Section (6)

Physics(I)

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- Motion in two dimensions

Equations of kinematics in two dimensions

- A particle is moving with a constant acceleration

$a = \text{const}$

$a_x = \text{const}$ — One dimensional motion in *x-direction* with constant acceleration

INDEPENDENT

$a_y = \text{const}$ — One dimensional motion in *y-direction* with constant acceleration

- The motion in two dimensions with constant acceleration can be considered as two independent motions:
  - One along x-direction with constant acceleration (a_x)
  - One along y-direction with constant acceleration (a_y)

$$\Delta x = v_{0x}t + \frac{1}{2}a_x t^2$$

$$v_x = v_{0x} + a_x t$$

$$v_x{}^2 = v_{0x}{}^2 + 2a_x \Delta x$$

$$\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$$

$$v_y = v_{0y} + a_y t$$

$$v_y{}^2 = v_{0y}{}^2 + 2a_y \Delta y$$

1) A plane drops a package of supplies to a party of explorers, as shown in Figure 1. If the plane is traveling horizontally at 40 m/s and is 100 m above the ground, where does the package strike the ground relative to the point at which it is released?

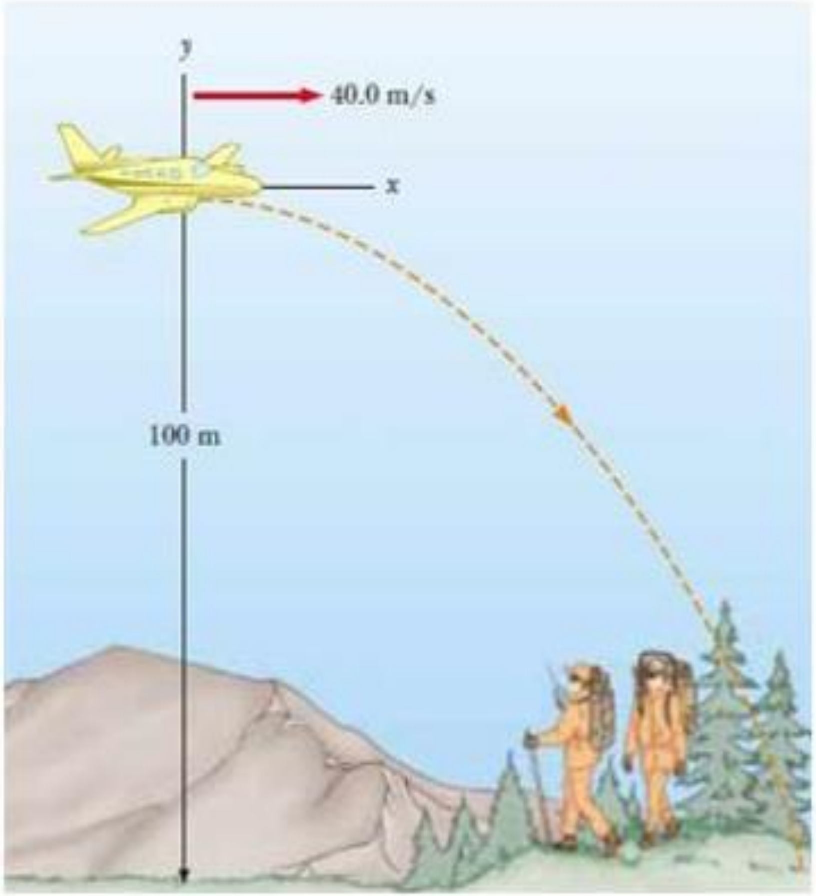
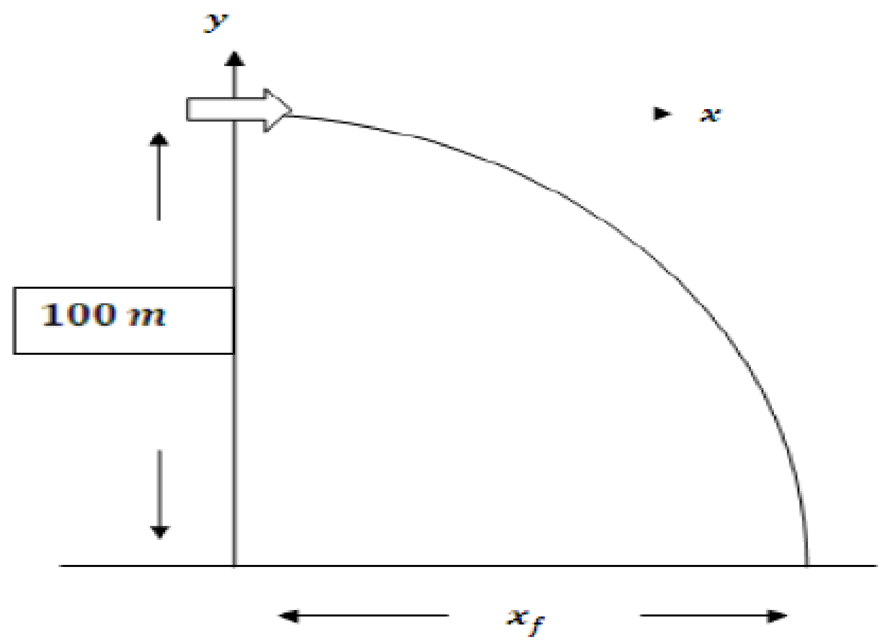


Figure 1

Solution Q1



$$v_{xi} = 40 \ m/s$$

$$x_i = y_i = 0$$

$$y_f = -100 \ , \qquad v_{yi} = 0$$

$$\because x_f = x_i + v_{xi} t$$

$$\therefore x_f = 0 + 40\,t = 40\,t \longleftarrow \boxed{1}$$

$$\because y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$$

$$\because a_y = -g$$

$$\because y_f = y_i + v_{yi}t - \frac{1}{2}gt^2$$

$$-100 = 0 + 0 - \frac{1}{2}(9.8)t^2$$

$$\therefore t = 4.52 \ s$$

In (1):

$$\therefore x_f = 40\,t = 40 \times 4.53 = 181\,m$$

A ball is tossed from an upper-story window of a building. The ball is given an initial velocity of 8.00 m/s at an angle of 20.0° below the horizontal. It strikes the ground 3.00 s later. (a) How far horizontally from the base of the building does the ball strike the ground? (b) Find the height from which the ball was thrown.

Solution:

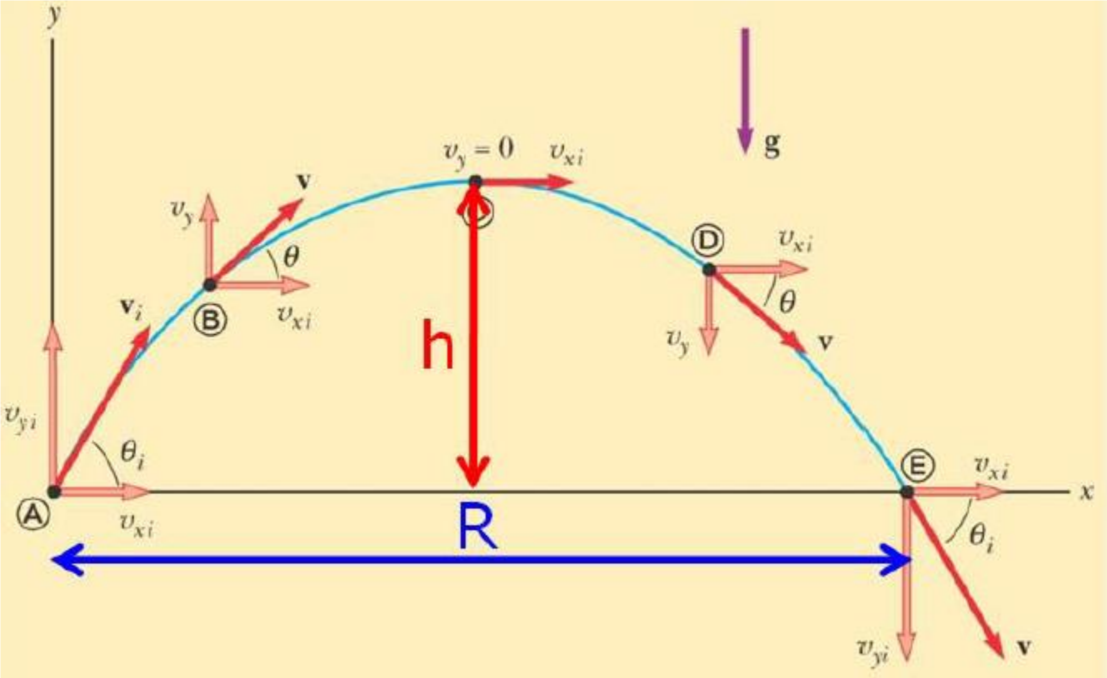
(a) $x_f = v_{xi}\, t = 8.00 \cos 20.0° (3.00) = \boxed{22.6 \text{ m}}$

(b) Taking y positive downwards,

$$y_f = v_{yi} t + \frac{1}{2} g t^2$$

$$y_f = 8.00 \sin 20.0° (3.00) + \frac{1}{2}(9.80)(3.00)^2 = \boxed{52.3 \text{ m}}.$$

Projectile motion



$$t_{\max} = \frac{v_0 \sin \theta_0}{g}$$

$$h_{\max} = \frac{v_0^2 \sin^2 \theta_0}{2g} = \frac{v_{oy}^2}{2g}$$

$$R = \frac{v_0^2 \sin 2\theta_0}{g}$$

15. A projectile is fired in such a way that its horizontal range is equal to three times its maximum height. What is the angle of projection?

P4.15 $h = \dfrac{v_i^2 \sin^2\theta_i}{2g}$; $R = \dfrac{v_i^2(\sin 2\theta_i)}{g}$; $3h = R$,

so $\dfrac{3v_i^2 \sin^2\theta_i}{2g} = \dfrac{v_i^2(\sin 2\theta_i)}{g}$

or $\dfrac{2}{3} = \dfrac{\sin^2\theta_i}{\sin 2\theta_i} = \dfrac{\tan\theta_i}{2}$

thus $\theta_i = \tan^{-1}\left(\dfrac{4}{3}\right) = \boxed{53.1°}$.

2) An object is launched at a velocity of 20 m/s in a direction making an angle of 25° upward with the horizontal.

a) What is the maximum height reached by the object?
b) What is the total flight time (between launch and touching the ground) of the object?
c) What is the horizontal range (maximum x above ground) of the object?

Solution:

a)

$$h_{\max} = \frac{v_0^2 \sin^2 \theta_0}{2g} = \frac{v_{oy}^2}{2g}$$

h_{max} = (20)^2 sin^2(25)/(2*9.8) = 3.64 m

b)

$$t_{\max} = \frac{v_0 \sin \theta_0}{g}$$

t_{flight} = 2 t_{max} = 2 (20sin(25) /9.8) = 1.73 s

c)

$$R = \frac{v_0^2 \sin 2\theta_0}{g}$$

R= (20)^2 sin(50)/9.8 = 31.27 m

A rock is thrown from a bridge at an upward launch angle of 30° with an initial speed of 25 m/s. The bridge is 30 m above the river. How much time elapses before the rock hits the water.

Using the y coordinate equation for a projectile motion with the figure shown below:

$$y = v_{oy}t - \frac{1}{2}gt^2$$

With $y = -30$, $v_{oy} = v_o \sin\theta$ and $\theta = 30^o$ one can get:

$$-30 = 25 \sin 30\, t - \frac{1}{2}9.8t^2$$

Which gives:

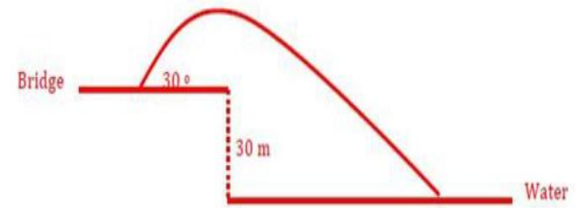
$$-30 = 12.5t - 4.9t^2$$

Solving for t:

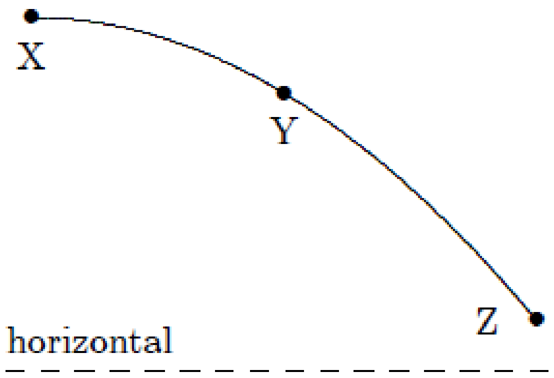
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{12.5 \pm \sqrt{-12.5^2 + 4\times4.9\times30}}{2\times4.9}$$

$$t = 4.06\ s$$



. A stone is thrown horizontally and follows the path XYZ shown. The direction of the acceleration of the stone at point Y is:



A. ↓
B. →
C. ↘
D. ↙
E. ↗

Answer: **A**

A particle starts from the origin at $t = 0$ with a velocity of $(16\hat{\mathbf{i}} - 12\hat{\mathbf{j}})$ m/s and moves in the xy plane with a constant acceleration of $\vec{\mathbf{a}} = (3.0\hat{\mathbf{i}} - 6.0\hat{\mathbf{j}})$ m/s^2. What is the speed of the particle at $t = 2.0$ s?

52 m/s a.
39 m/s b.
46 m/s c.
33 m/s d.
43 m/s e.

A particle moves in the xy plane with a constant acceleration given by $\vec{\mathbf{a}} = -4.0\hat{\mathbf{j}}$ m/s^2. At $t = 0$, its position and velocity are $10\hat{\mathbf{i}}$ m and $(-2.0\hat{\mathbf{i}} + 8.0\hat{\mathbf{j}})$ m/s, respectively. What is the distance from the origin to the particle at $t = 2.0$ s?

6.4 m a.
10 m b.
8.9 m c.
2.0 m d.
6.2 m e.

A rock is projected from the edge of the top of a building with an initial velocity of 12.2 m/s at an angle of 53° above the horizontal. The rock strikes the ground a horizontal distance of 25 m from the base of the building. Assume that the ground is level and that the side of the building is vertical. How tall is the building?

25.3 m	a.
29.6 m	b.
27.4 m	c.
23.6 m	d.
18.9 m	e.