

# Superposition

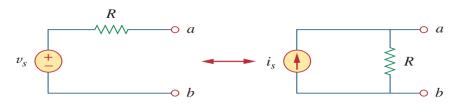
The **superposition** principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

#### **Steps to Apply Superposition Principle:**

- Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using the techniques covered in previous lectures.
- 2. Repeat step 1 for each of the other independent sources.
- 3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

# Source Transformation

A **source transformation** is the process of replacing a voltage source  $v_s$  in series with a resistor R by a current source  $i_s$  in parallel with a resistor R, or vice versa.





$$v_s = i_s R$$

or

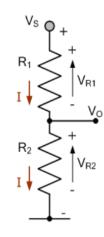
$$i_s = \frac{v_s}{R}$$

## Voltage Divider Rule

Voltage Divider circuits are used to produce different voltage levels from a common voltage source but the current is the same for all components in a series cicruit

$$\therefore V_{R2} = V_{S} \left( \frac{R_{2}}{R_{1} + R_{2}} \right)$$

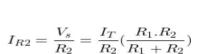
$$\therefore V_{R1} = V_{S} \left( \frac{R_{1}}{R_{1} + R_{2}} \right)$$



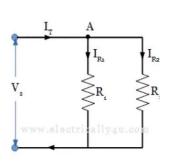
### Current Divider prove

$$I_{R1} = \frac{V_s}{R_1} = \frac{I_T}{R_1} (\frac{R_1.R_2}{R_1 + R_2})$$

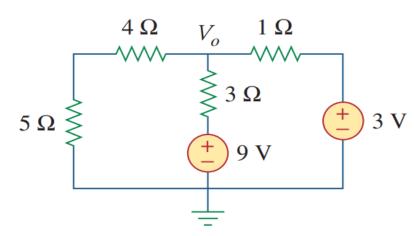
$$I_{R1} = I_T(\frac{R_2}{R_1 + R_2})$$



$$I_{R2} = I_T(\frac{R_1}{R_1 + R_2})$$



**Q1.** Using superposition, find  $V_0$  in the given circuit.



### **Solution**

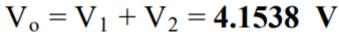
Let  $V_0 = V_1 + V_2$ , where  $V_1$  and  $V_2$  are due to 9-V and 3-V sources respectively. To find  $V_1$ , consider the circuit below.

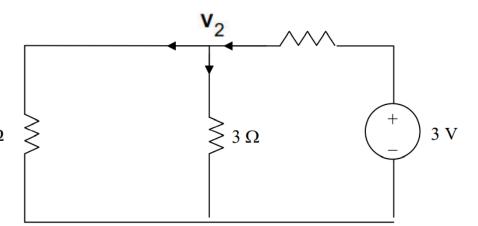
$$\frac{9 - V_1}{3} = \frac{V_1}{9} + \frac{V_1}{1} \longrightarrow V_1 = 27/13 = 2.0769 \ V^{9\Omega} \ge$$

To find  $V_2$ , consider the circuit below.

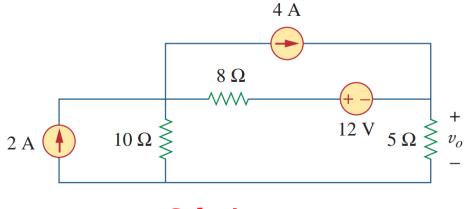
$$\frac{V_2}{9} + \frac{V_2}{3} = \frac{3 - V_2}{1} \longrightarrow V_2 = 27/13 = 2.0769 V$$
9 \( \Omega \text{9} \)







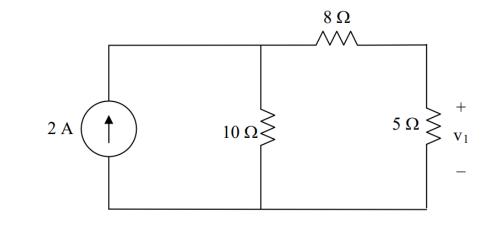
**Q2.** Use superposition to find  $V_0$  in the given circuit.



#### **Solution**

Let  $V_0 = V_1 + V_2 + V_3$ , where  $V_1$ ,  $V_2$ , and  $V_3$  are due to the independent sources. To find  $V_1$ , consider the circuit below.

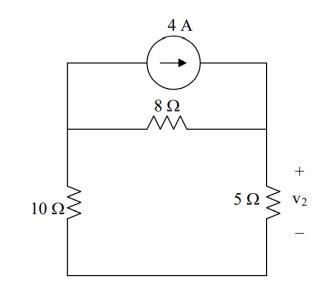
$$V_1 = I_{5\Omega}R = 5I_{5\Omega} = 5x \frac{10}{10 + 8 + 5}x2 = 4.3478 V$$





To find  $v_2$ , consider the circuit below.

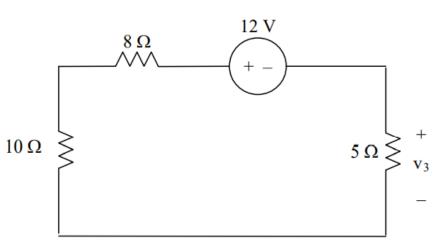
$$V_2 = I_{5\Omega}R = 5I_{5\Omega} = 5x \frac{8}{8+10+5} \times 4 = 6.9565 V$$



To find  $v_3$ , consider the circuit below.

$$v_3 = -12\left(\frac{5}{5+10+8}\right) = -2.6087 V$$

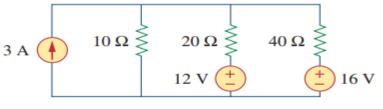
(The –Ve sign here indicates that the current in the resistance due to this source, is opposite to the current due to the other sources)



$$V_0 = V_1 + V_2 + V_3 = 8.6956 \text{ V}$$

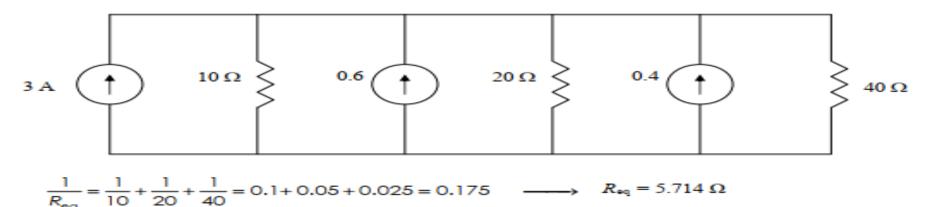


Q3. Use source transformation to reduce the circuit in Figure to a single voltage source in series with a single resistor.



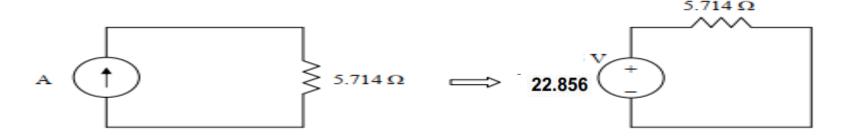
#### **Solution**

Convert the voltage sources to current sources and obtain the circuit shown below.



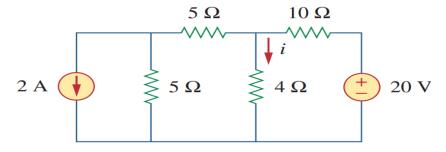
$$I_{eq} = 3 + 0.6 + 0.4 = 4$$

Thus, the circuit is reduced as shown below. Please note, we that this is merely an exercise in combining sources and resistors. The circuit we have is an equivalent circuit which has no real purpose other than to demonstrate source transformation. In a practical situation, this would need some kind of reference and a use to an external circuit to be of real value.



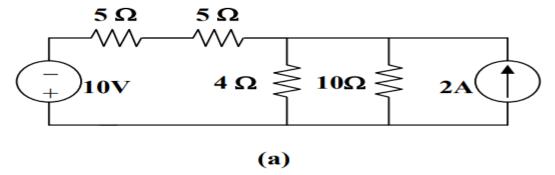


**Q4.** For the circuit in the given figure, use source transformation to find i.

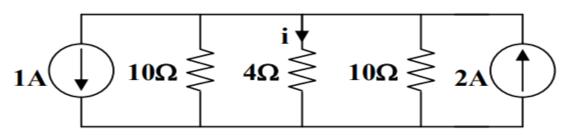


### **Solution**

We transform the two sources to get the circuit shown in Fig. (a).



We now transform only the voltage source to obtain the circuit in Fig. (b).

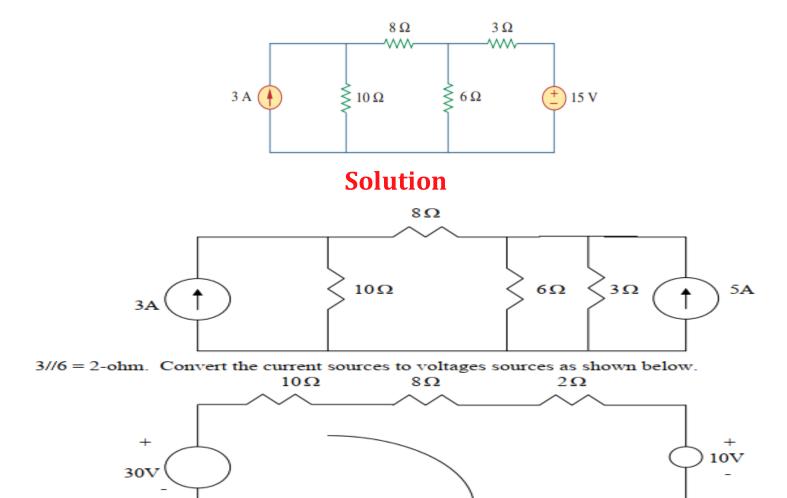


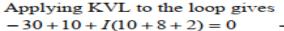
**(b)** 

$$10||10 = 5 \text{ ohms}, i = [5/(5+4)](2-1) = 5/9 = 555.5 \text{ mA}$$



### **Q5.** Referring to Figure, use source transformation to determine the current and power absorbed by the $8-\Omega$ resistor.



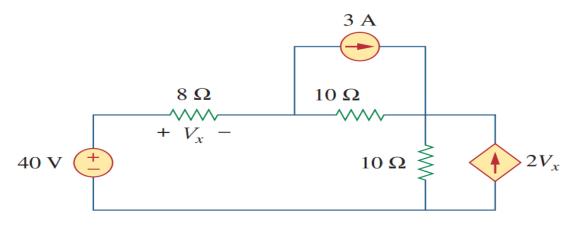


$$\longrightarrow$$
 I = 1 A

$$p = VI = I^2 R = 8 W$$



**Q6.** Use source transformation to determine  $V_x$ .



#### **Solution**

Transform the two current sources in parallel with the resistors into their voltage source equivalents yield,

a 30-V source in series with a 10- $\Omega$  resistor and a 20V<sub>x</sub>-V sources in series with a 10- $\Omega$  resistor.

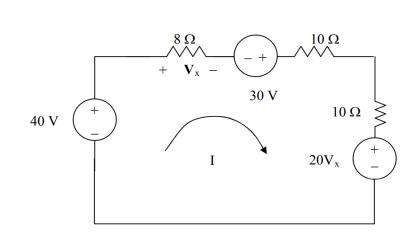
### We now have the following circuit,

We now write the following mesh equation and constraint equation which will lead to a solution for  $V_x$ ,

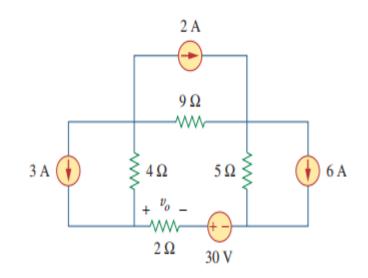
$$28I - 70 + 20V_x = 0$$
 or  $28I + 20V_x = 70$ , but  $V_x = 8I$  which leads to

$$28I + 160I = 70 \text{ or } I = 0.3723 \text{ A or } V_x = 2.978 \text{ V}.$$



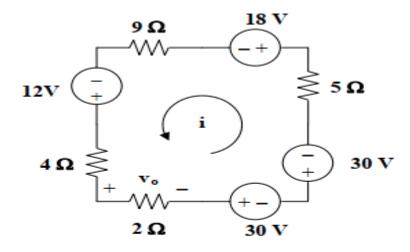


**Q7.** Obtain  $V_0$  in the circuit of Figure using source transformation.



### **Solution**

Transforming only the current source gives the circuit below.



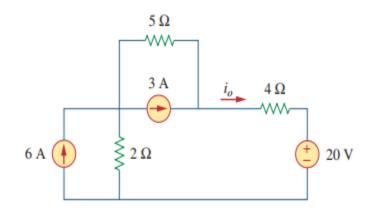
Applying KVL to the loop gives,

$$-(4+9+5+2)i + 12 - 18 - 30 - 30 = 0$$
  
 $20i = -66$  which leads to  $i = -3.3$ 



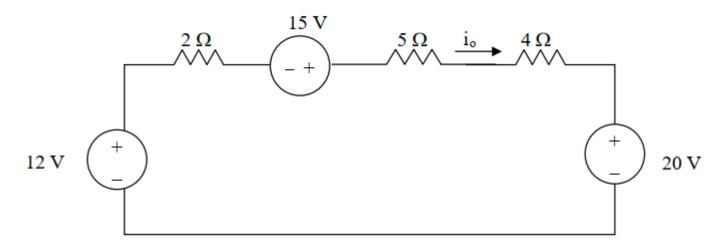
$$v_0 = 2i = -6.6 V$$

**Q8.** Use source transformation to find  $i_0$  in the circuit of the Figure.

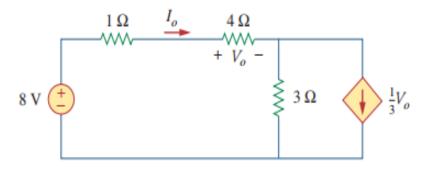


### **Solution**

Transforming the current sources gives the circuit below.

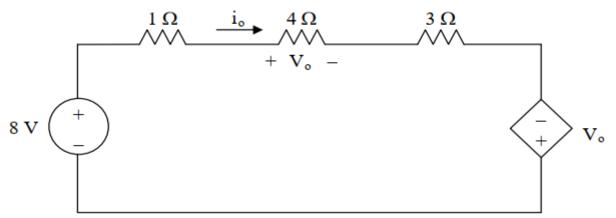


### **Q9.** Use source transformation to find $i_0$ in Figure.



### **Solution**

Convert the dependent current source to a dependent voltage source as shown below.



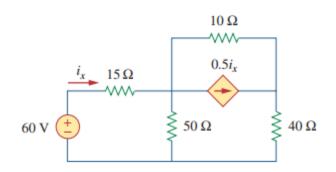
$$-8 + i_{\circ}(1 + 4 + 3) - V_{\circ} = 0$$

But 
$$V_{\circ} = 4i_{\circ}$$

$$-8 + 8i_{\circ} - 4i_{\circ} = 0$$
  $\longrightarrow$   $i_{\circ} = 2 \text{ A}$ 

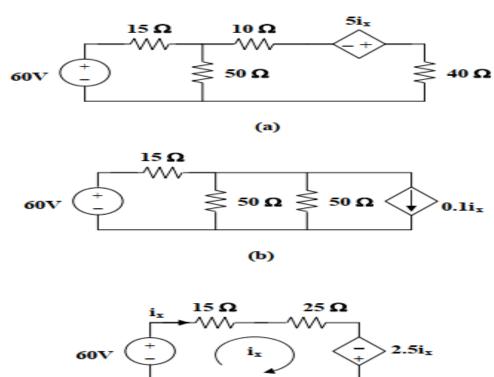


### **Q10.** Use source transformation to find in the circuit of Figure.



### **Solution**

As shown in Fig. (a), we transform the dependent current source to a voltage source,



(c)



In Fig. (b), 50||50 = 25 ohms. Applying KVL in Fig. (c),  $-60 + 40i_x - 2.5i_x = 0$ , or  $i_x = 1.6$  A