

# Question One: Multiple Choice. (10 Marks)

$P$

1. Let  $P$ : This is a great website,  $Q$ : You should not come back here. Then 'This is a great website and you should come back here.' is best represented by?

A.  $\neg P \vee \neg Q$

☒ B.  $P \wedge \neg Q$

C.  $P \vee Q$

D.  $P \wedge Q$

$P$

2.  $P \rightarrow R$  is

A. Tautology

☒ B. Contradiction

C. Contingency

D. none of the above

3.  $A \subseteq B$  if and only if the quantification

☒ A.  $\forall x(x \in A \rightarrow x \in B)$

B.  $\forall x(x \in B \rightarrow x \in A)$

\* 4. Let  $f$  and  $g$  be the functions from the set of integers to the set of integers defined by  $f(x) = 2x - 3$  and  $g(x) = 3x + 2$ . What is the composition of  $f$  and  $g$ ? What is the composition of  $g$  and  $f$ ?

A.  $6x + 7, 6x + 11$

B.  $6x + 7, 3x + 11$

C.  $6x + 11, 6x + 7$  ✗

D. Non of the above ✗

$2x - 3$

$3x + 2$

$6x$

5. let  $p(x)$  " $x+1=3$ " and the truth value is true in domain natural numbers. What is the type of quantifier?

☒ A. Existential ✗

B. Uniqueness

C. Both a and b

D. Universal ✗

6. Find the bitwise "and" 0110 1101, 1001 0011

A. 1000 0010

B. 0001 0000

C. 1000 0000

D. 0000 0001

$$\begin{array}{r} 0110 \ 1101 \\ 1001 \ 0011 \\ \hline 0000 \ 0001 \end{array}$$

And

7. Let  $p$  and  $q$  be the propositions  $p$ : It is below freezing,  $q$ : It is snowing. Then the statement "It is not below freezing and it is not snowing" is equivalent to which propositions

A.  $\neg p \rightarrow \neg q$

☒ B.  $\neg p \wedge \neg q$

C.  $\neg p \wedge q$

D.  $\neg p \vee \neg q$  ✗

8. Let  $P$  statement "Maria takes a English course" and  $q$  Maria will find a new job. Express the statement  $P \rightarrow q$  as English statement.

- ☒ A. If Maria takes an English course, then She will find a new job
- ☐ B. Maria takes an English course and she will find a new job  $\times$
- ☐ C. Maria takes an English course but She will not find a new job  $\times$

9. Let  $P(x)$  be the statement "x spends more than five hours every weekday in class," where the domain for  $x$  consists of all students. Express  $\exists x P(x)$  quantifications in English

- ☒ A. There is a student who spends more than five hours every weekday in class.
- ☐ B. Every student spends more than five hours every weekday in class.  $\times$
- ☐ C. There is a student who does not spend more than five hours every weekday in class.
- ☐ D. No student spends more than five hours every weekday in class.  $\times$

10. Let  $p$ ,  $q$ , and  $r$  be the propositions:  $p$ : You have the flu,  $q$ : You miss the final examination,  $r$ : You pass the course. If you have the flu then you'll not pass the course OR If you miss the final examination then you'll fail the course

- ☒ A.  $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$
- ☐ B.  $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$   $\times$
- ☐ C.  $(p \rightarrow \neg r) \wedge (q \rightarrow \neg r)$   $\times$
- ☐ D.  $(p \rightarrow \neg r) \vee (q \rightarrow r)$

$$(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$$

11. Which of the following propositions is tautology?

- ☐ A.  $(p \vee q) \rightarrow q$
- ☐ B.  $p \vee (q \rightarrow p)$   $\times$
- ☒ C.  $p \vee (p \rightarrow q)$
- ☐ D. Both (b) & (c)

$p$	$\neg p$	$q$	
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	F

12.  $p \vee q$  is logically equivalent to

- ☐ A.  $\neg q \rightarrow \neg p$
- ☐ B.  $q \rightarrow p$
- ☐ C.  $\neg p \rightarrow \neg q$
- ☒ D.  $\neg p \rightarrow q$

T
T
T
F

13. Let  $p$  and  $q$  be the propositions  $p$ : It is below freezing,  $q$ : It is snowing. Then the statement "It is either snowing or below freezing (or both)" is equivalent to which propositions

- ☐ A.  $\neg p \rightarrow \neg q$
- ☐ B.  $\neg p \wedge q$
- ☒ C.  $p \vee q$
- ☐ D.  $p \rightarrow q$

14.  $(p \rightarrow q) \wedge (p \rightarrow r)$  is logically equivalent to

- ☒ A.  $p \rightarrow (q \wedge r)$
- ☐ B.  $p \rightarrow (q \vee r)$
- ☐ C.  $p \wedge (q \vee r)$
- ☐ D.  $p \vee (q \wedge r)$

15. Let's consider a propositional language where  $\bullet p$  means "Paola is happy",  $\bullet q$  means "Paola paints a picture",  $\bullet r$  means "Renzo is happy". Formalize the following sentence: "if Paola is happy and paints a picture then Renzo isn't happy" Then which of these choices express that:

- ☒ A.  $p \wedge q \rightarrow \neg r$
- ☐ B.  $p \wedge \neg q \rightarrow \neg r$
- ☐ C.  $p \vee q \rightarrow \neg r$
- ☐ D.  $\neg p \vee q \rightarrow \neg r$

$$(p \wedge q) \rightarrow \neg r$$

**Question Two: True or False. (10 Marks)**

1.  $\neg(p \rightarrow q)$  and  $p \wedge \neg q$  are logically equivalent  $\top$
2. Let  $Q(x, y)$  denote the statement "X is greater than Y." What are the truth values of  $Q(6, -6)$   $\top$
3.  $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$   $\top$
4. If A and B are sets with  $A \subseteq B$ , then  $A \cup B = A$   $\text{f}$
5. The conditional statement  $p \rightarrow (p \vee q)$  is a tautology  $\top$
6.  $\neg \exists x Q(x) \equiv \forall x Q(x)$   $\text{f}$
7.  $\neg \forall x P(x) \equiv \exists x P(x)$   $\text{f}$
8. The Quantifier  $\forall x P(x)$  is false, when there is an x for which  $P(x)$  is false.  $\top$
9.  $\neg \forall x P(x) \equiv \exists x \neg P(x)$   $\top$
10. The conditional statement  $(p \wedge q) \rightarrow p$  is a tautology  $(\top)$
11. Determine whether each of these conditional statements is true or false If  $3+2=5$ , then  $2+6=4$   $\text{f}$
12. If A and B are sets with  $A \subseteq B$ , then  $A \cup B = B$   $\top$
13. Let  $Q(x, y)$  denote the statement "x is greater than y." What are the truth values of  $Q(3, 1)$   $\top$
14. The conditional statement  $\neg p \rightarrow (p \rightarrow q)$  is a tautology  $(\text{f})$
15.  $\neg \exists x Q(x) \equiv \forall x \neg Q(x)$   $\top$

Good Luck and Best Wishes

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