الجامعة المصرية
للتعلم الإلكتروني الأهلية

EELU
THE EGYPTIAN E-LEARNING UNIVERSITY

GEN206
Discrete Mathematics

Section 2

Faculty of Information Technology
Egyptian E-Learning University

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1. Use truth tables to verify these equivalences.

a) $p \wedge \mathbf{T} \equiv p$
b) $p \vee \mathbf{F} \equiv p$
c) $p \wedge \mathbf{F} \equiv \mathbf{F}$
d) $p \vee \mathbf{T} \equiv \mathbf{T}$
e) $p \vee p \equiv p$
f) $p \wedge p \equiv p$

p	p ∧ T
1	1
0	0

p	p ∨ F
1	1
0	0

p	p ∧ F
1	0
0	0

p	p ∨ T
1	1
0	1

p	p ∨ p
1	1
0	0

p	p∧ p
1	1
0	0

17. Use truth tables to verify the absorption laws.

a) $p \vee (p \wedge q) \equiv p$ **b)** $p \wedge (p \vee q) \equiv p$

Solution

a) $p \vee (p \wedge q) \equiv p$

p	q	p ^q	pv(p^q)
1	1	1	1
1	0	0	1
0	1	0	0
0	0	0	0

Solution

b) $p \wedge (p \vee q) \equiv p$

p	q	p∨q	p∧(p∨q)
1	1	1	1
1	0	1	1
0	1	1	0
0	0	0	0

11. Show that each of these conditional statements is a tautology by using truth tables. And without truth tables

a) $(p \wedge q) \rightarrow p$

e) $\neg(p \rightarrow q) \rightarrow p$

Solution

a) $(p \wedge q) \rightarrow p$

p	q	p ^q	(p^q) -> p
1	1	1	1
1	0	0	1
0	1	0	1
0	0	0	1

Without truth table

Let p^q =e

then e ⇒ p ≡ ¬e ∨ p

So ¬(p^q) ∨ p ≡ (¬p∨¬q)∨p ≡ (¬p ∨p)∨¬q ≡ True∨¬q ≡ True

Solution

e) $\neg(p \to q) \to p$

p	q	p→q	¬(p→q)	¬(p→q) →p
1	1	1	0	1
1	0	0	0	1
0	1	1	0	1
0	0	1	0	1

Without truth table

$\neg(p \to q) \to p \equiv \neg(\neg p \vee q) \to p \equiv p \wedge \neg q \to p \equiv \neg(p \wedge \neg q) \vee p$
$\equiv \neg p \vee q \vee p \equiv (\neg p \vee p) \vee q \equiv (\text{True}) \vee q = \text{True}$

Predicates and Quantifiers

Solution

2. Let $P(x)$ be the statement "The word x contains the letter a." What are these truth values?

a) $P(\text{orange})$ T **b)** $P(\text{lemon})$ F
c) $P(\text{true})$ F **d)** $P(\text{false})$ T

5. Let $P(x)$ be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these quantifications in English.

a) $\exists x P(x)$ **b)** $\forall x P(x)$
c) $\exists x \neg P(x)$ **d)** $\forall x \neg P(x)$

Solution

(a) There exists a student that spends more than five hours every weekday in class.
(b) All students spend more than five hours every weekday in class.
(c) There exists a student that does not spend more than five hours every weekday in class.
(d) All students do not spend more than five hours every weekday in class.

Solution

7. Translate these statements into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consists of all people.

a) $\forall x(C(x) \rightarrow F(x))$

b) $\forall x(C(x) \wedge F(x))$

c) $\exists x(C(x) \rightarrow F(x))$

d) $\exists x(C(x) \wedge F(x))$

(a) All comedians are funny.
(b) Every person is a comedian and funny.
(c) There exists a person such that, if the person is a comedian, then the person is funny.
(d) There exists a person that is a comedian and funny.

Solution

11. Let $P(x)$ be the statement "$x = x^2$." If the domain consists of the integers, what are these truth values?

a) $P(0)$ **b)** $P(1)$ **c)** $P(2)$

d) $P(-1)$ **e)** $\exists x P(x)$ **f)** $\forall x P(x)$

(a) True
(b) True
(c) False
(d) False
(e) True
(f) False

Thank You