$$\frac{(x+2)^2}{4}+\frac{(y-1)^2}{3}=1$$

$$a^2=4, b^2=3 \Rightarrow c^2=a^2-b^2=4-3=1$$
$$\therefore a=2, b=\sqrt{3}, c=1$$

Eccentricity: $e=\frac{c}{a}=\frac{1}{2}$

Center: $(-2,1)$

Major axis: parallel to x-axis, its equation is $y=1$ and its length is $2a=4$

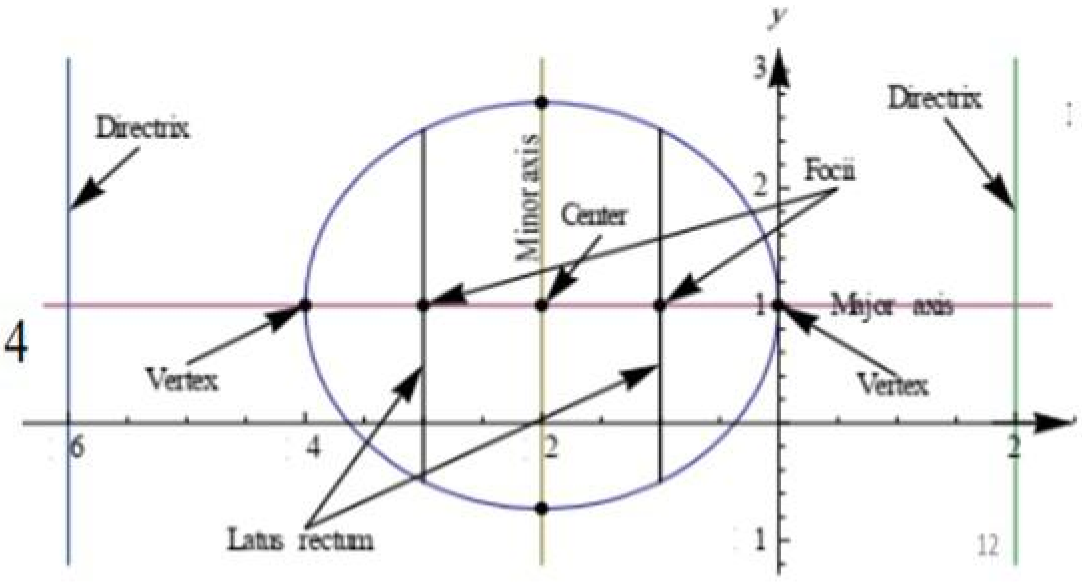
Minor axis: parallel to y-axis, its equation is $x=-2$ and its length is $2b=2\sqrt{3}$

Vertices : $V_1(0,1), V_2(-4,1)$

Foci : $F_1(-1,1), F_2(-3,1)$

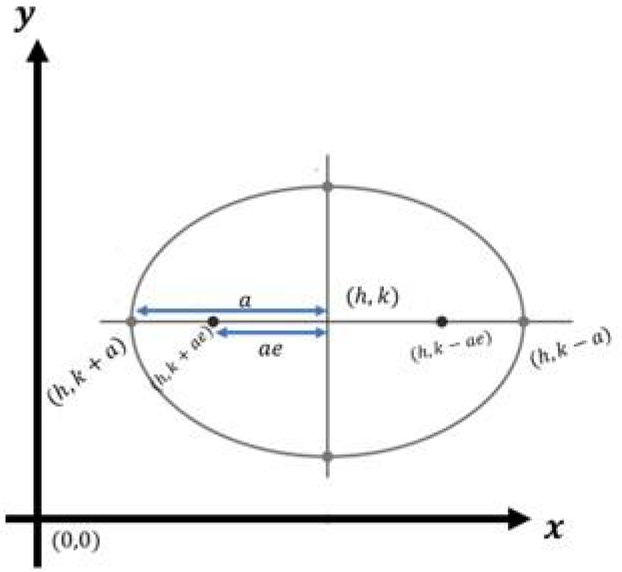
Directrices : $x=-2\pm\frac{a}{e}=-2\pm 4$

$x=2, x=-6$



Case 2 – Eclipse having the major axes parallel to the x-axis

Standard Equation form $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}+=1$	
Center	(h, k)
Focus points	(h+ae , k) , (h-ae , k)
Vertices	(h+a, k) , (h-a, k)
Directrix equations	$x=\frac{a}{e}+h$, $x=-\frac{a}{e}+h$
Equation of the major axis	$y=k$
The length of the major axis	Length = 2a
Equation of the minor axis	$x=h$
The length of the minor axis	Length = 2b



Example 4

Find the elements of the ellipse $9x^2+16y^2-54x+32y-47=0$

Solution

$$\left(9x^2-54x\right)+\left(16y^2+32y\right)-47=0,$$

$$9\left(x^2-6x\right)+16\left(y^2+2y\right)-47=0,$$

By completing square of the L.H.S, we get

Perfect Square Quadratic Expression
(with leading coefficient of 1)

$$x^2+Bx=\left(x+\frac{B}{2}\right)^2-\left(\frac{B}{2}\right)^2$$

$$9\left[(x-3)^2-9\right]+16\left[(y+1)^2-1\right]-47=0,$$

$$9(x-3)^2-81+16(y+1)^2-16-47=0,$$

$$9(x-3)^2+16(y+1)^2=81+16+47=144, \qquad \div 144$$

$$\frac{(x-3)^2}{16}+\frac{(y+1)^2}{9}=1$$

$$a^2 = 16, b^2 = 9 \Rightarrow c^2 = a^2 - b^2 = 16 - 9 = 7$$

$$\therefore a = 4, b = 3, c = \sqrt{7}$$

Eccentricity: $e = \frac{c}{a} = \frac{\sqrt{7}}{4}$

Center: $(3, -1)$

Major axis: parallel to x-axis, its equation is $y = -1$ and its length is $2a = 8$

Minor axis: parallel to y-axis, its equation is $x = 3$ and its length is $2b = 6$

Vertices : $V_1(7, -1), V_2(-1, -1)$

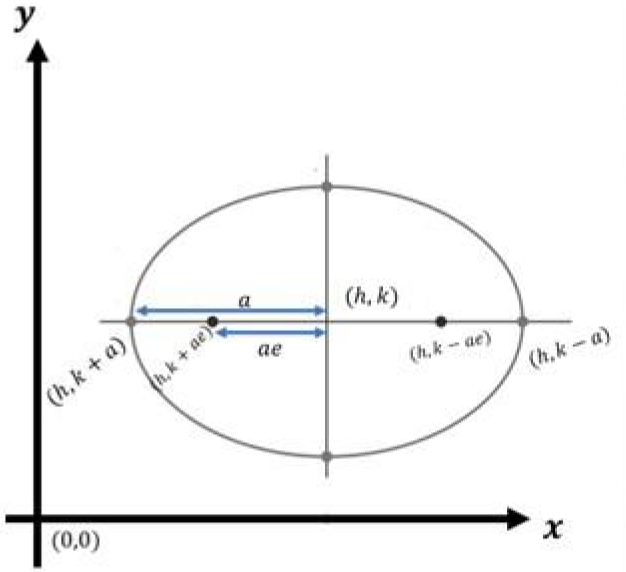
Foci : $F_1\left(3 + \sqrt{7}, -1\right), F_2\left(3 - \sqrt{7}, -1\right)$

Directrices : $x = 3 \pm \frac{a}{e} = 3 \pm \frac{16}{\sqrt{7}}$

$$x = 3 + \frac{16}{\sqrt{7}}, x = 3 - \frac{16}{\sqrt{7}}$$

Case 2 – Eclipse having the major axes parallel to the x-axis

Standard Equation form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} += 1$	
Center	(h, k)
Focus points	(h+ae , k) , (h-ae , k)
Vertices	(h+a, k) , (h-a, k)
Directrix equations	$x = \frac{a}{e} + h$, $x = -\frac{a}{e} + h$
Equation of the major axis	$y = k$
The length of the major axis	Length = 2a
Equation of the minor axis	$x = h$
The length of the minor axis	Length = 2b
	



Example 5

Find the elements of the ellipse $2x^2 + y^2 - 4x + 4y - 10 = 0.$

Solution

$$\left(2x^2 - 4x\right) + \left(y^2 + 4y\right) - 10 = 0,$$

$$2\left(x^2 - 2x\right) + \left(y^2 + 4y\right) - 10 = 0,$$

By completing square of the L.H.S,

$$2\left[(x-1)^2 - 1\right] + \left[(y+2)^2 - 4\right] - 10 = 0,$$

$$2(x-1)^2 - 2 + (y+2)^2 - 4 - 10 = 0,$$

$$2(x-1)^2 + (y+2)^2 = 2 + 4 + 16 = 16, \qquad \div 16$$

$$\frac{(x-1)^2}{8} + \frac{(y+2)^2}{16} = 1$$

$$a^2 = 16,\ b^2 = 8 \quad \Rightarrow c^2 = a^2 - b^2 = 16 - 8 = 8$$

$$\therefore a = 4,\ \ b = 2\sqrt{2},\ \ c = 2\sqrt{2}$$

Eccentricity: $e = \dfrac{c}{a} = \dfrac{2\sqrt{2}}{4} = \dfrac{\sqrt{2}}{2} = \dfrac{1}{\sqrt{2}}$

Center: $(1, -2)$

Major axis: parallel to y-axis, its equation is $x = 1$ and its length is $2a = 8$

Minor axis: parallel to x-axis, its equation is $y = -2$ and its length is $2b = 4\sqrt{2}$

Vertices : $V_1(1, -2+4), V_2(1, -2-4)$

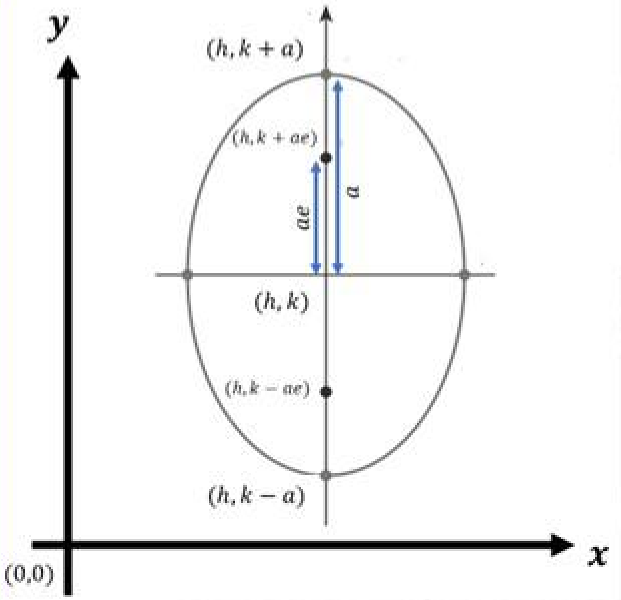
Foci : $F_1\left(1, -2+2\sqrt{2}\right), F_2\left(1, -2-2\sqrt{2}\right)$

Directrices : $y = -2 \pm \dfrac{a}{e} = -2 \pm 4\sqrt{2}$

$$y = -2 + 4\sqrt{2}, x = -2 - 4\sqrt{2}$$

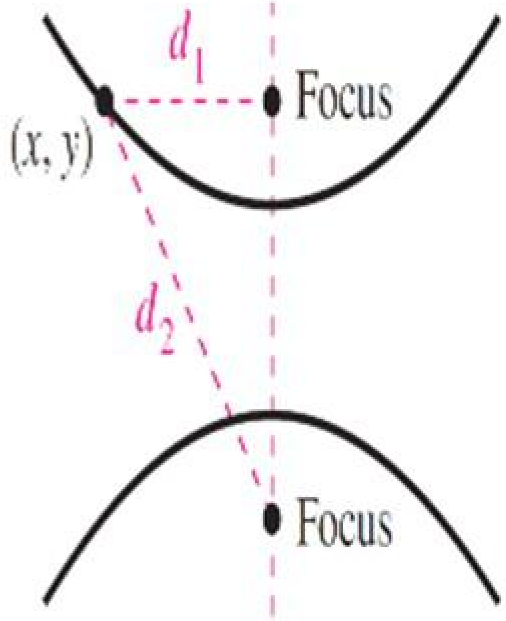
Case 1 – Eclipse having the major axes parallel to the y-axis

Standard Equation form $\dfrac{(y-k)^2}{a^2} + \dfrac{(x-h)^2}{b^2} = 1$	
Center	(h, k)
Focus points	(h, k+ae) , (h, k-ae)
Vertices	(h, k+a) , (h, k-a)
Directrix equations	$y = \frac{a}{e} + k$, $y = -\frac{a}{e} + k$
Equation of the major axis	$x = h$
The length of the major axis	Length = 2a
Equation of the minor axis	$y = k$
The length of the minor axis	Length = 2b



Hyperbola

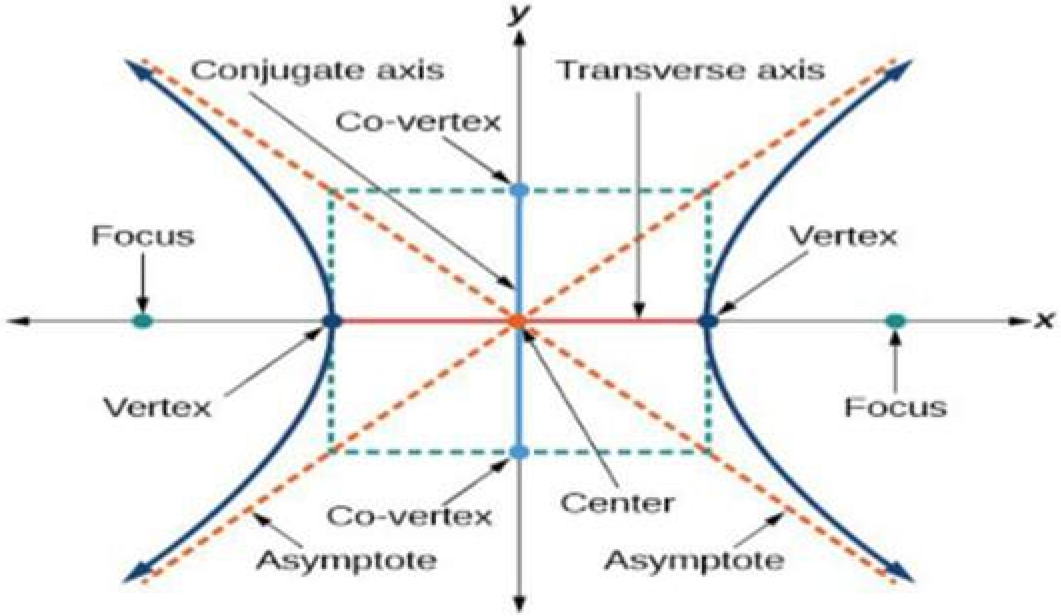
Definition: A hyperbola is the set of all points (x, y) in the plane, such that the *difference* of their distances from two distinct fixed points, called the *foci* (plural of focus) of the hyperbola, is a constant=$2a$.



$d_2 - d_1$ is a positive constant.

- The distance between the two foci equals **2c.**

- The chord through the foci is called ***transverse axis*** and its length = ***2a***.
- The ***transverse axis*** intersects the hyperbola at two points called ***vertices***.
- The ***center*** is the midpoint between the vertices (or the midpoint between the foci) .
- The line segment, of length ***2b***, perpendicular to the transverse axis at the center is ***the conjugate axis.***
- The endpoints of the conjugate axis are called ***co-vertices***.

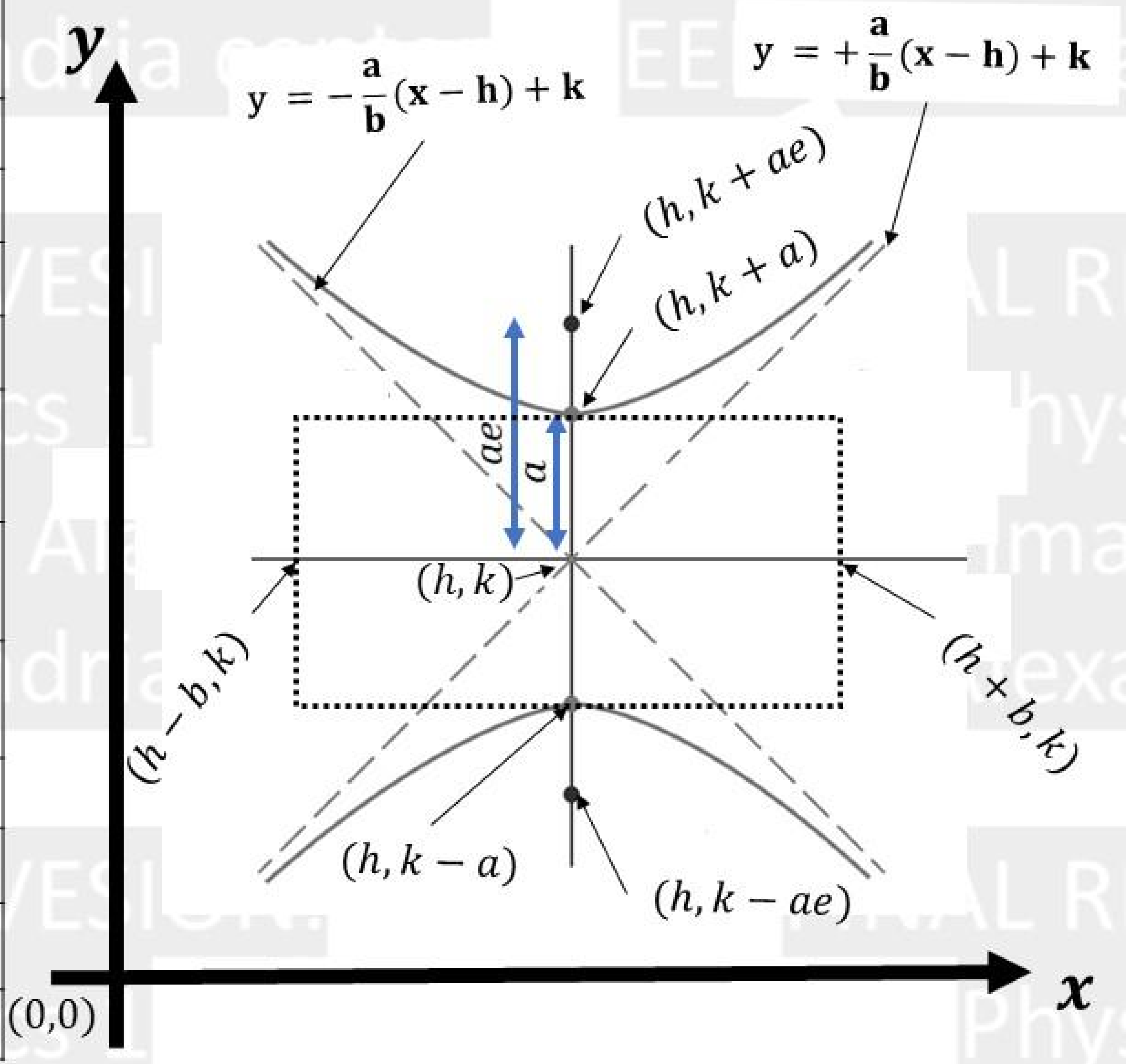


- For the hyperbola, we always have $c > a, c > b, c^2 = a^2 + b^2$ and the eccentricity $e = \dfrac{c}{a}$.
- Each hyperbola has ***two asymptotes*** that intersect at the center of the hyperbola. The *asymptotes* pass through the corners of a rectangle of dimensions 2*a* by 2*b*, with its center at (*h*, *k*).

Case 1 – Hyper-parabola having the transverse axes parallel to the y-axis (vertical)

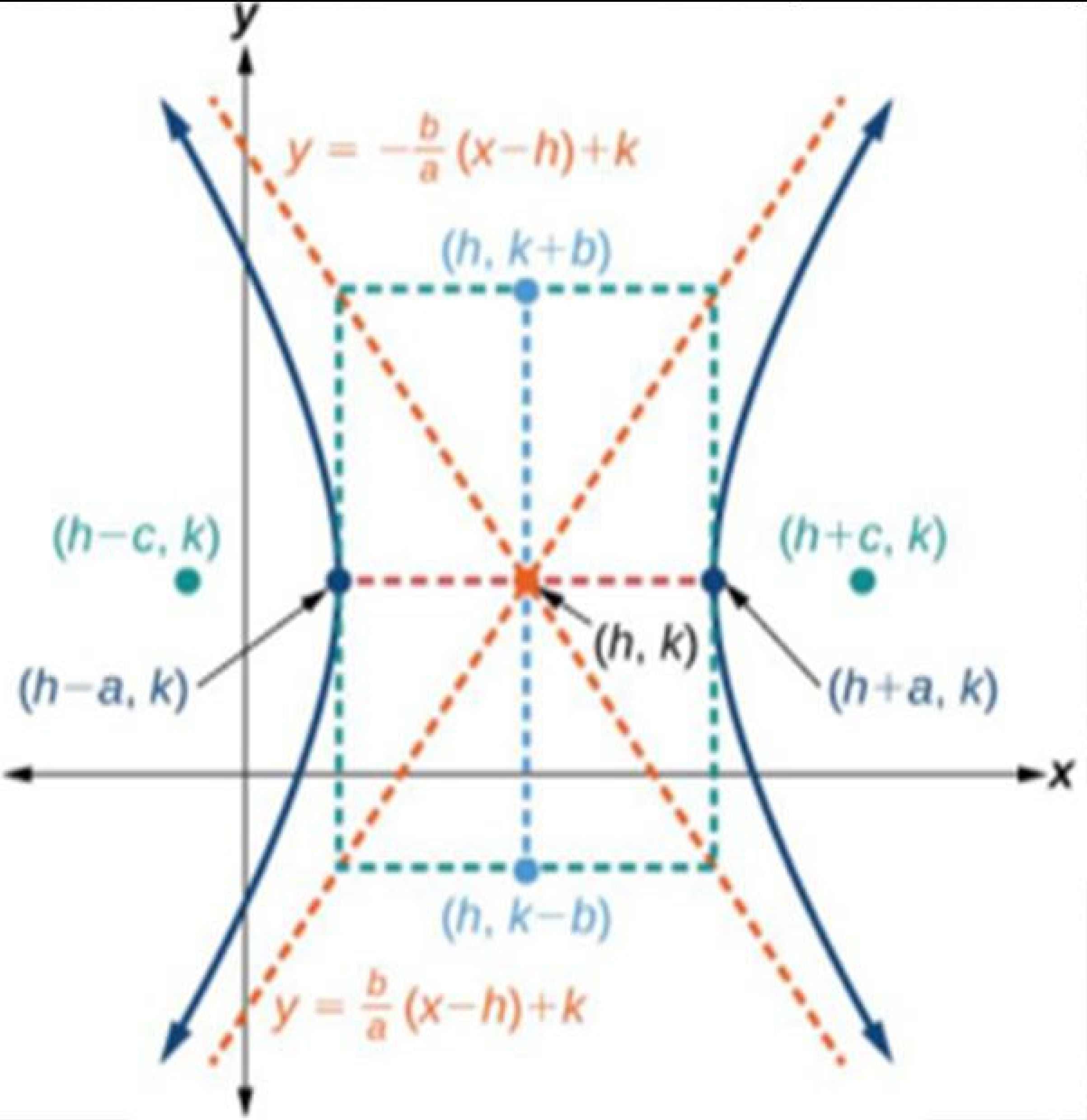
Standard Equation form : $\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$

Center	(h, k)
Focus points	(h, k+ae) , (h, k-ae)
Vertices	(h, k+a) , (h, k-a)
Co-vertices	(h+b, k) , (h-b, k)
Directrix equations	$y=\frac{a}{e}+k$ | $y=-\frac{a}{e}+k$
Asymptotes' equations	$y=+\frac{a}{b}(x-h)+k$
	$y=-\frac{a}{b}(x-h)+k$
Equation of the transverse axis	$x=h$
The length of the transverse axis	Length = 2a
Equation of the conjugate axis	$y=k$
The length of the conjugate axis	Length = 2b



Case 2 – Hyper-parabola having the transverse axes parallel to the x-axis (horizontal)

Standard Equation form : $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$		
Center	(h, k)	
Focus points	(h+ae, k) , (h-ae, k)	
Vertices	(h+a, k) , (h-a, k)	
Co-vertices	(h, k+b) , (h, k-b)	
Directrix equations	$x = \frac{a}{e} + h$	$x = -\frac{a}{e} + h$
Asymptotes' equations	$y = +\frac{b}{a}(x-h)+k$	
	$y = -\frac{b}{a}(x-h)+k$	
Equation of the transverse axis	$y = k$	
The length of the transverse axis	Length = 2a	
Equation of the conjugate axis	$x = h$	
The length of the conjugate axis	Length = 2b	



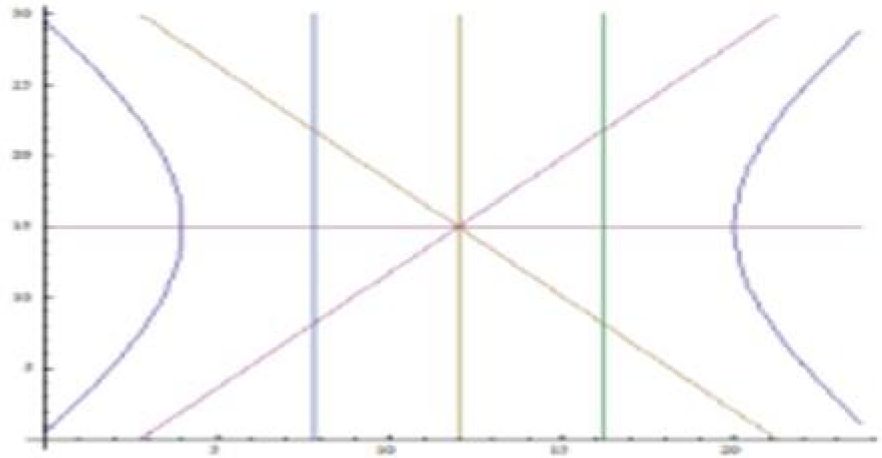
Example 6

Find the elements and sketch the hyperbola $\frac{(x-12)^2}{64}-\frac{(y-15)^2}{169}=1$

Solution

$$a^2=64,\ b^2=169 \Rightarrow c^2=a^2+b^2=64+169=233$$

$$\therefore a=8,\ b=13,\ c=\sqrt{233}$$



Eccentricity: $e=\dfrac{c}{a}=\dfrac{\sqrt{233}}{8}$

Center: $(12,15)$

Transverse axis: parallel to x-axis, its equation is $y=15$

Conjugate axis: parallel to y-axis, its equation is $x=12$

Case 2 – Hyper-parabola having the transverse axes parallel to the x-axis (horizontal)

Center: $(12,15)$

Vertices : $V_1(20,15), V_2(4,15)$

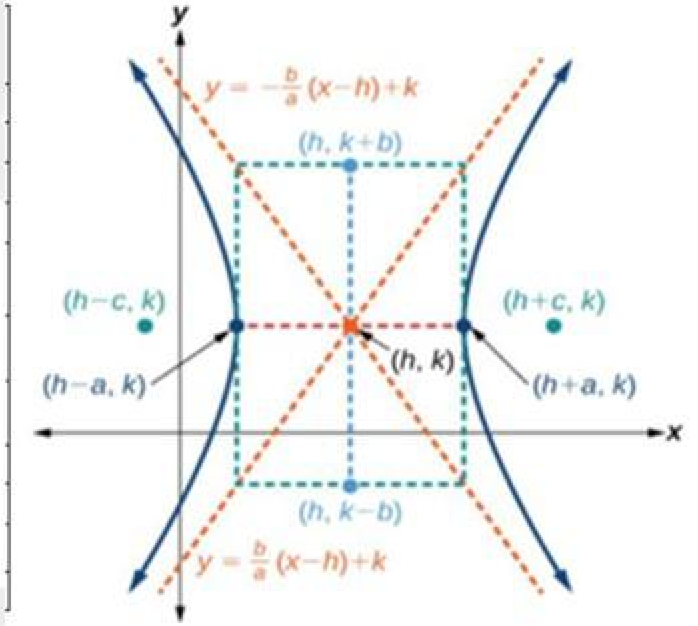
Foci : $F_1\left(12+\sqrt{233},15\right), F_2\left(12-\sqrt{233},15\right)$

Directrices : $x = 12 \pm \dfrac{a}{e} = 12 \pm \dfrac{64}{\sqrt{233}}$

$$x = 12 + \frac{64}{\sqrt{233}}, x = 12 - \frac{64}{\sqrt{233}}$$

Asymptotes : $y-15 = \pm\dfrac{13}{8}(x-12) \Rightarrow y = \pm\dfrac{13}{8}(x-12)+15$

Standard Equation form : $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$		
Center	(h, k)	
Focus points	(h+ae, k) , (h-ae, k)	
Vertices	(h+a, k) , (h-a, k)	
Co-vertices	(h, k+b) , (h, k-b)	
Directrix equations	$x = \frac{a}{e} + h$	$x = -\frac{a}{e} + h$
Asymptotes' equations	$y = +\frac{b}{a}(x-h)+k$	
	$y = -\frac{b}{a}(x-h)+k$	
Equation of the transverse axis	$y = k$	
The length of the transverse axis	Length = 2a	
Equation of the conjugate axis	$x = h$	
The length of the conjugate axis	Length = 2b	



Example 7

Find the elements and sketch the hyperbola $2x^2 - 3y^2 + 8x + 6y + 17 = 0$

Solution

$$\left(2x^2 + 8x\right) + \left(-3y^2 + 6y\right) + 17 = 0,$$

$$2\left(x^2 + 4x\right) - 3\left(y^2 - 2y\right) + 17 = 0,$$

By completing square of the L.H.S, we find that

$$2\left[(x+2)^2 - 4\right] - 3\left[(y-1)^2 - 1\right] + 17 = 0,$$

$$2(x+2)^2 - 8 - 3(y-1)^2 + 3 + 17 = 0,$$

$$2(x+2)^2 - 3(y-1)^2 = -12, \qquad \div -12$$

$$\frac{(y-1)^2}{4} - \frac{(x+2)^2}{6} = 1.$$

$$a^2 = 4,\ b^2 = 6 \Rightarrow c^2 = a^2 + b^2 = 4 + 6 = 10$$

$$\therefore a = 2,\ b = \sqrt{6},\ c = \sqrt{10}$$

Eccentricity: $e = \frac{c}{a} = \frac{\sqrt{10}}{2} = \sqrt{\frac{5}{2}}$

Center: $(-2,1)$

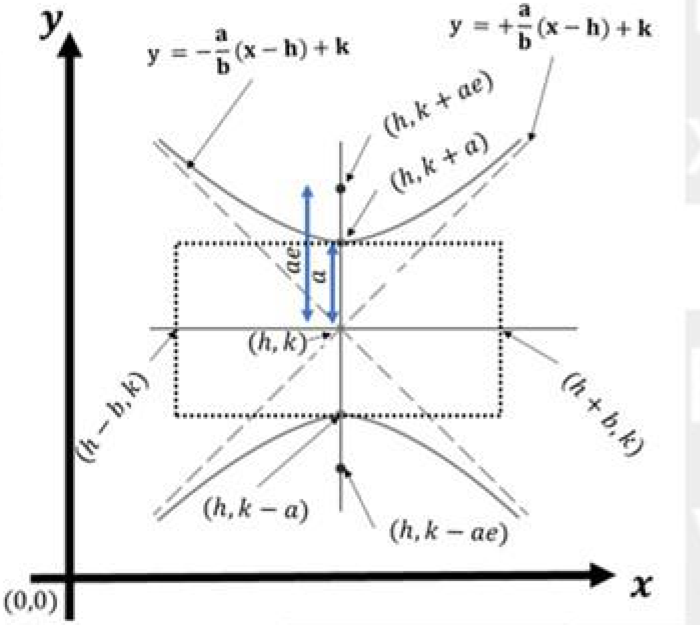
Transverse axis: parallel to y-axis, its equation is $x = -2$

Conjugate axis: parallel to x-axis, its equation is $y = 1$

Vertices : $V_1(-2,3), V_2(-2,-1)$

Case 1 – Hyper-parabola having the transverse axes parallel to the y-axis (vertical)

Standard Equation form : $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$		
Center	(h, k)	
Focus points	(h, k+ae) , (h, k-ae)	
Vertices	(h, k+a) , (h, k-a)	
Co-vertices	(h+b, k) , (h-b, k)	
Directrix equations	$y = \frac{a}{e} + k$	$y = -\frac{a}{e} + k$
Asymptotes' equations	$y = +\frac{a}{b}(x-h) + k$	
	$y = -\frac{a}{b}(x-h) + k$	
Equation of the transverse axis	$x = h$	
The length of the transverse axis	Length = 2a	
Equation of the conjugate axis	$y = k$	
The length of the conjugate axis	Length = 2b	



Foci : $$F_1\left(-2,1+\sqrt{10}\right),F_2\left(-2,1-\sqrt{10}\right)$$

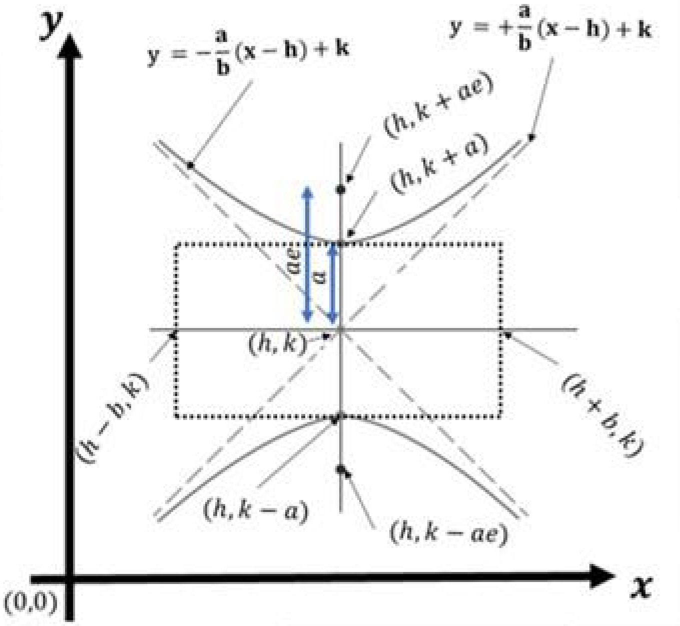
Directrices : $$y=1\pm\frac{a}{e}=1\pm\frac{4}{\sqrt{10}}$$

$$y=1+\frac{4}{\sqrt{10}},y=1-\frac{4}{\sqrt{10}}$$

Asymptotes : $$y=\pm\frac{2}{\sqrt{6}}(x+2)+1.$$

Case 1 – Hyper-parabola having the transverse axes parallel to the y-axis (vertical)

Standard Equation form : $\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$	
Center	(h, k)
Focus points	(h, k+ae) , (h, k-ae)
Vertices	(h, k+a) , (h, k-a)
Co-vertices	(h+b, k) , (h-b, k)
Directrix equations	$y=\frac{a}{e}+k$, $y=-\frac{a}{e}+k$
Asymptotes' equations	$y=+\frac{a}{b}(x-h)+k$ $y=-\frac{a}{b}(x-h)+k$
Equation of the transverse axis	$x=h$
The length of the transverse axis	Length = 2a
Equation of the conjugate axis	$y=k$
The length of the conjugate axis	Length = 2b



Example 8

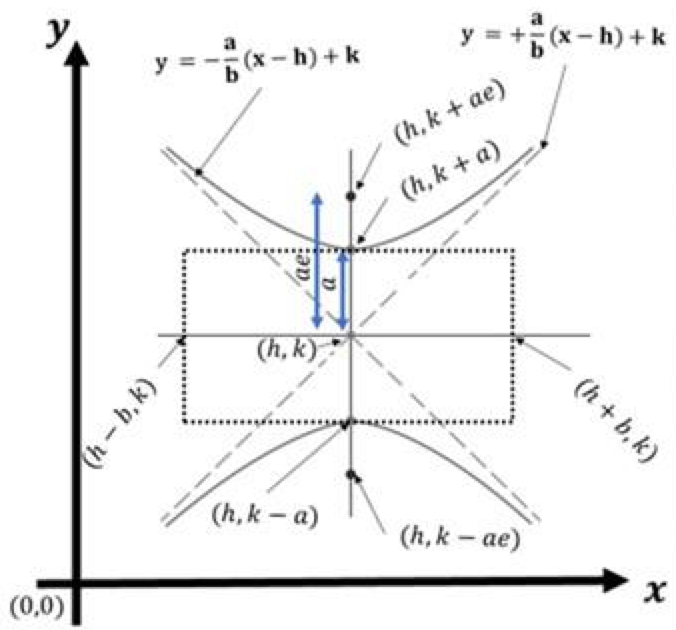
Write the equation of a hyperbola with vertices (0, -6) and (0,6) and asymptote $y = 3/4x$

Example 8

Write the equation of a hyperbola with vertices $\boxed{(0,-6) \text{ and } (0,6)}$ and asymptote having an equation : y=(3/4)x

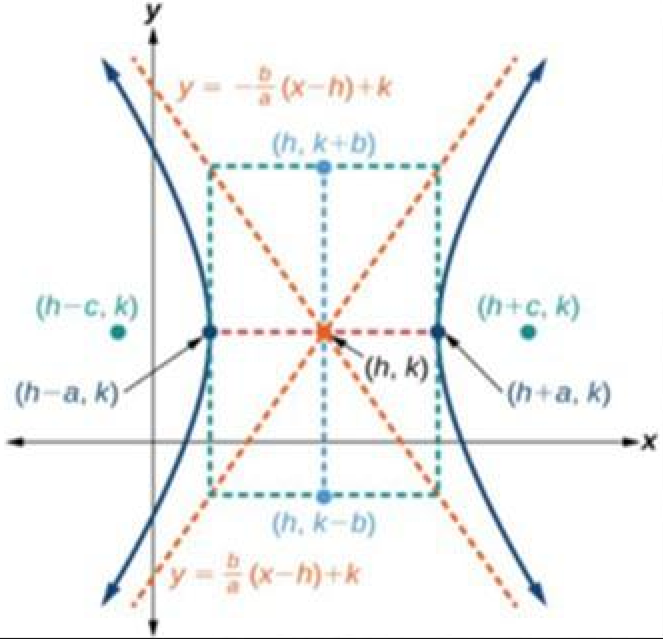
Case 1 – Hyper-parabola having the transverse axes parallel to the y-axis (vertical)

Standard Equation form : $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$		
Center	(h, k)	
Focus points	(h, k+ae) , (h, k-ae)	
Vertices	(h, k+a) , (h, k-a)	
Co-vertices	(h+b, k) , (h-b, k)	
Directrix equations	$y = \frac{a}{e} + k$	$y = -\frac{a}{e} + k$
Asymptotes' equations	$y = +\frac{a}{b}(x-h) + k$	
	$y = -\frac{a}{b}(x-h) + k$	
Equation of the transverse axis	$x = h$	
The length of the transverse axis	Length = 2a	
Equation of the conjugate axis	$y = k$	
The length of the conjugate axis	Length = 2b	



Case 2 – Hyper-parabola having the transverse axes parallel to the x-axis (horizontal)

Standard Equation form : $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$		
Center	(h, k)	
Focus points	(h+ae, k) , (h-ae, k)	
Vertices	(h+a, k) , (h-a, k)	
Co-vertices	(h, k+b) , (h, k-b)	
Directrix equations	$x = \frac{a}{e} + h$	$x = -\frac{a}{e} + h$
Asymptotes' equations	$y = +\frac{b}{a}(x-h) + k$	
	$y = -\frac{b}{a}(x-h) + k$	
Equation of the transverse axis	$y = k$	
The length of the transverse axis	Length = 2a	
Equation of the conjugate axis	$x = h$	
The length of the conjugate axis	Length = 2b	



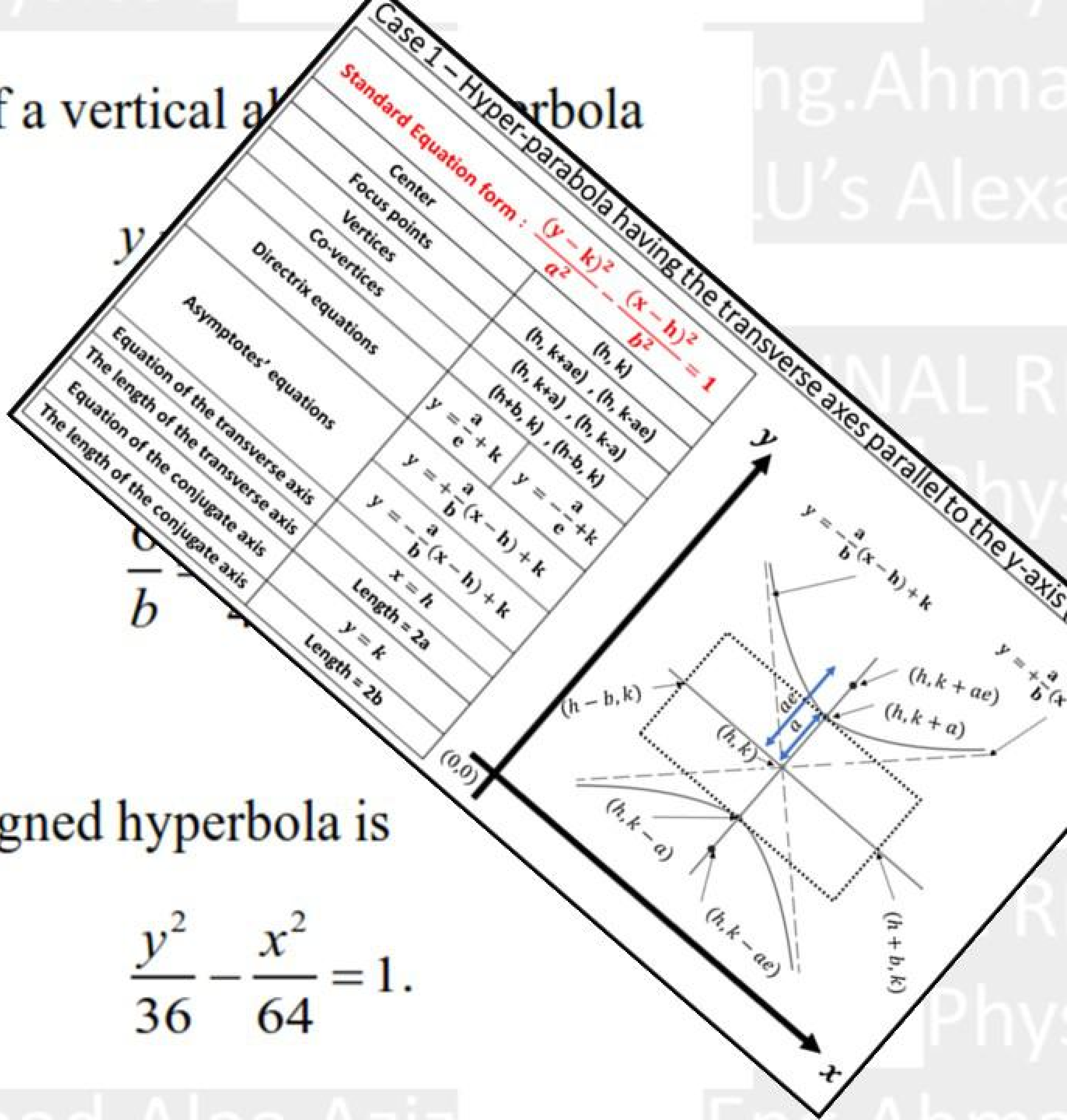
- The equation for the asymptote of a vertical al… …rbola

It is given that

- Therefore, b=8.

- The formula for this vertically aligned hyperbola is

$$\frac{y^2}{36}-\frac{x^2}{64}=1.$$


Case 1 – Hyper-parabola having the transverse axes parallel to the y-axis

Standard Equation form : $\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$

Center	(h, k)	
Focus points	(h, k+ae) , (h, k-ae)	
Vertices	(h, k+a) , (h, k-a)	
Co-vertices	(h+b, k) , (h-b, k)	
Directrix equations	$y=\frac{a}{e}+k$	$y=-\frac{a}{e}+k$
Asymptotes' equations	$y=+\frac{a}{b}(x-h)+k$	$y=-\frac{a}{b}(x-h)+k$
Equation of the transverse axis	$x=h$	
The length of the transverse axis	Length = 2a	
Equation of the conjugate axis	$y=k$	
The length of the conjugate axis	Length = 2b	

Exercises

1- Determine the elements of the following conic sections

(a) $2x^2 - 3y^2 + 8x + 6y + 17 = 0.$

(b) $x^2 + 5y^2 + 6x - 40y + 84 = 0.$

(c) $y^2 + 8x + 6y + 1 = 0.$

(d) $25x^2 + 16y^2 + 150x - 128y - 1119 = 0.$

2- Find the equation of the ellipse that passes through origin and has its foci at the points (1,0) and (3,0)?

3- Find the standard form of the equation of the hyperbola with vertices (–2, –4), (–2, 6) and foci (–2, –5), (–2, 7) ?

Special thanks to Eng,Waleed Khaled Sohag center for his efforts

Thank you