* Predicate :- (فرض)

$\boxed{X}$ $\boxed{\text{is grater than } 3}$

$X \rightarrow$ subject

is grater than 3 $\rightarrow$ Predicate

$\rightarrow P(X)$

EX(1): Let $P(X)$ denote that statement " $X>3$ " what are the truth value of $P(4)$ and $P(2)$?

ans.

$\because X=4 \quad \therefore 4>3 \quad \therefore P(4)=T$

$\because X=2 \quad \therefore 2 \not> 3 \quad \therefore P(2)=F$

EX(2): Let $Q(x,y)$ denote that statement " $X=y+3$ ". what are the truth values of the propositions $Q(1,2)$ and $Q(3,0)$?

Ans.

* $Q(1,2) \qquad x=1 \;,\; y=2$

$\because x=y+3 \Rightarrow 1 \neq 2+3$

$\therefore Q(1,2)=F$

* $Q(3,0) \qquad x=3 \;,\; y=0$

$\because x=y+3 \Rightarrow 3=0+3$

$3=3$

$\therefore Q(3,0)=T$

...

* Quantifiers :- "تعبير الكمية"

Quantifier

- Universal: $\forall x\, P(x)$
- Existential: $\exists x\, P(x)$
- Uniqueness: $\exists !\, x\, P(x)$, $\exists_1\, x\, P(x)$

لازم كل قيم x تحقق الشرط T

لكل قيم من x

*** Universal :-**

Every

• " $P(x)$ for all values of x in the doma

على الأقل يوجد قيمة x تحقق شرط T

*** Existential :-**

• " There exists an element x in the domain such that $P(x)$ ".

at least one / there is at least one.

*** Uniqueness :-** قيمة معينة

• " There exists a unique x such that $P(x)$ is true ".

*** Example (1):**

Express the statement :-

" Every student in this class has studied Calculus ".

Ans.

$P(x)$: " x has studied Calculus ".

$S(x)$: " x in this class ".

$\therefore$ The statement : $\forall x\, (S(x) \rightarrow P(x))$.

(2):

Let $P(x)$ be the statement " $x+1 > x$ " what is the truth value of the quantification $\forall x\, P(x)$, where the domain consists of all real numbers?

Ans.

• As $P(x)$ is true for all real numbers.

$\therefore \forall x\, P(x)$ is true.

~*~

Let $x = 3$

$\because x+1 > x$

$\therefore 3+1 > 3$

$4 > 3$.

$\therefore \forall x\, P(x)$ is true.

###

* EX(3):

Let $Q(x)$ be the statement " $x < 2$ " what is the truth value of the quantification $\forall x\, Q(x)$, where the domain consists of all real numbers?

Ans.

$\because Q(x)$ is not true for all/every real number x.

for example $x=3$, $Q(3)$ false

$\therefore \forall x\, Q(x)$ is false.

لو فيه واحدة غلط تكون false

* EX(4):

Let $P(x)$ denote the statement " $x > 3$ " what is the truth value of the quantification $\exists x\, P(x)$, where the domain consists of all real numbers?

Ans.

$\because P(x)$ is ~~[illegible]~~ sometimes true.

For example: $x = 4$, $\therefore P(4)$ true

$\therefore \exists x\, P(x)$ is true.

###

* EX(5): what is the truth value of $\exists x\, P(x)$, where $P(x)$ is the statement " $x^2 > 10$ " and the universe of discourse consists of the positive integers not exceeding 4?

إعداد صحيحة
موجبة لا تزيد عن 4

Ans.

$\because$ domain is $\{1, 2, 3, 4\}$

$\because P(x)$ is sometimes true.

For example $x = 4$, then $P(4)$ is true as " $4^2 > 10$ "

$\therefore \exists x\, P(x)$ is true.

$\forall x\, P(x)$ False

...slate into English :- هام

* Negating Quantified Expressions :-

Ex(1):
$P(x)$ is the statement " x has taken a course in Calculus " and the domain consists of the students in your class.

* $\forall x\, P(x)$:

" Every student in your class has taken a course in Calculus ".

➔ The negation of this statement is :-

$$\neg \forall x\, P(x) \equiv \exists x\, \neg P(x)$$

" There is at least one student in your class who has not taken a course in calculus ".

* $\exists x\, P(x)$:

" At least one student in your class has taken a course in calculus "

➔ The negation of this statement is :-

$$\neg \exists x\, P(x) \equiv \forall x\, \neg P(x) .$$

" Every student in this class has not taken Calculus ".

A(2), Q(3)   Q(4)

Q(5)

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* Proofs :- "البراهين"

- Proofs techniques
  - Direct proof
  - Indirect proof
    - contraposition
    - contradiction.

" معلومات أساسية لازم تكون حافظها "

* Even Integer : عدد زوجي

$\boxed{a = 2n \text{ , where } n \text{ is integer.}}$

* odd Integer : عدد فردي

$\boxed{a = 2n + 1 \text{ , where } n \text{ is Integer}}$

*

Even + 1 = odd	$\neg$ Even = odd
odd + 1 = Even	$\neg$ odd = Even

* Prefect square :-

$\boxed{a = (n)^2}$

* Rational number :- الأعداد الكسرية

$a = \frac{n}{m}$, where n, m are integers with no common factor and $m \neq 0$

*

$\neg$ Rational = Irrational

$\neg$ Irrational = Rational

...t Proof:

$p \to q$

خطوات الحل

1. we assume that p is true.
2. we try to prove that q is also true.
3. Then $p \to q$ is true.

EX(1): Give a direct proof of the theorem " If n is an odd integer, then n^2 is odd "

Ans:

p: " n is an odd integer "

q: " n^2 is odd "

1. we assume that p is true:

   $n = 2m+1$, m is integer.

2. we try to prove that q is also true:

$$n^2 = (2m+1)^2$$
$$= 4m^2 + 4m + 1$$
$$= \boxed{2(2m^2+2m)} + 1$$
$$= \text{even} \quad + 1$$
$$= \text{odd}.$$

3. $\therefore$ $p \to q$ is true.

###

$A(2)$, $\Phi(6)$

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* Indirect Proof (Contra-Position) :-

$p \to q$

$\neg q \to \neg p$

خطوات الحل

1. we assume that $\neg q$ is true.
2. we try to prove that $\neg p$ is also true.
3. Then $\neg q \to \neg p$ is true.
4. Then $p \to q$ is also true.

***EX(1):** Prove by contraposition that if n is an integer and $3n+2$ is odd, then n is odd.

ans.

p: " $3n+2$ is odd "

q: " n is odd "

1. we assume that $\neg q$ is true (n is even).

   $n = 2m$

2. we try to prove that $\neg p$ is also true.

$$3n+2 = 3(2m)+2$$
$$= 6m+2$$
$$= 2(3m+1)$$
$$= \text{even}.$$

$\therefore$ $3n+2$ is even.

3. $\therefore$ $\neg q \to \neg p$ is true.
4. $\therefore$ $p \to q$ is also true.

###

..rect Proof (Contradiction):-

(A) - we want to prove p.

1. $\neg p \rightarrow F$

2. $\therefore$ p is true.

(B) - we want to show $p \rightarrow q$

1. $\neg q$

2. Show that $(p \wedge \neg q) \rightarrow F$

3. Then $p \rightarrow q$ is true.

Ex: Given a proof by contradiction of theorem " If $3n+2$ is odd, then n is odd " .

Ans.

1. we assume that (n is even).

$n = 2m$

2. $3n+2 = 3(2m)+2$

$= 6m+2$

$= 2(3m+1)$

$= 2 * (\text{any integer})$

$=$ even.

$\therefore$ $(3n+2)$ is even , then p is false.

3. $\therefore$ $p \wedge \neg q$ is false.

$\therefore$ $p \rightarrow q$ is true. #

A(2), Φ(7)