

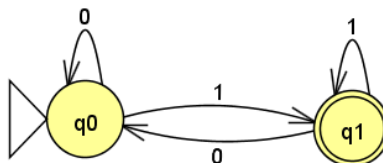
Model Answer

Question 1: Choose the correct answer:

(10 Marks)

1. If $A = \{1, 2, 3, 4, 5\}$ and $B = \{5, 6, 7\}$ then $|A - B|$ equals:
 - a. 5
 - b. 4**
 - c. 3
 - d. 7
2. If $B = \{\{1\}, \{3, 4, 5\}\}$, then $|B|$ equals:
 - a. 4
 - b. 2**
 - c. 3
 - d. 1
3. If $A = \{3, 4, 6, 7\}$ and $B = \{5, 7\}$, which of the following is in $A \times B$:
 - a. 7
 - b. (7, 7)**
 - c. (5, 6)
 - d. {7, 7}
4. If $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3\}$, which of the following is correct?
 - a. A is a subset of B.
 - c. B is a proper subset of A.**
 - b. A and B are disjoint.
 - d. $A - B = \emptyset$
5. Consider the function $f, f: \{a, b, c\} \rightarrow \{1, 2, 3\}$, such that $f(a) = 1, f(b) = 3$ and $f(c) = 3$ then the domain of f is:
 - a. {3}
 - b. {1, 2, 3}
 - c. {a, b, c}**
 - d. {1, 3}
6. Consider the function $h, h: Z \rightarrow \{0, 1\}$ defined by $h(x) = \begin{cases} 0 & \text{if } x \text{ is even} \\ 1 & \text{otherwise} \end{cases}$, then:
 - a. h is one to one
 - b. h is onto**
 - c. h is bijective
 - d. h is invertible
7. If $\Sigma = \{a, b, c\}$, w is a string over Σ and $|www| = 18$, what is the length of w ?
 - a. 18
 - b. 12
 - c. 6**
 - d. 3
8. If $L = \{aba, ba\}$, which of the following strings is NOT in L^+ ?
 - a. λ**
 - b. baba
 - c. baaba
 - d. ba
9. If $L = \{\lambda, 011, 00\}$, which of the following strings is NOT in L^+ ?
 - a. λ
 - b. 11**
 - c. 01100
 - d. 00011
10. If $L = \{aa, ab\}$, which of the following strings is NOT in L^* ?
 - a. aba**
 - b. aaaa
 - c. aaab
 - d. abaa
11. If $L = \{a, aa, aaa, \dots\}$ and $M = \{\lambda, b, bb, bbb, \dots\}$, which of the following strings is NOT in $L.M$:
 - a. aaa
 - b. λ**
 - c. abb
 - d. aab
12. If $L_1 = \{a^n b^n : n > 0\}$ and $L_2 = \{c^m : m > 0\}$, which of the following strings NOT in $L_1 \cdot L_2$?
 - a. λ
 - b. ccc
 - c. ab
 - d. All of the previous**
13. Find L in $L.\{a, b\} = \{a, baa, b, bab\}$
 - a. $L = \{a, ba\}$
 - c. $L = \{ba, \lambda\}$**
 - b. $L = \{b, ba\}$
 - d. $L = \{ab, ba\}$
14. Select a correct grammar generating the language $\{b, abc, aabcc, \dots\}$
 - a. $S \rightarrow aAc, A \rightarrow bA | \lambda$
 - b. $S \rightarrow b | aSc$**
 - c. $S \rightarrow \lambda | b | aSc$
 - d. $S \rightarrow aAc | \lambda, A \rightarrow bA$

15. Select a *correct* grammar generating the language $\{ac, abc, abbc, abbbc, \dots\}$
- a. $S \rightarrow aAc, A \rightarrow bA \mid \lambda$ c. $S \rightarrow acS \mid a \mid \lambda$
b. $S \rightarrow aAc, A \rightarrow bA$ d. $S \rightarrow aAc \mid \lambda, A \rightarrow bA$
16. Select a correct grammar to define the language $\{0, 01, 011, 0111, \dots\}$
- a. $S \rightarrow 0A, A \rightarrow 1 \mid 1A$ c. $S \rightarrow 0A, A \rightarrow \lambda \mid 1A$
b. $S \rightarrow 0 \mid 1S$ d. $S \rightarrow 1 \mid 0S$
17. Which of the following grammar is regular?
- a. $G = \langle \{S\}, \{a, b\}, \{S \rightarrow \lambda \mid abS\}, S \rangle$
b. $F = \langle \{S\}, \{a, b\}, \{S \rightarrow \lambda \mid b \mid aSa\}, S \rangle$
c. $H = \langle \{S, A, B\}, \{a, b\}, \{S \rightarrow AB, A \rightarrow aA \mid a, B \rightarrow bB \mid b\}, S \rangle$
d. $I = \langle \{S, A\}, \{a, +\}, \{S \rightarrow S+A \mid A, A \rightarrow a\}, S \rangle$
18. Let $G = \langle \{D, S\}, \{0, 1, 2, \dots, 9\}, P, S \rangle$, where P is:
 $S \rightarrow D1 \mid D3 \mid D5 \mid D7 \mid D9$ $D \rightarrow \lambda \mid D0 \mid D1 \mid D2 \mid D3 \mid D4 \mid D5 \mid D7 \mid D8 \mid D9$
Then the string $2018 \in L(G)$
- a. True b. False
19. If a given function f is bijective, then it is invertible.
- a. True b. False
20. Select the language defined by the following automaton:



- a. $\{1w : w \in \{0, 1\}^*\}$ c. $\{1w : w \in \{1\}^*\}$
b. $\{w1 : w \in \{0, 1\}^*\}$ d. $\{w1 : w \in \{0\}^*\}$

Question 2:

(10 Marks)

- 1) Construct a grammar to describe the language $\{10^n 1 : n \geq 1\}$, and then show **by either a derivation or a parse tree** that the string 10001 can be derived from the constructed grammar.

Answer:

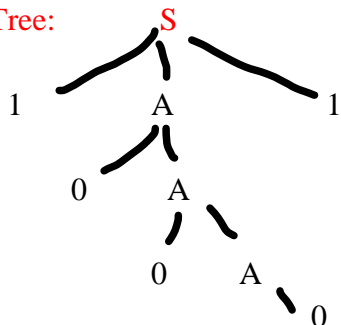
$$S \rightarrow 1A1, A \rightarrow 0 \mid 0A$$

(2 Marks)

Derivation: $S \Rightarrow 1A1 \Rightarrow 10A1 \Rightarrow 100A1 \Rightarrow 10001$

(2 Marks)

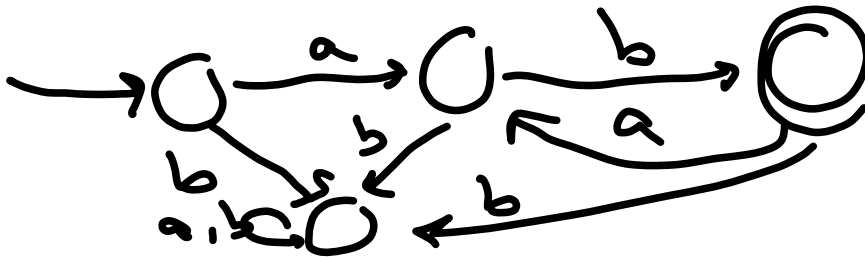
Parse Tree:



2) Show that the language $\{ab, abab, ababab, \dots\}$ is regular. (2 Marks)

Answer: only one of the following representations is enough (regular grammar or finite machine)

1. This language can be represented by the regular grammar: $S \rightarrow ab \mid abS$
2. This language can be represented by the machine:



3) Design a finite state machine to describe each of the following languages: (4 Marks)

a. $L = \{01, 011\}$



b. $M = \{aaw : w \in \{a, b\}^*\}$

