

Answer the following questions:

Question 1: Choose the correct answer:

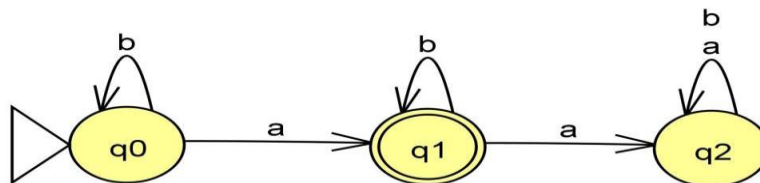
(10 Marks)

1. If $A = \{1, 2, 3, 4, 5\}$ and $B = \{5, 6, 7\}$, then $|A - B|$ equals
 - a. 4
 - b. 5
 - c. 7
 - d. 3
2. Select a bijective function:
 - a. $f: \mathbb{Z} \rightarrow \mathbb{Z}: f(x) = 2x$
 - b. $g: \mathbb{R} \rightarrow \mathbb{R}: g(x) = x^2$
 - c. $I: \mathbb{R} \rightarrow \mathbb{R}: I(x) = x$
 - d. $h: \mathbb{Z} \rightarrow \{0, 1\}: h(x) = 0$ if x is even and $h(x) = 1$ otherwise
3. If the function f is bijective, then f is invertible.
 - a. True
 - b. False
4. If $|A| = 3$ and $|B| = 2$, then $|A \times B|$ equals
 - a. 6
 - b. 9
 - c. 4
 - d. 8
5. If $a = c$ and $b = d$ then $(a, b) = (c, d)$
 - a. True
 - b. False
6. Let $A = \{a, b, c\}$, if w is a string over A and $|www| = 36$, what is the length of w ?
 - a. 6
 - b. 12
 - c. 18
 - d. 36
7. If $L = \{aba, ba\}$, which of the following strings is NOT in L^* :
 - a. aa
 - b. bb
 - c. ab
 - d. All of the previous
8. Find L in $L \cdot \{\lambda, a\} = \{\lambda, a, b, ab, ba, aba\}$
 - a. $L = \{a, b\}$
 - b. $L = \{\lambda, b, ab\}$
 - c. $L = \{\lambda, a, b\}$
 - d. $L = \{b, ab\}$
9. Let $G = \langle \{D, S\}, \{0, 1, 2, \dots, 9\}, P, S \rangle$, where P is:

$$S \rightarrow D0 \mid D2 \mid D4 \mid D6 \mid D8$$

$$D \rightarrow \lambda \mid D0 \mid D1 \mid D2 \mid D3 \mid D4 \mid D5 \mid D7 \mid D8 \mid D9$$
 Which of the following strings is NOT in $L(G)$:
 - a. 13
 - b. 22
 - c. 24
 - d. 28
10. Choose a *correct* grammar generating the language $\{a, aaa, aaaaa, \dots, a^{2n+1}, \dots\}$
 - a. $S \rightarrow a \mid aaS$
 - b. $S \rightarrow \lambda \mid a \mid aaS$
 - c. $S \rightarrow a \mid aS$
 - d. $S \rightarrow \lambda \mid aS$
11. The language $L = \{a^{n^2} : 1 \leq n \leq 4\}$ is regular.
 - a. True
 - b. False
12. Select a regular language:
 - a. $\{1^n : n \geq 0, n \neq 1\}$
 - b. $\{a^n b^p : n \neq p\}$
 - c. $\{a^n b^m : n \geq m\}$
 - d. $\{a^n b^n : n \geq 0\}$

13. Regular expressions are algebraic notations used to describe regular languages.
- True
 - False
14. Choose a regular expression to describe the language $\{\lambda, a, b, ca, bc, cca, bcc, \dots, c^n a, bc^n, \dots\}$
- $\lambda + bc^*a$
 - $\lambda + c^*a + bc^*$
 - $\lambda + ac^* + bc^*$
 - $\lambda + ac^* + cb^*$
15. Choose the correct language described by the regular expression $a^*(a + b)$
- $\{\lambda, a, b, aa, ab, aaa, aab, \dots\}$
 - $\{a, b, aa, ab, aaa, aab, \dots\}$
 - $\{a, b, aa, ba, bb, ab, aaa, baa, \dots\}$
 - $\{\lambda, a, b, aa, ba, aaa, baa, \dots\}$
16. Let $G = \langle \{S\}, \{a\}, P, S \rangle$, where P is: $S \rightarrow \lambda \mid aS$, then G is a regular grammar.
- True
 - False
17. The language $L = \{a^n b^n c^n : n \geq 0\}$ is context-free.
- True
 - False
18. The concatenation of two context-free languages is context-free.
- True
 - False
19. Two finite machines M_1 and M_2 are said to be **equivalent** if $L(M_1) = L(M_2)$.
- True
 - False
20. The language accepted by the following DFA is:
- $\{a\}$
 - $\{b^n ab^n : n \geq 0\}$
 - $\{b^n ab^m : m, n \geq 0\}$
 - $\{b^n ab^m a : m, n \geq 0\}$



Question 2:

(10 Marks)

- Construct the following machines over the alphabet $\{0, 1\}$:
 - DFA that accepts the language $\{00, 01\}$.
 - DFA that accepts the language $L = \{01w : w \in \{0, 1\}^*\}$
 - NFA that accepts the language $L = \{u00v : u, v \in \{0, 1\}^*\}$
 - NFA without λ -transition and with a single final state that accepts the language: $\{0\} \cup \{1^n : n \geq 1\}$
 - A Mealy machine that produces an output 1 for any input.

Question 3:

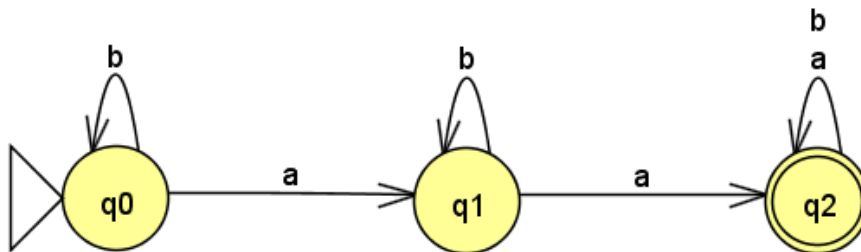
- Why the NFA is less efficient in recognizing strings than DFA? **(2 Marks)**
- Consider the NFA given by the following table: **(4 Marks)**

| | | λ | a | b |
|-------|---|-----------|--------|--------|
| Start | 0 | {1, 2} | {1} | ϕ |
| | 1 | ϕ | ϕ | {2} |
| Final | 2 | ϕ | {2} | ϕ |

- Find the lambda closure for all states.
- Convert the machine into DFA and draw the graph for the resulting machine.
- Construct NFA for the following regular expression using RE to FA algorithm: **(4 Marks)**
 $a^*b c^* + ac$

Question 4:

- Using Pumping lemma for regular languages, show that the language $\{a^n b c^n : n \geq 0\}$ is Not regular. **(3 Marks)**
- Step by step find a regular expression describes the language accepted by the following automaton using FA to RE algorithm: **(4 Marks)**



- Construct grammars for the following languages: **(3 Marks)**
 - $L = \{ b a^n b : n \geq 0 \}$
 - $M = \{ a^n b c^n : n \geq 0 \}$

Question 5:

- Show that the function $f(n) = n + 1$, $n \geq 0$ is *Turing Computable* where the number n is represented in binary form. **(3 Marks)**
- Construct PDA to describe the language $\{a^{n+2} c b^n : n \geq 0\}$ **(4 Marks)**
- Write regular expression to describe each of the following languages: **(3 Marks)**
 - $L = \{a^n b^m, n, m \geq 1\}$
 - $M = \{a^n b^m, n \geq 2, m \geq 3\}$
 - The set of all strings over the alphabet $\{a, b\}$ that begin and end with the same letter.

Good Luck