
بسم الله الرحمن الرحيم

EELU

الجامعة المصرية للتعلم الإلكترونى

Egyptian E-Learning University

MATH – 1

BY

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CHAPTER 4

Integration

(Continued)

LECTURE 8.

Definite and Indefinite integrals

Aims and Objectives:

(1) Learn the definite and indefinite integrals.

(2) Understand the methods of evaluating integrals.

(3) Apply rules of integrals.

(4) Have a strong intuitive feeling for these important concepts.

Exact Value Of Definite Integral:

Definition (4.4):

If $c > d$, then $\int_c^d f(x)dx = -\int_d^c f(x)dx$

Definition (4.5):

If $f(a)$ exists, then $\int_a^a f(x)dx = 0$

Theorem (4.4): $\int_a^b k\,dx = k(b-a)$

Theorem(4.5): If f is integrable on $[a, b]$ and k is any real umber, then kf is integrable on $[a, b]$ and

$$\int_a^b kf(x)dx = k\int_a^b f(x)dx$$

Theorem (4.6):

If f and are integrable on $[a, b]$, then $f + g$ and $f - g$ are integrable on $[a, b]$

and $i) \int_a^b [f(x)+g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

$ii) \int_a^b [f(x)-g(x)]dx = \int_a^b f(x)dx - \int_a^b g(x)dx$

Theorem (4.7):

If $a < c < b$, and if f is integrable on both $[a,c]$ and $[c,b]$, then f is integrable on $[a,b]$ and

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

Example 5:

1- $\int_{-2}^{3} 7dx$

Solution:

Using Theorem(4.5)

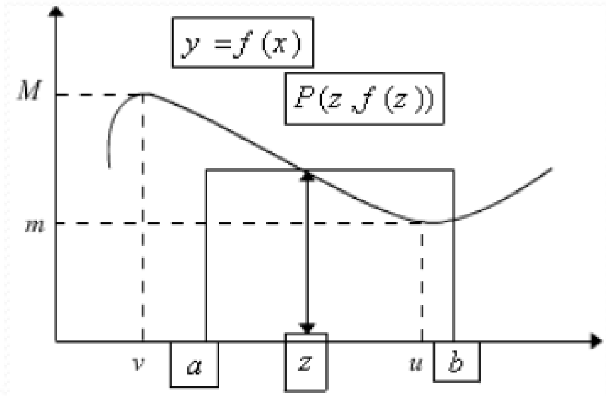
$$\int_{-2}^{3} 7dx = 7[3-(-2)] = 7(5) = 35$$

Example 6:

2- $\int_{-1}^{1} dx = 1-(-1) = 2$

The mean value theorem:

If f is continuous on a closed interval $[a, b]$, then there is a number z in the open interval (a, b) such that $\int_a^b f(x)dx = f(z)(b-a)$



Let $f(u) = m$ and $f(v) = M$, where u and v are in $[a , b]$ since f is not a constant function $m < f(x) < M$, for some x in $[a , b]$

$$\int_a^b m\,dx < \int_a^b f(x)dx < \int_a^b M\,dx$$

Employing theorem

$$m(b-a) < \int_a^b f(x)dx < M(b-a)$$

Dividing by $b - a$ and replacing m and M by $f(u)$ and $f(v)$,

$$f(z) = \frac{1}{b-a}\int_a^b f(x)dx$$

Multiplying both sides by $b - a$ gives us the conclusion of the theorem.

Example 8:

It can proved that $\int_0^3 [4-(x^2/4)]dx = \frac{39}{4}$.

Find a number that satisfies the conclusion of the mean value theorem for this integral.

Solution:

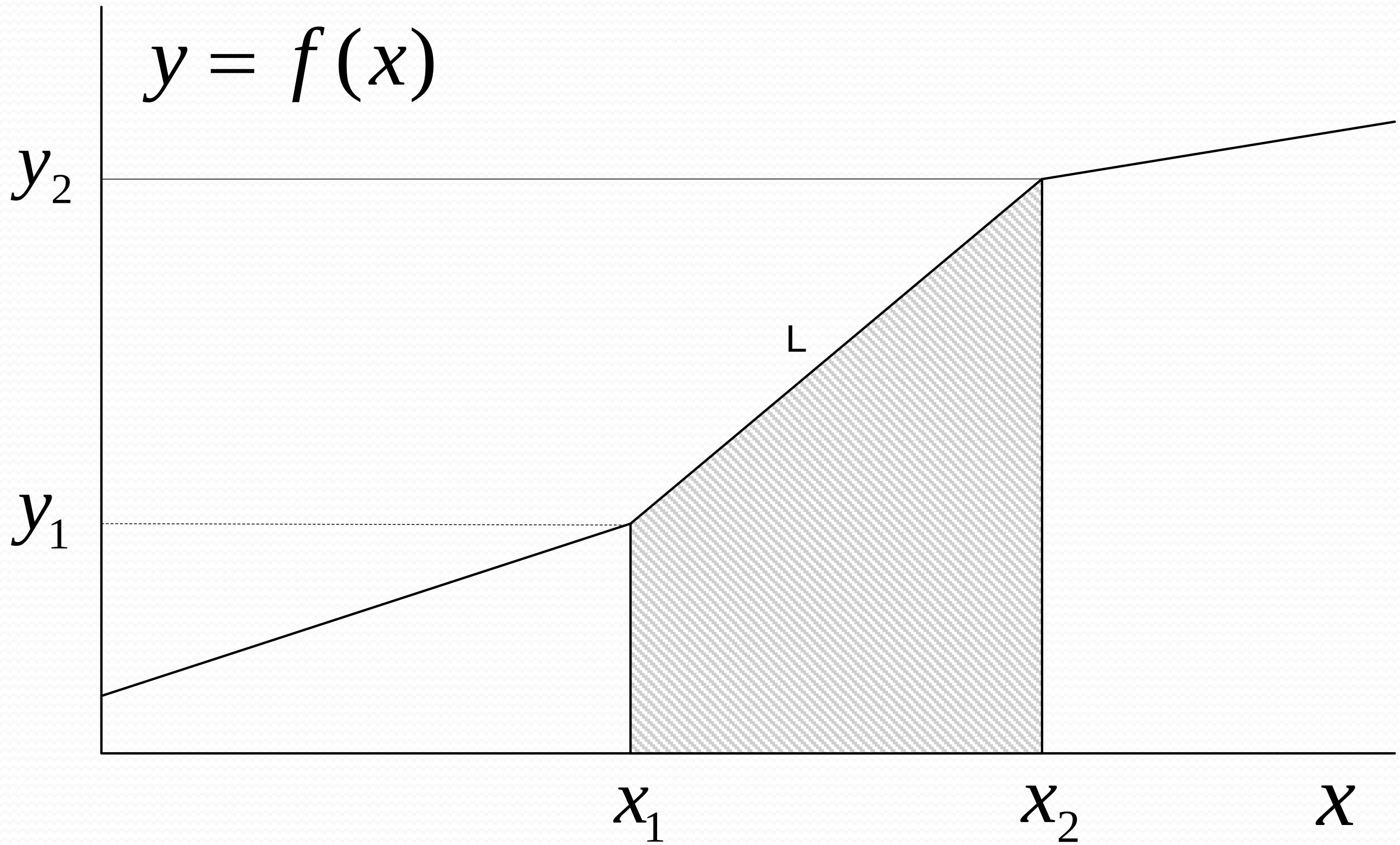
According to the mean value theorem for definite integrals, there is a number z between 0 and 3 such that $\int_0^3 (4-\frac{x^2}{4})dx = (4-\frac{z^2}{4})(3-0)$

Or, equivalently, $\frac{39}{4} = (\frac{16-z^2}{4})(3)$

Multiplying both sides of the last equation by $\frac{4}{3}$ leads to $13 = 16 - z^2$ and, therefore , $z^2 = 3$.

Consequently , $\sqrt{3}$ satisfies the condition of theorem

Area Under a Straight-Line Segment



$$\int_{x_1}^{x_2} y\,dx = \frac{1}{2}\left(y_2 + y_1\right)\left(x_2 - x_1\right)$$

Tabulation of Integrals

$$F(x) = \int f(x)dx$$

$$I = \int_a^b f(x)dx$$

$$I = F(x)\Big]_a^b = F(b) - F(a)$$

Table 1. Common Integrals.

$f(x)$	$F(x)=\int f(x)dx$	Integral Number
$af(x)$	$aF(x)$	I-1
$u(x)+v(x)$	$\int u(x)dx+\int v(x)dx$	I-2
a	ax	I-3
$x^n \qquad (n \neq -1)$	$\frac{x^{n+1}}{n+1}$	I-4
e^{ax}	$\frac{e^{ax}}{a}$	I-5
$\frac{1}{x}$	$\ln x$	I-6
$\sin ax$	$-\frac{1}{a}\cos ax$	I-7
$\cos ax$	$\frac{1}{a}\sin ax$	I-8
$\sin^2 ax$	$\frac{1}{2}x-\frac{1}{4a}\sin 2ax$	I-9

$\cos^2 ax$	$\frac{1}{2}x+\frac{1}{4a}\sin 2ax$	I-10
$x\sin ax$	$\frac{1}{a^2}\sin ax-\frac{x}{a}\cos ax$	I-11
$x\cos ax$	$\frac{1}{a^2}\cos ax+\frac{x}{a}\sin ax$	I-12
$\sin ax\cos ax$	$\frac{1}{2a}\sin^2 ax$	I-13
$\sin ax\cos bx$ for $a^2\neq b^2$	$-\frac{\cos(a-b)x}{2(a-b)}-\frac{\cos(a+b)x}{2(a+b)}$	I-14
xe^{ax}	$\frac{e^{ax}}{a^2}(ax-1)$	I-15
$\ln x$	$x(\ln x-1)$	I-16
$\frac{1}{ax^2+b}$	$\frac{1}{\sqrt{ab}}\tan^{-1}\left(x\sqrt{\frac{a}{b}}\right)$	I-17

Example 1.

$$y = 12e^{4x}$$

$$z = \int 12e^{4x}dx = 12\frac{e^{4x}}{4} + C$$

$$= 3e^{4x} + C$$

Example 2.

$$y = 6x^2 + \frac{3}{x}$$

$$z = \int \left(6x^2 + \frac{3}{x} \right) dx$$

$$= \int 6x^2 dx + \int \frac{3}{x} dx$$

$$= \frac{6x^3}{3} + 3\ln x + C$$

$$= 2x^3 + 3\ln x + C$$

Example 3.

$$I = \int_0^{\pi} \sin x dx$$

$$I = \int_0^{\pi} \sin x dx = -\cos x \Big]_0^{\pi}$$

$$= -\cos \pi - \left(-\cos 0\right)$$

$$= -(-1) - (-1) = 2$$

THANK YOU