
بسم الله الرحمن الرحيم

EELU

الجامعة المصرية للتعلم الإلكترونى

Egyptian E-Learning University

MATH – 1

BY

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CHAPTER 1

Functions and Their Graphs

Outlines

(1) FUNCTIONS, ESSENTIAL FUNCTIONS (LINEAR, POWER, POLYNOMIAL, RATIONAL, ALGEBRAIC, TRIGONOMETRIC).

(2) NEW FUNCTIONS FROM OLD FUNCTIONS (GRAPHING, COMBINED, COMPOSITE FUNCTIONS).

(3) EXPONENTIAL FUNCTIONS, INVERSE FUNCTIONS, LOGARITHMIC AND INVERSE TRIGONOMETRIC FUNCTIONS.

Aims and Objectives:

(1) DEFINE FUNCTIONS AND TO GIVE SOME ILLUSTRATING EXAMPLES.

(2) REPRESENT THE FUNCTION SYMBOLICALLY.

(3) DEMONSTRATE A FUNCTION AS AN INPUT OUTPUT PROCESS.

(4) INTRODUCE THE NOTION OF DOMAIN AND RANGE OF A FUNCTION.

(5) EXPLAIN THE NOTION OF PIECEWISE FUNCTIONS.

(6) DEFINE THE GRAPH OF A FUNCTION.

(7) SKETCH THE GRAPH OF A FUNCTION.

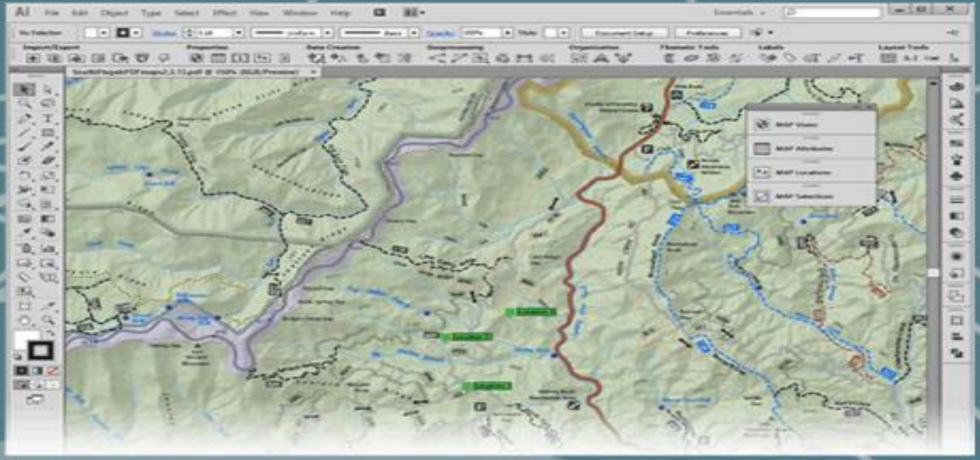
Lecture 1

ESSENTIAL FUNCTIONS

(LINEAR, POWER, POLYNOMIAL, RATIONAL, ALGEBRAIC, TRIGONOMETRIC).

- **APPLICATIONS OF FUNCTION REPRESENTATION IN COMPUTER SCIENCE**

Communication networks analysis, social network analysis, cartography, bioinformatics, and many more.



WHAT DOES THE FUNCTION MEAN?

A function is an equation which shows the relationship between the input x and the output y and where there is exactly one output for each input (inputs is domain and output is the range).

To say that braking distance is a function of speed we can write $d = f(s)$. To say that the concentration of a medicine in the bloodstream is a function of time between doses we can write $c = g(t)$.

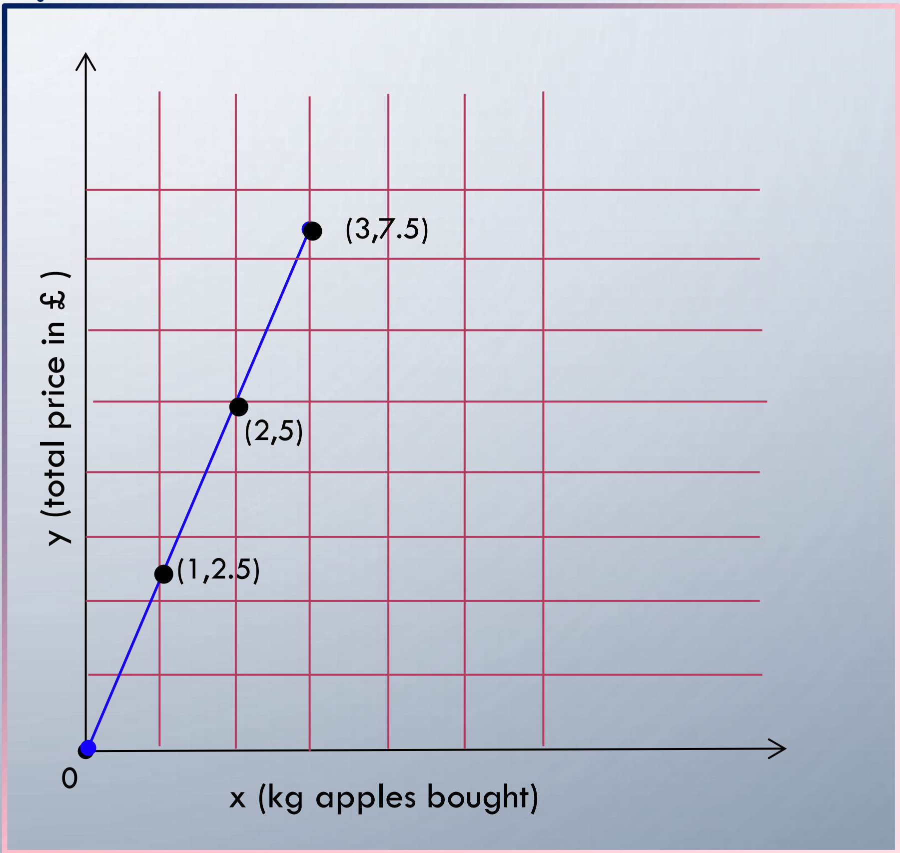
EXAMPLE The costumer has to pay, y, which is dependent on how many kg of apples, x, that he buys: The number of kg bought is called the independent variable since that's what we're changing whereas the total price is called the dependent variable since it is dependent on how many kg we actually buy:

WHAT DOES THE FUNCTION MEAN?

We can thus write our values as ordered pairs: $y = 2.50\ x$

- (0, 0) - Origin
- (1, 2.5)
- (2, 5)
- (3, 7.5)

Input, x (kg)	Output, y (£)
0	0
1	2.50
2	5.00
3	7.50
Input, x (kg)	**Output, y (£)**

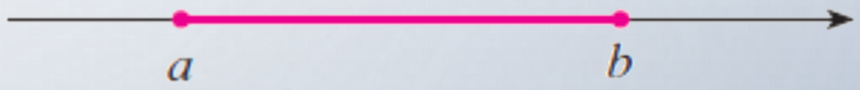
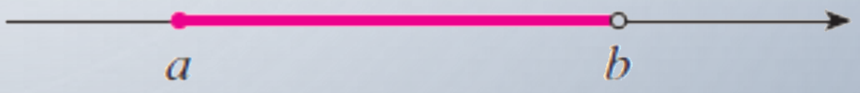
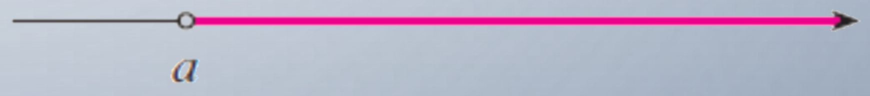
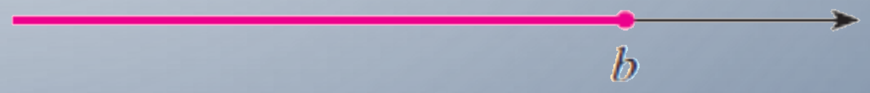


These ordered pairs can then be plotted into a graph as a function $y = 2.50x$.

Set of Numbers:

Natural numbers	$\mathbb{N}=\{1,2,3,4,\ldots\}$
Integers	$\mathbb{Z}=\{\ldots,-2,-1,0,1,2,\ldots\}$
Rational numbers	$\mathbb{Q}=\left\{\frac{p}{q}\,\middle\|\,p,q\in\mathbb{Z}\,;\,q\neq 0\right\}$
Irrational numbers	$\mathbb{Q}'=\left\{\ldots,\sqrt{2},\sqrt[3]{-5},\pi,e,\ldots\right\}$
Real numbers	$\mathbb{R}=\mathbb{Q}\cup\mathbb{Q}'$

Intervals :

Notation	***Set description***	***Picture***
(a,b)	$\{x \mid a < x < b\}$	
$[a,b]$	$\{x \mid a \le x \le b\}$	
$[a,b)$	$\{x \mid a \le x < b\}$	
(a,∞)	$\{x \mid x > a\}$	
$(-\infty,b]$	$\{x \mid x \le b\}$	
$(-\infty,\infty)$	$\mathbb{R} = \{x \mid -\infty < x < \infty\}$	

PROPERTIES OF REAL NUMBERS

I. Algebraic properties of $\mathbb{R}$:

1) $a+b=b+a\ \forall\ a,b\in\mathbb{R}$ (commutative law for addition)

2) $(a+b)+c=a+(b+c)\ \forall\ a,b,c\in\mathbb{R}$ (associative law for addition)

3) $\exists\ 0\in\mathbb{R}:\ a+0=0+a=\mathrm{a}\ \forall\ a\in\mathbb{R}$ (existence of a zero element)

4) $\forall\ a\in\mathbb{R},\exists(-\mathrm{a})\in\mathbb{R}:\ a+(-a)=0$ (existence of negative elements)

5) $ab=ba\ \forall\ a,b\in\mathbb{R}$ (commutative law for multiplication)

6) $(ab)c=a(bc)\ \forall\ a,b,c\in\mathbb{R}$ (associative law for multiplication)

7) $\exists\ 1\in\mathbb{R},1\neq 0:\ a\cdot 1=\mathrm{a}\ \&\ 1\cdot a=a$ (existence of a unit element)

8) $\forall\ a\in\mathbb{R},\ \ a\neq 0,\exists\left(\frac{1}{a}\right)\in\mathbb{R}:a\left(\frac{1}{a}\right)=1\ \&\ \left(\frac{1}{a}\right)a=1$ (existence of negative reciprocals)

9) $a(b+c)=ab+ac\ \forall\ a,b,c\in\mathbb{R}$ (distributive law of multiplication over addition)

PROPERTIES OF REAL NUMBERS

II. Rules for Inequalities :

If a, b and c are real numbers, then

(i) If $a < b$ then $a \pm c < b \pm c$

(ii) If $a < b$ and $c > 0$ then $\mathrm{ac} < \mathrm{bc}$.

(iii) If $a < b$ and $c < 0$ then $\mathrm{ac} > \mathrm{bc}$.

(iii) If $0 < a < b$ and $\frac{1}{a} > \frac{1}{b}$.

III. Absolute Value

The absolute value of a real number a, denoted by $|a|$, is defined by:

$$\sqrt{a^2} = |\mathrm{a}| = \begin{cases} \mathrm{a} & if\ \mathrm{a} \geq 0 \\ -\mathrm{a} & if\ \mathrm{a} < 0 \end{cases}$$

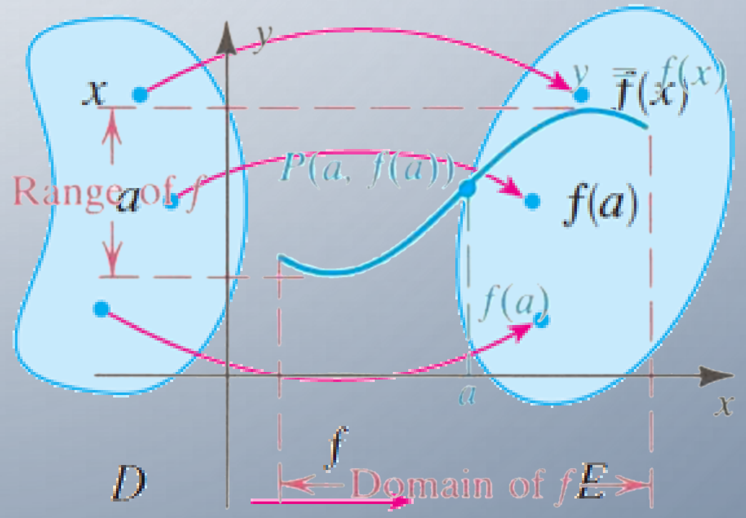
1. $|a\ b| = |a|\ |b|\ \forall\ a, b\epsilon\mathbb{R}$.
2. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|},\quad b \neq 0\ \forall\ a, b\epsilon\mathbb{R}$.
3. $|a+b|\ \leq\ |a| + |b| \forall\ \ a, b\epsilon\mathbb{R}$.
4. $\big||a| - |b|\big|\ \leq |a-b|\ \ \forall\ a, b\epsilon\mathbb{R}$.
5. $|\mathrm{x}| = a \Leftrightarrow x = \pm a$.
6. $|\mathrm{x}| \leq a \Leftrightarrow -a \leq x \leq a$.
7. $|\mathrm{x}| \geq a \Leftrightarrow -a \geq x \text{ or } x \geq a$.

Functions and Their Graphs

Definition :

Let D & E be subsets of $\mathbb{R}$. A real valued function f on D is a rule that assigns for each value of the variable $x \in D$ exactly one real value of the variable $y \in E$ such that $y = f(x)$.

* The element x is called **independent** variable and y is called **dependent** variable.



$\{(x, f(x)) \mid x \in D\}$

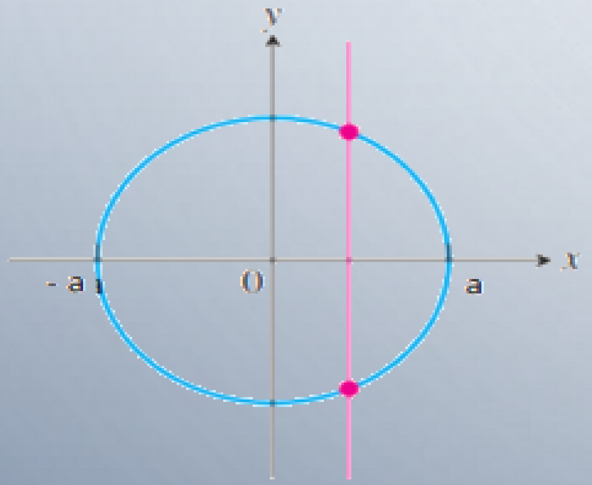
* **The domain** of the function is the set D of all possible input values of x.

* **The range** of the function is the set of all values of $y = f(x)$ as x varies through D.

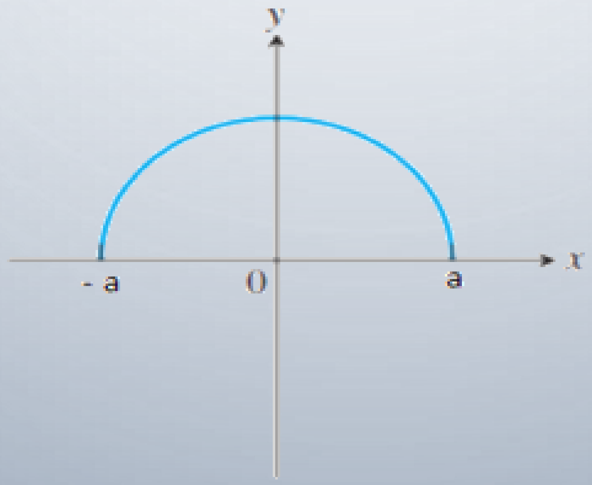
FUNCTIONS AND THEIR GRAPHS

The graph of a function f is the graph of the equation $y = f(x)$ for x in the domain of f.

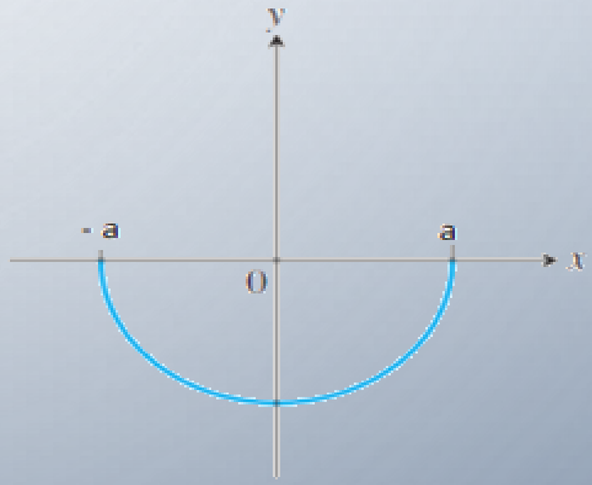
The Vertical Line Test: The graph of a function intersects any vertical line at most once.



(a) $x^2 + y^2 = a^2$
Circle
Not a function



(b) $y = \sqrt{a^2 - x^2}$
Upper semi-circle
Domain [-a,a]
Range [0,a]



(c) $y = -\sqrt{a^2 - x^2}$
Lower semi-circle
Domain [-a,a]
Range [-a,0]

FUNCTIONS AND THEIR GRAPHS

- **GRAPH OF FUNCTIONS:**

If f is a function with domain $\mathcal{D}$, then the graph of f is the set of all points $\mathcal{P}\,(x, f\,(x))$ in the plane, where $x \in \mathcal{D}$. That is, the graph of f is the graph of the equation $y = f\,(x)$:

Graph of f = $\{(x,y) : x \in \mathcal{D}$ and $y = f\,(x)\}$.

EXAMPLE:

Graph the function $f\,(x) = x^2$ over the interval $-2 \le x \le 2$

SOLUTION:

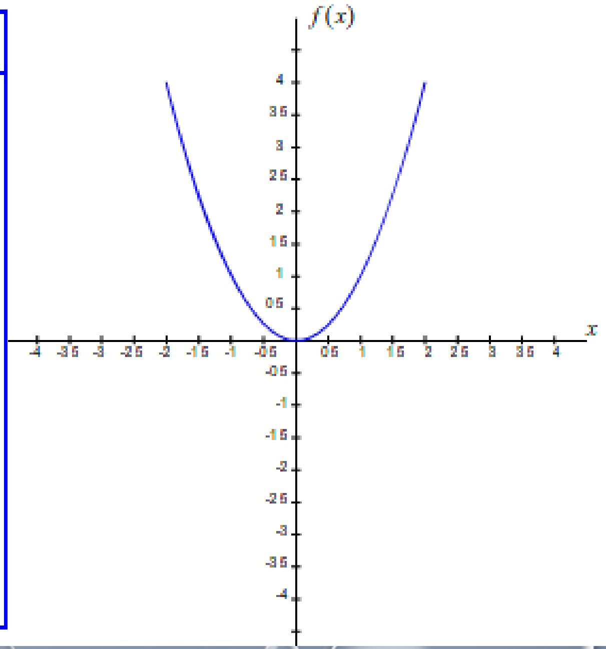
To graph the function, we carry out the following steps:

1- We make a table of input-output pairs for the function.

2- We plot the corresponding points to learn the shape.

3- We sketch the graph by connecting the point.

x	$f(x) = x^2$
-2.0	4.0
-1.75	3.0625
-1.5	2.25
-1.0	1.0
-0.5	0.25
0	0
0.5	0.25
1.0	1.0
1.5	2.25
1.75	3.0625
2	4



Functions and Their Graphs

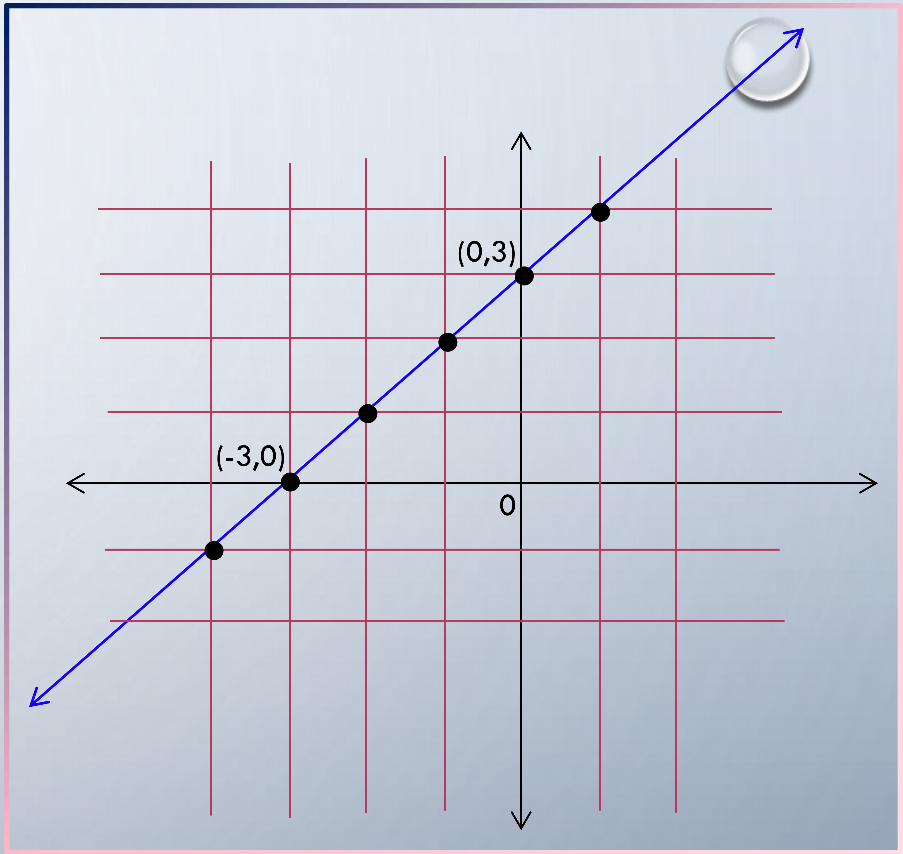
(a) Linear Functions :

A graph of a linear function is a straight line.

The slope intercept form of the equation of a straight line is

$$y = f(x) = mx + b,$$

where m is the slope of the line and b is the y-intercept.



x	0	-1	-2	-3	1	2	3
y	3	2	1	0	4	5	6

EXAMPLE $y = x + 3, m = 1 \; and \; b = 3$

$$Domain = \mathbb{R} = (-\infty, \infty)$$

$$Range = \mathbb{R} = (-\infty, \infty)$$

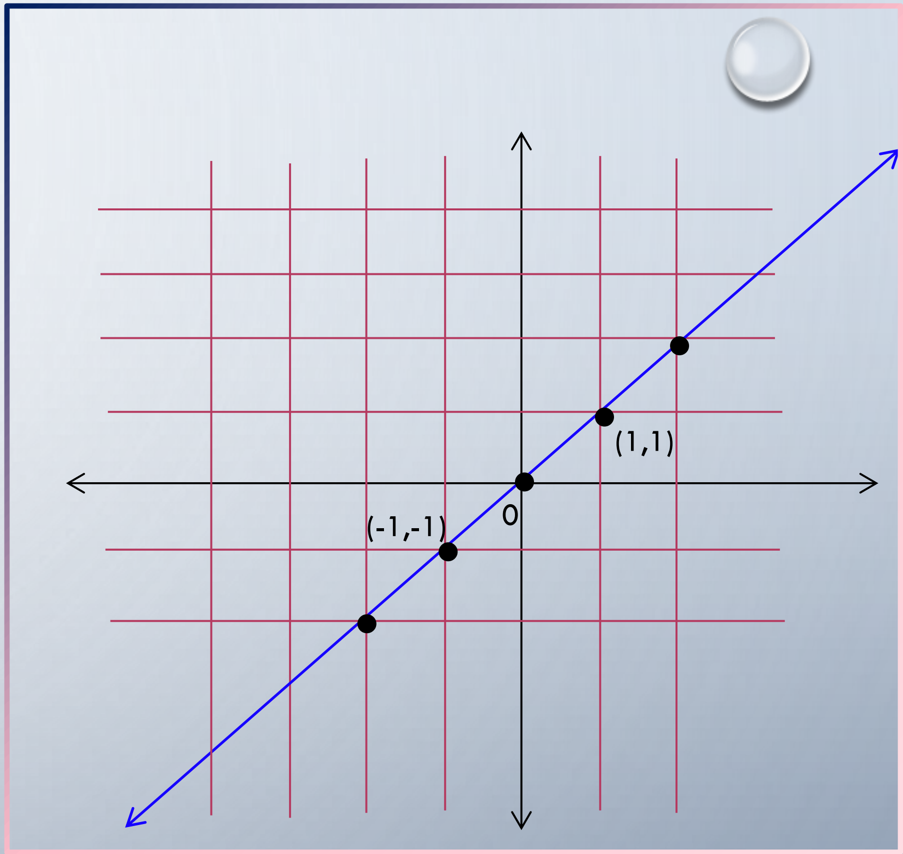
(b) Power Functions :

A function of the form $f(x) = x^n$, where n is a constant, is called a power function.

(i) n=1 $\Rightarrow$ $Domain = \mathbb{R}$, $Range = \mathbb{R}$

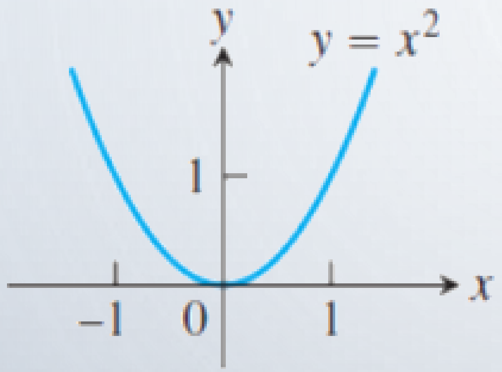
EXAMPLE

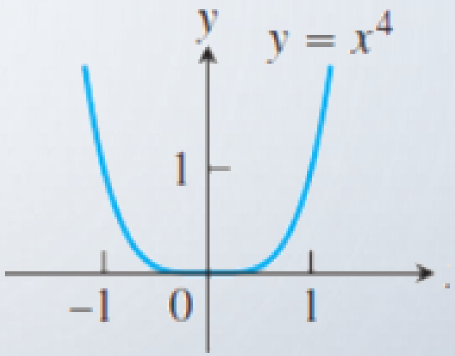
$$y = x,$$
$$m = 1 \; and \; b = 0$$
$$Domain = R = (-\infty, \infty)$$
$$Range = R = (-\infty, \infty)$$

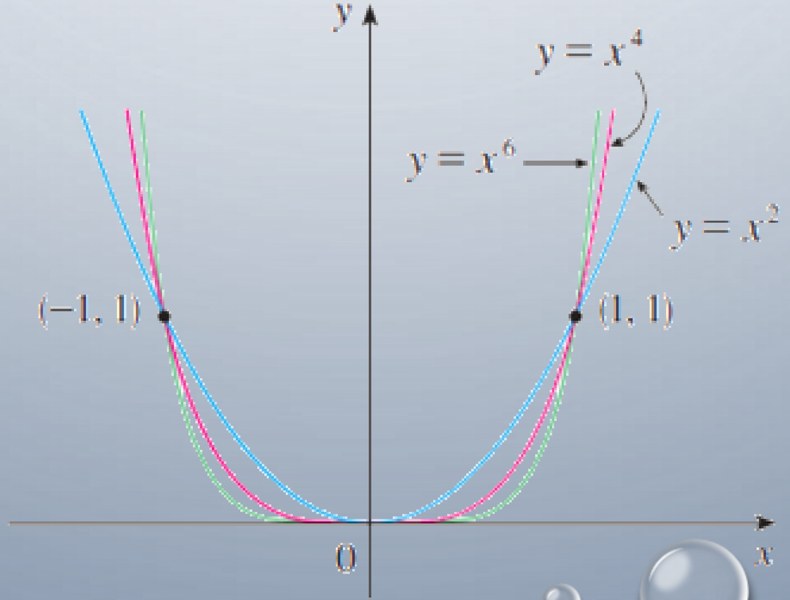


x	0	-1	-2	-3	1	2	3
y	0	-1	-2	-3	1	2	3

(ii) $\mathbf{n = 2, 4, 6, \ldots \quad \Rightarrow \quad \mathit{Domain} = \mathbb{R}, \;\; \mathit{Range} = [0, \infty)}$







EXAMPLE:

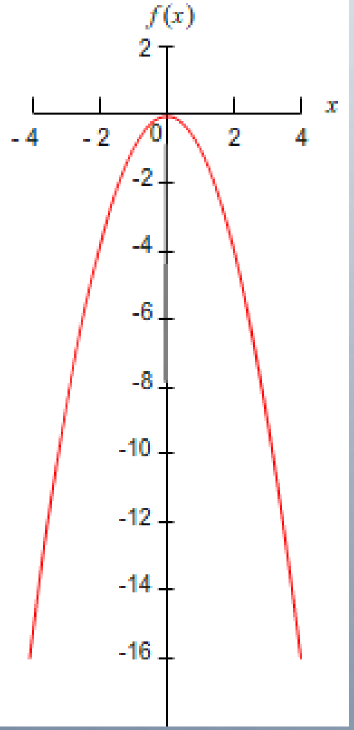
Graph the function

$f(x) = -x^2$

over the interval $-4 \le x \le 4$

SOLUTION:

x	$f(x) = -x^2$
- 4.0	-16
- 3.0	-9
- 2.5	-6.25
- 2.0	-4
- 1.5	-2.25
0	0
1.5	-2.25
2.0	-4
2.5	-6.25
3.0	-9
4.0	-16



EXAMPLE:

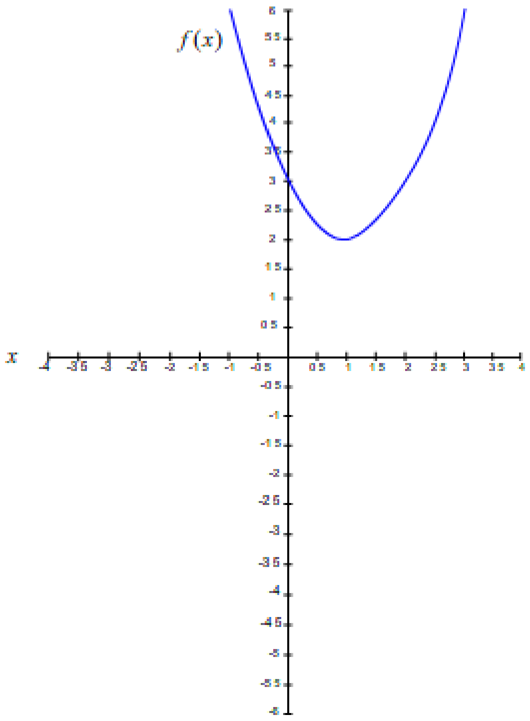
Graph the function

$f(x) = x^2 - 2x + 3$

over the interval $-1 \le x \le 3$

SOLUTION:

x	$f(x)$
-1	6
0	3
0.5	2.25
1	2
2	3
3	6



EXAMPLE:

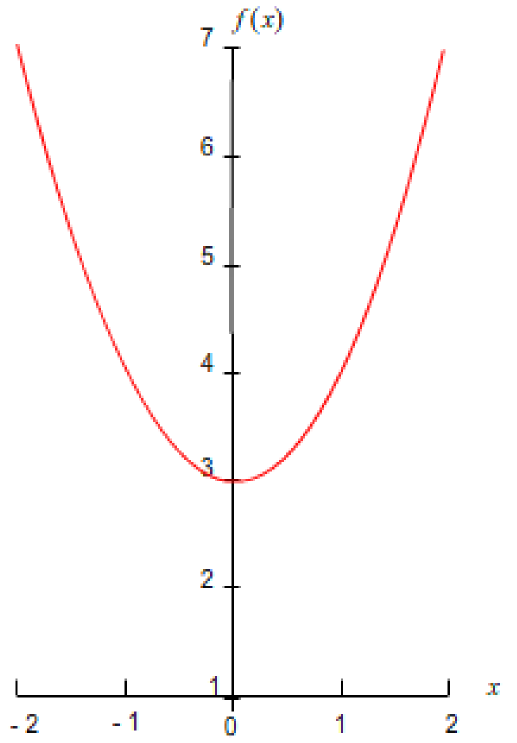
Graph the function

$f(x) = x^2 + 3$

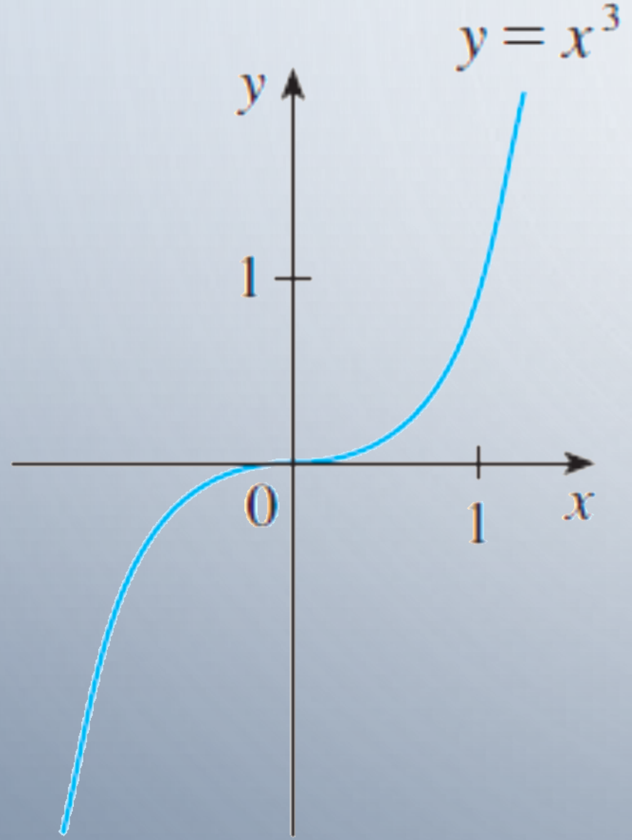
over the interval $-2 \le x \le 2$

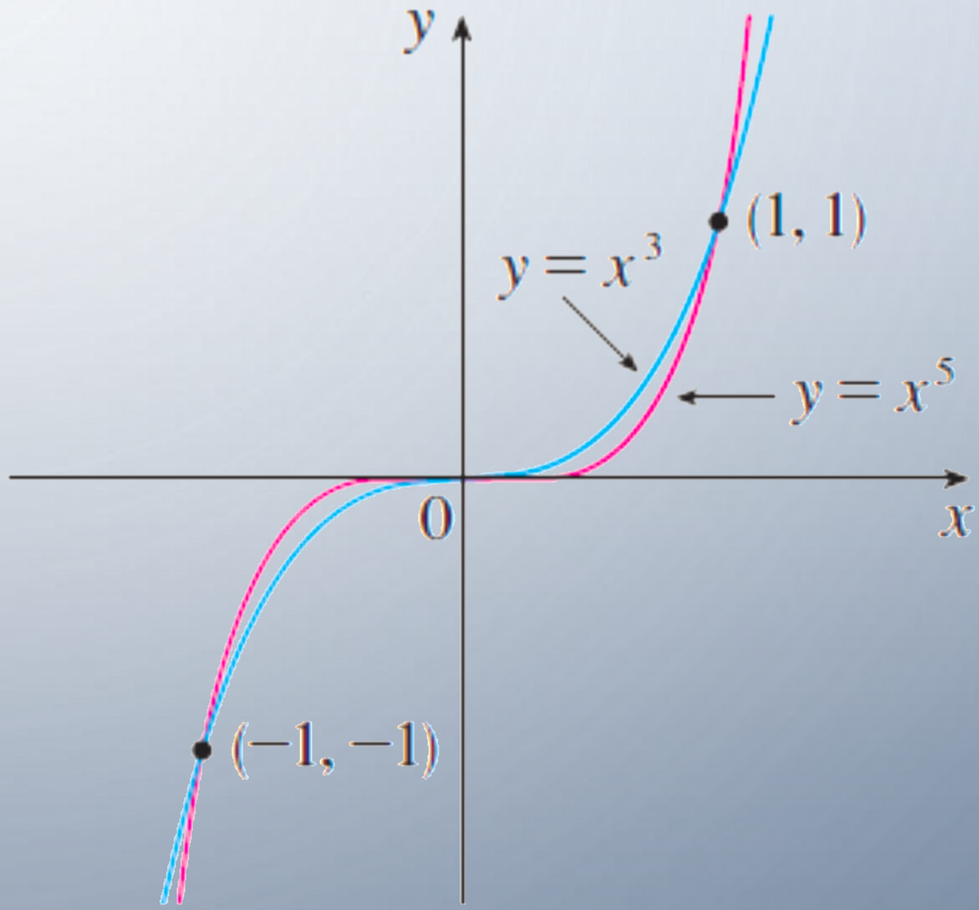
SOLUTION:

x	$f(x) = x^2 + 3$
- 2.0	7
-1.75	6.5
- 1.5	5.25
- 1.0	4
0.5	3.25
0	3
0.5	3.25
1.0	1
1.5	5.25
1.75	6.5
2.0	7

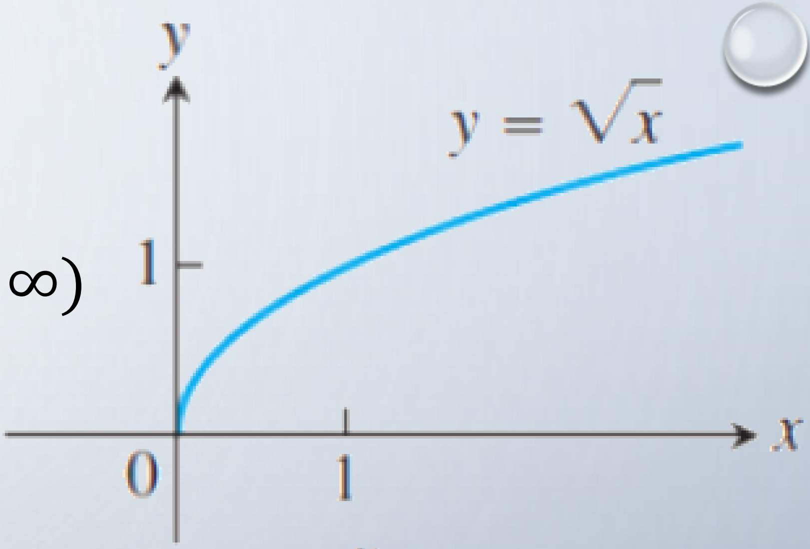


(iii) $\mathbf{n = 3, 5, 7, \ldots} \quad \Rightarrow \quad \boldsymbol{Domain = \mathbb{R},\ Range = \mathbb{R}}$

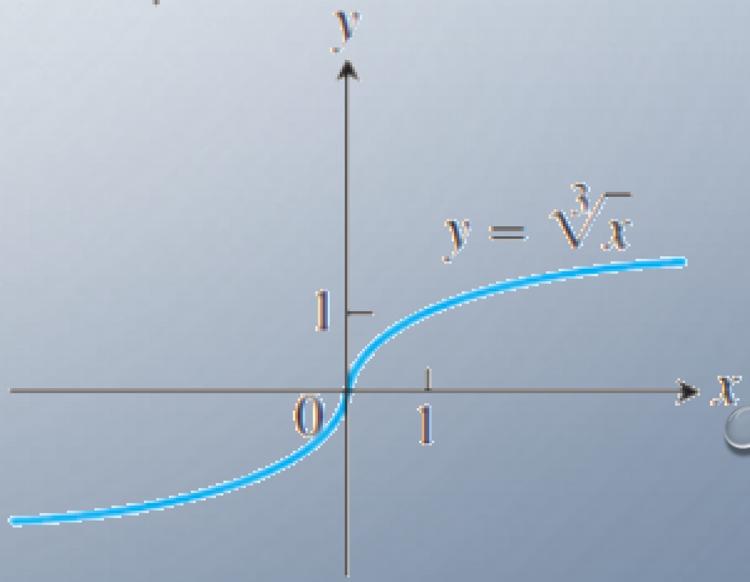




(iii) $\mathbf{n = \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \ldots \quad \Rightarrow \; \boldsymbol{Domain} = [0, \infty), \; \boldsymbol{Range} = [0, \infty)}$



(iv) $\mathbf{n = \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots \Rightarrow \; \boldsymbol{Domain} = \mathbb{R}, \; \boldsymbol{Range} = \mathbb{R}}$



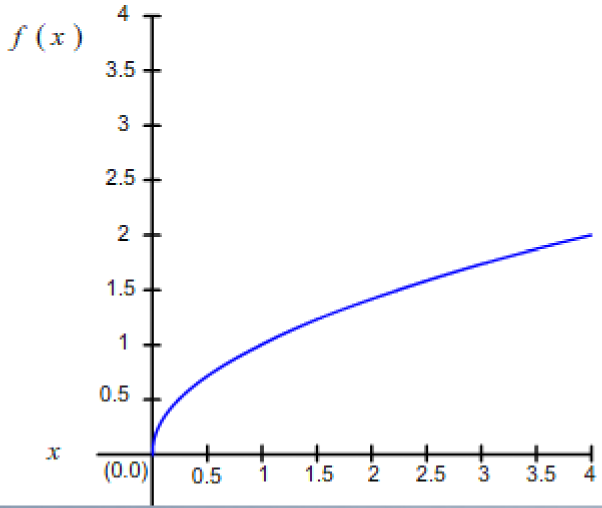
EXAMPLE:

Graph the function $f(x) = \sqrt{x}$

over the interval $0 \le x \le 4$

SOLUTION:

x	$f(x) = \sqrt{x}$
0	0
0.25	0.5
1	1
2	$\sqrt{2}$
4	2



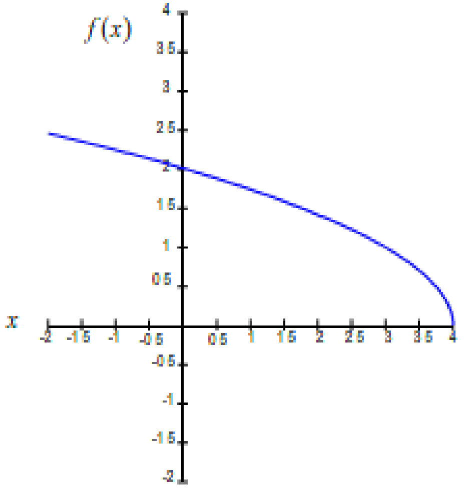
EXAMPLE:

Graph the function $f(x) = \sqrt{x - 4}$

over the interval $-2 \le x \le 3$

SOLUTION:

x	$f(x) = \sqrt{4-x}$
-2	$\sqrt{6}$
0	2
2	$\sqrt{2}$
3.75	0.5
4	0



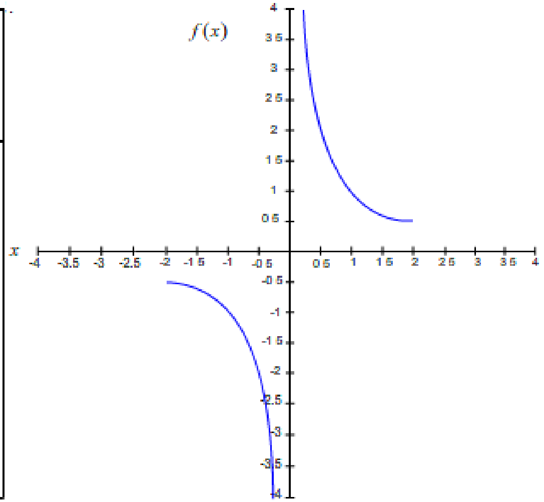
EXAMPLE:

Graph the function $f(x) = \frac{1}{x}$

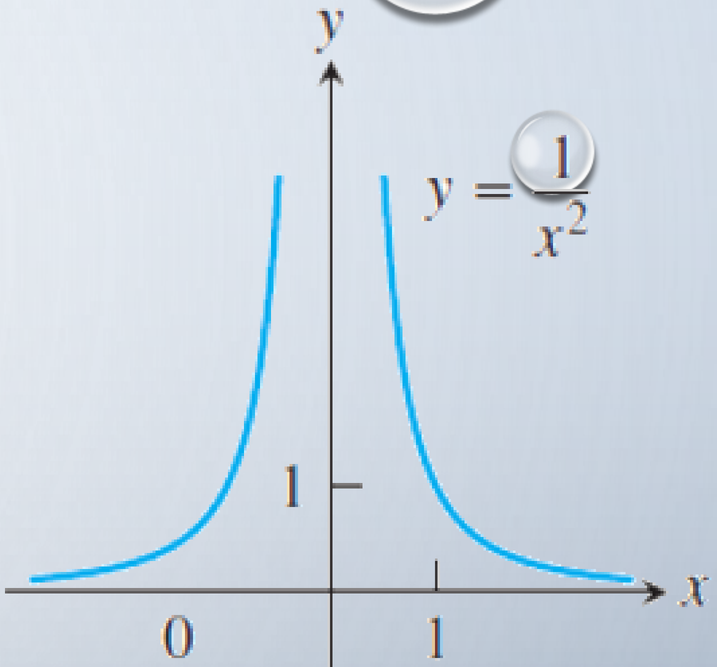
over the interval $-2 \le x \le 2$

SOLUTION:

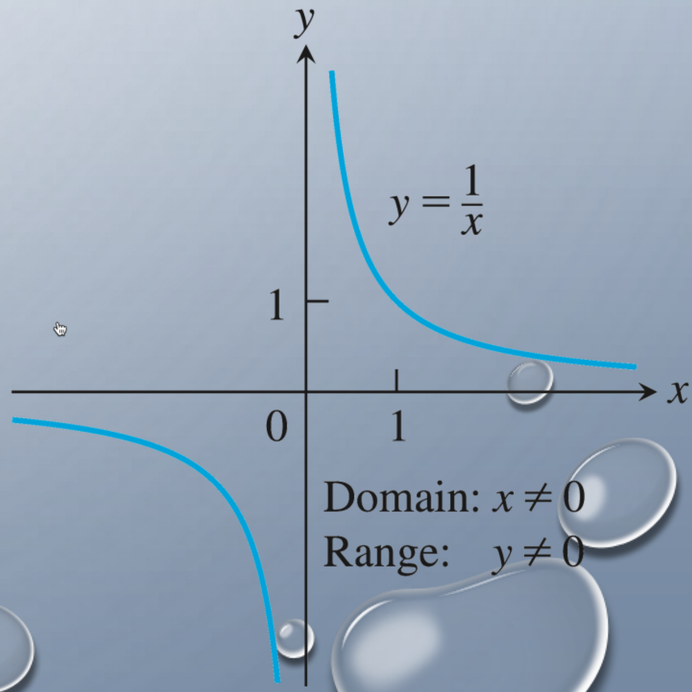
x	$f(x) = \frac{1}{x}$
-2	-0.5
-1	-1
-0.5	-2
-0.25	-4
0.25	4
0.5	2
1	1
2	0.5



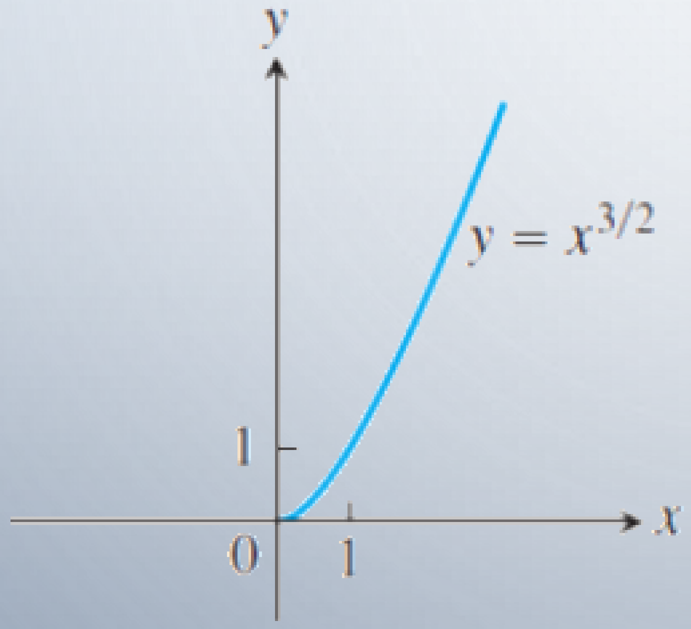
(v) $\mathbf{n = -2, -4, -6, \ldots \Rightarrow \boldsymbol{Domain} = \mathbb{R} - \{0\},\ \boldsymbol{Range} = (0, \infty)}$



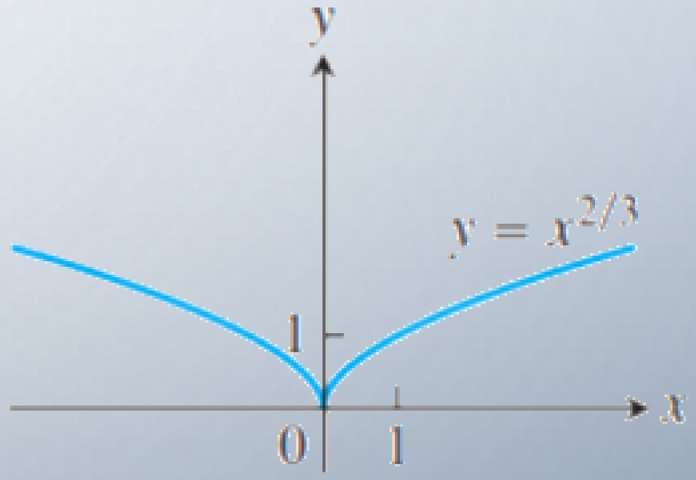
(vi) $\mathbf{n = -1, -3, -5, \ldots \Rightarrow \boldsymbol{Domain} = \mathbb{R} - \{0\},\ \boldsymbol{Range} = \mathbb{R} - \{0\}}$



(vii) $\mathbf{n} = \frac{3}{2}, \frac{2}{3} \ldots \quad \Rightarrow \quad \boldsymbol{Range} = [\mathbf{0}, \infty)$

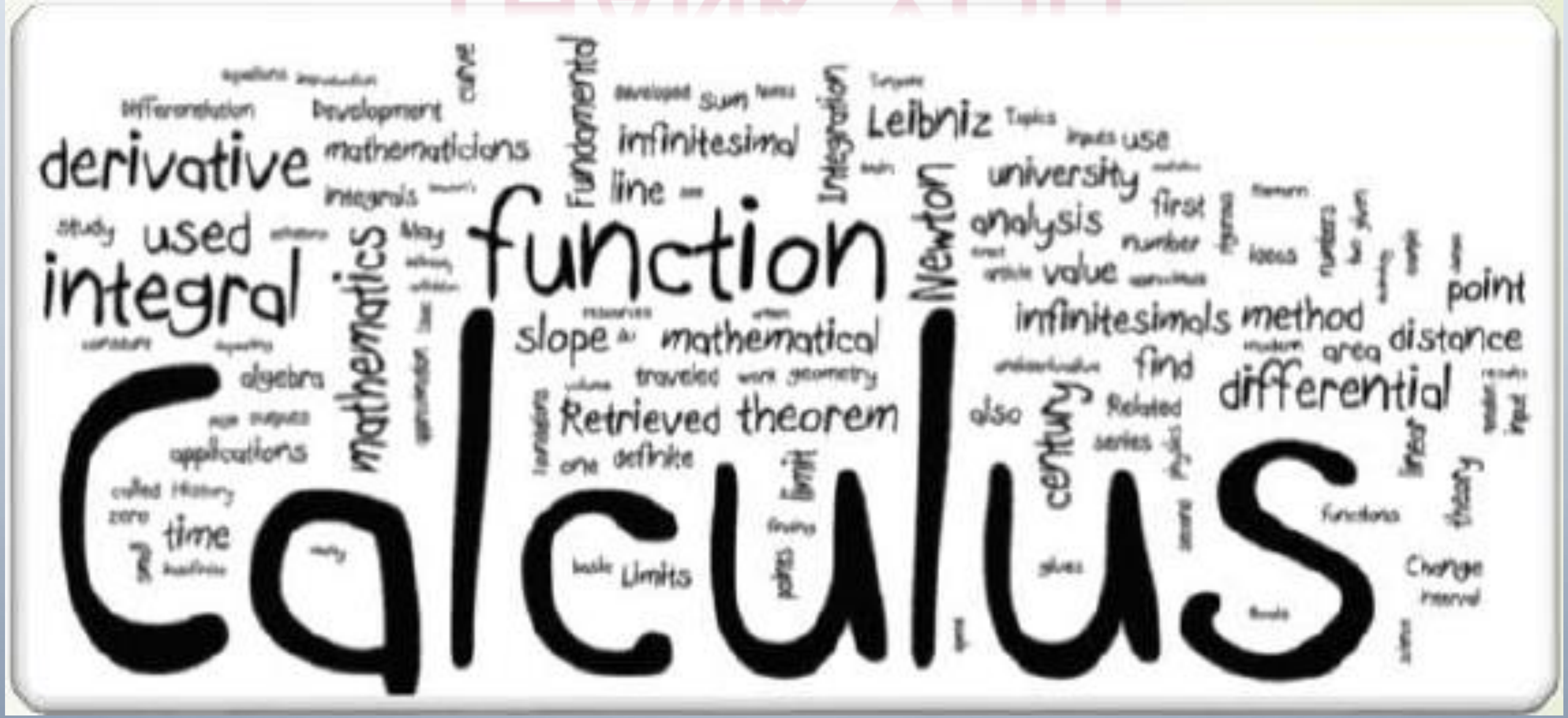


$\boldsymbol{Domain} = [\mathbf{0}, \infty)$



$\boldsymbol{Domain} = \mathbb{R}$

THANK YOU


Calculus
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