Revision
SOLUTIONS

1) Let p be "It is cold" and let q "It is raining". Give a simple verbal sentence which describes each of the following statements:
   a) $\neg p$
   b) $p \wedge q$
   c) $p \vee q$
   d) $q \vee \neg p$

   answer. a) It is not cold. b) It is cold and raining. c) It is cold or it is raining. d) It is raining or it is not cold.

2) Repeat problem one for:
   a) $\neg p \wedge \neg q$
   b) $\neg(\neg q)$

   answer: a) "It is not cold and it is not raining" or " It is neither cold nor raining". b) "It is not true that it is not raining" or " It is false that it is nor raining".

3) Let p be "Ahmed speaks French" and let q be "Ahmed speaks Danish". Give a simple verbal sentence which describes each of the following:
   a) $p \vee q$
   b) $p \wedge q$
   c) $p \wedge \neg q$
   d) $\neg p \vee \neg q$

   answer: a) Ahmed speaks French or Danish. b) Ahmed speaks French and Danish. c) Ahmed speaks French but not Danish. d) Ahmed does not speak French or she does not speak Danish.

4) Repeat problem three for:
   a) $\neg\neg p$
   b) $\neg(\neg p \wedge \neg q)$

   answer: a) It is not true that Ahmed does not speak French.
   b) It is not true that Ahmed speaks neither French nor Danish.

5) Determine the true value of each of the following statements:

a) 4 + 2 = 5 and 6 + 3 = 9
b) 3 + 2 = 5 and 6 + 1 = 7
c) 4 + 5 = 9 and 1 + 2 = 4
d) 3 + 2 = 5 and 4 + 7 = 11

answer: a) false b) true c) false d) true

6) Determine the true value of each of the following statement:

a) 1 + 1 = 5 and 2 + 2 = 4
b) 1 + 1 = 5 and 3 + 3 = 4
c) 2 + 5 = 9 and 3 + 7 = 8
d) 2 + 5 = 9 and 1 + 7 = 8

answer: a) false b) false c) false d) false

7) Determine the true value of each of the following statements:

a) It is false that 2 + 2 = 4 and 1 + 1 = 5.
b) It is false that 2 + 2 = 4 or London is in France.

answer: a) true b) false

8) Find the truth table of $\neg p \wedge q$.

answer:

p	q	$\neg p$	$\neg p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

9) Find the truth table of $\neg(p \vee \neg q)$.

answer:

p	q	$\neg p$	$\neg q$	$\neg(p \vee \neg q)$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	F

10) Find the truth table of the following:

a) $p \wedge (q \vee r)$

b) $(p \wedge q) \vee (p \wedge r)$

answer:

p	q	r	$q \vee r$	$p \wedge (q \vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

$p \wedge (q \vee r)$

p	q	r	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	F	F
F	T	T	F	F	F
F	T	F	F	F	F
F	F	T	F	F	F
F	F	F	F	F	F

$(p \wedge q) \vee (p \wedge r)$

11) Verify that the proposition $p \vee \neg(p \wedge q)$ is a tautology.

answer:

p	q	$p \wedge q$	$\neg(p \wedge q)$	$p \vee \neg(p \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

12) Verify that the proposition $(p \wedge q) \wedge \neg(p \vee q)$ is a contradiction.

answer:

p	q	$p \wedge q$	$(p \vee q)$	$\neg(p \vee q)$	$(p \wedge q) \wedge \neg(p \vee q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

13) Show that the propositions $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are logically equivalent.

answer:

p	q	$p \wedge q$	$\neg(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

14) Simplify each of the following propositions:

a) $\neg(\neg p \vee q)$

b) $\neg(\neg p \wedge \neg q)$

answer: a) $\neg(\neg p \vee q) = \neg\neg p \wedge \neg q = p \wedge \neg q$.

b) $\neg(\neg p \wedge \neg q) = \neg\neg p \vee \neg\neg q = p \vee q$

15) Simplify each of the following statements:

a) It is not true that sales are decreasing and prices are rising.

b) It is not true that it is not cold or it is raining.

answer: a) Since $\neg(p \wedge q) = \neg p \vee \neg q$, the given statement is logically equivalent to the statement "Sales are increasing or prices are falling"

b) Since $\neg(\neg p \vee q) = p \wedge \neg q$, the given statement is logically equivalent to the statement "It is cold and it is not raining".

16) Write the negation of each the following statements as simply as possible.

a) He is tall but handsome.

b) He has blond hair or blue eyes.

answer: a) Use $\neg(p \wedge q) = \neg p \vee \neg q$ to obtain " He is not tall or not handsome".

b) $\neg(p \vee q) = \neg p \wedge \neg q$ to obtain "He has neither blond hair nor blue eyes".

17) Write the negation of each the following statements as simply as possible.

a) He is neither rich nor happy

b) He has lost his job or he did not go to work today.

answer: a) Use $\neg(\neg p \wedge \neg q) = p \vee q$ to obtain "He is rich or happy".

b) Use $\neg(p \vee \neg q) = \neg p \wedge q$ to obtain "He has not lost his job and he did go to work today".

18) Show that $p \to q$ is logically equivalent to $\neg p \vee q$. that is,

$p \to q = \neg p \vee q$

answer:

p	q	$\neg p$	$\neg p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

19) Rewrite the following statements without using the conditional.

a) If it is cold, he wears a hat.
b) If productivity increases, then wages rise.

answer: Recall that "If p then q" is equivalent to "Not p or q", that is,
$p \to q = \neg p \vee q$

a) It is not cold or he wears a hat.
b) Productivity does not increase or wages rise.

20) Construct the truth table of $\neg p \to (q \to p)$.

answer:

p	q	$q \to p$	$\neg p$	$\neg p \to (q \to p)$
T	T	T	F	T
T	F	T	F	T
F	T	F	T	F
F	F	T	T	T

21) Which, if any of the propositions: $q \to p$, $\neg p \to \neg q$, and $\neg q \to \neg p$ is logically equivalent to $p \to q$?

answer:

p	q	$\neg p$	$\neg q$	$p \to q$	$q \to p$	$\neg p \to \neg q$	$\neg q \to \neg p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

22) Show that the biconditional $p \leftrightarrow q$ can be written in terms of the original three connectives $\vee, \wedge,$ and $\neg$.

answer:

$p \to q = \neg p \vee q$ and $q \to p = \neg q \vee p$ and

p	q	$p \leftrightarrow q$	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

23) Find the truth table for $(p \to q) \vee \neg(p \leftrightarrow \neg q)$

answer:

p	q	$\neg q$	$p \to q$	$\neg(p \leftrightarrow \neg q)$	$(p \to q) \vee \neg(p \leftrightarrow \neg q)$
T	T	F	T	T	T
T	F	T	F	F	F
F	T	F	T	F	T
F	F	T	T	T	T

24) Determine the truth value of each of the following statements where $\mathbb{R}$ is the universal set.

a) $\forall x, |x| = x.$

b) $\exists x,\ x^2 = x.$

c) $\forall x,\ x+1 > x.$

d) $\exists x,\ x+2 = x.$

answer: a) false b) true c) true d) false

25) Let A = {1,2,3,4,5}. Determine the truth value of each of the following statements.:

a) $(\exists x \in A)(x+3=10)$

b) $(\forall x \in A)(x+3<10)$

c) $(\exists x \in A)(x+3<5)$

d) $(\forall x \in A)(x+3 \le 7)$

answer: a) false b) true c) true d) false

26) Determine the truth value of the following statement where $\mathbb{R}$ is the universal set.

a) $\forall x, x^2 = x.$

b) $\exists x, 2x = x.$

c) $\forall x, x-3 < x.$

d) $\exists x, x^2 - 2x + 5 = 0.$

answer: a) false b) true c) true d) false

27) Negate each of the statements:

a) $\forall x p(x) \wedge \exists y q(y).$

b) $\exists x p(x) \vee \forall y q(y)$

answer: a) Note that $\neg(p \wedge q) = \neg p \vee \neg q$ hence

$\neg(\forall x p(x) \wedge \exists y q(y)) = \neg\forall x p(x) \vee \neg\exists y q(y) = \exists x \neg p(x) \vee \forall y \neg q(y)$

b) Note that $\neg(p \vee q) = \neg p \wedge \neg q$ hence

$\neg(\exists x p(x) \vee \forall y q(y)) = \neg\exists x p(x) \wedge \neg\forall y q(y) = \forall x \neg p(x) \wedge \exists y \neg q(y)$