الجامعة المصرية
للتعلم الإلكتروني الأهلية

EELU
THE EGYPTIAN E-LEARNING UNIVERSITY

GEN206
Discrete Mathematics

Section 05

Faculty of Information Technology
Egyptian E-Learning University

Fall 2020

8. Find these values.

a) $\lfloor 1.1 \rfloor$

b) $\lceil 1.1 \rceil$

c) $\lfloor -0.1 \rfloor$

d) $\lceil -0.1 \rceil$

e) $\lceil 2.99 \rceil$

f) $\lceil -2.99 \rceil$

g) $\lfloor \frac{1}{2} + \lceil \frac{1}{2} \rceil \rfloor$

h) $\lceil \lfloor \frac{1}{2} \rfloor + \lceil \frac{1}{2} \rceil + \frac{1}{2} \rceil$

a) 1 b) 2 c) -1 d) 0 e) 3 f) -2 g) 1 h) 2

9. Find these values.

a) $\lceil \frac{3}{4} \rceil$

b) $\lfloor \frac{7}{8} \rfloor$

c) $\lceil -\frac{3}{4} \rceil$

d) $\lfloor -\frac{7}{8} \rfloor$

e) $\lceil 3 \rceil$

f) $\lfloor -1 \rfloor$

g) $\lfloor \frac{1}{2} + \lceil \frac{3}{2} \rceil \rfloor$

h) $\lfloor \frac{1}{2} \cdot \lfloor \frac{5}{2} \rfloor \rfloor$

(a) 1 ; (b) 0 ; (c) 0 ; (d) -1 ; (e) 3 ; (f) -1 ; (g) Floor (0.5 + 2) = 2 ; (h) Floor (0.5 x 2) = 1

38. Find $f \circ g$ and $g \circ f$, where $f(x) = x^2 + 1$ and $g(x) = x + 2$, are functions from **R** to **R**.

$$(f \circ g)(x) = x^2 + 4x + 5$$

$$(g \circ f)(x) = x^2 + 3$$

Recall

Sequences

Geometric

A *geometric progression* is a sequence of the form

$$a, ar, ar^2, \ldots, ar^n, \ldots$$

where the $\boxed{\textit{initial term } a}$ and the $\boxed{\textit{common ratio } r}$ are real numbers.

$$2, 10, 50, 250, \ldots$$

Recall

Sequences

Arithmetic

An *arithmetic progression* is a sequence of the form

$$a, a + d, a + 2d, \ldots, a + nd, \ldots$$

where the $\boxed{\textit{initial term } a}$ and
the $\boxed{\textit{common difference } d}$ are real numbers.

1. Find these terms of the sequence $\{a_n\}$, where $a_n = 2 \cdot (-3)^n + 5^n$.

a) a_0 **b)** a_1 **c)** a_4 **d)** a_5

Solution

1. Find these terms of the sequence $\{a_n\}$, where $a_n = 2 \cdot (-3)^n + 5^n$.

a) a_0 **b)** a_1 **c)** a_4 **d)** a_5

3 -1 787 2639

5. List the first 10 terms of each of these sequences.

a) the sequence that begins with 2 and in which each successive term is 3 more than the preceding term

b) the sequence that lists each positive integer three times, in increasing order

c) the sequence that lists the odd positive integers in increasing order, listing each odd integer twice

d) the sequence whose nth term is $n! - 2^n$

e) the sequence that begins with 3, where each succeeding term is twice the preceding term

f) the sequence whose first term is 2, second term is 4, and each succeeding term is the sum of the two preceding terms

Solution

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(a) 2, 5, 8, 11, 14, 17, 20, 23, 26, 29
(b) 1, 1, 1, 2, 2, 2, 3, 3, 3, 4
(c) 1, 1, 3, 3, 5, 5, 7, 7, 9, 9
(d) $-1, -2, -2, 8, 88, 656, 4912, 40064, 362368, 3627776$
(e) 3, 6, 12, 24, 48, 96, 192, 384, 768, 1536
(f) 2, 4, 6, 10, 16, 26, 42, 68, 110, 178

Recall

Summation notation

$$\sum_{j=1}^{5} j^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$$

$$= 1 + 4 + 9 + 16 + 25$$

$$= 55.$$

Recall

Summation notation

$$\sum_{j=1}^{5} j^2 = \sum_{k=0}^{4} (k+1)^2$$

Recall

Summation notation

THEOREM

If a and r are real numbers and $r \neq 0$, then

$$\sum_{j=0}^{n} ar^j = \begin{cases} \dfrac{ar^{n+1} - a}{r - 1} & \text{if } r \neq 1 \\ (n+1)a & \text{if } r = 1. \end{cases}$$

Recall

Summation notation

TABLE 2 Some Useful Summation Formulae.

Sum	*Closed Form*
$\sum_{k=0}^{n} ar^k \ (r \neq 0)$	$\frac{ar^{n+1} - a}{r - 1}, r \neq 1$
$\sum_{k=1}^{n} k$	$\frac{n(n+1)}{2}$
$\sum_{k=1}^{n} k^2$	$\frac{n(n+1)(2n+1)}{6}$
$\sum_{k=1}^{n} k^3$	$\frac{n^2(n+1)^2}{4}$
$\sum_{k=0}^{\infty} x^k, \lvert x \rvert < 1$	$\frac{1}{1-x}$
$\sum_{k=1}^{\infty} kx^{k-1}, \lvert x \rvert < 1$	$\frac{1}{(1-x)^2}$

29. What are the values of these sums?

a) $\sum_{k=1}^{5}(k+1)$

b) $\sum_{j=0}^{4}(-2)^{j}$

20

11

c) $\sum_{i=1}^{3} \sum_{j=0}^{2} j$

9

d) $\sum_{i=0}^{2} \sum_{j=0}^{3} i^2 j^3$

180

35. Show that $\sum_{j=1}^{n}(a_j - a_{j-1}) = a_n - a_0$, where $a_0, a_1, \ldots, a_n$ is a sequence of real numbers. This type of sum is called **telescoping**.

$$\sum_{j=1}^{n} a_j a_{j-1} \quad = \sum_{j=1}^{n} a_j - \sum_{j=1}^{n} a_{j-1} \;= \sum_{j=1}^{n} a_j - \sum_{k=0}^{n-1} a_k$$

$$= (a_n + \cancel{a_{n-1}} + \cancel{a_{n-2}} + \cdots + \cancel{a_1}) - (\cancel{a_{n-1}} + \cancel{a_{n-2}} + \cdots + \cancel{a_1} + a_0)$$

$$= \boldsymbol{a_n - a_0}$$

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39. Find $\sum_{k=100}^{200} k$. (Use Table 2.)

$\sum_{k=1}^{n} k$	$\frac{n(n+1)}{2}$

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$$\sum_{k=100}^{200} k$$

$$= \sum_{k=1}^{200} k - \sum_{k=1}^{99} k$$

$$= \frac{200(200+1)}{2} - \frac{99(99+1)}{2}$$

$$= 20100 - 4950$$

$$= 15150$$

40. Find $\sum_{k=99}^{200} k^3$. (Use Table 2.)

$\sum_{k=1}^{n} k^3$	$\frac{n^2(n+1)^2}{4}$

40. Find $\sum_{k=99}^{200} k^3$. (Use Table 2.)

$$\sum_{k=99}^{200} k^3$$

$$= \sum_{k=1}^{200} k^3 - \sum_{k=1}^{98} k^3$$

$$= \frac{200^2(200+1)^2}{4} - \frac{98^2(98+1)^2}{4}$$

$$= 404,010,000 - 23,532,201$$

$$= 380477799$$