Lecture 2

Kinematics in 1D Continued

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Course Instructor:

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Lecture 2: Kinematics

"**Kinematics** is a subfield of physics, developed in classical mechanics, that describes the **motion** of points, bodies (objects), and systems of bodies (groups of objects) **without considering the forces** that cause them to move."

Key concepts

- **Displacement, velocity, and acceleration**
- **Equations of motion**

Lecture 2: What to expect from kinematics

What does kinematics tell us?

- Position of an object or a system of objects at any arbitrary time given initial conditions
- Time needed to travel from one point to another
- Velocity needed for finishing a trip within certain time

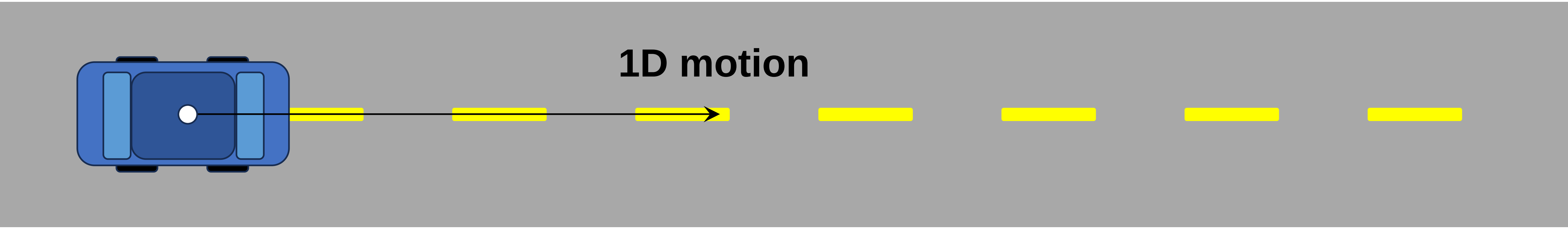
Key concepts

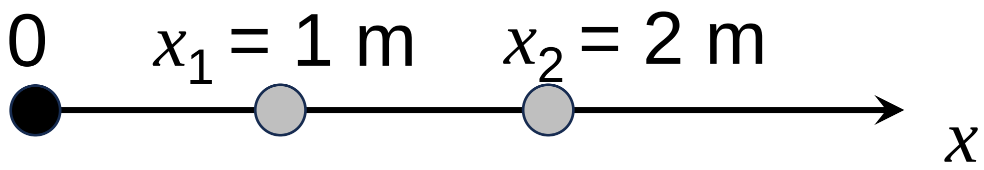
- **Displacement, velocity, and acceleration**
- **Equations of motion**

Displacement

A particle is a point object; has mass but infinitesimal size

- Only cares about **translation motion**
- Ignores **rotational motion**





The object's position is its location with respect to a chosen reference point.

So, what is x?

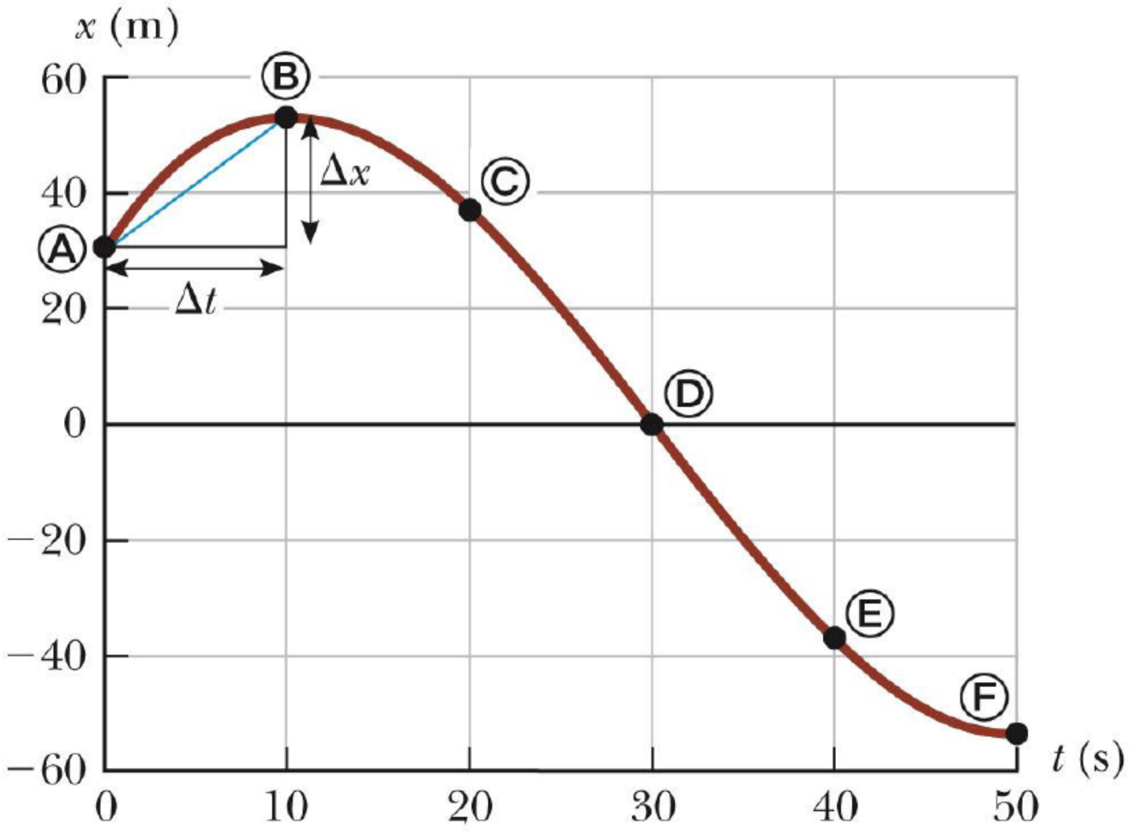
Speed and Velocity

Speed is a scalar, velocity is a vector

- Average speed = distance car driven/time taken
- Average velocity = displacement/time taken

$$v_{\text{av-}x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

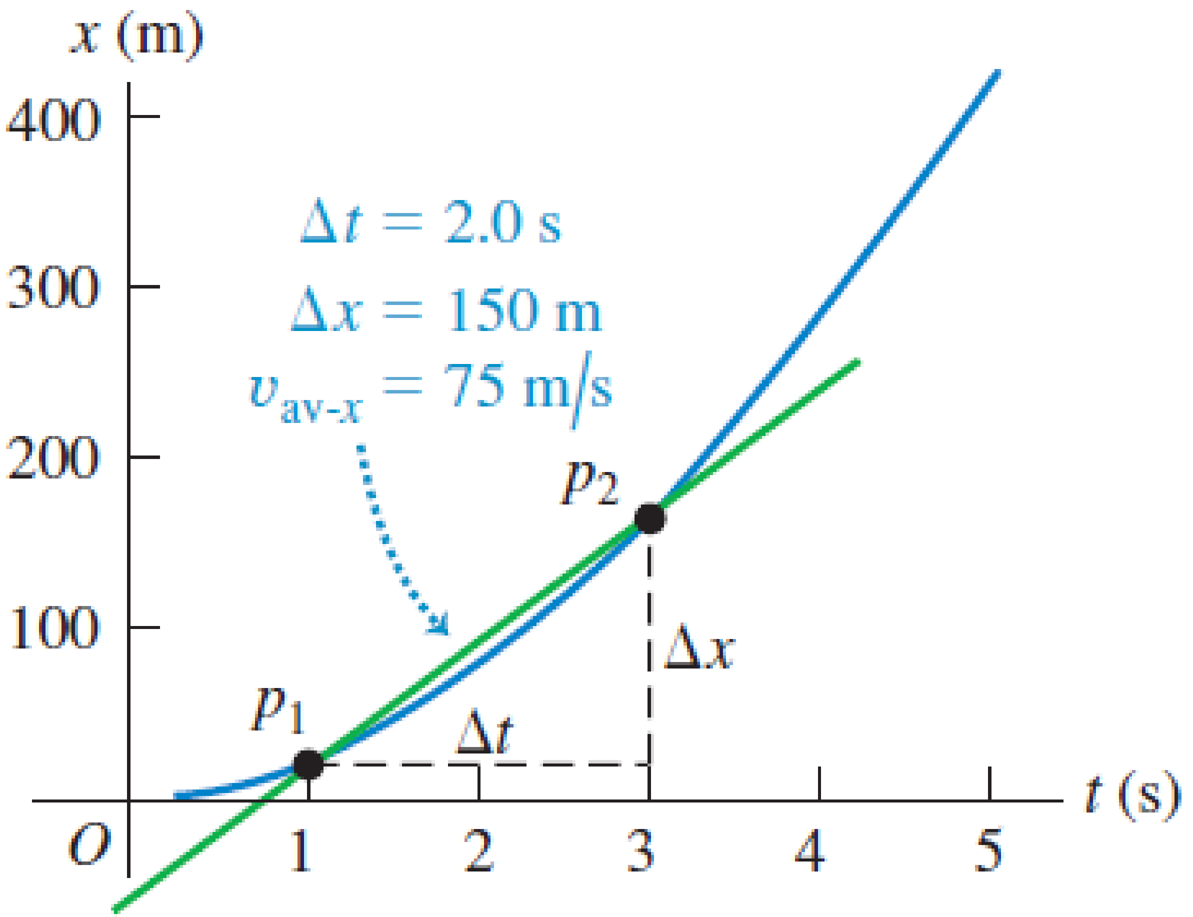


Instantaneous velocity at one moment of time:

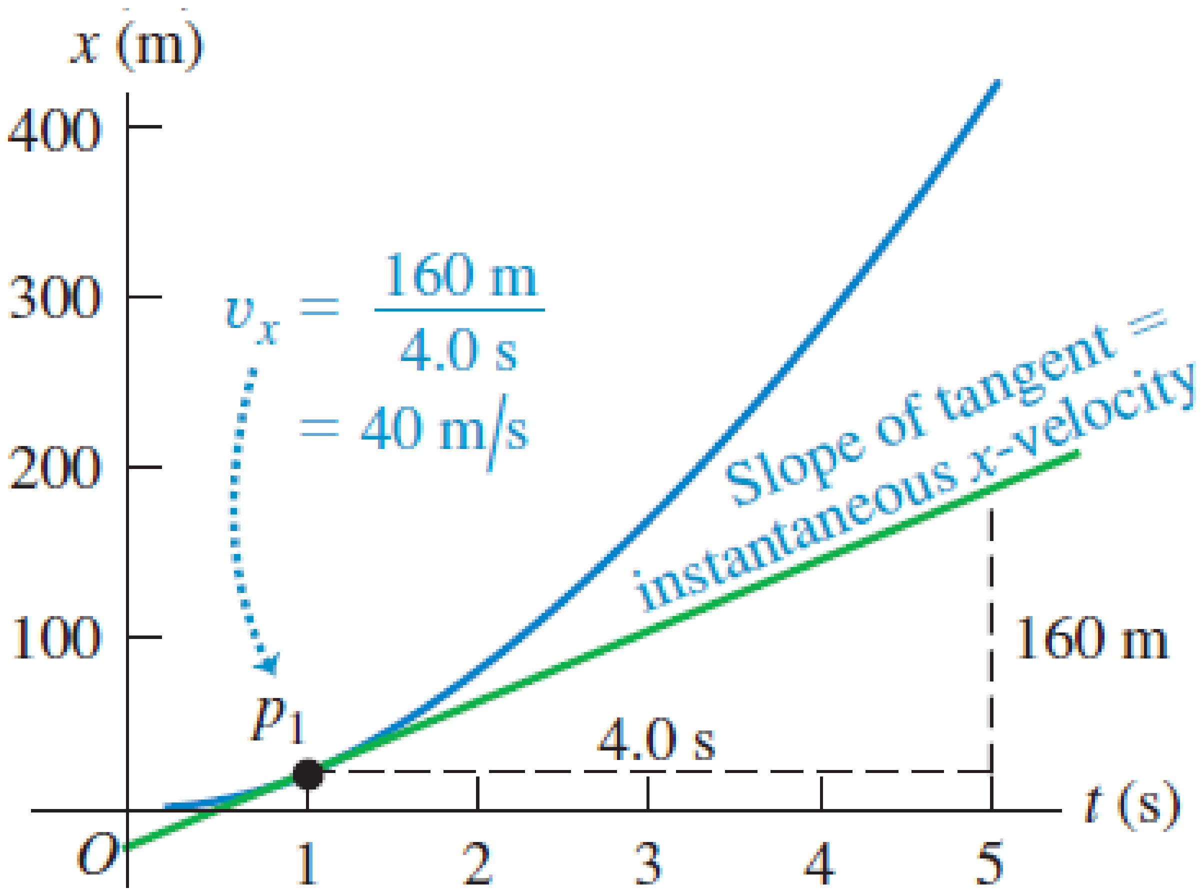
$$v = \underset{\Delta t \to 0}{limit} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Car speedometer shows the Instantaneous speed (magnitude of velocity

Instantaneous Velocity Sketched



As the average x-velocity $v_{av\text{-}x}$ is calculated over shorter and shorter time intervals ...

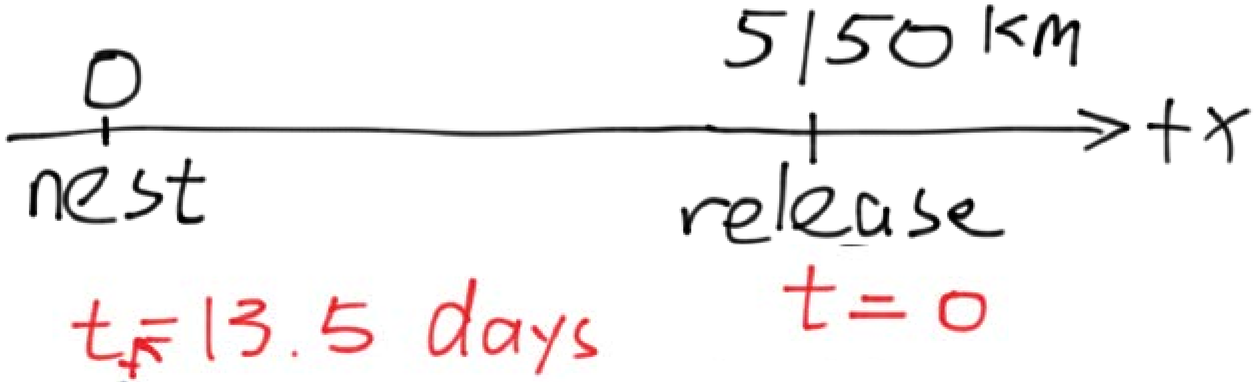


The instantaneous x-velocity v_x at any given point equals the slope of the tangent to the x-t curve at that point.

Exercise

2.2 .. In an experiment, a shearwater (a seabird) was taken from its nest, flown 5150 km away, and released. The bird found its way back to its nest 13.5 days after release. If we place the origin in the nest and extend the to the release point, what was the bird's average velocity in (a) for the return flight, and (b) for the whole episode, from leaving the nest to returning?

(a) If you don't know what to do, try sketching the process!



For the return flight

$$\Delta\vec{x} = \vec{x}(t_f) - \vec{x}(t=0)$$

$$= [0 - 5150\,\text{km}]\,\hat{i}$$

$$= -5150\,\text{km}\,\hat{i}$$

$$\vec{v} = \frac{\Delta\vec{x}}{\Delta t} = \frac{-5150\,\text{km}}{13.5\,\text{d}}\,\hat{i}$$

$$= \frac{-5150\,\text{km}\cdot 1000\,\text{m/km}}{13.5\,\text{d}\cdot(24\times 3600)\,\text{s/d}}\,\hat{i}$$

$$= -4.42\,\text{m/s}\,\hat{i}$$

Exercise

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(b) For the whole trip?

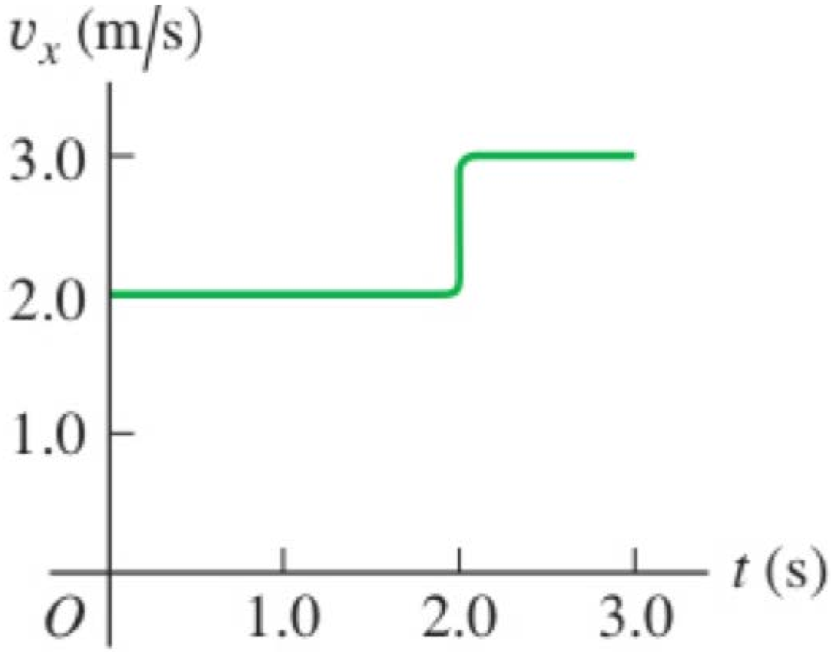
what is Δx?

$$\Delta \vec{x} = (0 - 0)\,\hat{i}$$

$$= 0\,\hat{i} = \vec{0}$$

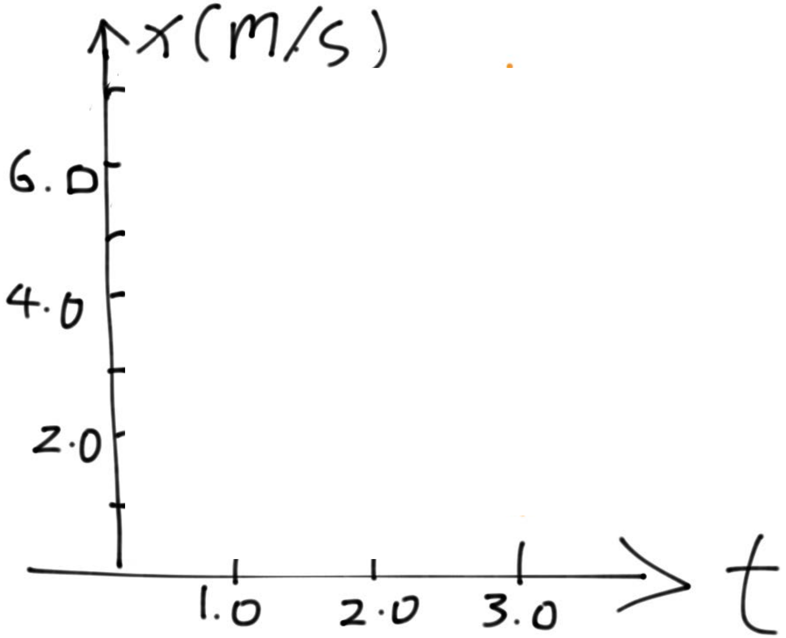
$$\text{so } \vec{v} = \frac{\Delta \vec{x}}{\Delta t} = \vec{0}$$

2.9 •• A ball moves in a straight line (the x-axis). The graph in Fig. E2.9 shows this ball's velocity as a function of time. (a) What are the ball's average speed and average velocity during the first 3.0 s? (b) Suppose that the ball moved in such a way that the graph segment after 2.0 s was -3.0 m/s instead of $+3.0$ m/s. Find the ball's average speed and average velocity in this case.



$$\bar{v} = \frac{\Delta x}{\Delta t}$$

$$\Delta x = \bar{v}\Delta t$$



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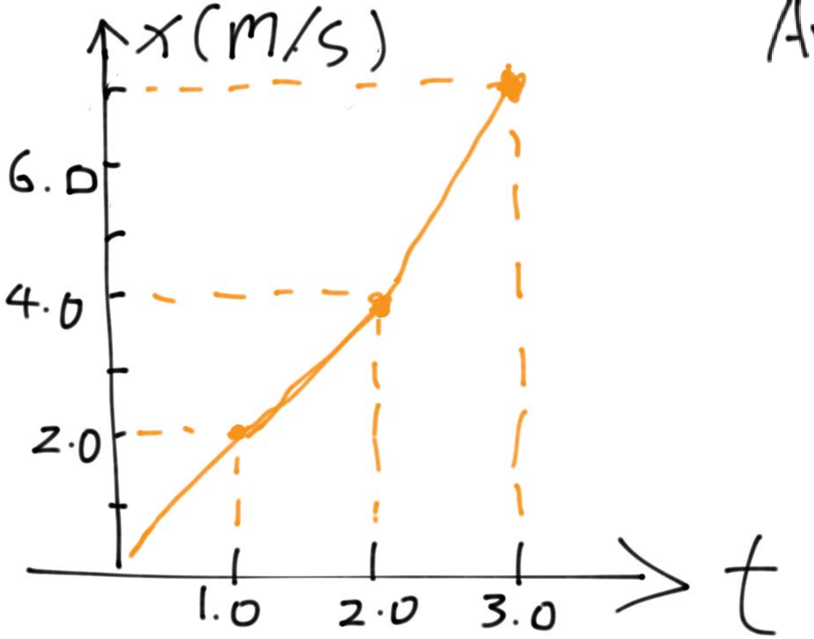


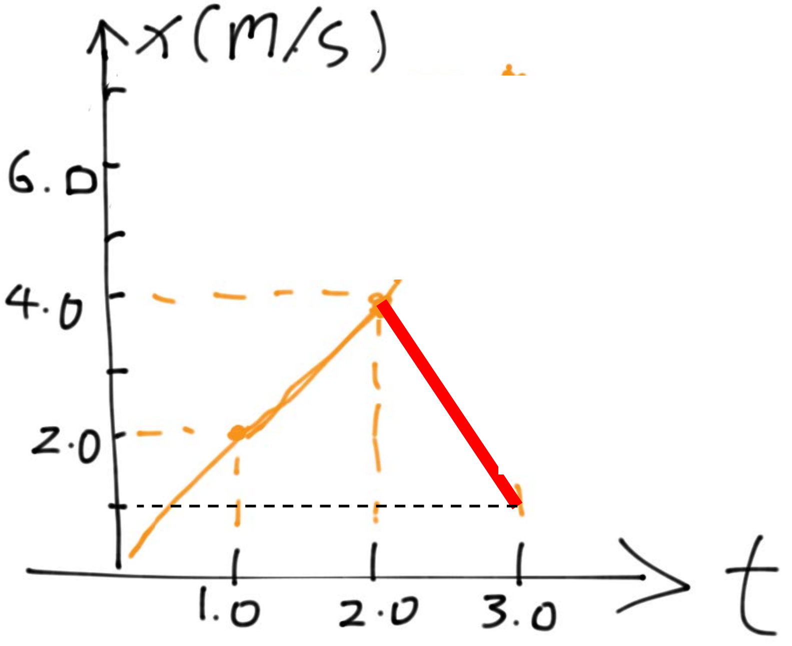
Average velocity

$$\bar{V} = \frac{\Delta x}{\Delta t} = \frac{7-0}{3}\ \text{m/s}\ \hat{i}$$

$$\approx 2.33\ \text{m/s}\ \hat{i}$$

$$\text{Speed} = \frac{7}{3} = 2.33\ \text{m/s}$$

(b) Suppose that the ball moved in such a way that the graph segment after 2.0 s was $-3.0\ \text{m/s}$ instead of $+3.0\ \text{m/s}$. Find the ball's average speed and average velocity in this case.



(b) $\Delta\vec{x} = 2\,\text{m/s}\cdot 2\,\text{s}\ \hat{i} + (-3\,\text{m/s})\cdot 1\,\text{s}\ \hat{i}$

$= 1\,\text{m}\ \hat{i}$

distance $= 2\,\text{m/s}\cdot 2\,\text{s} + 3\,\text{m/s}\cdot 1\,\text{s}$

$= 7\,\text{m}$

$\bar{V} = \dfrac{\Delta\vec{x}}{\Delta t} = \dfrac{1\,\text{m}}{3\,\text{s}}\hat{i} \cong 0.33\,\text{m/s}\ \hat{i}$

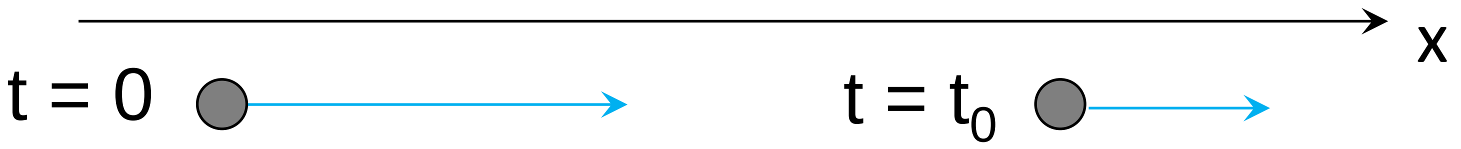
speed $= 2.33\ \text{m/s}$

Acceleration

Average acceleration = velocity change/time taken

$$a_{av\text{-}x} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \frac{\Delta v_x}{\Delta t}$$

Velocity is a vector, so acceleration is also a vector.



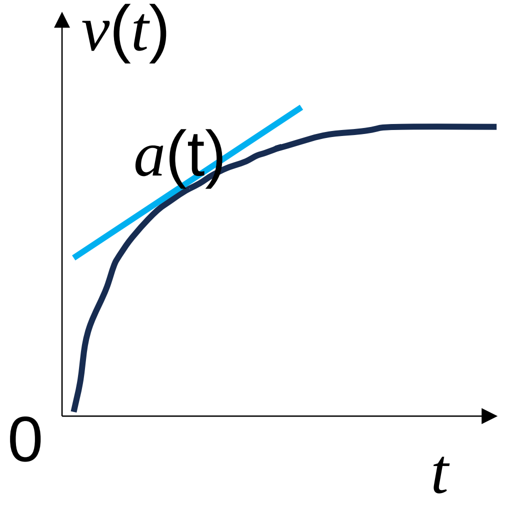
a < 0 if the particle is moving in the +x and decelerating

a < 0 also if the particle is moving in the -x and accelerating

Instantaneous Acceleration

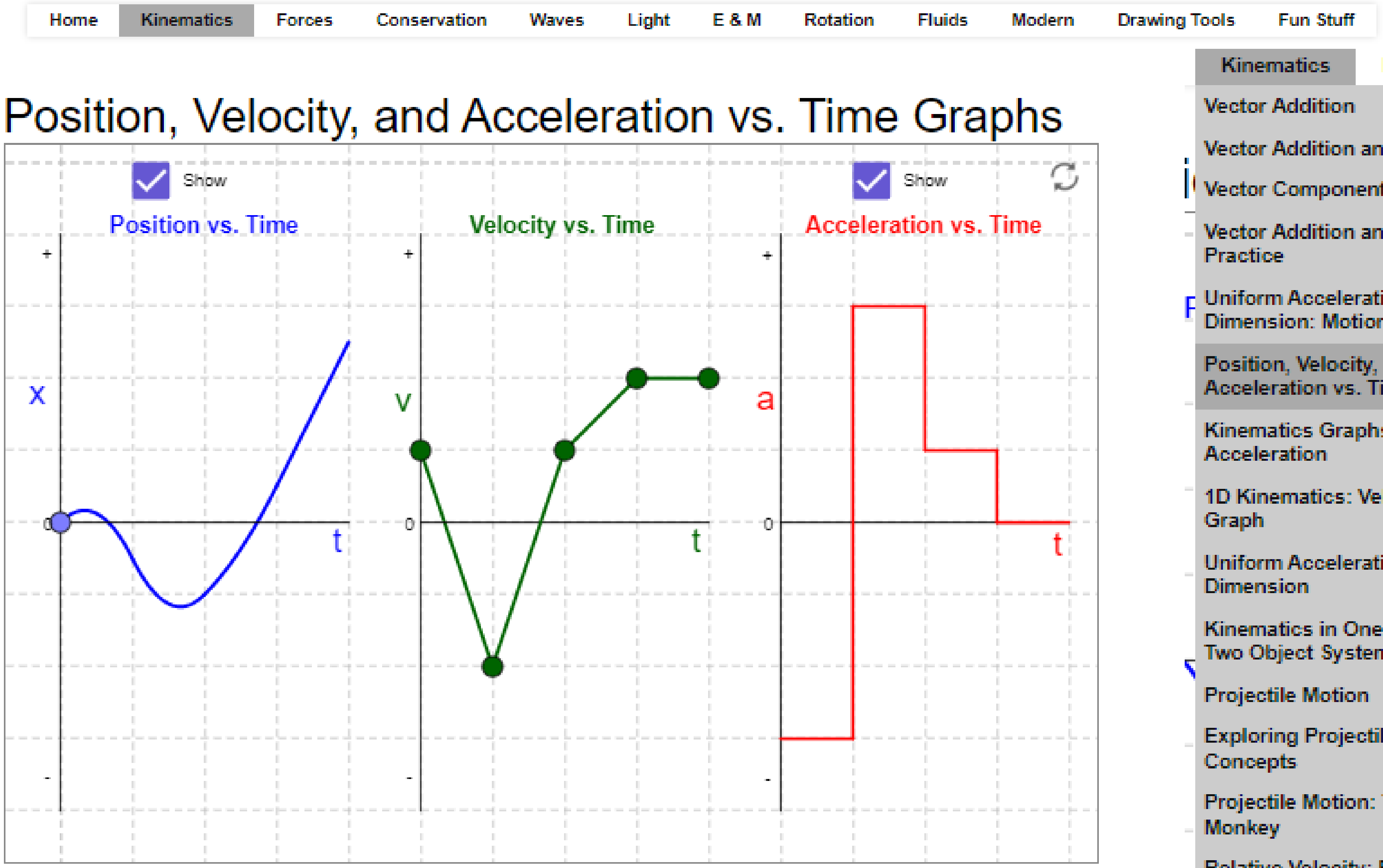
This is just like the definition of instantaneous velocity

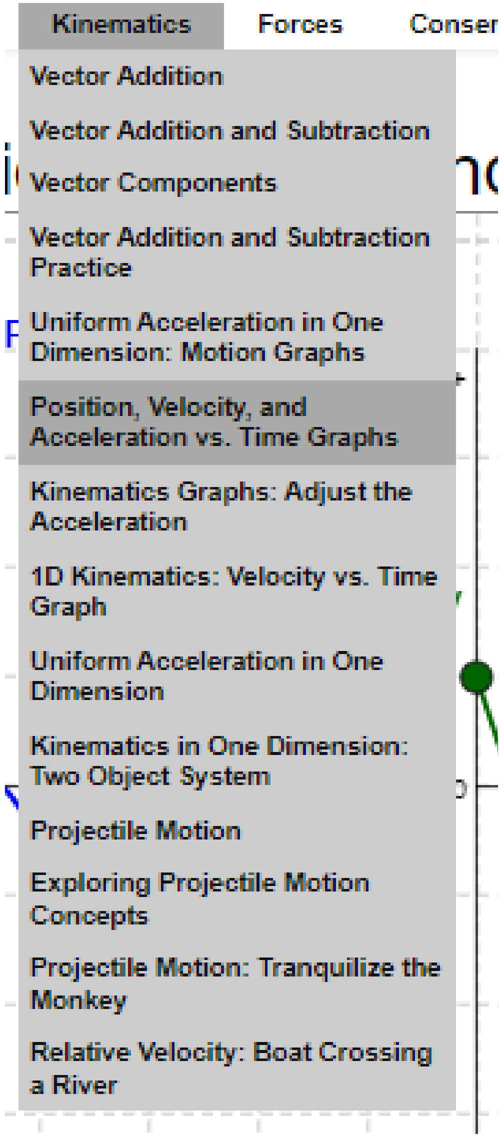
$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$



The acceleration at time t_1 is the slope of the velocity graph $\boldsymbol{v}(t)$ at that time

O-physics again!





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SI Units Motion

Displacement: meters (**m**), can be positive or negative

Velocity: rate of change of displacement, units: Meters per second (**m/s**)

Acceleration: rate of change of velocity, units: Meters per second square (**m/s^2**)

Constant Acceleration

Change of velocity at a constant rate:

$$\frac{d\boldsymbol{v}}{d\boldsymbol{t}} = \boldsymbol{a} = \boldsymbol{constant} = \frac{d^2\boldsymbol{x}}{d\boldsymbol{t}^2}$$

So velocity over time is:

$$\boldsymbol{v} = \boldsymbol{v_0} + \boldsymbol{at}$$

Where v_0 is the velocity at $t = 0$

So how far does a particle move under constant acceleration?

Displacement is related to time by:

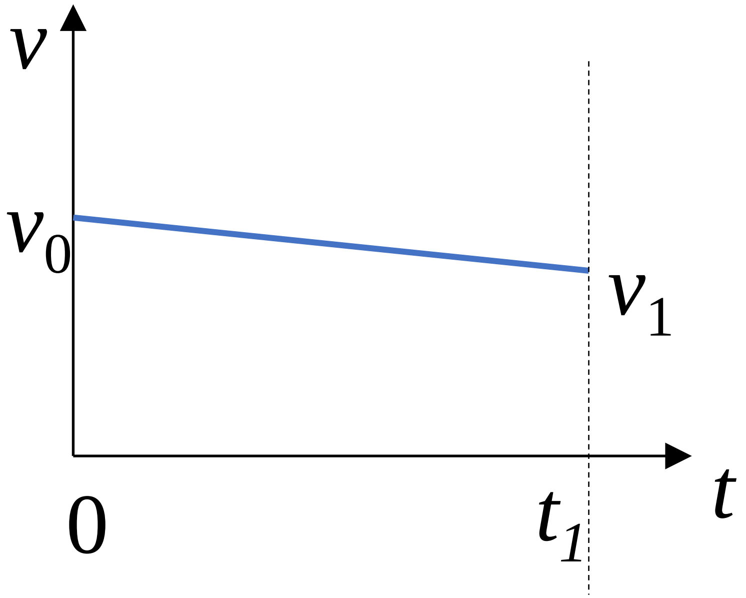
$$\frac{dx}{dt} = v = v_0 + at$$

Taking the integral on t both sides (still remember calculus?)

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

Where x_0 is the displacement at $t = 0$

Constant acceleration, continued with a curve view



Average velocity from 0 to t_1 is

$$\bar{v} = \frac{v_0 + v_1}{2}$$

Only applicable to **constant acceleration**

What if we don't have a timer?

$$x = x_0 + \boxed{v_0} t + \frac{1}{2} \boxed{a} t^2$$

$$v = v_0 + at \longrightarrow t = \frac{v - v_0}{a}$$

$$x = x_0 + v_0 \frac{v - v_0}{a} + \frac{1}{2} a \left(\frac{v - v_0}{a}\right)^2$$

$$x - x_0 = v_0 \frac{v - v_0}{a} + \frac{1}{2} a \left(\frac{v - v_0}{a}\right)^2$$

$$= \frac{v_0 v - {v_0}^2}{a} + \frac{v^2 - 2 v_0 v + {v_0}^2}{2a}$$

$$= \frac{v^2 - {v_0}^2}{2a}$$

$$v^2 - {v_0}^2 = 2a(x - x_0)$$

Key formulas for constant acceleration

$$v = v_0 + at$$

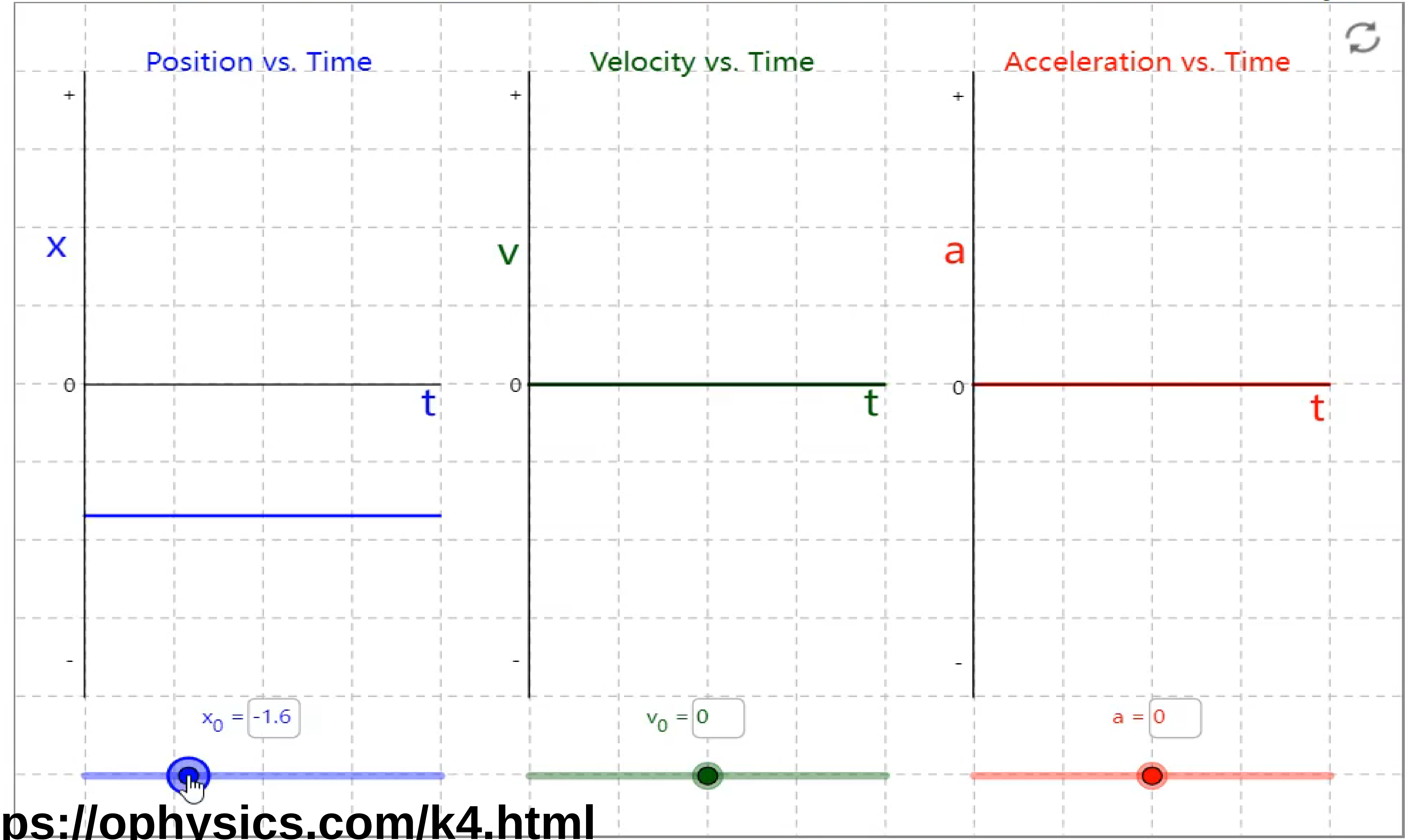
$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$\overline{v} = \frac{v_0 + v_1}{2}$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

No need for memorization.
I will provide an equation sheet for you in the exams,
But, without interpretations!
You need to be able to identify which one applies only to constant acceleration.
We will learn a lot more equations soon!

Uniform Acceleration in One Dimension: Motion Graphs



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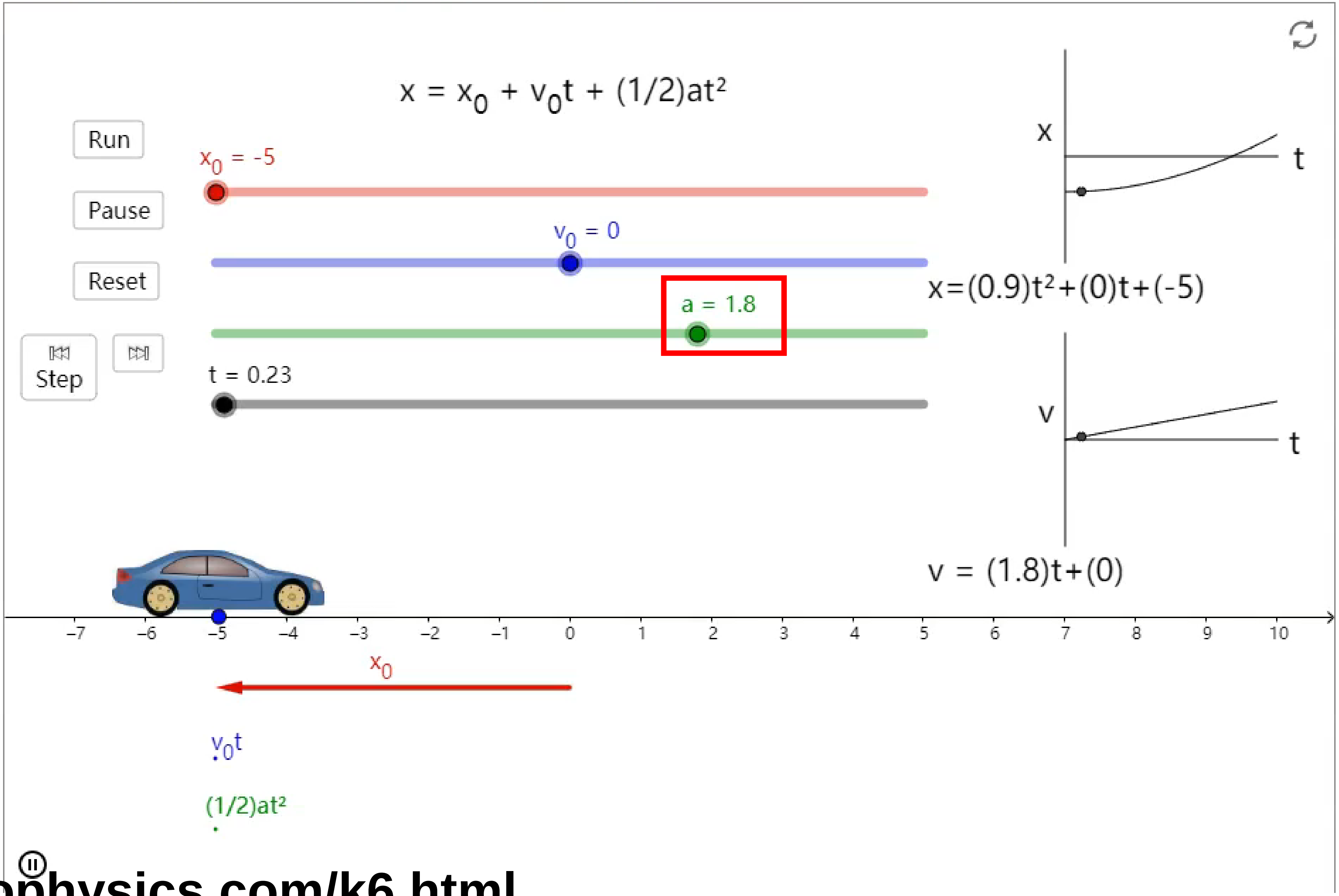
Uniform Acceleration in One Dimension



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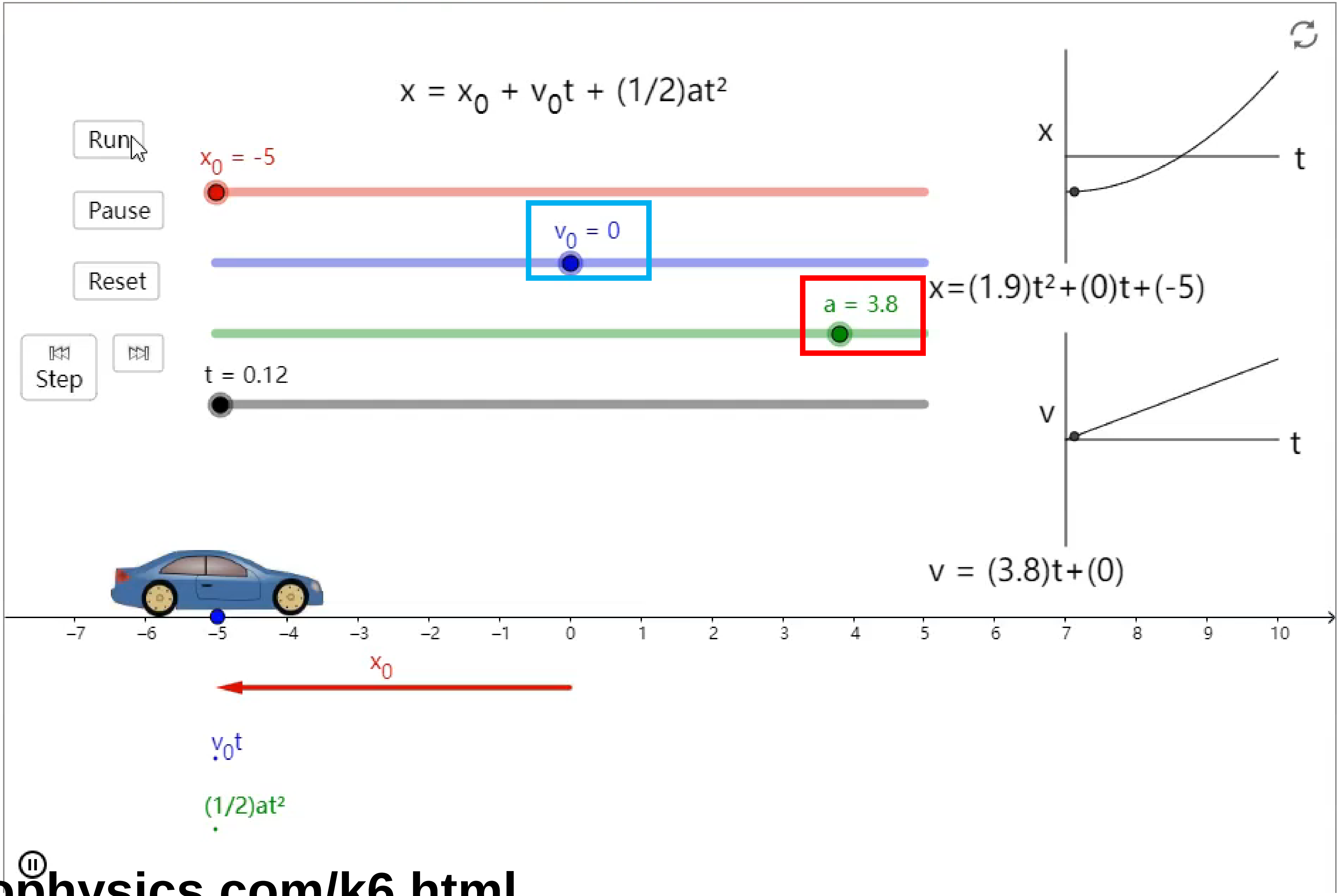
Uniform Acceleration in One Dimension



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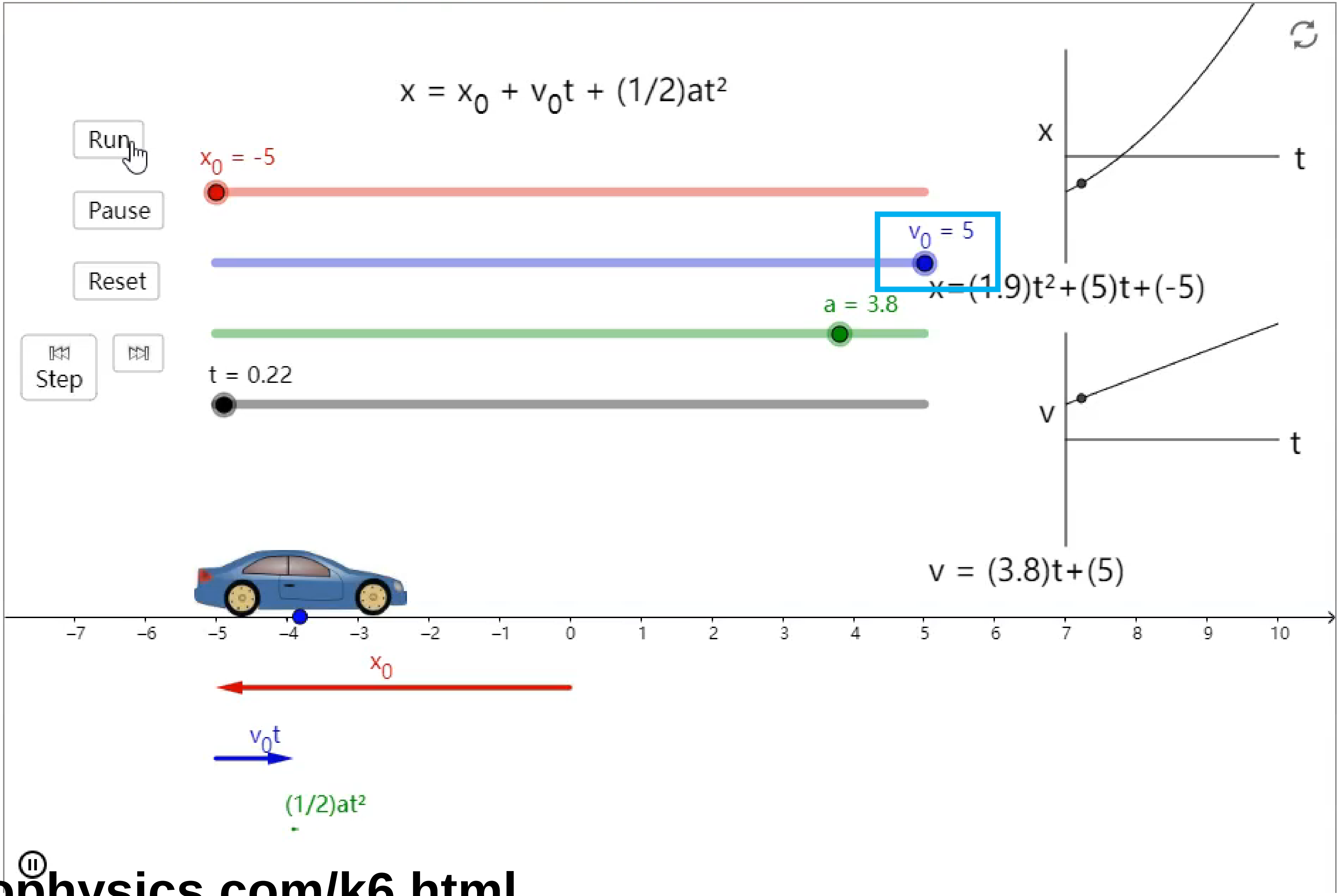

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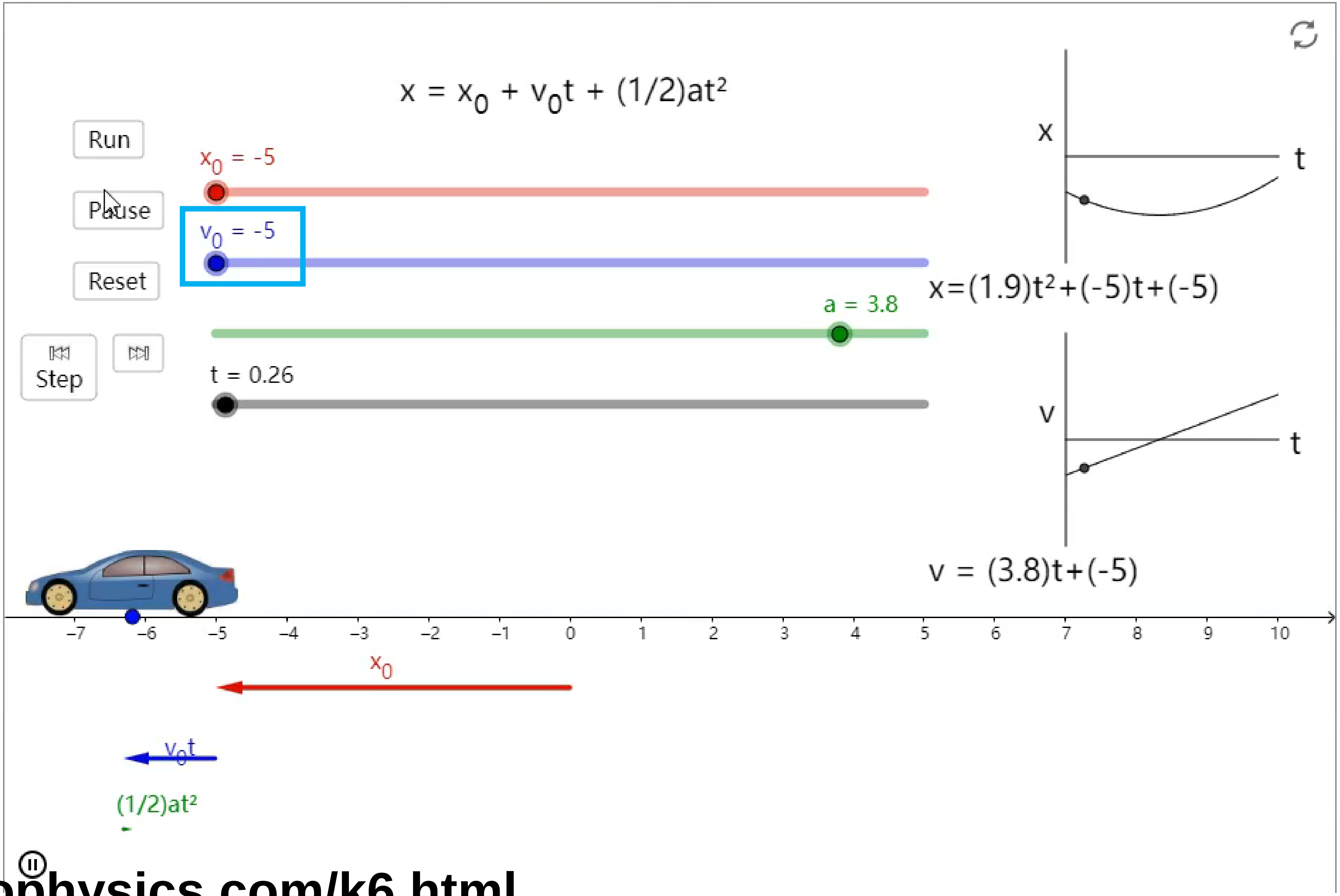
Uniform Acceleration in One Dimension



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Questions so far?

5 minutes of Q&A time

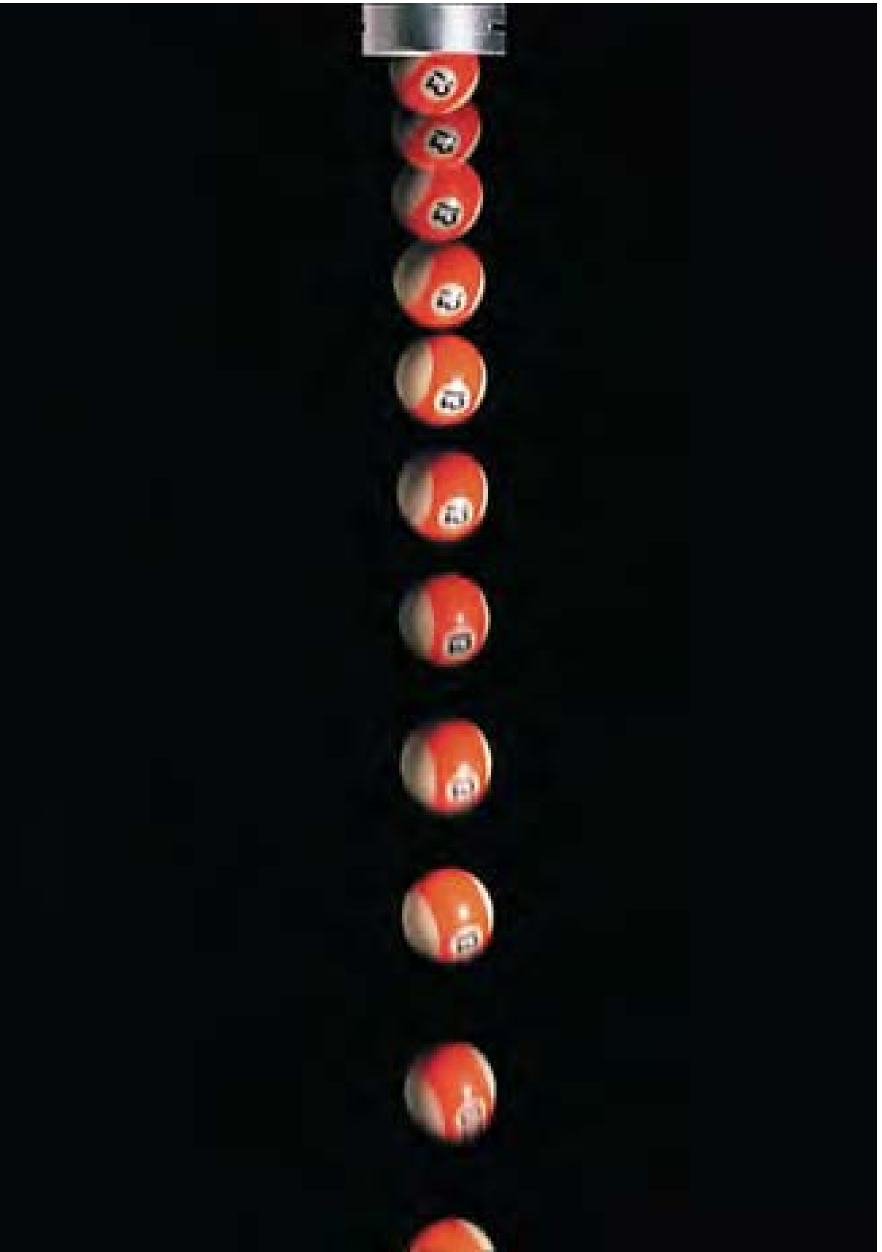
We derived a lot of equations for constant acceleration

So What is next?

TV Show: Fan Hua (繁花）

From the conversation: how tall is the Empire State Building?

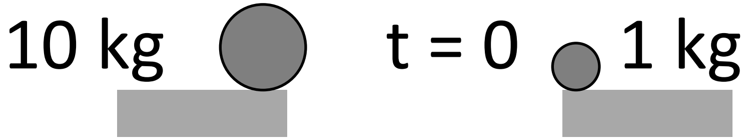
Freely Falling Bodies



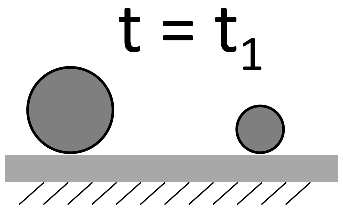
The most familiar example of motion with (nearly) constant acceleration is a body falling under the influence of the earth's gravitational attraction. Such motion has held the attention of philosophers and scientists since ancient times. In the fourth century B.C., Aristotle thought (erroneously) that heavy bodies fall faster than light bodies, in proportion to their weight. Nineteen centuries later, Galileo (see Section 1.1) argued that a body should fall with a downward acceleration that is constant and independent of its weight.

Gravity and Two Falling Spheres

Before Galileo, it was believed that falling objects quickly reached a natural speed, proportional to weight, then fell at that speed

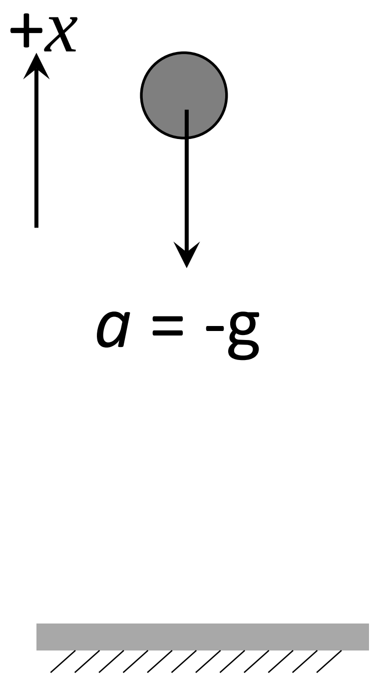


Galileo proves



Ignores resistance by air

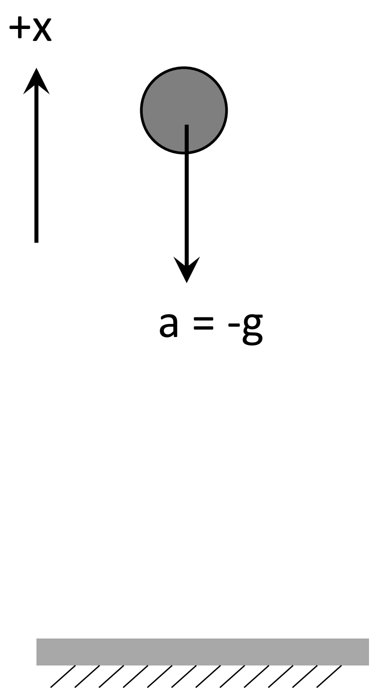
Gravitational acceleration



- We start with this simplified 1D problem
  - Acceleration a = **-**g, where $\boldsymbol{g}$ **~= 9.8 m/s²** around sea level
  - Notice the minus sign, because we usually take the direction in which height (h) increases as $+x$ direction

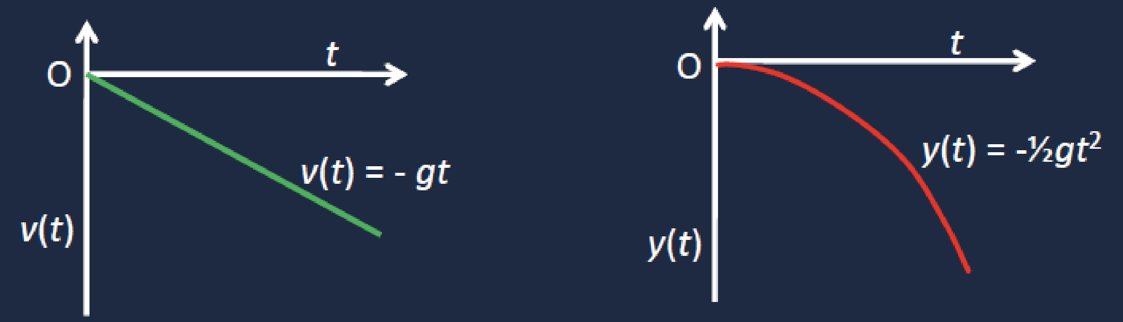
Gravitational Acceleration (Graphic View)

- Acceleration a = -g, where $\boldsymbol{g}$ **~= 9.8 m/s²** around sea level



- Notice the minus sign, because we usually take the direction in which height (h) increases as +x direction

- Taking upwards as positive, velocity and position as functions of time will look like this:



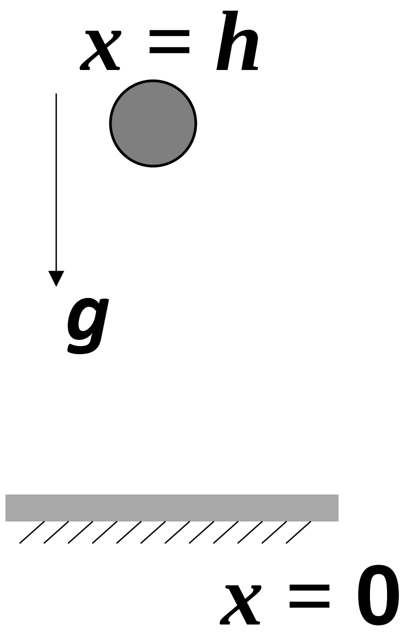
Case 1: Falling from height h

Q: how long does it take for a particle to fall back from a height *h*?
Derive an ***analytic equation***.

Assume: air resistance is negligible and initial velocity is 0

g ~= 9.8 m/s^2

$$v = v_0 + at$$
$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$
$$\bar{v} = \frac{v_0 + v_1}{2}$$
$$v^2 = v_0^2 + 2a(x - x_0)$$



Answer:

$$\boldsymbol{x} = \boldsymbol{x_0} + \boldsymbol{v_0} t + \frac{1}{2} a t^2$$

0 h 0 -g

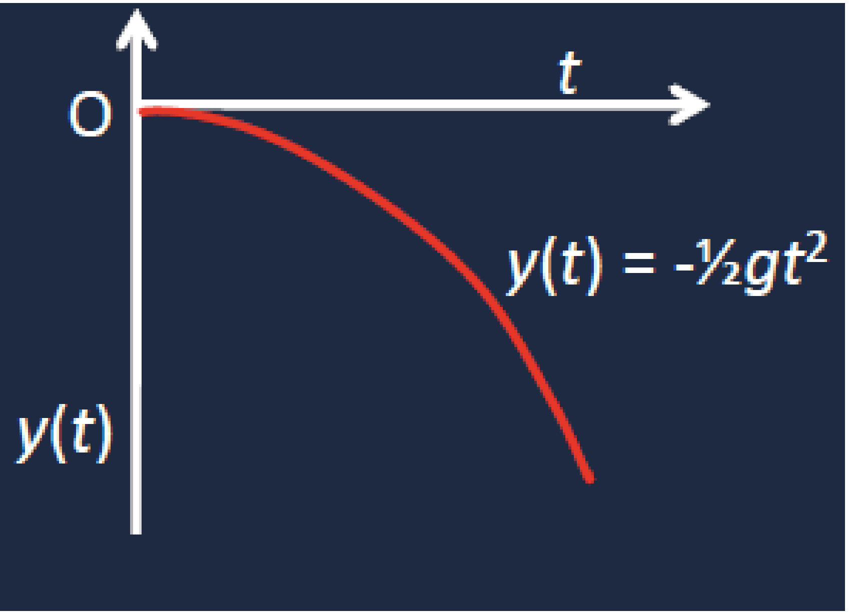
$$0 = h - \frac{1}{2} g t^2$$

Rearrange

$$t = \sqrt{\frac{2h}{g}}$$

So how tall is the State Empire Building?

Given that it takes 8.8 s to fall from the highest floor?



- If you work out the math, you should get $h = 0.5 \cdot 9.8\ m/s^2 \cdot (8.8\ s)^2 = 379.5\ m$

How tall is the building actually? Exactly 380 m. So the old gentleman probably estimated the number with exactly the same equation.

Should be longer in a real experiment!

How tall is the Empire State Building? It depends on how you measure it! At its top floor, the Empire State Building stands 1,250 feet (380 meters) tall. Counting the spire and antenna, the building clocks in at a mighty 1,454 feet (443 meters).

Case 2: Throwing a ball upward

Question: how long does it take for a particle to fall back if we throw it upward from $z = 0$ with an initial velocity $v = v_0$

Derive an analytic equation.

Approach 1:

$$\boxed{x_1 = x_0 = 0}$$

$$\boxed{v^2 - {v_0}^2 = 2a(x_1 - x_0)}$$

$$v^2 = {v_0}^2$$

$$v = -v_0$$

$$\boxed{-v_0 = v_0 - gt}$$

$$t = 2v_0/g$$

Approach 2: still use:

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

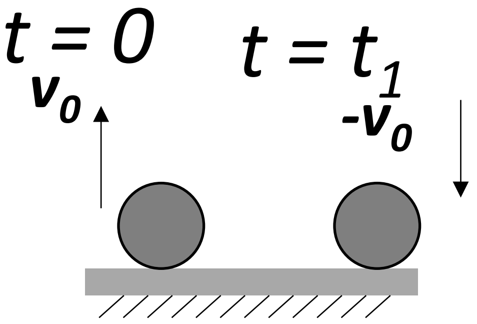
g ~= 9.8 m/s²

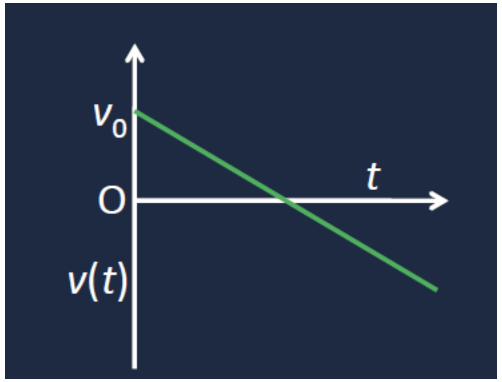
$$\boxed{v = v_0 + at}$$

$$\boxed{x = x_0 + v_0 t + \tfrac{1}{2}at^2}$$

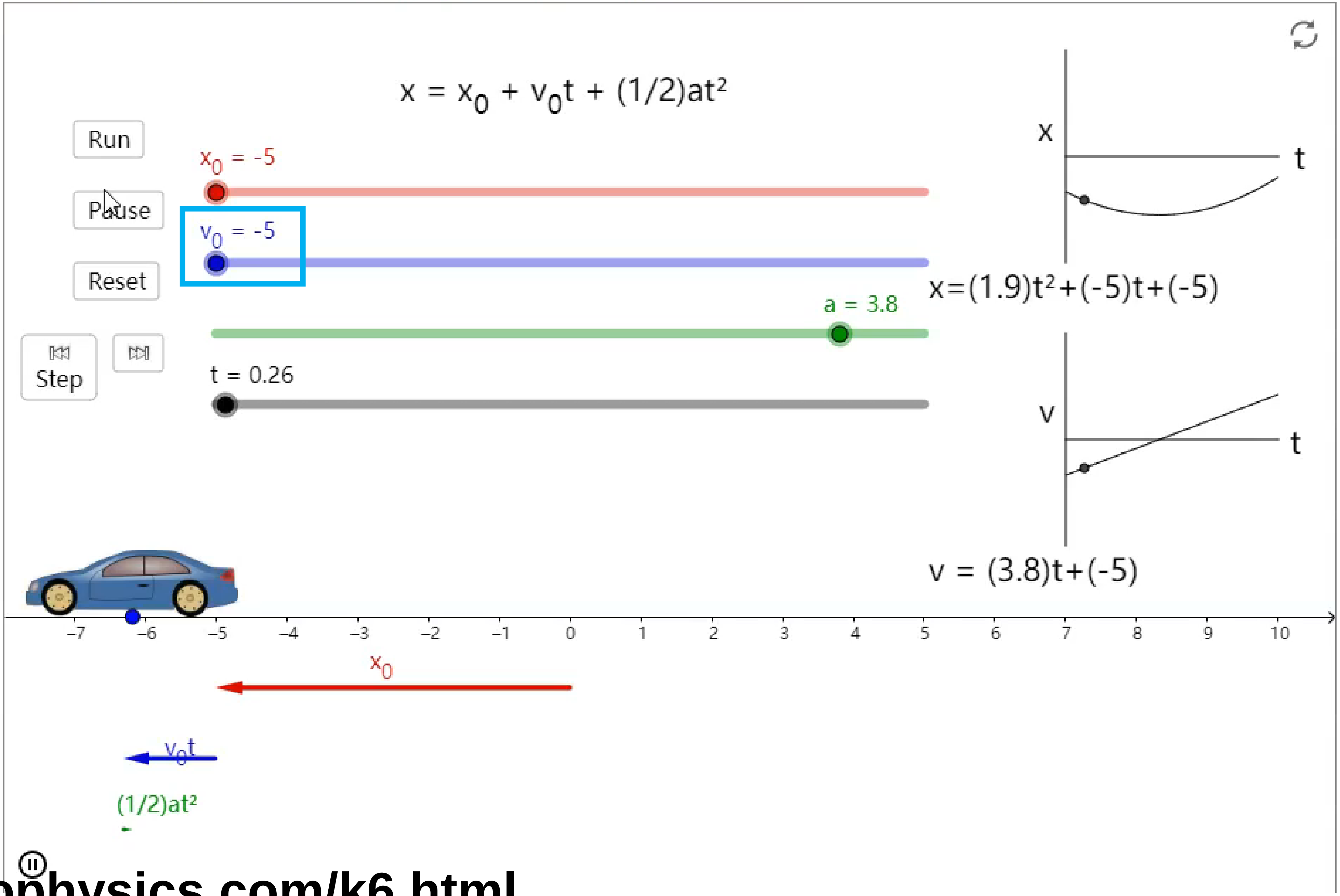
$$\bar{v} = \frac{v_0 + v_1}{2}$$

$$\boxed{v^2 = v_0^2 + 2a(x - x_0)}$$





Uniform Acceleration in One Dimension



https://ophysics.com/k6.html

Exercise 1: Flea

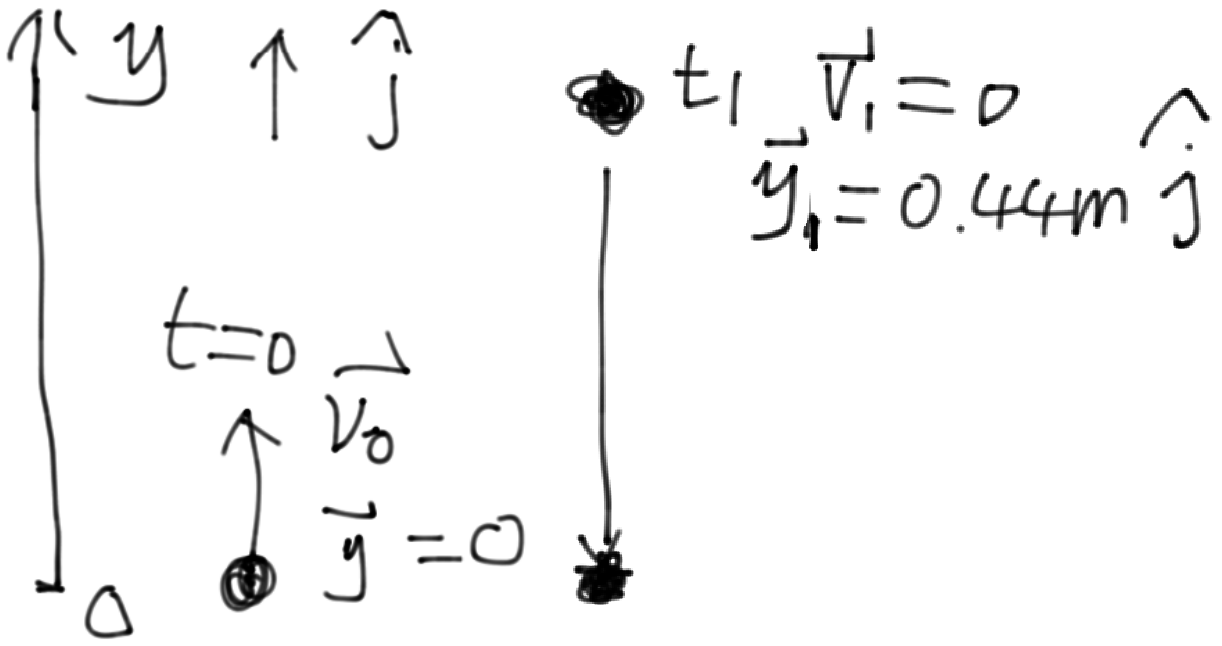
2.35 •• (a) If a flea can jump straight up to a height of 0.440 m, what is its initial speed as it leaves the ground? (b) How long is it in the air?

What is not mentioned?

$|g| \simeq 9.8 m/s$

Assume that the flea is on earth ~sea level

$\vec{a} = -|g|\,\hat{j}$



Exercise 1: Flea

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$$\vec{v}(t) = \vec{v}_0 - \vec{g}t$$

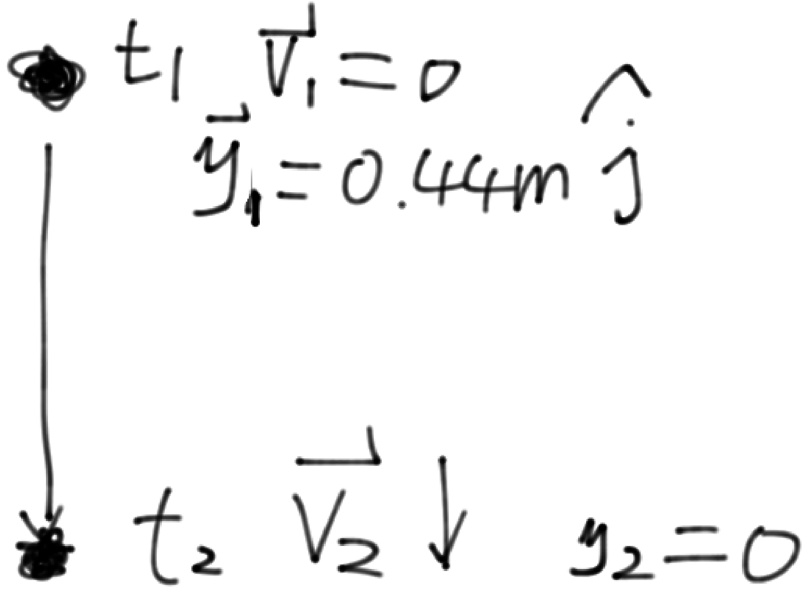
?

$$\vec{y}(t) = \int \vec{v}(t)\,dt = \underbrace{\vec{y}_0}_{0} + \vec{v}_0 t - \frac{1}{2}\vec{g}\,t^2$$

$$\vec{y}(t_1) = 0.44\,m\,\hat{j} = \vec{v}_0 t_1 - \frac{1}{2}\vec{g}\,t_1^2$$

$$\vec{v}(t_1) = 0 = \vec{V}_0 - \vec{g}\,t_1$$

$$\boxed{t_1 = \frac{\vec{v}_0}{\vec{g}}}$$



Exercise 1: Flea

2.35 •• (a) If a flea can jump straight up to a height of 0.440 m, what is its initial speed as it leaves the ground? (b) How long is it in the air?

$$\vec{y}(t_1) = 0.44\,\mathrm{m}\,\hat{j} = \vec{v}_0 t_1 - \frac{1}{2}\vec{g}t_1^2$$

$$\text{so: } 0.44\,\mathrm{m} = |\vec{v}_0| \cdot \frac{|\vec{v}_0|}{|\vec{g}|} - \frac{1}{2}|\vec{g}|\left(\frac{|\vec{v}_0|}{|\vec{g}|}\right)^2$$

$$|\vec{v}_0| = \sqrt{2|\vec{g}|\cdot|\vec{y}_1|} \simeq 2.9\,\mathrm{m/s}$$

Exercise 1: Flea

2.35 •• (a) If a flea can jump straight up to a height of 0.440 m, what is its initial speed as it leaves the ground? (b) How long is it in the air?

$$|\vec{v}_0| = \sqrt{2|\vec{g}| \cdot |\vec{y}_1|} \simeq 2.9 \text{ m/s}$$

$$\boxed{t_1 = \frac{\vec{v}_0}{\vec{g}}}$$

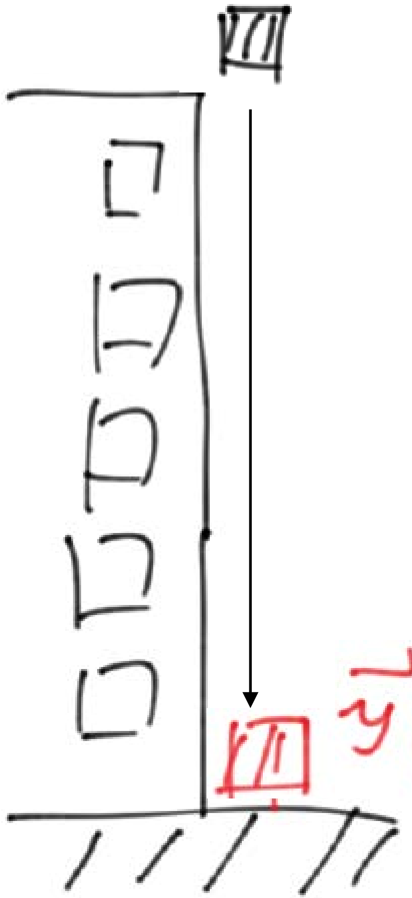
$t_2 = 2\, t_1 = 0.59$ s

Exercise 2: Drop Brick (Don't do it in real life)

$\vec{y}(t=0) = \boxed{\vec{y}_0}$?

$\vec{V}(t=0) = 0$

2.42 •• A brick is dropped (zero initial speed) from the roof of a building. The brick strikes the ground in 2.50 s. You may ignore air resistance, so the brick is in free fall. (a) How tall, in meters, is the building? (b) What is the magnitude of the brick's velocity just before it reaches the ground? (c) Sketch a_y-t, v_y-t, and y-t graphs for the motion of the brick.



$\vec{y}(t_1 = 2.5s) = 0$

$\vec{V}(t_1 = 2.5s) = ?$

$$\vec{y} = \underbrace{\vec{y}_0}_{?} + \underbrace{\vec{V}_0}_{0} t + \frac{1}{2}\underbrace{\vec{a}}_{-g\hat{j}} t^2$$

At $t = 2.5\,s$

$$0 = \vec{y}_0 - \frac{1}{2} g t^2 \hat{j}$$

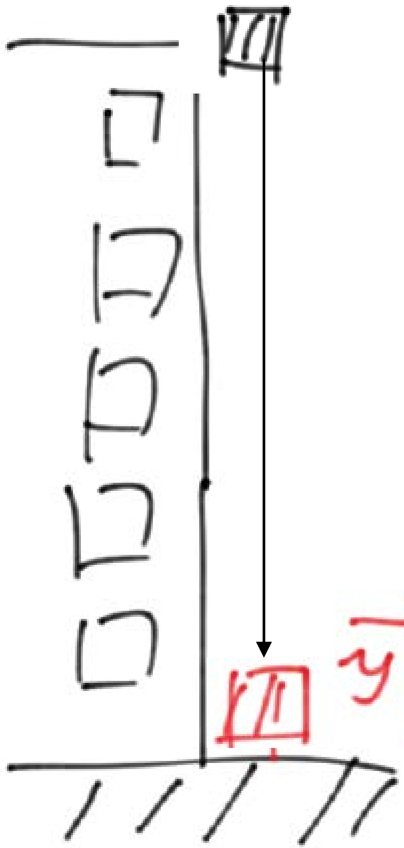
$$\Rightarrow |\vec{y}_0| = \frac{1}{2} g t^2 = 30.625\,m$$

Exercise 2: Drop Brick (Don't do it in real life)

$\vec{y}(t=0)=\vec{y}_0$

$\vec{V}(t=0)=0$

2.42 •• A brick is dropped (zero initial speed) from the roof of a building. The brick strikes the ground in 2.50 s. You may ignore air resistance, so the brick is in free fall. (a) How tall, in meters, is the building? (b) What is the magnitude of the brick's velocity just before it reaches the ground? (c) Sketch a_y-t, v_y-t, and y-t graphs for the motion of the brick.



$\vec{y}(t_1=2.5s)=0$

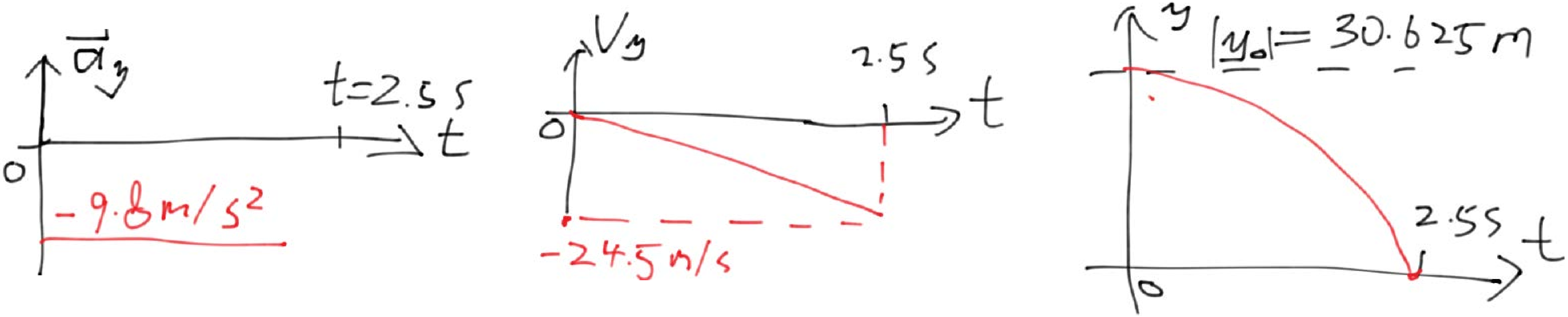
$\vec{V}(t_1=2.5s)=?$

$$\vec{V}=\underbrace{\vec{V_0}}_{0}-gt\,\hat{j}$$

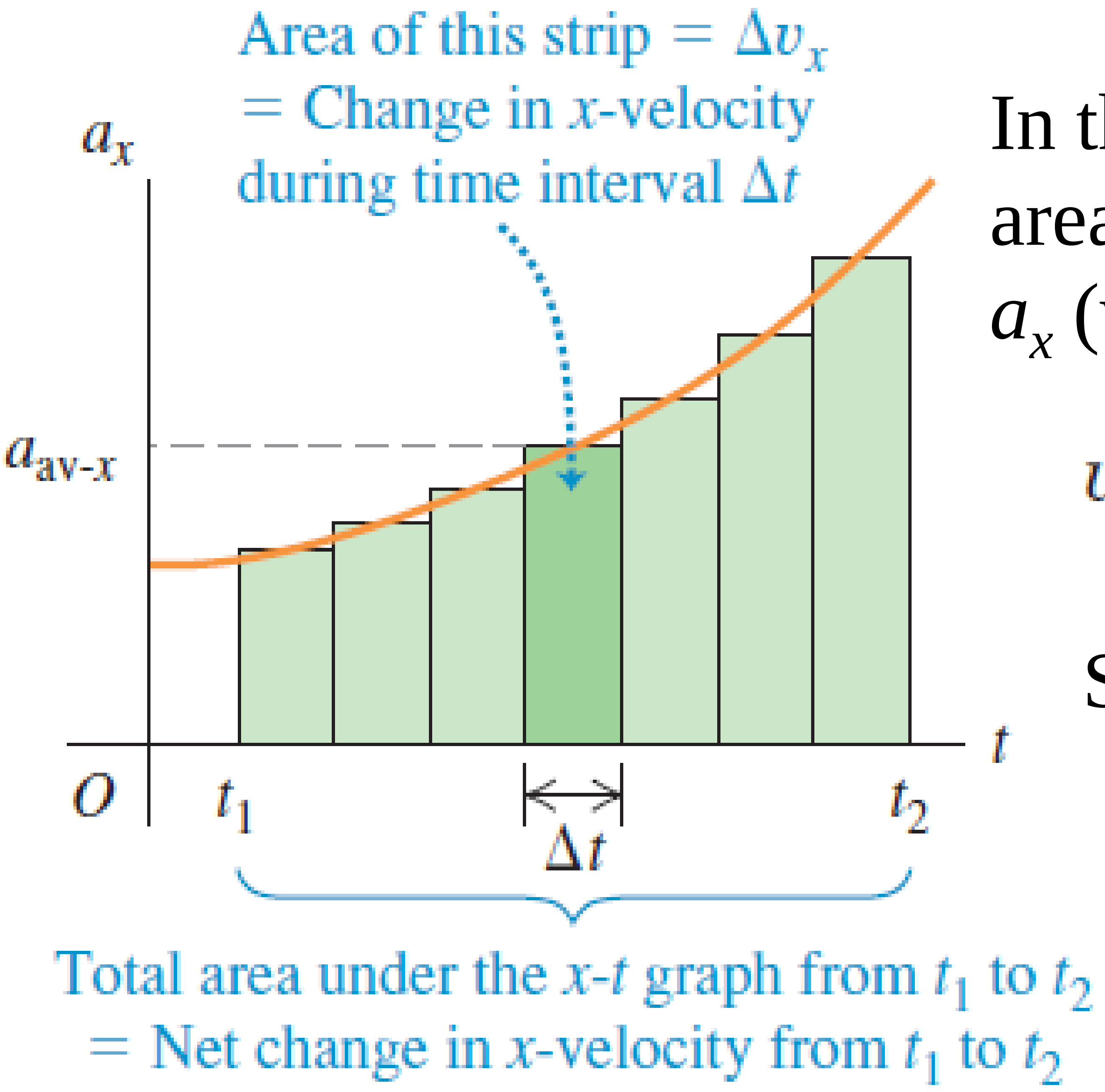
$$\Rightarrow \vec{V}=-9.8\,m/s^2\cdot 2.5\,s\,\hat{j}$$

Exercise 2: Drop Brick (Don’t do it in real life)

2.42 •• A brick is dropped (zero initial speed) from the roof of a building. The brick strikes the ground in 2.50 s. You may ignore air resistance, so the brick is in free fall. (a) How tall, in meters, is the building? (b) What is the magnitude of the brick’s velocity just before it reaches the ground? (c) Sketch a_y-t, v_y-t, and y-t graphs for the motion of the brick.



Velocity and Position by Integration



For each Δt $\quad \Delta v_x = a_{\text{av-}x}\,\Delta t$

In the limit when Δt becomes small, the area under the $a_x t$ curve is the *integral* of a_x (which is in general a function of t)

$$v_{2x} - v_{1x} = \int_{v_{1x}}^{v_{2x}} dv_x = \int_{t_1}^{t_2} a_x\,dt$$

Similarly, for displacement

$$x_2 - x_1 = \int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v_x\,dt$$

Velocity and Position by Integration

If $t_1 = 0$ and t_2 is any later time t, and if x_0 and v_{0x} are the position and velocity, respectively, at time $t = 0$

$$v_x = v_{0x} + \int_0^t a_x \, dt \qquad (2.17)$$

$$x = x_0 + \int_0^t v_x \, dt \qquad (2.18)$$

Example 2.9 Motion with changing acceleration

Sally is driving along a straight highway in her 1965 Mustang. At $t = 0$ when she is moving at 10 m/s in the positive x-direction, she passes a signpost at $x = 50$ m. Her x-acceleration as a function of time is:

$$a_x = 2.0 \text{ m/s}^2 - (0.10 \text{ m/s}^3)t$$

(a) Find her x-velocity and position x as functions of time. (b) When is her x-velocity greatest? (c) What is that maximum x-velocity? (d) Where is the car when it reaches that maximum x-velocity?

Analysis: The x-acceleration is **a function of time**, so we *cannot* use the **constant-acceleration** formulas of Section 2.4. Instead, we use Eq. (2.17) to obtain an expression for v_x as a function of time, and then use that result in Eq. (2.18) to find an expression for x as a function of t.

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$$a_x = 2.0\ \text{m/s}^2 - (0.10\ \text{m/s}^3)t$$

(a) Find her ***x*-velocity** and position x as functions of time.

$$v_x = 10\ \text{m/s} + \int_0^t [2.0\ \text{m/s}^2 - (0.10\ \text{m/s}^3)t]\,dt$$

$$= 10\ \text{m/s} + (2.0\ \text{m/s}^2)t - \tfrac{1}{2}(0.10\ \text{m/s}^3)t^2$$

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$$a_x = 2.0\ \mathrm{m/s^2} - (0.10\ \mathrm{m/s^3})t$$

(a) Find her x-velocity and **position *x*** as functions of time.

$$v_x = 10\ \mathrm{m/s} + (2.0\ \mathrm{m/s^2})t - \tfrac{1}{2}(0.10\ \mathrm{m/s^3})t^2$$

$$\begin{aligned} x &= 50\ \mathrm{m} + \int_0^t \left[10\ \mathrm{m/s} + (2.0\ \mathrm{m/s^2})t - \tfrac{1}{2}(0.10\ \mathrm{m/s^3})t^2\right] dt \\ &= 50\ \mathrm{m} + (10\ \mathrm{m/s})t + \tfrac{1}{2}(2.0\ \mathrm{m/s^2})t^2 - \tfrac{1}{6}(0.10\ \mathrm{m/s^3})t^3 \end{aligned}$$

Example 2.9 Motion with changing acceleration

Sally is driving along a straight highway in her 1965 Mustang. At $t = 0$ when she is moving at 10 m/s in the positive x-direction, she passes a signpost at $x = 50$ m. Her x-acceleration as a function of time is:

$$\boxed{a_x} = 2.0\ \mathrm{m/s^2} - (0.10\ \mathrm{m/s^3})t$$

(b) When is her x-velocity greatest? ***Greatest when acceleration*** $\boxed{\boldsymbol{a_x} \textbf{\textit{ is 0!}}}$

$$\boxed{0} = 2.0\ \mathrm{m/s^2} - (0.10\ \mathrm{m/s^3})t$$

$$t = \frac{2.0\ \mathrm{m/s^2}}{0.10\ \mathrm{m/s^3}} = 20\ \mathrm{s}$$

Example 2.9 Motion with changing acceleration

Sally is driving along a straight highway in her 1965 Mustang. At $t = 0$ when she is moving at 10 m/s in the positive x-direction, she passes a signpost at $x = 50$ m. Her x-acceleration as a function of time is:

$$a_x = 2.0\ \text{m/s}^2 - (0.10\ \text{m/s}^3)t$$

(c) What is that maximum x-velocity? Plug in the time we just calculated

$$t = \frac{2.0\ \text{m/s}^2}{0.10\ \text{m/s}^3} = 20\ \text{s}$$

$$v_{\text{max-}x} = 10\ \text{m/s} + (2.0\ \text{m/s}^2)(20\ \text{s}) - \tfrac{1}{2}(0.10\ \text{m/s}^3)(20\ \text{s})^2$$

$$= 30\ \text{m/s}$$

Example 2.9 Motion with changing acceleration

Sally is driving along a straight highway in her 1965 Mustang. At $t = 0$ when she is moving at 10 m/s in the positive x-direction, she passes a signpost at $x = 50$ m. Her x-acceleration as a function of time is:

$$a_x = 2.0\ \mathrm{m/s^2} - (0.10\ \mathrm{m/s^3})t$$

(c) Where is the car when it reaches that maximum x-velocity

Asking about x when v_x reaches maximum

We substitute $t = 20$ s into the expression for x from part (a):

$$x = 50\ \mathrm{m} + (10\ \mathrm{m/s})(20\ \mathrm{s}) + \tfrac{1}{2}(2.0\ \mathrm{m/s^2})(20\ \mathrm{s})^2$$

$$- \tfrac{1}{6}(0.10\ \mathrm{m/s^3})(20\ \mathrm{s})^3 \quad = 517\ \mathrm{m}$$

Summary: Kinematics in 1D

$$v_{\text{av-}x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$$a_{\text{av-}x} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \frac{\Delta v_x}{\Delta t}$$

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

$$v_x = v_{0x} + \int_0^t a_x \, dt$$

$$x = x_0 + \int_0^t v_x \, dt$$

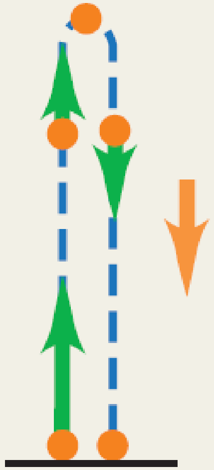
Constant x-acceleration only:

$$v_x = v_{0x} + a_x t \qquad (2.8)$$

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2 \qquad (2.12)$$

$$v_x{}^2 = v_{0x}{}^2 + 2a_x(x - x_0) \qquad (2.13)$$

$$x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t$$



$$a_y = -g = -9.80\ \text{m/s}^2$$