Fundamentals of Electric Circuits

CHAPTER 2 Basic Laws

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CHAPTER 2 Basic laws

2.2 Ohm’s law

2.3 Nodes, Branches, and Loops

2.4 Kirchhoff’s law

2.5 Series Resistors and Voltage Division

2.6 Parallel Resistors and Current Division

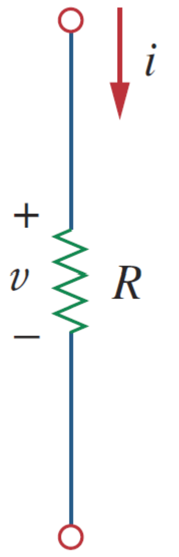
2.7 Wye-Delta Transformations

2.2 Ohm's law

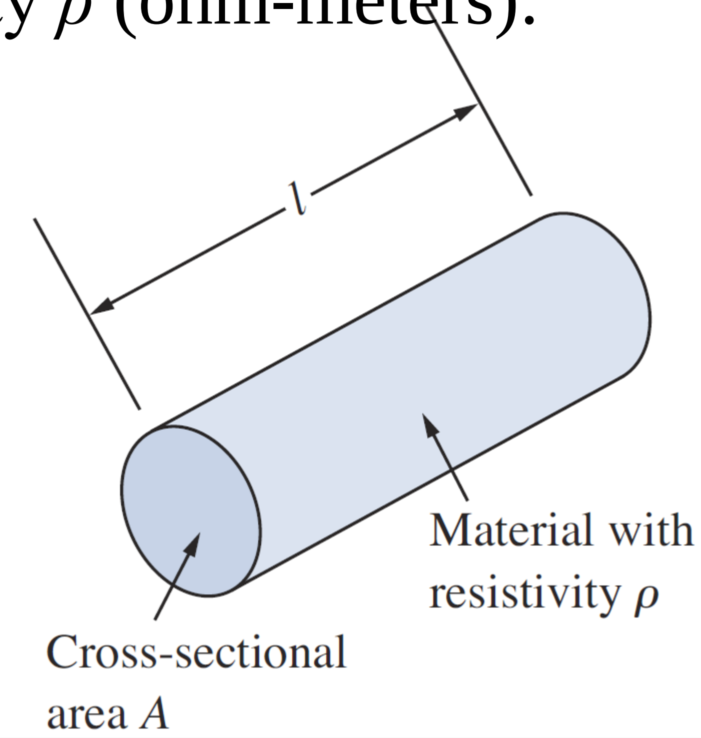
Resistor

- Materials tend to resist the flow of electric charge through them.
- This physical property is called "resistance", R
- The resistance of any material is a function of its length l, and cross-sectional area A, and the material's resistivity ρ (ohm-meters):

$$R = \rho \frac{l}{A}$$



The circuit element used to model the current-resisting behavior of a material is the resistor.



Resistivity ρ of Common Materials

TABLE 2.1
Resistivities of common materials.

Material	Resistivity ρ $(\Omega.m)$	Usage	
Silver	1.64×10^{-8}	Conductor	
Copper	1.72×10^{-8}	Conductor	
Aluminum	2.8×10^{-8}	Conductor	Low resistivity
Gold	2.45×10^{-8}	Conductor	
Carbon	4×10^{-5}	Semiconductor	
Germanium	47×10^{-2}	Semiconductor	
Silicon	6.4×10^{2}	Semiconductor	
Paper	10^{10}	Insulator	
Mica	5×10^{11}	Insulator	High resistivity
Glass	10^{12}	Insulator	
Teflon	3×10^{12}	Insulator	

Ohm's Law

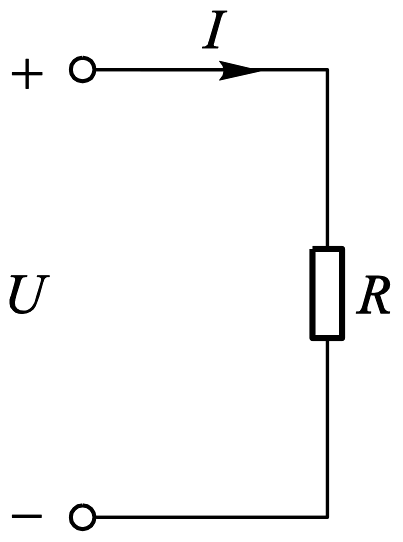
- ***In a resistor, the voltage V across a resistor is directly proportional to the current I flowing through it.***

$$\boldsymbol{V = IR}$$

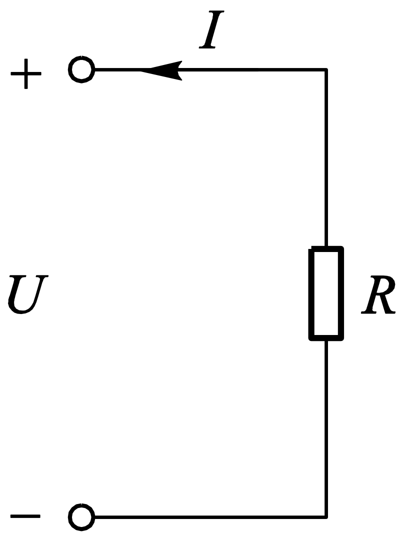
- The resistance of an element is measured in units of Ohms, Ω, (V/A)
- The higher the resistance, the less current will flow through for a given voltage.

Ohm’s Law

Ohm’s law requires conforming with the **passive sign convention**.



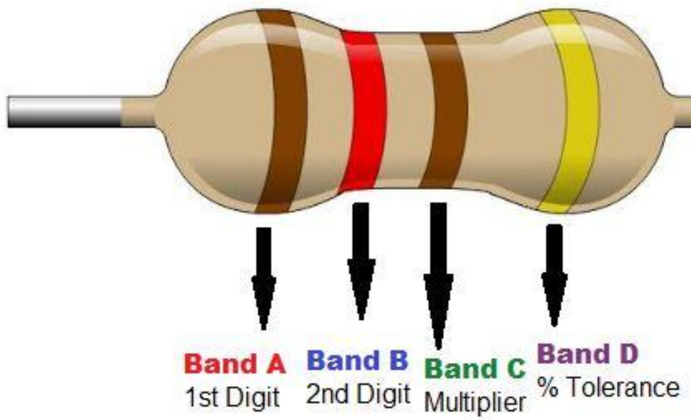
$$U = IR$$

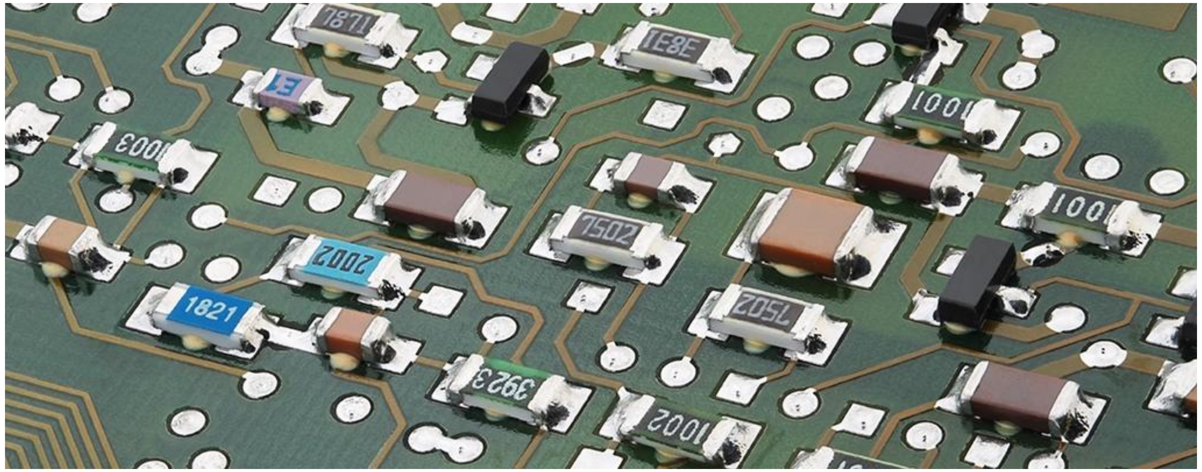


$$U = -IR$$

Resistor

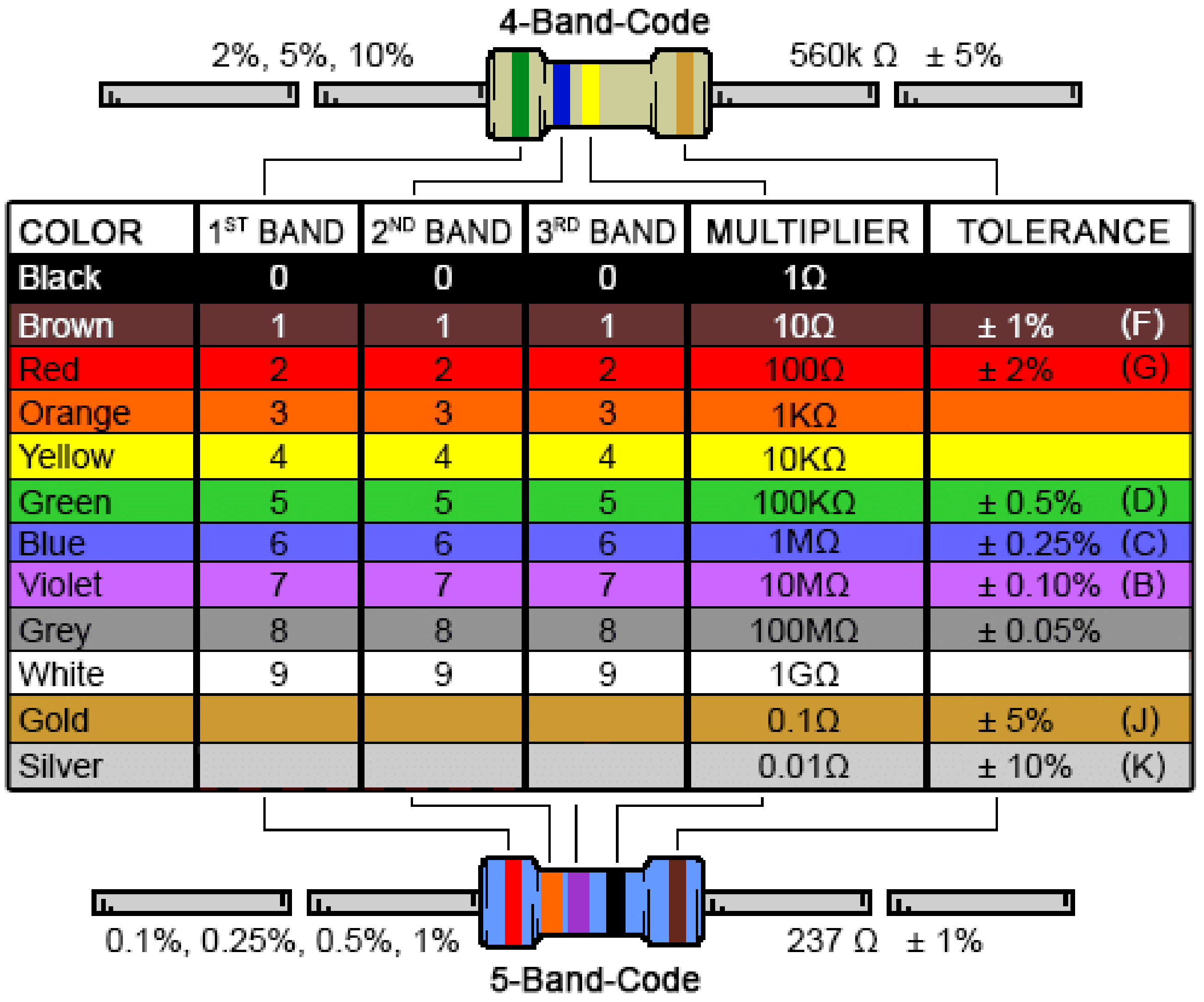
- Through-hole/ Surface mount resistor
- $R = \rho \frac{l}{A}$





Resistor

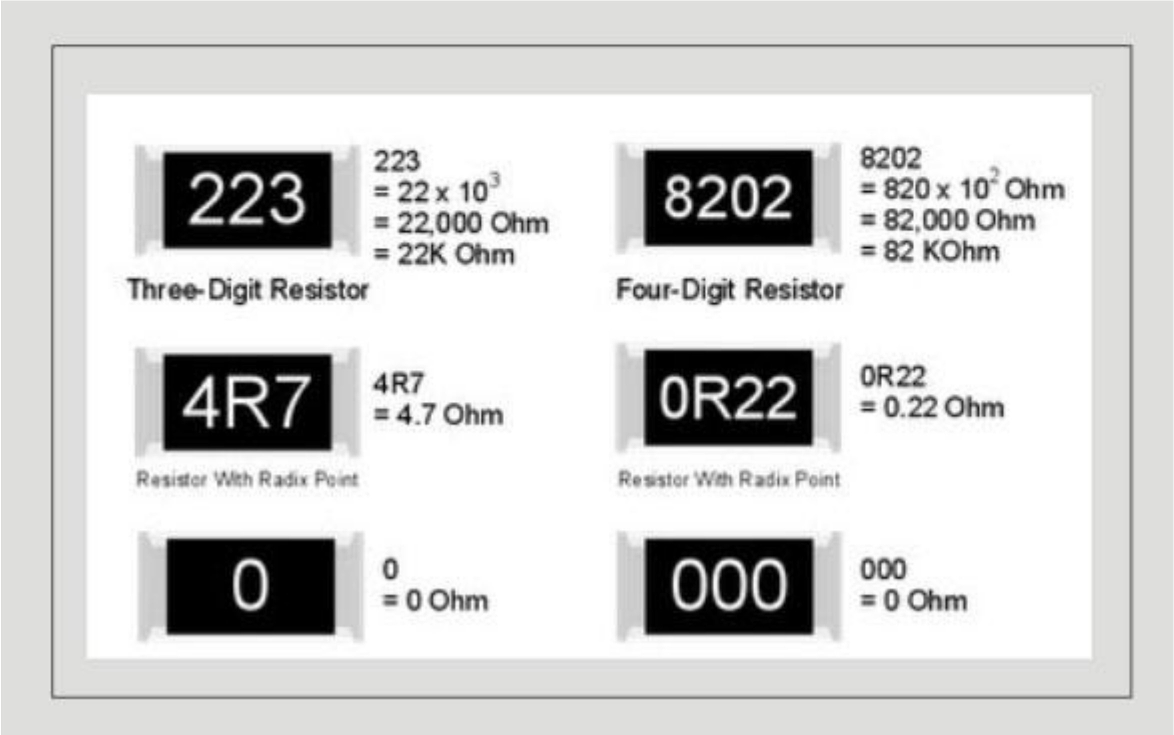
How to identify through-hole resistor value?



COLOR	1ST BAND	2ND BAND	3RD BAND	MULTIPLIER	TOLERANCE
Black	0	0	0	1Ω	
Brown	1	1	1	10Ω	± 1% (F)
Red	2	2	2	100Ω	± 2% (G)
Orange	3	3	3	1KΩ	
Yellow	4	4	4	10KΩ	
Green	5	5	5	100KΩ	± 0.5% (D)
Blue	6	6	6	1MΩ	± 0.25% (C)
Violet	7	7	7	10MΩ	± 0.10% (B)
Grey	8	8	8	100MΩ	± 0.05%
White	9	9	9	1GΩ	
Gold				0.1Ω	± 5% (J)
Silver				0.01Ω	± 10% (K)

Resistor

How to identify SMD resistor value?



Short and Open Circuits (two extreme possible values of *R*)

- A connection with resistance approaching zero (R=0) is called a **short** circuit. V=IR=0
- Ideally, any current may flow through the short circuit.
- In practice, a short circuit is usually a connecting wire.

- A connection with resistance approaching infinity (R=∝ is called an **open** circuit.
- The current is zero, and the voltage could be anything.

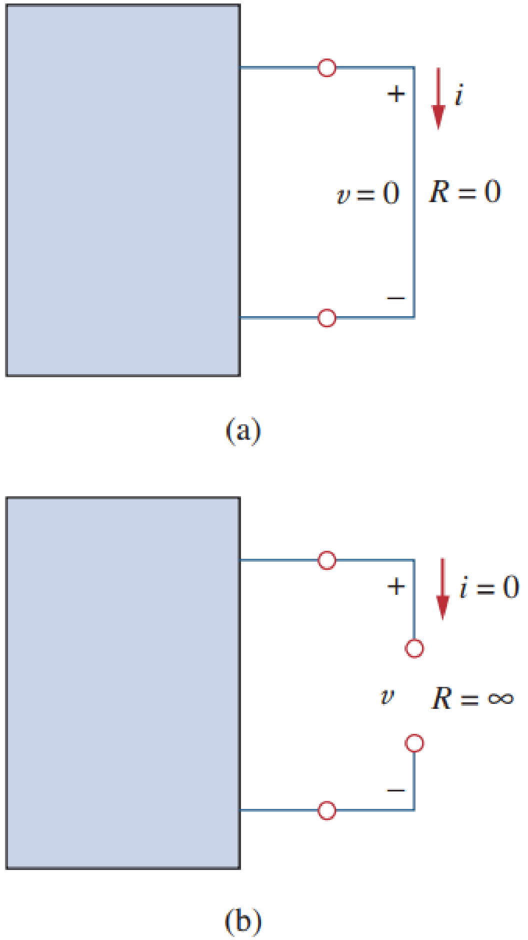


Figure 2.2
(a) Short circuit ($R = 0$), (b) Open circuit ($R = \infty$).

Variable resistor

- Variable resistor: adjustable resistance
- Potentiometer (pot): three terminal element with a sliding contact or wiper

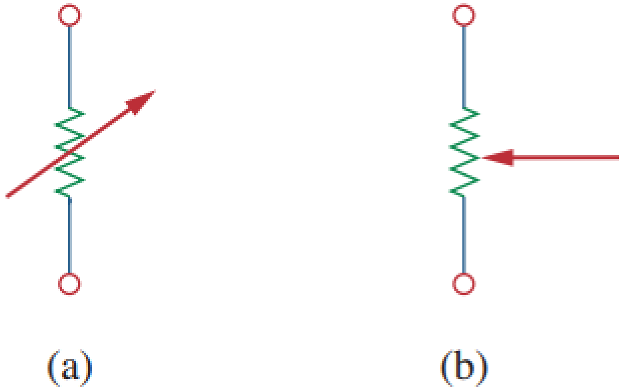
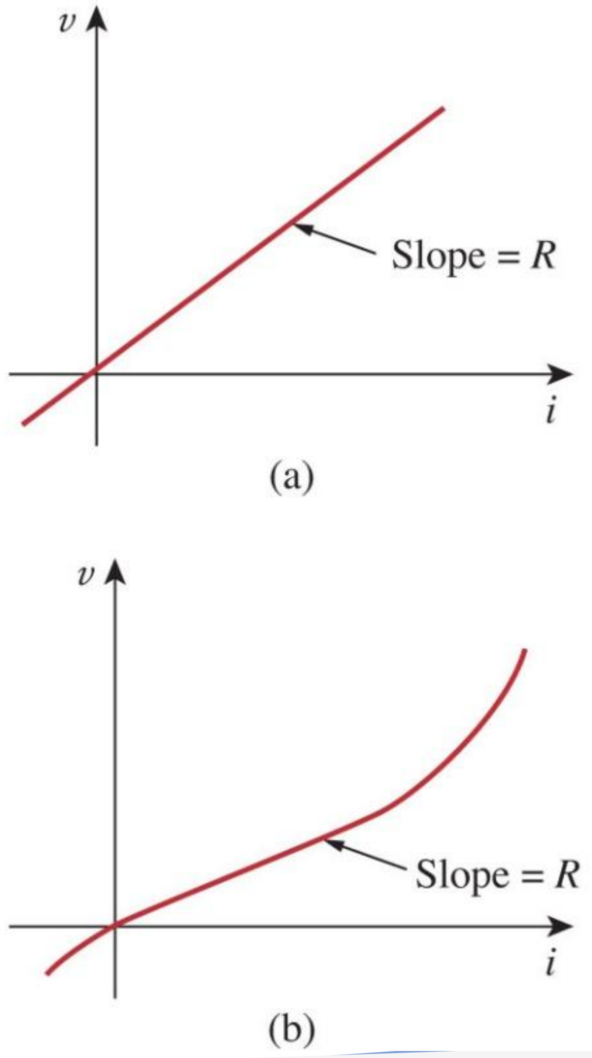


Figure 2.4
Circuit symbol for: (a) a variable resistor in general, (b) a potentiometer.

Linearity

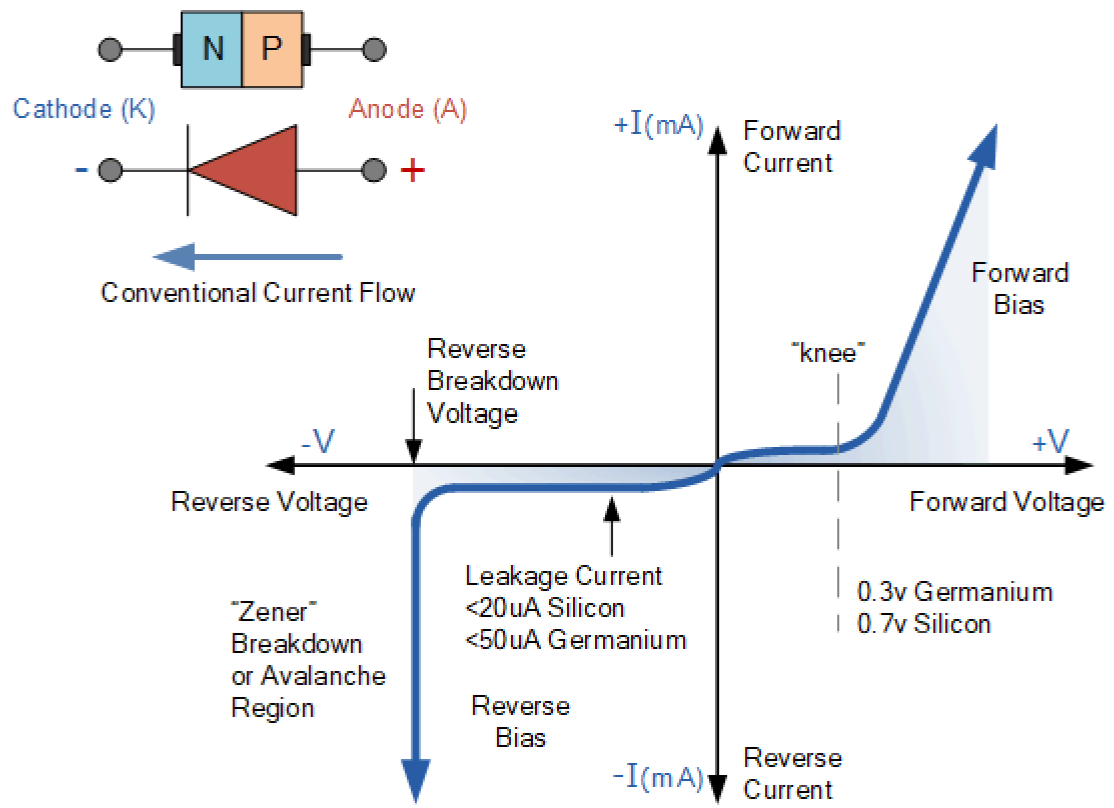
- Not all resistors obey Ohm's Law.
- Linear resistor: constant resistance and its current-voltage characteristic is always linearly proportional.
- Nonlinear resistor: variable resistance, (Diodes)



$V = IR$

Diode

- Pn junction.
- Diodes can be used as an one-way resistor.



Conductance

- Reciprocal of resistance R, denoted by G, describing the ability of an element to conduct electric current.

$$\boxed{G = \frac{1}{R} = \frac{i}{v}}$$

- The unit of conductance is siemens (S).
- The same resistor can be expressed by resistance or conductance, 10 Ω is the same as 0.1S.

Power Dissipation by a resistor

- Current through a resistor dissipates power

$$p = vi = i^2R = \frac{v^2}{R}$$

- The power dissipated is a non-linear function of current or voltage
- Power dissipated is always positive
- A resistor always absorbs power from the circuit
- A resistor is a passive element, incapable of generating energy

Example

Example 2.1

An electric iron draws 2 A at 120 V. Find its resistance.

Solution:

From Ohm's law,

$$R = \frac{v}{i} = \frac{120}{2} = 60\ \Omega$$

Practice Problem 2.1

The essential component of a toaster is an electrical element (a resistor) that converts electrical energy to heat energy. How much current is drawn by a toaster with resistance 15 Ω at 110 V?

Answer: 7.333 A.

Example

Example 2.2

In the circuit shown in Fig. 2.8, calculate the current i, the conductance G, and the power p.

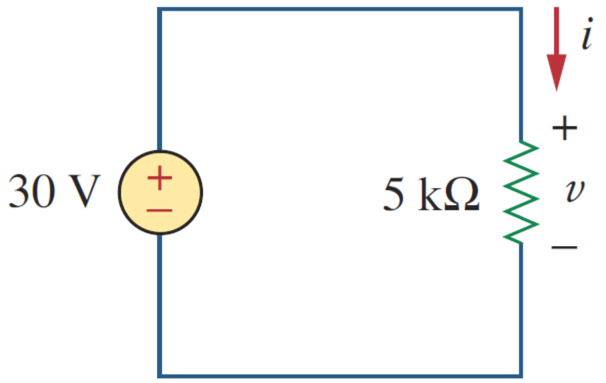


Figure 2.8
For Example 2.2.

Solution:
The voltage across the resistor is the same as the source voltage (30 V) because the resistor and the voltage source are connected to the same pair of terminals. Hence, the current is

$$i = \frac{v}{R} = \frac{30}{5 \times 10^3} = 6 \text{ mA}$$

The conductance is

$$G = \frac{1}{R} = \frac{1}{5 \times 10^3} = 0.2 \text{ mS}$$

We can calculate the power in various ways using either Eqs. (1.7), (2.10), or (2.11).

$$p = vi = 30(6 \times 10^{-3}) = 180 \text{ mW}$$

or

$$p = i^2R = (6 \times 10^{-3})^2 5 \times 10^3 = 180 \text{ mW}$$

or

$$p = v^2G = (30)^2 0.2 \times 10^{-3} = 180 \text{ mW}$$

Example

Practice Problem 2.2

For the circuit shown in Fig. 2.9, calculate the voltage v, the conductance G, and the power p.

Answer: 30 V, 100 μS, 90 mW.

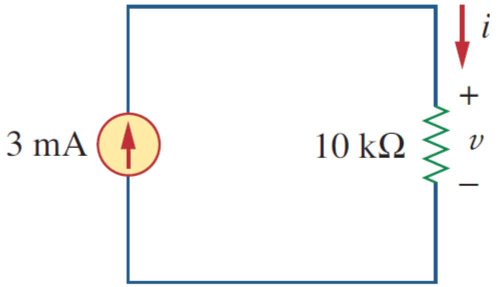


Figure 2.9
For Practice Prob. 2.2

2.3 Nodes, Branches, and Loops

Some basic network topology concepts:

- A **branch** represents a single element such as a voltage source or a resistor.
- A **node** is the point of connection between two or more branches.

  If a short circuit (connecting wire) connects two nodes, the two nodes constitute a single node.

- A **loop** is any closed path in a circuit.
- A loop is said to be ***independent (mesh)*** if it does not contain any other loop.

  Independent loops (meshes) result in independent sets of equations.

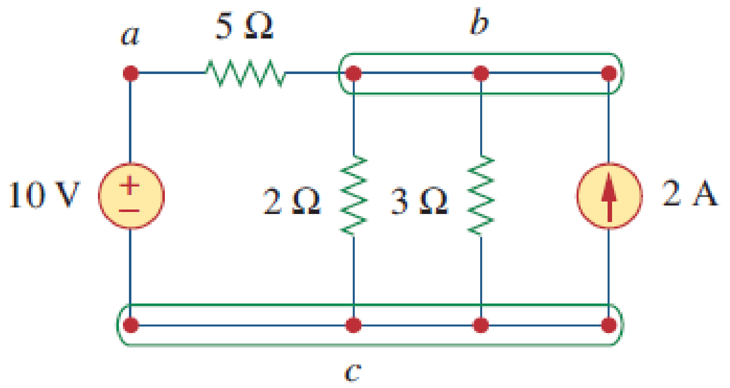


Figure 2.10
Nodes, branches, and loops.

5 branches, 3 nodes and 3 independent loops

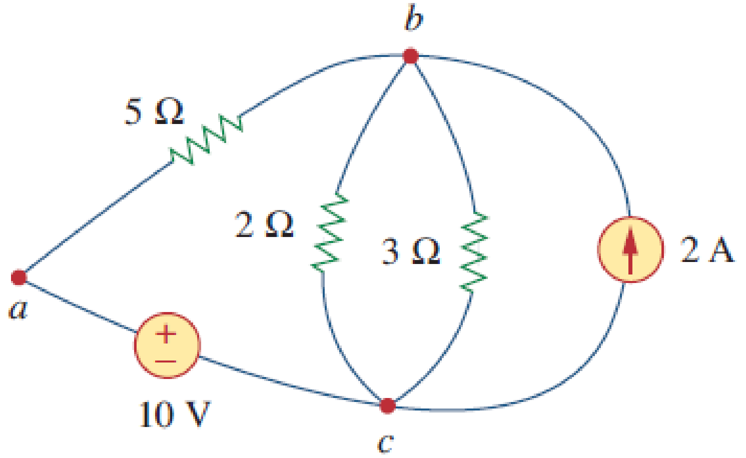


Figure 2.11
The three-node circuit of Fig. 2.10 is redrawn.

2.3 Nodes, Branches, and Loops

Some basic network topology concepts:

- A **branch** represents a single element such as a voltage source or a resistor. ***(b)***

- A **node** is the point of connection between two or more branches. ***(n)***

- A **loop** is any closed path in a circuit.

- A loop is said to be ***independent*** if it contains at least one branch which is not a part of any other independent loop. ***(l)***

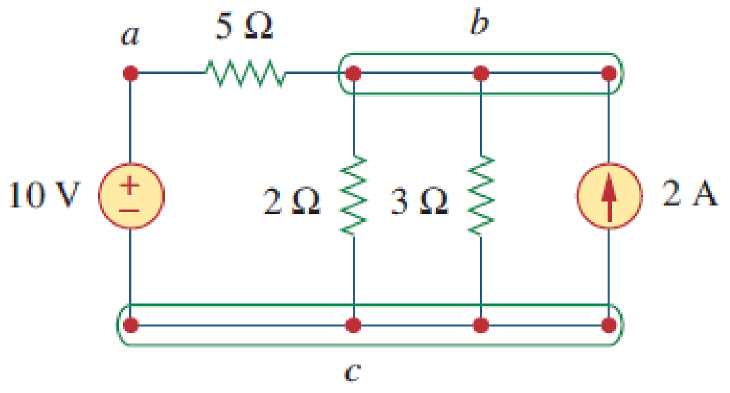
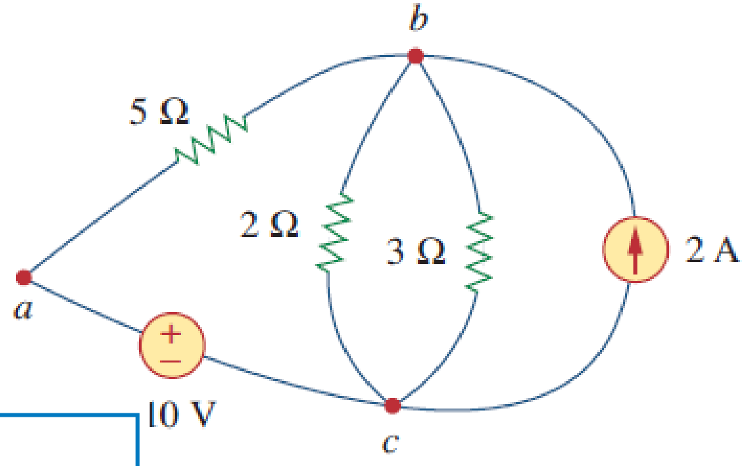


Figure 2.10
Nodes, branches, and loops.

5 branches, 3 nodes and 3 independent loops



2.11
e-node circuit of Fig. 2.10 is
.

The fundamental theorem of network topology:

$$\boxed{b = l + n - 1}$$

Two more definitions (series & parallel):

- Two or more elements are in ***series*** if they share a single node and carry the same current
- Two or more elements are in ***parallel*** if they are connected to the same two nodes and thus have the same voltage across them.

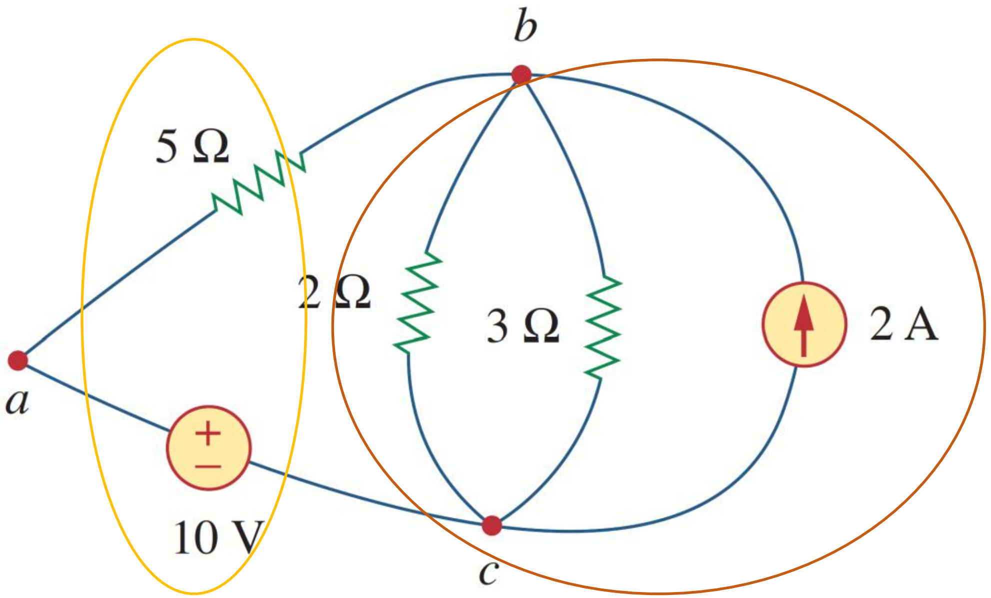


Figure 2.11

Example

Example 2.4

Determine the number of branches and nodes in the circuit shown in Fig. 2.12. Identify which elements are in series and which are in parallel.

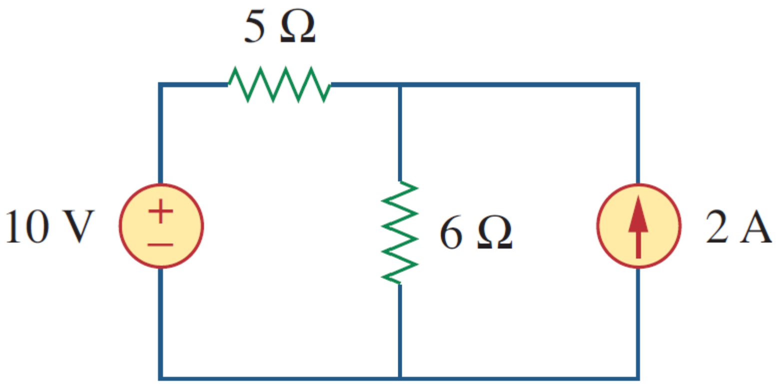
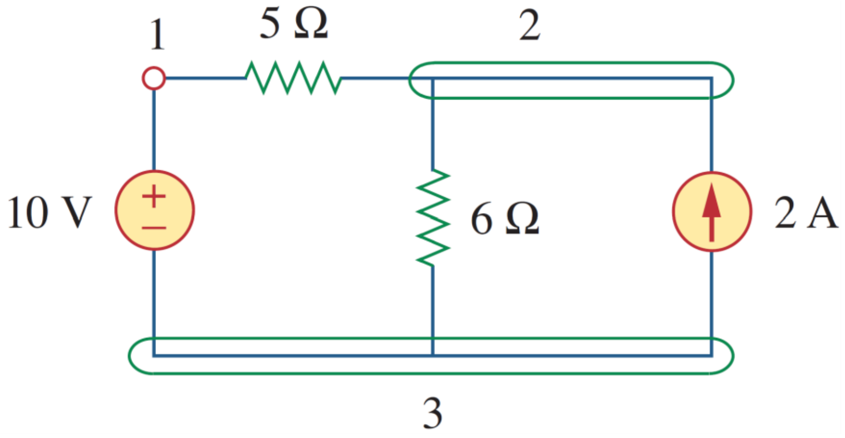


Figure 2.12
For Example 2.4.



Solution:

Since there are four elements in the circuit, the circuit has four branches: 10 V, 5 Ω, 6 Ω, and 2 A. The circuit has three nodes as identified in Fig. 2.13. The 5-Ω resistor is in series with the 10-V voltage source because the same current would flow in both. The 6-Ω resistor is in parallel with the 2-A current source because both are connected to the same nodes 2 and 3.

Example

How many branches and nodes does the circuit in Fig. 2.14 have? Identify the elements that are in series and in parallel.

Practice Problem 2.4

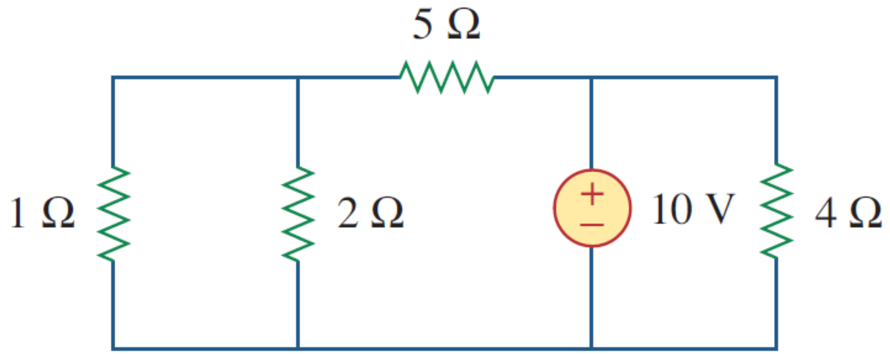


Figure 2.14
For Practice Prob. 2.4.

Answer: Five branches and three nodes are identified in Fig. 2.15. The 1-Ω and 2-Ω resistors are in parallel. The 4-Ω resistor and 10-V source are also in parallel.

2.4 Kirchhoff's Laws

- Ohm's law & Kirchhoff's laws: sufficient, and powerful tools
- There are two laws:
  - Kirchhoff's current law (KCL)
  - Kirchhoff's voltage law (KVL)

Kirchhoff's Current Law (KCL)

- Kirchhoff's current law is based on the conservation of charge. The algebraic sum of charges within a system cannot change, that is, the node stores no net charge.

$$0 = q_T(t) = q_1(t) + q_2(t) + \ldots = \int i_1(t)dt + \int i_2(t)dt + \ldots$$

- **Definition1**: KCL states that the algebraic sum of currents entering a node (or a closed boundary) is zero.

$$\sum_{n=1}^{N} i_n = 0$$

N is the number of branches connected to the node

- Currents entering a node may be regarded as **positive**, while currents leaving the node may be taken as **negative**

Kirchhoff’s Current Law (KCL)

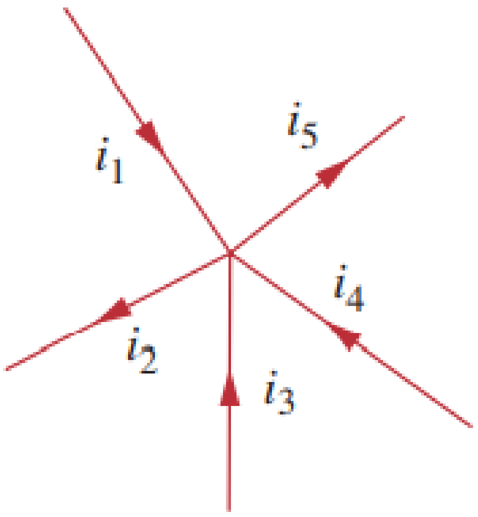


Figure 2.16
Currents at a node illustrating KCL.

Definition1: The algebraic sum of currents entering a node is zero.

$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$$

Definition2: The sum of the currents entering a node is equal to the sum of the currents leaving the node.

$$i_1 + i_3 + i_4 = i_2 + i_5$$

Kirchhoff's Current Law (KCL)

- **Definition1**: KCL states that the algebraic sum of currents entering a node (or **a closed boundary or supernode**) is zero.

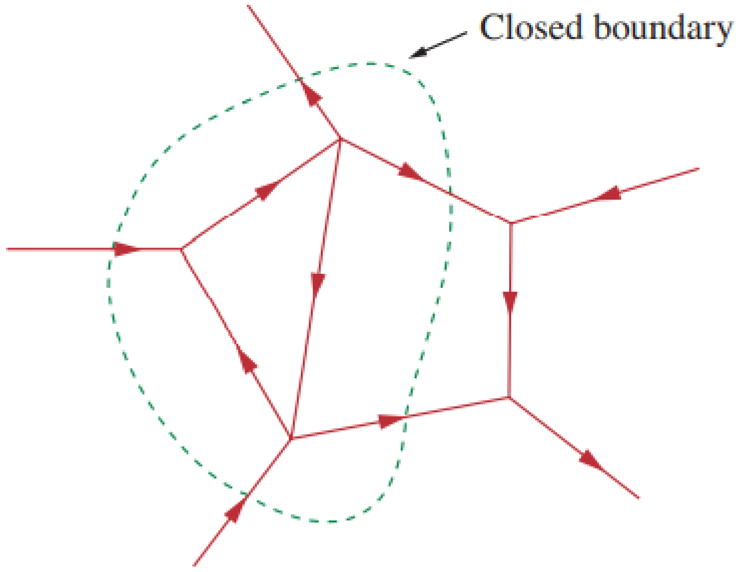
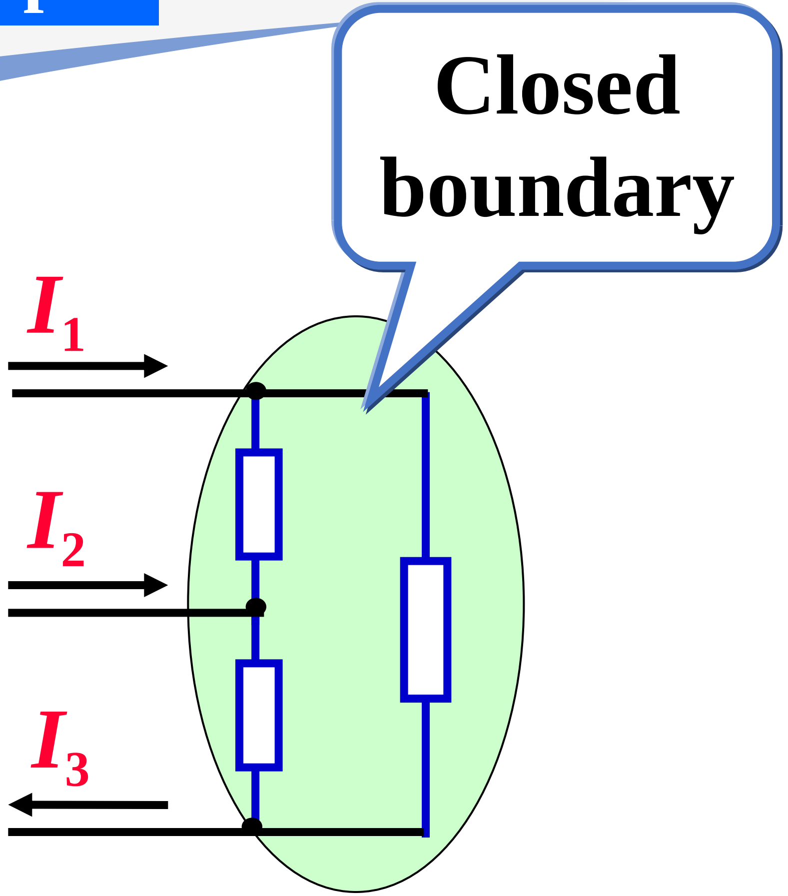


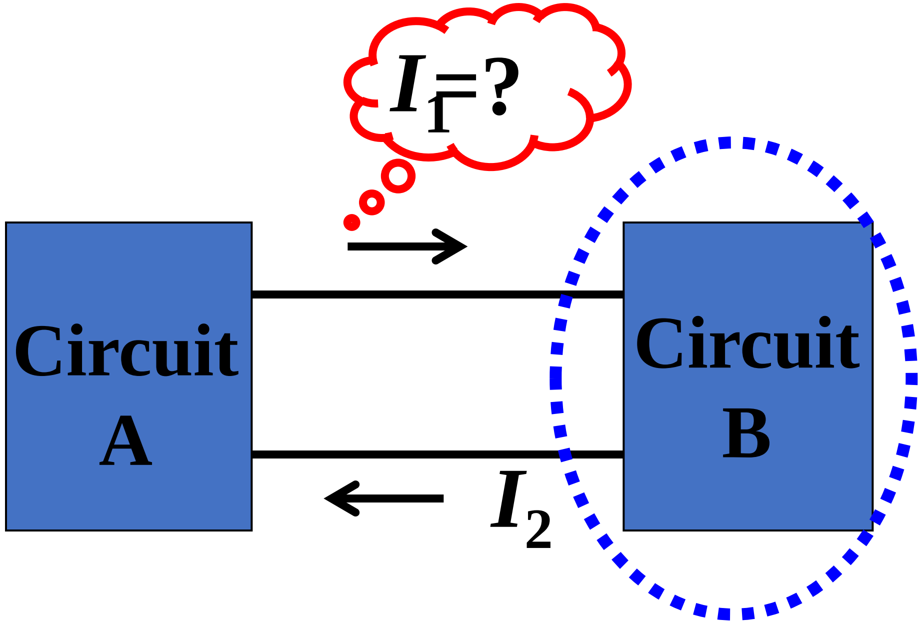
Figure 2.17
Applying KCL to a closed boundary.

The total current entering the closed surface is equal to the total current leaving the surface.

Example

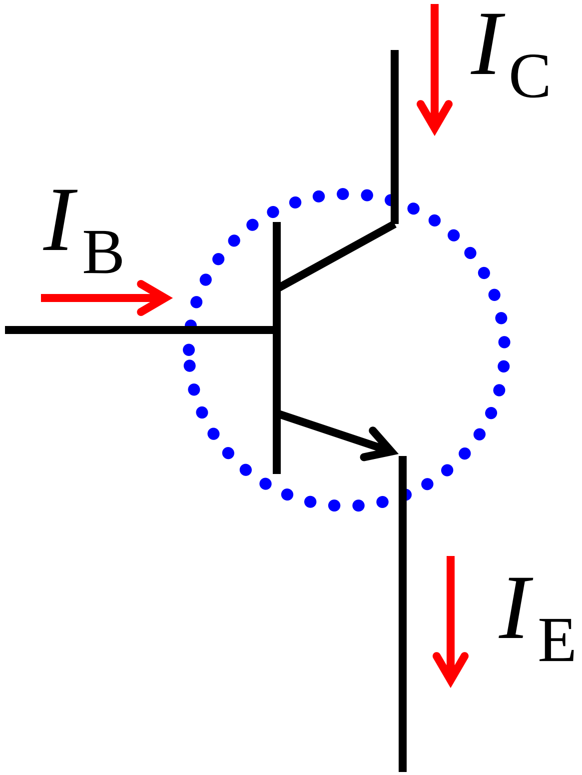


$I_1+I_2=I_3$



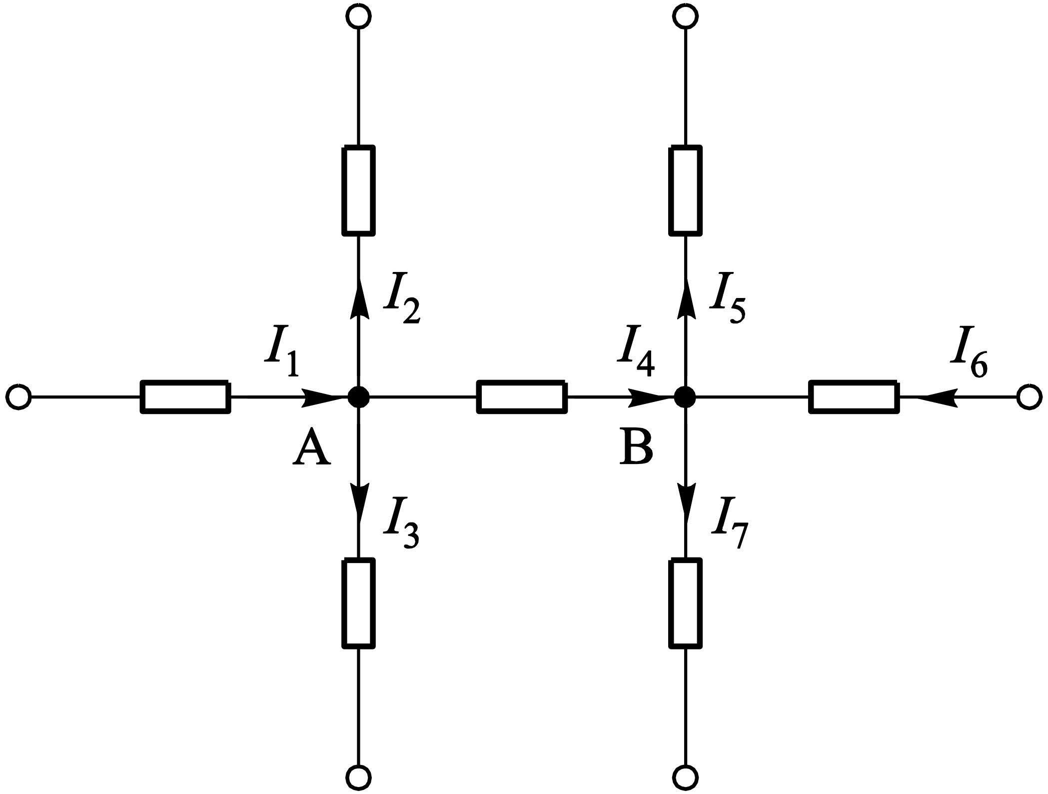
$I_1 = I_2$

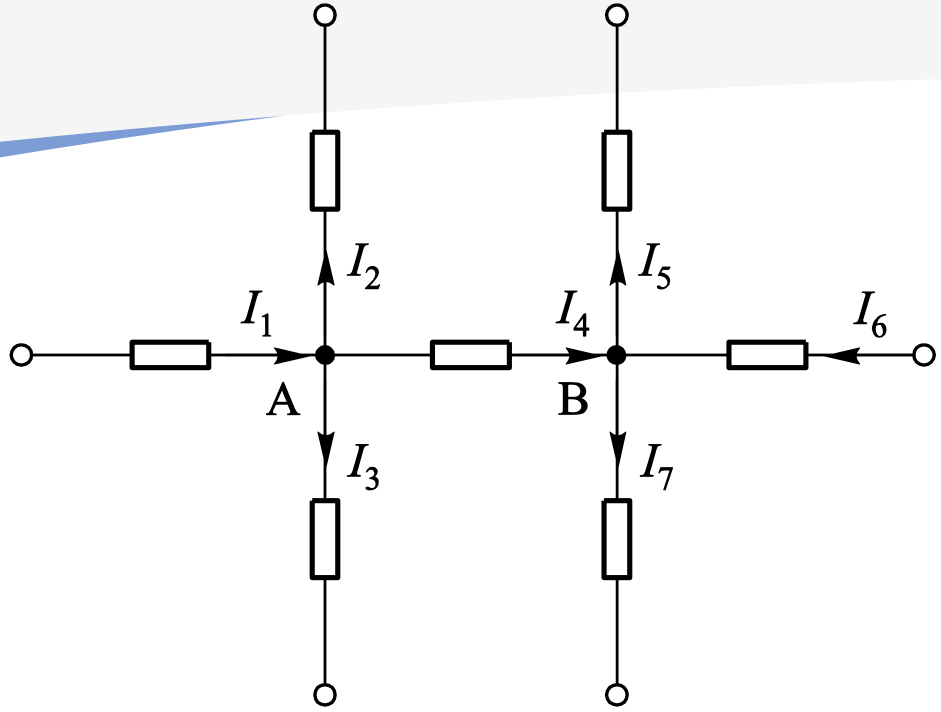
Example **Transistor**



$$\boxed{I_B + I_C = I_E}$$

Example **As shown in the figure，** I_1**=5A,** I_2**= -1A,** I_3**=2A,** I_5**=3A,** I_6**=-5A, find** I_4 **and** I_7**.**





Find I_4 and I_7

$$I_1 = 5\text{A}\,, I_2 = -1\text{A}\,,$$
$$I_3 = 2\text{A}\,, I_5 = 3\text{A}\,,$$
$$I_6 = -5\text{A}$$

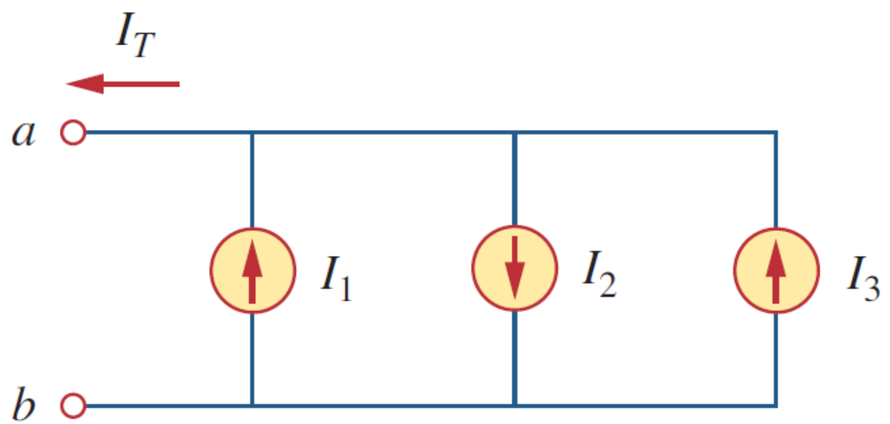
【Solution】 Applying KCL to node A

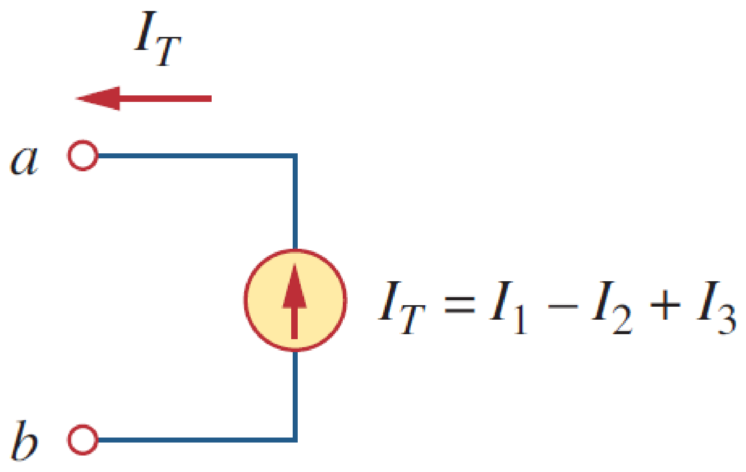
$$I_4 = I_1 - I_2 - I_3 = 5 - (-1) - 2 = 4\text{A}$$

Applying KCL to node B

$$I_7 = I_4 + I_6 - I_5 = 4 + (-5) - 3 = -4\text{A}$$

Example: Current sources in parallel





Applying KCL to node a:

$$I_T + I_2 = I_1 + I_3$$

The combined current is the algebraic sum of the current supplied by the individual sources.

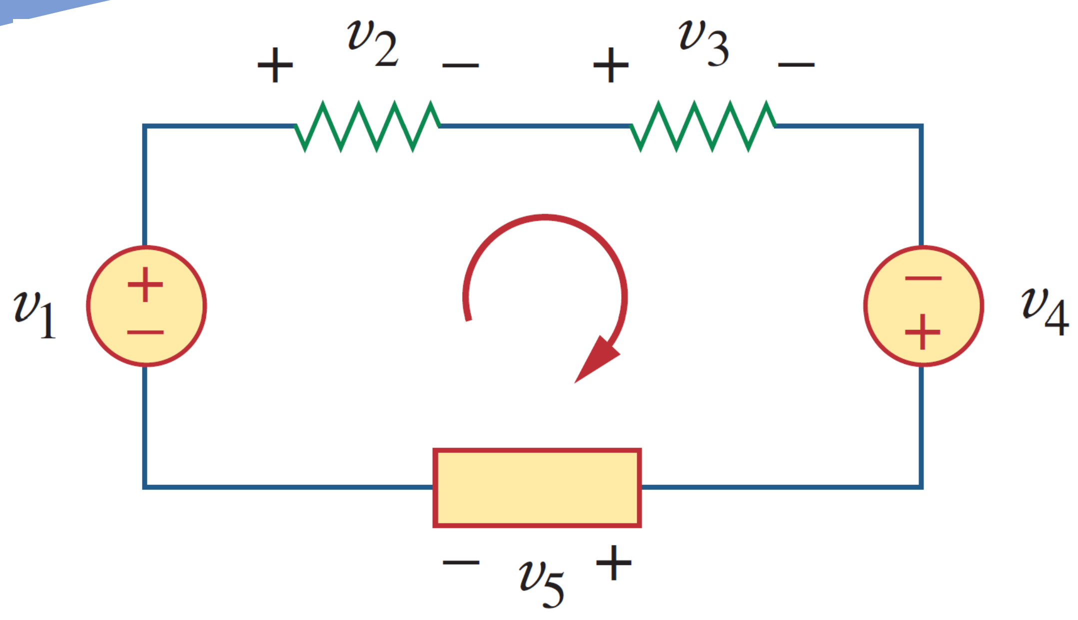
Kirchhoff's Voltage Law (KVL)

- Kirchhoff's voltage law is based on the conservation of energy.
- **Definition 1**: By taking either a clockwise or a counter-clockwise travel around the closed path (or loop), the algebraic sum of all voltages around a loop is zero.
- Voltage drop may be regarded as **positive**, while voltage rise may be taken as **negative**
- It can be expressed as:

$$\sum_{m=1}^{M} v_m = 0$$

where M is the number of voltages in the loop (or the number of branches in the loop) and v_m is the m_{th} voltage

Kirchhoff's Voltage Law (KVL)



Definition 1: The algebraic sum of all voltages around a loop is zero, voltage drop **positive**, voltage rise **negative**

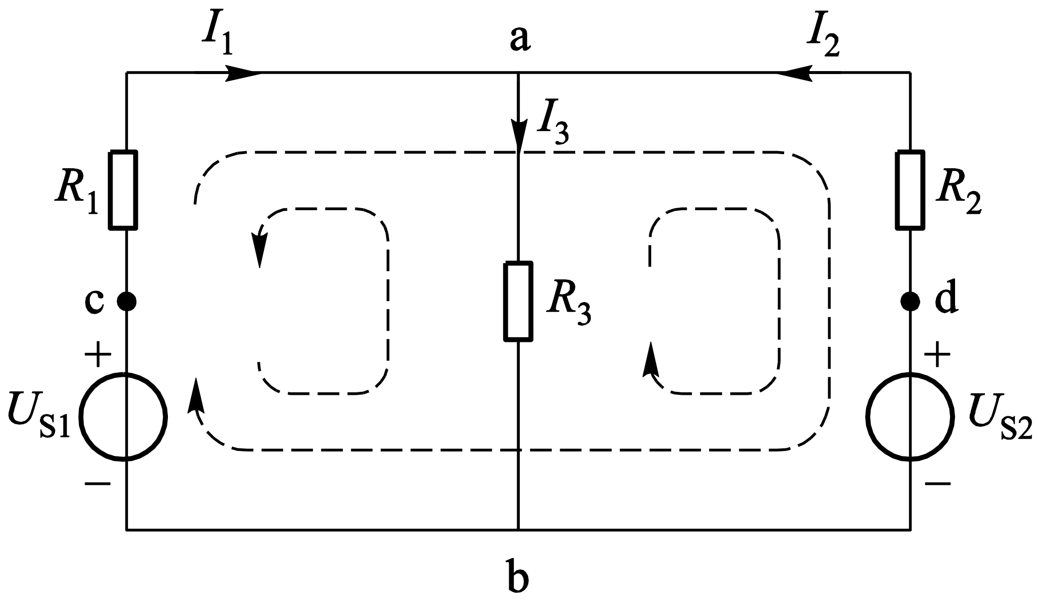
$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

Definition 2:
Sum of voltage drops=sum of voltage rises

$$v_2 + v_3 + v_5 = v_1 + v_4$$

Example:

Definition 1: The algebraic sum of all voltages around a loop is zero, voltage drop **positive**, voltage rise **negative**

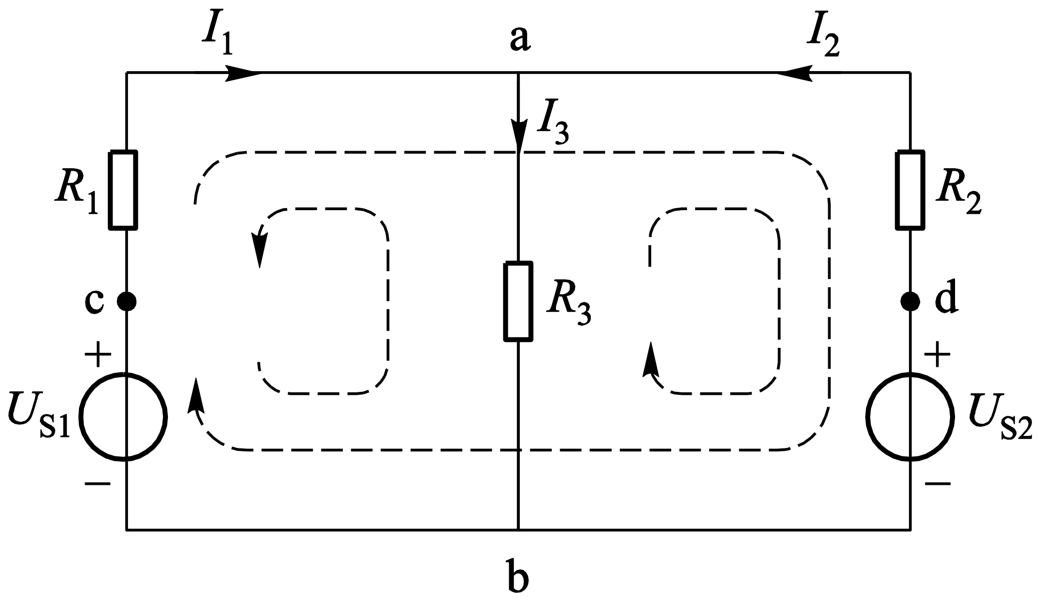


Applying KVL around loop "adbca":

$$-I_2R_2 + U_{S2} - U_{S1} + I_1R_1 = 0$$

Example:

Definition 1: The algebraic sum of all voltages around a loop is zero, voltage drop **positive**, voltage rise **negative**



A Better way to memorize!

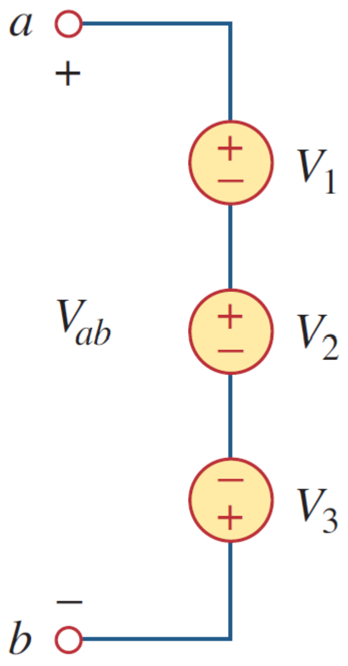
$$-I_2R_2 + U_{S2} - U_{S1} + I_1R_1 = 0$$

The sign of voltage depends on:

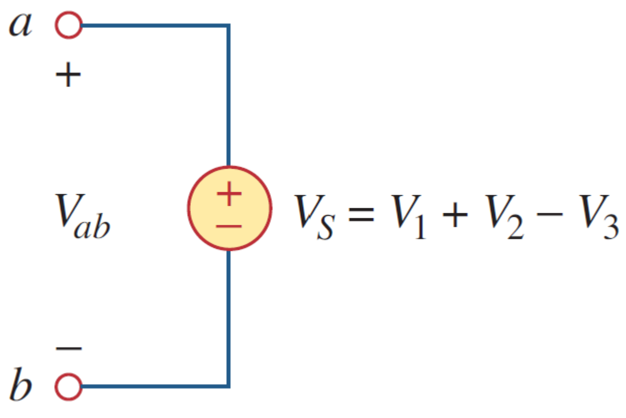
If the voltage or current reference is consistent with the travel direction, positive

If the voltage or current reference is not consistent with the travel direction, negative

Example: Voltage sources in series



$$V_{ab} = V_1 + V_2 - V_3$$

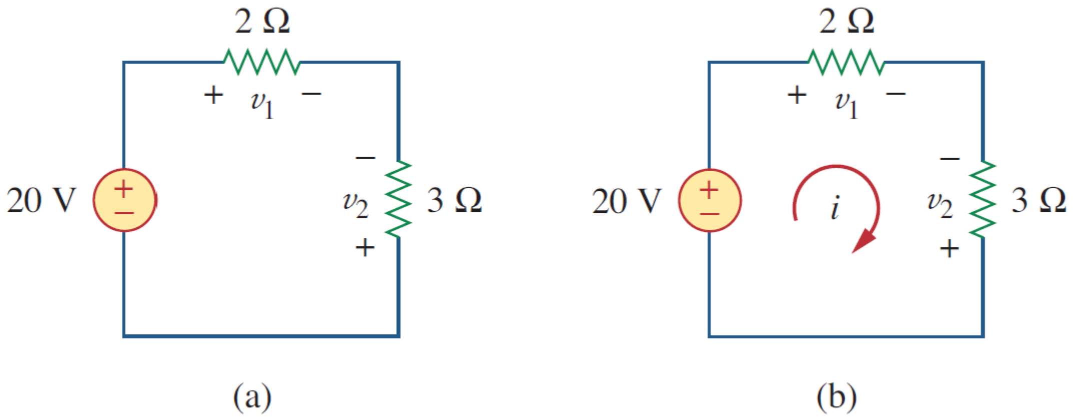


The combined voltage is the algebraic sum of the voltage of the individual sources.

Example 2.5

Example

For the circuit in Fig. 2.21(a), find voltages v_1 and v_2.



Solution:

To find v_1 and v_2, we apply Ohm's law and Kirchhoff's voltage law. Assume that current i flows through the loop as shown in Fig. 2.21(b). From Ohm's law,

$$v_1 = 2i, \qquad v_2 = -3i \tag{2.5.1}$$

Applying KVL around the loop gives

$$-20 + v_1 - v_2 = 0 \tag{2.5.2}$$

Substituting Eq. (2.5.1) into Eq. (2.5.2), we obtain

$$-20 + 2i + 3i = 0 \qquad \text{or} \qquad 5i = 20 \qquad \Rightarrow \qquad i = 4 \text{ A}$$

Substituting i in Eq. (2.5.1) finally gives

$$v_1 = 8 \text{ V}, \qquad v_2 = -12 \text{ V}$$

Example

Find v_1 and v_2 in the circuit of Fig. 2.22.

Practice Problem 2.5

Answer: 16 V, −8 V.

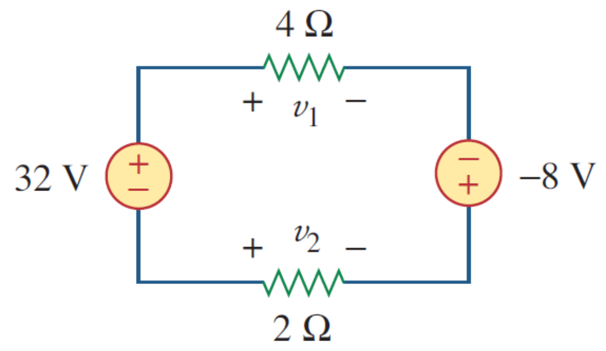


Figure 2.22
For Practice Prob. 2.5.

Example

Find v_x and v_o in the circuit of Fig. 2.24.

Practice Problem 2.6

Answer: 20 V, −10 V.

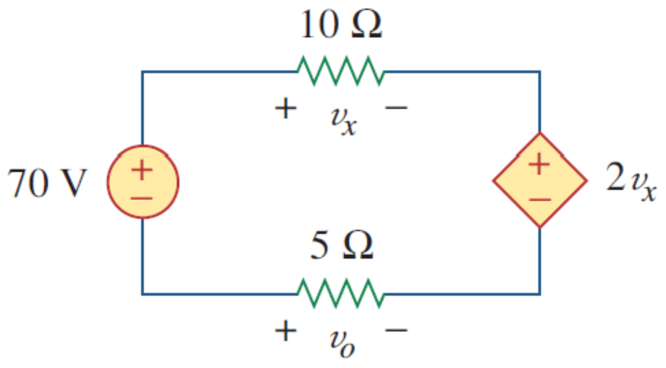


Figure 2.24
For Practice Prob. 2.6.

Example

Example 2.7 Find current i_o and voltage v_o in the circuit shown in Fig. 2.25.

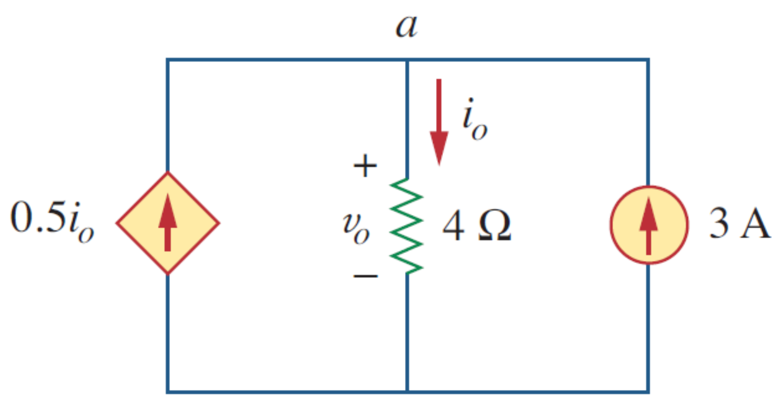


Figure 2.25
For Example 2.7.

Solution:
Applying KCL to node a, we obtain

$$3 + 0.5i_o = i_o \qquad \Rightarrow \qquad i_o = 6\text{ A}$$

For the 4-Ω resistor, Ohm's law gives

$$v_o = 4i_o = 24\text{ V}$$

Example

Practice Problem 2.7 Find v_o and i_o in the circuit of Fig. 2.26.

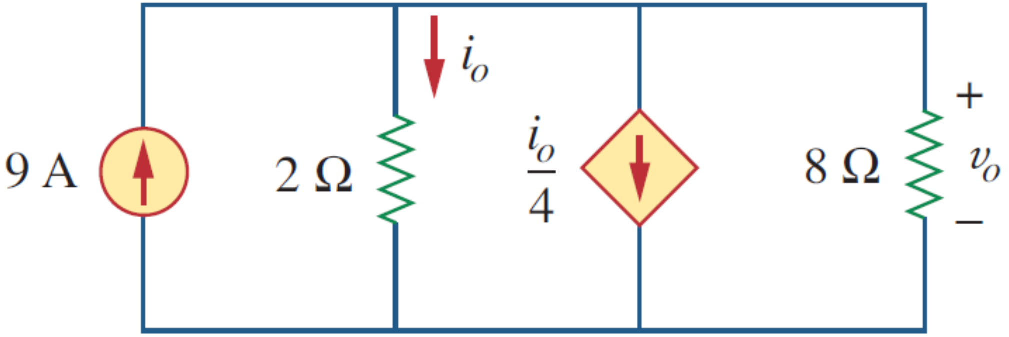
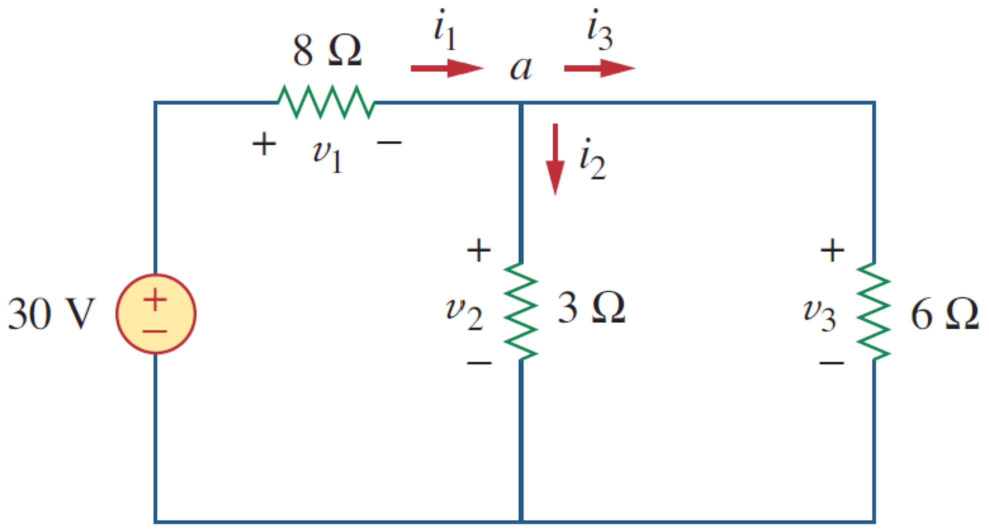


Figure 2.26
For Practice Prob. 2.7.

Answer: 12 V, 6 A.

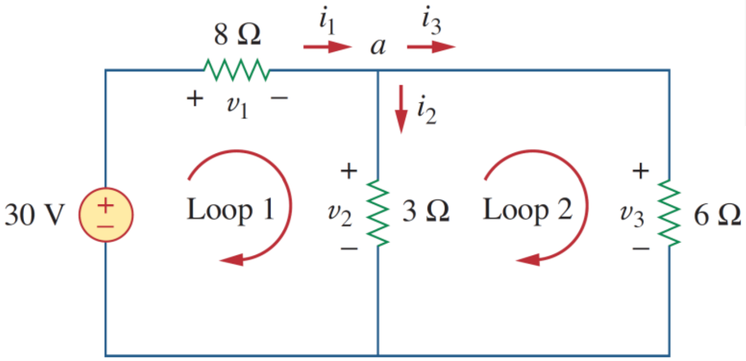
Example

Example 2.8 Find currents and voltages in the circuit shown in Fig. 2.27(a).



(a)

Figure 2.27
For Example 2.8.



Solution:

We apply Ohm's law and Kirchhoff's laws. By Ohm's law,

$$v_1 = 8i_1, \qquad v_2 = 3i_2, \qquad v_3 = 6i_3 \tag{2.8.1}$$

Since the voltage and current of each resistor are related by Ohm's law as shown, we are really looking for three things: (v_1, v_2, v_3) or (i_1, i_2, i_3). At node a, KCL gives

$$i_1 - i_2 - i_3 = 0 \tag{2.8.2}$$

Applying KVL to loop 1 as in Fig. 2.27(b),

$$-30 + v_1 + v_2 = 0$$

We express this in terms of i_1 and i_2 as in Eq. (2.8.1) to obtain

$$-30 + 8i_1 + 3i_2 = 0$$

or

$$i_1 = \frac{(30 - 3i_2)}{8} \tag{2.8.3}$$

Applying KVL to loop 2,

$$-v_2 + v_3 = 0 \quad \Rightarrow \quad v_3 = v_2 \tag{2.8.4}$$

as expected since the two resistors are in parallel. We express v_1 and v_2 in terms of i_1 and i_2 as in Eq. (2.8.1). Equation (2.8.4) becomes

$$6i_3 = 3i_2 \quad \Rightarrow \quad i_3 = \frac{i_2}{2} \tag{2.8.5}$$

Substituting Eqs. (2.8.3) and (2.8.5) into (2.8.2) gives

$$\frac{30 - 3i_2}{8} - i_2 - \frac{i_2}{2} = 0$$

or $i_2 = 2$ A. From the value of i_2, we now use Eqs. (2.8.1) to (2.8.5) to obtain

$$i_1 = 3 \text{ A}, \qquad i_3 = 1 \text{ A}, \qquad v_1 = 24 \text{ V}, \qquad v_2 = 6 \text{ V}, \qquad v_3 = 6 \text{ V}$$

Example

Find the currents and voltages in the circuit shown in Fig. 2.28.

Practice Problem 2.8

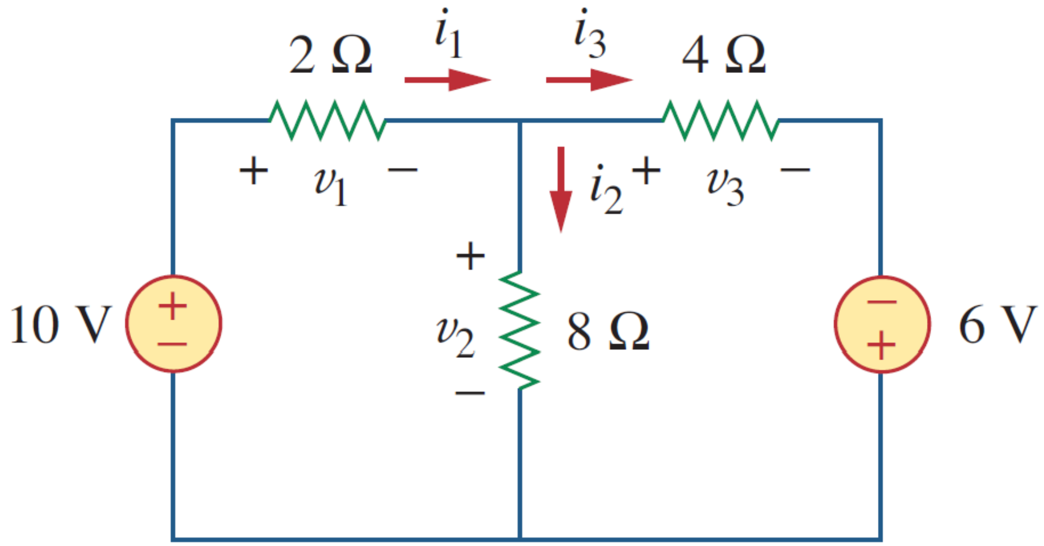
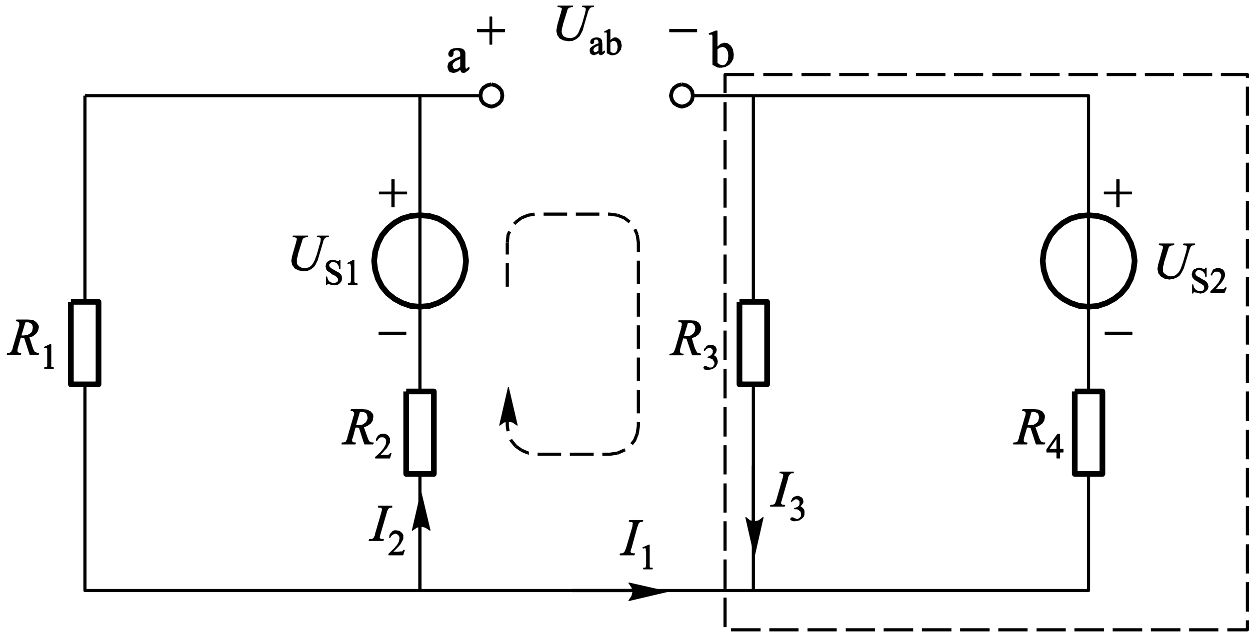


Figure 2.28
For Practice Prob. 2.8.

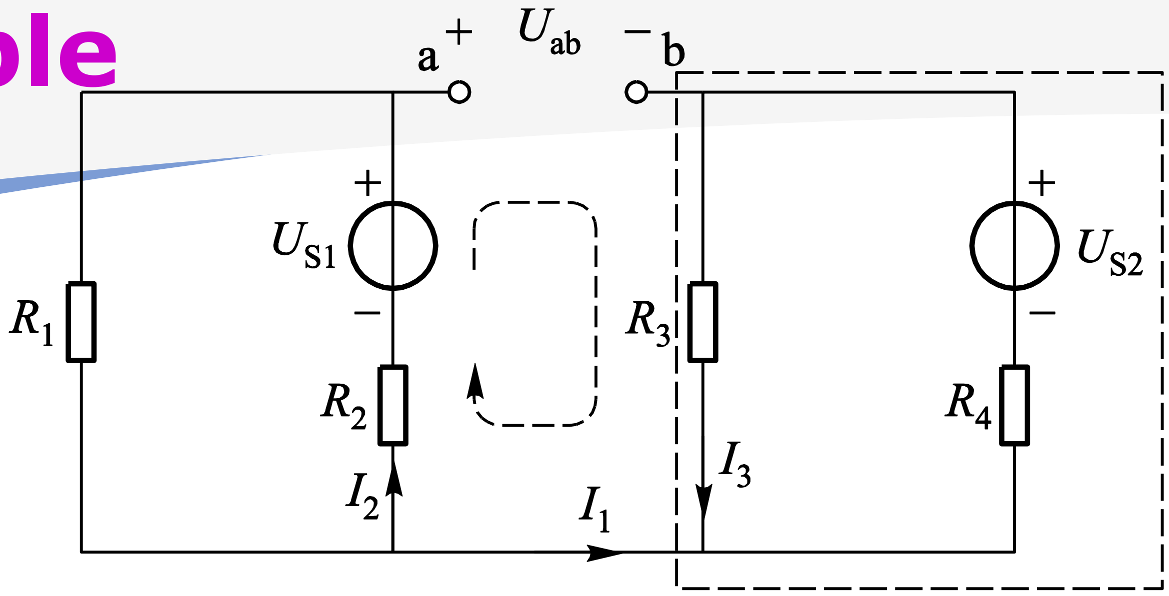
Answer: $v_1 = 6$ V, $v_2 = 4$ V, $v_3 = 10$ V, $i_1 = 3$ A, $i_2 = 500$ mA, $i_3 = 2.5$A

Example

As shown in the figure, Us1=12V, Us2=5V, R_1=12Ω, R_2=18 Ω, R_3=10 Ω, R_4=15 Ω, find the open-circuit voltage U_{ab}



Example



$U_{S1} = 12\text{V}, U_{S2} = 5\text{V},$
$R_1 = 12\Omega, R_2 = 18\Omega,$
$R_3 = 10\Omega, R_4 = 15\Omega$

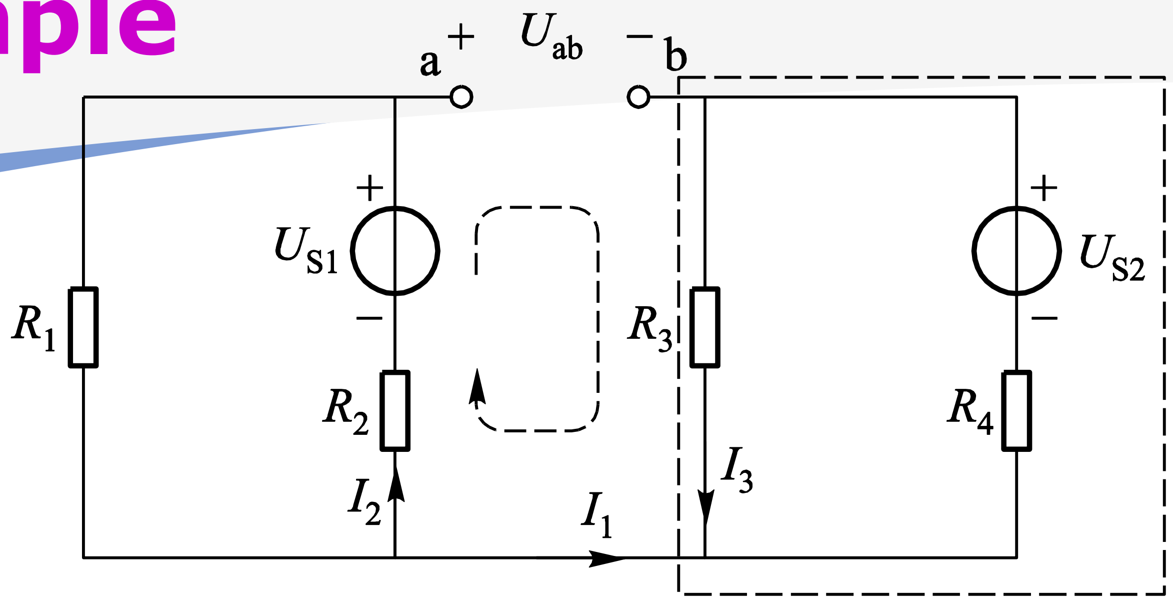
【Solution】According to KCL:

$$I_1 = 0\text{A}$$

According to Ohm's law and KVL：

$$I_2 = \frac{U_{S1}}{R_1 + R_2} = \frac{12}{12+18} = 0.4\text{A} \qquad I_3 = \frac{U_{S2}}{R_3 + R_4} = \frac{5}{10+15} = 0.2\text{A}$$

Example



$U_{S1} = 12\text{V}, U_{S2} = 5\text{V},$

$R_1 = 12\Omega, R_2 = 18\Omega,$

$R_3 = 10\Omega, R_4 = 15\Omega$

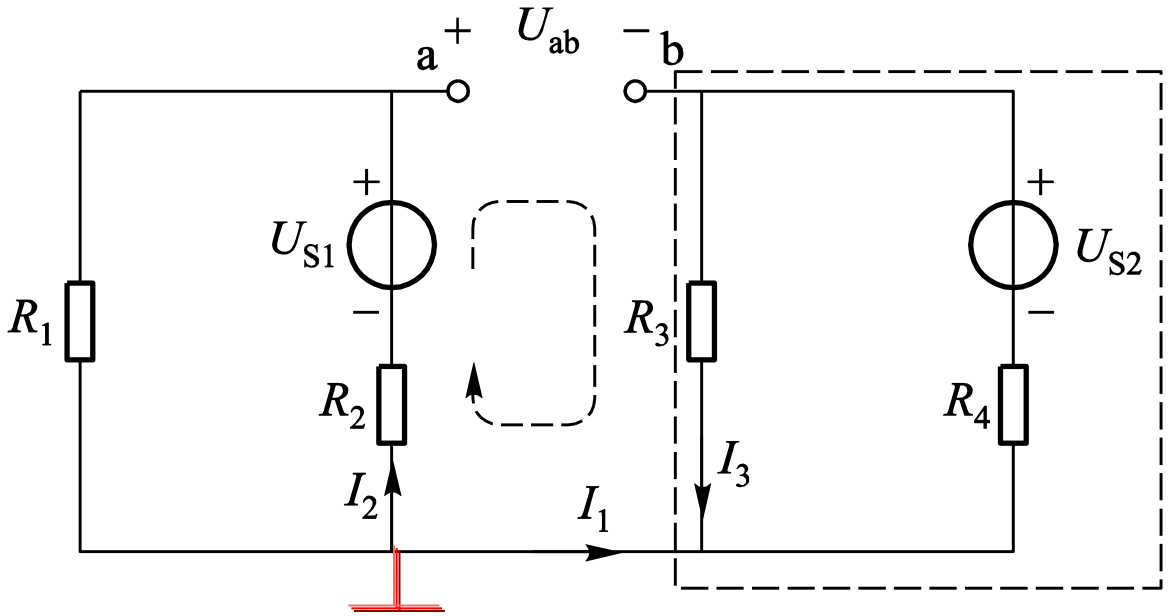
Applying KVL to the middle loop:

$$U_{ab} + I_3R_3 + I_2R_2 - U_{S1} = 0$$

$$U_{ab} = U_{S1} - I_3R_3 - I_3R_3 = 12 - 0.2\times10 - 0.4\times18 = 2.8\text{V}$$

Example

Is there any other method to find U_{ab} ?



$$V_a = I_2 R_1 = 0.4 \times 12 = 4.8\text{V}$$

$$V_b = I_3 R_3 = 0.2 \times 10 = 2\text{V}$$

$$U_{ab} = V_a - V_b = 4.8 - 2 = 2.8\text{V}$$

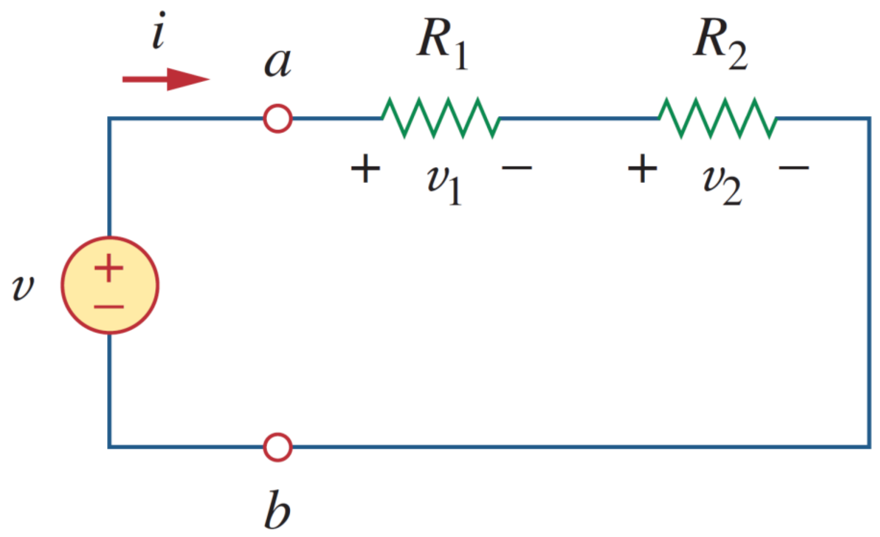
2.5 Series Resistors and Voltage division

- Two resistors are considered in series *if the same current flows through them*
- Apply Ohm's law to both resistors

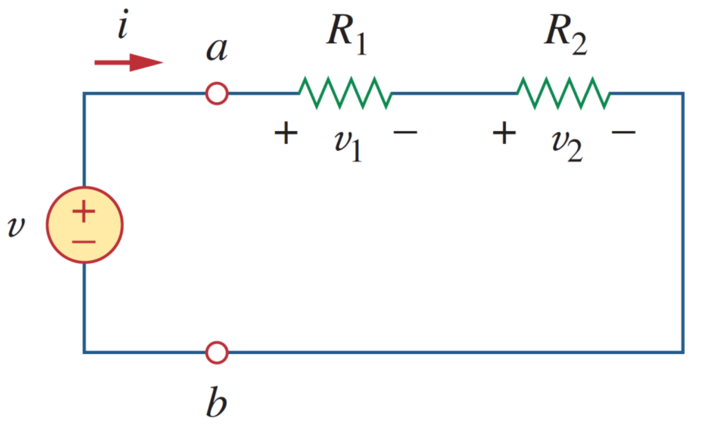
$$v_1 = iR_1 \qquad v_2 = iR_2$$

- Apply KVL to the loop

$$-v + v_1 + v_2 = 0$$



Series Resistors



- Combining the two equations:

$$v = v_1 + v_2 = i(R_1 + R_2)$$

- The two resistors can be replaced by an equivalent resistor:

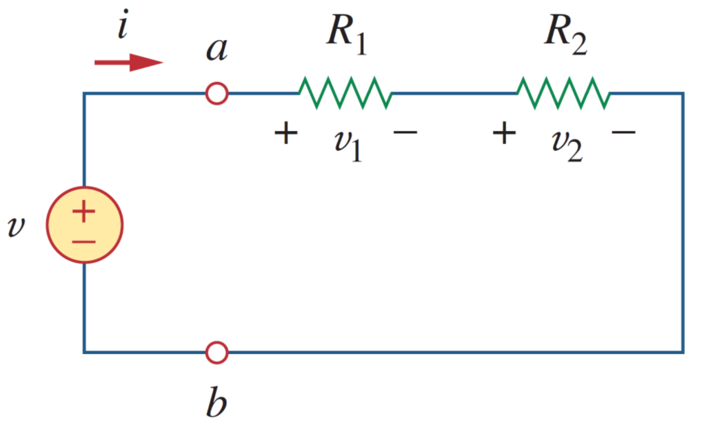
$$R_{eq} = R_1 + R_2$$

- For *N* resistors in series:

$$R_{eq} = R_1 + R_2 + \cdots + RN = \sum_{n=1}^{N} R_n$$

- Generally, the equivalent resistance of series-connected resistors is the sum of the individual resistances.

Voltage Division



$$v = v_1 + v_2 = i(R_1 + R_2)$$

- The voltage drop across any one resistor can be known.
- The current through all the resistors is the same, so using Ohm's law:

$$v_1 = \frac{R_1}{R_1+R_2} v \qquad v_2 = \frac{R_2}{R_1+R_2} v$$

- ***The principle of voltage division***:

  The voltages divided among the resistors are proportional to their resistances

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v$$

2.6 Parallel Resistors and Current Division

- When resistors are in parallel, the voltage drop across them is the same

$$v = i_1 R_1 = i_2 R_2$$

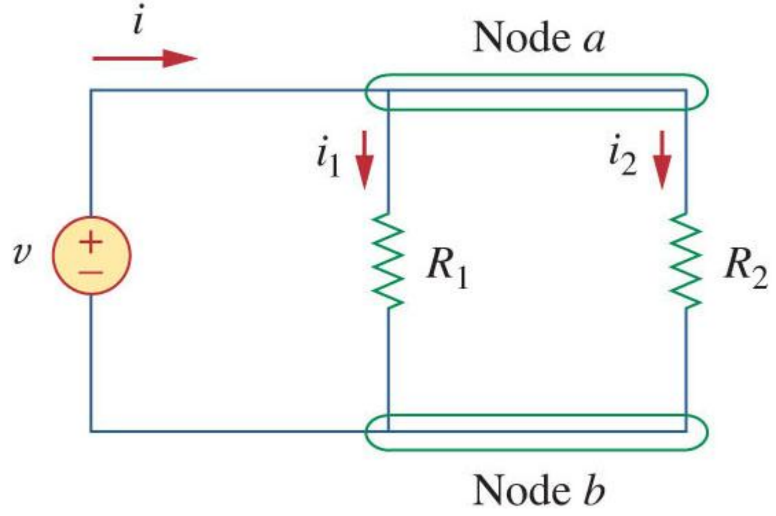
- By KCL, the current at node *a* is

$$i = i_1 + i_2$$

- The equivalent resistance is:

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

- Generally, the equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum.



2.6 Parallel Resistors and Current Division

When N resistors in parallel, the equivalent resistance is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

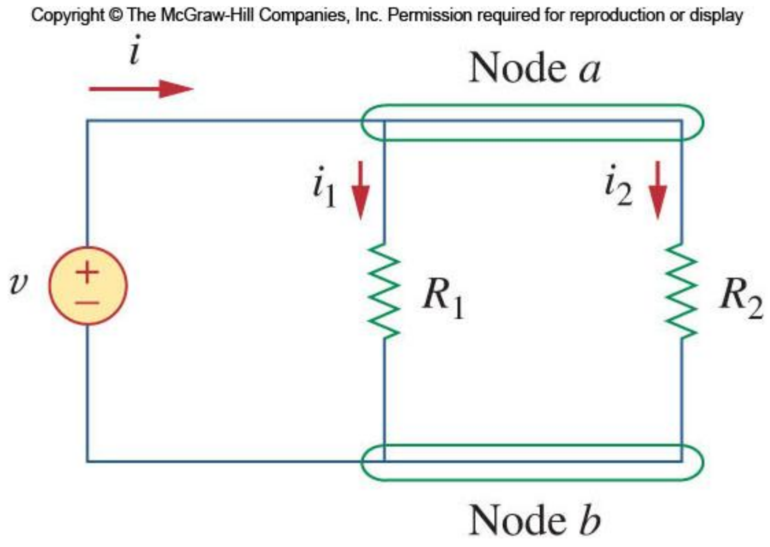
It is more often described by conductance

$$G_{eq} = G_1 + G_2 + G_3 + \cdots + G_N$$

The equivalent conductance of resistors connected in parallel is the sum of their individual conductances

Current Division

- Given the current *i* entering the node, the voltage drop across the equivalent resistance will be the same as that for the individual resistors



$$v = iR_{eq} = \frac{iR_1R_2}{R_1+R_2}$$

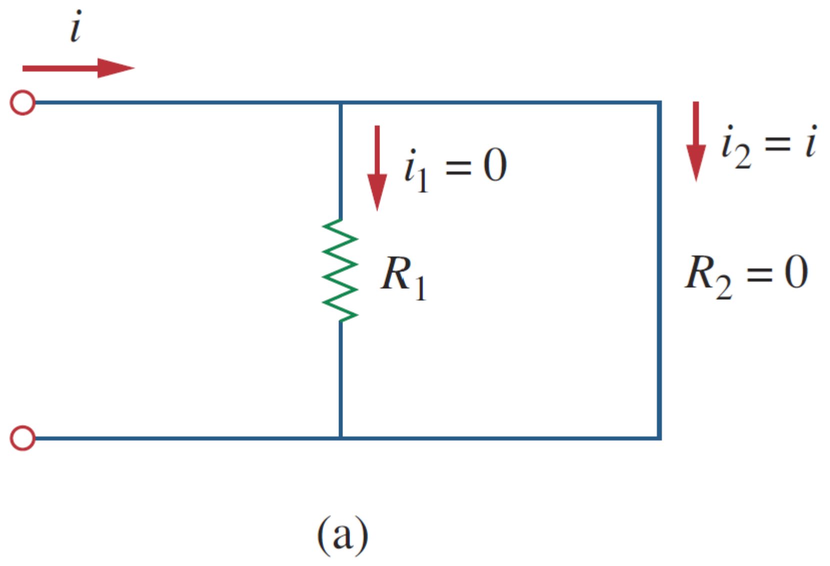
- The current through each resistor:

$$i_1 = \frac{R_2}{R_1+R_2}i \qquad i_2 = \frac{R_1}{R_1+R_2}i$$

- ***The principle of current division***:

  The total current is shared by the resistors in inverse proportional to their resistances

Current Division (extreme case)



(a)

Short circuit

If R_2 is a short circuit

$$i_1 = \frac{R_2}{R_1 + R_2} i \qquad i_2 = \frac{R_1}{R_1 + R_2} i$$

$i_1 = 0, \quad i_2 = i$

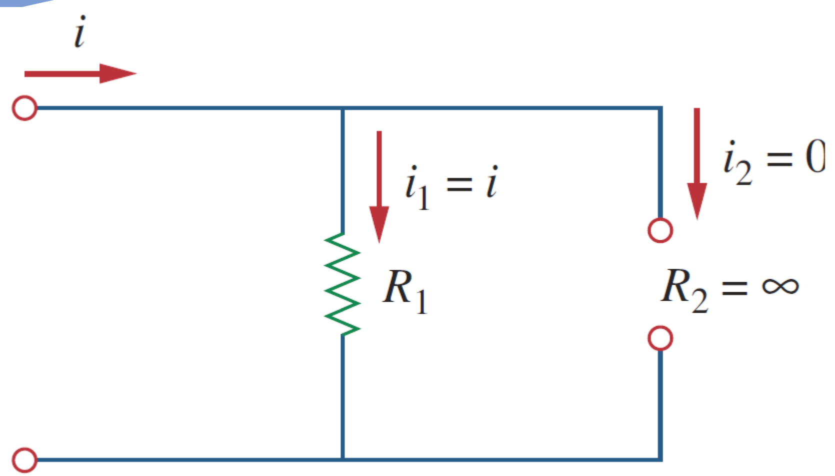
- **The entire current flows through the short circuit**

$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$

- $\boldsymbol{R_{eq} = 0}$

- **The equivalent resistance is 0**

Current Division (extreme case)



(b)

Open circuit

If R_2 is a open circuit

$$i_1 = \frac{R_2}{R_1+R_2} i \qquad i_2 = \frac{R_1}{R_1+R_2} i$$

$i_1 = i, \quad i_2 = 0$

- **The entire current flows through the R_1**

$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$

- $\boldsymbol{R_{eq} = R_1}$

- **The equivalent resistance is R_1**

Example: Equivalent resistance Derivation

Find R_{eq} for the circuit shown in Fig. 2.34.

Example 2.9

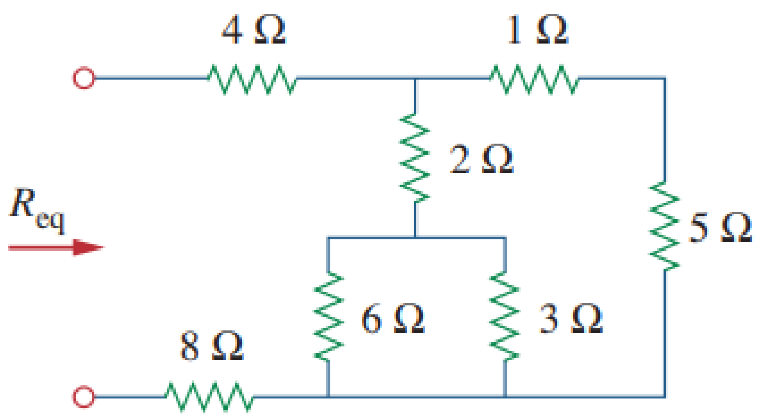


Figure 2.34
For Example 2.9.

Example

Practice Problem 2.9 By combining the resistors in Fig. 2.36, find R_{eq}.

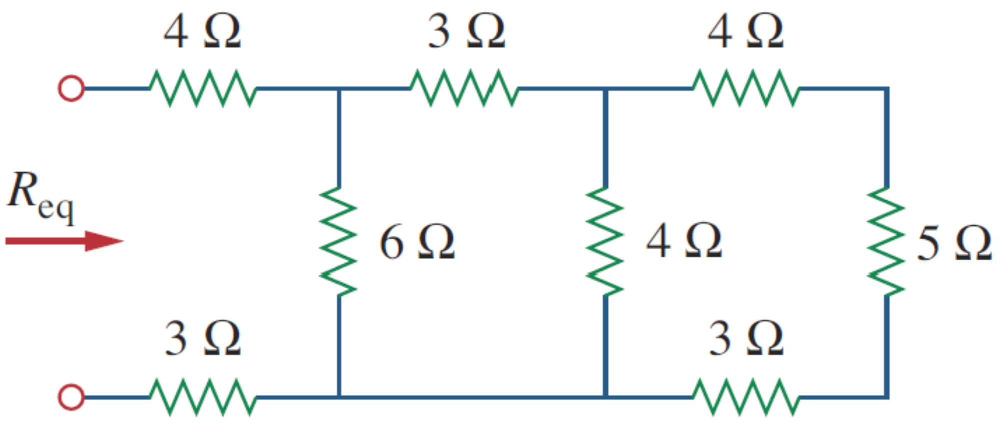


Figure 2.36
For Practice Prob. 2.9.

Answer: 10 Ω.

Example

Example 2.10 Calculate the equivalent resistance R_{ab} in the circuit in Fig. 2.37.

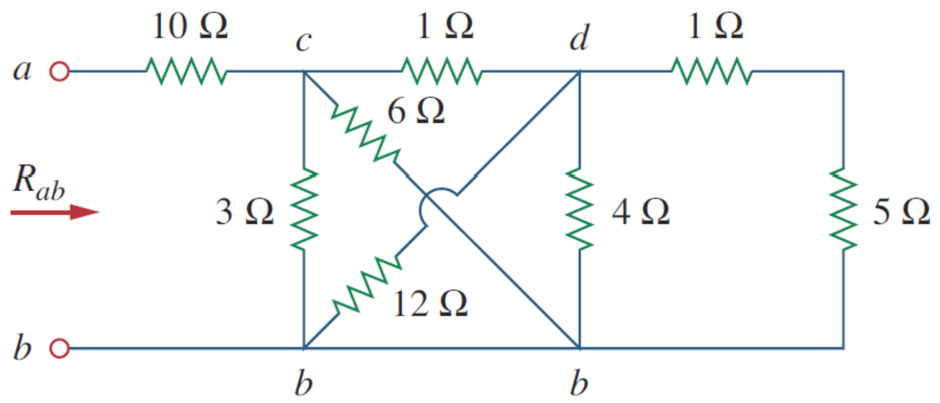


Figure 2.37
For Example 2.10.

Example

Find R_{ab} for the circuit in Fig. 2.39.

Practice Problem 2.10

Answer: 19 Ω.

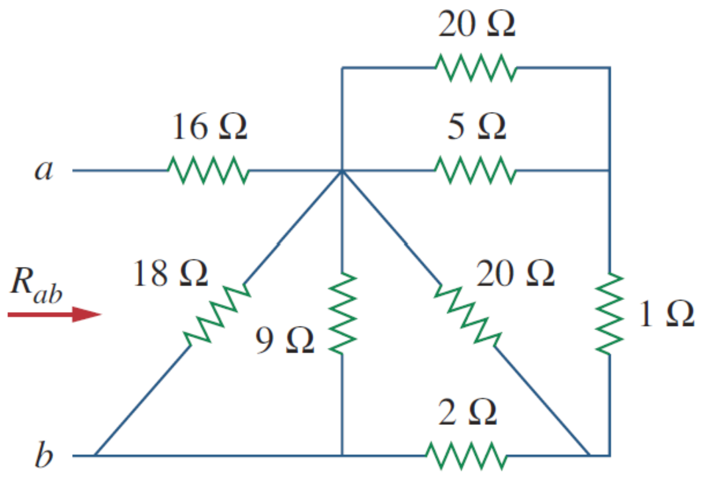
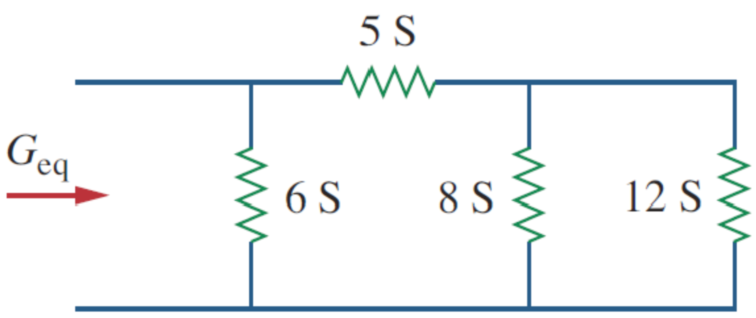


Figure 2.39
For Practice Prob. 2.10.

Example

Find the equivalent conductance G_{eq} for the circuit in Fig. 2.40(a).

Example 2.11

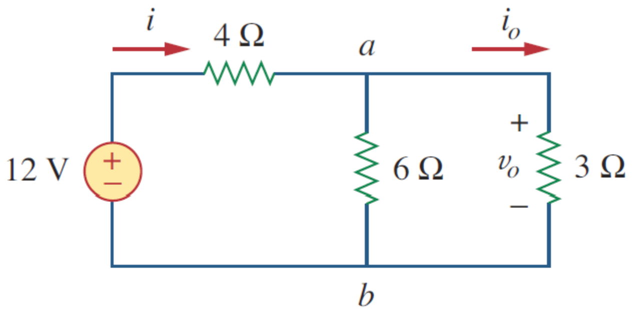


(a)

Example

Example 2.12

Find i_o and v_o in the circuit shown in Fig. 2.42(a). Calculate the power dissipated in the 3-Ω resistor.



(a)

Example

Find v_1 and v_2 in the circuit shown in Fig. 2.43. Also calculate i_1 and i_2 and the power dissipated in the 12-Ω and 40-Ω resistors.

Practice Problem 2.12

Answer: $v_1 = 10$ V, $i_1 = 833.3$ mA, $p_1 = 8.333$ W, $v_2 = 20$ V, $i_2 = 500$ mA, $p_2 = 10$ W.

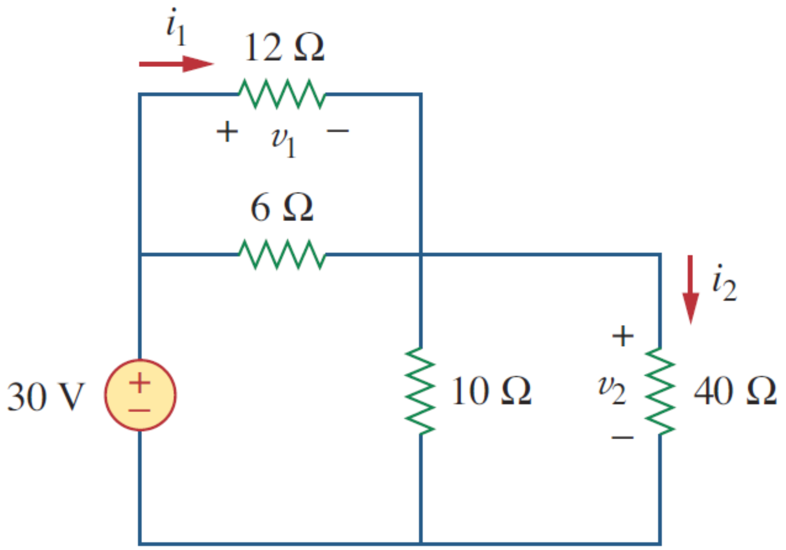
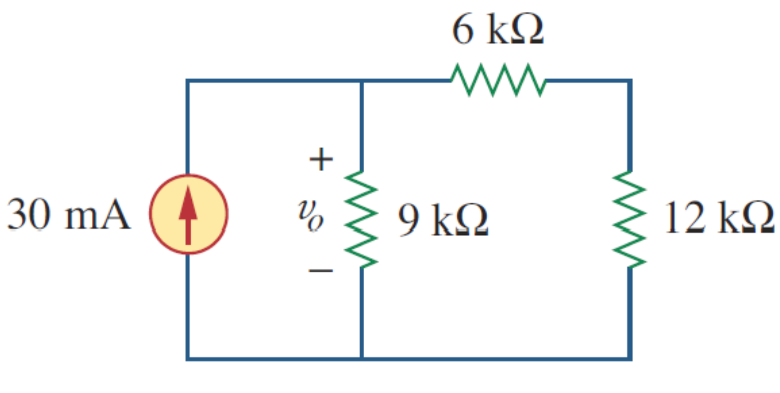


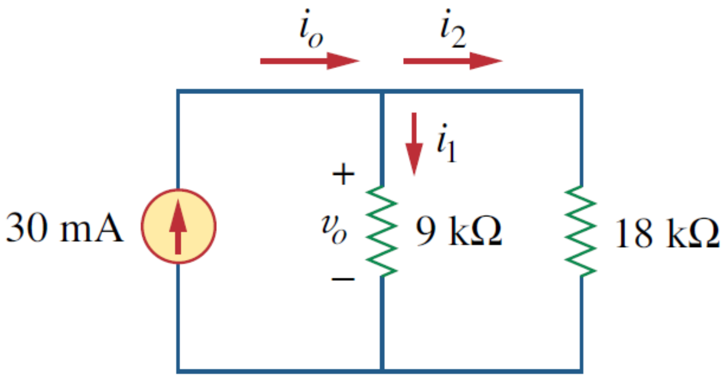
Figure 2.43
For Practice Prob. 2.12.

For the circuit shown in Fig. 2.44(a), determine: (a) the voltage v_o, (b) the power supplied by the current source, (c) the power absorbed by each resistor.

Example 2.13



(a)



(b)

Solution:

(a) The 6-kΩ and 12-kΩ resistors are in series so that their combined value is 6 + 12 = 18 kΩ. Thus the circuit in Fig. 2.44(a) reduces to that shown in Fig. 2.44(b). We now apply the current division technique to find i_1 and i_2.

$$i_1 = \frac{18{,}000}{9{,}000 + 18{,}000}(30 \text{ mA}) = 20 \text{ mA}$$

$$i_2 = \frac{9{,}000}{9{,}000 + 18{,}000}(30 \text{ mA}) = 10 \text{ mA}$$

Notice that the voltage across the 9-kΩ and 18-kΩ resistors is the same, and $v_o = 9{,}000 i_1 = 18{,}000 i_2 = 180$ V, as expected.

(b) Power supplied by the source is

$$p_o = -v_o i_o = -180(30) \text{ mW} = -5.4 \text{ W}$$

(c) Power absorbed by the 12-kΩ resistor is

$$p = iv = i_2(i_2 R) = i_2^2 R = (10 \times 10^{-3})^2 \, (12{,}000) = 1.2 \text{ W}$$

Power absorbed by the 6-kΩ resistor is

$$p = i_2^2 R = (10 \times 10^{-3})^2 \, (6{,}000) = 0.6 \text{ W}$$

Power absorbed by the 9-kΩ resistor is

$$p = \frac{v_o^2}{R} = \frac{(180)^2}{9{,}000} = 3.6 \text{ W}$$

or

$$p = v_o i_1 = 180(20) \text{ mW} = 3.6 \text{ W}$$

Notice that the power supplied (-5.4 W) equals the power absorbed (1.2 + 0.6 + 3.6 = 5.4 W). This is one way of checking results.

Example

Practice Problem 2.13

For the circuit shown in Fig. 2.45, find: (a) v_1 and v_2, (b) the power dissipated in the 3-kΩ and 20-kΩ resistors, and (c) the power supplied by the current source.

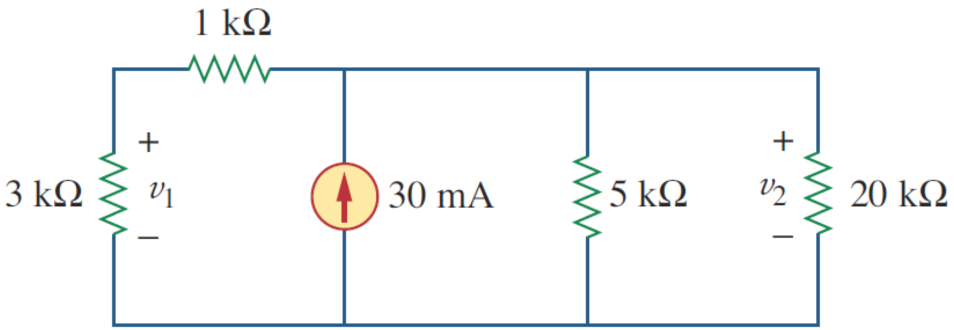
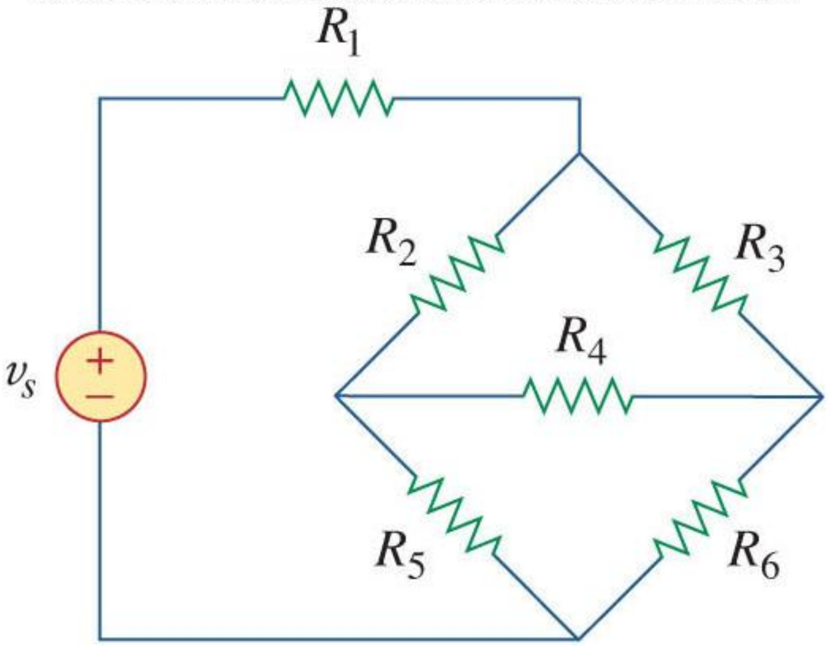


Figure 2.45
For Practice Prob. 2.13.

Answer: (a) 45 V, 60 V, (b) 675 mW, 180 mW, (c) -1.8 W.

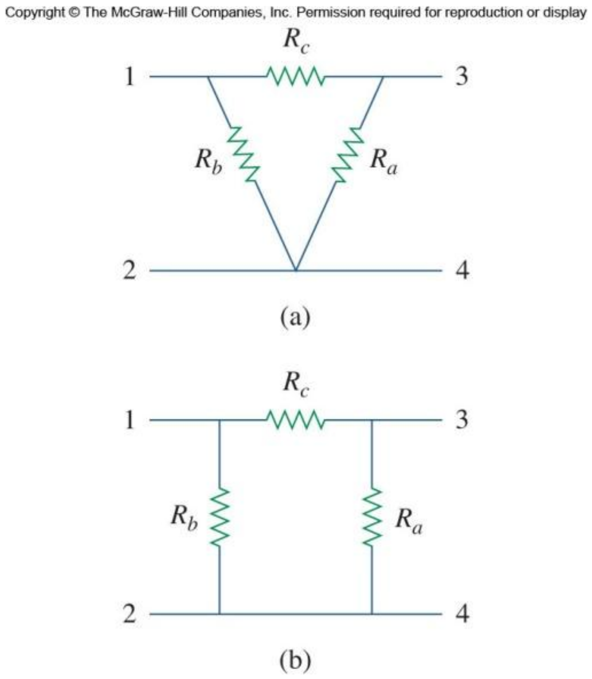
2.7 Wye-Delta Transformations

- There are cases where resistors are neither parallel nor series
- Consider the bridge circuit shown here
- This circuit can be simplified by using three-terminal equivalent networks

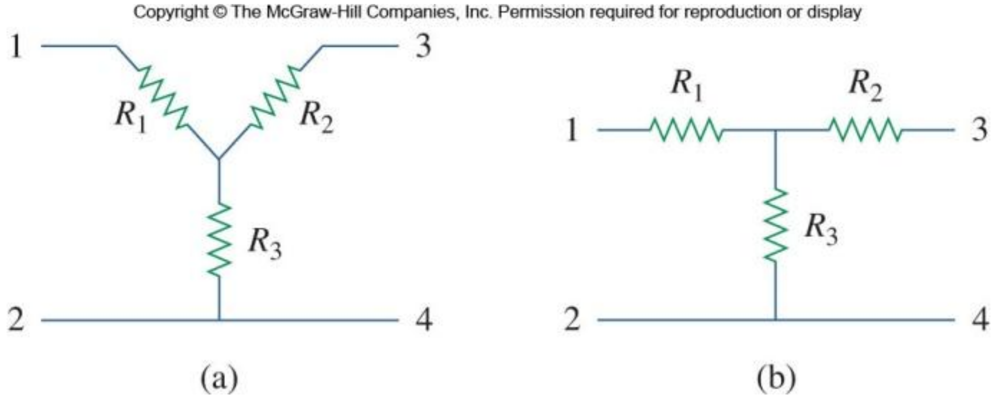


2.7 Wye-Delta Transformations

- Two types of three-terminal equivalent networks
- **Delta (Δ) or pi (Π) networks**

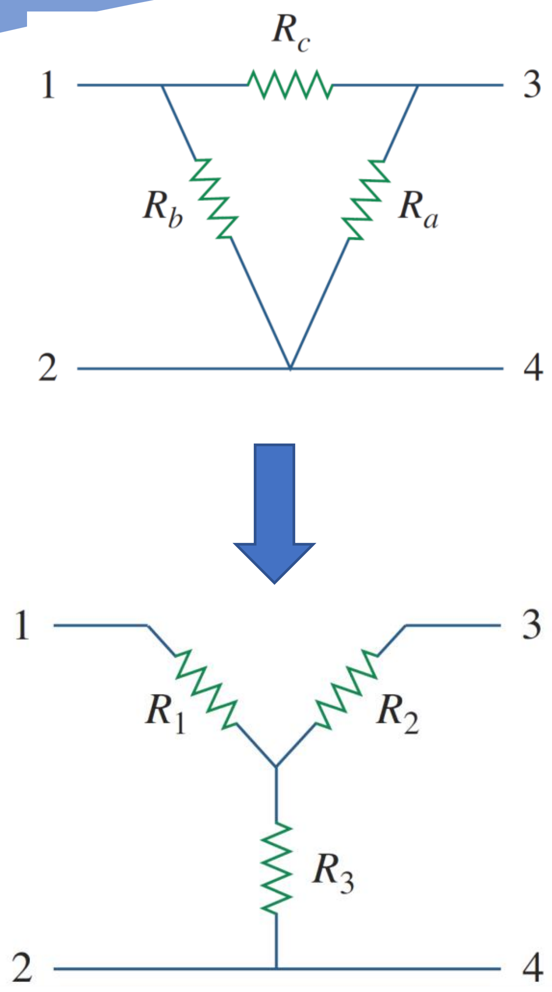


- **Wye (Y) or tee (T) networks**



Delta to Wye Conversion

The equivalent resistance between each pair of nodes in the Δ (or Π) network is the same as the resistance between the same nodes in the Y(or T) network



$$R_{12}(\mathrm{Y}) = R_1 + R_3 \tag{2.46}$$

$$R_{12}(\Delta) = R_b \parallel (R_a + R_c)$$

Setting $R_{12}(\mathrm{Y}) = R_{12}(\Delta)$ gives

$$R_{12} = R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} \tag{2.47a}$$

Similarly,

$$R_{13} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} \tag{2.47b}$$

$$R_{34} = R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \tag{2.47c}$$

Subtracting Eq. (2.47c) from Eq. (2.47a), we get

$$R_1 - R_2 = \frac{R_c(R_b - R_a)}{R_a + R_b + R_c} \tag{2.48}$$

Adding Eqs. (2.47b) and (2.48) gives

$$\boxed{R_1 = \frac{R_b R_c}{R_a + R_b + R_c}} \tag{2.49}$$

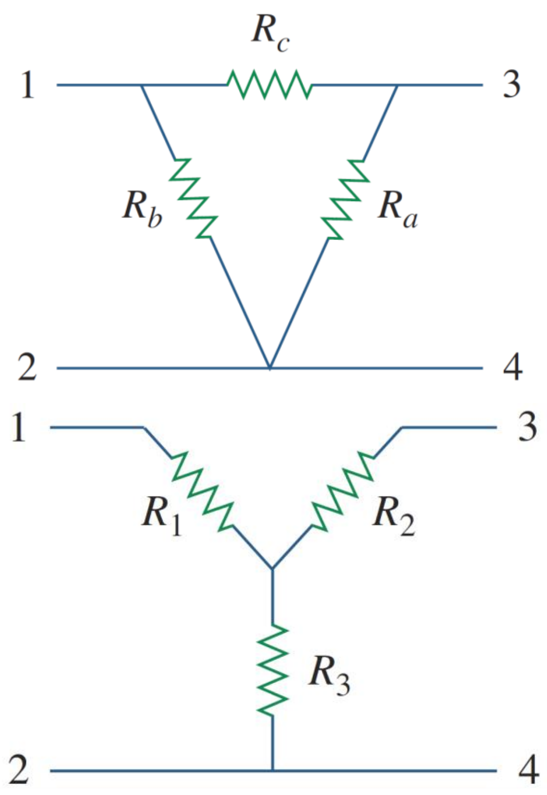
Delta to Wye Conversion

- The conversion formula from delta to wye transformation:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$



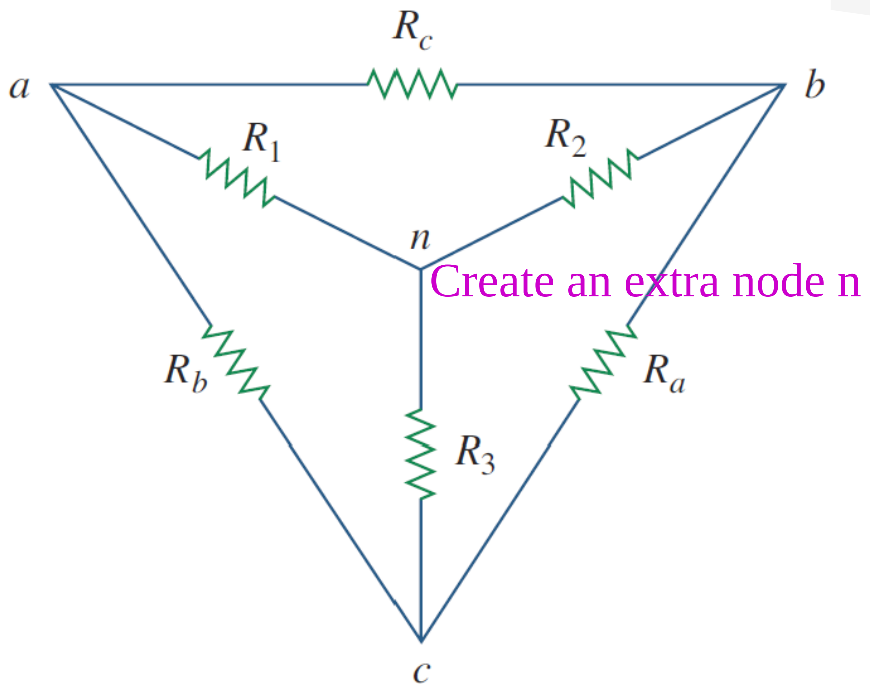


Figure 2.49
Superposition of Y and Δ networks as an aid in transforming one to the other.

The delta network consists of the outer resistors, labeled a, b, and c
The wye network consists of the inside resistors, labeled 1, 2, and 3

Conversion rule (from delta to wye):
Each resistor in the Y network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three Δ resistors.

Wye to Delta Conversion

The conversion formula for wye to delta transformation are:

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

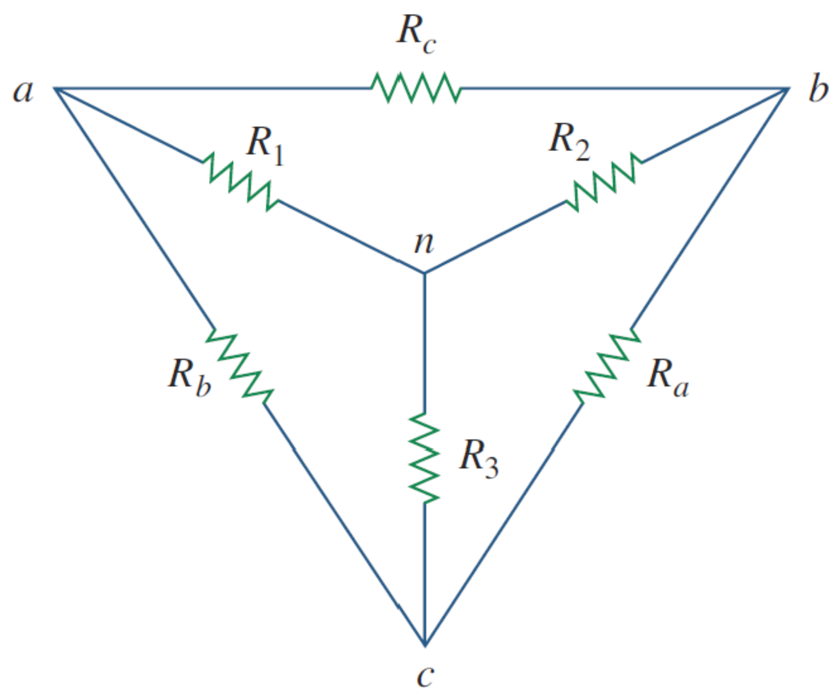


Figure 2.49
Superposition of Y and Δ networks as an aid in transforming one to the other.

Conversion rule (from wye to delta):
Each resistor in the Δ network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor.

Wye to Delta Conversion

The Y and Δ networks are said to be ***balanced*** when

$$R_1 = R_2 = R_3 = R_Y, \qquad R_a = R_b = R_c = R_\Delta$$

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \qquad R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} \qquad R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} \qquad R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

Under these conditions, conversion formulas become

$$\boxed{R_Y = \frac{R_\Delta}{3} \qquad \text{or} \qquad R_\Delta = 3R_Y}$$

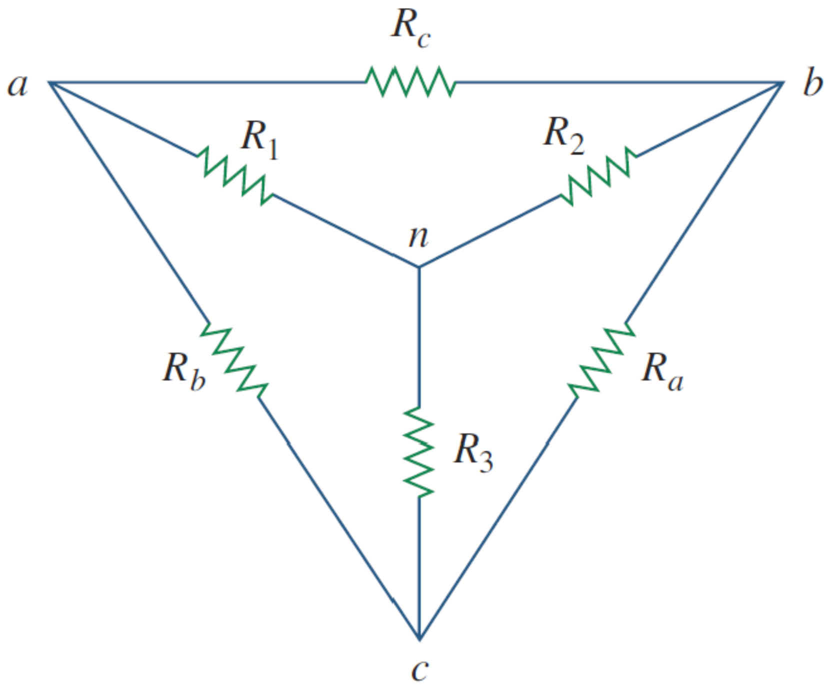


Figure 2.49
Superposition of Y and Δ networks as an aid in transforming one to the other.

Example

Convert the Δ network in Fig. 2.50(a) to an equivalent Y network.

Example 2.14

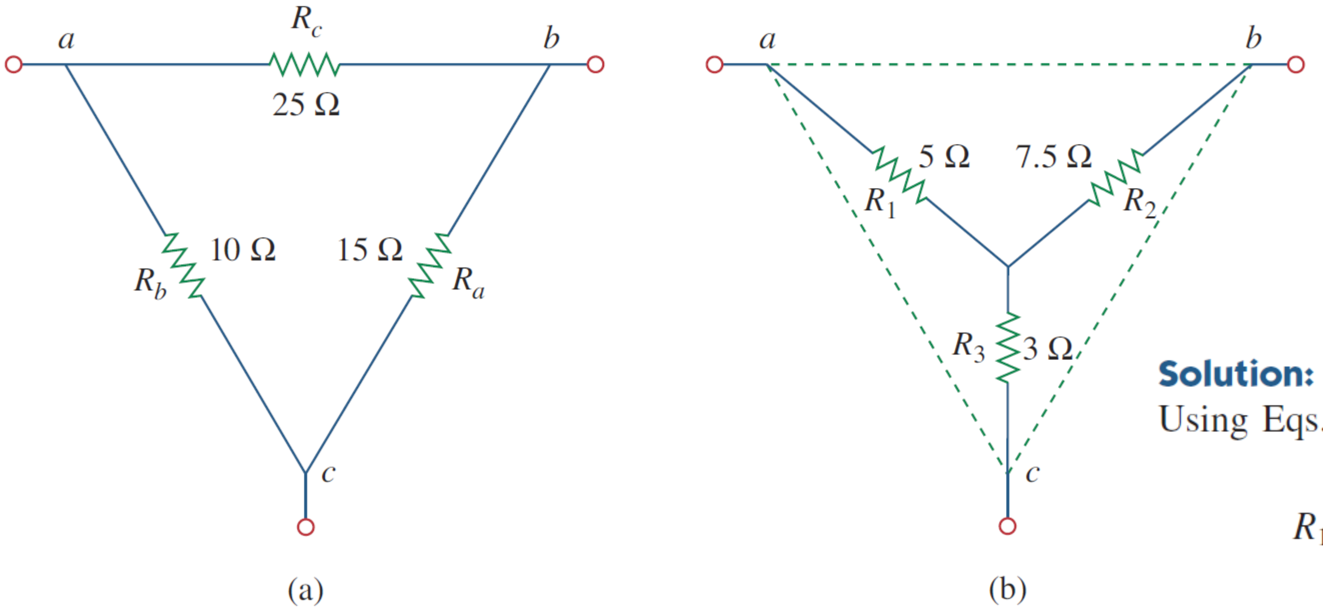


Figure 2.50
For Example 2.14: (a) original Δ network, (b) Y equivalent network.

Solution:
Using Eqs. (2.49) to (2.51), we obtain

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = \frac{250}{50} = 5\ \Omega$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\ \Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3\ \Omega$$

The equivalent Y network is shown in Fig. 2.50(b).

Example

Practice Problem 2.14 Transform the wye network in Fig. 2.51 to a delta network.

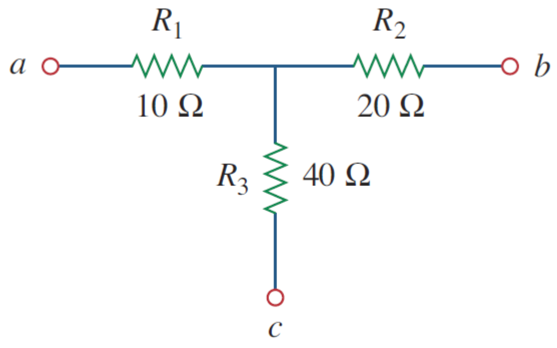


Figure 2.51
For Practice Prob. 2.14.

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

Answer: $R_a = 140\ \Omega$, $R_b = 70\ \Omega$, $R_c = 35\ \Omega$.

Example

Example 2.15

Obtain the equivalent resistance R_{ab} for the circuit in Fig. 2.52 and use it to find current i.

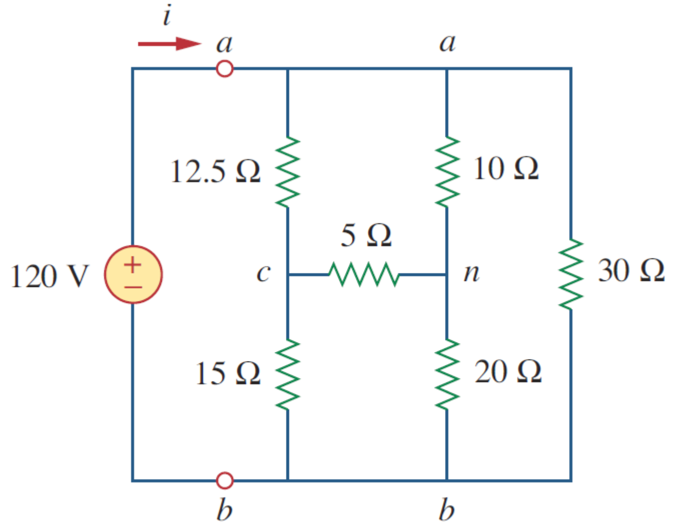


Figure 2.52
For Example 2.15.

Attempt. In this circuit, there are two Y networks and three Δ networks. Transforming just one of these will simplify the circuit. If we convert the Y network comprising the 5-Ω, 10-Ω, and 20-Ω resistors, we may select

$$R_1 = 10\ \Omega, \qquad R_2 = 20\ \Omega, \qquad R_3 = 5\ \Omega$$

Thus from Eqs. (2.53) to (2.55) we have

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{10}$$

$$= \frac{350}{10} = 35\ \Omega$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = \frac{350}{20} = 17.5\ \Omega$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \frac{350}{5} = 70\ \Omega$$

With the Y converted to Δ, the equivalent circuit (with the voltage source removed for now) is shown in Fig. 2.53(a). Combining the three pairs of resistors in parallel, we obtain

$$70 \parallel 30 = \frac{70 \times 30}{70 + 30} = 21\ \Omega$$

$$12.5 \parallel 17.5 = \frac{12.5 \times 17.5}{12.5 + 17.5} = 7.292\ \Omega$$

$$15 \parallel 35 = \frac{15 \times 35}{15 + 35} = 10.5\ \Omega$$

so that the equivalent circuit is shown in Fig. 2.53(b). Hence, we find

$$R_{ab} = (7.292 + 10.5) \parallel 21 = \frac{17.792 \times 21}{17.792 + 21} = \mathbf{9.632\ \Omega}$$

Then

$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.632} = \mathbf{12.458\ A}$$

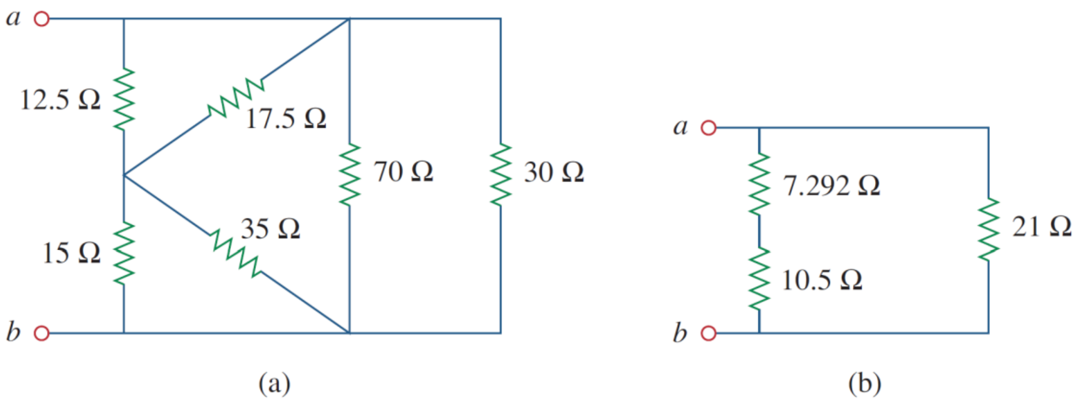


Figure 2.53
Equivalent circuits to Fig. 2.52, with the voltage source removed.

Example

Practice Problem 2.15 For the bridge network in Fig. 2.54, find R_{ab} and i.

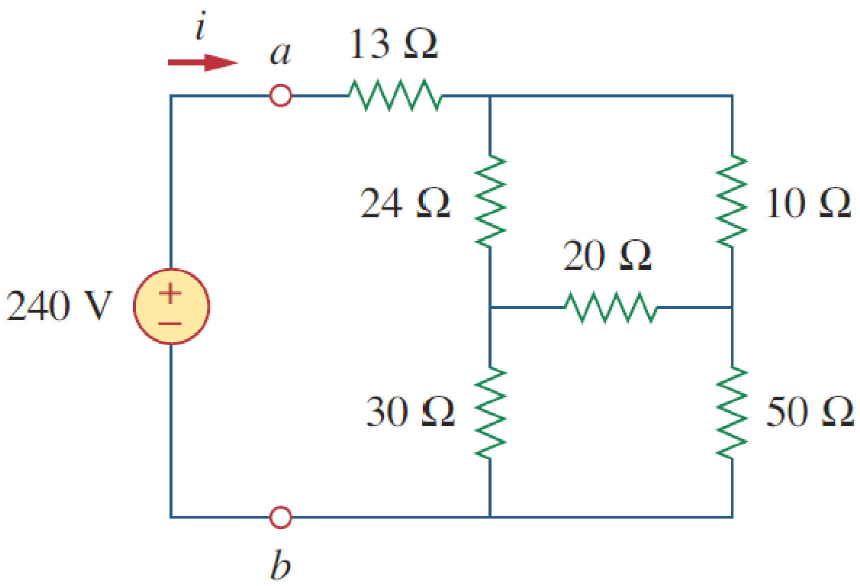


Figure 2.54
For Practice Prob. 2.15.

Answer: 40 Ω, 6 A.

Summary

1. A resistor is a passive element in which the voltage v across it is directly proportional to the current i through it. That is, a resistor is a device that obeys Ohm's law,

$$v = iR$$

where R is the resistance of the resistor.

2. A short circuit is a resistor (a perfectly, conducting wire) with zero resistance ($R = 0$). An open circuit is a resistor with infinite resistance ($R = \infty$).
3. The conductance G of a resistor is the reciprocal of its resistance:

$$G = \frac{1}{R}$$

4. A branch is a single two-terminal element in an electric circuit. A node is the point of connection between two or more branches. A loop is a closed path in a circuit. The number of branches b, the number of nodes n, and the number of independent loops l in a network are related as

$$b = l + n - 1$$

5. Kirchhoff's current law (KCL) states that the currents at any node algebraically sum to zero. In other words, the sum of the currents entering a node equals the sum of currents leaving the node.
6. Kirchhoff's voltage law (KVL) states that the voltages around a closed path algebraically sum to zero. In other words, the sum of voltage rises equals the sum of voltage drops.
7. Two elements are in series when they are connected sequentially, end to end. When elements are in series, the same current flows through them ($i_1 = i_2$). They are in parallel if they are connected to the same two nodes. Elements in parallel always have the same voltage across them ($v_1 = v_2$).
8. When two resistors $R_1\ (=1/G_1)$ and $R_2\ (=1/G_2)$ are in series, their equivalent resistance R_{eq} and equivalent conductance G_{eq} are

$$R_{eq} = R_1 + R_2, \qquad G_{eq} = \frac{G_1 G_2}{G_1 + G_2}$$

9. When two resistors R_1 ($=1/G_1$) and R_2 ($=1/G_2$) are in parallel, their equivalent resistance R_{eq} and equivalent conductance G_{eq} are

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}, \qquad G_{eq} = G_1 + G_2$$

10. The voltage division principle for two resistors in series is

$$v_1 = \frac{R_1}{R_1 + R_2} v, \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$

11. The current division principle for two resistors in parallel is

$$i_1 = \frac{R_2}{R_1 + R_2} i, \qquad i_2 = \frac{R_1}{R_1 + R_2} i$$

12. The formulas for a delta-to-wye transformation are

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}, \qquad R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

13. The formulas for a wye-to-delta transformation are

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}, \qquad R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

Assignment 2

2.15 Calculate v and i_x in the circuit of Fig. 2.79.

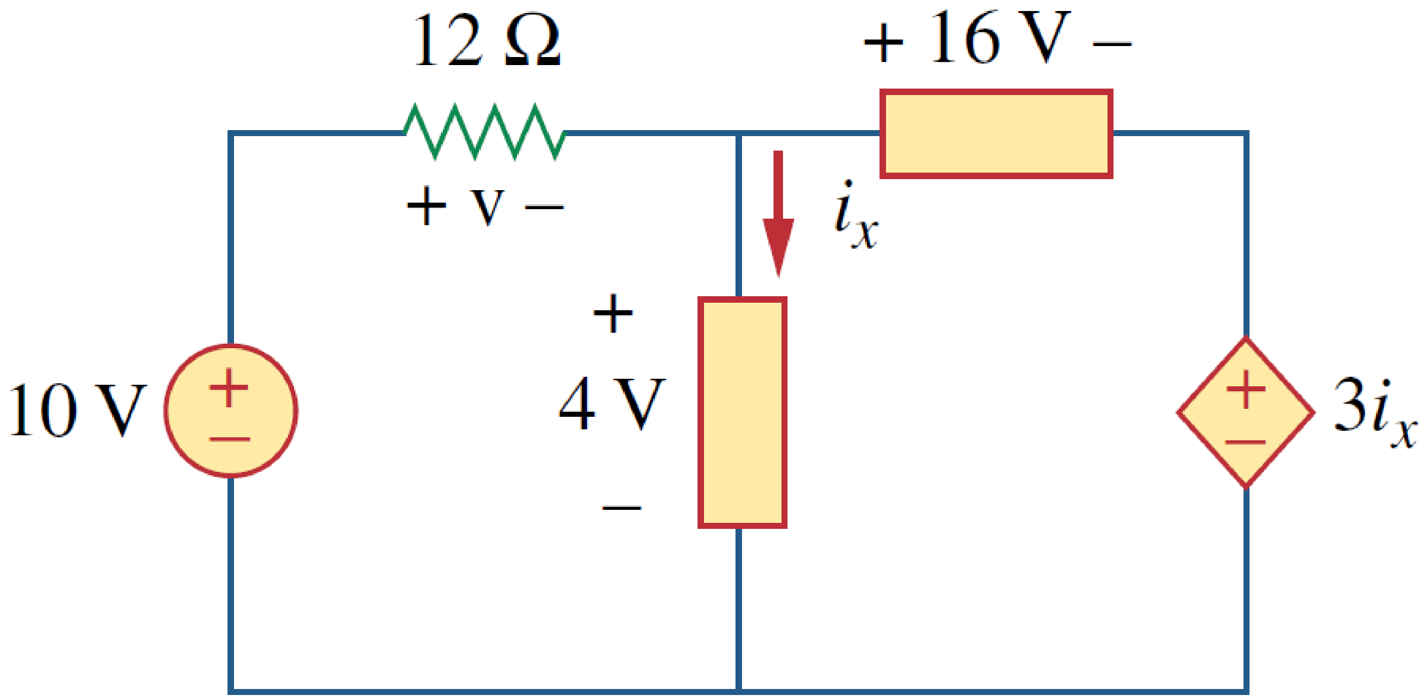


Figure 2.79
For Prob. 2.15.

2.22 Find V_o in the circuit in Fig. 2.86 and the power absorbed by the dependent source.

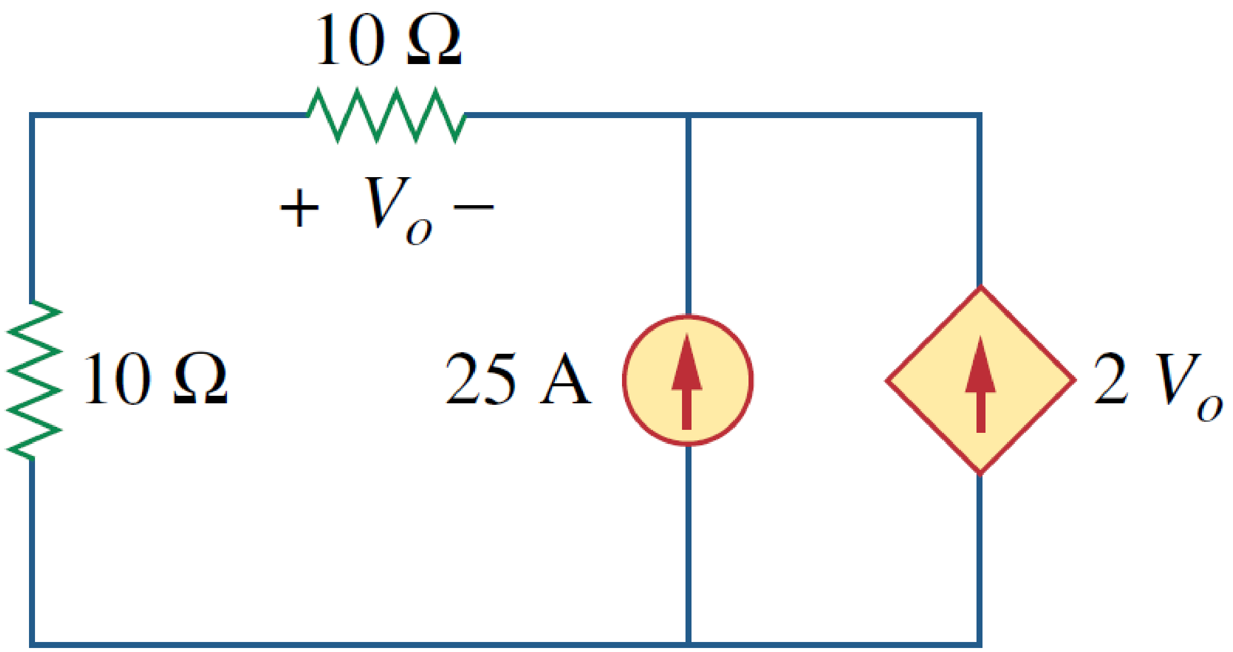


Figure 2.86
For Prob. 2.22.

2.25 For the network in Fig. 2.89, find the current, voltage, and power associated with the 20-kΩ resistor.

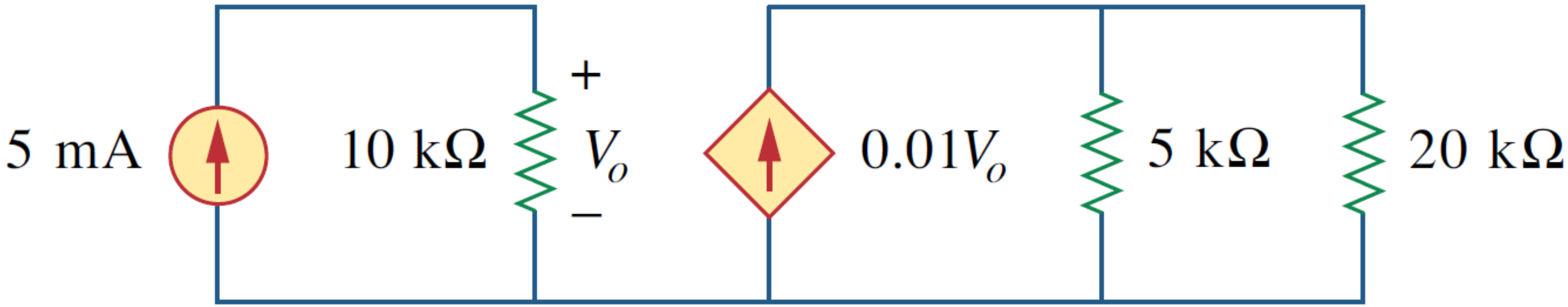


Figure 2.89
For Prob. 2.25.

2.36 Find i and V_o in the circuit of Fig. 2.100.

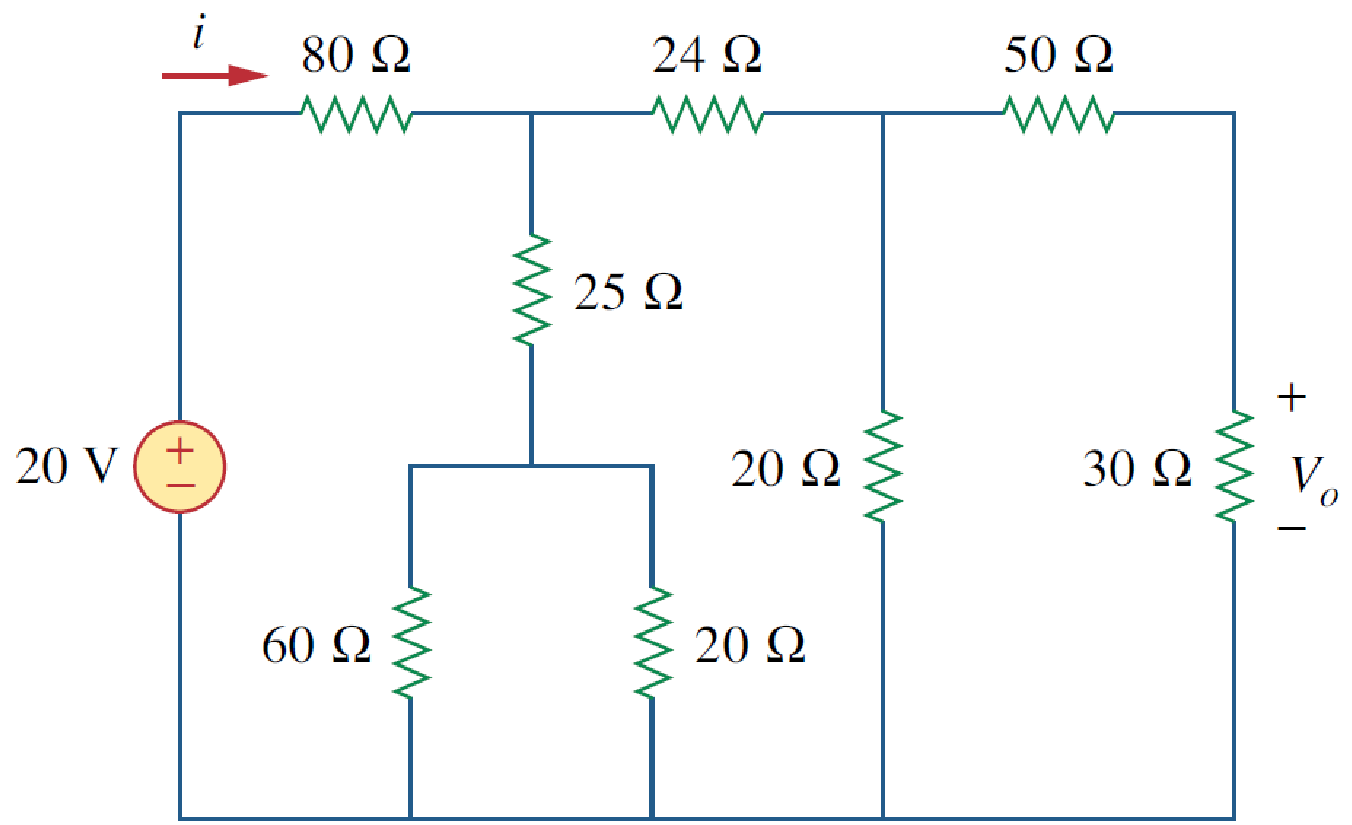


Figure 2.100
For Prob. 2.36.

2.41 If $R_{eq} = 50\ \Omega$ in the circuit of Fig. 2.105, find R.

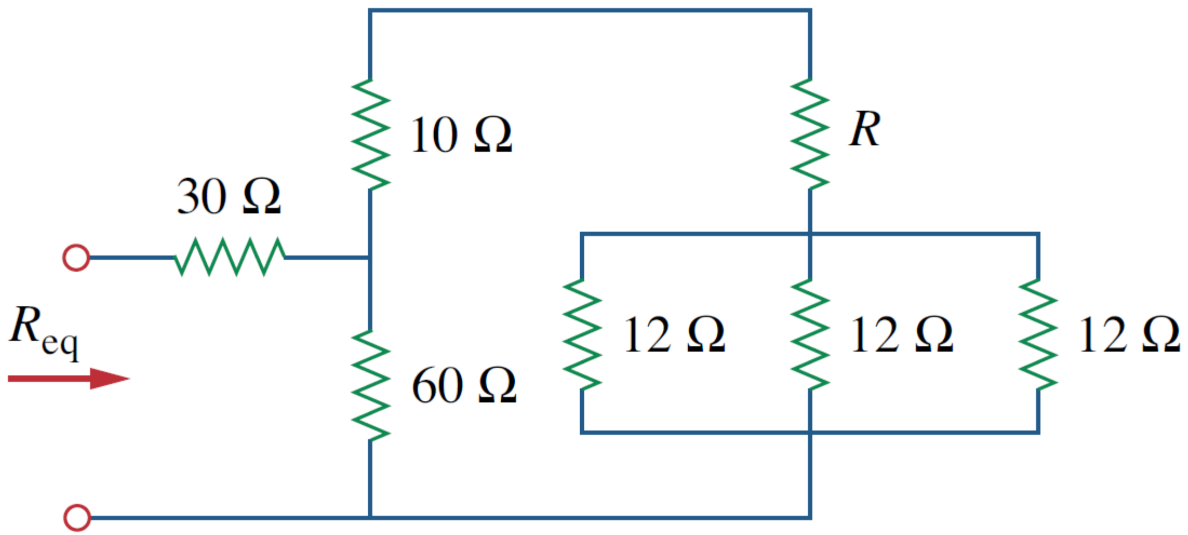


Figure 2.105
For Prob. 2.41.

2.51 Obtain the equivalent resistance at the terminals *a*-*b* for each of the circuits in Fig. 2.115.

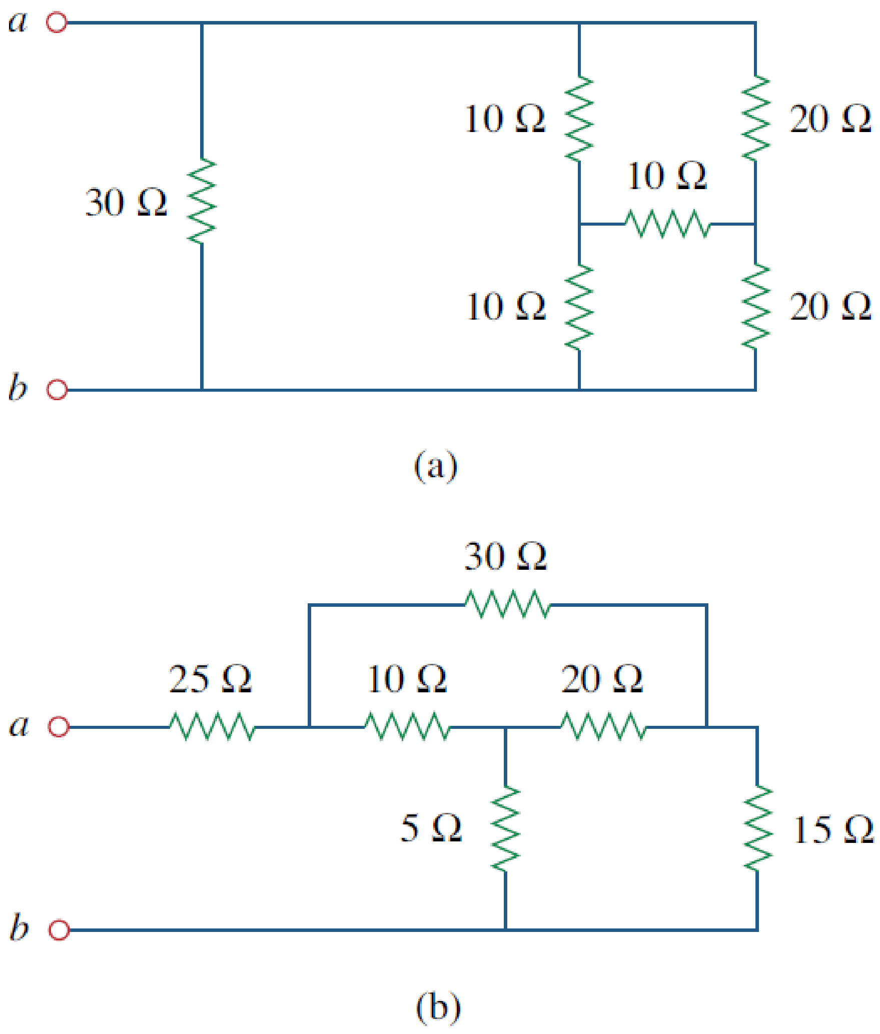


Figure 2.115
For Prob. 2.51.

Thank You!