
بسم الله الرحمن الرحيم

EELU

الجامعة المصرية للتعلم الإلكترونى

Egyptian E-Learning University

MATH – 1

BY

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CHAPTER 5

Applications of Definite Integrals

LECTURE 9.

Area Between Curves

Aims and Objectives:

(1) Introduce the notion of the area.

(2) Understand methods of finding area.

(3) Explain the concepts of volume of a solid.

(4) Show how the volume of the solid can be generated.

(5) Evaluate volumes of solid of revolution.

(6) Have a strong intuitive feeling for these important concepts.

Notion of the Area:

Applications of Definite Integrals:

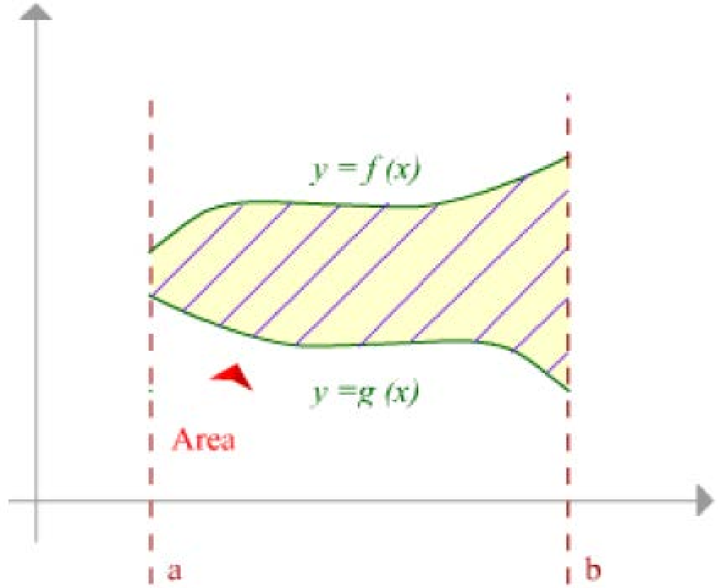
The definite integral is useful for solving a large variety of applied problems. In this chapter we shall discuss area, volume, and lengths of curves.

Here you find a brief introduction to applications of definite integrals and area between curves. We introduce first the notion of area bounded by the curve of the function, the x axis, the lines $x = a$ and $x=b$.

This is mathematical and graphical illustration of area between curves.

Theorem:

If f and g are continuous and $f(x) \geq g(x)$ for all x in $[a, b]$, then the area A of the region bounded by the graphs of f, g, $x = a$ and $x = b$ is $A = \int_a^b [f(x) - g(x)]dx$



$$A = \int_a^b f(x)dx - \int_a^b g(x)dx = \int_a^b [f(x) - g(x)]dx$$

This formula for A can be extended to the case in which f or g is negative for some x in $[a,b]$.

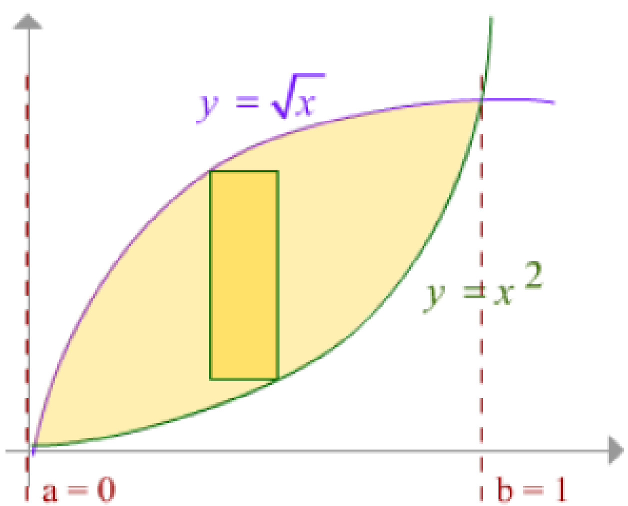
Example 1:

Find the area of the region bounded by the graphs of the equations $y = x^2$ and $y = \sqrt{x}$.

Solution:

We shall employ the Riemann sum approach.

The region and a typical rectangle are sketched in the following figure.



length = $\sqrt{w_i} - w_i^2$

Width = Δx_i

Area = $(\sqrt{w_i} - w_i^2)\Delta x_i$

As indicated in the figure, the length of typical rectangle is $\sqrt{w_i} - w_i^2$ and its area is $(\sqrt{w_i} - w_i^2)\Delta x_i$.

Using the theorem with $a = 0$ and $b = 1$ we obtain

$$A = \lim_{\|p\| \to 0} \sum_i (\sqrt{w_i} - w_i^2)\Delta x_i \qquad = \int_0^1 (\sqrt{x} - x^2)dx$$

$$= [\frac{2}{3}x^{3/2} - \frac{1}{3}x^3]_0^1 \quad = \quad \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

The area can be found by direct substitution in the theorem with $f(x) = \sqrt{x}$ and $g(x) = x^2$

$$A = \int_a^b [f(x) - g(x)]dx$$

Example 2:

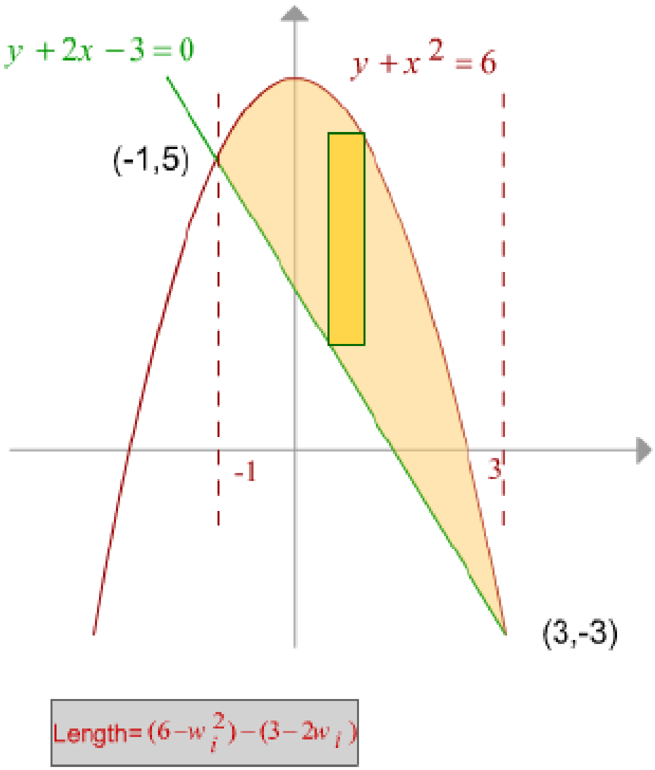
Find the area of the region bounded by the graphs of

$y + \chi^2 = 6$ and $y + 2\chi - 3 = 0$

Solution:

The region and a typical rectangle are sketched in the figure.

The points of intersection $(-1,5)$ and $(3,-3)$ of the two graphs may be found by solving the two given equations simultaneously.



It is necessary to solve each equation for y terms of χ,

obtaining $y = 6 - x^2$ and $y = 3 - 2x$

The function $f(x) = 6 - x^2$ and $g(x) = 3 - 2x$

As shown in the figure the length of a typical

rectangle is $(6 - w_i^2) - (3 - 2w_i)$

Where is some number in the subinterval of a

partition $\mathcal{P}$ of $[-1,3]$ the area of this rectangle is

$$A = \lim_{\|p\| \to 0} \sum_i [(6 - w_i^2) - (3 - 2w_i)]\Delta x_i$$

$$= \int_{-1}^{3} [(6 - x^2) - (3 - 2x)]dx$$

Then $$= \int_{-1}^{3} (3 - x^2 + 2x)dx$$

$$= [3x - \frac{x^3}{3} + x^2]_{-1}^{3}$$

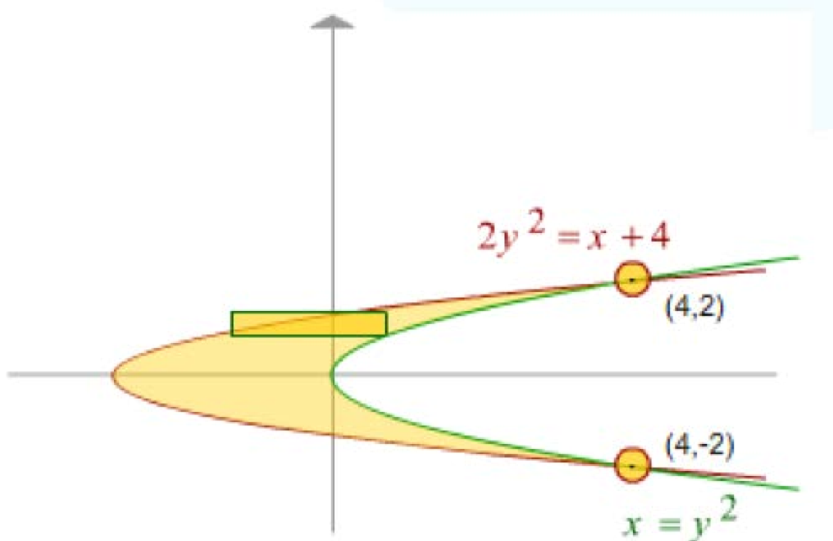
$$= [9 - \frac{27}{3} + 9] - [-3 - (-\frac{1}{3}) + 1] = \frac{32}{3}$$

Example 3:

Find the area of the region bounded by the graphs of the equations $2y^2 = \chi + 4$ and $\chi = y^2$

Solution:

One of two sketches of the region can be used to find the area, we use the integration with respect to y to find the area with only one integration.



Length = $w_i^2 - (2w_i^2 - 4)$

Width = Δy_i

Letting $f(y)=y^2,\ \ g(y)=2y^2-4$, the length $f(w_i)-g(w_i)$ of a horizontal rectangle is $w_i^2-(2w_i^2-4)$

since the width is Δy the area of the rectangle is

Hence, the area of $\mathcal{R}$ is $[w_i^2-(2w_i^2-4)]\Delta y_i$

$$A = \lim_{\Delta y \to 0} \sum_i [w_i^2-(2w_i^2-4)\Delta y_i$$

$$= \int_{-2}^{2} [y^2-(2y^2-4)]dy$$

$$= \int_{-2}^{2} (4-y^2)dy$$

$$= [4y-\frac{y^3}{3}]_{-2}^{2} = [8-\frac{8}{3}]-[-8-(-\frac{8}{3})] = \frac{32}{3}$$

THANK YOU