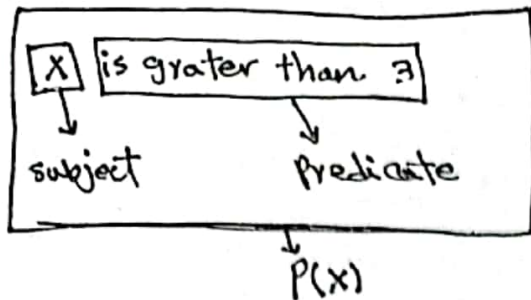


1. Predicate :- <sup>فرض</sup>



EX(1): Let  $P(x)$  denote that statement " $x > 3$ " what are the truth value of  $P(4)$  and  $P(2)$ ?

ans.

$$\because x=4 \quad \because 4 > 3 \quad \therefore P(4) = T$$

$$\because x=2 \quad \because 2 \not> 3 \quad \therefore P(2) = F$$

EX(2): Let  $\Phi(x, y)$  denote that statement " $x = y + 3$ ". what are the truth values of the propositions  $\Phi(1, 2)$  and  $\Phi(3, 0)$ ?

Ans.

$$* \Phi(1, 2) \quad x=1, y=2$$

$$\because x = y + 3 \Rightarrow 1 \neq 2 + 3$$

$$\therefore \Phi(1, 2) = F$$

$$* \Phi(3, 0) \quad x=3, y=0$$

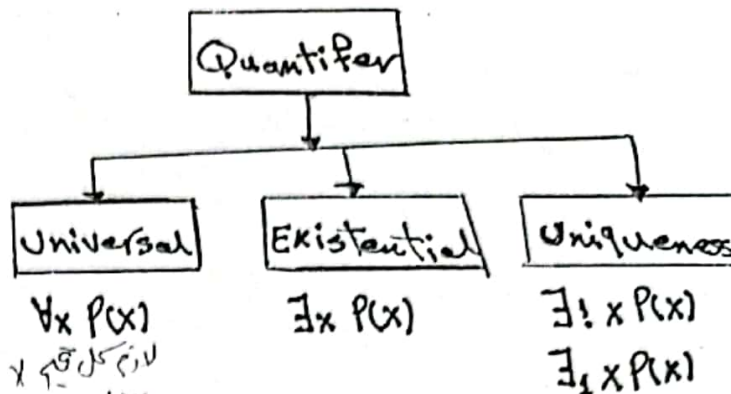
$$\because x = y + 3 \Rightarrow 3 = 0 + 3 \\ 3 = 3$$

$$\therefore \Phi(3, 0) = T$$

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\* Quantifiers :-

"كُلِّ، بعض"



\* Universal :-

EVERY

" $P(x)$  for all values of  $x$  in the domain"

لكل قيم x في المجال

\* Existential :-

"There exists an element  $x$  in the domain such that  $P(x)$ "

at least one / there is at least one.

\* Uniqueness :-

فقط واحد

"There exists a unique  $x$  such that  $P(x)$  is true"

\* Example (1):

Express the statement :-

"Every student in this class has studied Calculus"

Ans.

$P(x)$  : " $x$  has studied Calculus"

$S(x)$  : " $x$  in this class"

"The statement :  $\forall x (S(x) \rightarrow P(x))$ "

Let  $P(x)$  be the statement " $x+1 > x$ "  
 what is the truth value of the  
 quantification  $\forall x P(x)$ , where  
 the domain consists of all  
real numbers?

Ans.

As  $P(x)$  is true for all real numbers.

$\therefore \forall x P(x)$  is true.

~\*~

Let  $x=3$

$\therefore x+1 > x$

$\therefore 3+1 > 3$

$4 > 3$

$\therefore \forall x P(x)$  is true.

##

\* EX(3):

Let  $Q(x)$  be the statement " $x \leq 2$ "  
 what is the truth value of the  
 quantification  $\forall x Q(x)$ , where  
 the domain consists of all real  
 numbers?

Ans.

$\therefore Q(x)$  is not true for all/every  
 real number  $x$ .

for example  $x=3$ ,  $Q(3)$  false

$\therefore \forall x Q(x)$  is false.  
 false is not true

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\* EX(4):

Let  $P(x)$  denote the statement " $x > 3$ "  
 what is the truth value of the  
 quantification  $\exists x P(x)$ , where  
 the domain consists of all real  
 numbers?

Ans.

$\therefore P(x)$  is ~~true~~ sometimes true.

For example:  $x=4$ ,  $\therefore P(4)$  true

$\therefore \exists x P(x)$  is true.

##

\* EX(5): what is the truth value of  
 $\exists x P(x)$ , where  $P(x)$  is the  
 statement " $x^2 > 10$ " and the  
 universe of discourse consists  
 of the positive integers not  
 exceeding 4? جواباً على السؤال  
 4 is not chosen

Ans.

$\therefore$  domain is  $\{1, 2, 3, 4\}$

$\therefore P(x)$  is sometimes true.

For example  $x=4$ , then  $P(4)$  is true  
 as " $4^2 > 10$ "

$\therefore \exists x P(x)$  is true.

$\forall x P(x)$  False

...

State into English:-

### \* Negating Quantified Expressions:-

$P(x)$  is the statement "x has taken a course in Calculus" and the domain consists of the students in your class.

\*  $\forall x P(x)$  :

"Every student in your class has taken a course in Calculus".

\* The negation of this statement is:-

$$\neg \forall x P(x) \equiv \exists x \neg P(x)$$

"There is at least one student in your class who has not taken a course in Calculus".

\*  $\exists x P(x)$  :

"At least one student in your class has taken a course in Calculus".

\* The negation of this statement is:-

$$\neg \exists x P(x) \equiv \forall x \neg P(x)$$

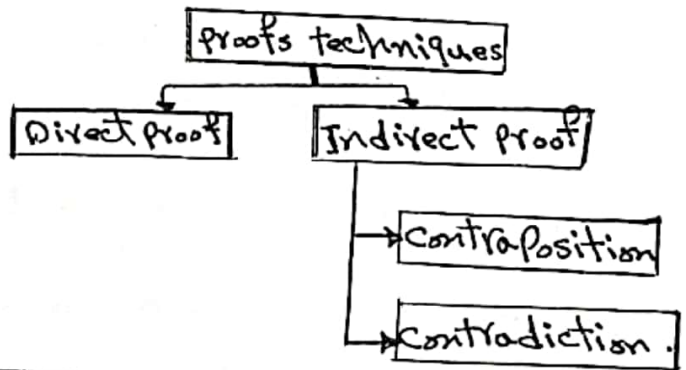
"Every student in this class has not taken Calculus".

A(2),  $\phi(3)$

$\phi(4)$

$\phi(5)$

\* Proofs:- البراهين



"كل طالب في صفي قد درس الكالculus"

\* Even Integer: زوجي

$$a = 2n, \text{ where } n \text{ is integer.}$$

\* Odd Integer: فردي

$$a = 2n + 1, \text{ where } n \text{ is integer.}$$

\*

$$\text{Even} + 1 = \text{odd}$$

$$\text{Odd} + 1 = \text{Even}$$

$$\neg \text{Even} = \text{odd}$$

$$\neg \text{odd} = \text{Even}$$

\* Perfect square:-

$$a = (n)^2$$

\* Rational number:-

عدد نسبي

$$a = \frac{n}{m}, \text{ where } n, m \text{ are integers with no common factor and } m \neq 0$$

\*

$$\neg \text{Rational} = \text{Irrational}$$

$$\neg \text{Irrational} = \text{Rational}$$



Proof:

$$P \rightarrow q$$

1. We assume that  $P$  is true.
2. We try to prove that  $q$  is also true.
3. Then  $P \rightarrow q$  is true.

Ex(1): Give a direct proof of the theorem "If  $n$  is an odd integer, then  $n^2$  is odd"

Ans:

$P$ : " $n$  is an odd integer"

$q$ : " $n^2$  is odd"

1. we assume that  $P$  is true:  
 $n = 2m+1$ ,  $m$  is integer.

2. we try to prove that  $q$  is also true:

$$\begin{aligned} n^2 &= (2m+1)^2 \\ &= 4m^2 + 4m + 1 \\ &= 2(2m^2 + 2m) + 1 \\ &= \text{even} + 1 \\ &= \text{odd}. \end{aligned}$$

3.  $\therefore P \rightarrow q$  is true.

##

$$A(2), \Phi(6)$$

\*Indirect proof (Contraposition):-

$$P \rightarrow q$$

$$\neg q \rightarrow \neg p$$

1. we assume that  $\neg q$  is true.
2. we try to prove that  $\neg p$  is also true.
3. Then  $\neg q \rightarrow \neg p$  is true.
4. Then  $P \rightarrow q$  is <sup>also</sup> true.

\*Ex(1): Prove by Contraposition that if  $n$  is an integer and  $3n+2$  is odd, then  $n$  is odd.

ans:

$P$ : " $3n+2$  is odd"

$q$ : " $n$  is odd"

1. we assume that  $\neg q$  is true  
( $n$  is even).

$$n = 2m$$

2. we try to prove that  $\neg p$  is also true.

$$\begin{aligned} 3n+2 &= 3(2m)+2 \\ &= 6m+2 \\ &= 2(3m+1) \\ &= \text{even}. \end{aligned}$$

$$\therefore 3n+2 \text{ is even.}$$

3.  $\therefore \neg q \rightarrow \neg p$  is true.

4.  $\therefore P \rightarrow q$  is <sup>also</sup> true.

##

## Direct proof (Contradiction):-

① - we want to prove  $P$ .

1.  $\neg P \rightarrow F$

2.  $\therefore P$  is true.

③ - we want to show  $P \rightarrow Q$ .

1.  $\neg Q$

2. Show that  $(P \wedge \neg Q) \rightarrow F$

3. Then  $P \rightarrow Q$  is true.

Ex: Given a proof by contradiction of theorem "If  $3n+2$  is odd, then  $n$  is odd".

Ans:

1. we assume that ( $n$  is even).

$$n = 2m$$

2.  $3n+2 = 3(2m)+2$

$$= 6m+2$$

$$= 2(3m+1)$$

$$= 2 \times (\text{any integer})$$

$$= \text{even.}$$

$\therefore (3n+2)$  is even, then  $P$  is false.

3.  $\therefore P \wedge \neg Q$  is false.

$\therefore P \rightarrow Q$  is true. ###

$$A(2), \Phi(7)$$