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EELU
THE EGYPTIAN E-LEARNING UNIVERSITY

Physics2

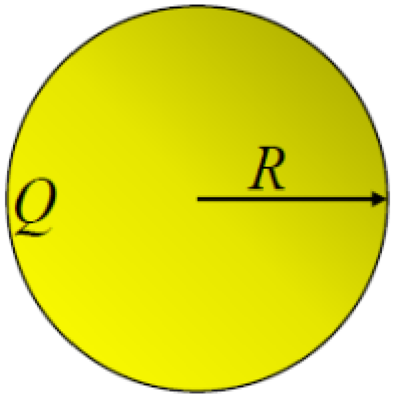
Section 07

Faculty of Information Technology
Egyptian E-Learning University

Spring 2021

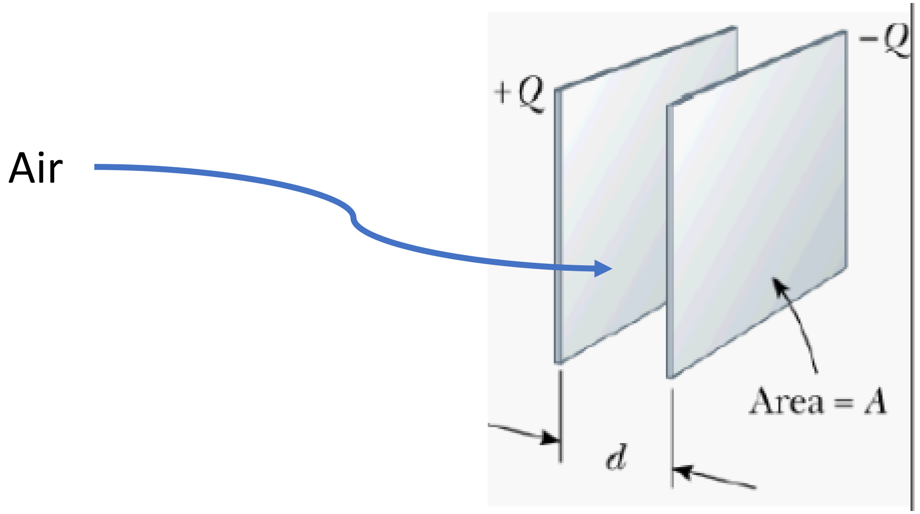
Calculation of capacitance

Isolated charged sphere



$$C_0 = \frac{R}{k_e} = 4\pi\varepsilon_0 R$$

Parallel Plates



$$C_0 = \frac{\varepsilon_0 A}{d}$$

Q1.

An air-filled spherical capacitor is constructed with inner- and outer-shell radii of **7.00cm** and **14.0cm**, respectively.

(a) Calculate the capacitance of the device.
(b) What potential difference between the spheres results in a **4mC** charge on the capacitor?

Q1.a Solution:

$$C = \frac{|Q|}{|\Delta V|}$$

$$|\Delta V| = |V_1 - V_2| = \left|\frac{k_e Q}{R_1} - \frac{k_e Q}{R_2}\right| = \frac{k_e Q|R_2 - R_1|}{R_2 R_1}$$

$$C = \frac{R_2 R_1}{k_e|R_2 - R_1|} = \frac{0.07 \times 0.14}{8.99 \times 10^9(0.14 - 0.07)} = 15.58\ PF$$

Q1.b Solution:

$$C = \frac{|Q|}{|\Delta V|}$$

$$Q = 4mC, \qquad |\Delta V| = \frac{|Q|}{C} = \frac{4 \times 10^{-3}}{15.58 \times 10^{-12}} = 256.86\ MV$$

Q2.

An air-filled parallel-plate capacitor has plates of area **2.30 cm^2** separated by **1.50 mm**.

(a) Find the value of its capacitance.
(b) The capacitor is connected to a 12.0-V battery, What is the charge on the capacitor?
(c) What is the magnitude of the uniform electric field between the plates?

Q2 solution.

An air-filled parallel-plate capacitor has plates of area **2.30 cm^2** separated by **1.50 mm**.

$$(a) C = \frac{\varepsilon_o A}{d} = \frac{8.85\times10^{-12}\times2.3\times10^{-4}}{0.0015} = 1.357\ pF$$

$$(b) Q = C \times \Delta V = 1.35 \times 10^{-12} \times 12 = 16.28 \times 10^{-12}\ C$$

$$(c) \Delta V = E.d$$

$$E = \frac{\Delta V}{d} = \frac{12}{0.0015} = 8000\frac{V}{m}$$

OR

$$E = \frac{\sigma}{\varepsilon_o} = \frac{\frac{Q}{A}}{\varepsilon_o} = 70.783 \times \frac{10^{-9}}{8.85 \times 10^{-12}} = 7998.03 \approx 8000 v/m$$

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Combinations of capacitors

In parallel, the potential difference is the same across all capacitors.

$$C_{eq} = C_1 + C_2 + C_3 + \ \ldots$$

in series, the charge is the same on all capacitors.

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

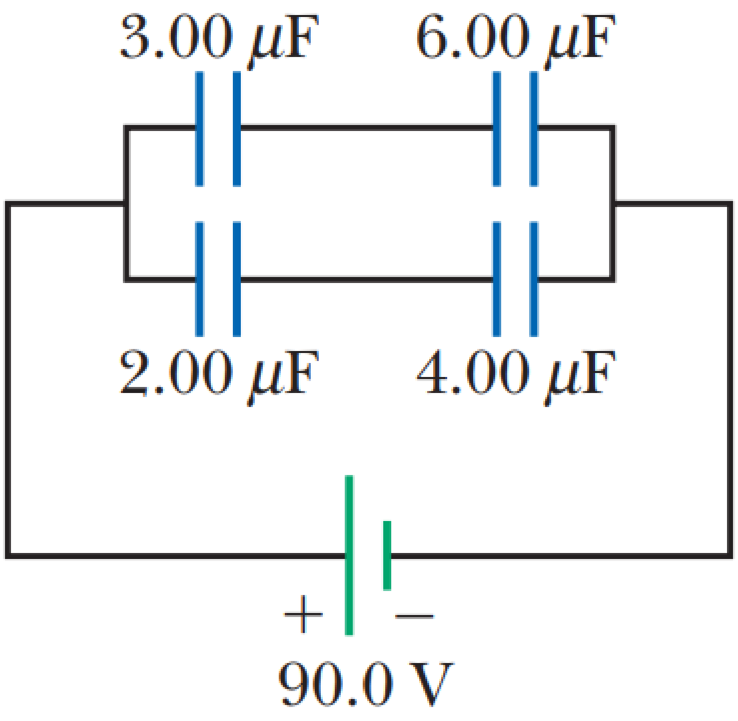
Q1

For the system of four capacitors shown in the Figure, find

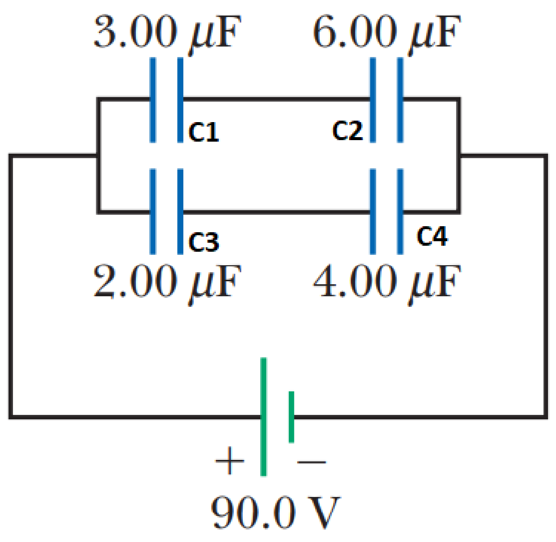
a) the equivalent capacitance of the system.

b) The charge on each capacitor.

c) the potential difference across each capacitor



Q1.solution



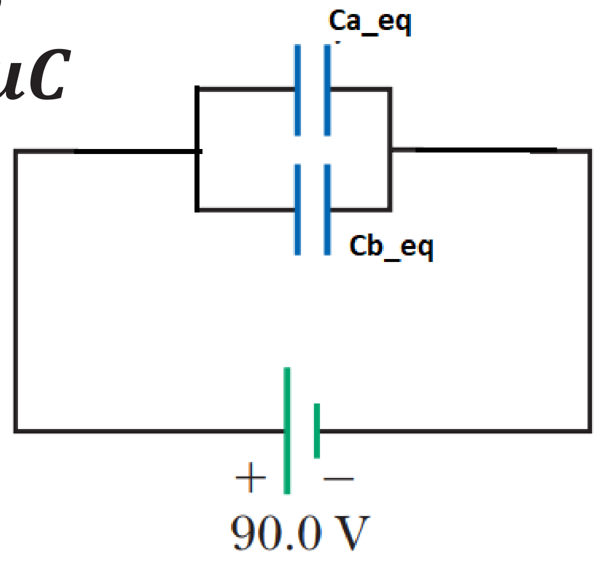
a) $$C_a = \frac{C_1 C_2}{C_1 + C_2} = 2\mu F$$

$$C_b = \frac{C_3 C_4}{C_3 + C_4} = \frac{4}{3} = 1.33\ \mu F$$

$$\boldsymbol{C_{eq}} = C_a + C_b = \mathbf{3.33\ \mu F}$$

b) $$Q_a = \boldsymbol{Q_1} = \boldsymbol{Q_2} = \Delta V \times C_a = 90 \times 2 \times 10^{-6} = \mathbf{180\ \mu C}$$

$$Q_b = \boldsymbol{Q_3} = \boldsymbol{Q_4} = \Delta V \times C_b = 90 \times \frac{4}{3} \times 10^{-6} = \mathbf{120\ \mu C}$$



C) $$\boldsymbol{\Delta V_1} = \frac{Q_1}{C_1} = \frac{180\mu C}{3\ \mu F} = 60V, \boldsymbol{\Delta V_2} = \frac{180}{6} = 30V$$

$$\boldsymbol{\Delta V_3} = \frac{120}{2} = 60V, \boldsymbol{\Delta V_4} = \frac{120}{4} = 30V$$

Capacitors with any dielectric

$$C = kC_o$$

K is a dimensionless factor , called the **dielectric constant**

where C_0 is the capacitance in the absence of the dielectric

Q1

How much charge can be placed on a capacitor with air between the plates before it breaks down(Air: $E_{max} = 3 \times 10^6\ V/m$)if the area of each plate is **5.00** $\boldsymbol{cm^2}$?

What If? Find the maximum charge if **polystyrene** ($E_{max} = 24 \times 10^6\ V/m$, k=2.56) is used between the plates instead of air.

Q1.solution

a) $C_{air} = \frac{Q}{\Delta V} = \frac{\varepsilon_0 A}{d} = \frac{Q}{E \times d}$,

$so\ Q_{air(max)} = \varepsilon_0 A E_{max} = 8.85 \times 10^{-12} \times 5 \times 10^{-4} \times 3 \times 10^{6}$

$= 13.3\ nC$

b) $C_{poly} = \frac{k_{poly}.\varepsilon_0.A}{d} = \frac{\mathrm{Q_{max}}}{\mathrm{E_{max}.d}}$

$Q_{poly(max)} = k_{poly}.\,\varepsilon_0.\,A.\,E_{max} = 2.56 \times 8.85 \times 10^{-12} \times 5 \times 10^{-4}$

$\times 24 \times 10^{6} = 271.9\ nC$

Energy stored in charged capacitor

$$U_E = \frac{Q^2}{2C} = \tfrac{1}{2}Q\,\Delta V = \tfrac{1}{2}C(\Delta V)^2$$

Q1

Two capacitors, $C1 = 18.0\ \mu F$ and $C2 = 36.0\ \mu F$, are connected in series, and a 12.0-V battery is connected across the two capacitors. Find

(a) the equivalent capacitance

(b) the energy stored in this equivalent capacitance.

(c) Find the energy stored in each individual capacitor.

(d) Show that the sum of these two energies is the same as the energy found in part (b).

(e) If the same capacitors were connected in parallel, what potential difference would be required across them so that the combination stores the same energy as in part (a)?

(f) Which capacitor stores more energy in this situation, C1 or C2?

Q1 .Solution

$(a)\, C_{eq(Series)} = \frac{C_1 C_2}{C_1+C_2} = \frac{18\times36}{18+36} = 12\ \mu F$

$(b)\, U = 0.5 \times C_{eq} \times V^2 = 864\ \mu J$

$(c)\, U_1 = 0.5 \times \frac{Q_1^2}{C_1}$

$$Q_1 = Q_{eq} = C_{eq} \times \Delta V = 12 \times 10^{-6} \times 12 = 144\ \mu C$$

$$U_1 = \frac{(144 \times 10^{-6})^2}{2 \times 18 \times 10^{-6}} = 675\ \mu J, \qquad U_2 = \frac{(144 \times 10^{-6})^2}{2 \times 36 \times 10^{-6}} = 288 \mu J.$$

d) $U_1 + U_2 = 864 \mu J = U_{total} (from\ b)$.

e) U=$U = 0.5 \times C_{eq} \times V^2 = 864\ \mu J$

$$C_{eq} = C_1 + C_2 = 54\ \mu F, V(required) = \sqrt{\frac{2 \times 864}{54}} = 5.66\ V$$

f) In parallel connection $\Delta V_1 = \Delta V_2$,

$since\ U = 0.5 \times C \times \Delta V^2, U_2 will\ be\ greater\ than\ U_1 as\ C_2 > C_1$

MCQ

1) By what factor is the capacitance of a metal sphere multiplied if its volume is tripled?

(a) 3

(b) $3^{1/3}$

(c) 1

(d) $3^{-\frac{1}{3}}$

$(e)\frac{1}{3}$

Ans.(b)