Q.1 If the derivative of a function is zero everywhere, the function must necessarily be?

A -linear
B – Constant
C – Quadratic
D – Everywhere zero

$\because f(x) = C \quad \forall\ C$ is constant.
$\therefore f'(x) = \frac{dy}{dx} =$ Zero.

Ans : b

Q.2 The derivative of a function at a point is:

a. An equation
b. A function
c. A number
d. Zero

$\because f(x) = nx$
$\therefore f'(x) = n \quad (n$ is numb$)$

Ans : C

Q.3 Find the slope of $\frac{d}{dx}(x^2 + 5x)$

a. 2x+5
b. 2x+5x
c. 2x
d. $x^2 + 5$

$\because y = x^2 + 5x$
$\therefore \frac{dy}{dx} = 2x + 5$

Ans : A

Q.4 Find the slope of $f(x) = -3x^2 - 6x$ at x = 1?

a. $f'(x) = 6x$
b. $m = 0$
c. $f'(x) = -6x - 6$
d. $m = -12$

$\because f(x) = -3x^2 - 6x$
$\therefore f'(x) = -6x - 6$
$\because$ slope $= m = f'(1)$
$= -6(1) - 6 = -12$ ###

Ans : d

Q.5 find the derivatives of the function $f(x) = \sqrt{x^2 - 1}$

a. $\frac{1}{2x\sqrt{x^2-1}}$
b. $\frac{1}{2\sqrt{x^2-1}}$
c. $\frac{x}{\sqrt{x^2-1}}$
d. $\frac{1}{\sqrt{x^2-1}}$

$\because f(x) = \sqrt{x^2-1}$
$\therefore f'(x) = \frac{\text{مشتقة ما بداخل الجذر}}{2 * \text{الجذر}}$
$= \frac{2x}{2\sqrt{x^2-1}} = \frac{x}{\sqrt{x^2-1}}$ ###

Ans : C

Q.6 If $f(x) = sin^2(x)$, find $f'(\frac{\pi}{4})$.

a. -1
b. 1
c. 0.5
d. -2

$\because f(x) = \sin^2(x) = [\sin(x)]^2$
$\therefore f'(x) =$ مشتقة ما بداخل القوس * مشتقة القوس
$f'(x) = 2 * \sin(x) * \cos(x)$
$\therefore f'(\frac{\pi}{4}) = 2 * \sin(\frac{\pi}{4}) * \cos(\frac{\pi}{4})$
$= 2 * \frac{\sqrt{2}}{2} * \frac{\sqrt{2}}{2} = 1$ ###

Ans : B

خلي بالك تعوض عن قيمة $\pi = 180$ مش بـ 3.14 لأنه دي زاوية.

Q.7 The graph of $y = 2x^3 - x^4$ has how many local maximums:

a. 1
b. 2
c. 3
d. 4

$\because y = 2x^3 - x^4$

$\therefore y' = 6x^2 - 4x^3$

$\because y' = 0$

$\therefore 6x^2 - 4x^3 = 0$

$2x^2(3 - 2x) = 0$

$2x^2 = 0 \Rightarrow x = 0$

$3 - 2x = 0 \Rightarrow x = \frac{3}{2}$

$\because f(0) = 2(0)^3 - (0)^4 = 0$

$\because f(\frac{3}{2}) = 2(\frac{3}{2})^3 - (\frac{3}{2})^4 = \frac{27}{16}$ → max

Ans : A

Q.8 The point of local maxima for the function (0.46,2.87)

a. $\frac{\pi}{3}$
b. $\frac{\pi}{2}$
c. $\frac{\pi}{4}$
d. $\frac{\pi}{6}$

Ans : C

Q.9 Find the value of a such that the function $f(x) = xe^{ax}$ has a critical point at $x = 3$

a. $\frac{1}{3}$
b. 3
c. $\frac{-1}{3}$
d. -3

$\because f(x) = xe^{ax}$

$\therefore f'(x) = x.e^{ax}.a + e^{ax}.1$

$\therefore f'(x) = axe^{ax} + e^{ax}$ $\because f'(x) = 0$ and $x = 3$

$\therefore 3ae^{3a} + e^{3a} = 0 \Rightarrow e^{3a}(3a + 1) = 0$

$\therefore 3a + 1 = 0 \Rightarrow 3a = -1 \Rightarrow a = \frac{-1}{3}$ ##

Ans : C

Q.10 The minimum value of the function $f(x)=x^3-3x^2-24x+100$ in the interval $[-3, 3]$ is

a. 20
b. 28
c. 16
d. 32

12 الحل في صفحة

Ans : B

Q.11 Find the derivative of $\theta = \frac{3r+2}{2r+3}$

a) $\frac{d\theta}{dr} = \frac{5}{(2r+3)^2}$
b) $\frac{d\theta}{dr} = \frac{12r+5}{(2r+3)^2}$
c) $\frac{d\theta}{dr} = \frac{12r+13}{(2r+3)^2}$
d) $\frac{d\theta}{dr} = \frac{5}{2r+3}$

12 الحل في صفحة

Ans : A

Q.12 Find the derivative of $y = \sqrt{1+\sqrt{2x}}$

a) $\frac{1}{4\sqrt{x+x\sqrt{x}}}$
b) $\frac{1}{2\sqrt{2x+2x\sqrt{2x}}}$
c) $\frac{1}{\sqrt{x+x\sqrt{x}}}$
d) $\frac{1}{4\sqrt{2x+2x\sqrt{2x}}}$

12 الحل في صفحة

Ans : B

Q.13 Ans : A

$y = \sqrt{\frac{x-1}{x+1}}$

a) $\frac{1}{(x+1)\sqrt{x^2-1}}$

b) $\frac{1}{(x+1)\sqrt{x-1}}$

c) $\frac{1}{(x-1)\sqrt{x^2-1}}$

d) $\frac{1}{(x+1)\sqrt{x^2+1}}$

الحل ص 13

Q.14 Ans : B

$y = x^4 + 2x + 5$ y'' is :

a) $3x^2 + 2$

b) $12x^2$

c) $x^3 + 2$

d) 0

$\because y = x^4 + 2x + 5$

$\therefore y' = 4x^3 + 2$

$\therefore y'' = 12x^2$

Q.15 Ans : d

$y = e^{ax}$, $\frac{d^n y}{dx^n}$ is:

a) e^{ax}

b) e^{ax-n}

c) ae^{ax-n}

d) $a^n e^a x$

$\because y = e^{ax}$

$\therefore \frac{dy}{dx} = a e^{ax}$

$\therefore \frac{d^n y}{dx^n} = a^n e^{ax}$

Q.16 Ans : c

$y = (2-x)^3$, *has critical points at* $x =$:

a) {0}

b) ± 2

c) 2

d) None

$\because y = (2-x)^3$

$\therefore y' = 3(2-x)^2 * -1$ $\because y' = 0$

$\therefore -3(2-x)^2 = 0 \Rightarrow (2-x)^2 = 0 \Rightarrow 2-x = 0 \Rightarrow \boxed{x=2}$

Q.17 Ans : d

$y = (2-x)^3$ has maximum value at x=

a) 0

b) 2

c) {0,2}

d) None

هذا السؤال السابق حلها كنا قيمة واحدة للنقاط الحرجة minimum عليها يعني قيمة واحدة لـ x سينفقش أقول عليها maximum أو none . لذلك إجابة هذا السؤال none.

Q.18 Ans : d

$y = 3 + 2x + x^2$ has maximum value at :

a) -1

b) 0

c) 2

d) None

$\because y = 3 + 2x + x^2$

$\therefore y' = 2 + 2x$ $\because y' = 0$

$\therefore 2 + 2x = 0 \Rightarrow 2x = -2 \Rightarrow \boxed{x=-1}$ critical Point

$\therefore$ maximum value at x = none.

Q.19	$y = x^3 - 6x^2 + 9x - 8$ has maximum value at: a) X=1 b) X=3 c) A,b d) None الحل ص 13	Ans : a
Q.20	$y = x^3 - 6x^2 + 9x - 8$, has minimum value at: a) X=1 b) X=3 c) A,b d) None الحل ص 13	Ans : b
Q.21	If functions f and g are such that f(x) = g(x) + k where k is a constant, then (A) f '(x) = g '(x) + k (B) f '(x) = g '(x) (C) None of the above ∵ f(x) = g(x) + k ∴ f'(x) = g'(x) + 0 ∴ f'(x) = g'(x) ### ∴ Answer B	Ans : b
Q.22	If $\sin(xy) + \cos(xy) = 0$ then $\frac{dy}{dx} =$ (A) $\frac{y}{x}$ (B) $-\frac{y}{x}$ (C) $-\frac{x}{y}$ (D) $\frac{x}{y}$ الحل ص 14	Ans : b
Q.23	If $y = x + e^x$ then $\frac{d^2x}{dy^2}$ is : (A) e^x (B) $-\frac{e^x}{(1+e^x)^3}$ (C) $-\frac{e^x}{(1+e^x)^2}$ (D) $\frac{-1}{(1+e^x)^3}$ الحل ص 14	Ans : b
Q.24	If $x^2y + y^3 = 2$ then the value of $\frac{d^2y}{dx^2}$ at the point (1, 1) is : (A) $-\frac{3}{4}$ (B) $-\frac{3}{8}$ (C) $-\frac{5}{12}$ (D) none الحل ص 14	Ans : b

Q.25 If $y = e^{\sqrt{x}} + e^{-\sqrt{x}}$ then $\frac{dy}{dx}$ equals

(A) $\frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2\sqrt{x}}$ (B) $\frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2x}$ (C) $\frac{1}{2\sqrt{x}}\sqrt{y^2-4}$ (D) $\frac{1}{2\sqrt{x}}\sqrt{y^2+4}$

Ans : A,C

Q.26

(A) $f'(x) = -\frac{1}{2\sqrt{x}}$
(B) $f'(x) = \frac{1}{\sqrt{x}}$
(C) $f'(x) = \frac{1}{2\sqrt{x}}$
(D) $f'(x) = \sqrt{x}$

$f(x) = x^{\frac{1}{2}}$

$\therefore f(x) = x^{\frac{1}{2}} = \sqrt{x}$

$\therefore f'(x) = \frac{1}{2\sqrt{x}}$

$\therefore$ Answer [C]

Ans : C

Q.27

(A) $f'(x) = 15x^2 + 470x$
(B) $f'(x) = 5x^2 + 470x$
(C) $f'(x) = 10x$
(D) $f'(x) = 15x^2 - 470x$

$f(x) = 5x^2(x+47)$

$\therefore f(x) = 5x^2(x+47)$

$f(x) = 5x^3 + 235x^2$

$\therefore f'(x) = 15x^2 + 470x$. $\therefore$ Answer [A]

Ans : a

Q.28

(A) $f'(x) = \frac{5x^2-470x}{(x+47)^2}$
(B) $f'(x) = \frac{10x^2+470x}{(x+47)}$
(C) $f'(x) = 10x$
(D) $f'(x) = \frac{5x^2+470}{(x+47)^2}$
(E) None of the above

$f(x) = \frac{5x^2}{x+47}$

$\therefore f(x) = \frac{5x^2}{x+47}$

$\therefore f'(x) = \frac{10x(x+47) - (5x^2 \cdot 1)}{(x+47)^2}$

$= \frac{10x^2+470x-5x^2}{(x+47)^2}$

$= \frac{5x^2+470x}{(x+47)^2}$ $\therefore$ Answer [E]

Ans : e

Q.29

(A) $f'(x) = 15x^2 + 470x$
(B) $f'(x) = 10x - 470$
(C) $f'(x) = 10x + 470$
(D) $f'(x) = 15x^2 - 470x$

$f(x) = 5(x+47)^2$

$\therefore f(x) = 5(x+47)^2$

$\therefore f'(x) = 5 * 2(x+47)^1 \cdot 1$

$= 10(x+47)$

$= 10x + 470$. $\therefore$ Answer [C]

Ans : c

Q.30 Find the second derivative of the following function:

(A) $f''(x) = 30x - 470$
(B) $f''(x) = 30x + 470$
(C) $f''(x) = 15x^2 + 235$
(D) $f''(x) = 15x^2 + 470x$

$f(x) = 5x^2(x+47)$

$\therefore f(x) = 5x^2(x+47) = 5x^3 + 235x^2$

$\therefore f'(x) = 15x^2 + 470x$

$\therefore f''(x) = 30x + 470$

$\therefore$ Answer [B]

Ans : b

Q.31	$u(x) = \sqrt{x}$ Find $-\frac{u''(x)}{u'(x)}$. (A) $\frac{1}{2x}$ (B) $-\frac{1}{2x}$ (C) $2x$ (D) $-2x$	Ans : A
Q.32	$\frac{d}{dx}(x^2 e^x \sin x) =$ A) $x e^x(2\sin x + x\sin x + x\cos x)$ B) $x e^x(2\sin x + x\sin x - \cos x)$ C) $x e^x(2\sin x + x\sin x + \cos x)$ D) None of these	Ans : a
Q.33	If $y = x\sin x$, then A) $\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \cot x$ B) $\frac{dy}{dx} = \frac{1}{x} + \cot x$ C) $\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} - \cot x$ D) None of these	Ans : A
Q.34	If $y = \frac{1}{a-z}$, then $\frac{dz}{dy} =$ A) $(z-a)^2$ B) $-(z-a)^2$ C) $(z+a)^2$ D) $-(z+a)^2$	Ans : a
Q.35	If $y = x + \frac{1}{x}$, then A) $x^2\frac{dy}{dx} + xy = 0$ B) $x^2\frac{dy}{dx} + xy + 2 = 0$ C) $x^2\frac{dy}{dx} - xy + 2 = 0$ D) None of these	Ans : c

Q.36 $\frac{d}{dx}\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 =$ — Ans : A

A) $1-\frac{1}{x^2}$

B) $1+\frac{1}{x^2}$

C) $1-\frac{1}{2x}$

D) None of these

16 صفحة

Q.37 If $pv = 81$, then $\frac{dp}{dv}$ is at v = 9 equal to — Ans : b

A) 1

B) -1 (-1)

C) 2

D) None of these

$\because pv = 81 \Rightarrow p = \frac{81}{v}$

$\therefore \frac{dp}{dv} = \frac{81 * -1}{v^2} = \frac{-81}{v^2}$

$\therefore \left.\frac{dp}{dv}\right|_{v=9} = \frac{-81}{(9)^2} = \frac{-81}{81} = -1$

∴ Answer B

Q.38 For the function $f(x) = x^2 - 6x + 8, 2 \le x \le 4$, the value of x for which $f'(x)$ vanishes, is — Ans : c

A) $\frac{9}{4}$

B) $\frac{5}{2}$

C) 3

D) $\frac{7}{2}$

$\because f(x) = x^2 - 6x + 8$

$\therefore f'(x) = 2x - 6$

$\because f'(x)$ vanishes $\Rightarrow f'(x) = 0$

$\therefore 2x - 6 = 0$

$2x = 6 \Rightarrow x = \frac{6}{2} = 3$ ∴ Answer C

Q.39 For the curve $\sqrt{x} + \sqrt{y} = 1$, $\frac{dy}{dx}$ at $\left(\frac{1}{4}, \frac{1}{4}\right)$ is — Ans : c

A) ½

B) 1

C) -1

D) 2

17 صفحة

Q.40 If $f(x) = 3e^{x^2}$, then $f'(x) - 2xf(x) + \frac{1}{3}f(0) - f'(0) =$ — Ans : b

A) 0

B) 1

C) $\frac{7}{3}e^{x^2}$

D) None of these

17 صفحة

Q.41 The derivative of $\tan x - x$ with respect to x is

A) $1-\tan^2 x$

B) $\tan x$

C) $-\tan^2 x$

D) $\tan^2 x$

$\therefore \tan(x) - x$

$\therefore \frac{d}{dx}[\tan(x)] - \frac{d}{dx}[x]$

$= \sec^2(x) - 1$

$= \tan^2 x$.

$\therefore$ answer [D]

قانون: $\tan^2 x = \sec^2 x - 1$

Ans : d

Q.42 $\frac{d}{dx}(e^{x^3})$ is equal to

A) $3xe^{x^3}$

B) $3x^2e^{x^3}$

C) $3x\left(e^{x^2}\right)^2$

D) $2x^2e^{x^3}$

$\frac{d}{dx}(e^{x^3}) = e^{x^3} * 3x^2$

مشتقة الدالة × مشتقة الأس

$= 3x^2 e^{x^3}$.

$\therefore$ answer [B]

Ans : b

Q.43 $\frac{d}{dx}\{\cos(\sin x^2)\} =$

A) $\sin(\sin x^2).\cos x^2.2x$

B) $-\sin(\sin x^2).\cos x^2.2x$

C) $-\sin(\sin x^2).\cos^2 x.2x$

D) None of these

$\therefore \frac{d}{dx}[\cos(\sin^2 x^2)]$

$= -\sin(\sin^2 x^2) * \frac{d}{dx}(\sin^2 x^2)$

$= -\sin(\sin x^2) * \cos(x)^2 * 2x$

$\therefore$ answer [B]

Ans : b

Q.44 If $y = t^{4/3} - 3t^{-2/3}$, then $dy/dt =$

A) $\frac{2t^2+3}{3t^{5/3}}$

B) $\frac{2t^2+3}{t^{5/3}}$

C) $\frac{2(2t^2+3)}{t^{5/3}}$

D) $\frac{2(2t^2+3)}{3t^{5/3}}$

[17] كلاهما

Ans : d

Q.45 If $y = \sin[\cos(\sin x)]$, then $dy/dx =$

A) $-\cos[\cos(\sin x)]\sin(\cos x).\cos x$

B) $-\cos[\cos(\sin x)]\sin(\sin x).\cos x$

C) $\cos[\cos(\sin x)]\sin(\cos x).\cos x$

D) $\cos[\cos(\sin x)]\sin(\sin x).\cos x$

[17] كلاهما

Ans : b

Q.46	If $x^{2/3}+y^{2/3}=a^{2/3}$, then $\frac{dy}{dx}=$ A) $\left(\frac{y}{x}\right)^{1/3}$ B) $-\left(\frac{y}{x}\right)^{1/3}$ C) $\left(\frac{x}{y}\right)^{1/3}$ D) $-\left(\frac{x}{y}\right)^{1/3}$	Ans : b
Q.47	If $y=\sqrt{(1-x)(1+x)}$, then A) $(1-x^2)\frac{dy}{dx}-xy=0$ B) $(1-x^2)\frac{dy}{dx}+xy=0$ C) $(1-x^2)\frac{dy}{dx}-2xy=0$ D) $(1-x^2)\frac{dy}{dx}+2xy=0$	Ans : b
Q.48	If $y=3x^5+4x^4+2x+3$ then A) $y_4=0$ B) $y_5=0$ C) $y_6=0$ D) None of these	Ans : c
Q.49	If $f(x)=mx+c, f(0)=f'(0)=1$ then $f(2)=$ A) 1 B) 2 C) 3	Ans : c
Q.50	If $y=a\sin x+b\cos x$, then $y^2+\left(\frac{dy}{dx}\right)^2$ is a A) Function of x B) Function of y C) Function of x and y D) Constant	Ans : d

Q.51	$\frac{d}{dx}\left(x^2 \sin\frac{1}{x}\right) =$ A) $\cos\left(\frac{1}{x}\right) + 2x\sin\left(\frac{1}{x}\right)$ B) $2x\sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right)$ C) $\cos\left(\frac{1}{x}\right) - 2x\sin\left(\frac{1}{x}\right)$ D) None of these	Ans : b
Q.52	$\frac{d}{dx}[\cos(1-x^2)^2] =$ A) $-2x(1-x^2)\sin(1-x^2)^2$ B) $-4x(1-x^2)\sin(1-x^2)^2$ C) $4x(1-x^2)\sin(1-x^2)^2$ D) $-2(1-x^2)\sin(1-x^2)^2$ D) $\overline{2}$	Ans : c
Q.53	If the function $y(x) = x^3 - 3x + 1$, then a) y(x) is minimum at (-1,3) and maximum at (1,-1) b) y(x) is maximum at (-1,3) and minimum at (1,-1) c) y(x) is maximum at (-1,3) and maximum at (1,-1) d) None of the above	Ans : b
Q.54	If the function $y(x) = x^3 + 8$, then we have a) minimum point at (0,8) b) negative point of inflection at (0,8) c) positive point of inflection at (0,8) d) None of the above	Ans : c
Q.55	If the position of a particle is given by the equation of motion $f(t) = 1/(t+1) = (t+1)^{-1}$, then at t = 2 seconds a) the velocity is 1/9 m/s and speed is -1/9	Ans : c

b) the velocity is 1/9 and speed is 1/9 m/s

c) the velocity is -1/9 m/s and speed is 1/9

d) None of the above

[20] الحل

Ans. Chapter (3).

Q(10): $\because f(x) = x^3 - 3x^2 - 24x + 100$

$\therefore f'(x) = 3x^2 - 6x - 24$

$\because f'(x) = 0$

$\therefore 3x^2 - 6x - 24 = 0 \quad (\div 3)$

$\therefore x^2 - 2x - 8 = 0$

$(x + 2)(x - 4) = 0$

$x + 2 = 0 \quad \therefore \boxed{x = -2}$ xx

$x - 4 = 0 \quad \therefore \boxed{x = 4}$

$\because$ interval is $[-3, 3]$

$\therefore \boxed{x = -3} \quad \boxed{x = 3}$

$\therefore f(-2) = (-2)^3 - 3(-2)^2 - 24(-2) + 100 = 128$

$\therefore f(-3) = (-3)^3 - 3(-3)^2 - 24(-3) + 100 = 118$

$\therefore f(3) = (3)^3 - 3(3)^2 - 24(3) + 100 = \boxed{28}$

$\therefore f(4) = (4)^3 - 3(4)^2 - 24(4) + 100 = 20$

not used as $4 > 3$ as $[-3, 3]$

$\therefore$ minimum value $= 28$ at $x = 3$.

$\therefore$ Answer $\boxed{B}$.

###

Q(11): $\because \theta = \dfrac{3r + 2}{2r + 3}$

$\therefore \dfrac{d\theta}{dr} = \dfrac{[(2r+3)*3] - [2*(3r+2)]}{(2r+3)^2}$

$= \dfrac{(6r + 9) - (6r + 4)}{(2r+3)^2}$

$= \dfrac{6r + 9 - 6r - 4}{(2r+3)^2}$

$= \dfrac{5}{(2r+3)^2}$.

$\therefore$ Answer $\boxed{A}$ ###

Q(12):

$\because y = \sqrt{1 + \sqrt{2x}}$

$\therefore y' = \dfrac{\text{مشتقة الجذر}}{2 * \text{الجذر}} = \dfrac{\frac{2}{2\sqrt{2x}} \leftarrow 2 * \text{الجذر}}{2\sqrt{1 + \sqrt{2x}}}$ مشتقة ما تحت الجذر

$= \dfrac{\frac{1}{\sqrt{2x}}}{2\sqrt{1 + \sqrt{2x}}}$

$= \dfrac{1}{\sqrt{2x} * 2\sqrt{1 + \sqrt{2x}}}$

$= \dfrac{1}{2\sqrt{2x(1 + \sqrt{2x})}}$

$= \dfrac{1}{2\sqrt{2x + 2x\sqrt{2x}}}$.

$\therefore$ Answer $\boxed{B}$.

###

Q(18):

$$\because y = \sqrt{\frac{x-1}{x+1}}$$

$$\therefore y' = \frac{\frac{(x+1)-(x-1)}{(x+1)^2}}{2\sqrt{\frac{x-1}{x+1}}}$$

$$= \frac{\frac{x+1-x+1}{(x+1)^2}}{2\sqrt{\frac{x-1}{x+1}}}$$

$$= \frac{\frac{2}{(x+1)^2}}{2\sqrt{\frac{x-1}{x+1}}}$$

$$= \frac{1}{(x+1)^2\sqrt{\frac{x-1}{x+1}}}$$

$$= \frac{1}{\sqrt{(x+1)^4 * \frac{x-1}{x+1}}}$$

$$= \frac{1}{\sqrt{(x+1)^3(x-1)}}$$

$$= \frac{1}{\sqrt{(x+1)^2(x+1)(x-1)}}$$

$$= \frac{1}{\sqrt{(x+1)^2}\sqrt{(x+1)(x-1)}}$$

$$= \frac{1}{(x+1)\sqrt{x^2-x+x-1}}$$

$$= \frac{1}{(x+1)\sqrt{x^2-1}}.$$

∴ Answer A

###

Q(19):

$$\because y = x^3 - 6x^2 + 9x - 8$$

$$\therefore y' = 3x^2 - 12x + 9$$

$$\because y' = 0$$

$$\therefore 3x^2 - 12x + 9 = 0 \quad (\div 3)$$

$$x^2 - 4x + 3 = 0$$

$$(x-1)(x-3) = 0$$

$x - 1 = 0$ | $x - 3 = 0$

$x = 1$ | $x = 3$

$$\because f(1) = (1)^3 - 6(1)^2 + 9(1) - 8 = -4$$

$$\because f(3) = (3)^3 - 6(3)^2 + 9(3) - 8 = -8$$

∴ maximum value $= -4$ at $x = 1$

∴ Answer A.

Q(20):

نفس خطوات حل السؤال رقم Q(19)

∴ minimum value $= -8$ at $x = 3$.

∴ Answer B

(22):

$\because \sin(xy) + \cos(xy) = 0$

اسم لاستقاق الضمني

Implicit Differentiation

$x \xrightarrow[\div]{\text{تفاضل}} 1$

$y \longrightarrow y'$

$\because \sin(xy) + \cos(xy) = 0$

$\therefore \cos(xy)(xy' + y \cdot 1) - \sin(xy)(xy' + y \cdot 1) = 0$

(تفاضل الدالة، تفاضل الناتج، تفاضل الدالة، تفاضل الناتج)

$xy \longrightarrow xy' + y \cdot 1$

(تفاضل أول، تفاضل ثاني، تفاضل ثاني، تفاضل أول)

$\therefore \cos(xy)(xy' + y) - \sin(xy)(xy' + y) = 0$

$(xy' + y)[\cos(xy) - \sin(xy)] = 0$

$\therefore xy' + y = 0$

$xy' = -y$

$\therefore y' = \frac{-y}{x}$.

$\therefore$ Answer B

Q(23):

$\because y = x + e^x$

$\therefore \frac{dy}{dx} = 1 + e^x$

$dy = (1 + e^x)\,dx$

$\therefore \frac{dx}{dy} = \frac{1}{1+e^x} \longrightarrow **$

$\therefore \frac{d^2x}{dy^2} = \frac{0(1+e^x) - 1(e^x)}{(1+e^x)^2} * \frac{dx}{dy}$

$= \frac{-e^x}{(1+e^x)^2} * \frac{1}{1+e^x}$

$= \frac{-e^x}{(1+e^x)^3}$.

$\therefore$ Answer B .

Q(24):

$\because x^2y + y^3 = 2$

$\therefore x^2y' + 2xy + 3y^2y' = 0$

$x^2y' + 3y^2y' = -2xy$

$y'(x^2 + 3y^2) = -2xy$

$\therefore y' = \frac{-2xy}{x^2 + 3y^2}$

$\therefore y' = \left.\frac{dy}{dx}\right|_{(1,1)} = \frac{-2(1)(1)}{(1)^2 + 3(1)^2} = \frac{-1}{2}$.

$\therefore y'' = \frac{(x^2+3y^2)[-2(xy'+y)] - [-2(xy)(2x + 6y^2y')]}{(x^2+3y^2)^2}$

$\therefore y'' = \left.\frac{d^2y}{dx^2}\right|_{(1,1)} = \frac{(1+3)[-2(\frac{-1}{2}+1)] - [-2(2 + 6(\frac{1}{2})]}{(1+3)^2}$

$= \frac{-3}{2}$ $\therefore$ Answer B

Q:

$$y = e^{\sqrt{x}} + e^{-\sqrt{x}}$$

$$\therefore \frac{dy}{dx} = e^{\sqrt{x}} * \frac{1}{2\sqrt{x}} + e^{-\sqrt{x}} * \frac{-1}{2\sqrt{x}}$$

$$= \frac{e^{\sqrt{x}}}{2\sqrt{x}} + \frac{-e^{-\sqrt{x}}}{2\sqrt{x}}$$

$$= \frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2\sqrt{x}}$$

∴ Answer [A] .

Q(32):

$$\frac{d}{dx}(x^2 e^x \sin x) =$$

$$= x^2(e^x \cos x + e^x \sin x) + 2x \sin x - e^x$$

$$= x^2 e^x \cos x + x^2 e^x \sin x + 2x e^x \sin x$$

$$= x e^x [x \cos x + x \sin x + 2 \sin x]$$

∴ Answer [A]

Q(31):

$$\because u(x) = \sqrt{x}$$

$$\therefore u'(x) = \frac{1}{2\sqrt{x}}$$

$$\therefore u''(x) = \frac{[0(2\sqrt{x})] - [1 * 2\frac{1}{2\sqrt{x}}]}{(2\sqrt{x})^2}$$

$$= \frac{\frac{-1}{\sqrt{x}}}{4x} = \frac{-1}{4x\sqrt{x}}$$

$$\therefore -\frac{u''(x)}{u'(x)} = -\left[\frac{\frac{-1}{4x\sqrt{x}}}{\frac{1}{2\sqrt{x}}}\right]$$

$$= \frac{1}{2x} .$$

∴ Answer [A]

Q(33):

$$\because y = x \sin x$$

$$\therefore \frac{dy}{dx} = x \cos x + \sin x \cdot 1$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{x \cos x}{y} + \frac{\sin x}{y}$$

$$= \frac{x \cos x}{x \sin x} + \frac{\sin x}{x \sin x}$$

$$= \frac{\cos x}{\sin x} + \frac{1}{x}$$

$$= \cot(x) + \frac{1}{x}$$

∴ Answer [A]

$$y = \frac{1}{a - z}$$

$\therefore ya - yz = 1$

$\therefore yz = ya - 1 \quad (\div y)$

$\therefore z = \frac{ya}{y} - \frac{1}{y}$

$z = a - \frac{1}{y}$

$\therefore \frac{dz}{dy} = 0 - \frac{-1}{y^2}$

$= \frac{1}{y^2}$

$= \frac{1}{\frac{1}{(a-z)^2}} = (a-z)^2$

$\therefore$ Answer [A]

Q(35):

$\because \boxed{y = x + \frac{1}{x}}$

$\therefore \frac{dy}{dx} = 1 + \frac{-1}{x^2}$

$\frac{dy}{dx} = 1 - \frac{1}{x^2}$

$\boxed{\frac{dy}{dx} = \frac{x^2 - 1}{x^2}}$

(a) $x^2 \frac{dy}{dx} + xy = 0$

$x^2\left(\frac{x^2-1}{x^2}\right) + x\left(x + \frac{1}{x}\right) = 0$

$x^2 - 1 + x^2 + 1 = 0$

$2x^2 \neq 0$

خطأ. الإجابة ليست [A]

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(b) $x^2 \frac{dy}{dx} + xy + 2 = 0$

$x^2\left(\frac{x^2-1}{x^2}\right) + x\left(x + \frac{1}{x}\right) + 2 = 0$

$x^2 - 1 + x^2 + 1 + 2 = 0$

$2x^2 + 2 \neq 0$

خطأ

(c) $x^2 \frac{dy}{dx} - xy + 2 = 0$

$x^2\left(\frac{x^2-1}{x^2}\right) - x\left(x + \frac{1}{x}\right) + 2 = 0$

$x^2 - 1 - [x^2 + 1] + 2 = 0$

$x^2 - 1 - x^2 - 1 + 2 = 0$

$-2 + 2 = 0$

$0 = 0$ ✓

$\therefore$ Answer [C].

Q(36):

$\because y = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$

$= (\sqrt{x})^2 + 2\sqrt{x}\,\frac{1}{\sqrt{x}} + \left(\frac{1}{\sqrt{x}}\right)^2$

$y = x + 2 + \frac{1}{x}$

$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$

$= \frac{d}{dx}\left(x + 2 + \frac{1}{x}\right)$

$= 1 + 0 + \frac{0-1}{x^2}$

$= 1 - \frac{1}{x^2}$

$\therefore$ Answer [A]

$\therefore \sqrt{x} + \sqrt{y} = 1$

$\therefore \frac{d}{dx}(\sqrt{x}) + \frac{d}{dx}(\sqrt{y}) = \frac{d}{dx}(1)$

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = 0$

$\frac{1}{2\sqrt{y}} \frac{dy}{dx} = \frac{-1}{2\sqrt{x}}$

$\frac{1}{\sqrt{y}} \frac{dy}{dx} = \frac{-1}{\sqrt{x}}$

$\therefore \frac{dy}{dx} = \frac{\frac{-1}{\sqrt{x}}}{\frac{1}{\sqrt{y}}}$

$\therefore \left.\frac{dy}{dx}\right|_{(\frac{1}{4},\frac{1}{4})} = \frac{\frac{-1}{\sqrt{1/4}}}{\frac{1}{\sqrt{1/4}}} = -1 .$

$\therefore$ answer C .

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Q(44):

$\because y = t^{4/3} - 3t^{-2/3}$

$\therefore \frac{dy}{dt} = \frac{4}{3}t^{\frac{1}{3}} - 3(\frac{-2}{3})t^{\frac{-5}{3}}$

$= \frac{4t^{\frac{1}{3}}}{3} + \frac{2}{t^{\frac{5}{3}}}$

$= \frac{4t^{\frac{1}{3}}t^{\frac{5}{3}} + 6}{3t^{5/3}}$

$= \frac{4t^{\frac{1}{3}+\frac{5}{3}} + 6}{3t^{5/3}}$

$= \frac{2(2t^2+3)}{3t^{5/3}} .$ ###

$\therefore$ Answer D

Q(40):

$\because f(x) = 3e^{x^2}$

$\therefore f'(x) = 3e^{x^2} * 2x$

$\therefore f'(x) = 6xe^{x^2}$

$\therefore f'(0) = 6(0)e^{(0)^2} = 0 .$

$\therefore \frac{1}{3}f(0) = \frac{1}{3} * 3e^{(0)^2} = 1 .$

$\therefore f'(x) = 2xf(x) + \frac{1}{3}f(0) - f'(0)$

$= 6xe^{x^2} - 2x * 3e^{x^2} + 1 - 0$

$= 6xe^{x^2} - 6xe^{x^2} + 1$

$= 1 .$

$\therefore$ Answer B

Q(45):

$\because y = \sin[\cos(\sin x)]$

$\therefore \frac{dy}{dx} = \cos[\cos(\sin x)] * \frac{d}{dx}[\cos(\sin x)]$

(الدالة الأولى) (الدالة الثانية)

$= \cos[\cos(\sin x)] * [-\sin(\sin x) * \cos(x)]$

(الدالة الأولى) (الدالة الثانية)

$= -\cos[\cos(\sin x)]\sin(\sin x)\cdot\cos(x) .$

$\therefore$ Answer B

Q(46):

$$x^{2/3} + y^{2/3} = a^{2/3}$$

$$\therefore \frac{d}{dx}[x^{2/3}] + \frac{d}{dx}[y^{2/3}] = \frac{d}{dx}[a^{2/3}]$$

$$\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}}\frac{dy}{dx} = 0 \quad (\div \tfrac{2}{3})$$

$$x^{-\frac{1}{3}} + y^{-\frac{1}{3}}\frac{dy}{dx} = 0$$

$$y^{-\frac{1}{3}}\frac{dy}{dx} = -x^{-\frac{1}{3}}$$

$$\therefore \frac{dy}{dx} = -\frac{x^{-\frac{1}{3}}}{y^{-\frac{1}{3}}} = -\left(\frac{x}{y}\right)^{-\frac{1}{3}} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$$

∴ Answer B

$$\frac{x^{-\frac{1}{3}}}{y^{-\frac{1}{3}}} = \frac{\frac{1}{x^{1/3}}}{\frac{1}{y^{1/3}}} = \frac{y^{1/3}}{x^{1/3}} = \left(\frac{y}{x}\right)^{\frac{1}{3}}$$

Q(47):

$$\therefore y = \sqrt{(1-x)(1+x)}$$

$$\therefore y = \sqrt{1-x^2} \rightarrow ***$$

$$\therefore \frac{dy}{dx} = \frac{-2x}{2\sqrt{1-x^2}} = \frac{-x}{\sqrt{1-x^2}} \rightarrow **$$

وتجرب كل واحدة من الخيارات وتشوف
مين النتيجة بتساوي الصفر.

ⓐ $(1-x^2)\frac{dy}{dx} - xy = 0$

$$\therefore (1-x^2)*\frac{-x}{\sqrt{1-x^2}} - x\sqrt{1-x^2} = 0$$

$$-x\sqrt{1-x^2} - x\sqrt{1-x^2} \neq 0$$

∴ لا

ⓑ $(1-x^2)\frac{dy}{dx} + xy = 0$

$$(1-x^2)*\frac{-x}{\sqrt{1-x^2}} + x\sqrt{1-x^2} = 0$$

$$\frac{-x(1-x^2)}{\sqrt{1-x^2}} + x\sqrt{1-x^2} = 0$$

$$\frac{-x(1-x^2)}{(1-x^2)^{1/2}} + x\sqrt{1-x^2} = 0$$

$$-x(1-x^2)^{1/2} + x\sqrt{1-x^2} = 0$$

$$-x\sqrt{1-x^2} + x\sqrt{1-x^2} = 0$$

$$\therefore 0 = 0. \quad \#\#$$

∴ Answer B

Q(48):

$$\therefore y = 3x^5 + 4x^4 + 2x + 3$$

$$\therefore y' = 15x^4 + 16x^3 + 2$$

$$\therefore y'' = 60x^3 + 48x^2$$

$$\therefore y''' = 180x^2 + 96x$$

$$\therefore y^{(4)} = 360x + 96$$

$$\therefore y^{(5)} = 360$$

$$\therefore y^{(6)} = \text{Zero}. \quad \#\#$$

∴ Answer C

19):

$\therefore f(x) = mx + c$

$\therefore f'(x) = m$

$\therefore f(0) = f'(0) = 1$

$\therefore m(0) + c = 1 \Rightarrow \boxed{c = 1}$

$\therefore \boxed{m = 1}$

$\therefore f(2) = m(2) + c$

$= 1(2) + 1 = 3.$

$\therefore$ Answer $\boxed{C}$

Q(51):

$\frac{d}{dx}(x^2 \sin\frac{1}{x}) =$

(الثانية دالة مشتقة، الثانية مشتقة)

$= 2x \sin(\frac{1}{x}) + x^2 * \cos(\frac{1}{x}) * \frac{-1}{x^2}$

(مشتقة الأولى، الثانية، الأولى، مشتقة الثانية)

$= 2x \sin(\frac{1}{x}) - \cos(\frac{1}{x})$

$\therefore$ Answer $\boxed{13}$

Q(50):

$\therefore y = a \sin x + b \cos x$

$\therefore \frac{dy}{dx} = a \cos x - b \sin x$

$\therefore y^2 + (\frac{dy}{dx})^2 = (a \sin x + b \cos x)^2 +$

$(a \cos x - b \sin x)^2$

$= a^2 \sin^2 x + 2ab \sin x \cos x + b^2 \cos^2 x$

$+ a^2 \cos^2 x - 2ab \cos x \sin x + b^2 \sin^2 x$

$= a^2 \sin^2 x + b^2 \cos^2 x + a^2 \cos^2 x + b^2 \sin^2 x$

$= a^2 (\sin^2 x + \cos^2 x) + b^2 (\sin^2 x + \cos^2 x)$

$= a^2 \cdot 1 + b^2 \cdot 1$

$= a^2 + b^2 =$ constant ثابت

$\therefore$ Answer $\boxed{D}$

Q(52):

$\frac{d}{dx}[\cos(1-x^2)^2] =$

$= -\sin(1-x^2)^2 * \frac{d}{dx}[(1-x^2)^2]$

(مشتقة الدالة، مشتقة الداخلية)

$= -\sin(1-x^2)^2 * 2(1-x^2) * -2x$

(مشتقة القوس، مشتقة ما بداخل القوس)

$= 4x(1-x^2) \sin(1-x^2)^2 .\#\#$

$\therefore$ answer $\boxed{C}$

Q(53):

$\therefore y(x) = x^3 - 3x + 1$

$\therefore y'(x) = 3x^2 - 3$

$\therefore y'(x) = 0$

$\therefore 3x^2 - 3 = 0$

$3x^2 = 3 \quad (\div 3)$

$x^2 = 1 \quad (\sqrt{\ })$

$\therefore \boxed{x = \pm 1}$ or $\boxed{x = 1}$, $\boxed{x = -1}$

$\therefore y(-1) = (-1)^3 - 3(-1) + 1 = 3 \rightarrow$ max

$\therefore y(1) = (1)^3 - 3(1) + 1 = -1 \rightarrow$ min

$\therefore$ maximum value = 3 at $x = -1$

minimum value = -1 at $x = 1$

$\therefore$ Answer $\boxed{B}$

Q(54):

$\therefore y = x^3 + 8$

$\therefore y' = 3x^2$

$\therefore y' = 0$

$\therefore 3x^2 = 0 \quad (\div 3)$

$\therefore x^2 = 0 \quad (\sqrt{\ })$

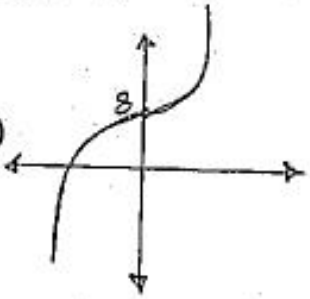
$\therefore \boxed{x = 0}$

$\therefore y = y(0) = (0)^3 + 8 = 8 \Rightarrow \boxed{y = 8}$

$\therefore y'' = 6x$

$y''(0) = 6(0)$

$= 0$



$\therefore$ Point $(0, 8)$ is Positive Point of inflection.

$\therefore$ Answer $\boxed{C}$

Q(55):

$\therefore f(t) = \frac{1}{1+t} = (1+t)^{-1}$

$\therefore f'(t) = -(1+t)^{-2}$

$\therefore t = 2$

$\therefore f'(2) = -(1+2)^{-2} = \frac{-1}{9}$

$\therefore$ The velocity at $t = 2$ is $\frac{-1}{9}$ m/s

$\therefore$ The speed at $t = 2$ is $\frac{1}{9}$ m/s.

$\therefore$ Answer $\boxed{C}$